BSRLM Geometry Working Group

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Perspectives on the Design of the School Geometry Curriculum

A report based on the meeting at the University of Exeter, 26th February 2000

by

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The question of how to construct an appropriate geometry curriculum is a long-standing one. A recent estimate suggests that there are more than 50 geometries. This creates a fundamental problem in devising a geometry curriculum: there are just too many interesting things to include so some decision has to be made as to what to include and what to exclude. This report features three perspectives on the issue of the design of the school geometry curriculum.

Introduction

A crucial question in mathematics education is the design of the mathematics curriculum. Nowhere in the mathematics curriculum has this been felt more keenly than in geometry. In the UK as long ago as 1871, an Association for the Improvement of Geometry Teaching was formed, which later evolved into the Mathematical Association. Since then there have probably been more than a dozen major inquiries into the teaching and learning of geometry (some UK-based, some international), all of which have had to consider the issue of the design of the geometry curriculum.

As part of the recent study by the International Commission on Mathematical Instruction (ICMI) on the teaching of geometry, the group considering changes and trends in geometry curricula observed that "In recent years a problem of lack of coherence in the curriculum in geometry has been felt more and more. The teaching of geometry contains small bits of polygon classification, some formulae for measuring various shapes, some incidence geometry, a little mention of transformations, a few constructions, an introduction to vectors, and finally some analytic geometry. From the point of view of pupils it might well look like a kind of inconsistent ‘bazaar’, but we have only vague empirical evidence for this” (Hansen 1998 p 238).

While it remains unclear whether or not the pupil experience of geometry is actually inconsistent and muddled, the design of the geometry curriculum remains a critical question. In the sections that follow, three perspectives are presented on what is important in an appropriate geometry curriculum for schools (the sections are in alphabetic order by surname of author).

What is an appropriate geometry curriculum for schools?

Tandi Clausen-May, National Foundation for Educational Research

An appropriate geometry curriculum is one:

- which depends upon shape, space and movement, not on paper and print.
- which is based upon the three dimensional world we live in, not the two dimensional world of the printed page.
- with a minimum of writing, a modicum of drawing, and a maximum of making.
- in which concepts, rather than the use of language, are developed and assessed.
- which develops the spatial ability of all pupils.
- which enables all pupils, including (in fact, especially) the stupid, non-academic ones (like Einstein was) to think about and work with shape and space.

Some implications of the above:

Starting points: For some reason, we teach two dimensional geometry before we teach three dimensional geometry. (Well, actually, it's because we got caught in the print trap, which says that all teaching has to be out of books, so it has to be two dimensional.) That is patently silly. Three dimensions comes first. So at least in the primary school, geometry should be something you make and do, and talk about and imagine; it should not be something you write.

Teaching resources: A 'geometry text book' is a contradiction in terms, like a 'swimming textbook'. You might have a book about swimming – giving its history, a description of techniques and styles, and so on – but swimming is something you do, not study. So is spatial thinking, and that is what the geometry curriculum should be designed to develop.

Teachers: Many teachers, even mathematics teachers, are not good spatial thinkers. They will need a lot of support to enable them to develop their own spatial ability to the point where they can teach others effectively.

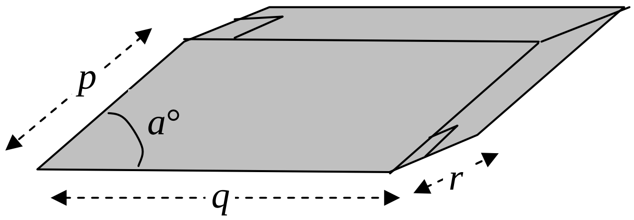
Using ICT: ICT has already opened a lot of doors to learners who cannot access print, but are much better at thinking geometrically than their teachers ever could be. This alternative means of communication must be developed.

Assessment: Assessment drives the curriculum. Here is a good geometry question:

> Provide the pupil with a paper circle.
>
> Ask her to find the centre of the circle.

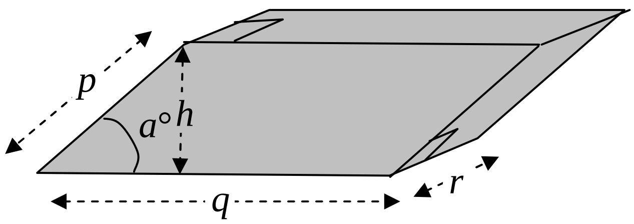
This question asks the pupil to think spatially. She does not need to use language at all to respond appropriately.

Here is another good question. It is harder, and it could be in a formal, written test:



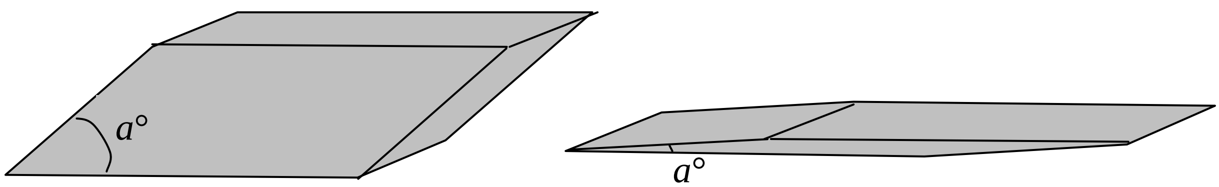
The rectangular based parallelopiped in the diagram has edges of length p, q and r.
An angle of a° is marked on the diagram.
What value of a will make the volume of the parallelopiped as large as possible?

This problem can be solved in at least two qualitatively different ways. It can be solved analytically:

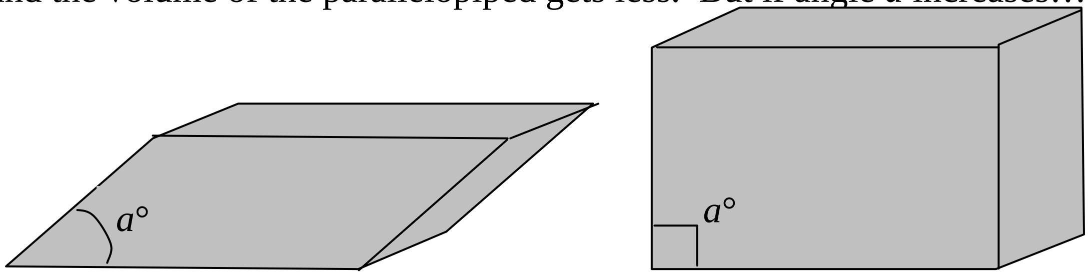


Volume = base × height = $q\ r \times h$
sin a° = h/p
$h = p$ sin a°
Volume = $q\ r \times p$ sin a°
Maximum sin a° is 1, when $a = 90$
So $a = 90$

Or it can be solved rather more efficiently by imagining the parallelopiped, and visualising what happen as angle a decreases…..

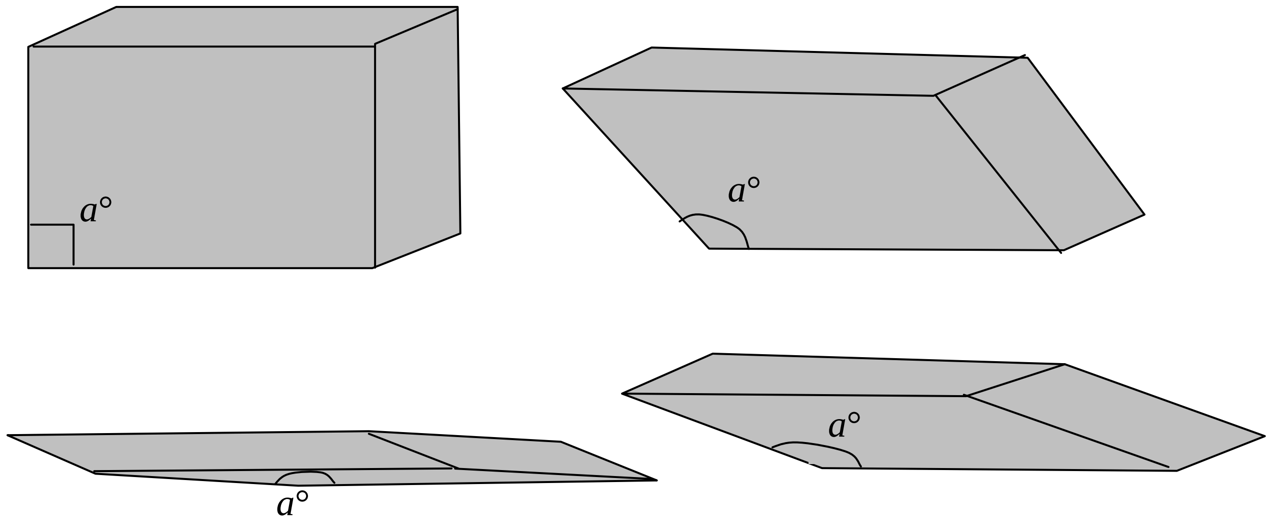


….. and the volume of the parallelopiped gets less. But if angle *a* increases….



… then the volume increases as well.

But if it increases beyond the right angle, then the parallelopiped flattens out the other way and the volume gets less again…..

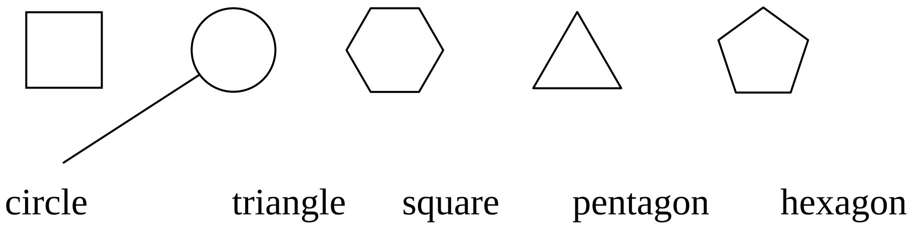


So one can ***see*** that the volume is at it greatest when the angle is a right angle.

In contrast, here is a bad question:

Match each shape to its name.

The first is done for you.



This is a pure language test, and has precious little to do with geometry.

We need assessment structures that assess holistic geometrical thinking, not analytical thinking, and not language. ICT-delivered assessments may make this easier, but I think we will need more activities (like finding the centre of the paper circle by folding it) as well.

What is an appropriate geometry curriculum?

Alan Mclean, Rolle School of Education, University of Plymouth

Last week I started teaching an introductory course in Shape & Space to a year 1 BEd group and was struck yet again with what an odd collection of facts, vocabulary, research, activities, good ideas and government proscription it appeared to be. While I attempt to provide an overarching structure, e.g. using the strands of the NC Using & Applying, relating activities to the Van Hiele levels etc., I do feel there is a real lack of clarity as to what the important ideas actually are. This confusion is reflected in DfEE publications; it would be difficult to see the new National Curriculum or the Framework for the National Numeracy Strategy as having been designed with any clear sense of the purpose of the geometry curriculum or of the connections inherent within the content presented.

I am not suggesting that structure should be restored to the curriculum by a return to Euclidean Geometry but the kind of connectionist understanding now being suggested for the teaching of number appears to have been lost in Shape & Space. Nor am I suggesting that a purely serialist approach is appropriate. Much of the richness of this area of the curriculum lies in the possibilities for exploration. In fact it is difficult to see what parts of the current school curriculum could not be covered through a holistic exploration of the three themes of

- Tiling and tessellation
- Explorations of invariance using dynamic geometry software
- Investigations based around Logo.

Given the potential richness of geometry we have to choose the topics which are most worthwhile and to do this we do need to get much clearer about purpose. This is after all what we tell ITE students on a daily basis. What is it we are trying to achieve in teaching geometry? The aims for mathematics teaching in general are often listed in terms of

- the needs of life and work
- the need to develop logical thinking
- mathematics as a form of communication
- the development of an awareness of mathematics as part of our culture.

Aims which might be appropriate for the geometry curriculum could be a subset or elaboration of these. We might think that geometry is an area of mathematics in which it is particularly appropriate for pupils to

- develop the skills needed for the world of work, e.g. learn to cooperate in groups in an extended Logo project

- develop logical thinking, e.g. in designing a macro using dynamic geometry software
- clarify the precise use of language, e.g. through classifying shapes in a study of transformations
- see the links between mathematics and other subjects, e.g. in a project on fractals
- begin to understand the nature of proof, e.g. through exploring what "being convinced" has meant at different points in history
- developing an understanding of the effect of initial assumptions on the theories or models developed, e.g. in considering what "different" means in a simple investigation or, at a higher level, in reviewing the stories of mathematicians involved in the search for the fifth postulate
- understand the central place of problem solving in modern culture, e.g. in a mathematically based design project.

There may be a great richness in the possibilities for teaching geometry but it is difficult to see how that can be accessed in the current educational climate without a clear justification in terms of aims and objectives. Unless we can develop such a specification I suspect that the geometry curriculum will, by default, remain as the rather unconnected list of properties and skills which make up the current statutory orders.

The Need to Teach Geometry as a Historic, Cultural and Cognitive Heuristic

Stuart Rowlands, University of Plymouth and Robert Carson, Montana State University

Until the late 1950's, Euclidean geometry was the main *mathematical* subject taught in ('selective' sector) schools. It was the primary means by which deductive reasoning had been taught for over 2200 years. It was sustained in this role for many centuries by the historical, social, cultural, and philosophical context of classical studies that were taught at the same time. The learner was in a reasonably good position to understand the role geometry had played in the enculturation of Greek intellectuals and its influence on all other domains of intellectual culture from art and philosophy to science and the law. They might expect to replicate that influence in their own minds by participating in a similar induction process.

However, instead of a dynamic enculturation into reason, logical argumentation, and other high level cognitive skills, the teaching of mathematics decayed into a worship of dead letters. Pedagogical hacks forced students to learn Euclid by rote and to memorize the sequence *of axiom, theorem, proof,* without learning how to work proofs or to think mathematically. By the time the Mathematics Association was formed in 1897 and Heaviside gave his famous 'fudge and fiddlesticks' paper in 1901 (that most boys learn by 'doing' and are too immature for abstract reasoning), it was possible to ask why anyone should be forced to endure this kind of 'classical liberal education'. Unfortunately, it was neither classical nor liberal – it had become a mindless tradition of memorization and regurgitation.

What we have now is 'shape and space' and the banishment of proof. The use of geometry as an educational heuristic for the cultivation of reason has been eliminated from the experience of an entire generation of learners. There are ways to patch around its absence, even to teach logic and argumentation, but the wisdom of severing all ties with the entire cultural foundation represented by classical civilization and its unique modes of thought is questionable, and has been since the process began over a century ago.

Greek geometry ought to be accessible to every secondary school pupil, not as an exercise in rote learning the sequence of propositions and proofs, nor as a worship of some bygone tradition, but as an induction into one of the foundational disciplines of the mind. In order for students to understand its value, it must somehow be acquired in context, through knowledge of its history and its historical consequences. Does the textbook need to be Euclid, or the approach Euclidean? Not necessarily. The Pythagoreans never heard of Euclid. Nor did Plato's students. But they did enter into a systematic conversation that generated the ideas and processes Euclid later systematized. It is that conversation we wish to call attention to.

How we can capture for the pupil all of the insights, energy, excitement, imagery and intellectual vividness of those early thinkers? By restoring the contextual frames that at one time helped establish the significance of geometry. By a more imaginative and artful curricular design we could place students "on location" in a simulation of those historical moments when each significant developmental step occurred in the evolution of abstract geometry. Let them relive those profound developmental moments. In this way they could experience the growth of one of humankind's most profound cultural tools – reason. Every traditional culture has its epic tale. Where is ours? And why are we failing to tell it?

Concluding Comments

In an elegant essay on geometry, Sawyer (1977 p12) concludes that "In the subject matter of geometry we suffer from an embarrassment of riches. We have so many tools for the discussion of geometric problems - Euclid, transformations, coordinates, matrices, calculus. However, it is noticeable that no one of these is the magic key that unlocks all doors. An eclectic approach seems in order; pupils should learn the power and limitations of each". Such an "eclectic" approach does not necessarily have to result in pupils experiencing geometry as a kind of "inconsistent bazaar". Both Barbeau (1988) and Nissen (2000) point to approaches to geometry that, through maintaining multiple perspectives (see, also, Coxford 1993), *can* provide pupils with experience of the variety of ways of solving spatial problems and proving geometric results while, at the same time, appreciating the versatility and inventiveness of mathematics.

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BSRLM Geometry Working Group

The BSRLM geometry working group focuses on the teaching and learning of geometrical ideas in its widest sense. The aim of the group is to share perspectives on a range of research questions that could become the basis for further collaborative work. Suggestions of topics for discussion are always welcome. The group is open to all.

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