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COMING TO KNOW ABOUT 'DEPENDENCY' WITHIN A DYNAMIC GEOMETRY ENVIRONMENT

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The ability to define relationships between objects is one of the most powerful features of a dynamic geometry package such as Cabri-Géomètre*. In this paper I document how one pair of 12 year old students begin to come to know about this form of functional dependency within this particular computer environment. I suggest that this process of coming to know about dependency may be understood as an interweaving between the 'voices' of the students and the teacher within the socially organised activity taking place in the classroom.*

La capacidad para definir relaciones entre objetos es una de las características mas potentes de un paquete integrado de geometría dinámica como es el Cabri Geometría*. En este artículo, presento como un par de alumnos de 12 años empiezan a comprender el concepto de dependencia funcional dentro de este contexto informático particular. Sugiero que este proceso de iniciación a la comprensión de la dependencia puede interpretarse como un entretejerse las 'voces' de los alumnos y el profesor dentro de una actividad socialmente organizada que tiene lugar en el aula.*

Introduction

One of the most powerful features of a dynamic geometry package such as *Cabri-Géomètre* is the ability to define relationships between objects and to explore graphically the implications (Laborde 1993 p 53). The drag facility allows a figure to be continuously transformed while the relationships between the objects remain invariant. The idea of dependency (and independency) can be explored by, for instance, observing the nature of the relationships between the objects used to construct the figure in question when a chosen object such as a point is dragged. Another way of observing dependency is when an object is deleted. Then all dependent objects are also deleted.

As Holzl *et al* (1994) discovered when they observed pupils attempting to construct a rectangle, the students had to come to terms with "the very essence of Cabri; that *a figure consists of relationships* and that there is *a hierarchy of dependencies*" (emphasis in original). An example of this hierarchy of dependencies is the difference (in *Cabri 1* for the PC) between *basic point*, *point on object* and *point of intersection*. While all three types of point look identical on the screen, basic points and points on objects are moveable (with obvious restrictions on the latter). Yet a point of intersection cannot be dragged. This is because a point of intersection depends on the position of the basic objects which intersect. In their study, Holzl *et* *al* found that students need to develop an awareness of such functional dependency if they are to be successful with non-trivial geometrical construction tasks using *Cabri*. The experience of Holzl *et al* is that "Not surprisingly, the idea of functional dependency has proved difficult [for students] to grasp".

In this paper I describe how one pair of 12 year old students begin to come to know about dependency within the dynamic geometry environment *Cabri*. The data comes from a project designed to trace the transition of student conceptions of some chosen geometrical objects from informal notions towards formal mathematical definitions. I begin with an outline of the theoretical framework with which I will interpret the data.

Theoretical framework

It is evident that "coming to know" is a complex process and that an understanding of such a process cannot be explored in a framework that detaches that learning from its sociocultural setting. Mercer (1995 and with Edwards 1987), for instance, has expounded on the guided construction of knowledge within the classroom by stressing the importance of talk between teachers and learners. Wertsch (1991), too, has built on the work of Vygotsky and others with the claim that "human action typically employs 'mediated means' such as tools and language, and that these mediated means shape the action in essential ways" (p 12). Yet as Confrey (1995a) points out, there are a number of limitations to employing an overly narrow Vygotskian perspective (or its interpretation) in mathematics education. These include:

1. Vygotskian theory (or its interpretation) may encourage the neglect or devaluation of concrete activity
2. Advocates (or interpreters) of Vygotskian theory may focus on, and privilege, language to the detriment of other forms of intellectual interaction and inquiry.

Indeed as Cobb (1993 and 1995) has shown, classroom learning of mathematics is not always consistent with the sociocultural view that social and cultural processes drive individual thought. Nevertheless, both Confrey and Cobb point to ways of moving beyond the tensions that are apparent between a Piagetian (individualistic) and a Vygotskian (sociocultural) viewpoint. They point to an interweaving of a student's own cognising activity within the socially organised activity in which the student is a participant. As Cobb (1995) says "it is impossible to understand how students could construct an intellectual inheritance that took millennia to create unless we understand how their negotiation and use of symbolic means supports their mathematical development".

Confrey (1995b) employs a distinction between 'voice' and 'perspective' to signal the two kinds of learning that result from a reciprocal interaction between a student and a teacher (reciprocal in that both parties learn). 'Voice' refers to the student's conceptions while ‘perspective’ can be used to describes the teacher’s viewpoint. This has echoes with Wertsch’s (1991) concept of the ‘voice’ of the mind and how learning through talking and thinking involves ‘ventriloquating’ through the voices of others . These ideas are based on the work of Bakhtin who stressed that voices always exist in a social milieu so that there is no such thing as a voice that exists in total isolation from other voices. ‘Ventriloquating’ is the process whereby one voice speaks *through* another voice. As a student begins employing a term such as ‘dependency’ it is initially only half theirs. “It becomes one’s own only when the speaker populates it with his own intention, his own accent, when he appropriates the word, adapting it for his own semantic and expressive intention” (Bakhtin 1981 pp 293-294).

Confrey (1995b) proposes that “classrooms can be described as places in which children engage in grounded activities and systematic enquiry”. Grounded activities are, according to Confrey, “actions involving practical activity which are mediated by one’s interactions with others”. In contrast, systematic enquiry involves “communication through the use of signs” which “can be viewed as social activity mediated by one’s experience in grounded activity”. Confrey suggests that “looking at the interactions between these two forms of mediated activity may yield some useful insights into how we might successfully educate people in mathematics”.

In the case study that follows, I document how one pair of 12 year old pupils interact with the teacher/researcher regarding the notion of dependency during four 50-minute mathematics lessons that took place at intervals over a period of six months. I suggest an interpretation of the data from this case study making use of the notions of the interweaving between voice and perspective and of the process of ‘ventriloquating’.

The Case Study

The pair of students reported on here are 12 year olds with no previous experience of using a dynamic geometry package, although they have used various drawing packages and other IT resources. The class is an above-average mathematics class in a city comprehensive school whose results in mathematics at age 16 are at the national average. The mathematics teachers use a resource-based approach to teaching mathematics and the students usually work in pairs or small groups. The class has three 50-minute mathematics lessons per week. For this part of the study, computer use for *Cabri* was restricted to one computer in the classroom (the students have access to computer laboratories for other computer applications). This meant that, as student pairs took it in turn to use the computer, it was often several weeks between sessions for particular pairs. The version of *Cabri* used was *Cabri I* for the PC.

In an introductory session the pair of students were introduced to some of the menu items in *Cabri* and then allowed to choose their own goal. The notion of "messing up" (a term suggested by Healy *et al* 1994 to refer to whether a figure could be dragged to see if it became unrecognisable) was introduced, with the students being encouraged to formulate *mathematically* challenging goals. Following this introductory session, the students worked through a series of tasks on quadrilaterals (Jones 1995). Each of the classroom tasks required the students to analyse a figure presented on paper and to construct the figure using *Cabri* such that the figure is invariant when any basic point used in the construction is dragged. This means that the students have to focus on the *relationship* between the basic objects (points, lines and circles) necessary to construct the figure.

The Exploratory Session

There are three explicit references to the notion of dependency during the initial exploratory session. The first comes from the students when they have created a circle by centre and radius point. They find that dragging the centre point changes the size of the circle. I ask them what will happen if they drag the radius point.

C: It [the circle] will get smaller or bigger depending which way you moved it [the radius point].

This indicates the students have some idea of functional dependency. Later in the session, during the drawing of a 2D representation of a cube (which they refer to as a box), the students want to delete a point. When attempting to do so they get the following message from *Cabri*: "Delete this object and its dependents?"

C: Dependents? Is that the whole box [ie cube]?
Me: Why don't you see, because you can undo it.

They delete the point and two line segments are also deleted. This gives me the opportunity to explicitly refer to dependency.

Me: So that bit of line depended on that point, and that bit of line did, so they both went.

Near the end of the lesson, the students construct a triangle and its three angle bisectors. They construct points of intersection but find that these points cannot be dragged.

Me: These points [pointing at the points of intersection] depend on these points [the points used to create the triangle].

After a little thought and dragging, one of the students says:

C: You can't drag that point [a point of intersection] because it is dependent on them [indicating the points used to create the triangle].

Session 2

In session 2 the students complete two tasks involving lines and circles. At one point, one of the students asks:

C: What's the intersection doing? Does it keep the dot [the point] there?

Me: What you are finding is the point *here*, where the circle crosses the line.

C: Right, so if it was like *that* [indicating a different arrangement of lines], then it [the point of intersection] would be *there*.

Me: It is always where the lines cross.

(note that, in this exchange, I did not mention dependency. I will comment on this later in the paper). The students complete the task and I ask them why the figure cannot be "messed up". One of the students replies:

H: They stay together because of the intersections.

Session 3

During this session the students are asked first to construct another pattern of circles and then to construct Figure 1, below.

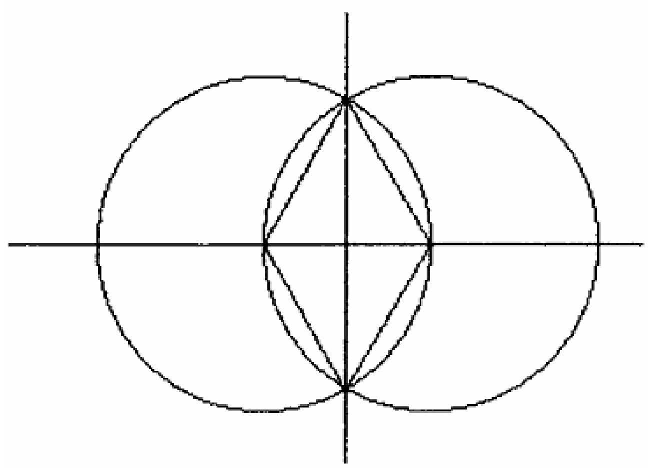

Figure 1

Referring to points of intersection, one of the students comments:

H: A bit like glue really. It's just glued them together.

A little later, the other student asks why you can't drag points of intersection.

Me: Because the intersection points just show you where two things cross.

C: So how come it keeps it together if it's just a dot to show you where they cross?

Me: You can move *that* point because it's the centre of the first circle that you drew.
So if you move that, then because you are changing the size of the first circle, the point where it crosses the other circle changes so that changes the other circle.

H: So that changes everything.

Me: Because the other circle depends on that.

C: So because it depends on it, it moves.

Session 4

The students successfully construct the required figure during this session and explore, with me, which objects can be dragged and what is the effect of dragging them.

C: So it's all about depending on stuff, isn't it?

Me: It's like a function. When one thing is a function of another it depends on the other.

C: So there's a rule in Cabri ....
... if things don't depend on each other you have to make them depend on each other to know what moves, because ..

H: to make...

C: so everything ...

H: to make the pattern

C: depends on one thing

H: to make the pattern and then it's non-messupable.

C: and then it can move. But because everything is dependent on one thing then it will always be the same, related to each other.

Discussion

The above extracts of student/teacher dialogue illustrate how one pair of students began to come to understand the notion of dependency within the context of the dynamic geometry package *Cabri*. They begin with an existing notion of dependency, knowing, for instance that the size of a circle depends on its radius. They also readily understand that when an object is deleted its *immediate* dependents are also deleted. As the students encounter points of intersection and need to construct objects dependent on these points, hence creating chains (or hierarchies) of dependency, then a way of explaining what is going on can be based on the theoretical framework introduced earlier.

This explanation involves viewing the interaction as an interweaving between individual sense-making and the social situation of a pair of students jointly working on a task and being able to refer to me whenever they thought it necessary. This interweaving is so strong that it is probably unwise to attempt to separate out, to too great an extent, any of the individual constituents (Wertsch refers to this as speaking of "individual(s)-acting-with mediated means" 1991 p 12). In this way, the case study can be viewed as an example of the interaction between the two forms of mediated activity (grounded activity and systematic inquiry) proposed by Confrey (1995b). There is a sense in which the students are both borrowing terms from Cabri (for example, dependents) and modes of expression from me *and speaking*, at least in the early stages, *as if they were me*. For example, the statement by one of the students towards the end of the initial exploratory session can be interpreted as an instance of 'ventriloquating'.

C: You can't drag that point [a point of intersection] because it is dependent on them [indicating the points used to create the triangle].

In the second session, when the students ask for clarification of the nature of a point of intersection, I do not refer to notions of dependency. I merely state that a point of intersection "is where the lines cross". As it transpires, the students have developed their own interpretation of the nature of points of intersection.

H: A bit like glue really. It's just glued them together.

Ainley and Pratt (1995) have noted the same sort of interpretation of points of intersection. During session 3, I become aware of the students' interpretation and this time I do refer explicitly to dependency. During session 4, it is the students who raise the issue of dependency. By this session they seem to be recognising its central importance and are beginning to offer their own explanation of dependency. In Bakhtian terms, it could be said that the students are beginning to populate the notion with their own intentions. In terms of Confrey's (1995b) notions, the students' solving of some geometrical problems can be viewed as "grounded activity" while their coming to know about dependency is "systematic inquiry".

In this paper I only document the explicit, and necessarily mostly verbal, uses of the notion of 'dependency'. Nevertheless, these explicit references combine verbal statements with practical activity in a way that cannot be separated. The verbal statements all refer to *action*. Wertsch (1995 p 71) maintains that "some notion of action holds the key to avoiding potential dead ends in sociocultural research", although he admits "I am less certain that the notion of mediated action I have outlined here [and in Wertsch 1991] will ultimately fill the bill".

There is, in addition, within the sessions briefly described in this paper, a continuous movement between teacher/researcher goals and student-orientated goals. For example, in the initial session the students are able to choose their own goals but for me these had to be *mathematically* challenging goals. In later sessions, although tasks were set, sub-goals were student-chosen and interventions were kept to a minimum.

Finally, throughout the sessions with this case study pair, there are also numerous examples of the implicit use of the idea of dependency. These are, for the most part, captured on videotape. It may be that when an analysis of these is added to the account, a fuller picture of the interweaving between the 'voices' of the students and the teacher within the socially organised activity taking place in the classroom will result.

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