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Learning Geometrical Concepts using Dynamic Geometry Software

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Dynamic geometry software promises direct manipulation of geometrical objects and relations. This paper reports aspects of a research study deigned to examine the impact of using such software on student conceptions. Analysis of the data from the study indicates that, while the use of dynamic geometry software can assist students in making progress towards more mathematical explanation (and thereby provide a foundation on which to build further notions of deductive reasoning in mathematics), the 'dynamic' nature of the software influences the form of explanation, especially in the early stages.

Computer-based learning environments continue to be a seductive notion in mathematics education. The promise is that through using particular software in carefully-designed ways, it is possible for learners simultaneously to use and come to understand important aspects of mathematics, something that in other circumstances can be particularly elusive.

One type of promising computer-based learning environment features what is commonly referred to as the "direct manipulation" of mathematical objects and relations. In the domain of geometry, examples of such software include *Cabri-géomètre, Sketchpad, Inventor, Thales, Cinderella, Dr Geo,* and others. Such software is often called dynamic geometry software.

This paper reports on data taken from a longitudinal study of lower secondary (junior high) school students (aged 12 years old) learning aspects of geometry in a particular DGE (in this case *Cabri-géomètre*). The focus for the analysis is the students' evolving mathematical explanations as they tackle problems involving the construction of various quadrilaterals. The analysis indicates that, while the use of dynamic geometry software can assist students in making progress towards more mathematical explanation (and thereby provide a foundation on which to build further notions of deductive reasoning in mathematics), the 'dynamic' nature of the software influences the form of explanation, especially in the early stages. This underlines the vital role of the teacher in ensuring that the students' ability to devise explanations moves from what Hölzl (1996) calls reasoning "in a *Cabri* – specific style" to more general mathematical language.

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The Nature of Dynamic Geometry Software

The various different forms of dynamic geometry software typically share certain crucial attributes. One attribute that distinguishes such packages from simple drawing programmes is the ability to specify the geometrical relationships between objects created on the computer screen, such as points, lines, and circles. This is primarily done by specifying that, for instance, a particular point is on a line, or that one line is parallel to another line. A second attribute of such software, and probably the defining one, is the ability to then explore graphically the implications of the geometrical relationships established in constructing a figure. This is usually achieved through use of the 'drag' facility. This is the ability to 'grab' elements of the geometrical figure, using the computer mouse, and observe how the various parts of the figure respond dynamically as the chosen element is 'dragged' around the screen. As this dragging takes place, the display gives the impression that the geometrical figure is being continuously deformed, while, at the same time, maintaining the geometrical relationships that were specified in the original construction. This means that when one line is dragged, any line which has been specified to be parallel to the line being dragged also moves, but in such a way that it always remains parallel to the first line.

By operating in this fashion, dynamic geometry environments appear to have the potential to:

- provide students with 'direct experience' of geometrical theory and thereby break down what can all too often be an unfortunate separation between geometrical construction and deduction

- make it possible for students to focus on what varies and what is invariant in a geometric figure

- enable students to gain more a meaningful idea of proof and proving

It is possible to identify, however, aspects of the nature of the software environment that are likely to impact on geometrical conceptions developed by those using the software to learn aspects of geometry. For example, within any particular DGE there are likely to be some or all of the following:

- points that cannot be dragged (in general, a DGE distinguishes 'basic' points from, for instance, points of intersection – the former can be dragged, the latter can not – even though they cannot be distinguished visibly)

- dragging effects that are determined by the software designer (for example, the decision about what happens to a point placed arbitrarily on line segment when one end of the line segment is dragged)

- objects that look identical but behave differently (for example, a circle with a point on its circumference can dialate when the point is dragged (keeping its centre point fixed), or, if constructed in a different way, can be dragged around the screen while maintaining the same radius)

These aspects of the behaviour of the software are likely to impact on the learner. It is also important for such learners to be able to discern properly between an image that just "looks right" and one that includes the necessary geometrical construction for its particular geometrical properties to remain invariant when any element used in its construction is dragged. Laborde (1993 p49) makes the useful distinction between what she calls *drawing* and *figure* in the following way: "drawing refers to the material entity while figure refers to a theoretical object". Other concerns about the software relate to the opportunity afforded by the software of testing a myriad of diagrams through use of the 'drag' function provided by the DGE, or of confirming conjectures through measurements (that also adjust as the figure is dragged). These latter concerns may mean that use of the software, rather than enhancing the learning of proof and proving, may actually *reduce* the perceived need for deductive proof (Hoyles and Jones, 1998).

The Research Study

Theoretical framework

The theoretical framework for this study was derived from research in the following areas:

- theoretical models of the teaching and learning of geometrical concepts, especially the van Hiele model (see, for example, Fuys, Geddes,and Tischer, 1988)

- theoretical perspectives on the teaching and learning of proof and proving (for example, Hanna, 1998)

- socio-cultural perspectives on learning, especially the idea of the mediation of tools (Wertsch, 1998)

- theoretical perspectives on the role of technological tools in the learning process (Pea, 1993)

Progressing from level 2 to 3 of the van Hiele framework involves, amongst other things, moving from identifying geometrical figures by their properties (which are seen as independent) to recognising that a particular property of a figure precedes or follows from other properties and that relationships exist between different figures. For proof and proving to be meaningful activities for students, the various functions of proof and proving have to be communicated to the students in an effective way. For students in lower secondary school a focus on explanation, taken as a discourse establishing the validity of statements about suitable geometrical objects, seems likely to be productive.

Using dynamic geometry software, from a sociocultural perspective, is more than utilising a physical artifact. As students interact when tackling geometrical problems using a DGE in the social setting of the mathematics classroom, they talk the language of geometry even before being introduced to the technical terminology. In this way they "tune to the constraints and affordances by negotiating the situated environment established by the symbolic representation system. In doing so, they develop explanations of why objects behave in the way they do" (Resnick, Pontecorvo, and Säljö, 1997 pp14-15).

The research focus and empirical study

The focus of the research is on the following:

- the impact of using dynamic geometry software on the interpretation that students give to geometrical objects encountered using the software
- how learners learn to express explanations and verifications of geometrical theorems, properties and classifications

In this paper the intention is to examine how the use of the dynamic geometry software both enables and constrains students who are learning to explain the relationships between the properties of quadrilaterals through tackling tasks using a particular DGE (in this case *Cabri-géomètre*). The data comes from a longitudinal case study of lower secondary (junior high) school students (aged 12 years old) carried out in the UK where the students, typically:

- know some of the properties of certain plane geometrical figures
- have some experience of conjecturing and describing observations in open-ended problem situations
- but have not been introduced to the formal aspects of proof and proving

Design choices were made with a view to the typicality of the setting. The school selected for the empirical work was an urban comprehensive school whose results in mathematics at age 16 were at the national average (there is a national system of testing in the UK that allows such judgements to be made). The mathematics teachers in the school used a problem-based approach to teaching mathematics and the students usually worked in pairs or small groups on mathematical problems and occasionally used computers. Throughout their mathematics work the students were expected to be able to explain the mathematics they were doing, either orally or in writing.

All the students in the class were tested using a van Hiele test (Usiskin 1982) at the start of the unit of work and on its completion. The teaching unit was prepared to form three phases, and designed to fit around other mathematics work for the class. During each of the phases, the students worked in pairs (usually the pairs they worked in for all their mathematics work).

Phase 1: preliminary experience with *Cabri-géomètre* while working through a short series of tasks involving lines and circles.

Phase 2: a series of three tasks that involved constructing the following quadrilaterals: a rhombus, a square, and a kite.

Phase 3: a series of six tasks that involved relationships between various quadrilaterals

For more details of the study, see Jones 1996, 1997, 1998 and 1999.

Data analysis

The data below comes from one pair of students (pair A) during phases 2 and 3 of the empirical study.

Phase 2, Task 1: construct a rhombus and explain why the shape is a rhombus.

Pair A written explanation:

> The radius is the same for the circle and the diamond [the rhombus] and we made the diamond from the help of the first construction. The sides are all the same because if the centre is in the right place the sides are bound to be the same. The diagonals of the diamond cross in the middle though they are different size (length). They cross at the middle through the line. Their diagonals bisect each other. The angles [at the intersection of the diagonals] are all the same. They are 90^0. The opposite angles [of the rhombus] are the same. Two are more than 90^0 but less than 180^0 and the others are less than 90^0 but more than 0^0.

> This shape is a rhombus because the sides are the same, the diagonals bisect at right angles and the opposites have the same angles.

Phase 2, Task 2: construct a square and explain why the shape is a square.

Pair A written explanation:

> It is a square because the sides are equal and the diagonals intersect. The diagonals are [at] right angles (90^0).

Phase 3, Task 5: constructing a rectangle that can be modified to a square and explaining why all squares are rectangles

Pair A written explanation:

> A rectangle …… becomes a square when the diagonals become right angles where they meet.

Phase 3, Task 7: constructing a trapezium that can be modified to a parallelogram and explaining why all parallelograms are trapeziums

Pair A written explanation:

> It is a trapezium because it has one pair of parallel lines. A parallelogram is parallel both ways.

Phase 3, Task 9: a task to show the relationships between the 'family' of quadrilaterals

Extracts from Pair A session transcript (pseudonyms Harri and Russell):

Teacher: Why is a square a special sort of rectangle?
Russell: Because they've both got right angles [at the vertices] but with a rectangle one of the sides is bigger than the other.
Teacher: Why is a rectangle a special case of a parallelogram?
Harri: The two opposite [sides] are the same length but [in a parallelogram] they [the angles at the vertices] are not right angles.

By the end of the teaching unit the students were reasonably competent with the hierarchical classification of quadrilaterals. The students accepted that particular quadrilaterals could be special cases of other quadrilaterals and could provide reasonable explanations of why this is the case. This is in some contrast to previous research that has found that many students have significant problems with the hierarchical classification of quadrilaterals (see, for example, Fuys *et al*, 1988).

The qualitative development of the students' explanations can be summarised as follows:

- initially, an emphasis on description rather than explanation. Some reliance on perception rather than mathematical reasoning. Lack of capability with precise mathematical language (similar to that found in other studies, for example Fuys *et al* 1988 p135-6).

- at an interim stage, explanations become more mathematically precise but are influenced (mediated) by the nature of the dynamic geometry software (for example by the use of the term 'dragging' or by other phrases linked to the dynamic nature of the software).

- at the end of the teaching unit, explanations related entirely to the mathematical context.

Conclusions

Some of the value and function of the hierarchical classification of quadrilaterals, as de Villiers (1994 p15-16) makes clear, come from the following:

- it leads to more economical definitions of concepts and formulation of theorems

- it simplifies the deductive systematisation and derivation of the properties of more special concepts

- it often provides a useful conceptual schema during problem solving

- it sometimes suggests alternative definitions and new propositions

- it provides a useful global perspective

Given the significant problems mentioned above that many students have with the hierarchical classification of quadrilaterals, de Villiers (1994 p17) suggests that computer microworlds such as dynamic geometry software "offer great potential for conceptually enabling many children to see and accept the possibility of hierarchical inclusions". The evidence reported in this paper supports such a suggestion but documents the mediational impact of using such software. As documented by this study and other research, this mediational impact involves at least three aspects.

First, the students need to come to terms with the notion of a hierarchy of functional dependency within a figure (see, for example, Jones 1996). Secondly, the students need to gain an appreciation of the notion of the constraint of robustness of a figure under drag as a *mathematical* feature, rather than, say, as 'mechanical glue' (Jones 1998). Thirdly, the 'dynamic' nature of the software influences the form of explanation given by the students (what Hölzl 1996 p184 calls reasoning "in a *Cabr*i-specific style").

Much previous research with dynamic geometry software has focused on students in upper secondary school where the students have received considerable teaching input in plane geometry, including the proving of elementary theorems, but are new to the particular software tool. The study reported in this paper focuses on students in lower secondary school where students have quite limited experience of the formal aspects of geometry (and have certainly never seen a proof or been asked to prove a theorem). The evidence shows that:

- when using dynamic geometry software, students can make progress towards mathematical explanations, which, a range of research suggests, should provide a foundation on which to build further notions of deductive reasoning in mathematics

- the 'dynamic' nature of the software influences the form of explanation given by the students, particularly in the early stages; what Hölzl (ibid) calls reasoning "in a Cabri-specific style"

- there is a vital role for the teacher in ensuring that students move from reasoning "in a Cabri-specific style" to more general mathematical language

In the mathematics classroom, the practical issues of when and how to use dynamic geometry software are very important. Knowing more about the impact that the software has on student conceptions should help the software to be used in a more effective way.

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Note: Keith Jones is a founding member and convenor of the geometry working group of the British Society for Research into Learning Mathematics (BSRLM). The papers of the working group can be found in the BSRLM proceedings.