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The Mediation of Mathematical Learning through the use of Pedagogical Tools: a sociocultural analysis

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Abstract: A sociocultural analysis suggests that pedagogical artifacts employed in the teaching and learning of mathematics both enable and constrain learning. This paper summarises three classroom studies of mathematics learning that have utilised a sociocultural approach. Each of the studies indicates how insight can be gained into the ways in which students attempt to make sense of the mathematics they encounter.

Introduction

Mathematics deals with abstract objects. The use of concrete materials such as manipulatives, and tools such as calculators and computer software packages, to support mathematics learning is reasonably well-established and widely encouraged (in the case of calculators, at least in moderation). The intention of using such artifacts is to enable learning to take place through encounters with embodiments and representations of abstract mathematical objects. This paper focuses on how we might understand the mathematical learning taking place when students use such artifacts.

A key claim of a sociocultural analysis is that in order to understand the learning process we need to consider carefully what stands between the learners and the 'knowledge' that they are intended to learn; that is, we need to focus on the learning mediated through employing such resources (see, for example, Wertsch 1991, 1998). From a sociocultural perspective, artifacts do not serve simply to facilitate mental processes that would otherwise exist. Instead, they fundamentally shape and transform them. From this perspective, understanding the process of learning mathematics entails understanding how the use of pedagogical artifacts impacts on the learners conceptions of mathematical objects encountered through the use of such artifacts. As Cobb (1997 p170) says, "anticipating how students might act with particular tools and what they might learn as they do so is central to our attempts to support their mathematical development"

The purpose of this paper is to outline the results of some of the studies that have adopted a sociocultural approach to the mediation of mathematical learning by

tools. In the next section a brief overview of the sociocultural perspective on tool mediation is presented. This is followed by a review of empirical studies of mathematical learning that have adopted such a perspective. The paper finishes with a summary of what a sociocultural analysis of mathematical learning as mediated by artifacts might have to offer.

A Sociocultural Analysis of Tool Mediation

From a sociocultural viewpoint, the appropriation of mediational means is central to characterising the achievements of learning. Also crucial is social interaction aimed at supporting the effort towards such achievements. The nature of the appropriation of the mediational means depends upon the nature of the contexts in which these means are encountered - and on efforts related to the guided coordination of those contexts. The mediated nature of the encounters of the learner with the subject matter, in this case mathematics, is usually presented in triangular form (for examples see Cole 1996 p119, Ohtani 1994).

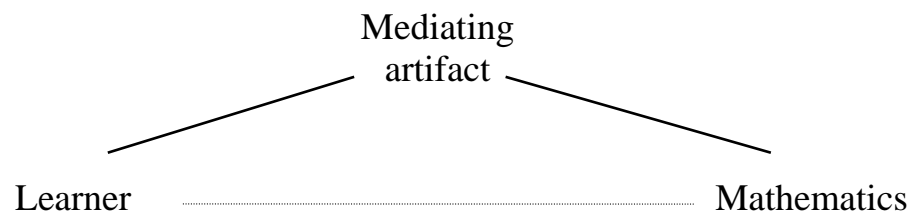


Figure 1

From this perspective, external tool-mediated action is transformed into internal mental action. Yet the impact is not solely one way. Internal mental action both enables and constrains tool use just as tool use both enables and constrains internal mental action.

The elements of tool mediation (from a sociocultural perspective) can be summarised as follows:

1. Tools are instruments of access to the knowledge, activities and practices of a community.
2. The types of tools existent within a practice are interrelated in intricate ways with the understandings that participants in the practice can construct.
3. Tools do not serve simply to facilitate mental processes that would otherwise exist, rather they fundamentally shape and transform them.
4. Tools mediate the user's action - they exist between the user and the world and transform the user's activity upon the world.
5. Action can not be reduced or mechanistically determined by such tools, rather such action always involves an inherent tension between the mediational

means and the individual or individuals using them in unique, concrete instances.

It is to some unique, concrete instances that we turn next.

Studies of the Tool Mediation of Mathematical Learning

In this section a number of studies of mathematical learning that have adopted a sociocultural approach to tool mediation are summarised. In such studies the focus is not so much on how using particular artifacts enables mathematics to be learnt more effectively or efficiently (necessarily) but on how such tools are actually used and transformed by students in activity and what characterises such devices as instruments of access to the knowledge and practices of school mathematics.

Using physical devices to instantiate linear functions (Meira 1998).

In this study, pairs of 13 year olds worked on problems involving linear functions. Nine pairs of students were studied. They were divided into three groups each with three pairs. All pairs were presented with equivalent problems, but each group had access to only one physical device that instantiated linear functions: a winch, a spring, or a computer-based ‘number machine’. At first, the students who worked with the winch and springs showed a more immediate and practical grasp of those mechanisms than did the students working with the computer environment. Gradually, however, the input-output displays generated by the number machine gave the students access to mathematical practices emanating from their prior study of functions and patterns.

As the work progressed, the students using the winch and the springs developed conceptual difficulties with aspects of each device (the size of the spool at the top of the winch, and the stiffness of the spring as a salient feature, respectively). The claim Meira makes is that using and making sense of physical devices in mathematical activity involves situated processes not predicted or explained by the epistemic view, close correspondence between representation and knowledge structure giving the greater transparency. Yet the number machine was not straightforwardly more transparent than the other devices. As Meira’s analysis shows, “the ability to see through the device was mediated by the students mutual appropriation of each other’s actions and discursive contributions, which allowed transparency in the making”.

Using devices to draw circles (Chassapis 1999)

In this study, a group of six 8 year old pupils were involved in a task in which they each had to create on paper and then cut out two circles (the circles were later joined together to form a ‘snake’). The pupils did this on three occasions, first with no drawing instrument available (ie by freehand drawing), secondly

with circle templates available, and finally pairs of compasses were introduced and the children instructed in their use.

After the first activity, the children were unhappy with their freehand drawing and Chassapis (1999) interprets this as indicating that their drawings did not correspond to their mental images of circles as ‘round’ objects. The availability of circle templates enabled the children to exercise greater motor control and hence produce more rounded circles, but, Chassapis maintains, this did not radically change the pupils’ spontaneous concept of a circle (at the age of 8 pupils have yet to be given a formal definition of a circle). The reason for this is that the template provides an image of the circle ‘ready-made’. The use of a pair of compasses did result in the children describing what they had drawn using language akin to that used in a mathematical definition of a circle. Chassapis argues that when a pair of compasses is used to draw a circle, “the action-bound practical thinking that occurs imposes the consideration of the centre and radius of the circle as crucial elements”.

Using dynamic geometry software to construct quadrilaterals (Jones 1997, 1998)

In this study 6 pairs of 12 year olds worked on a series of tasks involving constructing various quadrilaterals over a nine month period using a particular piece of dynamic geometry software. Analysis of the data (transcripts of videotapes) points to a number of instances of tool mediation. First, it appears that learners find the need to invent terms. For example, one pair of pupils employed the phrase “all stay together” to refer to invariance and coined the term “edge point” to refer to a point on the circumference of a circle. To some extent this parallels the need of the software designers to provide descriptors for the various different forms of point they are forced to use. Nevertheless, research on pupil learning with Logo suggests that learners use a hybrid of Logo and natural language when talking through problem solving strategies (for example, Hoyles 1996).

A second instance of tool mediation of learning is when students appear to understand a particular aspect of the computer environment but in fact have entirely their own perspective. For example, one student thought of points of intersection as ‘glue’ which would bind together geometrical objects such as lines and circles (a phenomenon observed by other researchers Ainley and Pratt 1995). A third illustration of the mediation of learning is how earlier experiences of successfully constructing figures can tend to structure later constructions. For example, one pair, who had successfully used intersecting circles to construct figures that were invariant under drag, would keep returning to this approach despite there being a number of different, though equally valid, alternatives.

Following from this last point, a further mediation effect can be that the dynamic geometry software might encourage a procedural effect with children focusing on the sequence of construction rather than on analysing the geometrical structure of the problem. A fifth illustration of the mediation of learning within the dynamic geometry software is that even if the drag mode allows a focus on invariance, pupils may not necessarily appreciate the significance of this. Thus hoping points of intersection will ‘glue’ a figure together, or that constructing a figure in a particular order will ensure it is invariant under drag, does not necessarily imply a particularly sophisticated notion of invariance.

Summary

These three research studies indicate that adopting a sociocultural perspective on the use of pedagogical devices allows some insight to be gained into the ways in which students attempt to make sense of the mathematics they encounter. Each study provides evidence of how the process by which learners create meaning is embedded within the setting or context and is mediated by the forms of interaction and by the tools being used. While research on the use of concrete manipulatives, for example, is, according to Thompson (1992), “equivocal at best”, research adopting a sociocultural approach may help to identify the ways in which pedagogical devices both provide the means and constrain mathematical learning, and thus make possible their more informed use in the classroom.

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