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RADIO PULSE SEARCH AND X-RAY MONITORING OF SAX J1808.4−3658 : WHAT CAUSES ITS ORBITAL EVOLUTION?

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ABSTRACT

The accreting millisecond X-ray pulsar (AMXP) SAX J1808.4−3658 shows a peculiar orbital evolution that proceeds at a much faster pace than predicted by conservative binary evolution models. It is important to identify the underlying mechanism responsible for this behavior because it can help to understand how this system evolves. It has also been suggested that, when in quiescence, SAX J1808.4-3658 turns on as a radio pulsar, a circumstance that might provide a link between AMXPs and black-widow radio pulsars. In this work we report the results of a deep radio pulsation search at 2 GHz using the Green Bank Telescope in August 2014 and an X-ray monitoring of the 2015 outburst with Chandra, Swift, and INTEGRAL. In particular, we present the X-ray timing analysis of a 30-ks Chandra observation executed during the 2015 outburst. We detect no radio pulsations, and place the strongest limit to date on the pulsed radio flux density of any AMXP. We also find that the orbit of SAX J1808.4-3658 continues evolving at a fast pace and we compare it to the bhevior of other accreting and non-accreting binaries. We discuss two scenarios: either the neutron star has a large moment of inertia ($I > 1.7x10^{45}$ g cm^2) and is ablating the donor (by using its spin-down power) thus generating mass-loss with an efficiency of 40% or the donor star is undergoing quasi-cyclic variations due to a varying mass-quadrupole induced by either a strong (1 kG) field or by some unidentified mechanism probably linked to irradiation.

Subject headings: binaries: general — stars: individual (SAX J1808.4−3658) — stars: neutron — stars: rotation — X-rays: binaries — X-rays: stars — stars: pulsar

1. INTRODUCTION

The accreting millisecond X-ray pulsar (AMXP) SAX J1808.4−3658 is an accreting neutron star located at a distance of 3.5 kpc (Galloway & Cumming 2006) that is spinning at 401 Hz (Wijnands & van der Klis 1998) and orbiting its 0.05-0.08 $M_\odot$ companion in 2.01 hours (Chakrabarty & Morgan 1998; Deloye et al. 2008; Wang et al. 2013). This source was discovered by *BeppoSAX* in 1996 (in 't Zand et al. 1998) and is the best studied AMXP of all 18 known members (see Patruno & Watts 2012 for a review). It has shown eight outbursts so far, observed with a recurrence time of approximately 3–4 years. The high time and/or spectral resolution of X-ray telescopes like *RXTE*, *XMM-Newton*, *INTEGRAL*, *Chandra*, *Swift* and *Suzaku* has allowed a thorough study of the pulsations (see e.g., Hartman et al. 2008; Burderi et al. 2009; Patruno et al. 2012), its aperiodic timing variability (Wijnands et al. 2001, 2003; Patruno et al. 2009c; Bult & van der Klis 2015) and X-ray spectral properties (Gierliński et al. 2002; Poutanen & Gierliński 2003; Cackett et al. 2009; Papitto et al. 2009; Patruno et al. 2009b).

The coherent timing of the pulsations has revealed the lack of a strong spin up during the outbursts (Hartman et al. 2008, 2009) and a constant spin-down in quiescence that is compatible with magnetic dipole energy loss (surface magnetic field $B \approx 10^8$ G, see Hartman et al. 2008; di Salvo et al. 2008; Hartman et al. 2009; Patruno et al. 2012). There is also indirect observational evidence that SAX J1808.4−3658 turns on as a radio pulsar during quiescence, although no radio pulsations have been detected so far (Homer et al. 2001; Burderi et al. 2003; Campana et al. 2004). Indeed, the optical counterpart of SAX J1808.4−3658 is overluminous with respect to a non-irradiated brown-dwarf model (Bildsten & Chakrabarty 2001) during this phase. A source of irradiation is required to explain this behavior, but the feeble X-ray irradiation coming from the accretion disk/neutron star surface during quiescence (Homer et al. 2001; Heinke et al. 2009) cannot account for the donor luminosity. It has been speculated that a pulsar wind impinging on the donor surface (Burderi et al. 2003; Campana et al. 2004) might be responsible for the observed excess luminosity[8]. Optical modulation at the orbital period is now well established in black-widow and redback radio pulsar systems (Breton et al. 2013) and something similar has been found for SAX J1808.4−3658 too during quiescence (Deloye et al. 2008; Wang et al. 2013). In recent works, Xing et al. (2015) and de Oña Wilhelmi et al. (2016) have identified a possible gamma-ray counterpart of SAX J1808.4−3658 and spectral modeling of the *FERMI*/LAT data imply that (if the counterpart is confirmed) about 30% of the spin-down energy is trans-

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[8] Although the requirement of a pulsar wind is often mentioned as indirect evidence for a radio pulsar turning on, it is also possible that a pulsar wind is active without the radio pulsar mechanism operating in the system (or at least without radio pulsations being observable; they may be obscured by intra-binary material; see e.g., Jaodand et al. 2016)

formed into gamma-rays, providing further evidence in favor of this scenario, although no gamma-ray pulsations have been found so far.

The orbital evolution of SAX J1808.4−3658 shows an increase of the orbital period on a relatively short timescale of ~70 Myr (Hartman et al. 2008; di Salvo et al. 2008) and an acceleration of the rate of expansion of the orbit up to 2011 (Patruno et al. 2012). The driving mechanism for the evolution of a binary with a ~2 hr orbital period like SAX J1808.4−3658 is expected to be angular momentum loss due to gravitational wave emission and the expected orbital evolution timescale in this case is ~4 Gyr (Hartman et al. 2008), which is almost two orders of magnitude longer than the observed one. This behavior might require that an additional mechanism has a strong influence on the orbit, beside the gravitational wave emission.

Anomalously fast orbital evolution is not unique to SAX J1808.4−3658. Several other low-mass X-ray binaries (LMXBs), comprising both neutron star and black hole accretors, are also observed to show a faster evolution than expected (see Section 6 for an in-depth discussion). There is also a similar behavior in many (non-accreting) binary radio millisecond pulsars with orbital parameters similar to SAX J1808.4−3658, known as "black-widows" (BWs), where the rotational power emitted in form of wind and radiation by the pulsar is impinging and ablating the semi-degenerate[9] donor companion (Nice et al. 2000; Doroshenko et al. 2001; Lazaridis et al. 2011). These systems have a companion star with a typical mass of $\lesssim 0.1\,M_{\odot}$ and several (but not all) of them have orbital periods of about 1–3 hours. Most BWs have very short binary evolution timescales too, orders of magnitude shorter than the expected theoretical values for their secular evolution. These short timescales variations are believed to reflect some short-term effects rather than the secular evolution of the binary.

For SAX J1808.4−3658 di Salvo et al. (2008) proposed a scenario in which the pulsar wind, powered by the rotational spin-down of the neutron star in quiescence, causes the ejection of the gas flowing through the inner Lagrangian point L_1 (radio-ejection scenario; see also Burderi et al. 2001, 2009). Hartman et al. (2008, 2009) and Patruno et al. (2012) proposed an alternative mechanism where the binary evolution is not necessarily driven by the matter expulsion, but it is rather a quasi-stochastic process due to the development of a significant mass quadrupole in the donor star that results in a coupling between the donor spin and the orbital period of the binary. Patruno et al. (2012) in particular showed that the so-called Applegate mechanism (Applegate 1992; Applegate & Shaham 1994), which assumes that a mass quadrupole develops in the donor star due to quasi-periodic magnetic cycles, seems to be a promising candidate surviving the observational scrutiny. However, there is not yet any conclusive evidence about the exact operating mechanism behind the orbital evolution of SAX J1808.4−3658.

Motivated by these facts we have conducted a deep radio pulse search of SAX J1808.4−3658 during quiescence and an X-ray monitoring during the last 2015 outburst. The deep radio pulse search was done with the Greenbank Radio Telescope in August 2014 (during quiescence) to answer the question: does SAX J1808.4−3658 really turn on as a radio pulsar during quiescence? A detection could help to solve two problems: the first is that it would allow a continuous monitoring of the orbital evolution not only during outbursts but also during quiescence. The second is that it can allow the precise measurement of the spin-down power of the pulsar which might play a fundamental role in the ablation of the companion. The X-ray monitoring campaign was made with the *Swift* X-Ray Telescope (XRT), the *INTEGRAL* and *Chandra* telescopes during the 2015 outburst. We used the *Chandra* telescope to monitor the pulsations of the source and track the long term orbital evolution of SAX J1808.4−3658. The question we seek to answer is whether the determination of a new orbital solution that includes also the 2015 outburst can provide hints on the exact mechanism behind the rapid orbital evolution of the system.

[9] In this paper we use the term "semi-degenerate" instead of brown-dwarf because the mass transfer process alters significantly the internal structure of these stars which are quite different from isolated brown dwarfs (Tauris 2011).

2. X-RAY OBSERVATION AND DATA REDUCTION

To construct the outburst light curve we analyzed all pointed *Swift*/XRT observations taken between April 1st, 2015 (MJD 57113) and August 26, 2015 (MJD 57260). During this period 62 observations were taken (ProgamIDs 33737, 33737 and 33801) in either window-timing mode (1.76-ms resolution) or photon-counting mode (2.5-s). We extracted X-ray count rates averaged per spacecraft orbit using the online *Swift*/XRT data products generator (Evans et al. 2007). To estimate the count rate to flux conversion ratio we also used this tool to create and fit energy spectra (Evans et al. 2009), using the 0.3 – 10 keV energy and the default event grades. One Type I (i.e., thermonuclear) X-ray burst was detected and excluded from our analysis (see Figure 1).

We also used data recorded with the *INTEGRAL* spacecraft, which carries three high-energy instruments: a high-angular resolution imager IBIS, a high-energy resolution spectrometer SPI and an X-ray monitoring instrument JEM-X. These instruments are equipped with coded aperture masks enabling image reconstruction in the hard X-ray/soft γ-ray band. Driven by sensitivity considerations, we used only data from the *INTEGRAL* Soft Gamma-Ray Imager ISGRI (Lebrun et al. 2003), the upper detector layer of IBIS (Ubertini et al. 2003), sensitive to photons with energies in the range ~20 keV – 1 MeV (effectively ~300 keV). Typical integration times are in the range 1800–3600 s. We used the imaging software tools (Goldwurm et al. 2003) of the Offline Scientific Analysis (OSA) package version 10.1 distributed by the *INTEGRAL* Science Data Centre (ISDC, see e.g. Courvoisier et al. 2003). *INTEGRAL* had the Galactic center/bulge region often in its field of view during the April 2015 outburst of SAX J1808.4−3658 and the initial part of the outburst was properly sampled. The lightcurve for the 20-100 keV range during the early phase of outburst is shown in Figure 2. Each data point represents the averaged count rate of a combination of typical 3–5 Science Windows. The onset of the outburst, somewhat before MJD 57121, is clearly visible in this figure.

For the timing analysis we use *Chandra* data taken with the High Resolution Camera (HRC) with the HRC-S detector operating in timing mode. The observation started on May 24, 2015 at 22:23:18 UT (MJD 57166.9) and ended on May 25, 2015 at 07:15:30 UT (MJD 57167.3) for a total exposure time of 29.6-ks. In this configuration the data are collected with a time resolution of 16 μs and very limited energy resolution. The *Chandra* data were processed with the CIAO software (v 4.6) and were barycentered with the *faxbary* tool by using the most precise optical position available (Hartman et al. 2008) and the JPL DE405 solar system ephemeris. The pulse profiles are generated by folding the data in stretches of ∼ 2000s in pulse profiles composed by 32 bins.

The folding procedure uses the ephemeris reported in Patruno et al. (2012) and extrapolates the solution to the time of the *Chandra* observation. Given the low signal-to-noise of the observations we only measured the time of arrivals of the fundamental pulse frequency (ν) which also prevents that pulse shape variability affects the fiducial point defining the pulse time of arrival (ToA; see Hartman et al. 2008 and Patruno et al. 2010 for details of the procedure). To follow the evolution of the orbit and the pulsar spin we fit the ToAs with the software TEMPO2 (Hobbs et al. 2006) and after obtaining a new ephemeris we re-fold the data and repeat the procedure until convergence of the solution.

We also created power density spectra of the *Chandra* data. No background subtraction was applied to the data before calculating the power spectra. The Poissonian noise level was measured by taking the average power between 3000 and 4000 Hz, a region dominated by counting statistics noise alone. After obtaining the mean Poissonian value we subtracted it from the power spectra.

We used 128-s long segments to calculate the power spectra so that our frequency boundaries are 1/128 Hz and 4096 Hz. The powers were normalized in the rms normalization (van der Klis 1995) which gives the power density in units of $(\mathrm{rms/mean})^2\,\mathrm{Hz}^{-1}$. We define the fractional rms amplitude between the frequencies ν_1 and ν_2 as:

$$\mathrm{rms} = \left[\int_{\nu_1}^{\nu_2} P(\nu)d\nu\right]^{1/2} \tag{1}$$

and calculate the errors from the dispersion of the data points in the power spectra.

3. RADIO OBSERVATIONS

SAX J1808.4−3658 was observed on two different occasions, 2014 August 9 and 22 (MJD 56878 and MJD 56891, respectively), using the 110-m Robert C. Byrd Green Bank Telescope (GBT), in West Virginia. During this period, it was known to be in X-ray quiescence, with the previous outburst having ended in 2011, and the next outburst starting 2015 April. The data were recorded using the Green Bank Ultimate Pulsar Processing Instrument (GUPPI) backend. This combination of GBT with GUPPI provides a high sensitivity to faint millisecond radio pulsations – arguably the deepest search that can be done with current radio telescopes, given that the source is well outside the Arecibo-visible declination range.

The distance to the source (∼3 kpc) suggests a relatively high expected dispersion measure (DM$\gtrsim$ 100 pc cm^{-3}, based on the NE2001 model of Cordes & Lazio 2002, 2003). Furthermore, there is the potential for radio eclipses from intra-binary material – in analogy with the rotation-powered black widow and redback millisecond pulsar systems, where the eclipse duration is typically longer at lower radio frequencies, e.g. Archibald et al. (2013). Hence, our observations were conducted at a relatively high central observing frequency of 2 GHz to mitigate these effects, while still maintaining sensitivity to the typically steep spectra of radio pulsars ($f^{-1.4}$, where f is the frequency of the electromagnetic radiation; see Bates et al. (2013)). GUPPI provided 800 MHz of bandwidth, with 61.44 μs samples and 0.391 MHz channels recorded as 8-bit samples in `psrfits` format. The orthogonal polarizations were summed in quadrature, providing only total intensity. We acquired 60/30-min integration on 2014 August 9 and 22, respectively. The observational setup and offline data analysis (see §4) were tested using the millisecond pulsar PSR J1824–2452A (M28A). Radio pulsations from M28A were easily recovered at the known pulsar spin frequency of 327.4 Hz and DM = 119.9 pc cm^{-1}.

4. RADIO DATA ANALYSIS

We began the data analysis by sequentially combining groups of three observational sub-integration of 322 s each (except last sub-ints in both 9th and 22nd August observations which lasted for 55 s and 187 s, respectively) . This resulted in four independent raw data sets of ∼16 min each, i.e. 13% of SAX J1808.4−3658's binary orbital period of 2.01 hr in each case. Two raw datasets towards the end of 9th and 22nd August observations were ∼11 and ∼13 mins long, respectively. The total integration time was sub-divided in this way in order to enable linear acceleration searches (see, §4.1), and because of the potential for eclipsing, which in analogy with the black widow systems could last for at least 10% of the orbit. The observation start times, duration, and the corresponding orbital phases of SAX J1808.4−3658 are summarized in Table 1.

Initial data preparation and periodicity searching was realized using `PRESTO`, a comprehensive pulsar processing software developed by Scott Ransom (for details see, Ransom 2001; Ransom et al. 2002, 2003). Radio frequency interference (RFI) was excised using an RFI mask generated with `rfifind`. Given that the DM towards SAX J1808.4−3658 is unknown, we used `prepsubband` to generate RFI-masked, barycentered, and de-dispersed time series over trial DMs ranging from 0–1000 pc cm^{-3} (using a DM step size of 0.1 pc cm^{-3} up to a DM of 500 pc cm^{-3} and a step size of 0.3 from 500–1000 pc cm^{-3}, resulting in 6671 time series in total). For each time series we created a corresponding Fourier power spectrum using `realfft`. The residual intra-channel DM smearing was 41–81 μs (i.e. 1.6-3.2% of the pulse period) for DMs of 100 – 200 pc cm^{-3}, which corresponds to an approximate distance range of 3–6 kpc in the NE2001 model.

As described below, we searched the dedispersed time series for pulsations using both a blind Fourier-based periodicity search *and* by directly folding the data using an X-ray-derived rotational and orbital ephemeris.

4.1. *Blind Fourier-based periodicity search*

We first performed a blind periodicity search in the event that the X-ray derived ephemeris was inaccurate and to check the possibility for a serendipitous, and unrelated radio pulsar along the line of sight.

The apparent rotational period of binary pulsars is Doppler shifted by their binary motion. This results in spreading of spectral power over multiple Fourier bins as $z = aT^2/cP$, where z is the number of Fourier bins drifted, T is the integration length, c is the speed of light, and P is the spin period. For SAX J1808.4−3658, the maximum orbital acceleration is $a \approx$ 14 m/s (companion mass M_c∼0.05 – 0.08 $M_\odot$), corresponding to a drift of z = 18 bins in 16-min observations. As demonstrated by Ransom et al. (2001, 2002), such a signal can be successfully recovered by searching over multiple linear frequency derivatives. We employed this technique of Fourier-based acceleration searches, using `accelsearch` and searched z_{max} = 100 for all the 6671 Fourier power spectra (§4) in each 16-min sub-integration.

Table 1
Green Bank Telescope Summary of Observations

Obs No.	Sub-int No.	Obs. Start Date	Obs. Start MJD	Integration Time (Mins.)	Orbital Phase Coverage
S-band observations					
1	1	2014-08-09	56878.149456	16.1	0.97-0.10
1	2	2014-08-09	56878.160641	16.1	0.10-0.24
1	3	2014-08-09	56878.171826	16.1	0.24-0.37
1	4	2014-08-09	56878.183011	11.6	0.37-0.46
2	5	2014-08-22	56891.022211	16.1	0.39-0.53
2	6	2014-08-22	56891.033396	13.8	0.53-0.65

We then identified the best candidates from the above acceleration searches using the *ACCEL_sift* subroutine of `PRESTO`, which groups candidates found at different trial DMs. *ACCEL_sift* did not identify any candidates with a rotational period close to that of SAX J1808.4−3658 . Nonetheless, in case there was a serendipitous pulsar along the same line of sight, for each of the *ACCEL_sift* candidates, we folded the corresponding de-dispersed time series and selected those folds showing a reduced-$\chi^2 > 2$ (this is used as a proxy for signal-to-noise) to also fold the raw data. We then inspected the candidates by eye and used parameters such as signal to noise ratio, measured DM, pulse profile and, converged period and period derivative solution from *prepfold* output plot to make an informed selection. This inspection did not reveal any convincing pulsar candidate from the blind search.

4.2. *Direct Folding Search with X-ray-derived Ephemeris*

With *a priori* knowledge of the spin and orbital parameters, it is possible to perform a deeper search for radio pulsations compared to the blind search discussed above. Previous coherent timing analysis of SAX J1808.4−3658 , enabled by its X-ray pulsations during outbursts, provides such an ephemeris.

However, the short-orbital-period black widow and redback millisecond pulsar binaries are known to show non-deterministic orbital variations (see, Patruno et al. 2012; Breton et al. 2013; Archibald et al. 2013) and such variations should also be expected in the case of SAX J1808.4−3658, meaning that any previously derived ephemeris may not extrapolate well to future observations. X-ray pulsation searches in the redback transitional millisecond pulsar PSR J1023 + 0038 (e.g., Archibald et al. 2015; Jaodand et al. 2016) have established that one can successfully account for such non-deterministic orbital variations by searching over a small deviation in the time of ascending node ($T_{\rm asc}$). Therefore, when folding the GBT radio data with X-ray derived ephemerides, we searched both over DM and a $\Delta T_{\rm asc}$ value compared to the fiducial ephemeris value.

Given the integration times of 1 and 0.5 hr respectively during the first and the second observation epochs, we could ensure significant orbital coverage of SAX J1808.4−3658 's ~2 hr orbit (see Table 1). We used two known orbital ephemerides: the one obtained from coherent timing analysis up to 2011 (Patruno et al. 2012) and the one obtained by also including the 2015 outburst (§4.2). In addition, we varied $T_{\rm asc}$ over a range of ±30 s in steps of 0.1 s, resulting in 2×600 trial ephemerides per DM trial.

Each of the 6671 dedispersed time series for every 16-min sub-integration were then folded using `prepfold` and these 2×600 ephemerides. Moreover, the folding operation was conducted in two additional ways: by allowing `prepfold` to optimize the S/N in a narrow range of spin period and spin period derivative around the nominal ephemeris prediction and only allowing an optimization in spin period derivative. Hence, at the end of these ephemeris-based searches we obtained $2 \times 2 \times 6671 \times 600 = 16,010,400$ folded profiles. We filtered the profiles by creating histograms of the S/N of the folds in each 16-min sub-integration and choosing only candidates above a certain threshold to inspect by eye. We found no candidate profiles with sufficient S/N that clearly peaked in both trial DM and $\Delta T_{\rm asc}$.

5. RESULTS

5.1. *X-Ray Lightcurve*

SAX J1808.4−3658 was detected in outburst with *Swift*/BAT on April 9th, 2015 (MJD 57121, Sanna et al. 2015). During the closest previous *Swift*/XRT observation, which occurred on April 3th (MJD 57195), SAX J1808.4−3658 was still in quiescence (Campana et al. 2015). The 0.3–10 keV X-ray lightcurve of SAX J1808.4−3658 (see Figure 1) shows the very typical evolution that was also observed in the other outbursts. The outburst has started after approximately 3.5 years since the previous one, in line with the typical recurrence time of 3–4 years.

The *Swift*/XRT started monitoring SAX J1808.4−3658 after a Type I X-ray burst on April 11th (MJD 57123). The source showed the same evolution seen in previous outbursts, with an observed 0.3–10 keV peak flux of $\approx 3 \times 10^{-9}$ erg s^{-1} cm^{-2} that (assuming a distance of 3.5 kpc; Galloway & Cumming 2006) corresponds to a luminosity of 4×10^{36} erg s^{-1}. The outburst showed an initial near exponential decay (slow decay) lasting about 15 days. It then transitioned into a faster linear decay for about 5 days when the source reached a luminosity of $\approx 10^{35}$ erg s^{-1}, before entering a prolonged outburst reflaring tail that lasted another ~ 100 days.

During the outburst reflaring tail, typical of all previous outbursts (van der Klis 2000; Wijnands et al. 2001, 2003; Campana et al. 2008; Patruno et al. 2009c, 2016), the luminosity of the source oscillates between very faint states close to 2×10^{32} erg s^{-1} and relatively brighter ones of $\sim 10^{36}$ erg s^{-1}. Two bright reflares were seen in 2015 on May 13th (MJD

57155) and May 18th (MJD 57164) after which several progressively weaker reflares followed on a cadence of five to ten days. The *Chandra* observation we report in this work took place during the second bright reflare.

The power spectra of the *Chandra* data show no relevant feature at any frequency. We exclude the presence of a 1 Hz modulation (similar to that observed in several previous outbursts) with rms amplitude larger than 10% at the 95% confidence level. This upper limit is derived by looking at the power in the 0.05–10 Hz range as done for example in Patruno et al. (2009c).

5.2. *X-Ray Pulsations*

The X-ray pulsations are very clearly detected in each data segment at the 4-8σ level, where we define the significance as the ratio between the pulse amplitude and its statistical error. The sinusoidal fractional amplitude of the pulsations is, on average, around 2% (sinusoidal amplitude or semi-amplitude) and does not show any significant variation during the duration of the observations. The pulse time of arrivals are fitted with a constant pulse frequency plus a Keplerian circular orbit and the statistical errors on the fitted parameters are obtained with standard χ^2 minimization techniques.

Since we do not see any significant timing noise in the data (at the timescales of the observations) and the variance of the pulse ToAs is compatible with that expected from measurement errors alone, we take our statistical errors as a good representation of the true *statistical* ones. In previous work (e.g., Hartman et al. 2008; Patruno et al. 2012) it was shown that when observing the pulsations of SAX J1808.4−3658 a strong timing noise is always observed on timescales of the order of hours to days. Part of this noise is correlated to X-ray flux variations and introduce systematic errors on the determination of the spin frequency of the order of 10^{-8}–10^{-7} Hz. These systematic errors are particularly pronounced during the reflares when strong pulse shape variability is observed (Hartman et al. 2008, 2009). The magnitude of such systematic errors can be estimated by looking at long data stretches that are longer than the typical timing noise timescales. However, since the pulsations available in our analysis refer only to a short data span ($\approx$ 30-ks) we cannot determine the size of the systematic errors in our analysis. Indeed, Hartman et al. (2008, 2009) and Patruno et al. (2012) estimated the average systematic error on the pulse frequency over the entire baseline of the observations, which lasted for weeks/months. Here, instead, the much shorter data span implies that the systematic effect of timing noise can be substantially larger than average. For example, by looking at Figure 1 in Hartman et al. (2009) we see that on timescales of few hours the timing noise can induce pulse phase shifts of the order of 0.1–0.3 cycles. For this work such a phase shift would translate in systematic errors on the determination of the pulse frequency of up to a few 10^{-6}Hz.

Even if our statistical errors on the pulse frequency are large ($\sim 10^{-6}$ Hz) we cannot neglect the effect of systematic errors, although we can only use a rough estimate of its magnitude by looking at the behavior of the pulsations recorded during previous outbursts. The orbital and pulse frequency solution is reported in Table 2.

5.3. *Orbital Solution*

To determine the orbital evolution we follow the procedure already used in Patruno et al. (2012); Hartman et al. (2009, 2008) and fit the time of passage through the ascending node (which is equivalent to orbital phase zero) together with the measurements of the previous outbursts. The reference point $T_{\rm asc,ref}$ is taken from Table 1 of Hartman et al. (2009), and we use the quantity $\Delta T_{\rm asc} = T_{\rm asc,i} - (T_{\rm asc,ref} + N\,P_b)$, where $T_{\rm asc,i}$ refers to the i–th outburst and N is the closest integer to $(T_{\rm asc,i} - T_{\rm asc,ref})\,/P_b$. The reference orbital period P_b can also be found in Table 1 of Hartman et al. (2009).

Table 2
SAX J1808.4−3658 Timing Solution for the 2015 Outburst

Parameter	Value	Stat. Error	Syst. Error
ν [Hz]	400.9752067	1.1×10^{-6}	$\sim10^{-6}$
$T_{\rm asc}$ [MJD]	57167.025002	7×10^{-6}	
e	< 0.003	(95% c.l.)	
$a_1\ \sin i^*$ (lt-ms)	62.812	2×10^{-3}	
$P_{\rm b}^*$ (s)	7249.156980	4×10^{-6}	
Eccentricity e^* (95% c.l.)	$< 1.2\times10^{-4}$		
Epoch (MJD)	52499.9602472		

* these values are taken from Hartman et al. (2009) and are kept fixed during the fit.

Up to the 2008 outburst, the $\Delta T_{\rm asc}$ evolution showed a trend that was compatible with a quadratic polynomial representing an orbital expansion at a constant rate (Hartman et al. 2008; di Salvo et al. 2008). Indeed, the time of passage through the ascending node can be expressed as a polynomial expansion:

$$T_{\rm asc}(N) = T_{\rm asc,ref} + P_b N + \frac{1}{2} P_b \dot{P}_b N^2 + ... \qquad (2)$$

By adding the 2011 outburst data points, Patruno et al. 2012 showed that a quadratic polynomial was insufficient to describe the observed behavior of the $T_{\rm asc}$ variations, which were instead successfully described by a cubic polynomial. The physical interpretation given was that, on the observed baseline of 13 years, the orbit was expanding at an accelerated rate.

We now add the 2015 outburst data (see Figure 3) and we first try to fit the $T_{\rm asc}$ data points with a quadratic polynomial, which corresponds to the solution found in Hartman et al. (2008, 2009); di Salvo et al. (2008). The fit is statistically poor with a χ^2 of 492 for 4 degrees of freedom (dof). A cubic polynomial is also a poor description of the data with $\chi^2/\,\rm dof = 314/3$. To obtain a p-value above the canonical 5% threshold we need to fit the data with a fifth-order polynomial ($\chi^2/\,\rm dof = 3.4/1$, p-value 6%), which suggests that either the observed variability is governed by a stochastic process or, if a periodicity is present, it must be significantly longer than the observational baseline[10]. We stress that the concavity of the 5th order polynomial curve changes sign around 2011, which implies that the orbit has started to shrink after that time.

Next, we tried to fit the data with a sinusoid that could represent the effect of a slightly eccentric orbit with periastron advance. From our previous work (Patruno et al. 2012) we already know that a sinusoid is a statistically poor fit to the data. Indeed we find a formally bad fit with a $\chi^2 = 83.9$ for 2 dof. Furthermore the fit requires an eccentricity of about 0.004 which is much larger than the best upper limits available on SAX J1808.4−3658 ($e < 1.2\times10^{-4}$, Hartman et al. 2009). Finally we attempted to fit the data with a Keplerian

[10] As a cautionary test we also try to remove the 2011 point (assuming it is an outlier, even if there is no evidence or reason to believe that this is the case) and fit the data again with a quadratic polynomial. The data give also a poor fit with $\chi^2 = 11.1$ for 3 degrees of freedom (and p-value < 1%)

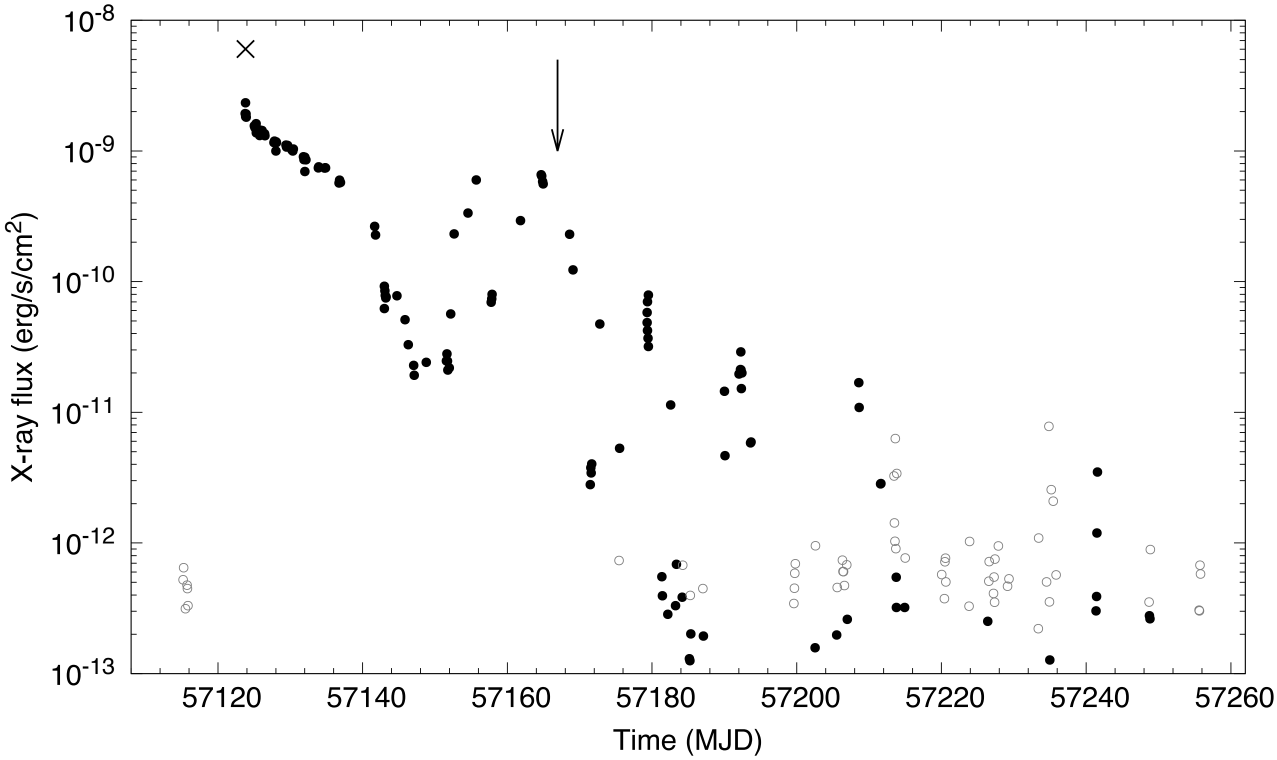

Figure 1. X-ray lightcurve (0.3–10 keV) of SAX J1808.4−3658 obtained with the *Swift*/XRT telescope. The cross marks the occurrence of a Type I X-ray burst, whereas open circles are non-detection. The arrow identifies the time of the *Chandra* observation.

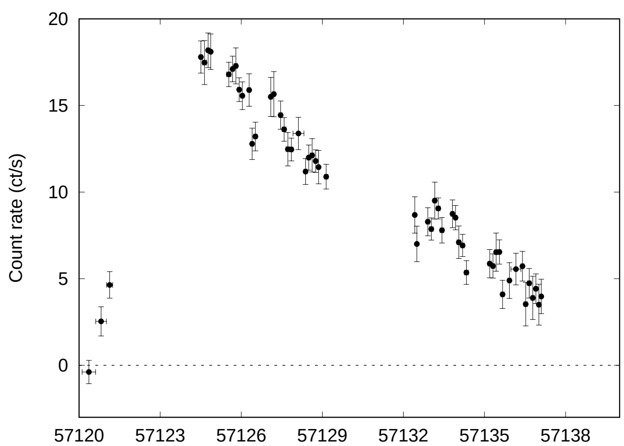

Figure 2. *INTEGRAL*/IBIS lightcurve of the 2015 outburst in the 20–100 keV energy band. The onset of the outburst is detected around MJD 57121. The data points have typical integration times of 1800 – 3600 s. The horizontal error bars (larger than the symbols only for a few points) define the time interval over which the data has been integrated.

orbital delay curve that could represent the effect of orbital motion caused by a third body in an eccentric orbit. We find this can fit the data (with $\chi^2 = 1.1$ for 1 dof) if the third body has a mass of about 8 Jupiter masses and is in a relatively wide (≈ 5 AU) orbit with an eccentricity of about 0.7 and an orbital period of about 17.4 years. We can test this scenario by looking at the pulse frequency derivative of SAX J1808.4−3658 since the pulsar would be accelerated along the orbit. Given the fitted orbital parameters, the orbital velocity would be $v_{orb} \approx 50\,\mathrm{m\,s^{-1}}$ with variations along the orbit due to the large eccentricity. To get a first order orbital acceleration we use $e = 0$ and we get $a_{orb} \approx 6 \times 10^{-7}\,\mathrm{m\,s^{-2}}$. This would imply a pulse frequency derivative of (Joshi & Rasio 1997):

$$\dot{\nu}_p = \nu_s \frac{\boldsymbol{a}_{orb} \cdot \boldsymbol{n}}{c} \tag{3}$$

where $\boldsymbol{n}$ is a unit vector along the line of sight and $\nu_s = 401$ Hz is the spin frequency of SAX J1808.4−3658. This gives $\dot{\nu}_p = 8 \times 10^{-13} \cos\theta$,where θ is the angle between the acceleration and line of sight vectors. Since we know from previous observations that SAX J1808.4−3658 is spinning down at a relatively constant rate of $\dot{\nu}_s \approx 10^{-15}\,\mathrm{Hz\,s^{-1}}$ we can confidently exclude this scenario.

5.4. *Radio Pulse Search*

Exhaustive searches using both a blind Fourier-based periodicity search, and folding with a range of perturbed ephemerides, failed to find radio pulsations from SAX J1808.4−3658 for any trial DM or $\Delta T_{\rm asc}$ §4.

In the absence of detectable radio pulsations, we can place a stringent upper limit on pulsed radio emission from SAX J1808.4−3658, with the notable caveat that an active radio pulsar could in principle be enshrouded by intra-binary material for a large fraction of the time (e.g. Archibald et al. 2015). In analogy with the black widow systems, however, it is reasonable to assume that SAX J1808.4−3658 would only be eclipsed for ~10% of its orbit at 2 GHz observing frequency.

To set an upper limit on the flux density, we use the modified radiometer equation (see Dewey et al. 1985; Bhattacharya 1998; Lorimer & Kramer 2012):

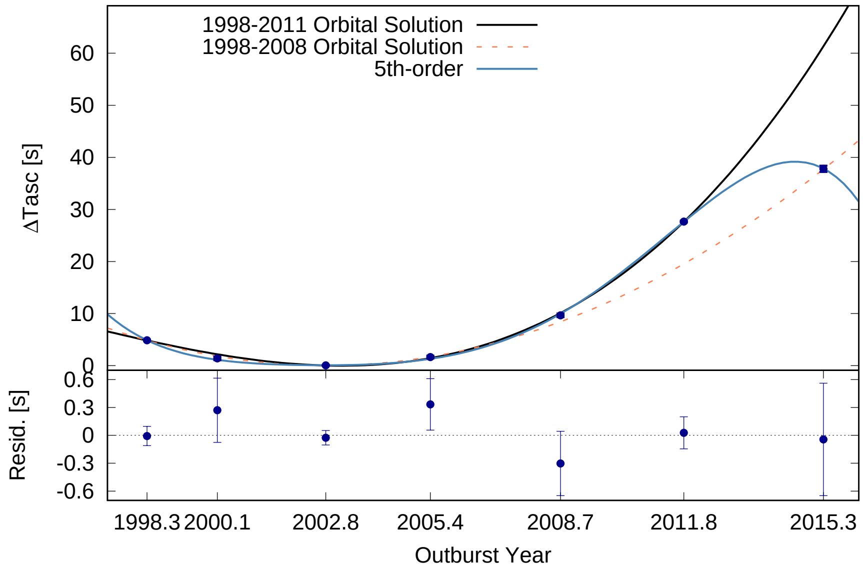

Figure 3. Orbital evolution of SAX J1808.4−3658 over 17 years. The $\Delta T_{\rm asc}$ cannot be fitted with a cubic (solid black line) or a quadratic (dotted orange line) polynomial. A 5-th order polynomial (solid blue line) is necessary to obtain a statistically acceptable fit, which suggests a complex behavior of the orbit. The panel in the bottom shows the residuals with respect to the 5-th order polynomial fit.

$$S_{min} = \frac{\left(\frac{S}{N}\right)\beta\, T_{sys}}{G\,\sqrt{n_p\, t_{obs}\,\triangle f}}\sqrt{\frac{W}{P-W}} \qquad (4)$$

We used the S-band receiver/frontend (Rcvr2_3) at GBT. For this receiver the system noise temperature T_{sys} is 22 K and the gain of the telescope G is 1.9 K Jy^{-1}. Here, $\triangle f$ is the 800 MHz bandwidth, the correction factor β is assumed ideal and close to 1, the number of polarisations n_p is two and finally the integration time t_{obs} corresponds to 16 min[11]. With the assumption that the pulse duty cycle is $\sim$ 10% and signal to noise ratio $\left(\frac{S}{N}\right)_{min}$ for candidate identification by eye is 8, we obtain a maximum flux density of 30 μJy at 2 GHz (equivalently $\sim$ 50 μJy at 1.4 GHz, for an assumed spectral index of $\alpha = -1.4$).

This limit can be used as an important input for future radio searches, and a point of comparison in the event that SAX J1808.4−3658 becomes a detectable radio pulsar in the future. We note that of the 106 Galactic field pulsars (outside of globular clusters) in the ATNF catalog (accessible at http://www.atnf.csiro.au/research/pulsar/psrcat/; See also Manchester et al. 2005) with quoted flux densities at 1.4 GHz, and with spin period < 10 ms, less than 10% have comparably low flux density to the upper limit we set on SAX J1808.4−3658. These low-flux-density millisecond pulsars are predominantly recent discoveries from the PALFA pulsar survey with Arecibo (Scholz et al. 2015; Lazarus et al. 2015).

[11] While in principle we could quote a 2x deeper limit by coherently folding the full 1-hr 9 August 2014 data set, we choose not to do so because some fraction of this integration is during orbital phases in which any radio pulsar is likely to be eclipsed. We therefore prefer to set a more conservative flux density limit using the 16-min sub-integration, which together span a wide range of orbital phases.

6. DISCUSSION

We have conducted the deepest radio pulse search for SAX J1808.4−3658 during its quiescent phase in August 2014. No radio pulsations have been detected, setting the strongest possible upper limit (30 μJ at 2 GHz) on the presence of radio pulsations that exist (to date) for any AMXP. The presence of a radio pulsar turning on during quiescence cannot be excluded with the present upper limits, but if a radio pulsar signal is present it has to be quite weak at high radio frequencies (2 GHz), substantially scattered by the intervening interstellar medium, or perpetually eclipsed to be still compatible with the current constraints. The beam width of millisecond radio pulsars is very large (typical values of $\sim$100°, e.g. Lorimer 2008) so that missing the pulsar because of beaming, although possible, is unlikely. The strongest evidence for a large beaming angle comes from X-ray observations of globular clusters, where very few unidentified X-ray sources have spectral properties compatible with unknown millisecond pulsars whose radio beam is not pointing towards Earth (Heinke et al. 2005). Even if SAX J1808.4−3658 is an active radio pulsar in quiescence, there is still a good chance that eclipses might appear for 10–50% of the orbit due to free-free absorption by intra-binary material, a common occurrence in black widow and redback pulsars (Nice et al. 2000; Roberts 2013). To avoid this problem we have observed at a high-enough radio frequency that a long eclipse duration is unlikely. We have also observed at a wide range of orbital phases, when the neutron star is not behind its companion.

After 17 years of X-ray monitoring, the orbital period evolution of SAX J1808.4−3658 shows a non-predictable behavior. The statistical fit to the data show that neither a parabolic nor a cubic polynomial can describe the data correctly. We found an ambiguity in the interpretation of the long term trend of $T_{\rm asc}$, since the observations can be explained in two ways. Either the orbit is expanding throughout the 17-years long observational window, with some fluctuations around the mean $\dot{P}_b$, or the orbit has expanded until ~2011 followed by a shrinkage (i.e., the fifth-order polynomial curve changes concavity). This is not a surprising behavior since many binary systems have shown a similar orbital evolution. However, identifying the precise short-term mechanism responsible for such orbital evolution is a relatively difficult task.

In the following we will proceed by first discussing some fundamental properties of binary evolution, then we will compare SAX J1808.4−3658 to other known binaries that show anomalous orbital evolution and finally we will review possible mechanisms to explain such an anomaly.

6.1. *Binary Evolution Timescales*

Looking at the binary evolution, it is useful to define a timescale $\tau_{\rm ev} = \frac{P_b}{\dot{P}_b}$ that can be compared to the expected evolutionary timescales from theoretical models. Differentiating the third Kepler law and assuming that all mass lost by the companion is accreted by the primary, one obtains the well known equation (see e.g., Frank et al. 2002):

$$\frac{\dot{a}}{a} = \frac{2\dot{J}}{J} + \frac{-2\dot{M}_{\rm c}}{M_{\rm c}}(1-q) \qquad (5)$$

where $q = M_{\rm c}/M_{\rm NS}$ is the mass ratio between the companion ($M_{\rm c}$) and neutron star mass ($M_{\rm NS}$), a is the orbital separation, J is the angular momentum and the dot refers to the first time derivative. In general the angular momentum loss of the binary ($\dot{J}$) can be decomposed in four terms (see e.g., Tauris & van den Heuvel 2006):

$$\frac{\dot{J}}{J} = \frac{\dot{J}_{\rm gw}}{J} + \frac{\dot{J}_{\rm mb}}{J} + \frac{\dot{J}_{\rm ml}}{J} + \frac{\dot{J}_{\rm soc}}{J} \qquad (6)$$

where the subscripts gw, mb, ml and soc refer to gravitational wave emission, magnetic braking, mass loss and spin-orbit coupling, respectively. When the binary is relatively compact, (orbital period of less than ~ 1 day), the evolution of the system is believed to be driven by angular momentum loss (encoded in the $\dot{J}$ term in the expression above; see e.g. Frank et al. 2002) rather than the nuclear evolution of the donor star. In ultra-compact ($P_b < 80$ min) and compact binaries (80 min< $P_b \lesssim$ 3.5 hr) the angular momentum loss is believed to be mainly due to emission of gravitational waves ($\dot{J}_{\rm gw}$) which becomes very efficient at short orbital separations (generally when $P_{\rm b} \lesssim$ 3hr, van der Sluys 2011). If there is no mass loss from the system, then the loss of angular momentum via gravitational waves drives the mass transfer and the orbital period changes according to the following expression (Rappaport et al. 1987; Verbunt 1993; di Salvo et al. 2008):

$$\dot{P}_b = -1.4 \times 10^{-14} M_{\rm NS}\, M_{\rm c}\, M^{-1/3} P_{\rm b,hr}^{-5/3} \times \\ \times (\zeta - 1/3)/(\zeta + 5/3 - 2q) \quad (7)$$

where all masses are expressed in solar units, $P_{\rm b,hr}$ is the orbital period in hours and ζ is the *effective* mass-radius index of the donor star ($R_{\rm c} \propto M_{\rm c}^{\zeta}$; see for example van Teeseling & King 1998).

When the orbital period of the binary is wider, in the range of 3.5 hr $\lesssim$ $P_{\rm b}$ $\lesssim$ 0.5–1 day, the dominant mechanism driving the binary evolution is thought to be angular momentum loss via magnetic braking ($\dot{J}_{\rm mb}$). There is currently considerable uncertainty about the details of magnetic braking because its efficiency depends on a number of poorly understood (and difficult to measure) stellar parameters (see e.g., Knigge et al. 2011 for a discussion). It also remains rather speculative whether the single-star braking laws can be extended, unaltered, in binary systems. Given these uncertainties the magnetic braking timescale can vary by up to an order of magnitude and indeed different recipes have been given in the literature (Skumanich 1972; Rappaport et al. 1983; Stepien 1995; see also Tauris & van den Heuvel 2006 and Appendix A in Knigge et al. 2011 for a review of several magnetic braking models). Nonetheless, moderately wide binaries where magnetic braking is dominant, are thought to lose angular momentum on a shorter timescale than those compact binaries where angular momentum loss is dominated by gravitational wave emission. The orbital parameters of SAX J1808.4−3658 imply that it should be a typical gravitational waves driven binary, since magnetic braking is believed to turn off (or become less efficient) once the donor becomes fully convective and/or semi-degenerate (Spruit & Ritter 1983; see however, Wright & Drake 2016 for recent results that suggest that a dynamo process might still occur in fully convective stars).

It is interesting at this point to compare what is observed in compact radio pulsar binaries (black widows and redbacks, see e.g. Roberts 2013) as well as other LMXBs and accreting white dwarfs which have similar orbital parameters as SAX J1808.4−3658 and have a measured $\dot{P}_b$. The reason why these systems might be relevant in this context is twofold: other LMXBs might be behaving in a similar way as SAX J1808.4−3658 since the same mechanisms might be at play, whereas black widows and redbacks are non-accreting systems and therefore Eq. (5) simplifies. In the last few years three redback pulsars have transitioned to an accreting LMXB state (Archibald et al. 2009; Papitto et al. 2013; Bassa et al. 2014; Roy et al. 2015). Therefore it is still possible that more of these systems (if not all) could display the same behavior and therefore the assumption that redbacks are always non-accreting might be invalid. As yet, no black widow system has been observed to transition from a rotation-powered to an accretion-powered state.

6.2. *Comparison with Other Interacting Binaries*

A large number of interacting binaries have a measured orbital period evolution which is too fast to be explained with simple binary evolution models involving only $\dot{J}_{\rm gw}$ and $\dot{J}_{\rm mb}$ and requires a number of additional effects. These systems include binary pulsars, cataclysmic variables and LMXBs with neutron star and black hole accretors. Among the binary pulsars, we discuss below only the cases of the black widows and redbacks because all binaries with a white-dwarf/neutron star companion are following the predictions of general relativity with exquisite precision (e.g., Taylor & Weisberg 1989; Weisberg & Taylor 2002). We also exclude from the sample those radio pulsars with a B-type star companion (not to be confused with the Be X-ray binaries, which are accreting systems and a subset of high mass X-ray binaries) since very different mechanisms involving the short nuclear evolution timescale of the massive companion need to be considered. This is also the reason why we do not include high mass X-ray binaries in our sample.

6.2.1. *Black Widows and RedBacks*

In black widows and redbacks the companion star is being ablated by the pulsar wind and high energy radiation,

Table 3
Black Widows and Redbacks with measured $\dot{P}_b$

Name (PSR)	Type	$P_b(hr)$	$\dot{P}_b$	Error
J1023+0038	RB	4.8	-7.3×10^{-11}	0.06×10^{-11}
J1227-4853	RB	6.9	-8.7×10^{-10}	0.1×10^{-10}
J1723-2837	RB	14.8	-3.5×10^{-9}	0.12×10^{-9}
J1731-1847	BW	7.5	-1.08×10^{10}	0.07×10^{-10}
J2051-0827	BW	2.4	-1.6×10^{-11}	0.08×10^{-11}
J1959+2048	BW	9.2	1.47×10^{-11}	0.08×10^{-11}

thus producing potential mass loss. Observational evidence of this phenomenon comes from the fact that radio pulsations are very often eclipsed by intra-binary material that induces free-free absorption of the pulsed signal. The orbital parameters of SAX J1808.4−3658 are compatible with those of a BW, and its 0.05-0.08 $M_\odot$ companion is also a semi-degenerate star (Bildsten & Chakrabarty 2001; Deloye et al. 2008; Wang et al. 2013). The only difference between BWs and SAX J1808.4−3658 is that in the latter system the companion is in Roche lobe overflow whereas BWs are thought, at least in some cases, to be detached systems Breton et al. (2013).

In black widows, as well as in redback pulsars, the *short-term* effects on the orbital evolution do occur on timescales which are generally orders of magnitude shorter than the predicted (secular) ones from angular momentum loss due to gravitational waves and/or magnetic braking. The six BWs and RBs which have a measured (and publicly available[12]) $\dot{P}_b$ show orbital evolution timescales from 100 Myr down to less than 1 Myr (see Table 3). Five of them have negative orbital period derivative (the orbit is shrinking), whereas only one (PSR B1957+20) has a positive value (the orbit is expanding). Similarly, in the case of SAX J1808.4−3658, $\tau_{\rm ev} = \frac{P_b}{\dot{P}_b} \approx 70$ Myr, whereas from Eq. (7) one would have expected a timescale of a few Gyr (varying slightly with the exact neutron star and companion mass chosen) thus meaning that the $\dot{J}_{\rm gw}$ term is not the dominant one. It is important to stress that these apparent orbital period derivatives are epoch dependent, and change on year-long timescales.

6.2.2. *Other Low Mass X-Ray Binaries*

In the few compact LMXBs where an orbital period derivative can be measured, we see a roughly equally distributed sign (6 positive, 4 negative signs and three upper-limits observed so far). The magnitude of the orbital period derivative is always much larger than expected from conservative binary evolution ($\dot{J} = 0$) and/or from angular momentum loss via gravitational waves and/or magnetic braking. In the literature a number of different mechanisms have been suggested to explain the large $\dot{P}_b$. These include mass loss from the companion (Burderi et al. 2010; Ponti et al. 2015), enhanced magnetic braking (González Hernández et al. 2014), the presence of a third body (Iaria et al. 2015), spin-orbit coupling (Wolff et al. 2009; Patruno et al. 2012) and in some cases even modified theories of gravity (Yagi 2012). The group of LMXBs appears to be the most heterogeneous among the different binaries that we are considering here. Indeed this group comprises transient LMXBs with black hole accretors (XTE J1118+480 and A0620–00), transient LMXBs with neutron star accretors (EXO 0748–676, MXB 1658–298, SAX J1748.9–2021, AX J1745.6–2901 and SAX J1808.4−3658) and persistent neutron star LMXBs (Her X–1, 2A 1822–37, XB 1916–053). Furthermore, some of these LMXBs are accreting pulsars and have orbital periods determined via timing of their pulsations (e.g., Her X–1, 2A 1822–37, SAX J1808.4−3658 SAX J1748.9–2021) whereas others are eclipsing systems and their period is determined via X-ray and/or optical photometry. No other orbital period derivative has been measured so far for any other LMXB. In Table 4 we list the LMXBs sample with their orbital period derivatives and the main proposed explanation given in the literature.

The binary EXO 0748–676 is an eclipsing binary with a 0.4 $M_\odot$ donor that shows sudden variations in $\dot{P}_b$, which were proposed to be due to spin-orbit coupling (Wolff et al. 2002, 2009). Wolff et al. (2002) analyzed two segments of X-ray data (1985–1990 and 1996–2000) and showed that the period had increases by about 8 ms. However, the period increase shows jitters and cannot be fit with a constant $\dot{P}_b$. Further work by Wolff et al. (2009) extended the analysis until 2008 and observed a similar behavior.

The eclipsing binary 2A 1822–37 is an accreting pulsar with a $\sim$0.3 – 0.4 $M_\odot$ companion and it has a relatively steady increase of the orbital period measured over a baseline of 30 years (see e.g., Burderi et al. 2010; Iaria et al. 2011; Chou et al. 2016). This system is an accretion-disk corona system showing extended partial eclipses of the central X-ray source. Burderi et al. (2010) and Iaria et al. (2011) have suggested that the binary contains an Eddington limited accreting neutron star whose irradiation of the donor is inducing severe mass loss that can explain the large orbital period derivative observed. The positive sign of $\dot{P}_b$ is ascribed to the response of the radius of the donor to mass-loss, with $R_c \propto M_c^\zeta$ and $n < 1/3$. Burderi et al. (2010) suggests that the donor in 2A 1822–37 has a deep convective envelope with $\zeta = -1/3$ (see e.g., Rappaport et al. 1982) thus justifying the positive $\dot{P}_b$.

The source AX J1745.6–2901 is an eclipsing binary with an accreting neutron star and a negative $\dot{P}_b$ (Ponti et al. 2015). The donor mass is constrained to be $M_c \lesssim 0.8\,M_\odot$. A strong mass loss is also suggested in this case, but since the system is shrinking the mass-radius index needs to be $n > 1/3$. The data on the $T_{\rm asc}$ collected over a baseline of about 30 years show significant scatter of up to several tens of seconds.

XB 1916–053 is an ultracompact ($P_b \approx 50$min) persistent dipping source monitored for over 37 years by X-ray satellites. Hu et al. (2008) studied the first 24 years of data and found that a quadratic function (i.e., a constant $\dot{P}_b$) was able to describe the data correctly. However, Iaria et al. (2015) found that, when considering the entire 37 years of observations, a quadratic function was unable to fit the orbital evolution and a model with a sinusoidal variation in addition to the quadratic component was required. A third body with mass of $\sim 0.06\,M_\odot$ was invoked to explain the observations with an orbital period of $\approx$ 26 years. It is instructive to notice that a deviation from a quadratic function was not apparent in the first 24 years of data, which suggests that a very long timescale periodicity (or quasi-periodicity) might still be present even in binaries where a constant $\dot{P}_b$ is observed over baselines of a few decades.

The globular cluster source SAX J1748.9–2021 is instead an intermittent AMXP (Altamirano et al. 2008; Gavriil et al. 2007; Patruno et al. 2009a; Sanna et al. 2016) and it has been observed in outburst five times. Its companion star is likely

[12] For a complete list of binary pulsars we refer to the ATNF pulsar catalog http://www.atnf.csiro.au/people/pulsar/psrcat/

to be a ~0.8 $M_\odot$ star close to the turnoff mass of the globular cluster NGC 6440, although much smaller masses down to 0.1 $M_\odot$ cannot be excluded (Altamirano et al. 2008). The orbital evolution has been studied by looking at the orbital ephemeris calculated with coherent timing in a way similar to what has been done in this work. Sanna et al. (2016) describes the orbital evolution with a quadratic function although the fit shows large deviations of the order of 100 seconds from the best fit function (which translated into a poor χ^2 of 78.4 for 1 dof). These authors interpret the large orbital expansion with a highly non-conservative mass loss scenario where the binary is losing more than 97% of the mass flowing through the inner Lagrangian point L_1.

Her X–1 shows instead a steady decrease of the orbit whose value is however compatible with both a conservative and a non-conservative mass transfer scenario (Staubert et al. 2009).

Finally, the orbital evolution of the two transient black hole LMXBs XTE J1118+480 and A0620–00 (González Hernández et al. 2012, 2014) was measured with radial velocity curves determined via optical spectroscopy for a period of time of ~ 10 and 20 years respectively. These observations were all carried out during the extended periods of quiescence of the binaries. The orbit shows a steady shrinkage interpreted as due to enhanced magnetic braking. The two binaries have a companion mass of 0.2 $M_\odot$ (XTE J1118+480; González Hernández et al. 2014) and 0.4 $M_\odot$ (A0620–00; Cantrell et al. 2010; González Hernández et al. 2014), respectively.

6.2.3. *Cataclysmic Variables*

It is well known that some cataclysmic variables show an anomalous orbital period derivative as well, with some of them proposed to be transferring mass at a higher rate than expected due to irradiation of the companion (Patterson et al. 2016, 2015; Knigge et al. 2000). Even some Algol type binaries (i.e., a semi-detached system composed by a detached early type main sequence star and a less massive subgiant/giant star in Roche lobe overflow) have been reported to evolve on a very short timescale (Erdem & Öztürk 2014). In this case non-conservative mass transfer scenario is expected to take place since the red giant will emit a significant wind. However, for a few of these systems (i.e., all the converging ones) the required mass loss is larger than the highest theoretical value for wind mass loss in giant stars. These observations might suggest that short term effects have some influence on the orbital evolution of accreting and non accreting neutron stars, persistent and transient systems and white dwarf/black hole/main sequence stellar accretors. It is worth noticing that no neutron star + white dwarf binary (both accreting and non-accreting) has been observed (as yet) to evolve on anomalous timescales. The only exception is the ultra-compact LMXB 4U 1822–30, which, however, is located in a globular cluster and therefore its large $\dot{P}_b$ might simply be due to the contamination induced by the gravitational potential well of the cluster (Jain et al. 2010). This suggests that, if there is a common reason behind this behavior for all type of binaries (which is of course not necessarily true), it must be related to the type of companion (main sequence or semi-degenerate star) rather than the type of accretor.

6.2.4. *Caveats*

A few cautionary words are necessary at this point on the CV, Algol type binaries and some LMXBs. For these type of systems, where the $\dot{P}_b$ is detected by looking at the eclipse times in optical data, some selection effects might be present. This means that those systems with a $\dot{P}_b$ in line with the theoretical predictions might be more difficult to measure/detect and therefore the reported values are invariably skewed towards large/anomalous $\dot{P}_b$ values. Something similar applies also to most LMXBs, with the exception of those systems where the $\dot{P}_b$ is measured via pulsar timing, in which case the sensitivity of the timing technique potentially allows the detection of values orders of magnitude smaller than in the CVs and Algol binaries. For the CVs there is ample literature on the topic and several different systems with a large $\dot{P}_b$ are reported. In this paper we include only the T Pyxidis and IM Normae systems which are the two best studied cases and have the highest orbital period variation (Patterson et al. 2015, 2016). We also include NN-Serpentis (Brinkworth et al. 2006) which is an eclipsing post common-envelope binary where no mass transfer is currently ongoing. We summarize the information on the orbital period evolution of all the binaries discussed in this work in Figure 4. From the figure, it is clear that all sources with short orbital periods that should be losing angular momentum via gravitational wave emission, are evolving on timescales which are at least an order of magnitude shorter than expected. The binaries with wider orbits, where magnetic braking should dominate, show also shorter evolutionary timescales than predicted, although a larger scattering is observed and some sources are close to the theoretical predictions.

6.3. *Models*

Since a large number of scenarios are invoked in the literature to explain the orbital evolution of different interacting binaries, it appears legitimate to ask whether it is still possible to find a common (and/or perhaps still unknown) mechanism behind the observed behavior. In the following discussion we proceed by considering all models proposed in the literature and try to apply each of them to the case of SAX J1808.4−3658. with the exception of the third body model, which has be already excluded (see Section 5.3.

6.3.1. *Mass Loss Model*

If the companion star experiences severe mass loss, for example because of ablation due to the irradiation from a pulsar wind or from the X-rays originating close to the compact object, then the orbital period of the binary changes dramatically (see Section 6.1).

In this case the orbital period derivative will depend on the amount of wind lost from the companion:

$$\frac{\dot{P}_b}{P_b} = -2\frac{\dot{M}_c}{M_c} \tag{8}$$

where M_c is the companion mass (see e.g., Frank et al. 2002; Postnov & Yungelson 2014). Applying this model to SAX J1808.4−3658 it is possible to explain the observed $\dot{P}_b$ if the donor is losing mass at a rate of about 10^{-9} $M_\odot$ yr^{-1} (di Salvo et al. 2008).

For the donor to lose a substantial amount of mass, there must be a way to efficiently inject energy into the donor star. Whatever mechanism is chosen, the amount of energy necessary to create such a strong mass loss must be consistent with the total energy budget available to the binary. In SAX J1808.4−3658 it has been proposed that the mass loss

Table 4
Binaries with Anomalous Orbital Period Derivatives

Name	$P_b[hr]$	$\dot{P}_b$	Transient	Companion Type	Sign	Proposed Model	References
Neutron Star LMXBs							
EXO 0748–676[a]	3.8	1.9×10^{-11}	Yes	MS	+	SOC	Wolff et al. (2009, 2002)
2A 1822–37	5.5	$1.51(8) \times 10^{-10}$	No	MS	+	Mass Loss	Burderi et al. (2010); Iaria et al. (2011)
SAX J1808.4–3658	2.0	$3.5(2) \times 10^{-12}$	Yes	SD	+	SOC/Mass Loss	Patruno et al. (2012); di Salvo et al. (2008)
MXB 1658–298	7.1	$8.4(9) \times 10^{-12}$	Yes	MS	+	Unknown	Paul & Jain (2010)
XB 1916–053	0.8	$1.5(3) \times 10^{-11}$	No	SD	+	Third Body	Iaria et al. (2015)
SAX J1748.9–2021	8.8	$1.1(3) \times 10^{-10}$	Yes	MS/Sub-G	+	Mass-Loss	Sanna et al. (2016)
AX J1745.6–2901	8.4	$-4.03(32) \times 10^{-11}$	Yes	MS/Sub-G	–	Mass Loss	Ponti et al. (2015)
Hercules X–1	40.8	$-4.85(13) \times 10^{-11}$	No	MS	–	Several	Staubert et al. (2009)
Black Hole LMXBs							
XTE J1118+480	4.0	$-6(1.8) \times 10^{-11}$	Yes	MS	–	Enhanced MB	González Hernández et al. (2012, 2014)
A0620–00	7.8	$-1.9(3) \times 10^{-11}$	Yes	MS	–	Enhanced MB	González Hernández et al. (2014)

MS = main sequence; sub-G = sub-giant; SD = semi degenerate; MB= magnetic braking; SOC = Spin-Orbit Coupling;
[a] This source shows segments of data where a constant P_b is required. The error on $\dot{P}_b$ is not given and confidence intervals are determined via Maximum Likelihood Method (Wolff et al. 2009).

is driven by a pulsar wind and high energy radiation impinging onto the donor surface (di Salvo et al. 2008; Burderi et al. 2009). For a circular binary orbit, the total angular momentum is (Frank et al. 2002):

$$J \propto M_{\rm NS} M_{\rm c} M^{-1/3} P_b^{1/3} \tag{9}$$

where P_b is the orbital period and $M = M_{\rm NS} + M_{\rm c}$ the total binary mass, and $M_{\rm c} < M_{\rm NS}$. The orbital energy $E_{\rm orb}$ is

$$-E_{\rm orb} \propto J/P_b \tag{10}$$

From Eq. 9, an increase in P_b requires $M_{\rm c}$ to decrease, since for an isolated system J and M cannot increase. And from Eq. 10 we see that $-E_{\rm orb}$ must decrease, making the orbit less tightly bound. In other words, an orbital period increase requires energy injection from somewhere. Marsh & Pringle (1990) show that energy injection by the secondary star is too slow for observed P_b changes as this is governed by the star's thermal timescale[13].

The only energy source left is the spin energy of the NS. This is:

$$E_{\rm spin} \sim k^2\, M_{\rm NS}\, v_K^2 \sim k^2\, G M_{\rm NS}^2 / R_{\rm NS} \tag{11}$$

where k is the radius of gyration and $R_{\rm NS}$ its physical radius, and $v_K = (G M_{\rm NS}/R_{\rm NS})^{1/2}$ is the breakup spin velocity. This is a huge reservoir, since

$$E_{\rm spin}/(-E) \sim k^2 \frac{M_{\rm NS}}{M_{\rm c}} \frac{a}{R} \sim 400 - 1000 \tag{12}$$

with $k^2 \sim 0.4$. But of course not all of $E_{\rm spin}$ can be used to drive mass loss from the system and so increase P_b. We can estimate the required minimum efficiency η for spin energy conversion into orbit energy for SAX J1808.4–3658 .

The total orbital binding energy of the binary is:

$$E_{\rm orb} = -\frac{G M_{\rm NS} M_{\rm c}}{2a} \tag{13}$$

If we use the third Kepler law, then we have that the orbital period is very well determined ($P_b = 2.01$ hr, from the X-ray pulse timing). The total mass of the binary is ill constrained, mostly because of the unknown neutron star mass. If we assume a range of total binary mass from 1.4 up to $3 M_\odot$, then the variation in a is of the order of 20% ($6-8\times10^{10}$ cm). Here we will assume $a = 6.4\times10^{10}$ cm ($M_{\rm NS} = 1.4\, M_\odot$, $M_{\rm c} = 0.08\, M_\odot$). The orbital energy is therefore:

$$E_{\rm orb} \sim 3 \times 10^{47}\ {\rm erg} \tag{14}$$

The semi-major axis of the binary changes according to the 3rd Kepler law:

$$\dot{a} = \frac{2a}{3P_b} \dot{P}_b \tag{15}$$

The value $\dot{P}_b$ is measured from observations and its value is $\sim 3.5 \times 10^{-12}$. Therefore $\dot{a} = 2 \times 10^{-5}\ {\rm cm\, s^{-1}}$. The orbital energy variation is (we assume $\dot{M}$ terms are negligible):

$$\dot{E}_{\rm orb} = \frac{G M_{\rm NS} M_{\rm c} \dot{a}}{2\, a^2} \sim 9 \times 10^{31}\ {\rm erg\, s^{-1}} \tag{16}$$

The total spin down power is:

$$\dot{E}_{\rm sd} = I \omega \dot{\omega} \tag{17}$$

where $\omega = 2\pi\,\nu$, $\dot{\omega} = 2\pi\dot{\nu}$ and ν and $\dot{\nu}$ are the spin frequency (401 Hz) and the spin down ($1.65(20)\times10^{-15}\ {\rm Hz\, s^{-1}}$) observed in SAX J1808.4–3658 (Patruno et al. 2012). Numerically, $\dot{E}_{\rm sd} \approx 2.6 \times 10^{34}\ {\rm erg\, s^{-1}}$.

We need an efficiency η of at least the ratio between the two powers:

$$\eta = \frac{\dot{E}_{\rm orb}}{\dot{E}_{\rm sd}} \sim 0.003 \tag{18}$$

For the parameters assumed above, the Roche lobe radius is $R_L = 0.16\, R_\odot$ (see e.g., Eggleton 1983) and the fraction of intercepted power is $f = (R_L/2a)^2$ which is approximately 0.8%. This value suggests that the donor star must be extremely efficient in converting the incident power into mass loss since $\epsilon \sim \eta/0.8 \approx 40\%$. Campana et al. (2004) estimated that the irradiating power[14] required to explain the bright optical counterpart of SAX J1808.4–3658 (observed with VLT data in 2002) amounted to $L_{\rm irr} = 8^{+3}_{-1} \times 10^{33}\ {\rm erg\, s^{-1}}$ (see also

[13] This is of course what happens when a very low-mass star expands on mass transfer ($M_{\rm c} > M_{\rm NS}$) with $P_b \propto 1/M_{\rm c}$. The stars thermal energy expands it adiabatically.

[14] The authors used a distance of 2.5 kpc, whereas here we rescale the luminosity for a distance of 3.5 ± 0.1 kpc as determined by Galloway & Cumming (2006)

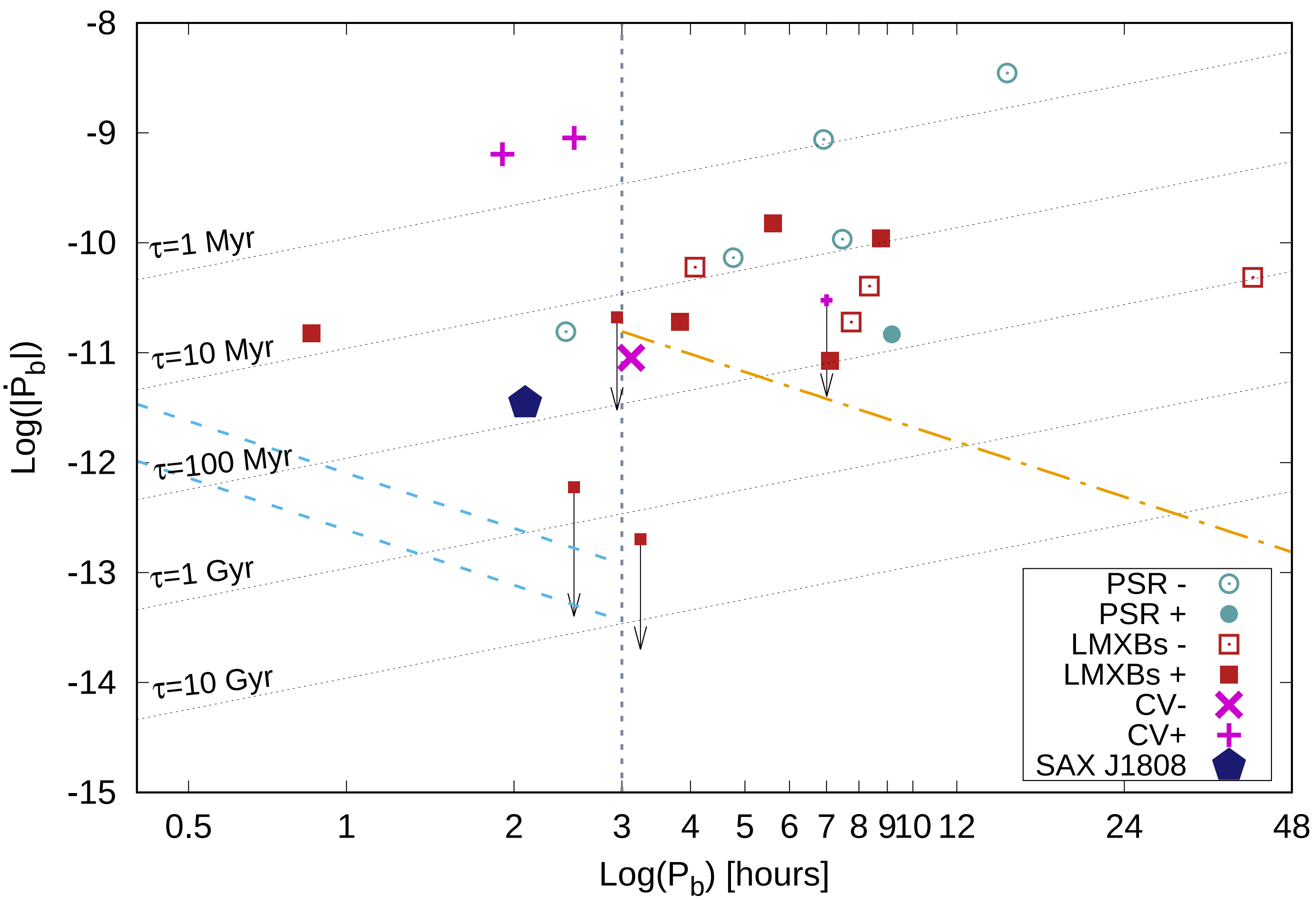

Figure 4. $|\dot{P}_b|$ vs. P_b diagram for SAX J1808.4−3658 (blue pentagon), LMXBs (red squares), binary pulsars (BW and RBs, blue circles) and some CVs (purple crosses). The open symbols identify converging systems (negative $\dot{P}_b$) whereas filled symbols are diverging ones. Only T PyX and IM Nor are plotted for CVs (plus NN-Ser which is drawn with the same symbol used for CVs). The oblique black dotted lines identify evolutionary timescales $\tau = P_b/|\dot{P}_b|$. The dashed vertical lines roughly separates the binaries where the dominant angular momentum loss should be gravitational radiation (left) from those where magnetic braking is expected to dominate (right) assuming no mass loss or spin-orbit coupling is present in the binary. The cyan oblique lines are theoretical values of $|\dot{P}_b|$ expected if gravitational wave emission is the main driver of orbital evolution. From top to bottom these lines are valid for a donor mass-radius relation $R_c \propto M_c$ and $R_c \propto M_c^{-1/3}$ respectively, assuming a donor mass of 0.08 $M_\odot$ and a neutron star mass of 1.4 $M_\odot$. A value of 0.08 $M_\odot$ has been chosen as it represents the most likely donor mass in SAX J1808.4−3658 . The orange oblique lines is instead the orbital period derivative as a function of orbital period when the maximum possible magnetic braking effect is considered (see main text for a discussion). A mass-radius index $\zeta = 1$ has been assumed for the magnetic braking case.

Homer et al. 2001 for a similar estimate made with observations taken in 1999). Therefore the optical data require a conversion of a fraction $\xi = L_{\rm irr}/\dot{E}_{\rm sd} \approx 0.2$–$0.6$ of incident power into thermal radiation by the donor star (the range provided takes into account the 1 σ error bars on the irradiation luminosity and the possibility that the distance is 2.5 kpc rather than 3.5 kpc). Since from the observational constraint we have that $\xi + \epsilon = 0.5 - 1.0$ and $\xi + \epsilon$ is bound to be equal to 1 by the conservation of energy, this scenario is energetically plausible for a range of parameters compatible with the observations.

If we assume that the mass loss from the donor of SAX J1808.4−3658 is constant, then one still needs to explain the $T_{\rm asc}$ of the 2011 outburst. This data point deviated by approximately 7 seconds from the predicted value that can be obtained in Eq. (2) by using a constant $\dot{P}_b = 3.5 \times 10^{-12}\,{\rm s\,s^{-1}}$. The 7-s deviation can be explained if the mass loss rate has increased by about $\sim 70\%$ during the 2008-2011 period, which would require a proportionally larger spin-down power than assumed above. Indeed the relation between the mass loss and the spin-down power is linear (see e.g. a discussion in Hartman et al. 2009):

$$\dot{M}_{\rm c} = \dot{E}_{\rm sd} \left(\frac{1}{2a}\right)^2 G\, M_{\rm c}\, R_{\rm c} \tag{19}$$

Since we have seen that the donor needs to convert the incident spin-down power into mass-loss with extraordinary efficiency, close to 40% ($\epsilon \sim 0.4$), then the spin-down power is larger than initially estimated either because of a larger moment of inertia of the neutron star or because the spin-down $\dot{\omega}$ is slightly larger than observed. In the first case one would need $I \gtrsim 1.7 \times 10^{45}\,{\rm g\,cm^2}$ and this can be used in principle to constrain the equation of state of ultra-dense matter (under the assumption that mass-loss is the main mechanism responsible for the binary evolution). In Figure 5 we plot an illustrative example of the type of constraints that can be obtained when using this method for a selection of equations of state (Fortin et al. 2016).

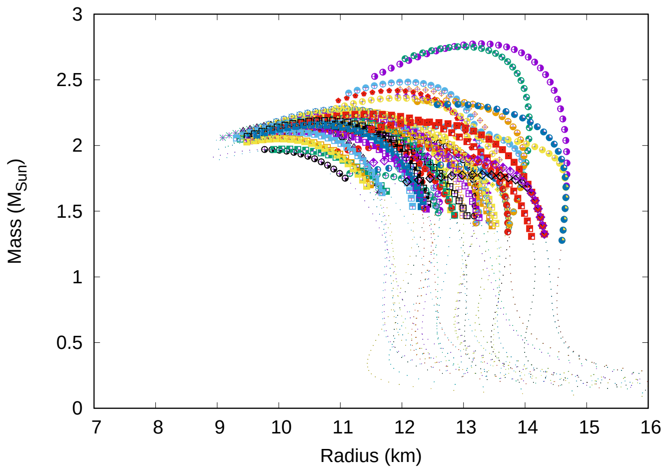

Figure 5. Constraints on the mass-radius relation of the neutron star in SAX J1808.4−3658 under the assumption that the binary is driven by mass loss. The different curves correspond to different equation of states of ultra-dense matter and are taken from Fortin et al. (2016). The thick part of the curves marks the segments for which the moment of inertia of the neutron star is $> 1.7 \times 10^{45}$ g cm^2 (see main text for a discussion).

In conclusion, this mechanism is energetically feasible if:

- SAX J1808.4−3658 has a neutron star with a large moment of inertia
- the incident pulsar power can be converted into mass loss with a $\approx$ 40% efficiency.

When looking at the whole sample of pulsar binaries, the behavior of other black-widow pulsars cannot be explained by this model since at least two of them are *shrinking* their orbit. Therefore if the mass-loss model is correct we need two different mechanisms to explain the population of binary pulsars. Furthermore, in a recent work (Patruno 2016) we have studied the orbital evolution of another AMXP, namely IGR J0029+5934. This system can be considered a "twin" system of SAX J1808.4−3658 since its orbital and physical parameters are extremely similar (see e.g., Patruno & Watts 2012). However, in that case the orbit of the binary is varying at a very slow pace ($\tau > 0.5$ Gyr), compatible with a conservative scenario where the binary evolution is driven by gravitational wave emission. In that case we constrained the efficiency of the pulsar spin-down to mass-loss conversion to be $\lesssim$ 5%. It is not clear therefore why IGR J0029+5934 is unable to convert spin-down power into mass-loss whereas SAX J1808.4−3658 is so efficient. We stress that the observations of *both* systems suggest a donor irradiated by a pulsar wind/high energy radiation and the donor mass is almost identical.

6.3.2. *Spin-Orbit Coupling*

An exchange of angular momentum between the stellar spin and the orbit can generate variations of the orbital period. This variations of the orbital angular momentum are encoded in the term $\dot{J}_{\rm soc}$ in Eq.(6).

The so-called Applegate model was developed by Applegate (1992) and Applegate & Shaham (1994) to explain the orbital variability observed in a sample of eclipsing variables and then later applied to the case of the black-widow pulsar PSR B1957+20 (Applegate & Shaham 1994). The model has been then further extended to Roche lobe filling systems like cataclysmic variables (Richman et al. 1994). The model can be briefly summarized as follows. If the donor star has internal deformations, then the gravitational potential outside of the active star is (terms higher than quadrupolar are ignored here):

$$\phi(x) = -\frac{GM}{r} - \frac{3}{2} G Q_{ik} \frac{x_i x_k}{r^5} \tag{20}$$

where x_i and x_k are Cartesian coordinates measured from the center of mass of the star and Q_{ik} is the quadrupole tensor (related to inertia tensor). For simplicity one can assume a circular orbit (as is legitimate to do in a binary like SAX J1808.4−3658), an alignment between the donor spin axis and orbital angular momentum, and that the stellar spin and orbit are synchronized. If the Cartesian system is chosen so that the z-axis is the angular momentum axis and the x-axis points from the center of mass to the companion star, then Q_{ik} reduces to $Q_{xx} = Q$. In a circular orbit, the relative velocity can be written as:

$$v^2 = r\frac{d\phi}{dt} \tag{21}$$

Therefore from Eq. (20) one can see that the relative velocity v is related to the time varying quadrupole Q. Since v is also related to the orbital period of the binary, it is clear that a time varying mass quadrupole term induces a variation of the orbital period. Applegate (1992) suggests that the cause of the time varying quadrupole Q might be related to magnetic cycles of period $P_{\rm mod}$ (in a way analogous to the familiar 11 years-long magnetic solar cycles). During these cycles, the magnetic field induces a redistribution of the angular momentum in different layers of the star and allows a transition between different equilibrium configurations. The strength of the surface magnetic field B required to explain a variation ΔP of the orbital period can be written as:

$$B^2 \sim 10 \frac{GM^2}{R^4} \left(\frac{A}{R}\right)^2 \frac{\Delta P}{P_{\rm mod}} \tag{22}$$

The transport of angular momentum inside the star needs of course some energy which Applegate (1992) suggests might come either from the donor internal nuclear burning reservoir or from tidal heating (Applegate & Shaham 1994). This is in essence the Applegate model which has been propose as a viable way to explain the behavior of many binary systems. It seems therefore natural to extend it to the case of LMXBs. For the case of SAX J1808.4−3658 one needs to assume a value for $P_{\rm mod}$ since the $T_{\rm asc}$ variations observed so far do not show a complete cycle. By assuming $P_{\rm mod} = 50$ yr, $R = 0.1\,R_\odot$, $M = 0.07\,M_\odot$, $A = 7 \times 10^{10}$ cm one finds $B \sim 1$ kG.

There are, however, two main problems with the Applegate model applied to SAX J1808.4−3658 and to many other compact binaries considered here. The first is that the required B field is of the order of 1 kG which is much larger than typical values thought to be present in fully convective stars. However, some isolated low mass stars and brown dwarfs have been observed with relatively strong surface B fields (Morin et al. 2010) and in some cases with fields larger than 1 kG (Reiners 2012). Furthermore, recent studies have provided observational evidence for the presence of magnetic activity in at least four fully convective stars, suggesting that the dynamo mechanism that produces stellar magnetic fields operates also through convection despite the absence of the tachocline, which is the boundary layer between radiative and convective envelope where the magnetic fields are generated (Wright & Drake 2016).

The second problem, which seems more difficult to circumvent, was discussed in a critical review by Brinkworth et al. (2006), who found that for very low mass stars like NN-Serpentis (which is a non-accreting post-common envelope binary with a 0.15 $M_{\odot}$ companion and an orbital period of 3 hours), the internal energy budget of the donor star might be insufficient to generate the required donor distortion. Even if one invokes the tidal heating mechanism proposed in Applegate & Shaham (1994) there would still be insufficient energy available to generate the required stellar distortions (see e.g. Burderi 2015). As was the case for the mass-loss model, the only source of energy left is the spin-down energy of the pulsar. In this case there needs to be a viable mechanism to transport energy deeper in the donor star which is able to generate a varying mass quadrupole. As noted by Applegate (1992), if the donor star becomes more oblate then the mass quadrupole $\Delta Q > 0$ and the orbital period decreases. The opposite happens if $\Delta Q < 0$: the orbital period increases. The observed behavior of most binaries considered in this work might then be explained if, for some reason, some of them (like SAX J1808.4−3658 and other diverging binaries) have $\Delta Q < 0$ whereas all other converging binaries have $\Delta Q > 0$. This idea remains highly speculative at the moment since the problem of what happens in the deep layers of irradiated stars has not been investigated yet.

6.3.3. *Enhanced Magnetic Braking*

This model could explain the sign and strength of $\dot{P}_b$ in SAX J1808.4−3658 only if the donor magnetic field is sufficiently strong. Indeed, the mass lost by the donor cannot be larger than the one estimated in Section 6.3.1 since the energy budget does not allow it. As an example, we follow the recipe provided by Justham et al. (2006), in which case the angular momentum lost by the binary via magnetic braking is:

$$\dot{J}_{\rm mb} = -\Omega_d\, B_s\, R_c^{13/4}\, \dot{M}_w^{1/2} (G\, M_c)^{-1/4} \qquad (23)$$

where Ω_d is the angular rotational frequency of the donor star, B_s is its dipolar magnetic field at the surface and $\dot{M}_w$ is the amount of wind loss rate. If we assume that SAX J1808.4−3658 is changing orbital parameters mainly because of angular momentum loss, then by rearranging Eq.(5) we obtain:

$$\dot{J} = \frac{\dot{a}}{a^{1/2}} M_{\rm NS} M_{\rm c} \left(\frac{G}{M}\right)^{1/2} \approx 2\times10^{35}\ {\rm g\,cm^2 s^{-2}} \qquad (24)$$

Assuming that the donor is tidally locked, then Eq.(23) gives the strength of the minimum B_s field required which is of the order of 10^3–10^4 G for a maximum wind loss rate of $10^{-9}\ {\rm M_{\odot}\,yr^{-1}}$, even stronger than the value calculated for the Applegate model. This shows that the magnetic braking model is unlikely to be the correct one. Furthermore, such explanation cannot work in several other binaries since at least in some neutron star LMXBs the sign of the observed orbital period derivative is opposite to that expected when magnetic braking is the main driver of binary evolution.

7. CONCLUSIONS

We have studied the AMXP SAX J1808.4−3658 at radio-wavelengths during quiescence in 2014 and during its last 2015 outburst. We have not detected radio pulsations and we place strong constraints on the flux density of the putative radio pulsar which, if really active, needs to be either among the 10% dimmest pulsars known or fully obscured by radio absorbing material (which would also be atypical at the relatively high radio observing frequencies we searched at). The study of the orbital evolution of the system has been extended to include the 2015 outburst, and we find two possible interpretations of the data: either the orbit is expanding with stochastic fluctuations around the mean or the system is shrinking with a change of sign around 2011.

In the first case the pulsar spin-down power is ablating the companion with an efficiency for the conversion of impinging power to mass-loss of the order of 40%.

Alternatively the Applegate model can explain the behavior of SAX J1808.4−3658 if a strong surface magnetic field of the order of a kG is present. The source of energy that powers this field needs to be the spin-down power of the pulsar but there is no evidence that such large fields exist in the donor star of SAX J1808.4−3658 or that they can be generated by the pulsar wind/high energy irradiation. This requires further theoretical investigation.

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