Exciting electromagnetic anapoles with Flying Doughnut pulses

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As was predicted in 1995 by Afanasiev and Stepanofsky, a superposition of electric and toroidal dipoles can lead to a non-trivial non-radiating charge current-configuration, the dynamic anapole. The anapoles were recently observed first in microwave metamaterials and then in dielectric nano-disks. However, spectroscopic studies of toroidal dipole and anapole excitations are challenging owing to their diminishing coupling to transverse electromagnetic waves. Here, we show that anapoles can be excited by electromagnetic Flying Doughnut (FD) pulses. First described by Helwarth and Nouchi in 1996, FD pulses are space-time inseparable exact solutions to Maxwells equations that have toroidal topology and propagate in free-space at the speed of light. We argue that FD pulses can be used as a diagnostic and spectroscopic tool for the anapole excitations in matter.

Flying Doughnut (FD) pulses were introduced by Hellwarth and Nouchi in 1996 [1] as exact solutions to Maxwells equations in free-space [2–9]. They are necessarily few-cycle, wideband electromagnetic perturbations with toroidal field topology that can exist in transverse electric (TE) and transverse magnetic (TM) configurations [1, 10]. FD pulses can be seen as propagating counterparts of the localized toroidal dipole excitations in matter [11]. The toroidal dipole is distinct from the conventional electric and magnetic dipoles [12, 13] and have attracted significant interest in recent years as important contributors to the electromagnetic properties of media with non-local response or elements of toroidal symmetry [11, 14–21]. Superposition of dynamic electric and toroidal dipoles leads to anapoles which, through destructive interference, exhibit vanishing radiated fields outside the source [20, 22]. Anapoles have been observed in microwave metamaterials [16] and dielectric nanoparticles [18], and their excitation has also been predicted in core-shell nanoparticles [23] and nanowires [24]. Moreover, anapoles have been employed to enhance nonlinear effects [25] and realize high-Q microwave metamaterials [26]. However, anapole modes are weakly coupled to free-space radiation, which renders experimental observations particularly challenging. In this paper, we demonstrate numerically the excitation of anapoles in a spherical dielectric particle driven by an FD pulse.

We consider a dispersionless spherical dielectric particle interacting with a TM FD pulse, as depicted in Fig. 1. The FD pulse is brought to focus on the dielectric sphere located at the origin of the coordinate system. The interactions between FD pulses and dielectric spherical particles are investigated by a finite element solver of Maxwell's equations in three dimensions. The simulations are conducted in the transient domain. We define the incident FD pulse as per the field prescriptions in ref. [1], where the TM FD pulse is defined in terms of the azimuthal magnetic field H_θ^{TM}, and radial and longitudinal electric fields, E_ρ^{TM} and E_z^{TM}:

$$H_\theta^{TM} = -4if_0 \frac{\rho\left(q_1+q_2-2ict\right)}{\left[\rho^2+\left(q_1+i\tau\right)\left(q_2-i\sigma\right)\right]^3} \tag{1}$$

$$E_\rho^{TM} = 4if_0\sqrt{\frac{\mu_0}{\epsilon_0}} \frac{\rho\left(q_2-q_1-2iz\right)}{\left[\rho^2+\left(q_1+i\tau\right)\left(q_2-i\sigma\right)\right]^3} \tag{2}$$

$$E_z^{TM} = -4f_0\sqrt{\frac{\mu_0}{\epsilon_0}} \frac{\rho^2-\left(q_1+i\tau\right)\left(q_2-i\sigma\right)}{\left[\rho^2+\left(q_1+i\tau\right)\left(q_2-i\sigma\right)\right]^3}, \tag{3}$$

where $\sigma = z+ct$ and $\tau = z-ct$, and f_0 is an arbitrary normalisation constant. The parameters q_1 and q_2 have dimensions of length and represent respectively the effective wavelength of the pulse and the focal region depth [1]. To maintain consistency with previous studies [10] we chose $q_2 = 100q_1$. The dielectric sphere is characterised by a radius $r = q_1$ and a non-dispersive refractive index of $n = 2$.

To evaluate the modes excited within the dielectric particle, the induced time-dependent displacement currents are extracted from the calculations and are Fourier-transformed into the frequency domain. We analyzed the interactions between the particles and the FD to a maximum value of $\nu = 0.8c/q_1$ as accuracy at higher fequencies is limited by the temporal resolution of the model.

The excited Cartesian multipoles can then be evaluated from the extracted displacement currents. The expressions for the lowest order multipoles are given below. These are the electric dipole $\mathbf{p}$, magnetic dipole $\mathbf{m}$, toroidal dipole $\mathbf{T}$, electric quadrupole $Q^e_{\alpha\beta}$, and magnetic quadrupole $Q^m_{\alpha\beta}$ [13, 27]:

$$\mathbf{p} = \frac{1}{i\omega}\int \mathbf{j} d^3r \tag{4}$$

$$\mathbf{m} = \frac{1}{2c}\int (\mathbf{r}\times\mathbf{j})\, d^3r \tag{5}$$

$$\mathbf{T} = \frac{1}{10c}\int \left[(\mathbf{r}\cdot\mathbf{j})\,\mathbf{r} - 2r^2\mathbf{j}\right] d^3r \tag{6}$$

$$Q^e_{\alpha\beta} = \frac{1}{2i\omega}\int \left[r_\alpha j_\beta + r_\beta j_\alpha - \frac{2}{3}\delta_{\alpha\beta}\left(\mathbf{r}\cdot\mathbf{j}\right)\right] d^3r, \tag{7}$$

$$Q^m_{\alpha\beta} = \frac{1}{3c}\int \left[(\mathbf{r}\times\mathbf{j})_\alpha\, \mathbf{r}_\beta + (\mathbf{r}\times\mathbf{j})_\beta\, \mathbf{r}_\alpha\right] d^3r, \tag{8}$$

where $\mathbf{j}$ is the induced current density, which is defined inside the dielectric sphere as:

$$\mathbf{j} = i\omega\epsilon_0\left(\epsilon_r - 1\right)\mathbf{E} \tag{9}$$

The scattering intensity of these multipoles can then be calculated using:

$$I_{total} = \frac{2\omega^4}{3c^3}\left|\mathbf{p}\right|^2 + \frac{2\omega^4}{3c^3}\left|\mathbf{m}\right|^2 + \frac{2\omega^6}{3c^5}\left|\mathbf{T}\right|^2 + \cdots$$
$$\cdots + \frac{4\omega^5}{3c^4}\mathrm{Im}\left(\mathbf{p}^\dagger\cdot\mathbf{T}\right) + \frac{\omega^6}{5c^5}Q^e_{\alpha\beta}Q^e_{\alpha\beta} + \frac{\omega^6}{20c^5}Q^m_{\alpha\beta}Q^m_{\alpha\beta}, \tag{10}$$

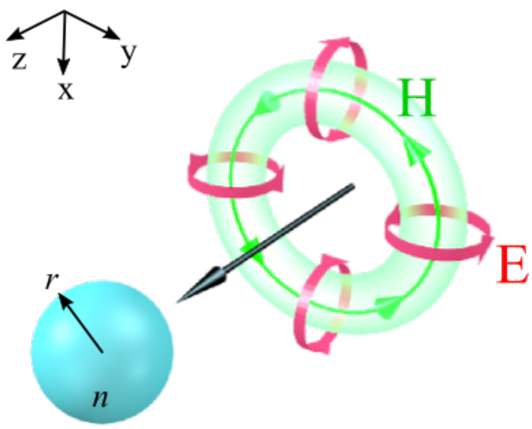

FIG. 1: Schematic illustrating a transverse magnetic (TM) FD pulse incident on a dielectric sphere of radius r and refractive index n. The pulse propagates along the z-axis and its topology is shown by the red and greeen arrows, representing E and H-fields, respectively.

where the term proportional to $\mathrm{Im}\left(\mathbf{p}^\dagger\cdot\mathbf{T}\right)$ reflects constructive or destructive interference between the electric and toroidal dipoles and as such can take either positive or negative values. Thus for co-located electric and toroidal dipoles of the correct amplitude and phase, their contributions to far-field scattering can completely cancel out resulting in an anapole excitation [16]. While an analysis of the multipole excitations in spherical nanoparticles interacting with TE and TM FD pulses can be found in Ref. [10], interference between different multipoles, which gives rise to anapole excitations, was not considered there. Here, we focus on such interference effects.

The calculated scattering intensity of the multipoles excited in the dielectric sphere are presented in Fig. 2a. At low frequencies, the scattering spectra are dominated by overlapping resonant electric dipole (blue line) and quadrupole (purple line) contributions. These result in a broad scattering peak as shown in Fig. 2(b), where the total scattering intensity spectra summed over all multipoles are presented. At $\nu \approx 0.46c/q_1$ the electric quadrupole contribution exhibits a minimum, whereas the electric (blue line) and toroidal (red line) dipoles present resonant peaks. However, the total scattering intensity spectrum (Fig. 2b) follows closely the trend of the electric quadrupole exhibiting suppressed scattering despite the presence of resonant electric and toroidal dipole contributions. Indeed, as indicated by the negative sign and large absolute value of the interference term (green line), $\mathrm{Im}\left(\mathbf{p}^\dagger\cdot\mathbf{T}\right)$, electric and toroidal dipoles interfere desctructively, which suppresses their total scattering intensity, as expected for a dynamic anapole. On the other hand, due to the cylindrical symmetry of the scattering problem and the TM polarization of the FD pulse, magnetic multipoles are suppressed over the spectral range of interest. Finally, we note that the electric octupole scattering contribution (not shown here) was found to be negligible over the spectral range of interest in accordance with recent studies on dielectric nanoparticles [28].

The presence of the anapole excitation can be confirmed by examining the near-field topology of the electromagnetic fields within the dielectric sphere (see Figs. 2c & 2d). At the anapole resonance ($\nu \approx 0.46c/q_1$), the x-component of the electric field (E_x) exhibits an antisymmetric pattern, which indicates the presence of a solenoid-like structure for the radial electric field component. Such an electromagnetic field configuration is typical for a toroidal dipole excitation oriented along the z-axis. At the same frequency, the longitudinal component of the electric field (E_z) reveals strong electric and toroidal dipole components along the z-axis. Both the electric and toroidal dipole excitation of the particle are confined mainly in the top half (along the $z-$axis) as illustrated in Fig. 2d. This asymmetry is imposed by the propagation direction of the FD pulse.

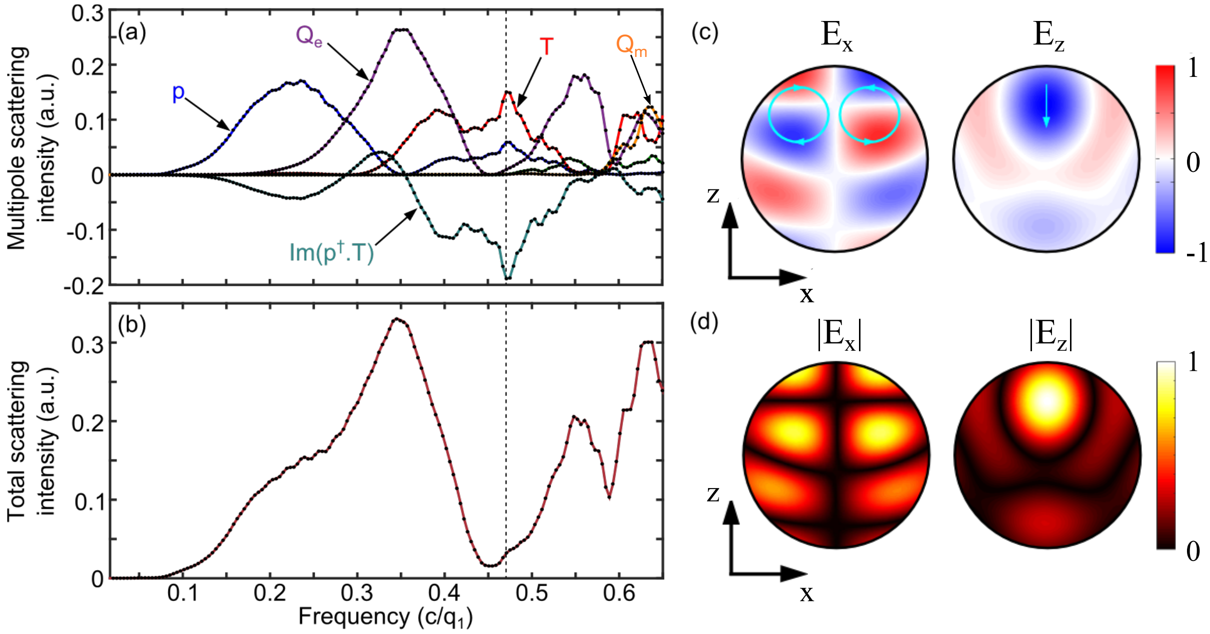

FIG. 2: (a) Scattering intensity of the dominant cartesian multipoles excited within the dielectric sphere when illuminated by a TM FD pulse as calculated by Eq. (9). (b) Total scattering intensity summed over the four leading multipoles. The point of suppressed scattering at $\nu \approx 0.46c/q_1$ is indicated by the dashed line. In both (a) and (b) dots correspond to simulation points, whilst lines serve as eye guides. (c) and (d) show the real parts and absolute values of the E_x and E_z components of the electric field on an xz cross-section of the dielectric particle at the anapole resonance ($\nu \approx 0.46c/q_1$). All fields are normalised to their maximum value. The direction of the vector fields are indicated by the light blue arrows in (c).

The anapole excitation observed in our numerical simulations is accompanied by a substantical decrease of the total scattering of the dielectric sphere, however it departs from an ideal anapole configuration for a number of reasons. Firstly, as the electric and toroidal dipoles are not equal in magnitude, destructive interference cannot completely cancel the far-field radiation, leaving some residual dipolar component. Furthermore, the scattering of other multipoles, such as the electric quadrupole Q_e, is non-negligible at the electric and toroidal dipole resonance. In addition, as the expansion here is only calculated up to the quadrupole order, it cannot be determined whether higher order multipoles e.g. electric octupole, will mask this dynamic anapole effect in the far-field. Nonetheless, this quasi-anapole is dominant up to the quadrupole order at $\nu \approx 0.46c/q_1$. This reinforces the necessity of including the toroidal contributions in the microscopic multipole analysis. We note that the minimum in scattering in the region of the electric quadrupole anti-resonance shall not be observed if the toroidal dipole is neglected and only the electric dipole is taken into account.

We argue that the excitation observed in our numerical simulations at $\nu \approx 0.46c/q_1$ has all characteristic features of the anapole, which is most notably manifested as a substantial decrease of the total scattering of the dielectric sphere resulting from destructive interference effects involving the toroidal dipole. However, the observed excitation departs from the canonical anapole configuration, which is characterized by perfect destructive interference of electric and toroidal dipoles, a configuration of particular interest due to the vector potential generated outside of the source that is not possible to remove by a change of gauge [20]. Indeed, an ideal, canonical anapole is not coupled to free-space electromagnetic waves, and only imperfect anapoles, where destructive interference is not complete, can be excited with free-space radiation. Here, it is exactly this imbalance between electric and toroidal dipoles that allows the excitation of the anapole mode. This situation is similar to "trapped-mode" excitations in metamaterials that are accessible through free-space only by introducing weak coupling to a radiating dipole [32].

Furthermore, in the case of the dielectric particle considered here, the scattering of higher multipoles, such as the electric quadrupole, is weak but non-negligible at the anapole resonance ($\nu \approx 0.46c/q_1$), and therefore toroidal dipole radiation interferes with a series of electric multipoles (multipoles higher then quadrupole are not considered in our analysis). However, we would like to note that the electric quadrupole contribution could be suppressed by increasing the refractive index of the particle [28, 33]. Finally, although we consider here dispersionless particles, we expect that TM FD pulses can still excite anapole modes in particles with weak dispersion. Indeed, since the anapole mode is excited over a narrow frequency band the presence of weak material dispersion will only slightly affect the excitation of anapoles.

In summary, we have demonstrated that TM FD pulses can excite resonant anapole modes in spherical dielectric particles, where scattering is significantly suppressed. In contrast to illumination with plane waves which leads to the excitation of higher order multipoles [18] and radially polarized light, where suppression of higher multipoles requires two tightly focused counter-propagating beams [34], a single FD pulse allows to excite anapole modes while suppressing higher multipoles in spherical dielectric particles. This result emphasizes the potential of Flying Doughnut pulses as a tool for sensing and spectroscopy of toroidal and anapole modes, even in systems that lack toroidal symmetry.

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