STUDENT INTERPRETATIONS OF A DYNAMIC GEOMETRY ENVIRONMENT

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Abstract: *It seems that aspects of student interpretations of computer-based learning environments may result from the idiosyncrasies of the software design rather than the characteristics of the mathematics. Yet, somewhat paradoxically, it is because the software demands an approach which is novel that its use can throw light on student interpretations. The analysis presented in this paper is offered as a contribution to understanding the relationship between the specific tool being used, in this case the dynamic geometry environment Cabri-Géomètre, and the kind of thinking that may develop as a result of interactions with the tool. Through this analysis a number of effects of the mediational role of this particular computer environment are suggested.*

Keywords: -

1. Introduction

There is considerable evidence that learners develop their own interpretations of the images they see and the words they hear. This evidence also suggests that, although individuals form their own meanings of a new phenomenon or idea, the process of creating these meanings is embedded within the setting or context and is mediated by the forms of interaction and by the tools being used. Such considerations have recently been turned to examining student learning within dynamic geometry environments (DGEs), as such tools have become more widely available (for example, Laborde and Capponi 1994, Hölzl 1996, Jones 1997).

An important issue in mathematical didactics, particularly given the abstract nature of mathematical ideas, is that student interpretations may not coincide with the intentions of the teacher. Such differences are sometimes referred to as “errors” (on behalf of the students) or “misconceptions”, although this is not the only possible interpretation (see, for example, Smith et al 1993). A key perspective on these differences in interpretation, and the theme of this paper, is highlighted by Brousseau (1997 p82), “errors ... are not erratic or unexpected ... As much in the teacher’s functioning as in that of the student, the error is a component of the meaning of the acquired piece of knowledge” (emphasis added). This indicates that we, as teachers, should expect students to form their own interpretations of the mathematical ideas they meet and that their ideas are a function of aspects of the learning environment in which they are working. Within a dynamic geometry environment, Ballachef and Kaput (1996 p 485) suggest, student errors could be a mixture of true geometric errors and errors related to the student’s understanding of the behaviours of the learning environment itself (based on an examination of work by Bellemain and Capponi 1992 and Hoyles 1995).

The focus for this paper is the interpretations students make when working with a dynamic geometry environment (DGE), in this case Cabri-Géomètre, particularly their understanding of the behaviours of the learning environment itself. One of the distinguishing features of a dynamic geometry package such as Cabri is the ability to construct geometrical objects and specify relationships between them. Within the computer environment, geometrical objects created on the screen can be manipulated by means of the mouse (a facility generally referred to as ‘dragging’). What is particular to DGEs is that when elements of a construction are dragged, all the geometric properties employed in constructing the figure are preserved. This encapsulates a central notion in geometry, the idea of invariance, as invariance under drag.

This paper reports on some data from a longitudinal study designed to examine how using the dynamic geometry package Cabri-Géomètre mediates the learning of certain geometrical concepts, specifically the geometrical properties of the ‘family’ of quadrilaterals. In what follows I illustrate how the interpretation of the DGE by students is a function of aspects of the computer environment. I begin by outlining the theoretical basis for this view of DGE use as tool mediation.

2. Theoretical framework

The concept of tool mediation is central to the Vygotskian perspective on the analyses of cognitive development (Wertsch 1991). The approach suggested below begins with the assumption that tools and artifacts are instruments of access to the knowledge, activities, and practices of a given social group (an example of such an approach is given by Lave and Wenger 1991). Such analyses indicate that the types of tools and forms of access existent within a practice are intricately interrelated with the understandings that the participants of the practice can construct.

This suggests that learning within a DGE involves what Brousseau refers to as a dialectical interaction, as students submit their previous knowings to revision, modification, completion or rejection, in forming new conceptions. The work of Meira (1998) on using gears to instantiate ratios, for example, challenges the artifact-as-bridge metaphor, in which material displays are considered a link between students' intuitive knowledge and their mathematical knowledge (taken as abstract). Meira notes that the sense-making process takes time and that even very familiar artifacts (such as money) are neither necessarily nor quickly well-integrated in the students' activities within school. Cobb (1997 p170) confirms that tool use is central to the process by which students mathematize their activity, concluding that "anticipating how students might act with particular tools, and what they might learn as they do so, is central to our attempts to support their mathematical development".

This theoretical framework takes the position that tools do not serve simply to facilitate mental processes that would otherwise exist, rather they fundamentally shape and transform them. Tools mediate the user's action - they exist between the user and the world and transform the user's activity upon the world. As a result, action can not be reduced or mechanistically determined by such tools, rather, such action always involves an inherent tension between the mediational means (in this case the tool DGE) and the individual or individuals using them in unique, concrete instances. Such theoretical work suggest some elements of tool mediation which can be summarised as follows:

1. Tools are instruments of access to the knowledge, activities and practices of a community.

2. The types of tools existent within a practice are interrelated in intricate ways with the understandings that participants in the practice can construct.
3. Tools do not serve simply to facilitate mental processes that would otherwise exist, rather they fundamentally shape and transform them.
4. Tools mediate the user's action - they exist between the user and the world and transform the user's activity upon the world.
5. Action can not be reduced or mechanistically determined by such tools, rather such action always involves an inherent tension between the mediational means and the individual or individuals using them in unique, concrete instances.

Examples of mathematics education research which make use of the notion of tool mediation include Cobb's study of the 100 board (Cobb 1995), Säljö's work on the rule of 3 for calculating ratios (Säljö 1991), and Meira's examination of using gears to instantiate ratios (Meira 1998).

Applying such notions to learning geometry within a DGE suggests that learning geometrical ideas using a DGE may not involve a fully 'direct' action on the geometrical theorems as inferred by the notion of 'direct manipulation', but an indirect action mediated by aspects of the computer environment. This is because the DGE has itself been shaped both by prior human practice and by aspects of computer architecture. This means that the learning taking place using the tool, while benefiting from the mental work that produced the particular form of software, is shaped by the tool in particular ways.

3. Empirical study

The empirical work on which the observations below are based is a longitudinal study examining how using the dynamic geometry package Cabri-géomètre mediates the learning of geometrical concepts. The focus for the study is how "instructional devices *are actually used and transformed by students in activity*" (Meira 1998, emphasis added) rather than simply asking whether the students learn particular aspects of geometry "better" by using a tool such as Cabri.

The data is in the form of case studies of five pairs of 12 years old pupils working through a sequence of specially designed tasks requiring the construction of various quadrilaterals using Cabri-géomètre in their regular classroom over a nine month period. Students were initially assessed at van Hiele level 1 (able to informally analyse figures) and the tasks designed to develop van Hiele level 2 thinking (able to logically interrelate properties of geometrical figures), see Fuys et al (1988). The version of Cabri in use was Cabri I for the PC. Sessions were video and audio recorded and then transcribed. Analysis of this data is proceeding in two phases. The first phases identified examples of student interpretation as a function of tool mediation, a number of which are illustrated below. The second phase, currently in progress, is designed to track the genesis of such tool mediation of learning.

4. Examples of student interpretations

Below are four examples of extracts from classroom transcripts which reveal student interpretations of the dynamic geometry environment.

4.1 Example 1

Student pair Ru and Ha are checking, part way through a construction, that the figure is invariant when any basic point is dragged.

Ru	Just see if they all stay together first.
Ha	OK.
Ru	Pick up by one of the edge points. [H drags a point]
Ha & Ru (together)	Yeah, it stays together!

In this example the students use the phrase "all stay together" to refer to invariance and the term "edge point", rather than either radius point (or rad pt as the drop-down menu calls it) or circle point (as the help file calls that form of point), to refer to a point on the circumference of a circle.

It is worth reflecting that in the implementation of Cabri I the designers found it necessary to utilise a number of different forms of point: basic point, point on object, (point of) intersection - not to mention midpoint, symmetrical point, and locus of points, plus centre of a circle and also rad pt (radius point) and circle point (a term used in the onscreen help). In addition, there are several forms of line: basic line, line segment, line by two pts (points) - not to mention parallel line, perpendicular line, plus perpendicular bisector, and (angle) bisector, and two different forms of circle: basic circle, circle by centre & rad pt. With such a multitude of terminology, it may not be totally unexpected that students invent their own terms.

4.2 Example 2

Pair Ho and Cl are in the process of constructing a rhombus which they need to ensure is invariant when any basic point used in its construction is dragged. As they go about constructing a number of points of intersection, one of the students comments:

Ho A bit like glue really. It's just glued them together.

This spontaneous use of the term "glue" to refer to points of intersection has been observed by other researchers (see Ainley and Pratt 1995) and is all the more striking given the fact that earlier on in the lesson the students had confidently referred to such points as points of intersection. Hoyles (1995 pp210-211) also provides evidence of the difficulty students have with interpreting points of intersection.

4.3 Example 3

Pair Ru and Ha are about to begin constructing a square using a diagram presented on paper as a starting point (see Appendix B). The pair argue about how to begin:

Ha If ...I .. erm ..
I reckon we should do that circle first [pointing to the diagram on paper].
Ru Do the line first.
Ha No, the circle. Then we can put a line from that centre point of the circle [pointing to the diagram on paper].

Ru Yeah, all right then.

Ha You can see one .. circle there, another there and another small one in the middle [pointing to various components of the diagram on paper].

The student pair had, in previous sessions, successfully constructed various figures that were invariant under drag including a rhombus and, prior to that, a number of arrangements of interlocking circles (see Appendix A). In particular they had successfully constructed a rhombus by starting with constructing two interlocking circles. Following the above interchange they followed a very similar procedure. The inference from the above extract of dialogue is that previous successful construction with the software package influences the way learners construct new figures.

An influence here might well be the sequential organisation of actions in producing a geometrical figure when using Cabri. This sequential organisation implies the introduction of explicit order of operation in a geometrical construction where, for most users, order is not normally expected or does not even matter. For example, Cabri-géomètre induces an orientation on the objects: a segment AB can seem orientated because A is created before B. This influences which points can be dragged and effectively produces a hierarchy of dependencies in a complex figure (something that has commented on by Balacheff 1996, Goldenberg and Cuoco 1998 and by Noss 1997, amongst others).

4.4 Example 4

Students Ru and Ha have constructed a square that is invariant under drag and are in the process of trying to formulate an argument as to why the figure is a square (and remains a square when dragged). I intervene to ask them what they can say about the diagonals of the shape (in the transcript Int. refers to me).

Ru They are all diagonals.

Int No, in geometry, diagonals are the lines that go from a vertex, from a corner, to another vertex.

Ru Yeah, but so’s that, from there to there [indicating a side of the square that, because of orientation, was oblique].

Int That's a side.
Ru Yeah, but if we were to pick it up like that .... like that. Then they're diagonals [indicating an orientation of 45 degrees to the bottom of the computer screen].

Student Ru is confounding diagonal with oblique, not an uncommon incident in lower secondary school mathematics (at least in the UK). What is more, the definition provided by me at the time does not help Ru to distinguish a diagonal from a side, while the drag facility allows Ru to orientate any side of the square so as to appear to be oblique (which in Ru's terms means that it is 'diagonal'). Of course, such oblique orientation is not invariant under drag, whereas a diagonal of a square is always a diagonal whatever the orientation. This example illustrates that, in terms of the specialised language of mathematics, the software can not hope to provide the range of terms required to argue why the figure is a square, nor could it be expected to do so. Such exchanges call for sensitive judgement by the teacher in terms of how such terminology is introduced, together with judicious use of the drag facility.

5. Some observations on the examples

The examples given above are representative of occurrences within the case studies arising from this research project. A number of comments can be made on these extracts which illustrate how student interpretations of the computer environment is shaped by the nature of the mediating tool. As Hoyles (1995 p211) explains, it is something of a paradoxical situation that student interpretations can be traced to the idiosyncrasies of the software design rather than the characteristics of the mathematics, yet it is just because the software demands an approach which is novel that its use can throw light on student interpretations.

First, it appears that learners find the need to invent terms. In example 1 above, the student pair employ the phrase "all stay together" to refer to invariance and coin the term "edge point" to refer to a point on the circumference of a circle. To some extent this parallels the need of the software designers to provide descriptors for the various different forms of point they are forced to use. Yet research on pupil learning with Logo suggests that learners use a hybrid of Logo and natural language when talking through problem solving strategies (for example, Hoyles 1996). This, I would argue, is one effect of tool mediation by the software environment. The software designers found it necessary to use hybrid terms. As a consequence, so may the students. Further analysis of the data from this study may shed some light on how this hybrid language may foster the construction of meaning for the student and to what extent it could become an obstacle for constructing an appropriate mathematical meaning.

A second instance of the mediation of learning is when children appear to understand a particular aspect of the computer environment but in fact they have entirely their own perspective. In example 2 above it is the notion of points of intersection, In this example, one student thinks of points of intersection as ‘glue’ which will bind together geometrical objects such as lines and circles. This, I would suggest, is an example of Wertsch’s (1991) ‘ventriloquating’, a term developed from the ideas of Bakhtin, where students employ a term such as intersection but, in the process, inhabit them with their own ideas. In other words, it can appear that when students are using the appropriate terms in appropriate ways, they understand such terms in the way the teacher expects. The evidence illustrated by example 2 suggests that students may just be borrowing the term for their own use.

A third illustration of the mediation of learning is how earlier experiences of successfully constructing figures can tend to structure later constructions. In example 3 above, the pair had successfully used intersecting circles to construct figures that are invariant under drag and would keep returning to this approach despite there being a number of different, though equally valid, alternatives.

Following from this last point, a further mediation effect can be that the DGE might encourage a procedural effect with children focusing on the sequence of construction rather than on analysing the geometrical structure of the problem. Thus pair Ru and Ha, rather than focusing on geometry might be focusing rather more on the procedure of construction. This may also be a consequence of the sequential organisation of actions implicit in a construction in Cabri-Géomètre.

A fifth illustration of the mediation of learning within the DGE is that even if the drag mode allows a focus on invariance, students may not necessarily appreciate the significance of this. Thus hoping points of intersection will ‘glue’ a figure together, or that constructing a figure in a particular order will ensure it is invariant under drag, does not necessarily imply a particularly sophisticated notion of invariance.

From the examples given above, a sixth illustration of the mediation of learning is provided by an analysis of the interactions with the teacher (in this case the researcher). The challenge for the teacher/researcher is to provide input that serves the learners' communicative needs. As Jones (1997 p127) remarks "the explanation of why the shape is a square is not simply and freely available within the computer environment". It needs to be sought out and, as such, it is mediated by aspects of the computer environment and by the approach adopted by the teacher.

6. Concluding remarks

In this paper I have suggested some outcomes of the mediational role of the DGE *Cabri-Géomètre*. While such outcomes refer to only one form of computer-based mathematics learning environment, these outcomes are similar to those emerging from research into pupils' learning with Logo (adapted from Hoyles 1996 pp103-107):

1. Children working with computers can become centrated on the screen product at the expense of reflection upon its construction
2. Students do not necessarily mobilise geometric understandings in the computer context
3. Students may modify the figure "to make it look right" rather than debug the construction process
4. Students do not necessarily appreciate how the computer tools they use constrain their behaviour
5. After making inductive generalisations, students frequently fail to apply them to a new situation
6. Students can have difficulty distinguishing their own conceptual problems from problems arising from the way the software happens to work
7. Manipulation of drawings on the screen does not necessarily mean that the conceptual properties of the geometrical figure are appreciated

As Hoyles remarks, such indications are intended to capture some of the general in the specific and thereby generate issues for further research.

None of the above is necessarily a criticism of Cabri. In the implementation of such software, decisions have to be made. Goldenberg and Cuoco (1998), for example, quote Jackiw, a principle designer of the DGE Geometer's Sketchpad as saying that "at its heart 'dynamic geometry' is not a well-formulated mathematical model of change, but rather a set of heuristic solutions provided by software developers and human-interface designers to the question 'how would people like geometry to behave in a dynamic universe?'" The point is that the decisions that are made mediate the learning and influence student interpretations. As Hoyles (1995 p210) writes: "the fact that the software constrains children's actions in novel ways can have rather positive consequences for constructivist teaching. The visibility of the software affords a window on to the way students build conceptions of subject matter". The finding from this study of the dynamic geometry package Cabri-Géomètre may well prove useful both to teachers using, or thinking about using, this form of software and to designers of such learning environments, as well as contribute to the further development of theoretical explanations of mathematics learning.

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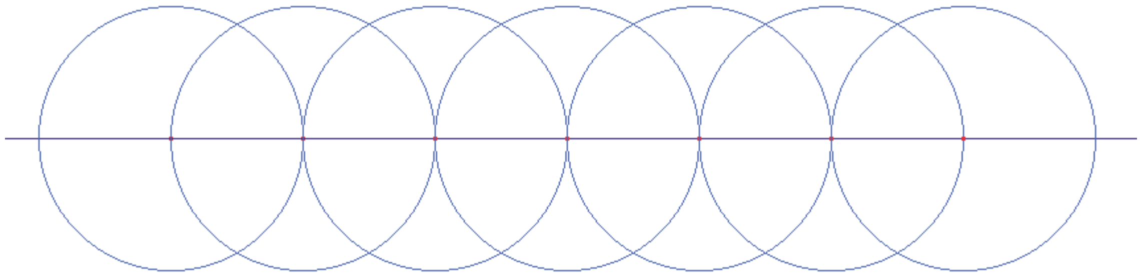
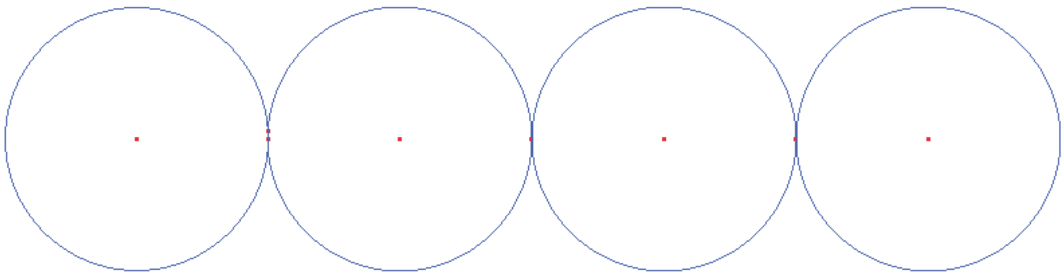
Note

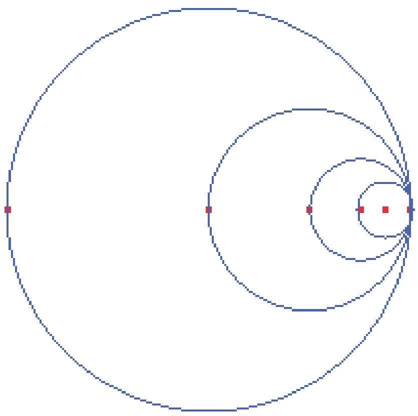
In the appendices that follow, the use of the phrase 'cannot be "messed up"' rather than 'invariant under drag' is based on the suggestion of Healy, L, Hoelzl, R, Hoyles, C, & Noss, R (1994). Messing Up. *Micromath*, 10(1), 14-16.

Appendix A: a task undertaken by pupils during their introduction to *Cabri-Géomètre*

Lines and Circles
Construct these patterns so that they cannot be "messed up".
In each case, write down how you constructed the pattern.





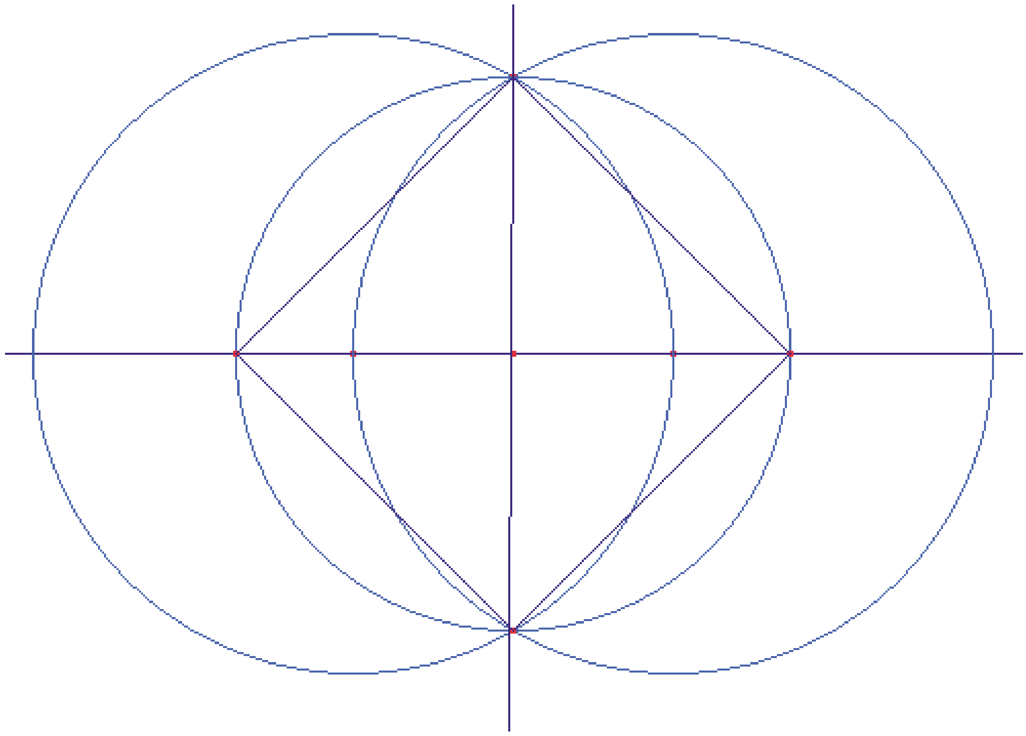


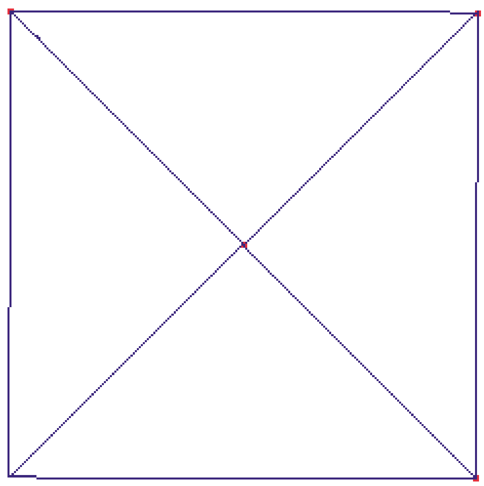
Now construct some patterns of your own using lines and circles.
Make sure you write down how you constructed them.

Appendix B: a task asking pupils to construct a square that is invariant under drag.

The Square

Construct these figures so that they cannot be "messed up".





What do you know about this shape from the way in which you constructed it?

Think about ..... sides
diagonals

Explain why the shape is a square.