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The Mediation of Learning within a Dynamic Geometry Environment

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Computer-based learning environments promise much in terms of enhancing mathematics learning. Yet much remains unclear about the relationship between the computer environment, the activities it might support, and the knowledge that might emerge from such activities. The analysis presented in this paper is offered as a contribution to understanding the relationship between the specific tool being used, in this case the dynamic geometry environment Cabri-Géomètre*, and the kind of thinking that may develop as a result of interactions with the tool. Through this analysis I suggest a number of effects of the mediational role of this particular computer environment.*

Introduction

Computer-based learning environments continue to be a seductive notion in mathematics education. As Balacheff and Kaput (1996 p469) explain, it is possible for such environments to have an intrinsic cognitive character which is unique when compared to other learning materials. This means that such environments may be able to offer “a channel of access to the world of formal [mathematical] systems” (Noss 1997 p30). The promise is that through using particular software in carefully designed ways, it is possible simultaneously to use and come to understand important aspects of mathematics, something that in other circumstances can be particularly elusive.

One type of promising environment identified by Balacheff and Kaput (1996 p492) features what is commonly referred to as the “direct manipulation” of mathematical objects and relations. Prime examples of this type of software development are dynamic geometry environments (DGEs), such as *Cabri-Géomètre*. Yet, as diSessa *et al* (1995 p2) point out, significant issues with important practical ramifications remain under-researched. A vital question, and the theme of this paper, concerns the relationship between the specific tool being used and the kind of thinking that may develop as a result of interactions with the tool.

This paper explores some early findings from a longitudinal study aimed at investigating how using the dynamic geometry package *Cabri-Géomètre* mediates the learning of certain geometrical concepts, specifically the geometrical properties of the ‘family’ of quadrilaterals. In what follows I suggest some aspects of the mediational role of the DGE *Cabri-Géomètre*. I begin by outlining the theoretical basis of this view of tool mediation.

Theoretical Framework

From a sociocultural viewpoint, the development of mathematical reasoning is viewed as culturally mediated, through language and through the use of artifacts, both of which are referred to as tools. For example, Rogoff (1990) argues that children's cognitive development must be understood not only as taking place with social support in interaction with others, but also as involving the development of skill with socio-historically developed tools that mediate intellectual activity. Such a perspective builds on the work of Vygotsky who stressed that "the central fact about our psychology is the fact of mediation" (Vygotsky 1982 p166). Wertsch too extends Vygotskian ideas by incorporating elements from the work of Bakhtin (Wertsch 1991, Wertsch *et al* 1995) demonstrating how language is a mediating influence which "lies on the borderline between oneself and the other .... [so that] expropriating it, forcing it to submit to one's own intentions and actions, is a difficult and complicated process" (Bakhtin 1981 pp 293-294). As Cobb (1997 p170) confirms "tool use is central to the process by which students mathematize their activity", and further that "anticipating how students might act with particular tools and what they might learn as they do so is central to our attempts to support their mathematical development"

Such theoretical work suggest some elements of tool mediation which can be summarised as follows:

1. Tools are instruments of access to the knowledge, activities and practices of a community.
2. The types of tools existent within a practice are interrelated in intricate ways with the understandings that participants in the practice can construct.
3. Tools do not serve simply to facilitate mental processes that would otherwise exist, rather they fundamentally shape and transform them.
4. Tools mediate the user's action - they exist between the user and the world and transform the user's activity upon the world.
5. Action can not be reduced or mechanistically determined by such tools, rather such action always involves an inherent tension between the mediational means and the individual or individuals using them in unique, concrete instances.

Examples of mathematics education research which make use of the notion of tool mediation include Cobb's study of the 100 board (Cobb 1993), Säljö's work on the rule of 3 for calculating ratios (Säljö 1991), and Meira's examination of using gears to instantiate ratios (Meira 1995).

Applying such notions to learning geometry within a DGE suggests that learning geometrical ideas using a DGE may not involve a fully 'direct' action on the geometrical theorems as inferred by the notion of 'direct manipulation', but an indirect action mediated by aspects of the computer environment. This is because the DGE has itself been shaped both by prior human practice and by aspects of computer architecture. This means that the learning taking place using the tool, while benefiting from the mental work that produced the particular form of software, is shaped by the tool in particular ways.

Some aspects of the mediational impact of *Cabri-géomètre*

Applying this theoretical perspective to the implementation of *Cabri 1* for the PC, together with a reading of previous research (for example: Goldenberg and Cuoco 1998, Hölzl 1996, Jones 1996, 1997), leads to the following being identified as possible mediating influences on learners using this particular tool:

1. The layout of the interface with the separation of the *creation* and *construction* menus.
2. The default cursor operation, which is drag rather than, say, the creation of a point.
3. The existence of a number of different forms of point in *Cabri I*: basic point, point on object, (point of) intersection - not to mention midpoint, symmetrical point, and locus of points, plus centre of a circle; also rad pt (radius point) and circle point.
4. The existence of a number of several forms of line: basic line, line segment, line by two pts (points) - not to mention parallel line, perpendicular line, plus perpendicular bisector, and (angle) bisector.
5. The existence of two different forms of circle: basic circle, circle by centre & rad pt.
6. The implementation of the drag-mode within the software which entails decisions being made about the behaviour of the geometrical objects when they are dragged. For example basic circle and circle by centre & rad pt behave differently under the drag-mode.
7. The fact that some points can be dragged while others cannot. For example, constructed points such as a (point of) intersection or a midpoint cannot be dragged, while others, such as a point on object can be dragged but only in a particular way.
8. The behaviour of a point placed arbitrarily on a line segment when an end-point of the segment is dragged.
9. The sequential organisation of actions in producing a geometrical figure. This implies the introduction of explicit order where, for most of the users, order is not normally expected or does not even matter. For example, *Cabri-géomètre* induces an orientation on the objects: a segment AB can seem orientated because A is created before B. This influences which points can be dragged and effectively produces a hierarchy of dependencies in a complex figure.

None of the above is necessarily a criticism of *Cabri*. In the implementation of such software, decisions have to be made. The point is that *the decisions that are made mediate the learning*. The remainder of this paper documents some examples of this shaping of learning within this particular DGE in an attempt to reveal possible tensions between the tool and the actions of the learners.

Empirical study

The empirical work on which the observations below are based is a longitudinal study examining how using the dynamic geometry package *Cabri-géomètre* mediates the learning of geometrical concepts. The focus for the study is how "instructional artifacts and representational systems are actually used and transformed by students *in activity*" (Meira 1995 p103, emphasis in original) rather than simply asking whether the students learn particular aspects of geometry "better" by using a tool such as *Cabri*.

The data is in the form of case studies of five pairs of 12 years old pupils working through a sequence of specially designed tasks requiring the construction of various quadrilaterals using *Cabri-géomètre* in their regular classroom over a nine month period. The version of *Cabri* in use was *Cabri I* for the PC. Sessions were video and audio recorded and then transcribed. In all, over 40 lesson transcripts are being analysed in two phases. The first phases identified examples of tool mediation, a number of which are illustrated below. The second phase, currently in progress, is intended to track the genesis of such tool mediation of learning.

Examples of tool mediation

Below are four examples of extracts from classroom transcripts which reveal aspects of the tool mediation of learning within the dynamic geometry environment.

Example 1

Pair Ru and Ha are checking, part way through a construction, that the figure is invariant when any basic point is dragged

Ru	Just see if they all stay together first.
Ha	OK.
Ru	Pick up by one of the edge points. [H drags a point]
Ha & Ru (together)	Yeah, it stays together!

Note the pupils' use of the phrase "all stay together" to refer to invariance and the term "edge point" rather than either radius point (or rad pt as the drop-down menu calls it) or circle point (as the help file calls that form of point).

Example 2

Pair Ho and Cl are in the process of constructing a rhombus. As they go about constructing a number of points of intersection, one of the students comments:

Ho:	A bit like glue really. It's just glued them together.

This spontaneous use of the term "glue" has been observed by other researchers (Ainley and Pratt 1995) and is all the more striking given the fact that earlier on in the lesson the pupils had confidently referred to points of intersection as just that.

Example 3

Pair Ru and Ha are constructing a square using a diagram presented on paper as a starting point. After a short discussion the pair begin by constructing two interlocking circles:

Ha	If ...I .. erm ..
	I reckon we should do that circle first.
Ru	Do the line first.
Ha	No, the circle. Then we can put a line from that centre point of the circle.
Ru	Yeah, all right then.
Ha	You can see one .. circle there, another there and another small one in the middle.

The inference from this extract is that previous successful construction with the software package influences the way learners construct new figures.

Example 4

Pupils Ru and Ha have constructed a square and are in the process of trying to formulate an argument as to why the figure is a square. I intervene by asking them what they can say about the diagonals of the shape (in the transcript I refers to me).

Ru	They are all diagonals.
I	No, in geometry, diagonals are the lines that go from a vertex, from a corner, to another vertex.
Ru	Yeah, but so's that, from there to there.
I	That's a side.
Ru	Yeah, but if we were to pick it up like that .... like that. Then they're diagonals

Pupil Ru is confounding diagonal with oblique, not an uncommon incident in lower secondary school mathematics. Here the software cannot provide any assistance to the student, indeed the drag facility allows any straight line to be moved to appear to the learner to have an oblique orientation. Furthermore, in terms of the specialised language of mathematics, such software can not hope to provide the range of terms required nor could it be expected to do so. Such exchanges call for sensitive judgement by the teacher.

Some observations on the examples

The examples given above are representative of occurrences within a number of the case studies. A number of comments can be made on these extracts which illustrate how learning within the computer environment is shaped by the nature of the mediating tool.

First, it appears that learners find the need to invent terms. In example 1 above, the pupil pair employ the phrase “all stay together” to refer to invariance and coin the term “edge point” to refer to a point on the circumference of a circle. To some extent this parallels the need of the software designers to provide descriptors for the various different forms of point they are forced to use. Yet research on pupil learning with Logo suggests that learners use a hybrid of Logo and natural language when talking through problem solving strategies (for example, Hoyles 1996). This, I would argue, is one effect of tool mediation by the software environment.

A second instance of the mediation of learning is when children appear to understand a particular aspect of the computer environment, in example 2 above it is the notion of points of intersection, but in fact they have entirely their own perspective. In this example, one student thinks of points of intersection as ‘glue’ which will bind together geometrical objects such as lines and circles. This, I would suggest, is an example of Wertsch’s (1991) ‘ventriloquating’, a term developed from the ideas of Bakhtin, where children employ a term such as intersection but, in the process, inhabit them with their own ideas.

A third illustration of the mediation of learning is how earlier experiences of successfully constructing figures can tend to structure later constructions. In example 3 above, the pair had successfully used intersecting circles to construct figures that are invariant under drag and would keep returning to this approach despite there being a number of different, though equally valid, alternatives.

Following from this last point, a further mediation effect can be that the DGE might encourage a procedural effect with children focusing on the sequence of construction rather than on analysing the geometrical structure of the problem. Thus pair Ru and Ha, rather than focusing on geometry might be focusing rather more on the procedure of construction. This may also be a consequence of the sequential organisation of actions implicit in a construction in *Cabri-Géomètre*.

A fifth illustration of the mediation of learning within the DGE is that even if the drag mode allows a focus on invariance, pupils may not necessarily appreciate the significance of this. Thus hoping points of intersection will ‘glue’ a figure together, or that constructing a figure in a particular order will ensure it is invariant under drag, does not necessarily imply a particularly sophisticated notion of invariance.

From the examples given above, a sixth illustration of the mediation of learning is provided by an analysis of the interactions with the teacher (in this case the researcher). The challenge for the teacher/researcher is to provide input that serves the learners’ communicative needs. As Jones (1997 p127) remarks “the explanation of why the shape is a square is not simply and freely available within the computer environment”. It needs to be sought out and, as such, it is *mediated* by aspects of the computer environment and by the approach adopted by the teacher.

Concluding remarks

In this paper I have suggested some outcomes of the mediational role of the DGE *Cabri-Géomètre*. While such outcomes refer to only one form of computer-based mathematics learning environment, these outcomes are similar to those emerging from research into pupils’ learning with Logo (see Hoyles 1996 p103-107):

1. Children working with computers become centrated on the screen product at the expense of reflection upon its construction
2. Students do not mobilise geometric understandings in the computer context
3. Students modify the figure “to make it look right” rather than debug the construction process
4. Students do not appreciate how the computer tools they use constrain their behaviour

5. After making inductive generalisations, students frequently fail to apply them to a new situation
6. Students have difficulty distinguishing their own conceptual problems from problems arising from the way the software happens to work
7. Manipulation of drawings on the screen does not necessarily mean that the conceptual properties of the geometrical figure are appreciated

As Hoyles remarks, such indications are intended to capture some of the general in the specific and thereby generate issues for further research. The finding from this study of the dynamic geometry package *Cabri-Géomètre* may well prove useful both to teachers using, or thinking about using, this form of software and to designers of such learning environments, as well as contribute to the further development of theoretical explanations of mathematics learning.

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