Flavourful Z' models for $R_{K^{(*)}}$

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Abstract

We show how any flavour conserving Z' model can be made flavour violating and non-universal by introducing mass mixing of quarks and leptons with a fourth family of vector-like fermions with non-universal Z' couplings. After developing a general formalism, we focus on two concrete examples, namely a fermiophobic model, and an $SO(10)$ GUT model, and show how they can account for the anomalous B decay ratios R_K and R_{K^*}. A similar analysis could be performed for $B-L$ models, E_6 models, composite models, and so on.

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1 Introduction

One of the simplest extensions of the Standard Model (SM) is to introduce an additional gauged $U(1)'$, which could emerge as a remnant of larger gauge group embeddings of the SM gauge group, with rank larger than 4. Such larger gauge groups include the left-right symmetric model, Pati-Salam, $SO(10)$, E_6. An extra gauged $U(1)'$ is common in string inspired models, where it is difficult to break the rank of the gauge group, or from alternative dynamical schemes such as composite models. For a review of Z' models and an extensive list of references see e.g. [1].

Most of the existing Z' models have universal couplings to the three families of quarks and leptons. The reason for this is both theoretical and phenomenological. Firstly many theoretical models naturally predict universal Z' couplings. Secondly, from a phenomenological point of view, having universal couplings avoids dangerous favour changing neutral currents (FCNCs) mediated by tree-level Z' exchange. The most sensitive processes involve the first two families, such as $K_0 - \bar{K}_0$ mixing, $\mu - e$ conversion in muonic atoms, and so on, leading to stingent bounds on the Z' mass and couplings [1].

Recently, the phenomenological motivation for considering non-universal Z' models has increased due to mounting evidence for semi-leptonic B decays which violate $\mu - e$ universality at rates which exceed those predicted by the SM [2]. In particular, the LHCb Collaboration and other experiments have reported a number of anomalies in $B \to K^{(*)} l^+ l^-$ decays such as the R_K [3] and R_{K^*} [4] ratios of $\mu^+\mu^-$ to e^+e^- final states, which are observed to be about 70% of their expected values with a 4σ deviation from the SM, and the P_5' angular variable, not to mention the $B \to \phi\mu^+\mu^-$ mass distribution in $m_{\mu^+\mu^-}$.

Following the recent measurement of R_{K^*} [4], a number of phenomenological analyses of these data, see e.g. [5], favour a operator of the left-handed (L) form [6], in the conventions of [7],

$$V_{tb}V_{ts}^* \frac{\alpha_{em}}{4\pi v^2} \left(C^{\rm SM}_{b_L \mu_L} + C^{\rm BSM}_{b_L \mu_L} \right) \bar{b}_L \gamma^\mu s_L \, \bar{\mu}_L \gamma_\mu \mu_L \tag{1}$$

where the SM operator arises from penguin diagrams and has a coefficient of $C^{\rm SM}_{b_L \mu_L} = 8.64$, while the beyond the SM (BSM) operator has a coefficient of $C^{\rm BSM}_{b_L \mu_L} \approx -1.3$. The analogous right-handed (R) operators must be significantly smaller [7]. The SM constants $V_{ts} = 0.040 \pm 0.001$ (predominantly real) and the Higgs vacuum expectation value (VEV) $v = 174$ GeV, set the scale of Eq.1,

$$V_{tb}V_{ts}^* \frac{\alpha_{em}}{4\pi v^2} \approx \frac{1}{(36 \text{ TeV})^2}. \tag{2}$$

This suggests a new physics operator of the form,

$$G^{\rm BSM}_{b_L \mu_L} \, \bar{b}_L \gamma^\mu s_L \, \bar{\mu}_L \gamma_\mu \mu_L \approx -\frac{1}{(33 \text{ TeV})^2} \, \bar{b}_L \gamma^\mu s_L \, \bar{\mu}_L \gamma_\mu \mu_L. \tag{3}$$

In a flavourful Z' model, the new physics operator in Eq.3 will arise from tree-level Z' exchange, where the Z' must dominantly couple to $\mu_L \mu_L$ over $\mu_R \mu_R$, $e_L e_L$, $e_R e_R$, and must also have the quark flavour changing coupling $b_L s_L$ which must dominate over $b_R s_R$. The coefficient of the tree-level Z' exchange operator will be typically of the form,

$$G_{b_L\mu_L}^{\rm BSM} = g_{b_L}^{Z'} g_{\mu_L}^{Z'} \left(\frac{g'^2}{M_{Z'}{}^2}\right) \approx -\frac{1}{(33\ \text{TeV})^2} \tag{4}$$

where the Feynman rule for the $Z'\bar{b}_L\gamma^\mu s_L$ coupling is $-i\gamma^\mu g_{b_L}^{Z'} g'$ and the $Z'\bar{\mu}_L\gamma^\mu\mu_L$ coupling is $-i\gamma^\mu g_{\mu_L}^{Z'} g'$, where g' is the Z' gauge coupling and M_Z' is the mass of the Z'. The required value of M_Z' will typically be much smaller than 33 TeV due to the model dependent coupling factors $g_{b_L}^{Z'}$ and $g_{\mu_L}^{Z'}$ which are anticipated to be quite small in realistic models. This means that the Z' in these models may be within reach of the LHC.

Motivated by the above considerations, there has been a large and growing body of literature which is concerned with flavour dependent Z' models (see e.g. [8]). Recent works on flavoured Z' approaches following the R_{K^*} measurement include those in [9]. One of the key challenges faced by these models is the requirement that they be anomaly free. This has motivated the phenomenological analysis of Z' models based on gauged $L_\mu - L_\tau$, possibly combined with vector-like quarks [10]. Without a Z', vector-like quarks directly mixing with ordinary quarks via the Higgs Yukawa couplings can lead to FCNCs [11]. However, vector-like quarks with a gauged $U(1)'$ typically forbids the Higgs coupling of vector-like quarks to ordinary quarks, but allows new possibilities [10]. For example, a simple idea is to have a dark $U(1)_X$ under which the SM quarks and leptons are neutral, but which is felt by vector-like fermions with the SM quantum numbers of the doublets Q_L and L_L, leading to a dark matter candidate and flavour-changing Z' operators after the vector-like fermion mass terms mix with SM fermions [12]. However adding such matter spoils the prospects for gauge coupling unification unless the vector-like matter comes in complete representations of $SU(5)$. The first example of mixing with vector-like fermions which preserves gauge unification and leads to flavour-changing Z' interactions was proposed some time ago by Langacker and London [13].

In this paper, motivated by the R_K and R_{K^*} anomalies, we show how any flavour conserving Z' model can be made flavour violating and non-universal by the mass mixing of quarks and leptons with a fourth family of vector-like fermions with non-universal Z' couplings. Unlike the original vector-like fermion models [11], having non-universal $U(1)'$ charges of the fourth vector-like family forbids mixing via the usual Higgs Yukawa couplings. Instead, new singlet scalars with appropriate $U(1)'$ charges are added to generate mass mixing of quarks and leptons with the vector-like family. Since we include a complete vector-like family, the mixing will include the doublets Q_L and L_L, leading to the left-handed new physics operators required for R_K and R_{K^*}. Since we consider a complete fourth vector-like family, unification is maintained. We develop a quite general formalism, which can be applied to any Z' model in the literature, including $B-L$ models, E_6 models, composite models, and so on. To illustrate the mechanism we consider two concrete examples, namely a fermiophobic model, and an $SO(10)$ Grand Unified Theory (GUT), and show how they can account for the anomalous B decay ratios R_K and R_{K^*}.

The layout of the remainder of the paper is as follows. In section 2 we consider the general class of models consisting of the usual three chiral families of left-handed quarks

Field	Representation/charge			
	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)'$
Q_{Li}	**3**	**2**	1/6	q_{Q_i}
u_{Ri}	**3**	**1**	2/3	q_{u_i}
d_{Ri}	**3**	**1**	−1/3	q_{d_i}
L_{Li}	**1**	**2**	−1/2	q_{L_i}
e_{Ri}	**1**	**1**	−1	q_{e_i}
ν_{Ri}	**1**	**1**	0	q_{ν_i}
H_u	**1**	**2**	−1/2	q_{H_u}
H_d	**1**	**2**	1/2	q_{H_d}
Q_{L4},$\tilde{Q}_{R4}$	**3**	**2**	1/6	q_{Q_4}
u_{R4},$\tilde{u}_{L4}$	**3**	**1**	2/3	q_{u_4}
d_{R4},$\tilde{d}_{L4}$	**3**	**1**	−1/3	q_{d_4}
L_{L4},$\tilde{L}_{R4}$	**1**	**2**	−1/2	q_{L_4}
e_{R4},$\tilde{e}_{L4}$	**1**	**1**	−1	q_{e_4}
ν_{R4},$\tilde{\nu}_{L4}$	**1**	**1**	0	q_{ν_4}
$\phi_{Q,u,d,L,e}$	**1**	**1**	0	$q_{\phi_{Q,u,d,L,e}}$

Table 1: The most general model we consider consists of the usual three chiral families of left-handed (L) and right-handed (R) quarks and leptons ψ_i $(i = 1, 2, 3)$ and one (or two) Higgs doublet(s) $H_{(u,d)}$, plus a fourth vector-like family of fermions ψ_4, $\tilde{\psi}_4$. There may be other exotics in addition to those shown in order to cancel anomalies, or the three chiral families may cancel anomalies by themselves without extra exotics. In any case, the vector-like fermion family are always anomaly free by themselves. The $U(1)'$ is broken by the VEVs of new Higgs singlets ϕ_ψ with charges $|q_{\phi_\psi}| = |q_{\psi_i} - q_{\psi_4}|$ to yield a massive Z'.

and leptons with one (or two) Higgs doublet(s) $H_{(u,d)}$, plus a fourth vector-like family of fermions, which has non-universal charges under a gauged $U(1)'$. We write down the Lagrangian for such a general class of models in the charge basis and the heavy mass basis, after diagonalisation of the heavy masses. In section 3, to illustrate the mechanism and how it may be applied in practice, we consider two concrete examples of well known Z' models which can be made flavourful via mixing with a non-universal fourth vector-like family, namely a namely a fermiophobic model, and an $SO(10)$ GUT model, and show how they can account for the anomalous B decay ratios R_K and R_{K^*}. Section 4 concludes the paper.

2 A class of Z' models with a vector-like family

In this section we analyse the general class of models defined in Table 1 consisting of the usual three chiral families of left-handed (L) and right-handed (R) quarks and leptons ψ_i $(i = 1, 2, 3)$ and one (or two) Higgs doublet(s) $H_{(u,d)}$, plus a fourth vector-like family of fermions ψ_4, $\tilde{\psi}_4$. The gauged $U(1)'$ charges q_{ψ_i} are universal up to the fourth family (i.e. $q_{\psi_1} = q_{\psi_2} = q_{\psi_3} \neq q_{\psi_4}$), although in general they need not be. The three chiral families must be anomaly free, since the vector-like family is anomaly free. The $U(1)'$ is broken by the VEVs of new Higgs singlets ϕ_ψ with charges $|q_{\phi_\psi}| = |q_{\psi_i} - q_{\psi_4}|$ to yield a massive

Z'.

The layout of this rather lengthy section is as follows. In the first subsection we present the Lagrangian of the general class of models in the charge basis. In the second subsection we show how the heavy masses may be diagonalised. In the third subsection we present the Lagrangian of the general class of models in the heavy mass basis.

2.1 The Lagrangian in the charge basis

In this subsection we present the Lagrangian of the general class of models in the charge basis. Including the fourth family, along with the usual three chiral families, the gauge part of the Lagrangian involving fermions is given by,

$$\begin{aligned}
\mathcal{L}^{gauge} &= i\overline{Q}_{L\alpha}\left(\partial_\mu - ig_3 G^A_\mu \frac{\lambda^A}{2} - ig_2 W^a_\mu \frac{\sigma^a}{2} - \frac{1}{6} i g_1 B_\mu - q_{Q\alpha} i g' B'_\mu\right)\gamma^\mu Q_{L\alpha} \\
&+ i\overline{u}_{R\alpha}\left(\partial_\mu - ig_3 G^A_\mu \frac{\lambda^A}{2} - \frac{2}{3} i g_1 B_\mu - q_{u\alpha} i g' B'_\mu\right)\gamma^\mu u_{R\alpha} \\
&+ i\overline{d}_{R\alpha}\left(\partial_\mu - ig_3 G^A_\mu \frac{\lambda^A}{2} + \frac{1}{3} i g_1 B_\mu - q_{d\alpha} i g' B'_\mu\right)\gamma^\mu d_{R\alpha} \\
&+ i\overline{L}_{L\alpha}\left(\partial_\mu - ig_2 W^a_\mu \frac{\sigma^a}{2} + \frac{1}{2} i g_1 B_\mu - q_{L\alpha} i g' B'_\mu\right)\gamma^\mu L_{L\alpha} \\
&+ i\overline{e}_{R\alpha}\left(\partial_\mu + i g_1 B_\mu - q_{e\alpha} i g' B'_\mu\right)\gamma^\mu e_{R\alpha} \\
&+ i\overline{\nu}_{R\alpha}\left(\partial_\mu - q_{\nu\alpha} i g' B'_\mu\right)\gamma^\mu \nu_{R\alpha}
\end{aligned} \tag{5}$$

where $\alpha = 1, \ldots, 4$ labels the four families of the same chirality, G^A_μ are $SU(3)_c$ gauge fields (the octet of gluons $A = 1, \ldots, 8$), W^a_μ are $SU(2)_L$ gauge fields ($a = 1, \ldots, 3$), B_μ are the $U(1)_Y$ gauge fields and B'_μ are the $U(1)'$ gauge fields, with the three usual gauge couplings g_i, as well as the $U(1)'$ gauge coupling g'. We denote the Pauli matrices as σ^a and the Gell-Mann matrices as λ^A.

In addition there is a similar gauge Lagrangian involving the fourth family of the opposite chirality $\tilde{\psi}_4$, obtained by the replacements, $Q_{L\alpha} \to \tilde{Q}_{R4}$, $u_{R\alpha} \to \tilde{u}_{L4}$, $d_{R\alpha} \to \tilde{d}_{L4}$, $L_{L\alpha} \to \tilde{L}_{R4}$, $e_{R\alpha} \to \tilde{e}_{L4}$, $\nu_{R\alpha} \to \tilde{\nu}_{R4}$.

The right-handed neutrinos are special, since the Standard Model gauge group allows large Majorana masses, although these may be forbidden by $U(1)'$. Henceforth, for simplicity, we shall ignore the right-handed neutrinos, and the associated vector-like fourth family, which is equivalent to ignoring neutrino mass.

We assume that the $U(1)'$ charges allow for Yukawa couplings of the first three chiral families ψ_i, but not the fourth vector-like family,

$$\mathcal{L}^{Yuk} = y^u_{ij} H_u \overline{Q}_{Li} u_{Rj} + y^d_{ij} H_d \overline{Q}_{Li} d_{Rj} + y^e_{ij} H_d \overline{L}_{Li} e_{Rj} + H.c. \tag{6}$$

where $i, j = 1, \ldots, 3$.

We assume that the $U(1)'$ charges allow for the fourth opposite chirality family $\tilde{\psi}_4$ to have interactions with the first three chiral families ψ_i via singlet fields ϕ which carry $U(1)'$ charge, in addition to explicit masses between opposite chirality fourth family fields $\tilde{\psi}_4$ and ψ_4 of the same charges,

$$\begin{aligned}\mathcal{L}^{mass} &= x_i^Q \phi_Q \overline{Q}_{Li} \tilde{Q}_{R4} + x_i^u \phi_u \overline{\tilde{u}}_{L4} u_{Ri} + x_i^d \phi_d \overline{\tilde{d}}_{L4} d_{Ri} + x_i^L \phi_L \overline{L}_{Li} \tilde{L}_{R4} + x_i^e \phi_e \overline{\tilde{e}}_{L4} e_{Ri} \\ &+ M_4^Q \overline{Q}_{L4} \tilde{Q}_{R4} + M_4^u \overline{\tilde{u}}_{L4} u_{R4} + M_4^d \overline{\tilde{d}}_{L4} d_{R4} + M_4^L \overline{L}_{L4} \tilde{L}_{R4} + M_4^e \overline{\tilde{e}}_{L4} e_{R4} + H.c. \quad (7)\end{aligned}$$

After the singlet fields ϕ develop vacuum expectation values (VEVs), we may define new mass parameters $M_i^Q = x_i^Q \langle \phi_Q \rangle$, and similarly for the other mass parameters, to give,

$$\mathcal{L}^{mass} = M_\alpha^Q \overline{Q}_{L\alpha} \tilde{Q}_{R4} + M_\alpha^u \overline{\tilde{u}}_{L4} u_{R\alpha} + M_\alpha^d \overline{\tilde{d}}_{L4} d_{R\alpha} + M_\alpha^L \overline{L}_{L\alpha} \tilde{L}_{R4} + M_\alpha^e \overline{\tilde{e}}_{L4} e_{R\alpha} + H.c. \quad (8)$$

where $\alpha = 1, \ldots, 4$.

2.2 Diagonalising the heavy masses

In this subsection we show how the heavy masses may be diagonalised, denoting the fields in this basis by primes. The idea is that, after diagonalisation, only the fourth family is massive (before electroweak symmetry breaking),

$$\mathcal{L}^{mass} = \tilde{M}_4^Q \overline{Q'}_{L4} \tilde{Q}_{R4} + \tilde{M}_4^u \overline{\tilde{u}}_{L4} u'_{R4} + \tilde{M}_4^d \overline{\tilde{d}}_{L4} d'_{R4} + \tilde{M}_4^L \overline{L'}_{L4} \tilde{L}_{R4} + \tilde{M}_4^e \overline{\tilde{e}}_{L4} e'_{R4} + H.c. \quad (9)$$

and the first three primed masses of each fermion type are zero. The original charge basis and the heavy mass basis are related by unitary mixing matrices,

$$Q'_L = V_{Q_L} Q_L,\ u'_R = V_{u_R} u_R,\ d'_R = V_{d_R} d_R,\ L'_L = V_{L_L} L_L,\ e'_R = V_{e_R} e_R. \quad (10)$$

The unitary mixing matrix which relates the column vector Q'_L of mass eigenstates (where the first three components are massless and the fourth component has a mass $\tilde{M}_4^Q$) to the original fields Q_L may be written as,

$$V_{Q_L} = V_{34}^{Q_L} V_{24}^{Q_L} V_{14}^{Q_L}, \quad (11)$$

where

$$V_{34}^{Q_L} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{34}^{Q_L} & s_{34}^{Q_L} e^{-i\delta_{34}^{Q_L}} \\ 0 & 0 & -s_{34}^{Q_L} e^{i\delta_{34}^{Q_L}} & c_{34}^{Q_L} \end{pmatrix}, \quad (12)$$

$$V_{24}^{Q_L} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{24}^{Q_L} & 0 & s_{24}^{Q_L} e^{-i\delta_{24}^{Q_L}} \\ 0 & 0 & 1 & 0 \\ 0 & -s_{24}^{Q_L} e^{i\delta_{24}^{Q_L}} & 0 & c_{24}^{Q_L} \end{pmatrix}, \quad (13)$$

$$V_{14}^{Q_L} = \begin{pmatrix} c_{14}^{Q_L} & 0 & 0 & s_{14}^{Q_L} e^{-i\delta_{14}^{Q_L}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14}^{Q_L} e^{i\delta_{14}^{Q_L}} & 0 & 0 & c_{14}^{Q_L} \end{pmatrix}. \tag{14}$$

Ignoring phases, the tangent of the mixing angles $t = \tan\theta$ are given by,

$$t_{14}^{Q_L} = \frac{M_1^Q}{M_4^Q}, \; t_{24}^{Q_L} = \frac{M_2^Q}{M_4'^Q}, \; t_{34}^{Q_L} = \frac{M_1^Q}{M_4''^Q}, \tag{15}$$

where

$$M_4'^Q = \sqrt{M_1^{Q^2} + M_4^{Q^2}}, \; M_4''^Q = \sqrt{M_2^{Q^2} + M_4'^{Q^2}}, \; \tilde{M}_4^Q = \sqrt{M_3^{Q^2} + M_4''^{Q^2}}. \tag{16}$$

Similar equations may be readily obtained in each of the other sectors u_R, d_R, L_L, e_R, with the trivial replacements, $Q_L \to u_R, d_R, L_L, e_R$.

2.3 The Lagrangian in the heavy mass basis

In this subsection we present the Lagrangian of the general class of models in the heavy mass basis, denoted by primes, in which only the fourth family is heavy (compared to the weak scale). In this basis the model involves the three massless chiral families ψ_i', where $i = 1, \ldots, 3$ which are massless before electroweak symmetry breaking, plus a heavy fourth family ψ_4', which have the same chirality as the first three families, with which they mix. In this basis, only the fourth family ψ_4' have explicit vector-like Dirac mass terms involving the opposite chirality heavy heavy fourth vector-like family $\tilde{\psi}_4$. The diagonal heavy mass (primed) basis is therefore the correct basis to work in if one wishes to study the interactions of the heavy vector-like fourth family states, ψ_4', $\tilde{\psi}_4$, or to integrate them out to leave the three massless (before electroweak symmetry breaking) families ψ_i'.

2.3.1 Yukawa couplings and CKM

In the original basis, the Yukawa couplings in Eq.6 may be written in terms of the three chiral families ψ_i plus the same chirality fourth family ψ_4 in a 4×4 matrix notation as,

$$\mathcal{L}^{Yuk} = H_u \overline{Q}_L \tilde{y}^u u_R + H_d \overline{Q}_L \tilde{y}^d d_R + H_d \overline{L}_L \tilde{y}^e e_R + H.c. \tag{17}$$

where $\tilde{y}^u, \tilde{y}^d, \tilde{y}^e$ are 4×4 matrices consisting of the original 3×3 matrices, y^u, y^d, y^e, but augmented by a fourth row and column consisting of all zeroes, since we have assumed that the fourth family ψ_4 does not couple to the Higgs doublets due to its non-universal $U(1)'$ charges.

In the primed basis in Eq.10, where only the fourth components of the fermions are very heavy, the Yukawa couplings become,

$$\mathcal{L}^{Yuk} = H_u \overline{Q'}_L \tilde{y}'^u u_R' + H_d \overline{Q'}_L \tilde{y}'^d d_R' + H_d \overline{L'}_L \tilde{y}'^e e_R' + H.c. \tag{18}$$

where

$$\tilde{y}'^u = V_{Q_L}\tilde{y}^u V^\dagger_{u_R}, \quad \tilde{y}'^d = V_{Q_L}\tilde{y}^d V^\dagger_{d_R}, \quad \tilde{y}'^e = V_{L_L}\tilde{y}^e V^\dagger_{e_R} \tag{19}$$

This shows that the fourth family states ψ'_4 with heavy vector-like masses in Eq.9 couple to the Higgs by virtue of their mixing with the first three chiral families.

The coupling of the heavy mass eigenstate ψ'_4 to the Higgs doublets is given by the fourth rows and columns of the primed Yukawa matrices in Eq.19,

$$\begin{aligned}\mathcal{L}^{Yuk}_{heavy} &= \tilde{y}'^u_{i4} H_u \overline{Q'}_{Li} u'_{R4} + \tilde{y}'^d_{i4} H_d \overline{Q'}_{Li} d'_{R4} + \tilde{y}'^e_{i4} H_d \overline{L'}_{Li} e'_{R4} \\ &+ \tilde{y}'^u_{4i} H_u \overline{Q'}_{L4} u'_{Ri} + \tilde{y}'^d_{4i} H_d \overline{Q'}_{L4} d'_{Ri} + \tilde{y}'^e_{4i} H_d \overline{L'}_{L4} e'_{Ri} \\ &+ \tilde{y}'^u_{44} H_u \overline{Q'}_{L4} u'_{R4} + \tilde{y}'^d_{44} H_d \overline{Q'}_{L4} d'_{R4} + \tilde{y}'^e_{44} H_d \overline{L'}_{L4} e'_{R4} + H.c.\end{aligned} \tag{20}$$

where

$$\begin{aligned}\tilde{y}'^u_{i4} &= (V_{Q_L}\tilde{y}^u V^\dagger_{u_R})_{i4}, \quad \tilde{y}'^d_{i4} = (V_{Q_L}\tilde{y}^d V^\dagger_{d_R})_{i4}, \quad \tilde{y}'^e_{i4} = (V_{L_L}\tilde{y}^e V^\dagger_{e_R})_{i4} \\ \tilde{y}'^u_{4i} &= (V_{Q_L}\tilde{y}^u V^\dagger_{u_R})_{4i}, \quad \tilde{y}'^d_{4i} = (V_{Q_L}\tilde{y}^d V^\dagger_{d_R})_{4i}, \quad \tilde{y}'^e_{4i} = (V_{L_L}\tilde{y}^e V^\dagger_{e_R})_{4i} \\ \tilde{y}'^u_{44} &= (V_{Q_L}\tilde{y}^u V^\dagger_{u_R})_{44}, \quad \tilde{y}'^d_{44} = (V_{Q_L}\tilde{y}^d V^\dagger_{d_R})_{44}, \quad \tilde{y}'^e_{44} = (V_{L_L}\tilde{y}^e V^\dagger_{e_R})_{44}\end{aligned} \tag{21}$$

which shows that there will be some Yukawa induced mass mixing between heavy fourth family fermions and light fermions. This Lagrangian also generates Feynman rules for Higgs bosons which couple the heavy fourth family to the three light chiral families. However the fourth family is too heavy to be produced in Higgs decays. There will be a contribution to the Standard Model Higgs production cross-section through gluon-gluon fusion triangle diagrams involving the fourth heavy family ψ'_4. This is unlike the case of a sequential fourth family, which is excluded by Higgs production being too large, due to the large Yukawa couplings of the fourth family to the Higgs boson. By contrast, in the case of the vector-like fourth family here, the Yukawa couplings to Higgs doublets in Eq.21 involving the fourth family will be smaller. This can be readily understood from Eq.21, since $\tilde{y}^u, \tilde{y}^d, \tilde{y}^e$ have zeroes in the fourth row and column, and so the couplings like $\tilde{y}'^u_{44}, \tilde{y}'^d_{44}$ will involve usual Yukawa couplings and will be mixing suppressed.

To calculate the CKM matrix, it would not be appropriate to diagonalise the primed Yukawa matrices in Eq.19 since this would re-mix the heavy vector-like masses throughout all the four families, and undo the heavy mass diagonalisation. The correct proceedure is to integrate out the heavy vector-like family ψ'_4, then calculate the CKM matrix in the low energy effective theory below the heavy vector mass scale. In the limit of large vector-like masses, ignoring the very small Higgs induced mixing between the heavy fourth family and the light three families, one may decouple the heavy states ψ'_4, by simply removing the fourth rows and colums of the primed Yukawa matrices in Eq.19, to leave the upper 3×3 blocks, which describe the three massless families, in the low energy effective theory involving the massless fermions ψ'_i,

$$\mathcal{L}^{Yuk}_{light} = y'^u_{ij} H_u \overline{Q'}_{Li} u'_{Rj} + y'^d_{ij} H_d \overline{Q'}_{Li} d'_{Rj} + y'^e_{ij} H_d \overline{L'}_{Li} e'_{Rj} + H.c. \tag{22}$$

where

$$y'^u_{ij} = (V_{Q_L}\tilde{y}^u V^\dagger_{u_R})_{ij}, \quad y'^d_{ij} = (V_{Q_L}\tilde{y}^d V^\dagger_{d_R})_{ij}, \quad y'^e_{ij} = (V_{L_L}\tilde{y}^e V^\dagger_{e_R})_{ij} \tag{23}$$

and $i, j = 1, \ldots, 3$. The physical three family quark and lepton masses in the low energy effective theory should be calculated using the 3×3 Yukawa matrices in Eq.23.

The CKM matrix for the quarks may be constructed in the usual way, by diagonaling the Yukawa matrices, y'^u, y'^d,

$$V'_{uL} y'^u V'^{\dagger}_{uR} = \mathrm{diag}(y_u, y_c, y_t), \quad V'_{dL} y'^d V'^{\dagger}_{dR} = \mathrm{diag}(y_d, y_s, y_b) \tag{24}$$

to yield the unitary 3×3 CKM matrix,

$$V_{\mathrm{CKM}} = V'_{uL} V'^{\dagger}_{dL}. \tag{25}$$

Note that there is no violation of unitarity of the CKM matrix due to the vector-like fourth family. Also there will be no tree-level Higgs mediated flavour changing neutral currents between the three light families (the usual GIM mechanism in the Higgs sector).

We emphasise that to calculate the CKM matrix and Yukawa eigenvalues one must diagonalise the Yukawa matrices y'^u, y'^d in Eq.23, which emerge after the fourth vector-like family has been correctly decoupled from the low energy effective theory. It is incorrect to calculate the CKM matrix from the original Yukawa matrices y^u, y^d in Eq.6, which do not take into account mixing with the fourth family.

2.3.2 Gauge couplings

Standard Model gauge couplings

In the diagonal heavy mass (primed) basis, given by the unitary transformations in Eq.10, the gauge Lagrangian in Eq.5 is invariant apart from the $U(1)'$ gauge part. This is because under the Standard Model gauge group all four families have the same charges, and so the unitary transformations cancel, as in the usual GIM mechanism. Thus the part of the Lagrangian involving gluons and electroweak gauge bosons remains flavour diagonal in the primed basis,

$$\begin{aligned}
\mathcal{L}^{gauge}_{SM} &= i\overline{Q}'_{L\alpha} \left(\partial_\mu - i g_3 G^A_\mu \frac{\lambda^A}{2} - i g_2 W^a_\mu \frac{\sigma^a}{2} - \frac{1}{6} i g_1 B_\mu \right) \gamma^\mu Q'_{L\alpha} \\
&+ i\overline{u}'_{R\alpha} \left(\partial_\mu - i g_3 G^A_\mu \frac{\lambda^A}{2} - \frac{2}{3} i g_1 B_\mu \right) \gamma^\mu u'_{R\alpha} \\
&+ i\overline{d}'_{R\alpha} \left(\partial_\mu - i g_3 G^A_\mu \frac{\lambda^A}{2} + \frac{1}{3} i g_1 B_\mu \right) \gamma^\mu d'_{R\alpha} \\
&+ i\overline{L}'_{L\alpha} \left(\partial_\mu - i g_2 W^a_\mu \frac{\sigma^a}{2} + \frac{1}{2} i g_1 B_\mu \right) \gamma^\mu L'_{L\alpha} \\
&+ i\overline{e}'_{R\alpha} \left(\partial_\mu + i g_1 B_\mu \right) \gamma^\mu e'_{R\alpha}
\end{aligned} \tag{26}$$

where $\alpha = 1, \ldots, 4$ labels the four families of the same chirality.

The $W^\pm$ gauge boson couplings are thus the same as in the Standard Model,

$$\mathcal{L}^{int}_W = \frac{g_2}{\sqrt{2}} \overline{u}'_{L\alpha} W^+_\mu \gamma^\mu d'_{L\alpha} + \frac{g_2}{\sqrt{2}} \overline{e}'_{L\alpha} W^+_\mu \gamma^\mu \nu'_{L\alpha} + H.c. \tag{27}$$

where $\alpha = 1,\ldots,4$ labels the four families of the same chirality. In addition there are the fourth family couplings involving the opposite chirality states $\tilde{\psi}'_4$. For the quarks, for example, these couplings may be divided into the light three families, and those involving the fourth family couplings,

$$\mathcal{L}_W^{int} = \frac{g_2}{\sqrt{2}}\overline{u}'_{Li}W_\mu^+\gamma^\mu d'_{Li} + \frac{g_2}{\sqrt{2}}\overline{u}'_{L4}W_\mu^+\gamma^\mu d'_{L4} + \frac{g_2}{\sqrt{2}}\overline{\tilde{u}}'_{R4}W_\mu^+\gamma^\mu \tilde{d}'_{R4} + H.c. \tag{28}$$

and similarly for the leptons. The above couplings allow a fourth family fermion to decay into W plus a light fermion, after including a small Higgs induced mass insertion between a fourth fermion and a light fermion, as shown in Eq.20.

In the low energy effective theory, after the heavy fourth family decouples, and electroweak symmetry is broken, and the light effective Yukawa matrices are diagonalised as in Eq.24, the W couplings become,

$$\mathcal{L}_W^{int} = \frac{g_2}{\sqrt{2}}\begin{pmatrix}\overline{u}_L & \overline{c}_L & \overline{t}_L\end{pmatrix} V_{\rm CKM} W_\mu^+\gamma^\mu \begin{pmatrix} d_L \\ s_L \\ b_L\end{pmatrix} + H.c. \tag{29}$$

where the CKM matrix is calculated as in Eq.25.

After electroweak symmetry breaking, the Z gauge boson couples in a flavour diagonal way to all the four families, both the light families ψ'_i and the heavy family ψ'_4, as in the usual GIM mechanism. This leads to the usual Z interaction Lagrangian

$$\mathcal{L}_Z^{int} = \frac{e}{2s_W c_W}\overline{\psi}'_\alpha Z_\mu \gamma^\mu (C_V^\psi - C_A^\psi \gamma_5)\psi'_\alpha \tag{30}$$

where

$$C_A^\psi = t_3, \qquad C_V^\psi = t_3 - 2s_W^2 Q \tag{31}$$

where $\psi'_\alpha = u'_\alpha, d'_\alpha, e'_\alpha, \nu'_\alpha$, where $\alpha = 1,\ldots,4$ labels the four families of the same chirality, and t_3 are eigenvalues of $\sigma_3/2$, while Q are the electric charges of the fermions. We emphasise that this is flavour diagonal, i.e. Z boson exchange does not change the flavour α of a fermion ψ'_α in the primed basis. In particular the heavy fourth family fermions ψ'_4 thus couple to Z bosons with exactly the same Feynman rules as the three light family fermions ψ'_i.

After the diagonalisation of the light fermion mass matrices, the Z boson couplings remain flavour diagonal, due to the unitary transformations cancelling, and are identical to those in the Standard Model, namely those in Eq.30, with the fields ψ'_α replaced by their three family mass eigenstates. The small Higgs induced mass mixing between ψ'_4 and ψ'_i will also not lead to any Z induced flavour changing since any mixing effect will be unitary and will cancel in Eq.30. We emphasise that such a Z exchange GIM mechanism is a consequence of the fact that all four families have the same electroweak charges.

<u>Z' gauge couplings</u>

The above GIM mechanism in the electroweak sector is in marked contast to the physics of Z' gauge bosons, where the $U(1)'$ charges depend on the family index α. This leads to flavour changing due to Z' gauge boson exchange, as we discuss. After $U(1)'$ breaking, we have a massive Z' gauge boson with diagonal gauge couplings to the four families of quarks and leptons, in the original basis,

$$\mathcal{L}^{gauge}_{Z'} = g' Z'_\mu \left(\overline{Q}_L D_Q \gamma^\mu Q_L + \overline{u}_R D_u \gamma^\mu u_R + \overline{d}_R D_d \gamma^\mu d_R + \overline{L}_L D_L \gamma^\mu L_L + \overline{e}_R D_e \gamma^\mu e_R \right) \quad (32)$$

where

$$\begin{aligned} &D_Q = \mathrm{diag}(q_{Q1}, q_{Q2}, q_{Q3}, q_{Q4}),\ D_u = \mathrm{diag}(q_{u1}, q_{u2}, q_{u3}, q_{u4}),\ D_d = \mathrm{diag}(q_{d1}, q_{d2}, q_{d3}, q_{d4}),\\ &D_L = \mathrm{diag}(q_{L1}, q_{L2}, q_{L3}, q_{L4}),\ D_e = \mathrm{diag}(q_{e1}, q_{e2}, q_{e3}, q_{e4}). \end{aligned} \quad (33)$$

In the diagonal heavy mass (primed) basis, given by the unitary transformations in Eq.10, the Z' couplings to the four families of quarks and leptons in Eq.32 becomes,

$$\mathcal{L}^{gauge}_{Z'} = g' Z'_\mu \left(\overline{Q}'_L D'_Q \gamma^\mu Q'_L + \overline{u}'_R D'_u \gamma^\mu u'_R + \overline{d}'_R D'_d \gamma^\mu d'_R + \overline{L}'_L D'_L \gamma^\mu L'_L + \overline{e}'_R D'_e \gamma^\mu e'_R \right) \quad (34)$$

where

$$\begin{aligned} &D'_Q = V_{Q_L} D_Q V^\dagger_{Q_L},\ D'_u = V_{u_R} D_u V^\dagger_{u_R},\ D'_d = V_{d_R} D_d V^\dagger_{d_R},\\ &D'_L = V_{L_L} D_L V^\dagger_{L_L},\ D'_e = V_{e_R} D_e V^\dagger_{e_R}. \end{aligned} \quad (35)$$

Although the 4×4 matrices D_Q, etc., are diagonal in flavour space, the 4×4 matrices D'_Q, etc., are not generally diagonal in flavour space, since the $U(1)'$ charges may be different for the four flavours. This is the case even if the $U(1)'$ charges are universal for the first three families, but differ only for the fourth family. Recall that in the primed basis the fourth family is very heavy while the first three are light. Then Eq.34 shows that, in general, Z' exchange can couple two light families of different flavour, or a heavy fourth family fermion to a light fermion of the first three families. For example, a Z' exchange diagram will allow the decay of a heavy fourth family fermion to three light fermions of different flavours. This decay mechanism will compete with the decay of a heavy fourth family fermion into a W plus a light fermion, which is suppressed by the small Higgs induced mass insertion arising from Eq.20.

In the low energy effective theory, after decoupling the fourth heavy family, Eq.34 gives the Z' couplings to the three massless families of quarks and leptons,

$$\mathcal{L}^{gauge}_{Z'} = g' Z'_\mu \left(\overline{Q}'_L \tilde{D}'_Q \gamma^\mu Q'_L + \overline{u}'_R \tilde{D}'_u \gamma^\mu u'_R + \overline{d}'_R \tilde{D}'_d \gamma^\mu d'_R + \overline{L}'_L \tilde{D}'_L \gamma^\mu L'_L + \overline{e}'_R \tilde{D}'_e \gamma^\mu e'_R \right) \quad (36)$$

where the 3×3 matrices $\tilde{D}'$ are given by,

$$\begin{aligned} &(\tilde{D}'_Q)_{ij} = (V_{Q_L} D_Q V^\dagger_{Q_L})_{ij},\ (\tilde{D}'_u)_{ij} = (V_{u_R} D_u V^\dagger_{u_R})_{ij},\ (\tilde{D}'_d)_{ij} = (V_{d_R} D_d V^\dagger_{d_R})_{ij},\\ &(\tilde{D}'_L)_{ij} = (V_{L_L} D_L V^\dagger_{L_L})_{ij},\ (\tilde{D}'_e)_{ij} = (V_{e_R} D_e V^\dagger_{e_R})_{ij}, \end{aligned} \quad (37)$$

where $i,j = 1,\ldots,3$. We emphasise that these matrices are not diagonal, leading to flavour changing neutral currents, mediated by tree-level Z' exchange. In the parametrisation in Eq.11, ignoring phases, each of the symmetric 3×3 matrices $\tilde{D}'$ schematically looks like,

$$\tilde{D}' = \begin{pmatrix} q_1 c_{14}^2 + q_4 s_{14}^2 & s_{14}s_{24}c_{14}(q_4 - q_1) & (s_{14}s_{34}c_{14}c_{24})(q_4 - q_1) \\ . & q_1 s_{14}^2 s_{24}^2 + q_2 c_{24}^2 + q_4 s_{24}^2 c_{14}^2 & q_1 s_{14}^2 s_{24} s_{34} c_{24} - q_2 s_{24} s_{34} c_{24} + q_4 s_{24} s_{34} c_{24} c_{14}^2 \\ . & . & q_1 s_{14}^2 s_{34}^2 c_{24}^2 + q_2 s_{24}^2 s_{34}^2 + q_3 c_{34}^2 + q_4 s_{34}^2 c_{14}^2 c_{24}^2 \end{pmatrix} \quad (38)$$

with different angles and charges for each matrix in Eq.37. When $q_1 = q_2 = q_3 = q_4$ these matrices are proportional to the unit matrix and there is no flavour changing due to Z' exchange. Also when $s_{i4} = \sin\theta_{i4} = 0$, these matrices are flavour diagonal.

After diagonalisation of the light quark Yukawa matrices, as in Eq.24, the Z' couplings to the physical quark mass eigenstates u, c, t, d, s, b are given from Eq.36 by,

$$\begin{aligned} \mathcal{L}^q_{Z'} &= g' Z'_\mu \begin{pmatrix} \overline{u}_L & \overline{c}_L & \overline{t}_L \end{pmatrix} V'_{uL} \tilde{D}'_Q V'^{\dagger}_{uL} \gamma^\mu \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} \\ &+ g' Z'_\mu \begin{pmatrix} \overline{d}_L & \overline{s}_L & \overline{b}_L \end{pmatrix} V'_{dL} \tilde{D}'_Q V'^{\dagger}_{dL} \gamma^\mu \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} \\ &+ g' Z'_\mu \begin{pmatrix} \overline{u}_R & \overline{c}_R & \overline{t}_R \end{pmatrix} V'_{uR} \tilde{D}'_u V'^{\dagger}_{uR} \gamma^\mu \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} \\ &+ g' Z'_\mu \begin{pmatrix} \overline{d}_R & \overline{s}_R & \overline{b}_R \end{pmatrix} V'_{dR} \tilde{D}'_d V'^{\dagger}_{dR} \gamma^\mu \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} \end{aligned} \tag{39}$$

Similarly the charged lepton couplings to Z' will be given by analogous results,

$$\begin{aligned} \mathcal{L}^e_{Z'} &= g' Z'_\mu \begin{pmatrix} \overline{e}_L & \overline{\mu}_L & \overline{\tau}_L \end{pmatrix} V'_{eL} \tilde{D}'_L V'^{\dagger}_{eL} \gamma^\mu \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} \\ &+ g' Z'_\mu \begin{pmatrix} \overline{e}_R & \overline{\mu}_R & \overline{\tau}_R \end{pmatrix} V'_{eR} \tilde{D}'_e V'^{\dagger}_{eR} \gamma^\mu \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \end{aligned} \tag{40}$$

Finally, ignoring neutrino mass, the Z' couplings to left-handed neutrinos are given by,

$$\mathcal{L}^\nu_{Z'} = g' Z'_\mu \begin{pmatrix} \overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}} \end{pmatrix} V'_{eL} \tilde{D}'_L V'^{\dagger}_{eL} \gamma^\mu \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} \tag{41}$$

These results show that, if the $\tilde{D}'$ term is proportional to the unit matrix, then this will not lead to flavour violation. However any non-universal part of $\tilde{D}'$ will lead to flavour changing in the physical mass basis of the light fermions. We shall see explicit examples of the application of this formalism in the next section.

3 Examples of flavourful Z' Models

The results in the previous section are of quite general applicability. However, to illustrate the mechanism and show how the formalism may be applied in practice, it is instructive to consider two concrete examples of well known Z' models which can be made flavourful via mixing with a non-universal fourth vector-like family and show how they can provide an explanation of R_K and R_{K^*}. Clearly the same method could be applied to any Z' model including $B-L$ models, E_6 models, composite models, and so on.

3.1 Fermiophobic model

The first example we consider is one in which the quarks and leptons start out not coupling to the Z' at all, as in fermiophobic models. We show that such fermiophobic Z' models may be converted to flavourful Z' models via mixing with a fourth vector-like family with Z' couplings. We then show how such a model is capable of accounting for R_K and R_{K^*}.

The starting point is a class of fermiophobic models, where none of the three chiral families of quarks and leptons (nor the Higgs doublets) carry the $U(1)'$ charges, together with a fourth vector-like family which carry $U(1)'$ charges, i.e. $q_1 = q_2 = q_3 = 0$ but $q_4 \neq 0$. The charges in Table 1 are therefore given by the diagonal matrices in Eq.33:

$$\begin{aligned} &D_Q = \mathrm{diag}(0,0,0,q_{Q4}),\ D_u = \mathrm{diag}(0,0,0,q_{u4}),\ D_d = \mathrm{diag}(0,0,0,q_{d4}),\\ &D_L = \mathrm{diag}(0,0,0,q_{L4}),\ D_e = \mathrm{diag}(0,0,0,q_{e4}). \end{aligned} \tag{42}$$

In addition we assume Higgs singlets ϕ_ψ with charges $|q_{\phi_\psi}| = |q_{\psi_4}|$ whose VEVs yield a massive Z', and whose couplings permit mixing of the fourth vector-like family with the three families of the same chirality. The mixing of quarks and leptons with the fourth vector-like family induces flavour violating Z' couplings to the three light families of quarks and leptons, as in Eq.36, which depend on 3×3 matrices $\tilde{D}'$ in Eq.38 of the form,

$$\tilde{D}'_Q = q_{Q4} \begin{pmatrix} (s^Q_{14})^2 & s^Q_{14}s^Q_{24}c^Q_{14} & s^Q_{14}s^Q_{34}c^Q_{14}c^Q_{24} \\ . & (s^Q_{24})^2(c^Q_{14})^2 & s^Q_{24}s^Q_{34}c^Q_{24}(c^Q_{14})^2 \\ . & . & (s^Q_{34})^2(c^Q_{14})^2(c^Q_{24})^2 \end{pmatrix}, \tag{43}$$

and similar matrices with $Q \to L$, and so on. The couplings of the quark and lepton mass eigenstates to the Z' are given by inserting Eq.43, and similar equations in each of the sectors Q_L, u_R, d_R, L_L, e_R, into Eqs. 39, 40, 41. This shows that the Z' will couple in a flavour violating way to the three light families, even though they carry no $U(1)'$ charges, because of their mixing with the fourth family which do carry $U(1)'$ charges. The mixing is controlled by three mixing angles θ_{i4} in each of the sectors Q_L, u_R, d_R, L_L, e_R, which involves 15 parameters.

Assuming that only $\theta^{Q_L}_{34}$ and $\theta^{L_L}_{34}$ are non-zero, with all other mixing angles being zero, the mixing matrices in Eq.43 become,

$$\tilde{D}'_Q = q_{Q4} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (s^Q_{34})^2 \end{pmatrix}, \quad \tilde{D}'_L = q_{L4} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (s^L_{34})^2 \end{pmatrix} \tag{44}$$

so that the Z' couplings from Eq.36 become,

$$\mathcal{L}^{gauge}_{Z'} = g' Z'_\lambda \left(q_{Q4}(s^Q_{34})^2 \overline{Q}'_{L3}\gamma^\lambda Q'_{L3} + q_{L4}(s^L_{34})^2 \overline{L}'_{L3}\gamma^\lambda L'_{L3} \right) \tag{45}$$

where the Z' couples to the third family left-handed quark and lepton doublets $Q'_{L3} = (t'_L, b'_L)$ and $L'_{L3} = (\nu'_{\tau L}, \tau'_L)$, where the primes indicate that these are the states before the Yukawa matrices are diagonalised. In particular this will lead the couplings,

$$\begin{aligned} \mathcal{L}^{gauge}_{Z'} &= g' Z'_\lambda \left(q_{Q4}(s^Q_{34})^2 \overline{b}'_L \gamma^\lambda b'_L + q_{L4}(s^L_{34})^2 \overline{\tau}'_L \gamma^\lambda \tau'_L + \dots \right), \\ &\approx g' Z'_\lambda \left(q_{Q4}(s^Q_{34})^2 (V'^\dagger_{dL})_{32} \overline{b}_L \gamma^\lambda s_L + q_{L4}(s^L_{34})^2 |(V'^\dagger_{eL})_{32}|^2 \overline{\mu}_L \gamma^\lambda \mu_L + \dots \right), \end{aligned} \quad (46)$$

where we have used Eq. 24 to expand the primed fields in terms of mass eigenstates,

$$\begin{aligned} b'_L &= (V'^\dagger_{dL})_{31} d_L + (V'^\dagger_{dL})_{32} s_L + (V'^\dagger_{dL})_{33} b_L, \\ \tau'_L &= (V'^\dagger_{eL})_{31} e_L + (V'^\dagger_{eL})_{32} \mu_L + (V'^\dagger_{eL})_{33} \tau_L, \end{aligned} \quad (47)$$

and assumed from the hierarchy of the CKM matrix that,

$$\begin{aligned} |(V'^\dagger_{dL})_{31}|^2 \ll |(V'^\dagger_{dL})_{32}|^2 \ll (V'^\dagger_{dL})^2_{33} &\approx 1, \\ |(V'^\dagger_{eL})_{31}|^2 \ll |(V'^\dagger_{eL})_{32}|^2 \ll (V'^\dagger_{eL})^2_{33} &\approx 1. \end{aligned} \quad (48)$$

From Eq. 46, Z' exchange generates the effective operator, as in Eq. 3,

$$G^{\rm BSM}_{b_L\mu_L}\, \bar{b}_L \gamma^\lambda s_L\, \bar{\mu}_L \gamma_\lambda \mu_L\,, \quad (49)$$

where we identify,

$$G^{\rm BSM}_{b_L\mu_L} = q_{Q4} q_{L4} (s^Q_{34})^2 (s^L_{34})^2 (V'^\dagger_{dL})_{32} |(V'^\dagger_{eL})_{32}|^2 \left(\frac{g'^2}{M'^2_Z} \right). \quad (50)$$

This operator dominates over the analogous operator with μ_L replaced by e_L, according to Eq.48. To explain the R_K and R_{K^*} anomalies we require $G^{\rm BSM}_{b_L\mu_L}$ to have the correct sign and magnitude, as discussed in Eqs. 3, 4. Motivated by the CKM matrix, we may assume $(V'^\dagger_{dL})_{32}$ and $(V'^\dagger_{eL})_{32}$ are both of order $V_{ts} \approx 0.04$. Then Eq. 4 requires,

$$G^{\rm BSM}_{b_L\mu_L} = q_{Q4} q_{L4} (s^Q_{34})^2 (s^L_{34})^2 (6 \times 10^{-5}) \left(\frac{g'^2}{M_{Z'}{}^2} \right) \approx -\frac{1}{(33 \text{ TeV})^2} \quad (51)$$

This suggests that in this model $M_{Z'} \lesssim (\sqrt{10^{-5}}) \times 33$ TeV $\lesssim 100$ GeV. Such a light Z' is not excluded since it does not couple to first generation quarks and leptons, so would not be produced at LEP, and its Drell-Yan production at the LHC would only proceed via $\bar{s}s$ annihilation through a coupling which is amplitude suppressed by $(V'^\dagger_{dL})^2_{32} \sim V^2_{ts} \sim 10^{-3}$.

3.2 $SO(10)$ model

The next example we consider is an $SO(10)$ model which breaks at the GUT scale,

$$SO(10) \to SU(5) \times U(1)_\chi \quad (52)$$

under which the three chiral 16 representations decompose as,

$$\mathbf{16}_i \to (\mathbf{10}, 1)_i + (\overline{\mathbf{5}}, -3)_i + (\mathbf{1}, 5)_i \quad (53)$$

where $U(1)_\chi$ charges should all be multipled by a normalisation factor of $\frac{1}{2\sqrt{10}}$. The $U(1)_\chi$ survives to low energy and is broken at the few TeV scale to provide an observable Z'. We assume that the low energy fourth vector-like family arises from incomplete surviving parts of the decompositions,

$$\begin{aligned} \mathbf{45} &\to (\mathbf{24},0)+(\mathbf{10},-4)+(\overline{\mathbf{10}},4)+(\mathbf{1},0) \\ \mathbf{10} &\to (\mathbf{5},-2)+(\overline{\mathbf{5}},2). \end{aligned} \tag{54}$$

The fourth vector-like family with masses near the few TeV scale consists of the following surviving parts of these multiplets,

$$(\mathbf{10},-4)+(\overline{\mathbf{10}},4)+(\mathbf{5},-2)+(\overline{\mathbf{5}},2), \tag{55}$$

where we assume that the other $(\mathbf{24},0)$ and $(\mathbf{1},0)$ parts get large GUT scale masses. The three chiral families and the fourth vector-like family are odd under a matter parity. The Higgs doublets emerge from a different $\mathbf{10}_H$ with even matter parity, allowing Higgs Yukawa couplings. In addition we will need the Higgs $\mathbf{16}_H$ and $\overline{\mathbf{16}}_H$ with even matter parity to mix the fourth vector-like family with the three chiral families. We do not address any doublet-triplet or other splitting problems here.

Then $SU(5)$ subsequently breaks to the Standard Model gauge group at the GUT scale,

$$\mathbf{10}\to Q,u^c,e^c, \qquad \overline{\mathbf{5}}\to L,d^c \tag{56}$$

We emphasise that the single vector-like family in Eq.55, includes quark and lepton doublets necessary to account for R_K and R_{K^*}. [2] In terms of the fields Q_L,u_R,d_R,L_L,e_R, the charges under $U(1)'=U(1)_\chi$ in Table 1 are therefore given by the diagonal matrices:

$$\begin{aligned} &D_Q=\mathrm{diag}\,(1,1,1,-4)\,,\;D_u=\mathrm{diag}\,(-1,-1,-1,4)\,,\;D_d=\mathrm{diag}\,(3,3,3,-2)\,,\\ &D_L=\mathrm{diag}\,(-3,-3,-3,2)\,,\;D_e=\mathrm{diag}\,(-1,-1,-1,4)\,, \end{aligned} \tag{57}$$

up to a normalisation factor of $\frac{1}{2\sqrt{10}}$ multiplying each matrix.

In addition we assume Higgs singlets whose VEVs yield a massive Z', and whose couplings permit mixing of the fourth vector-like family with the three families of the same chirality. The mixing of quarks and leptons with the fourth vector-like family induces flavour violating Z' couplings to the three light families of quarks and leptons, as in Eq.36, which depend on 3×3 matrices $\tilde{D}'$ from Eq.38 of the form,

$$\begin{aligned} \tilde{D}'_L &= -\frac{3}{2\sqrt{10}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}+\frac{5}{2\sqrt{10}}\begin{pmatrix}(s^L_{14})^2 & s^L_{14}s^L_{24}c^L_{14} & s^L_{14}s^L_{34}c^L_{14}c^L_{24}\\ . & (s^L_{24})^2(c^L_{14})^2 & s^L_{24}s^L_{34}c^L_{24}(c^L_{14})^2\\ . & . & (s^L_{34})^2(c^L_{14})^2(c^L_{24})^2\end{pmatrix}\\ \tilde{D}'_Q &= \frac{1}{2\sqrt{10}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}-\frac{5}{2\sqrt{10}}\begin{pmatrix}(s^Q_{14})^2 & s^Q_{14}s^Q_{24}c^Q_{14} & s^Q_{14}s^Q_{34}c^Q_{14}c^Q_{24}\\ . & (s^Q_{24})^2(c^Q_{14})^2 & s^Q_{24}s^Q_{34}c^Q_{24}(c^Q_{14})^2\\ . & . & (s^Q_{34})^2(c^Q_{14})^2(c^Q_{24})^2\end{pmatrix} \end{aligned} \tag{58}$$

[2] This may be compared the $SO(10)$ model in [14] where there are three low energy $(\mathbf{5},-2)+(\overline{\mathbf{5}},2)$ representations mixing with the three chiral families leading to flavour changing Z' interactions. However such a model is unable to account for R_K and R_{K^*}, in the absence of vector-like quark doublets.

Eq.58 consists of a universal matrix, proportional to the unit matrix, plus a non-universal matrix of the same form as Eq.43, but of opposite sign to that which appeared in the fermiophobic model. Similar matrices may be written down for each of the sectors Q_L, u_R, d_R, L_L, e_R. The couplings of the quark and lepton mass eigenstates to the Z' are given by inserting Eq.58, and similar equations in each of the sectors Q_L, u_R, d_R, L_L, e_R, into Eqs. 39, 40.

Assuming that only $\theta_{34}^{Q_L}$ and $\theta_{14}^{L_L}$ are non-zero, with all other mixing angles being zero, the mixing matrices in Eq.58 simplify,

$$
\begin{aligned}
\tilde{D}'_L &= -\frac{3}{2\sqrt{10}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + \frac{5}{2\sqrt{10}}\begin{pmatrix}(s^L_{14})^2&0&0\\0&0&0\\0&0&0\end{pmatrix}\\
\tilde{D}'_Q &= \frac{1}{2\sqrt{10}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} - \frac{5}{2\sqrt{10}}\begin{pmatrix}0&0&0\\0&0&0\\0&0&(s^Q_{34})^2\end{pmatrix}
\end{aligned}
\tag{59}
$$

The other matrices are universal, since we assume their mixing angles are zero,

$$
\tilde{D}'_e = \tilde{D}'_u = -\frac{1}{2\sqrt{10}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}, \qquad \tilde{D}'_d = \frac{3}{2\sqrt{10}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}
\tag{60}
$$

The universal (unit matrix) parts of Eq.59 and 60 when inserted into Eqs.39, 40, 41, lead to the universal Z' couplings for the quarks,

$$
\begin{aligned}
\mathcal{L}^{q,univ}_{Z'} &= \frac{1}{2\sqrt{10}}g'Z'_\mu \begin{pmatrix}\overline{u}_L & \overline{c}_L & \overline{t}_L\end{pmatrix}\gamma^\mu \begin{pmatrix}u_L\\c_L\\t_L\end{pmatrix}\\
&+ \frac{1}{2\sqrt{10}}g'Z'_\mu \begin{pmatrix}\overline{d}_L & \overline{s}_L & \overline{b}_L\end{pmatrix}\gamma^\mu \begin{pmatrix}d_L\\s_L\\b_L\end{pmatrix}\\
&- \frac{1}{2\sqrt{10}}g'Z'_\mu \begin{pmatrix}\overline{u}_R & \overline{c}_R & \overline{t}_R\end{pmatrix}\gamma^\mu \begin{pmatrix}u_R\\c_R\\t_R\end{pmatrix}\\
&+ \frac{3}{2\sqrt{10}}g'Z'_\mu \begin{pmatrix}\overline{d}_R & \overline{s}_R & \overline{b}_R\end{pmatrix}\gamma^\mu \begin{pmatrix}d_R\\s_R\\b_R\end{pmatrix}
\end{aligned}
\tag{61}
$$

Similarly the charged lepton couplings to Z' will be given by analogous results,

$$
\begin{aligned}
\mathcal{L}^{e,univ}_{Z'} &= -\frac{3}{2\sqrt{10}}g'Z'_\mu \begin{pmatrix}\overline{e}_L & \overline{\mu}_L & \overline{\tau}_L\end{pmatrix}\gamma^\mu \begin{pmatrix}e_L\\\mu_L\\\tau_L\end{pmatrix}\\
&- \frac{1}{2\sqrt{10}}g'Z'_\mu \begin{pmatrix}\overline{e}_R & \overline{\mu}_R & \overline{\tau}_R\end{pmatrix}\gamma^\mu \begin{pmatrix}e_R\\\mu_R\\\tau_R\end{pmatrix}
\end{aligned}
\tag{62}
$$

Finally, ignoring neutrino mass, the Z' couplings to left-handed neutrinos are given by,

$$\mathcal{L}^{\nu}_{Z'} = -\frac{3}{2\sqrt{10}} g' Z'_\mu \begin{pmatrix} \overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}} \end{pmatrix} \gamma^\mu \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} \tag{63}$$

There will be also be additional quark and lepton couplings from the non-universal parts of Eq.59, which, when inserted into Eq.36, leads to,

$$\begin{aligned} \mathcal{L}^{nonuniv}_{Z'} &= \frac{5}{2\sqrt{10}} g' Z'_\lambda \left((s^L_{14})^2 \bar{e}'_L \gamma^\lambda e'_L - (s^Q_{34})^2 \bar{b}'_L \gamma^\lambda b'_L + \ldots \right), \\ &\approx \frac{5}{2\sqrt{10}} g' Z'_\lambda \left((s^L_{14})^2 \bar{e}_L \gamma^\lambda e_L - (s^Q_{34})^2 \bar{b}_L \gamma^\lambda b_L - (s^Q_{34})^2 (V'^\dagger_{dL})_{32} \bar{b}_L \gamma^\lambda s_L + \ldots \right), \end{aligned} \tag{64}$$

where we have used Eq. 24 to expand the primed fields in terms of mass eigenstates,

$$\begin{aligned} b'_L &= (V'^\dagger_{dL})_{31} d_L + (V'^\dagger_{dL})_{32} s_L + (V'^\dagger_{dL})_{33} b_L \\ e'_L &= (V'^\dagger_{eL})_{11} e_L + (V'^\dagger_{eL})_{12} \mu_L + (V'^\dagger_{eL})_{13} \tau_L \end{aligned} \tag{65}$$

and assumed from the hierarchy of the CKM matrix that

$$\begin{aligned} |(V'^\dagger_{dL})_{31}|^2 \ll |(V'^\dagger_{dL})_{32}|^2 \ll (V'^\dagger_{dL})^2_{33} &\approx 1, \\ |(V'^\dagger_{eL})_{13}|^2 \ll |(V'^\dagger_{eL})_{12}|^2 \ll (V'^\dagger_{eL})^2_{11} &\approx 1. \end{aligned} \tag{66}$$

Combining the universal Z' couplings in Eq. 62 with the non-universal couplings in Eq. 64, leads to Z' mediated operators relevant for rare B decays,

$$G^{\rm BSM}_{b_L\mu_L}\ \bar{b}_L \gamma^\lambda s_L \left[\bar{\mu}_L \gamma_\lambda \mu_L + \left(1 - \frac{5}{3}(s^L_{14})^2\right) \bar{e}_L \gamma_\lambda e_L + \frac{1}{3} \bar{\mu}_R \gamma_\lambda \mu_R + \frac{1}{3} \bar{e}_R \gamma_\lambda e_R + \cdots \right] \tag{67}$$

where

$$G^{\rm BSM}_{b_L\mu_L} = \frac{3}{8} (s^Q_{34})^2 (V'^\dagger_{dL})_{32} \left(\frac{g'^2}{M'^{\,2}_Z} \right). \tag{68}$$

If $(s^L_{14})^2 \approx 3/5$ then the $\bar{e}_L e_L$ couplings will be suppressed. Also note that the $\bar{e}_R e_R$ and $\bar{\mu}_R \mu_R$ couplings are 1/3 times those of $\bar{\mu}_L \mu_L$, as predicted by $SO(10)$. Since the $\bar{\mu}_L \mu_L$ term dominates, then the model can explain the R_K and R_{K^*} anomalies, if $G^{\rm BSM}_{b_L\mu_L}$ has the correct sign and magnitude, as in Eqs. 3, 4. Assuming that $g' \approx 0.46$ [15], Eq.68 and Eq.3 then imply,

$$M'_Z \approx (s^Q_{34})\, (V'^\dagger_{dL})^{1/2}_{32}\ (9 \text{ TeV}) \tag{69}$$

Since the Z' in this model has flavour diagonal couplings to muons similar to the usual $U(1)_\chi$ model, the usual LHC limits apply, so we must have $M_{Z'} \gtrsim 3$ TeV [16], which implies $(s^Q_{34})\, (V'^\dagger_{dL})^{1/2}_{32} \gtrsim 1/3$. Actually $(s^Q_{34})\, (V'^\dagger_{dL})^{1/2}_{32} \gtrsim 1/3$ is quite a stringent limit, for example the usual CKM inspired expectation $(V'^\dagger_{dL})^{1/2}_{32} \sim \lambda \sim 0.22$ is already not viable, in agreement with the general results in [17]. However large mixings such as, for example, $(s^Q_{34}) \sim 1/\sqrt{2}$ and $(V'^\dagger_{dL})^{1/2}_{32} \sim 0.5$, would imply $M'_Z \sim 3.2$ TeV, just above the current limit. Note that the couplings of the Z' to electrons will be suppressed in this model relative to muons, which is the main LHC prediction of the model. Therefore the model predicts an imminent LHC discovery of a Z' in the muon channel, with a suppressed coupling in the electron channel. In addition, the model predicts $\bar{\mu}_L e_L$ and $\bar{e}_L \mu_L$ lepton flavour violating final states, with an amplitude suppressed by $(V'^{\dagger}_{eL})_{21}$, which is typically of order of a third of the Cabibbo angle in unified models.

4 Conclusion

In this paper we have shown how any flavour conserving Z' model can be made flavour violating and non-universal by introducing mass mixing of quarks and leptons with a fourth family of vector-like fermions with non-universal Z' couplings. We have developed a general formalism to achieve this for any Z' model, including $B-L$ models, E_6 models, composite models, and so on. All that is required is to specify the charges for the model in Table 1. These charges may be conveniently summarised in terms of the charge matrices defined in Eq.33. Once these charge matrices are written down for a particular model, the Lagrangian is completely specified using the general results given in the paper.

To illustrate the proceedure, we have considered two concrete examples, namely a fermiophobic model, and an $SO(10)$ GUT model, and shown how they can account for the anomalous B decay ratios R_K and R_{K^*}. In both examples, we have simply written down the charge matrices for the models, then applied the general results of the paper to calculate the Feynman rules for the Z' couplings to physical quark and lepton mass eigenstates, and isolated the flavour diagonal and off-diagonal parts resonsible for flavour violation and non-universality. The SM gauge couplings do not violate flavour in this models, since the three chiral quark and lepton families mix with the vector-like family with the same SM quantum numbers, only differing due to the non-universality of the $U(1)'$ charges.

The experimental predictions of such models are very rich and varied, and deserve a dedicated phenomenological study, beyond the scope of the present paper. Generally speaking, the phenomenological predictions may be divided into low energy flavour changing and rare processes, and high energy collider signatures. The low energy flavour changing will encompass lepton flavour violation, including τ decays [18], while the LHC predictions include a Z' as well as a complete fourth vector-like family, with interesting flavour dependent signatures. The Z' may be light and weakly coupled, for example around 100 GeV in the fermiophobic model, or heavier with non-universal couplings to electrons and muons, for example just above the current LHC limit of 3 TeV in the $SO(10)$ model.

In conclusion, we have proposed a new class of flavourful Z' models which may be obtained as a bolt-on or upgrade to any existing anomaly free Z' model, by adding a vector-like fourth family, with non-universal $U(1)'$ charges, together with scalar singlets which allow mass mixing to take place between the three chiral families and the vector-like family. We have shown that the resulting low energy Z' couplings will always violate flavour and may account for the anomalous B decay ratios R_K and R_{K^*}.

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