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Researching the Learning of Geometrical Concepts in the Secondary Classroom: problems and possibilities.

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Researching the learning of geometrical concepts in the secondary classroom presents both problems and opportunities. The specification of the geometry curriculum, the need to concretise abstract geometrical objects for classroom activities, the role of the teacher and the need to reconsider geometrical notions from different viewpoints are all factors which affect the acquisition of geometrical concepts by pupils. These factors can provide problems for the researcher. Yet there are also significant opportunities both to influence policy decisions and to contribute to both theoretical and practical debates regarding the teaching and learning of geometry.

In this paper I want to review some of the problems and possibilities that I am encountering in researching the acquiring of geometrical concepts by eleven and thirteen year olds. I will examine some of the background issues that influence the nature of what I am able to research. Amongst these influences will be the following: what geometry is being taught and what is not being taught, the reasons for teaching these aspects of geometry, the teaching methods and teaching materials being employed and the tasks the pupils complete. The problems and possibilities that I wish to discuss are linked to the research that I am conducting so it is there that I need to start.

The Research Project

In a review of the position of geometry teaching in UK schools in the mid 1980s, Fielker comments that it was "confusing and to some extent disappointing" (Fielker 1986). This was due, Fielker argues, to what he saw as confusion over variously, the different representations in Euclidean and transformation geometry, the place of vectors and the treatment of topology. It could be argued that this confusing and disappointing situation continues despite the recent review of the UK National Curriculum. Nevertheless, in both the UK National Curriculum for mathematics and in the US National Council of Teachers of Mathematics curriculum standards for school mathematics an underlying model of the teaching and learning of geometry can be discerned. This model is very similar to that proposed by the Mathematical Association in 1923 which recognised three stages in the teaching of geometry, briefly, Stage A: intuitive, experimental work; Stage B: 'Locally' deductive work in which formal symbolism and deductive reasoning is introduced, but where intuition and induction still have a place and will be used to bridge logically difficult gaps; and Stage C: Globally rigorous work (Mathematical Association 1923).

This model, interestingly, is remarkably similar to the van Hiele approach which has received some attention over recent years (see, for example, Van Hiele 1986 and Fuys *et al* 1988). Two brief extracts will illustrate the influence of this approach. The working party for the UK National Curriculum for mathematics suggests, for instance, that “the experimental approach [to geometry] needs to be complemented by due attention to intellectual rigour as the pupil progresses” (DES and Welsh Office 1988 p 32). Similarly, the US NCTM curriculum standards for school mathematics state that “the study of geometry in Grades 5-8 links the informal explorations begun in grades K-4 to the more formalised processes studied in Grades 9-12” (NCTM 1989 p 112).

It is just this transition from `informal explorations' to `formalised processes' that is the focus of my research. How do secondary school pupils handle the introduction of formal symbolism and deductive reasoning in geometry? What is the place of intuition and induction and how do they bridge logically difficult gaps for pupils? In particular, what is the relationship between abstract and intuitive thinking in learning geometry in the secondary school? Does intuitive thinking give way to formal thinking as the pupil progresses? How does this process take place?

I am currently undertaking fieldwork over a 12 month period in a secondary school. A year 7 class (11-12 year olds) and a year 9 class (13-14 year olds) have been identified on the basis of the class teachers' willingness to collaborate in the research. Within each class, pairs of pupils are being introduced to the dynamic geometry package *Cabri-geometre*, initially using an approach similar to that suggested by Healy *et al* (1994a, b and c). I now turn to the problems and possibilities that I am encountering. I shall begin with the problems.

Problems in Researching the Learning of Geometrical Concepts

On reflecting on the problems that I am encountering I should start by emphasising that none of them are due to current poor teaching. On the contrary, the class teacher with whom I am working is not only very experienced but, from the evidence of my observations of classroom practice and from appraisal and inspection reports, the approaches used in the classroom are exemplary. Nevertheless, classroom activities are, to a large extent, determined by the School's scheme of work which is itself determined by the UK National Curriculum. Here is the first problem. Geometry is not well-specified within the UK National Curriculum. It could be described as an odd mish-mash of relatively unconnected ideas. On the other hand, as the recent ICMI paper suggests, "there have been (and there persist even now) strong disagreements about the aims, contents and methods for the teaching of geometry at various levels, from primary school to university" (ICMI 1994). So, given that no `ideal' geometry curriculum has yet been designed (and perhaps one does not exist), we should not be surprised by the problems encountered with the UK curriculum.

Given the different aspects of geometry, synthetic, coordinate and transformation, it could be argued that there is very little synthetic (Euclidean) geometry in the UK curriculum, and only a modicum of coordinate and transformation geometry (for example, there is no 3D coordinate geometry and no mention of matrices; indeed, vectors were only put back into the curriculum following consultation over the recent changes). As a result, the curriculum contains little advice to support the classroom teacher in developing geometrical ideas with pupils. For example, consider the concept of a circle. I am finding that pupils view a circle as a disc. When they want to drag a circle in *Cabri*, pupils invariably point *inside* the circle rather than at the circle itself. But then their earlier experiences with circles involve working with solid discs or colouring in circles that have been drawn on worksheets. The notion of the circumference being the distance *around* the circle may also reinforce the idea that a circle *is* a disc. Of course any concretisation of an abstract concept like a circle will involve a certain degree of misrepresentation. It is the transition from such concrete notions to an abstract definition of the circle that I am interested in.

As Healy *et al* (1994a) point out, formal Euclidean geometry has not been a part of the UK school mathematics curriculum for some time. Yet *Cabri* is an ideal tool for the exploration of just such a geometry curriculum. So what happens when you introduce a tool which is ideal for exploring a particular aspect of geometry into a curriculum that does not contain that geometry? In beginning to use *Cabri* in a somewhat informal way, like Healy *et al* (although with the intention of provoking mathematical thinking), I have found that only a few pairs spontaneously choose overt mathematical goals. Instead, pupils have constructed a `crooked house', a `cat' and so on. What is more they have done so while at the same time resisting my interventions which have been intended to direct the pupils at perhaps what I consider to be more profitable geometrical areas.

A final problem concerns what constitutes progression in acquiring particular geometrical concepts. In considering the concept of a circle, one could argue that progressing from the notion of a circle as a disc to the locus definition should be considered as mathematical progress. The question then is what happens to the earlier notion. Is it replaced with a superior view? Or is it merely suppressed so that it resurfaces when an unusual problem is faced? If, particularly in geometry, certain notions have to be reconsidered from different viewpoints at different stages, what happens to these different viewpoints from the point of view of the learner?

Opportunities for Research in the Geometry Classroom

Many of the problems that I am encountering in my research are also opportunities. In terms of the overall geometry curriculum the lack of prescription allows some degree of freedom in terms of the design of classroom activities. Such opportunities can lead to a critical review, not only of the geometry curriculum but also of the models of learning geometry suggested by, for instance, van Hiele. Thus research into geometry in the classroom can inform both policy decisions, for instance regarding the review of the UK National Curriculum scheduled for five years hence, *and* theoretical considerations. The van Hiele model is already coming under critical review for not only having a somewhat flimsy theoretical basis but also for increasingly appearing unsatisfactory given pupil experience with computer tools such as *Cabri*.

For some considerable time we have viewed children's intellectual growth as proceeding from the concrete to the abstract, for example from Piaget's concrete operational stage to the more advanced stage of formal operations. Recently, Turkle and Papert (1991) have called for a "revaluation of the concrete". This involves "looking for psychological and intellectual development within rather than beyond the concrete and suggests the need for closer investigation of the diversity of ways in which the mind can use objects rather than the rules of logic to think with" (p 166). My feeling is that the geometry classroom could well be a good place to look.

The teacher has a crucial role in the mathematics classroom. When pupils are engaged in mathematical tasks, particularly, say, using a tool such as *Cabri* how, when and why should the teacher intervene? How are such subtle judgements to be made? Questions such this are especially pertinent in the geometry classroom.

Final Comments

In this brief paper I have tried to present some of the problems and possibilities that I am encountering in researching the learning of geometrical concepts in the secondary classroom. The problems are significant. But then so are the possibilities.

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