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Children Learning to Specify Geometrical Relationships using a Dynamic Geometry Package

Keith Jones
University of Southampton, UK

In order to understand the learning taking place when students use a dynamic geometry package such as Cabri-Géomètre*, a particular focus for study needs to be on the learning* mediated *through employing such a resource. In this paper I describe how one pair of 12 year old students begin learning how to specify geometrical relationships in* Cabri*. I argue that, while* Cabri *provides certain elements of the mathematical language necessary for the articulation of relevant mathematical ideas, significant aspects must be provided by the teacher.*

Introduction

The use of concrete materials such as manipulatives, and tools such as calculators and computers, to support mathematics learning is reasonably well-established and widely encouraged. In trying to understand the mathematics learning taking place when students use such devices, the work of Wertsch (1991), amongst others, suggests that we need to consider carefully what stands between the learners and the 'knowledge' that they are intended to learn; that is, we need to focus on the learning *mediated* through employing such resources. Ohtani (1994), for example, presents this in the usual triangular form (adapted slightly as Figure 1).

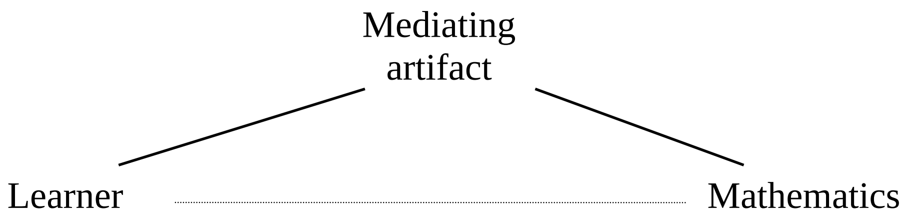

Figure 1

Dynamic geometry environments (DGEs), such as *Cabri-Géomètre*, are one example of such mediating artifacts. Such a package allows the user to experience the *direct* manipulation of geometrical objects (or, at least, the *appearance* of such direct manipulation). Within the computer environment, geometrical objects created on the screen can be manipulated by means of the mouse (a facility generally referred to as 'dragging'; for further details see Hölzl in press). What is particular to DGEs is that when elements of a construction are dragged, all the geometric properties employed in constructing the figure are preserved. This is because one of the significant features of a dynamic geometry package is the ability to specify relationships between geometrical objects (Laborde and Laborde 1995 p 240). In this way, the software provides the learner with a means of expressing mathematical ideas. As Noss and Hoyles (1996 p 54) argue: "It is this articulation which offers some purchase on what the learner is thinking, and it is in the process of articulation that a learner can create mathematics and simultaneously reveal this act of creation to an observer." Hence when students are using a DGE such as *Cabri* to tackle mathematical problems they are involved in both perceiving and specifying relationships between geometrical objects.

In this paper I focus on the transition from perceiving and specifying geometrical relationships when students are using *Cabri* and how this is mediated by the computer environment. In what follows I describe how one pair of 12 year old students begin learning how to specify geometrical relationships in *Cabri*. I argue that, while *Cabri* provides certain elements of the mathematical language necessary for the articulation of relevant mathematical ideas, significant aspects must be provided by the teacher. The data comes from a longitudinal research project designed to trace the transition of student conceptions of some chosen geometrical objects from informal notions towards formal mathematical definitions. I begin with a brief outline of the theoretical framework with which I will interpret the data.

The Mediation of Learning

One of the central concepts underlining the approach I adopt in this paper is Wertsch's notion of "individual(s)-acting-with mediated means" (Wertsch 1991 p 12) which is itself based on aspects of the work of Vygotsky and Bakhtin. From such a perspective there is an intimate relationship between psychological processes and the sociocultural setting such that all mental processes are considered to be mediated by communication that is inherently and complexedly situated. In this model, when we describe human action we can only do so in terms of the mediating artifact because "action and mediating means are mutually determined" (p 119).

A second central concept is the idea that the move from *perceiving* to *specifying* is at the heart of mathematics learning. In this context, *specifying* requires the use of elements of conventional mathematical language. With certain computer applications, such as Logo, spreadsheets and perhaps DGEs, the computer can become a special tool for mathematics learning because the actions of learners using such applications necessarily involves some formal use of mathematical language. Noss and Hoyles call such a computer environment "autoexpressive" when it contains elements of mathematical language "to talk about itself" (1996 p 69). For a DGE such as *Cabri*, some of the relevant elements of mathematical language (such as mid-point, bisector, perpendicular, and so on) can be considered to be explicitly available via the various menu items. Further elements are implicitly contained within the figure as it is constructed. I will return to this point later in this paper. Given these considerations, the central question here is how we can describe the learning of aspects of plane geometry when mediated by a computer application such as *Cabri*.

With the above in mind, particular foci for the presentation and analysis of the qualitative data from this study are:

- how particular geometric figures presented on paper are interpreted by the students when the aim is to construct them using *Cabri*
- how the figures are constructed; that is how they are specified in terms of the *Cabri* menu items
- what the response is to the feedback presented by the resulting image on the screen
- how the specification is checked
- what form of assistance is sought from the teacher/researcher and what the response is to interventions

I follow the example of Meira by focusing on how "instructional artifacts and representational systems are actually used and transformed by students *in activity*" (1995 p 103, emphasis in original) rather than simply asking whether the students learn particular aspects of geometry better by using a tool such as *Cabri*. This is because what I am interested in is both what the students learn *and* how they learn it.

Description of an Episode

This data comes from a research study in which pairs of students in their regular mathematics classroom tackle a series of tasks focusing on the geometrical properties of quadrilaterals. The pair of students in this extract are 12 year olds who have used *Cabri* on four previous occasions, each one lasting almost an hour, the last time being about four weeks earlier. The class is of above-average attainment in mathematics and from a UK city comprehensive school whose results in mathematics at age 16 are at the national average. The mathematics teacher employs a problem-based approach to teaching mathematics and the students usually work in pairs or small groups. The class has three 50-minute mathematics lessons per week. The version of *Cabri* in use was *Cabri I* for the PC.

The task the pair of students are undertaking is to construct the following diagram, Figure 2, using *Cabri* and hence obtain Figure 3.

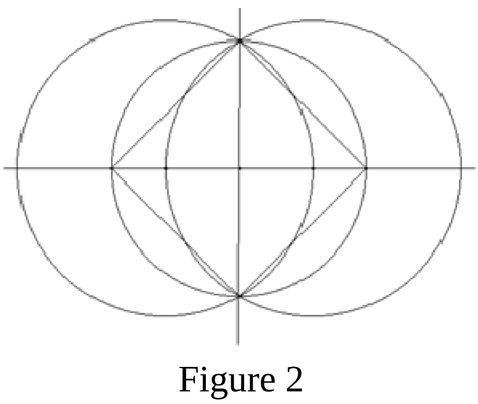

Figure 2

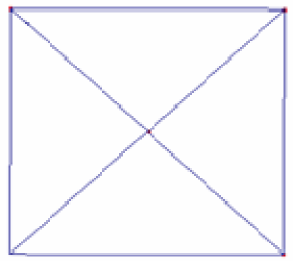

Figure 3

The task then asks the students to "explain why the shape is a square". The students know that they need to construct the figure in such a way that the figure is invariant when any basic object used in its construction is dragged. In the words of Healy *et al* (1994), the figure must be impossible to "mess up".

After a short discussion the pair begin by constructing two interlocking circles, as shown in Figure 4.

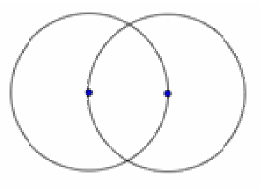

Figure 4

In order to draw the third circle they need to construct its centre. They realise that it has to be midway in between the centres of the two larger circles. In the extracts that follow, *R* and *H* are the students, *I* is myself as teacher/researcher.

28	R	You want to get that thing in between them, I can't remember what it's called
29		now.
30	H	Construction is it? No ..
31	R	Yes, on Construction, and it is ...
32	H & R (together)	Intersection!

The students attempt to use *intersection*, but, of course, it is not the correct choice. I decide to intervene.

39	I	What are you trying to do?
40	R	Make a point in between there.
41	I	An intersection will only give you the point where two lines cross. But there is
42		something else which will give you something which is halfway between.
43	R	Go under Construction.
44	I	Yes, have a look under Construction again.
45	H & R (together)	Yeah, Midpoint!

They create the third circle and check that their construction is correct by dragging one of the points on their figure.

69	R	Yeah, that's it Then we want like a diamond shape inside it.
70	H	So we need to ....

69 R Just see if they all stay together first.
70 H OK.
71 R Pick up by one of the edge point.
72 H & R (together) Yeah, it stays together!

The next step the students make is to draw two lines, see Figure 5, and again check, by dragging, that their construction is correct

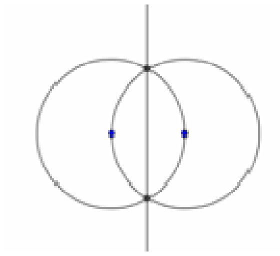

Figure 5

They complete their construction by drawing the four line segments forming the square and once more check, by dragging, that their figure cannot be "messed up". To construct the figure shown in Figure 2 they "erase" (or, more accurately, hide) the requisite lines and finish by constructing line segments as diagonals of the square.

One of the students comments:

167 R A square. Four triangles in it.
168 Or is it a rectangle? Those bits look longer.
169 H They do slightly.
170 R Should I get a ruler?

I intervene by asking them what they can say about the diagonals of the shape.

174 R They are all diagonals.
175 I No, in geometry diagonals are the lines that go from a vertex, from a corner, to
176 another vertex.
177 R Yeah, but so's that, from there to there.
178 I That's a side.
179 R Yeah, but if we were to pick it up like that .... like that. Then they're diagonals
180 I In mathematics, in geometry, a line that goes like that is called an oblique line.
181 It's not vertical, it's not horizontal. It's oblique.

Following this I prompt them into beginning to explain why the quadrilateral is a square. For example, I ask them to compare the lengths of the diagonals and how they intersect.

195 I and what can you say about that line and this line [referring to the diagonals]?
196 H They're the same distance.
197 I They're the same length?
198 H Length, yeah.
199 I OK, so the diagonals are the same length. And what can you say about the way in
200 which they cross?
201 H They cross exactly in the middle.
202 I So you're saying that from there to there is the same as from there to there.
203 H Yeah.

214 I At what angle do they cross?
215 H A right angle.
216 I (to R) Is it a right angle?
217 R No .. yeah.
218 I Yes? So this is a right angle here?
219 R Yeah.

The session finishes with my asking them:

271 I So what sort of shape has got diagonals that are the same, that cross in the
272 middle, so they bisect each other, that cross at 90 degrees, and has got 90 degree
273 corners? What sort of shape is it?
274 H A square.
275 I No other shape is like that?
276 H No.

Analysis and Discussion

The students successfully complete the task, but with particular input from myself as teacher/researcher. This is not altogether unexpected as, in every attempt to reveal the mathematical thinking of learners, the balance between exploration and guidance is always problematic. As Noss and Hoyles explain "This tension is not completely resolvable. We might be able to engineer situations in which a mathematical way of thinking is encouraged. But mathematics *per se* is not discovered by accident" (1996 p 71). What becomes of interest here is the *nature* of the interventions that were necessary.

The students begin confidently enough, although it soon transpires that they have forgotten the term *midpoint*. They know what they want to specify (the centre for the third, smaller, circle) but attempt to locate it using *intersection*, as the drop-down menu calls it (actually the item locates *points of intersection*). An intervention is sufficient to put the students on the right track again.

Lines 69 through 74 shows student R firstly referring to the square to be constructed as a *diamond* (presumably due to its orientation; see Hershkowitz 1990 p 82- 86) and later calling a point on the circumference of a circle an *edge point*. This latter choice of terminology is especially interesting as this particular form of point (and there are several forms of point in *Cabri I*) is referred to in *three different ways* on the screen in this version of *Cabri* (*Cabri I* for the PC). From the creation menu, one can construct a circle using the menu item *circle by centre and rad. pt*. (the user needs to know, presumably, that *rad. pt.* is a shortened version of *radius point*). The pop-up help offers the advice "select or create the centre of the circle, then *a point on the circle*", while the screen pointer uses the terms "this centre" and "this *circle point*" when creating such a circle. This particular student then invents their own term.

At this point, the students use the drag facility to check that their construction so far specifies the appropriate geometrical relationships. It does. By lines 167-170 in the transcript, student R is referring to the quadrilateral as a square, but queries the screen image. As I do not think measuring, particularly with a ruler, will resolve the matter I intervene by asking them to reflect on what they have done (transcript lines 174-276). In so doing, I have to introduce terminology that does not occur in any menu item in this version of *Cabri*. At various times I employ terms such as diagonal, vertex, oblique, bisect (note that bisector is a *Cabri* menu item), and right angle.

Finally, the students complete their construction, again checking by dragging that the construction can not be "messed up". They are convinced that the quadrilateral they have constructed is a square and they can articulate some of its geometrical properties.

Concluding Remarks

Overall, the episode portrayed here demonstrates that this particular pair of students had, at their disposal, sufficient technical fluency with *Cabri* to successfully complete the required task (albeit with some timely intervention). It was they who devised the strategy for the construction and consequently it was they who were able to specify their construction using the facilities offered by this particular dynamic geometry environment. They did not merely line up relevant objects by eye nor did they start guessing by randomly opening menus and trying out all the items in some false hope of hitting on the right one (phenomena observed by Noss *et al* 1994 and by Jones 1995).

Yet, at the same time, the computer environment alone was insufficient to allow the students to fully articulate their specification in conventional mathematical language. For one thing, the menu items cannot hope to provide the range of terms required (nor could they be expected to do so). For another, a full articulation of why the quadrilateral is a square requires some of those delicate chains of reasoning characteristic of the finer elements of mathematical proof. The explanation of why the shape is a square is not simply and freely available within the computer environment. It needs to be sought out and, as such, it is *mediated* by the computer environment.

On the other hand, the essence of the explanation *is* contained implicitly within the construction. The students' construction of the square is a *general* representation and not a copy of a particular concrete object. What is more, the properties of the figure are derived from definitions within the realm of the Euclidean axiomatic system. The UK mathematics curriculum expects students at this level (above average 12 year olds) to *begin* giving mathematical justification for their generalisations. An objective of the curriculum then is to develop their ability to use mathematical language effectively in presenting a convincing reasoned argument. As currently specified, it is only the more able 14 to 16 year old who are taught to "extend their mathematical reasoning into understanding and using more rigorous argument, leading to notions of proof" (DfEE 1995 p 20). It may be that experiences with a DGE such as *Cabri*, and tackling suitable tasks, will help to allow this objective to be realised.

The example provided in this paper shows some aspects of how it is interaction with more knowledgeable others that ensures that at least some of the explanation available with the DGE can become accessible to the student learners of mathematics. Hence, while *Cabri* provides certain elements of the mathematical language necessary for the articulation of relevant mathematical ideas, significant aspects must be provided by the teacher. This paper has attempted to document at least some of these aspects.

References

Department for Education (1995), *Mathematics in the National Curriculum*. London: HMSO.

Healy, L., Hoelzl, R., Hoyles, C., & Noss, R. (1994). “Messing Up”. *Micromath*, **10**(1), 14-16.

Hershkowitz, R. (1990). “Psychological Aspects of Learning Geometry”. In P Nesher & J Kilpatrick (Eds.), *Mathematics and Cognition*. (pp. 70-95). Cambridge: CUP.

Hölzl, R. (1996), ‘How does “dragging” affect the learning of geometry’, *International Journal of Computers for Mathematical Learning*, **1**(2), 169–187.

Jones, K. (1995), “Intuition and geometrical problem solving”. In C Mammana (Ed), *Proceedings of the ICMI Conference on Perspectives on the Teaching of Geometry for the 21st Century*. Catania, It: University of Catania. p 112-116

Laborde, C. and Laborde, J-M. (1995), “What about a Learning Environment where Euclidean Concepts are manipulated with a Mouse?”. In A. A. diSessa, C. Hoyles and R. Noss (Eds), *Computers and Exploratory Learning*. Berlin: Springer-Verlag.

Laborde, C. (1993). The Computer as Part of the Learning Environment: the case of geometry. In C. Keitel & K. Ruthven (Eds), *Learning from Computers: mathematics education and technology*. Berlin: Springer-Verlag. p48-67.

Meira, L. (1995), “Mediation by Tools in the Mathematics Classroom”. In L. Meira and D. Carraher (Eds), *Proceedings of the 19th Conference of the International Group for the Psychology of Mathematics Education*. Recife, Brazil. Vol I p102-111.

Noss, R. and Hoyles, C. (1996), *Windows on Mathematical Meanings: learning cultures and computers*. Dordrecht: Kluwer.

Noss, R., Healy, L, Hoyles, C., & Hoelzl, R., (1994), “Constructing Meanings for Construction”. In J P da Ponte and J F Matos (Eds), *Proceedings of the 18th Conference of the International Group for the Psychology of Mathematics Education*. Lisbon, Portugal. Vol III p360- 367.

Ohtani, M. (1994), “Sociocultural Mediateness of Mathematical Activity: analysis of “voices” in a seventh-grade mathematics classroom”. In J. P. da Ponte and J. F. Matos (Eds), *Proceedings of the 18th Conference of the International Group for the Psychology of Mathematics Education*. Lisbon, Portugal. Vol III p385- 391.

Wertsch, J. V. (1991), *Voices of the Mind: a sociocultural approach to mediated action*. London: Harvester.