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BSRLM Geometry Working Group

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Designing Dynamic Geometry Tasks that Support the Proving Process

A report based on the meeting at the University of Warwick, 13th November 1999

by

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A major challenge for mathematics education is to find ways in which proof in geometry has communicatory, exploratory, and explanatory functions alongside those of justification and verification. Ongoing research is suggesting that providing students with tasks which state "prove that…" might actually inhibit students' capacity for proving. In contrast, open tasks which favour a dynamic exploration of a statement and encourage the use of transformational reasoning may allow students to reconstruct, in terms of properties and relationships, all the elements needed in the proof. In this report we consider the transforming of closed problem into open ones and discuss the use of dynamic geometry software, such as Cabri, *in such a process.*

Introduction

Providing a mathematics curriculum that makes proof accessible to school students appears to be difficult. Proving, it seems, either appears as an obscure ritual or it disappears in a series of innocuous classroom tasks in which students learn to 'spot patterns' but may realise little more. For example, Schoenfeld (1989) reports that even when students can reproduce a formally taught Euclidean proof, a significant proportion conjecture a solution to the corresponding geometrical construction problem that "*flatly violates* the results they have just proven" (emphasis added). When the chosen proof contexts are data-driven, Coe and Ruthven (1994) found that "students' proof strategies were primarily empirical". In such a situation, the generation of numerical data becomes the object of the exercise and any notion of deductive argument appears to be abandoned. These fine-detailed studies are echoed in a large-scale survey of the views of and competencies in mathematical proof of almost 2500 high school students in the UK from which Healy and Hoyles (1998) report that even high attaining students in that age group "show a consistent pattern of poor performance in constructing proofs".

Yet proof in mathematics has a number of functions, including communicatory, exploratory, and explanatory ones alongside those of justification and verification (de Villiers 1990, Hanna and Jahnke 1996). A major challenge for mathematics education is to find ways in which proof in geometry reflects these wider functions (Hoyles and Jones 1998). In this report we consider issues in the design of suitable problems for use with dynamic geometry software, such as *Cabri*.

Designing Tasks for use with Dynamic Geometry Software

Established classroom tasks involving proof in geometry are typically of the following form (although this is a reasonably challenging example):

> *Problem 1*
> Let C and C' be two circles (with centres O and O' respectively) intersecting at two distinct points A and B. Let AD and AE be two diameters of C and C' respectively. Prove that D, B and E are collinear. Prove that DE and OO' are parallel segments.

Ongoing research is suggesting that providing students with tasks which state "prove that…" might actually inhibit students' capacity for proving. Extensive work by, for instance Boero and colleagues (for example, Boero *et al* 1996) and by Arzarello and colleagues (for example, Arzarello *et al* 1998) shows how the choice of context or framing of a classroom task ('frame of experience' in the work of Boero) is a crucial factor in activating mental processes involved in a 'dynamic' exploration of a problem situation by which students struggle mentally with the formulation of hypotheses and conjectures. Classroom tasks, such as problem 1 above, it seems, do not in general stimulate these essential mental processes. The research has found that students get locked into the theoretical aspect of geometry (what theorems do I know? What can I assume?) and tend not to try exploring the problem situation in a way that generates conjectures that can support the proving process.

It seems that to begin creating a more meaningful experience of proof for school students we need to bear in mind two considerations (Jones 1995). First we need contexts for proof with which students can engage. Secondly, we need ways of working in the classroom that provide opportunities for students to explain why they obtain a particular outcome. In particular, open tasks (Arsac *et al* 1988, Mogetta *et al* 1999), which favour both a dynamic exploration of a problem statement and a form of reasoning that Simon (1998) refers to as transformational, might allow students to construct and reconstruct, in terms of properties and relationships, all the elements needed in the proof whilst working on a problem.

An open task might be more like the form given in the examples below (Problem 2 adapted from Perham and Perham 1997, Problem 3 is from Arzarello *et al* 1998). The proposition is that the form of such tasks stimulates the types of reasoning processes associated with the transition from argumentation to proving. Such tasks also lend themselves to being tackled with the aid of dynamic geometry software in a way that also supports transformational reasoning. The particular contribution of dynamic geometry software is considered in the next section.

> *Problem 2*
> Inscribe a triangle in a circle. Draw a tangent at each vertex such that it intersects with the extended side of the triangle opposite the vertex. What do you observe about these points of intersection? Explain what you find in a way that would convince someone else.

> *Problem 3*
> ABCD is a quadrangle. The bisectors of the internal angles of the quadrangle intersect pairwise consecutively at points H, K, L and M. What do you observe about quadrangle HKLM ? Explain what you find in a way that would convince someone else.

The Role of the Dynamic Geometry Environment in the Proving Process

The first step in tackling a problem using dynamic geometry software involves interpreting the problem in terms of the menu items available within the software environment. Undertaking the construction involves making explicit the starting points and the relationships between them. This construction process, it is conjectured, should support the initial phase of the proving process.

Completing the construction and investigating its properties involves students enacting transformational reasoning processes. They can focus on invariant properties while dragging elements of the figure, and see the components of the figure in a relationship of functional dependence with each other. This leads to generating hypotheses under which a certain configuration has certain properties and, most crucially, involves a constant switching between the empirical level of the software screen image and the theoretical level of geometrical knowledge. It is this stimulation of knowledge in action that might stimulate the proving process through the need for explanation of observed geometrical properties and relationships.

A range of research has explored how the learning of geometry with dynamic geometry software involves transitions in the learning process between figures and concepts, between perceptual activity and mathematical knowledge. Typically, a geometrical problem cannot be solved while remaining only at the perceptual level of figures on the screen. Conceptual control is needed and this requires explicit knowledge. The use of the dragging function is in validating procedures and constructions and is the crucial instrument of mediation between figure and concepts, perception and knowledge. Arzarello *et al* (1998) present some features of such transitions in the move from conjecturing to proofs in geometry when using dynamic geometry software. Jones (under consideration) reports on students' evolving use of the language of mathematical argumentation, particularly that to do with justifying constructions, when students are using dynamic geometry software.

Concluding Comments

Classroom tasks which state 'prove that ..' may not, of necessity, inhibit students' capacity for proving. It is possible that exploring the problem statement given as Problem 1 above with dynamic geometry software could lead to the formation and solving of sub-problems that lead to insights into the form of the required proof. It might be that if the classroom or school environment in which the students are working cultivates what Goldenberg and colleagues call 'habits of mind' (Cuoco *et al* 1996, Goldenberg 1996) then it could be that even tasks which appear overly formal to students could be tackled in a way that utilises transformational reasoning and results in mathematics that is communicatory, exploratory, and explanatory. Hoyles (1997) has noted how the form of the curriculum shapes students' approaches to proof. A move to open problems may well be justified in situations where students have not developed appropriate 'habits of mind'. Where such habits are well-developed it could be that even so-called 'closed problems' might be treated in an open way. The question then becomes how to develop such 'habits of mind' as described by Goldenberg and colleagues. It is likely that exposure to open problems is essential in developing such habits. Hence the important task of developing open problems in geometry that fully utilise the potential of dynamic geometry software to provide a motivation to prove.

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BSRLM Geometry Working Group

The BSRLM geometry working group focuses on the teaching and learning of geometrical ideas in its widest sense. The aim of the group is to share perspectives on a range of research questions which could become the basis for further collaborative work. Suggestions of topics for discussion are always welcome. The group is open to all.

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