PRIMARY TRAINEE TEACHERS' UNDERSTANDING OF BASIC GEOMETRICAL FIGURES IN SCOTLAND

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Whilst teachers' mathematics knowledge is known to play a significant role in shaping the quality of their teaching, much less is known about the nature and extent of that knowledge, how it develops, and how such development can be supported through initial teacher training and continuing professional development. Earlier research has indicated that pre-service (trainee) primary teachers' subject knowledge of geometry is amongst their weakest knowledge of mathematics. This paper reports on an analysis of geometry subject knowledge data gathered in Scotland from undergraduate pre-service primary teachers, focusing on their ability to define and classify quadrilaterals. The results indicate that many trainee primary teachers have relatively poor command of these aspects of mathematics.

INTRODUCTION

It is well known that teachers' mathematics knowledge plays a significant role in shaping the quality of their teaching (Ball, Hill & Bass, 2005). Yet as Ball *et al* (*ibid*, p16) explain, "although many studies demonstrate that teachers' mathematical knowledge helps support increased student achievement, the actual nature and extent of that knowledge—whether it is simply basic skills at the grades they teach, or complex and professionally-specific mathematical knowledge—is largely unknown". This is not to downplay the studies of teachers' mathematical knowledge that have been, and are being, carried out. More it points to the complexity of the issues involved, especially since the context in which teachers gain their own mathematical knowledge, and the form of teacher training they receive (both pre- and in-service), can be so varied, not only across countries, but also within particular countries.

The data reported in this paper are from one component of a larger study being carried out in the UK. The over-arching focus is on teachers' knowledge of geometry since, at this time in the UK, the nature of the school curriculum is under review (QCA, 2005) and there are recommendations that the geometry component of the mathematics curriculum requires special attention and strengthening (RS/JMC, 2001).

What is particularly interesting, when focusing on teachers' mathematical knowledge, is the context in which the teachers learn mathematics themselves, and the context in which they are trained. In Scotland, one of the constituent countries of the UK, there is no statutory national curriculum; rather there are national 'Guidelines' for the teaching and learning of mathematics for students aged 5-14 (Scottish Office Education Department, 1991). In these guidelines, geometry (in the form of ‘Shape, position and movement’; *ibid*., 1991, p. 9) is one of four “attainment outcomes” (the others being ‘Problem-solving and enquiry’, ‘Information handling’, and ‘Number, money and measurement’). In contrast, in England, there *is* a statutory national curriculum, with geometry, in the form of “Shape, space and measures”, being part of the statutory specification for mathematics.

Preliminary analysis of data from a component of the wider study is finding that, in England, graduate pre-service (trainee) primary teachers’ subject knowledge of geometry is the area of mathematics in which they have the weakest knowledge (Jones, Mooney & Harries, 2002; Mooney, Fletcher & Jones, 2003). Their personal confidence in teaching geometry, gauged through a self-audit, is also low. This present paper reports on an analysis of geometry subject knowledge data gathered in Scotland from undergraduate pre-service (trainee) primary teachers. The chosen focus for this report is on their ability to define and classify quadrilaterals, partly because research studies have show that school students have difficulties with defining and classifying quadrilaterals (de Villers, 1994, p17; Jones, 2000), and partly because data from observing such trainee teachers has indicated that at least some of them cannot accept, for example, that ‘a square is a special type of a rectangle’.

THEORETICAL BACKGROUND

The terms ‘*concept image’* and ‘*concept definition*’ were introduced by Vinner and Hershkowitz (1980) in the context of the learning of some simple geometrical concepts and developed by Tall and Vinner (1981) in the context of more sophisticated mathematical ideas of limits and continuity. Given that *formal concept definitions* are definitions that are accepted as mathematical, Tall and Vinner (*ibid*, p. 152) defined a *concept definition* as ‘a form of words used to specify that concept’ and *concept image* as ‘the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and process’. In terms of geometrical figures a characteristic feature is their dual nature, in that both concept and image are closely inter-related. In this context, Fischbein (1993) proposed the notion of ‘*figural concept*’ in that, while a geometrical figure (such as a square) can be described as having intrinsic conceptual properties (in that it is controlled by geometrical theory), it is not solely a concept, it is an image too. (*ibid*, p. 141). Thus, when considering a square, it can be regarded as ‘a quadrilateral whose sides and angles are equal (a concept)’ as well as < □ > (an image) and not < □ >.

Taking this approach, on the one hand, individual students can be thought of as having their own *concept images* and their personal *concept definitions* of basic figures, all constructed through their own experiences of learning geometry. In this paper, for the purposes of analysis, we call examples of these a *personal figural concept*. On the other hand, there are formal concept images and definitions in geometry such that, when Euclidean definitions are used, a square, for example, is defined as a quadrilateral whose sides and angles are equal. We call such an example a *formal figural concept*. The research reported in this paper explores the nature of any gap between *personal figural concepts* and *formal figural concepts*.

Research on the teaching and learning of the classification of quadrilaterals illustrates these theoretical ideas. Following de Villers (1994), Heinze (2002) points out that mathematicians prefer a hierarchical classification for quadrilaterals (*ibid* pp. 83-4) and school curricula also follow this. One reason is its economical character, that is, if a statement is true for parallelograms, this means that it is also true for squares, rectangles and rhombuses. While this might seem straightforward to mathematicians, a number of studies have shown that many students have problems with a hierarchical classification of quadrilaterals (de Villers, 1994, p17; Jones, 2000), and this difficulty appears to persists with trainee teachers even though they are expected to have a sound knowledge of mathematics in order to teach this topic effectively. Kawasaki (1989), for example, found that only 5% could write a formal definition of a rectangle, and many of them defined it from their own image of rectangles, for example 'a rectangle is a quadrilateral whose sides are different'.

All this suggests that a gap exists between *personal figural concepts* and *formal figural concepts* for trainee teachers who have themselves undergone education in mathematics and therefore are supposed to understand mathematical topics up to at least secondary school level. It also suggests that images in their *personal figural concepts* have a strong influence over how they define/classify figures.

METHODOLOGICAL DESIGN

In order to explore this possible gap between the *formal figural concepts* and *personal figural concepts*, trainee primary teachers on a four-year teacher training course in Scotland were selected because the curriculum guidelines for Scotland specify that most pupils are expected to be able to define quadrilaterals and classify them in accordance with their properties by the time they are aged 14-15 (see also Fujita and Jones, 2003a). What is more, the expected level of understanding of mathematics for trainees on the course is that, to be allowed to commence the course, trainee have to have a level of mathematics indicating that they are able to classify quadrilaterals according to their definitions and properties (in Scotland this is called '*Standard Grade Credit level*').

Two sets of data are analysed below. One set of data comes from a survey of 158 trainee primary teachers in their first year of University study (most were 18 years old). After some taught input on the relationship between quadrilaterals, the following questions were presented to the trainee teachers:

Q1. Answer the following questions, and state your reasons briefly.

a. Is a square a trapezium?

b. Is a square rectangle?

c. Is a parallelogram a trapezium?

Q2. A kite is defined as ‘a quadrilateral, which has both pairs of adjacent sides equal’. Define the following quadrilaterals, and draw an image of each.

a. A parallelogram

b. A square

c. A rectangle

d. A trapezium

The design of this element of the study was informed by the research of Kawasaki referred to above.

The second set of data reported below is taken from a task used with 124 primary trainee teachers in their third year of University study (most were 20 years old). To show their understanding of hierarchical relationships in the classification of quadrilaterals, the trainees were asked to identify each quadrilateral in Figure 1 and draw arrows between particular pairs of quadrilaterals to show when one quadrilateral was a special case of another.

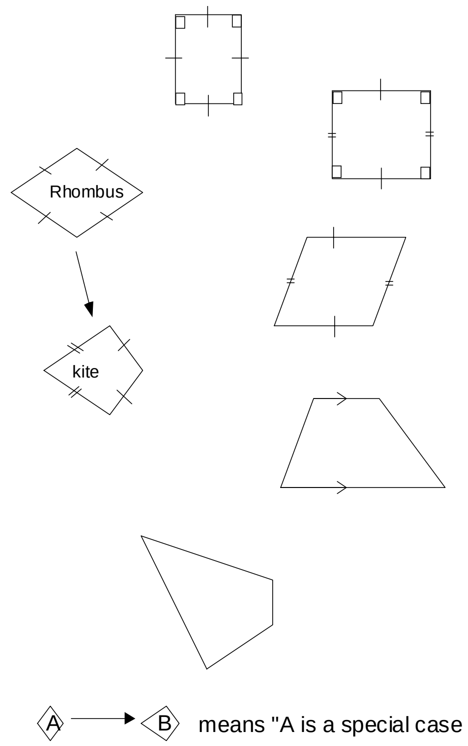


Figure 1: classifying quadrilaterals

For the analysis, we randomly selected 60 manuscripts, about 50%. Prior to the task, the trainees had a number of experiences of the teaching of simple geometrical shapes in primary school and had also studied ways of classifying quadrilaterals.

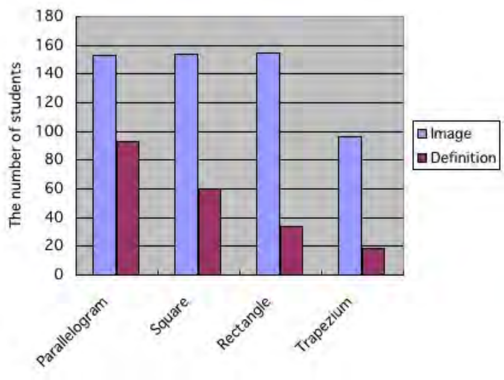
ANALYSIS

The results from the survey of the first year trainees are given in Figure 2 and Table 1 - the Table showing the results from the second question presented to the trainee teachers, and the Figure comparing the numbers of trainees providing the correct image compared to the number providing a correct definition.

This indicates that, for example, 14 trainees (8.9%) answered correctly the question about whether a square a trapezium, 20 trainees (12.7%) knew that a square is a rectangle, and 29 (18.4%) realised that a parallelogram is a trapezium. The latter result contrasts sharply with Kawasaki’s findings that 73% of Japanese trainee teachers can define a trapezium correctly.

Table 1 and Figure 2

Q2a Image Parallelogram	153 (96.8%)
Q2a Definition Parallelogram	93 (58.9%)
Q2b Image of a square	154 (97.5%)
Q2b Definition of a square	60 (38%)
Q2c Image of a rectangle	155 (98.1%)
Q2c Definition of a rectangle	34 (21.5%)
Q2d Image of a trapezium	96 (60.8%)
Q2d Definition of a trapezium	19 (12%)



Comparing image and definition in Figure 2, it can be seen that the majority of trainee teachers could at least draw a correct image of quadrilaterals (with the exception of a trapezium) but far less were able to provide their definitions. In the theoretical discussion in this paper, it was proposed that images in their *personal figural concepts* have strong influence when they define/classify figures, and this is appears to be bourn out in this study. For example, almost all trainees could draw a correct image of a square, while 62% (98 trainees) defined it incorrectly. Of these, 80 (about 82% of 98) wrote 'a quadrilateral whose sides are equal' and did not refer to 'angles'. If they had fully considered their *figural concepts*, they should have noticed that a rhombus can also satisfy this condition, and therefore it would be necessary to include something about the angles as well.

However, it seems that the image < □ > is so strong for them that many do not recognise the need to mention the angles being equal. Similarly, while 155 (98%) could draw an image of a rectangle, only 34 (21.5%) could define it correctly. Almost 70% (86 out of 124) defined a rectangle as 'a quadrilateral which has 2 longer sides and 2 shorter sides'. Again, they appear to be influenced by the image < □ >, and forgot to mention its angles. Moreover, 68 (43% of 158) defined both a square and a rectangle without mentioning angles. The results for parallelogram are slightly better, perhaps because the name 'parallelogram' is reminded them of 'parallel lines'.

Table 2 summarise an analysis of the third year trainee teachers' manuscripts, with the proportions obtained through counting the numbers of "correct" arrows from one quadrilateral to another (note that. some of the sample also drew additional "correct" arrows, such as, for example, from 'a square to a parallelogram - such arrows were not counted given the focus is on the efficient characteristics of the hierarchical classification for quadrilaterals).

Table 2

Arrows	Correct answer (%, n=60)
square -> rectangle	65%
square -> rhombus	40%
rectangle -> parallelogram	70%
rhombus -> parallelogram	16.7%
parallelogram -> trapezium	48.3%
trapezium -> quadrilateral	40%
kite -> quadrilateral	28.3%

Incorrect arrows were also found. For example, 13 trainees (about 21%) drew an arrow from ‘a rectangle’ to ‘a square’; that is, they regard that ‘a rectangle’ is a special case of ‘a square’. Similarly, 12 drew an arrow from ‘a rhombus’ to ‘a square’.

The weaker of the links shown in Table 2 occur in the relationships between ‘a rhombus’ and ‘a parallelogram’ (16.7%), and ‘a kite’ and ‘a quadrilateral’ (28.3%). The reason for these performances is uncertain, but it could be that trainee teachers persevere with their limited images of their *personal figural concepts* of, for example, parallelograms and rhombus and did not fully exercise their logical thinking skills. If they could flexibly ‘examine’ a rhombus, they might be able to notice that the opposite angles are equal in the rhombus and deduce that the rhombus has the pairs of parallel lines and therefore it is a parallelogram.

In summary, these results could be interpreted as relatively disappointing in that these trainee teachers do not seem to have a good understanding of the hierarchical relationship between quadrilaterals despite the entry requirements. Furthermore, even after two years or more years study on their course their understanding does not seem to improve. This suggests that a gap does exist between the *formal figural concepts* and their *personal figural concepts* such that their images are so influential in their personal figural concepts that they dominate their attempt to define basic quadrilaterals.

CONCLUDING COMMENTS

In Scotland there has been little study of the subject knowledge of trainee teachers. This paper presents in initial attempt to clarify what knowledge Scottish primary trainee teachers have. Further data is being collected of trainee teachers’ *personal figural concepts* and their understanding of hierarchical relationship between quadrilaterals. Meanwhile, the data is also just one component of a wider study that brings in data from England. As Ball *et al* (2005, p16) recommend "What is needed are more programs of research that complete the cycle, linking teachers' mathematical preparation and knowledge to their students' achievement".

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