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Convective Vector Wave Equation of Aeroacoustics

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Abstract

This paper extends the vector wave equation of aeroacoustics to consider the effect of uniform flow. Analytical time-domain and frequency-domain acoustic velocity integral formulations for the monopole source are deduced. Test cases for sound radiated from stationary and rotating sources in uniform flow are carried out to validate the developed acoustic velocity formulations.

Keywords: aeroacoustics; acoustic analogy; wave equation;acoustic velocity; uniform flow

1. Introduction

The scalar wave equation of aeroacoustics, i.e., Ffowcs Williams and Hawkings (FW-H) equation [1], and its integral formulations [2, 3] are usually used to compute the acoustic pressure radiated from flow and its interaction with solid surfaces. Recently, a vector wave equation of aeroacoustics and the corresponding acoustic velocity formulations have been deduced [4-6], which showed advantages in visualizing the acoustic energy flow path from sources [7] and around scattering surfaces [8]. The preceding investigations assumed sound propagation in quiescent medium, however, acoustic radiation and scattering phenomena widely exist in moving medium. For sound radiated from sources in uniform mean flow, the convective FW-H equation and the corresponding acoustic pressure formulations have been deduced [9-12], but acoustic power cannot be computed directly from these formulations because the acoustic velocity is not given. For sound scattered by rigid surfaces in uniform mean flow, the acoustic velocity boundary condition $u_n = 0$ rather than the acoustic pressure gradient boundary condition $\partial p'/\partial n = 0$ should be employed on the rigid scattering surface. Owing to the above requirements, an acoustic velocity formulation taking into account of the convective effect is meaningful to analyze the acoustic power output and the acoustic scattering in uniform flow.

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This paper presents a convective vector wave equation, which is an extension of the convective FW-H equation [9-11] and the vector wave equation of aeroacoustics [3], and the time-domain and frequency-domain acoustic velocity formulations for the monopole source in uniform flow are derived and validated.

2. Convective Vector Wave Equation and Its Integral Solutions

2.1. Convective vector wave equation

It is assumed that a data surface is defined by $f(\mathbf{x},t)=0$, which can be either an impermeable solid surface or a permeable surface inside the flow region. In order to consider the effect of uniform flow on sound generation and propagation, we make the following decomposition of the local flow velocity $\mathbf{u}=\mathbf{U}_\infty+\mathbf{u}'$, where $\mathbf{U}_\infty$ is the velocity of the uniform flow and $\mathbf{u}'$ is the perturbed component of the local flow velocity. Note that, in this situation, $\mathbf{u}'$ is the acoustic velocity outside the nonlinear flow region.

By substituting the above decomposition of the flow velocity into the generalized continuity and momentum equations derived by Ffowcs Williams and Hawkings [1] and performing mathematical manipulations, we can obtain another type of generalized continuity and momentum equations which account for the uniform flow as follows [9-11]:

$$\frac{D[\rho H(f)]}{Dt}+\frac{\partial[\rho u_j' H(f)]}{\partial x_j}=Q^M\delta(f) \tag{1}$$

$$\frac{D[\rho u_i' H(f)]}{Dt}+\frac{\partial[(\rho u_i' u_j' + p\delta_{ij}-\sigma_{ij})H(f)]}{\partial x_j}=L_i^M\delta(f) \tag{2}$$

with

$$\frac{D}{Dt}=\frac{\partial}{\partial t}+U_{\infty i}\frac{\partial}{\partial x_i} \tag{3}$$

$$Q^M=\rho_0(v_n-U_{\infty n})+\rho(u_n'-(v_n-U_{\infty n})) \tag{4}$$

$$L_i^M=[(p-p_0)\delta_{ij}-\sigma_{ij}]n_j+\rho u_i'[u_n'-(v_n-U_{\infty n})] \tag{5}$$

where superscript M denotes source terms in uniform flow. p is the static pressure; ρ is the density; σ_{ij} is the viscous stress tensor; ρ_0, p_0 and c_0 are, respectively, the density, pressure, and sound speed of the unperturbed fluid. u_i is the component of flow velocity in the i^{th} direction; u_n and v_n are the normal components of the flow and data surface velocities, respectively. $U_{\infty i}$ is the component of the uniform flow velocity in the i^{th} direction; $U_{\infty n}=U_{\infty i}n_i$. $\delta(.)$ and $H(.)$ are the Dirac delta and Heaviside functions, respectively; δ_{ij} is the Kronecker delta function. The convective FW-H equation and the corresponding time-domain [9-11] and frequency-domain [12] acoustic pressure formulations for sources in uniform flow can be deduced from Eqs. (1) and (2).

In a quiescent medium, the acoustic velocity formulations can be directly deduced from the acoustic pressure formulations by employing the linearized momentum equation [4, 5]. However, it is difficult to derive the analytical acoustic velocity formulations for sources in uniform flow by following this method because there is a convective term in the following linearized momentum equation:

$$\rho_0\left(\frac{\partial\mathbf{u}'}{\partial t}+\mathbf{U}_\infty\cdot\nabla\mathbf{u}'\right)=-\nabla p' \tag{6}$$

The vector wave equation of aeroacoustics [6], which was recently deduced from the generalized continuity and momentum equations, provides an alternative method to deduce the acoustic velocity formulation.

In this paper, we extend the vector wave equation to consider the effect of uniform flow. The classic and convective FW-H equations are derived by employing the perturbation of the density as the variable of the wave operator. Here, the vector $\rho\mathbf{u}'$ is employed as the variable of the wave operator to deduce the convective vector wave equation. A spatial derivative $\partial/\partial x_i$ of Eq. (1) is performed to obtain the following equation

$$\frac{D}{Dt}\frac{\partial[\rho H(f)]}{\partial x_i}+\frac{\partial^2[\rho u_j' H(f)]}{\partial x_i \partial x_j}=\frac{\partial[Q^M\delta(f)]}{\partial x_i} \tag{7}$$

Performing the material derivative D/Dt of Eq. (2) and multiplying by a constant $1/c_0^2$ yield

$$\frac{1}{c_0^2}\frac{D^2[\rho u_i' H(f)]}{Dt^2}+\frac{1}{c_0^2}\frac{D}{Dt}\frac{\partial[(\rho u_i'u_j'+p\delta_{ij}-\sigma_{ij})H(f)]}{\partial x_j}=\frac{1}{c_0^2}\frac{D[L_i^M\delta(f)]}{Dt} \tag{8}$$

Subtracting Eq. (7) from Eq. (8) gives the following equation

$$\frac{1}{c_0^2}\frac{D^2[\rho u_i' H(f)]}{Dt^2}-\frac{\partial^2[\rho u_j' H(f)]}{\partial x_i \partial x_j}=-\frac{\partial[Q^M\delta(f)]}{\partial x_i}+\frac{1}{c_0^2}\frac{D[L_i^M\delta(f)]}{Dt}-\frac{1}{c_0^2}\frac{D}{Dt}\left[\frac{\partial[T_{ij}^M H(f)]}{\partial x_j}\right] \tag{9}$$

with

$$T_{ij}^M=\rho u_i'u_j'+((p-p_0)-c_0^2(\rho-\rho_0))\delta_{ij}-\sigma_{ij} \tag{10}$$

Eq. (9) can also be equivalently expressed in the following vector form

$$\frac{1}{c_0^2}\frac{D^2[\rho\mathbf{u}'H(f)]}{Dt^2}-\nabla(\nabla\cdot[\rho\mathbf{u}'H(f)])=-\nabla[Q^M\delta(f)]+\frac{1}{c_0^2}\frac{D[\mathbf{L}^M\delta(f)]}{Dt}-\frac{1}{c_0^2}\frac{D[\nabla\cdot(H(f)\mathbf{T}^M)]}{Dt} \tag{11}$$

Note that the left hand side (LHS) of Eq. (11) is not a wave operator because the second term is not the Laplace operator. Applying the following identity to the vector $H(f)\rho\mathbf{u}'$

$$\nabla(\nabla\cdot[H(f)\rho\mathbf{u}'])=\nabla^2(H(f)\rho\mathbf{u}')+\nabla\times(\nabla\times[H(f)\rho\mathbf{u}']) \tag{12}$$

can turn the LHS of Eq. (11) into a convective wave operator with an additional source term $\nabla\times(\nabla\times[H(f)\rho\mathbf{u}'])$ on the right hand side (RHS). An equivalent expression of the generalized momentum equation (2) is as follows

$$\frac{\partial[\rho u_i' H(f)]}{\partial t}+\frac{\partial[\rho u_i' U_{\infty j} H(f)]}{\partial x_j}+\frac{\partial[(\rho u_i'u_j'+p\delta_{ij}-\sigma_{ij})H(f)]}{\partial x_j}=L_i^M\delta(f) \tag{13}$$

We can obtain the following expression from Eq. (13) by employing the identity $\nabla\times(\nabla\phi)=0$ where ϕ is an arbitrary scalar

$$\nabla\times(\nabla\times[H(f)\rho\mathbf{u}'])=\int_0^t(\nabla\times(\nabla\times[\mathbf{L}^M\delta(f)]))dt^*-\int_0^t(\nabla\times(\nabla\times(\nabla\cdot[H(f)\mathbf{X}^M])))dt^* \tag{14}$$

with

$$X_{ij}^M=T_{ij}^M+\rho u_i' U_{\infty j} \tag{15}$$

The expression of X_{ij}^M has a subtle difference from the expression of T_{ij}^M given in Eq. (10), there is an additional source term due to the interaction with mean flow. Substituting Eqs. (13) and (14) into Eq. (11) gives

$$\begin{aligned}\frac{1}{c_0^2}\frac{D^2[H(f)\rho\mathbf{u}']}{Dt^2}-\nabla^2[H(f)\rho\mathbf{u}']=&-\nabla[Q^M\delta(f)]\\&+\frac{1}{c_0^2}\frac{D[\mathbf{L}^M\delta(f)]}{Dt}+\int_0^t(\nabla\times(\nabla\times[\mathbf{L}^M\delta(f)]))dt^*\\&-\frac{1}{c_0^2}\frac{D[\nabla\cdot(H(f)\mathbf{T}^M)]}{Dt}-\int_0^t(\nabla\times(\nabla\times(\nabla\cdot[H(f)\mathbf{X}^M])))dt^*\end{aligned} \tag{16}$$

Eq. (16) is the convective vector wave equation of aeroacoustics, where Q^M, $\mathbf{L}^M$ and $\mathbf{T}^M$ are given earlier in Eqs. (4), (5) and (10), respectively. This wave equation is also an exact rearrangement of the generalized continuity and momentum equations.

The LHS of Eq. (16) is the convective wave operator to describe the effect of uniform flow on sound propagation, and the RHS of Eq. (16) contains the monopole, dipole and quadrupole sources but the expression of the quadrupole source is slightly different from those given in the convective FW-H equation.

Moreover, by employing the assumption of a small perturbation $\rho' << \rho_0$ and ignoring the second-order small quantity $\rho'\mathbf{u}'$, the convective vector wave equation (16) can be written as

$$\begin{aligned}\frac{1}{c_0^2}\frac{D^2[H(f)\rho_0\mathbf{u}']}{Dt^2}-\nabla^2[H(f)\rho_0\mathbf{u}'] = -\nabla[Q^M\delta(f)]+\frac{1}{c_0^2}\frac{D[\mathbf{L}^M\delta(f)]}{Dt}+\int_0^t(\nabla\times(\nabla\times[\mathbf{L}^M\delta(f)]))dt^* \\ -\frac{1}{c_0^2}\frac{D[\nabla\cdot(H(f)\mathbf{T}^M)]}{Dt}-\int_0^t(\nabla\times(\nabla\times(\nabla\cdot[H(f)\mathbf{X}^M])))dt^*\end{aligned} \tag{17}$$

Three-dimensional free space Green's function in time domain for the convective wave equation is expressed as follows [13]

$$g=\frac{\delta(t-\tau-R/c_0)}{4\pi R^*} \tag{18}$$

with

$$R^*=\frac{\sqrt{r^2+\gamma^2(\mathbf{M}_\infty\cdot\mathbf{r})^2}}{\gamma}=r\sqrt{1/\gamma^2+M_{\infty r}^2} \tag{19}$$

$$R=\gamma^2\left(R^*-rM_{\infty r}\right) \tag{20}$$

where $\gamma=\sqrt{1/(1-M_\infty^2)}$, $M_{\infty r}=M_{\infty i}\hat{r}_i$, $\hat{r}_i=r_i/r$, $M_{\infty i}=U_{\infty i}/c_0$; $r=|\mathbf{x}-\mathbf{y}|$ is the geometric distance between the source and the receiver; R is the distance of sound wave actually travelled from the source to the observer, which is different from the geometric distance r due to the effect of uniform flow. This Green's function is only suitable for subsonic uniform flow because the factor γ must be a real number [14]. Using the Fourier transformation, we can obtain the corresponding frequency-domain Green's function as follows:

$$G(\mathbf{x},\mathbf{y},\omega)=\int_{-\infty}^{\infty}g(\mathbf{x},\mathbf{y},t-\tau)\mathrm{e}^{\mathrm{i}\omega(t-\tau)}\mathrm{d}t=\frac{\mathrm{e}^{\mathrm{i}kR}}{4\pi R^*} \tag{21}$$

where k is the wavenumber and ω is the angular frequency of the sound received by the observer.

2.2. Acoustic velocity integral formulations for the monopole source

Starting from Eq. (17) and employing the convective Green's function (18), we can obtain the acoustic velocity formulation for the monopole source in uniform flow as follows:

$$4\pi\rho_0 u'_{Ti}(\mathbf{x},t)=-\frac{\partial}{\partial x_i}\int_{-\infty}^{\infty}\int_{f>0}\frac{Q^M\delta(f)\delta(t-\tau-R/c_0)}{R^*}d\mathbf{y}d\tau \tag{22}$$

where subscript i represents the component in the i^{th} direction. By following the derivation of the acoustic pressure formulation 1 of Farassat [2] and the acoustic velocity formulation V1 of Ghorbaniasl [4], we can obtain the following equation

$$4\pi\rho_0 u'_{Ti}(\mathbf{x},t)=\frac{1}{c_0}\frac{\partial}{\partial t}\int_{-\infty}^{\infty}\int_{f=0}\frac{Q^M\hat{R}_i\delta(t-\tau-R/c_0)}{R^*}dSd\tau+\int_{-\infty}^{\infty}\int_{f=0}\frac{Q^M\hat{R}_i^*\delta(t-\tau-R/c_0)}{R^{*2}}dSd\tau \tag{23}$$

with

$$\hat{R}_i^* = \frac{\partial R^*}{\partial x_i} = \frac{r(\hat{r}_i/\gamma^2 + M_{\infty r} M_{\infty i})}{R^*} \tag{24}$$

$$\hat{R}_i = \frac{\partial R}{\partial x_i} = \gamma^2 (\hat{R}_i^* - M_{\infty i}) \tag{25}$$

Furthermore, we can deduce the following time-domain acoustic velocity formulations by using the feature of Dirac delta function to eliminate the temporal integral:

$$4\pi\rho_0 u'_{Ti}(\mathbf{x},t) = \frac{1}{c_0}\frac{\partial}{\partial t}\int\limits_{f=0}\left[\frac{Q^M \hat{R}_i}{R^*(1-M_R)}\right]_{\text{ret}} dS + \int\limits_{f=0}\left[\frac{Q^M \hat{R}_i^*}{R^{*2}(1-M_R)}\right]_{\text{ret}} dS \tag{26}$$

where $[\cdot]_{\text{ret}}$ means all the quantities are calculated at the retarded time $\tau = t - R/c_0$, $M_R = M_i \hat{R}_i$. Eq. (26) is the extension of the time-domain acoustic velocity formulation V1 of Ghorbansial for the monopole source [4]. Finally, we obtain the following time-domain acoustic velocity formulation by transferring the derivative with respect to the observer time t into that with respect to the source time τ

$$\begin{aligned} 4\pi\rho_0 u'_{Ti}(\mathbf{x},t) = &\int\limits_{f=0}\left[\frac{\dot{Q}\hat{R}_i}{c_0 R^*(1-M_R)^2} + \frac{Q(\hat{R}_i^* - M_i)}{R^{*2}(1-M_R)^2} + \frac{Q\hat{R}_i(R^*\dot{M}_R + c_0(M_{R^*} - M^2))}{c_0 R^{*2}(1-M_R)^3}\right]_{\text{ret}} \mathrm{d}S \\ &- \int\limits_{f=0}\left[\frac{Q\hat{R}_i(M_{R^*}M_R + \gamma^2(M_{\infty M}^2 - M_{R^*}^2))}{R^{*2}(1-M_R)^3} + \frac{Q(M_R\hat{R}_i^* + \gamma^2(M_{\infty M}M_{\infty i} - M_{R^*}\hat{R}_i^*))}{R^{*2}(1-M_R)^2}\right]_{\text{ret}} \mathrm{d}S \end{aligned} \tag{27}$$

where the dot over quantities represents the derivative with respect to the source time. Eq (27) is the extension of the formulation V1A of Ghorbaniasl for the monopole source.

If the monopole source rotates with a constant angular speed or is in other periodic motions, performing the Fourier transform on Eq. (23) and employing the frequency-domain Green function given in Eq. (21), give the following frequency-domain acoustic velocity integral formulation:

$$4\pi\rho_0 \tilde{u}'_{Ti}(\mathbf{x},\omega) = \int\limits_{-\infty}^{\infty}\int\limits_{f=0}\left[-\frac{\mathrm{i}kQ^M\hat{R}_i}{R^*} + \frac{Q^M\hat{R}_i^*}{R^{*2}}\right]\mathrm{e}^{\mathrm{i}kR}\mathrm{e}^{\mathrm{i}\omega\tau}\mathrm{d}S\mathrm{d}\tau \tag{28}$$

where variable with a tilde ~ denotes the frequency-domain complex quantity. Eq. (28) is the extension of the frequency-domain acoustic velocity formulation FV1A of Mao [5].

Especially, when the source is stationary, we can obtain the following simplified time-domain formulation:

$$4\pi\rho_0 u'_{Ti}(\mathbf{x},t) = \frac{1}{c_0}\int\limits_{f=0}\left[\dot{Q}^M\hat{R}_i/R^*\right]_{\text{ret}} dS + \int\limits_{f=0}\left[Q^M\hat{R}_i^*/R^{*2}\right]_{\text{ret}} dS \tag{29}$$

The corresponding frequency-domain formulation can also be deduced as follows:

$$4\pi\rho_0 \tilde{u}'_{Ti}(\mathbf{x},\omega) = -\int\limits_{f=0}\frac{\mathrm{i}k\tilde{Q}^M\hat{R}_i}{R^*}\mathrm{e}^{\mathrm{i}kR}\mathrm{d}S + \int\limits_{f=0}\frac{\tilde{Q}^M\hat{R}_i^*}{R^{*2}}\mathrm{e}^{\mathrm{i}kR}\mathrm{d}S \tag{30}$$

A numerical validation of the developed acoustic velocity formulations will be carried out in Section 3.

3. Numerical Validation and Discussion

3.1. Method of numerical validation

In this section, numerical test cases are performed to validate the developed acoustic velocity formulations accounting for the effect of uniform flow. To the best knowledge of the authors, no benchmarking test case has been published for the acoustic velocity related to the sources in uniform flow. Therefore, we employ the following linearized momentum equation expressed in frequency domain to validate the acoustic velocity formulations

$$\mathrm{i}\omega\tilde{\mathbf{u}}' - \mathbf{U}_\infty \cdot \nabla\tilde{\mathbf{u}}' = \frac{\nabla\tilde{p}'}{\rho_0} \tag{31}$$

The terms on the LHS of Eq. (31) are calculated with the acoustic velocity formulations developed in this paper, while the term on the RHS of the Eq. (31) is calculated with the acoustic pressure formulations proposed in references [9, 10, 12]. A first-order discretization scheme is used to numerically compute the spatial derivatives related to the acoustic pressure and the acoustic velocity in Eq. (31), thus we have

$$\underbrace{i\omega\tilde{u}_i'(\mathbf{x}) - \sum_{j=1}^{3}\frac{U_{\infty j}[\tilde{u}_i'(\mathbf{x}+\Delta l\mathbf{e}_j) - \tilde{u}_i'(\mathbf{x})]}{\Delta l}}_{a_{i,LHS}} = \underbrace{\frac{1}{\rho_0}\frac{\tilde{p}'(\mathbf{x}+\Delta l\mathbf{e}_i) - \tilde{p}'(\mathbf{x})}{\Delta l}}_{a_{i,RHS}} \tag{32}$$

where Δl represents the distance of two stationary observers, and $\Delta l = 10^{-6} m$ is used in this paper.

We make the following definitions in all the numerical test cases presented in this Section. The density and sound speed of the ambient flow are $\rho_0 = 1.2\,\mathrm{kg \cdot m^{-3}}$ and $c_0 = 340\,\mathrm{m \cdot s^{-1}}$. The pulsating frequency of the acoustic source is $f_0 = 100\,\mathrm{Hz}$. The Mach number of uniform flow is selected as $\mathbf{M}_\infty = (0.3, 0.4, 0.5)$ to ensure that the uniform flow is subsonic.

For the sound radiated from a stationary source in uniform flow, the source is located at the coordinate origin. 36 observers are uniformly located on a circle in a plane at z=10m with a radius of 1m. The computational results will output the directivity pattern to validate the developed acoustic velocity formulations. For the sound radiated from a rotating source in uniform flow. The source rotates around the z axis in the plane of z=0 with the radius of rotation of 0.8m, and the frequency of source rotation is 50Hz. Only one observer is located on $(1.0\mathrm{m}, 1.0\mathrm{m}, 10.0\mathrm{m})$, and the computational results will output the spectra to validate the developed acoustic velocity formulations.

3.2. Case 1: stationary monopole point source in stationary medium

The strength of the monopole point source is $\int \tilde{Q} dS = 0.01\,\mathrm{kg \cdot s^{-1}}$. Eq. (30) and the frequency-domain acoustic pressure formulation are used, respectively, to compute the terms on the LHS and RHS of Eq. (32), which are compared in Fig. 1 for their real (Re) parts, imaginary (Im) parts and moduli (Abs) for all components in the three coordinate directions. The computational results obtained from the two methods are consistent with each other, validating the developed acoustic velocity formulation for the stationary monopole source in uniform flow.

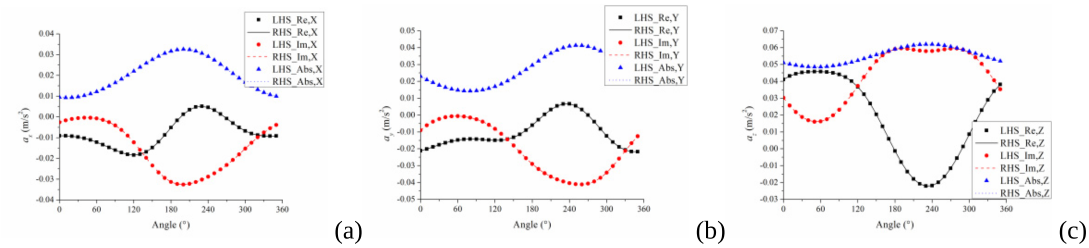

Fig. 1 Validation of acoustic velocity formulation for stationary monopole source via Eq. (32): (a) component in x direction; (b) component in y direction; (c) component in z direction.

3.3. Case 2: rotating monopole point source in stationary medium

The strength of a rotating monopole source is the same as that in the case of the stationary monopole source. Eq. (28) and the frequency-domain acoustic pressure formulation F1A are employed to compute the terms on the LHS and RHS of Eq. (32), respectively.

Fig. 2 displays the spectra obtained from the above-mentioned two methods, and a good agreement between these two results validates the frequency-domain acoustic velocity formulations for the rotating monopole source in uniform flow.

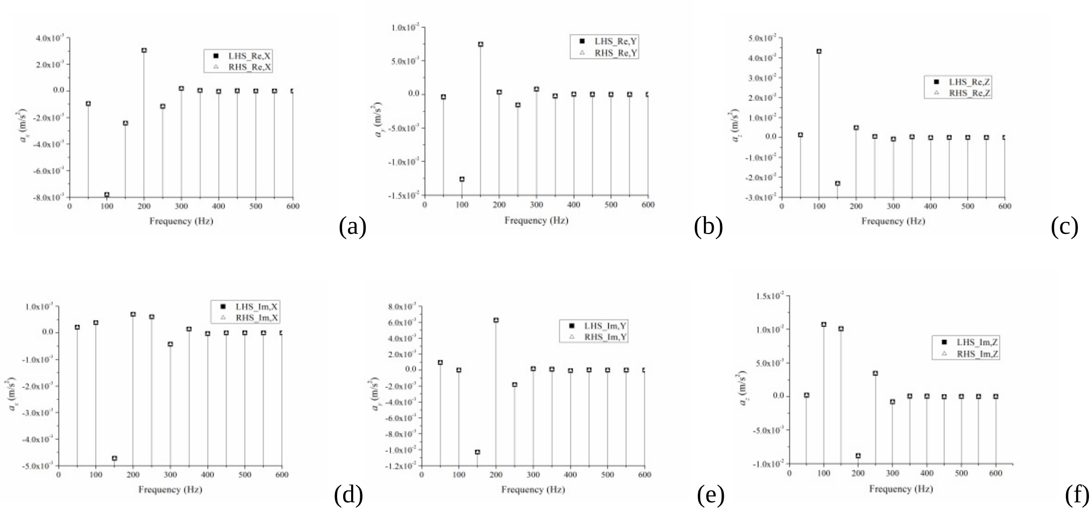

Fig. 2 Validation of acoustic velocity formulation for rotating monopole source via Eq. (32): (a) real part in *x* direction; (b) Real part in *y* direction; (c) real part in *z* direction; (d) imaginary part in *x* direction; (e) imaginary part in *y* direction; (f) imaginary part in *z* direction;

Moreover, we compute the acoustic velocity components with the time-domain and frequency-domain formulations, respectively. In solving the time-domain formulation Eq.(26), the number of samples in one revolution for the rotating source is 360; the source-time dominant algorithm [15, 16] is used to solve the retarded-time equation. The instantaneous acoustic velocity signals received by the stationary observer are linearly interpolated and then transferred into the frequency-domain signals by performing the fast Fourier transform, where the temporal derivative outside the integral is performed in frequency domain after the Fourier transform.

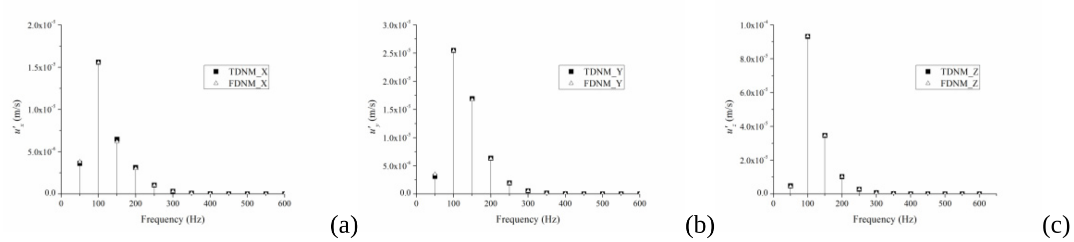

Fig. 3 Acoustic velocity components computed from the time-domain formulation (Eq. (26)) and frequency-domain formulation (Eq. (28)): (a) component in *x* direction; (b) component in *y* direction; (c) component in *z* direction

Fig. 3 compares the components of the acoustic velocity calculated with the time-domain and frequency-domain acoustic velocity formulations. The results obtained from these two methods achieve a good agreement, further validating that the proposed time-domain and frequency-domain acoustic velocity formulations accurately predict the acoustic velocity components of the noise radiated from the rotating monopole source in uniform flow.

4. Conclusion

In this paper, a convective vector wave equation is developed by starting from the generalized continuity and momentum equations. The analytical time-domain and frequency-domain acoustic velocity integral formulations for the monopole source in uniform flow are deduced and validated as well. By employing the developed acoustic velocity formulations, future research plans to analyze the effect of uniform flow on the acoustic power output from sources as well as sound scattered by solid surfaces.

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