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# Detecting damaged reinforcement bars in concrete structures using guided waves

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## Abstract

Many engineering structures must be inspected or monitored throughout their serviceable life to ensure their safe operation. In the case of reinforced concrete structures, the most common cause of premature failure is corrosion of the steel reinforcement bars that must be pre-empted. Modal based techniques are popular, since they can potentially detect damage using sensors placed remotely from the damage site. However, changes in modal parameters due to damage can be masked by their sensitivity to environmental factors and changes to the boundary conditions that are unrelated to damage. In this respect, wave based techniques provide a potential alternative. In this paper, wave propagation is modelled in a damaged steel reinforced concrete beam. The damaged section is modelled in conventional finite elements and this is coupled to wave finite element models (WFE) of the undamaged sections on either side. This hybrid modelling approach facilitates a wave based analysis of a one dimensional structure with potentially geometrically complex damage. A numerical case study is presented for a locally damaged beam represented by a loss of thickness of one reinforcement bar. It is shown that some wave modes, that feature deformation of the cross section, exhibit a strong reflection close to their cut-on frequency. This is due to the difference in cut-on frequency between the damaged and undamaged sections. A damage detection method is outlined in which the amplitudes of incident and reflected waves of low wave number are compared. No a priori knowledge of the dispersion curves is necessary. In numerical simulations, a reduction in the ratio of the reflected to incident wave amplitudes is seen in the vicinity of cut-on frequencies.

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## 1. Introduction

Most damage in reinforced concrete structures comprising steel bars is due to corrosion and delamination. Before any repair procedures can take place, this damage should be located and quantified. As repair and maintenance costs are considerable, an early detection of damage is needed. Non-destructive detection techniques (NDT) have been developed as both global and local approaches to detect damage. Vibration based methods can be divided into modal and wave based methods. In the latter, knowledge of the wave propagation characteristics and their reflection is often required. Since reinforced concrete structures are composite, analytical solutions do not exist

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and a numerical approach is needed. The Wave Finite Element method (WFE) can be adopted, where a small segment of the waveguide is modelled via FE. By applying periodicity, the free and forced wave propagation problems can be solved for a uniform waveguide whose cross section is modelled.

One of the major advantages of the WFE method is in coupling waveguides to predict the reflection and transmission coefficients due to a discontinuity. For instance, Renno and Mace calculated the reflection and transmission coefficients of joints using a hybrid wave finite element approach, where the joint is modelled via FE and a small portion of the waveguide is described via WFE [1]. Another major advantage of the WFE method is to compute the forced response due to dynamic excitation. This response can be used in deriving the wave amplitudes present in a structure. Zhang et al. previously identified joints from measured reflection coefficients in beam-like structures with application to a pipe support [2]. These coefficients were estimated based on the amplitudes of incident and reflected waves due to the presence of joint. The wave amplitudes were identified using the Wave Amplitude Decomposition (WAD) approach that relates the displacement at different points and the waves amplitudes. Also, Denis et al. have applied the WAD method to identify the reflection coefficient of a beam termination like an Acoustic Black Hole [3].

In this paper, WFE is applied to damaged and undamaged reinforced deep concrete beam sections. The forced displacement response is used to estimate the positive and negative going wave amplitudes via the WAD approach. Then, reflection coefficients associated with the presence of a damaged section are simulated to inspect and assess their sensitivity to an introduced defect representing the corrosion of a steel rebar. Subsequently, a novel algorithm is outlined for detecting the existence of damage. The performance of the method is illustrated using simulated data from the WFE model, although when applied to experimental data no model is required.

## 2. WFE application to damaged RC beam

The WFE method predicts the wave characteristics of a structure through analysing free wave propagation within a short section of the waveguide. By expressing the continuity of displacements and equilibrium of forces at the boundaries between successive segments, an eigenvalue problem is posed in terms of a transfer matrix across the section. The eigenvalues obtained relate the variables associated with the right and left sides of the section, and are a function of the wavenumbers. The eigenvectors are associated with the displacements and forces on the cross section of the short segment. This problem is solved at each specified frequency to obtain dispersion curves.

In order to apply the WFE method to a reinforced concrete beam, a section needs to be modelled in FE as shown in Figure 1-a in order to extract the associated mass and stiffness matrices. Using ANSYS, concrete is modelled using the SOLID65 element which is a 3D solid element. Each node of the element has three DOFs, which are translations in the X, Y and Z directions, and it is defined by eight nodes. Reinforcement rebars are modelled via the 3D discrete beam element REINF264 embedded in the SOLID65 element. The nodal locations, degrees of freedom and connectivity of the REINF264 element are identical to those of the base element which is the SOLID65. The location of the rebar is defined as an offset distance from the edges of the base element selected. Material properties are the same as those used in [4]. Then, the mass and stiffness matrices associated with the RC beam were extracted using ANSYS, and then post-processed using the WFE method. The wave modes were evaluated within the frequency range of 1 to 15 kHz with a frequency step of 50 Hz. The dispersion relations of modes 1 to 16 have been previously presented in [5].

Damage or discontinuities in structures can give rise to scattering of incident waves, which is potentially of use for damage inspection. When considering scattering caused by a discontinuity of finite length, coupling of a finite damaged section to undamaged waveguides is applied via the WFE-FE-WFE coupling approach [1]. The damaged and undamaged parts are modelled in FE with the dimensions and properties similar to the one presented before. The damaged reinforced concrete section is modelled in the same way as for the undamaged section, except that the area of the damaged rebar at the bottom right corner of the cross section is reduced to represent a loss of thickness due to corrosion as shown in 1-b. In this model, the corroded rebar is taken to have a diameter equivalent to a 40 percent reduction compared to the intact one.

The formulation to calculate the scattering matrix due to the damage is presented in [5]. The magnitudes of the reflection coefficients due to damage are plotted in Figure 2 for the least attenuated of the higher order modes, i.e. those that do not propagate at all frequencies. These particular modes show a sensitivity to a loss of area occurring in the steel reinforcements. This sensitivity is highlighted by their reflection coefficients due to damage at the mode cut-on frequencies. This is due to the shift of the cut-on frequencies between the undamaged and damaged waveguides,

i.e these evanescent modes start propagating in the undamaged waveguide at a lower frequency than the damaged one. Furthermore, the normalised nodal displacement in X, Y and Z directions can be plotted in order to identify the corresponding deformation shape for each wave mode of the undamaged waveguide. These nodal displacements are plotted at their cut-on frequencies in Figure 3. Displacements are mainly in the plane of the cross section.

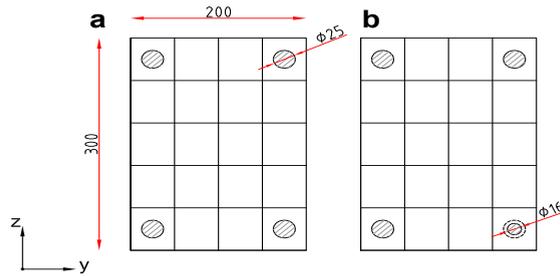


Fig. 1. RC section details, (a) undamaged beam; (a) damaged beam.

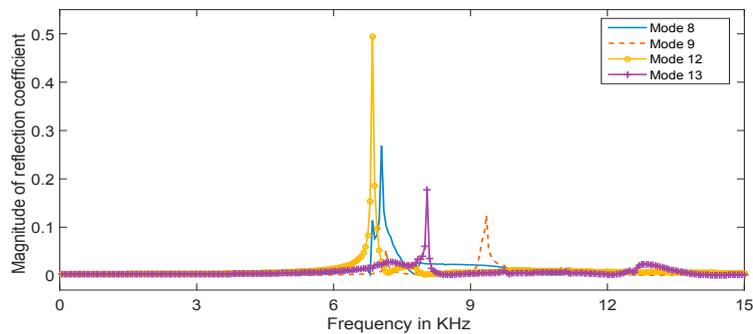


Fig. 2. Magnitude of the reflection coefficients for the least attenuated of the higher order modes in an RC beam due to damage in one rebar: 40 percent diameter reduction over a length of 0.05 m.

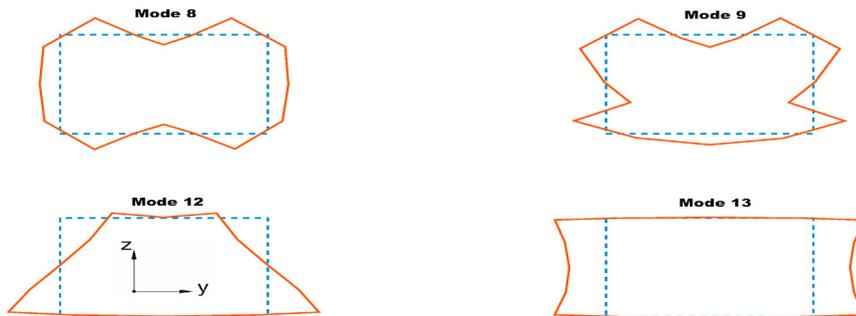


Fig. 3. Nodal displacements in the plane of the undamaged cross section for the least attenuated of higher order modes : Deformed section (—), Undeformed section (- - -).

### 3. Damage detection algorithm

Damage introduced as a diameter reduction has resulted in high magnitudes for the reflection coefficients near the cut-on frequencies of previously evanescent modes. In practice, one cannot reliably estimate the values of the full scattering matrix (for instance the magnitudes of reflection coefficients) from forced response measurements since all wave modes are excited and would need to be incorporated at the same time. However, one can estimate the amplitudes of the positive and negative going waves at a point in a waveguide via the Wave Amplitude Decomposition (WAD) method [6],[7]. One can assume a set of measurements from  $n$  transducers at  $n$  locations on a waveguide, where  $m$  wave components are of interest. The responses as outputs (displacement, velocity or acceleration) are related to the wave amplitudes by

$$\mathbf{W} = \mathbf{\Lambda} \mathbf{A} \tag{1}$$

where

$$\mathbf{W} = \begin{Bmatrix} W(x_1) \\ W(x_2) \\ \vdots \\ W(x_n) \end{Bmatrix} ; \mathbf{\Lambda} = \begin{bmatrix} e^{-ik_1 x_1} & \dots & e^{-ik_m x_1} & e^{ik_1 x_1} & \dots & e^{ik_m x_1} \\ e^{-ik_1 x_2} & \dots & e^{-ik_m x_2} & e^{ik_1 x_2} & \dots & e^{ik_m x_2} \\ \vdots & & \vdots & \vdots & & \vdots \\ e^{-ik_1 x_n} & \dots & e^{-ik_m x_n} & e^{ik_1 x_n} & \dots & e^{ik_m x_n} \end{bmatrix} ; \mathbf{A} = \begin{Bmatrix} A^+ \\ A^- \end{Bmatrix} \tag{2}$$

where  $\mathbf{W}$  is an  $n \times 1$  vector comprising the measured outputs at different points from  $x_1$  to  $x_n$  due to the forced excitation. In addition,  $\mathbf{\Lambda}$  is a propagation matrix of size  $n \times 2m$  which is dependent on the wavenumbers to be considered in the waveguide at each measurement location, and  $\mathbf{A}$  is a  $2m \times 1$  vector comprising  $A^+$  and  $A^-$  as the  $m$  positive and negative going wave amplitudes. In general, one needs to include all waves that contribute significantly to the observed response and their wavenumbers should be estimated beforehand and substituted accordingly into the WAD equation.

In an overdetermined system, where  $n > m$ , the wave components can be found in a least squares manner, where superscript  $H$  indicates the conjugate transpose of a matrix

$$\mathbf{A} = (\mathbf{\Lambda}^H \mathbf{\Lambda})^{-1} \mathbf{\Lambda}^H \mathbf{W} \tag{3}$$

However for this application, one is most interested in the response at the cut-on frequencies where the contribution of the mode cutting on may be dominant. For this reason, the pre-knowledge of the wavenumbers and cut-on frequencies is not required. One could assume the wavenumber values to be zero (real and imaginary parts), but then  $\mathbf{A}$  becomes singular. To overcome this problem, a small value of a real wavenumber can be selected. Subsequently, the system is reduced to an  $n \times 2$  matrix where  $A^+$  and  $A^-$  are scalar values at each frequency step. Some errors exist due to the assumption that the wavenumber used is small and real, which is an incorrect value before the cut on where the wavenumber is imaginary. In addition, at the cut-on frequency not only the desired mode is excited.

When no damage is present in the beam, the amplitude of the positive going waves should be equal to the negative going ones provided that the far boundary is perfectly reflecting. Hence, the amplitude ratio of the negative to positive going waves should be equal to 1. However, when damage exists, the amplitudes of the positive going waves (incident waves) are divided into reflected waves (negative going waves) and transmitted waves at the interface of the discontinuity. Therefore, the ratio should be less than 1 if damage exists. Subsequently, the damage detection algorithm contains the following steps: a point force is applied at a position  $x_e$  from one end of the beam, and the response is measured at different positions noted by  $x_r$  as shown in Figure 4. The measured values of the displacements  $W$  are substituted into Equation 3. The wavenumber values used in  $\mathbf{\Lambda}$  are chosen to be small representing the values of evanescent modes when they start cutting on. A value of 0.1 rad/m is used for the trial wavenumbers. Next, the wave amplitudes of the positive and negative going waves  $A^+$  and  $A^-$  are calculated at the mid point of the sensor array. Damage is presumed to exist when the value of the reflection ratio is less than 1.

### 4. Numerical simulations

The damage algorithm described in the previous section can be applied experimentally without recourse to a model. However, the WFE model is used here to obtain simulated response data which are used as a proxy for experimental data. In this section, the same RC beam section as in Section 2 was modelled and the forced response was calculated.

The forced response of a finite waveguide with a discontinuity includes the following steps: determine the magnitudes of directly excited waves due to external point excitation, calculate the reflection matrices due to the boundaries and discontinuities, assemble all the equations governing the propagating waves and determine the physical response at a specific point via superposition of the waves. In this model, one should excite the structure in the direction where the dominant displacement occurs; Figure 2 shows that the maximum displacement for modes 8 and 9 occurs in the Z direction in the middle of the top and bottom faces of the cross section. However, the maximum response is in the Y direction at the corners of the cross section for modes 12 and 13. Based on this observation, it is preferable to excite the beam in the Z-direction to excite both modes 8 and 9. Also, typically only the top and bottom faces of the RC beams are accessible in real structures. The transformations between the physical domain (where the motion is described in terms of displacements  $q$  and forces  $f$ ) and the wave domain (where the motion is described in terms of waves of amplitudes  $A^+$  and  $A^-$  travelling in the positive and negative directions respectively) are related and presented in [1].

The point force is in the Z-direction at position  $x_e = 0.3$  m and the receiver location  $x_r = 0.4$  to 1 m with a spacing of 0.05 m with a total beam length of 2m. The length of the damaged segment  $h$  is 0.2m located in the middle of the beam with 40% and 60% reduction in the diameter of one of the rebars for each beam model respectively. The displacement has been computed at each position and then substituted into Equation 3. The values of the positive and negative going waves are subsequently calculated at each frequency step to produce a plot of the ratio of reflected to incident wave amplitudes. In Figure 5, the undamaged beam model shows a constant reflection ratio value of 1 over the frequency range. However, damaged models show a reflection ratio less than 1 near the cut-on frequencies of modes 8 and 9. The results do not discriminate reliably between the two levels of damage, although the more severe damage case does result in a lower minimum value.

It can be seen in Figure 5 that the reflection ratio exceeds unity at some frequencies, particularly just before cut-on frequencies. This suggests that the magnitude of the negative going wave is larger than the magnitude of the positive going wave, which is physically incorrect. The error is likely attributable to invalidity of the underlying assumptions discussed in Section 3. Below a cut-on frequency the wave is highly evanescent and does not contribute significantly to the responses at the sensor array. Furthermore, the wavenumber is neither real nor small, as assumed. These data can be disregarded or, if found to be generally indicative of wave cut-on, may prove useful for data interpretation. More comprehensive parameter studies are required to investigate robustness of the technique to, for example, the value of wavenumber chosen near cut-on and the available frequency resolution.

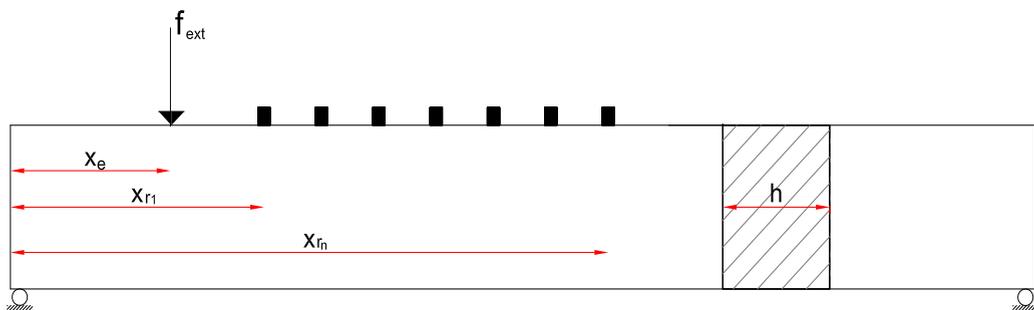


Fig. 4. Measurement array on a finite structure with a damaged section of length  $h$ .

## 5. Conclusions

A coupled FE and WFE approach for modelling a non-uniform section in an otherwise uniform waveguide has been used to model a reinforced concrete beam with localised damage. Simulations have shown that, close to a cut-off frequency, nascent waves can be reflected strongly due to loss of thickness of a single reinforcement bar. Measurement of these reflection coefficients is impracticable. However, a damage detection method has been proposed that uses an array of sensors on an accessible concrete surface to estimate just the amplitudes of right and left propagating waves

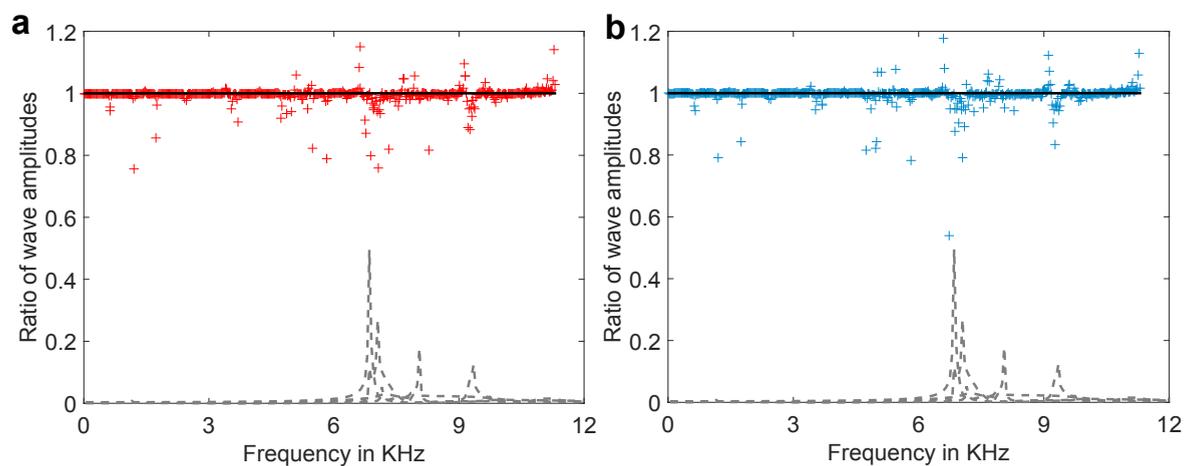


Fig. 5. Ratio of amplitudes of reflected to incident waves: Undamaged beam with continuous diameter rebar (—), (a) Damaged beam with 40 % reduction in diameter rebar over 20cm section (+); (b) Damaged beam with 60 % reduction in diameter over 20cm section (+). Magnitude of the reflection coefficients of an infinite damaged beam (- -) as predicted using Section 2.

at near-zero wave number. Localised damage manifests itself as a reduction in the ratio of these wave amplitudes. The sensor array needs to be located to one side of the potential damage site although more precise knowledge of its location is not required. Ongoing work aims to validate the technique experimentally for concrete beams with differing loss of thickness of a reinforcement bar.

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