OPTIMAL WELFARE PRICE FOR A HIGHWAY COMPETING WITH AN UNTOLLED ALTERNATIVE: THE INFLUENCE OF INCOME DISTRIBUTION

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ABSTRACT

In some countries it is fairly common to see two roads with the same origin and destination competing in the same corridor. One of them is usually a toll highway that offers a better quality to the users compared to its alternative: a free parallel single road. The users thus have to decide whether it is worth paying a toll for the advantages offered. This problem, known as the "untolled alternative", has been largely studied in the academic literature. Particular attention has been paid to calculate the optimal welfare toll that maximizes economic efficiency. However, there is a gap in the academic literature regarding how income distribution affects the optimal toll. The main objective of the paper is to add knowledge in the area by analyzing the influence of the distribution of the values of travel time (VTT) of the users —which is closely related to their income distribution— on the optimal toll price. To solve this problem, we define a mathematical model aimed at obtaining the optimal

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welfare price for this kind of corridor under the hypothesis that drivers assume that average flow conditions will prevail. Under the assumptions made in the paper and for the type of network studied, the results show that the higher the average VTT the higher the optimal price, and the higher the dispersion (variance) of this VTT the lower the optimal price.

Keywords: untolled alternative; optimal price; road; social welfare; income distribution.

INTRODUCTION

The problem of road pricing has been largely studied. It is well acknowledged that in order to achieve the maximum social welfare, users must internalize the externalities they produce and do not perceive. This is usually achieved through a toll or a tax. However, that toll can harm low income users. Depending on the objective function to be maximized (e.g. welfare, social equity, profit, etc.) the optimal toll might vary substantially. The academic literature about pricing and welfare is vast and diverse. However, as far as we are concerned, there is a gap regarding the optimal price in a corridor where a toll highway competes with a free conventional road for different distributions of the values of travel time (VTT). Particularly we did not find any research estimating the optimal welfare price when varying the conditions of the average value of travel time (VTT) and its dispersion. Apart from the introduction, the paper contains five more sections. Firstly, we conduct an extensive literature review on the subject and define the objectives of the paper. Secondly, we set the objective function to be optimized in this research, describe the methodology and explain the assumptions adopted. Thirdly, we define the parameters and variables to apply to a specific case study. Fourthly, we show the results responding to the objectives previously established. Finally, in the last section, we offer a set of conclusions and lessons.

LITERATURE REVIEW

The problem of the untolled alternative has been widely studied. One of the first research works dealing with the topic is carried out by Marchand (1968). He focuses on the case in which in practice there are constraints that prevent getting the first best (e.g. budget constrains). The author obtained a theoretical solution to the problem through a lagrangian approach. The model is static so it does not consider peak hour's demand. Verhoef, Nijkamp and Rietveld (1996) updated and improved the model previously mentioned by defining two functions to optimize: one to maximize welfare, and the other to get the maximum revenue from the toll. For the welfare case they showed that the lower the free-flow cost of using the tolled route compared to the untolled route, the higher is the second-best toll and the greater the efficiency of second-best tolling relative to the *first best*. That is, the greater the travel time difference between both roads, the higher the second-best toll.

Braid (1996) conducted a similar approach focusing on congestion. He assumed that users' costs depend on three parameters: schedule delay cost, the value of time waiting in the queue to access to the highway, and the toll. He proved that when the schedule delay cost is a V-shaped function and both roads have the same capacity, then two out of three users will drive through the toll highway and the gains of welfare will be two thirds of the hypothetical gains with the first best solution. However, the result depends on total trip demand being price inelastic.

Liu and McDonald (1998) studied the problem by adding peak and off-peak hours to a case study consisting of adding two toll lanes to an existing bridge made up of four lanes. They found welfare gains ninety percent greater when the whole bridge was tolled than when only two lanes were tolled.

The same authors (Liu and McDonald, 1999) improved their model by including a relationship between the peak and off-peak hour periods. They compared the behavior of a transportation network in three scenarios: without any toll, completely tolled (*first best*), and tolled only in some stretches (*second best*). They found important differences between the scenario without any toll and the *second best* scenario, as a decrease of total traffic or a switch from the toll road to the free road.

Verhoef (2002) set up a new model by introducing linear functions of cost and demand. This methodology is applicable to any network when searching for both the first and second best solution. A greater welfare was also achieved with the *first best* scheme.

Ferrari (2005) studied welfare variations for three types of users (public transportation users, car users and users who can shift from car to public transportation) after introducing a toll. Although in some scenarios a toll can be profitable for the society, with the combination of both no congestion and many users captive from the public transport, then the toll would imply a loss of welfare.

Due to the impossibility of achieving the *first best* solution, Yang and Zhang (2003) focused on finding the second best solution in two theoretical urban transport networks. The maximum welfare could be achieved through marginal cost pricing. However they found that tolling just 10 links out of 43 links was almost identical in terms of welfare.

Salas, Robusté and Saurí (2009) found the optimal price in the metropolitan area of Barcelona by calculating the social cost for several scenarios of traffic reduction. Verhoef, Koh and Shepherd (2010) also studied generalized costs in congested transport networks, and they proved that under certain assumptions the dimensionality of the problem can be reduced by deriving generalized price functions. In that type of congested urban networks, an increase in price can reduce the congestion (Xu, Ordóñez and Dessouky, 2015).

Regarding how heterogeneity influences the optimal toll, Verhoef and Small (2004) focused on the second best option with heterogeneity between public and private managers of infrastructure. Their research concludes that considering heterogeneity in users, i.e. drivers with different VTT, helps avoiding underestimation of the benefits stemming from second best pricing of one parallel link. Indeed, VTT was found to be a key factor influencing the response to pricing schemes (Yang and Huang, 2004). Consequently heterogeneity must be used to represent with more accuracy the different tradeoffs between time and money (Leurent, 1993).

Heterogeneity may have a great influence on welfare effects (Van den Berg, 2014). Depending on how the welfare is measured and whether toll revenue is included as a benefit the optimal toll would be different (Mayet and Hansen, 2000). Similarly, Yin and Yang (2004) found different optimal tolls when tolling at marginal cost, most equitable or minimal revenue schemes as well as different traffic flow patterns when minimizing total travel cost or total travel time. With discrete user classes and revenue refunding the pricing scheme can be Pareto-improving only if disutility of travel is reduced (Guo and Yang, 2010), although equity concerns about redistribution could turn out in inefficient toll levels (Diaz and Proost, 2014). Congestion pricing may be progressive in its welfare impact (De Palma and Lindsey, 2004), albeit under some circumstances private concessionaires may prefer a toll lower than the welfare optimal level (Guo, Yang and Liu, 2010). Finally, it is noteworthy that the intrinsic difference between VTT distributions may influence the fairness of the system (Xu et al. 2014).

Like the authors of this paper, Du and Wang (2014) have been unable to find many research works related to welfare and heterogeneous users, that is test different VTT distributions and explicitly derive conclusions from it. They considered park and ride facilities in the accesses to a city and introduced travel time reliability and heterogeneous commuters. However, they only tested the model with four different types of users, and did not look into differences for different VTT distributions. Mayet and Hansen (2000) developed the simplest possible analytical model to address the problem of road pricing with continuously distributed values of time, albeit they acknowledged that further work would be required to get a more realistic solution even though it would be harder to solve it analytically. Xiao and Yang (2008) extended that analytical model by involving both toll charges and capacity as variables, finding that when a private company manages the tolled highway, the optimal toll and capacity can hardly be determined.

It is remarkable that there is a gap in the literature regarding the optimal welfare price in a toll highway competing with a free road for different VTT distributions of the potential users. In other words, many studies have been surrounding the question, but none of them have directly solved it through a numerical model. Thus, the objective of the paper is to add knowledge in the topic by defining a model to obtain the optimal toll price in terms of the VTT distribution for an interurban corridor where a toll highway and an untolled conventional road compete. This type of network is common in many interurban corridors, for instance in Spain, where there are two options for travelling in a particular origin–destination pair. A numerical approach to the problem is used here since, as acknowledged in the literature review, due to the high complexity of the problem it would be harder to obtain a more realistic solution through an analytical model, even though to conduct a numerical approach a large number of assumptions must be taken.

ASSUMPTIONS AND METHODOLOGY

In order to pursue the objective of this paper, an optimization problem has been formulated which is aimed at calculating the optimal toll price that maximizes social welfare (i.e. minimize the social cost) in an interurban corridor of the above-mentioned characteristics, for a given number of potential users and different distributions of their VTT. For formulating this problem, some hypotheses were assumed which are described below:

The potential users will be divided into 100 groups. Each group will have a different VTT and a daily expenditure limit for transport related to their income. For this model each potential user is equivalent to one car, so if the group of potential users is made of fifty users that means fifty vehicles.

The potential users are supposed not to know traffic conditions in the corridor so their decisions will be based on expected travel time under average flow conditions. Therefore, they will decide whether they will travel or not, and through which road they will do it, on the basis of the expected travel time, their VTT, the expected fuel and the toll in the highway. For the model to be applicable to commuters in an urban transport network, it would be necessary to change this assumption.

In the selected transport network the total cost for the society is defined as follows:

$$SC = UC + EC + HOB + GB \qquad (1)$$

Where:

SC is the total social cost in €. This is the objective function that has to be minimized and accordingly the welfare will be maximized.

UC is the total cost that the users bear per trip in €. It is divided into four terms which are travel time, toll, fuel cost and maintenance of the vehicle.

EC represents the externalities produced by the vehicles in €. They are the summation of environmental cost – i.e. gas emissions, noise and so on – plus accidents.

HOB is the net operating balance for the road operator either public or private in €. Since equation (1) is represented in terms of social cost (and not social benefit) it consists of the maintenance cost in the toll highway minus the tolls paid in the highway.

GB is the result for the Government responsible for maintaining the conventional road in €. It is calculated as the maintenance cost of the road minus the taxes recovered from fuel.

Congestion costs do not appear explicitly. According to the methodology they automatically appear when traffic increases and the speed in the road decreases.

The potential users of the corridor are divided into groups according to their income and this is depicted by the index $i = 1, \ldots, 100$. The index j represents the different options for the potential users and it can be H (Toll Highway) or R (Conventional Road). The potential users have a daily expenditure limit for transport, which is defined as follows:

If $T^j + \varphi * FC^j > \Theta * I_i \; \forall j$, users of group i do not travel $N_{i,exp} = NPU_i$ and $N_i = 0$; otherwise $N_{i,exp} = 0$ and $N_i = NPU_i$ (2)

Where T^j is the toll to be paid in € in the option j, FC^j is the expected fuel cost in € under average traffic conditions in the option j, Θ is the limit of expenditure in transport (Litman, 2007) in per unit values, I_i is the daily income expressed in € of the group i, NPUi is the number of potential users of group i, N_i is the number of users of group i who decide to travel, and Ni,exp is the number of users of group i which are "pushed out" of the corridor because the cost they have to bear to travel is too high for them. φ is a coefficient that expresses the user's perception with respect to the cost of gasoline (Huang and Burris, 2013). All groups of users decide first whether they travel or not, and if they do it, then they decide through which road they will do it according to the cost they expect to bear in each alternative (3):

$$DC_i^j = T^j + \varphi * FC^j + VTT_i * ETT^j \quad (3)$$

Where DC_i^j is cost estimated by the group i for the option j expressed in €. VTT_i is the VTT in €/hour for the group of users i that is strongly related to their income, I_i (Ortúzar and

Willumsen, 2011). In order to simplify the model we assume that $VTT_i * 8 = I_i$, i.e. the daily income is equivalent to eight working hours which is the usual working day. Indeed, Verhoef and Small (2004) allow the demand curves of traveler types to vary while holding their VTTs fixed.

Finally, ETT^j is the expected travel time under average traffic conditions in the road j. DC_i^j is calculated for each group of potential users who have decided to travel; each of them will travel through the alternative (toll or untolled) with the lowest DC. Drivers assume average flow conditions will prevail while the road might be actually congested or with better flow conditions than expected. Annex 1 provides detailed information on how each cost has been calculated.

It is noteworthy that the problem of finding the toll that minimizes SC is nonlinear and nonconvex (Engelson and Lindberg, 2006; Yang and Huang, 2005), and therefore the use of commercial solvers based on mixed integer nonlinear programming does not guarantee that a global optimum can be obtained. Indeed, for the case study presented in the following section, it was found that the optimal solution obtained with BONMIN and COUENE algorithms, both available in GAMS and developed by IBM in collaboration with Carnegie Mellon University (see http://www.coin-or.org/index.html for more information), varies significantly depending on the initial toll from which these algorithms begin the search of the optimal solution. For this reason, and considering the moderate size of the problem under study, a resolution by simulating several scenarios is provided here since is a practical and rigorous option. Therefore, in order to analyze the influence of the average VTT and the dispersion of the VTT on the optimal toll that maximizes the social welfare, the above-described problem has been solved for different values of potential users and distributions of

VTT through a realistic case study, whose characteristics are described in the following section.

VARIABLES FOR THE CASE STUDY

In this section we define the characteristics of the variables of the case study where we apply our methodology. We distinguish three different types of variables: first, a set of input variables —VTT distribution, toll, and total number of potential users (TNPU)— that we study in greater detail; second, some parameters that we assume constant to facilitate the analysis —e.g. the slope of the roads, fuel taxes, etc.—; and third, a set of output variables —SC, number of users who do not travel and traffic distribution in the corridor— that result from the model. The toll is considered to be an input because it influences the output variables, that is, SC or traffic distribution will depend on the toll and not the other way around. The results are obtained for 25 Lognormal income distributions. Although we have focused our analysis on studying the effects of the VTT distribution and the TNPU on the optimal toll price, the methodology presented in the paper can be also used to study other variables. In Table 1 below, we show the ranges selected for the input variables, and the parameters that remain constant in the analysis:

TABLE 1. Parameters and variables selected for the case study

(Table 1. App. Here)

It is important to note that the total capacity of the corridor is 6,500 vehicles per hour. The conventional road does have the 26.15% of this total capacity. The ranges for the three variables, VTT_i, T^H and *TNPU* have been selected following the recommendations by several researches. Each *VTT* distribution is characterized by its average (μ), variance (σ2) and shape. The average was chosen as follows. For the case of long distance travelers in the California

State Route 91, values of time between 7.09$/hour and 29.42$/hour were found to be plausible (Small, Winston and Yan, 2005) and the high variability in the results was explained by the kind of survey conducted, i.e. Stated Preferences or Revealed Preferences. The values in the peak hour ranged from 20$/hour to 40$/hour for the case of the Insterstate 15 in California (Brownstone and Small, 2005). As in the previous research a high variability was found. For the case of Cost Benefit Analysis in Spain, a range from 9.18€/hour to 22.34€/hour is recommended (De Rus et al., 2010). Secondly, regarding the variance of the distribution, in the case of the Spanish regions this can vary up and down to 30% from its average (Calvo, Cortiñas and Sánchez, 2012). Furthermore, the standard economic measure of income inequality, based on Lorenz Curve (also known as GINI index) can range from around 0.20 to almost 0.70 across different countries (Ortiz and Cummins, 2012). Thirdly, the distribution's shape must be a Lognormal (Fosgerau, 2006), and the variances of this lognormal might range from 10 to 1000 (Nie and Liu, 2010). Finally, in an update of the valuations conducted by Abrantes and Wardman (2011) and by Shires and De Jong (2009) similar ranges, values and shape are recommended. According to the socioeconomic characteristics of Spain, its VTT distribution would correspond to a low average and high variance VTT distribution. Therefore, the 25 lognormal VTT distributions are considered as realistic for our case study.

RESULTS

This section is split into five subsections. The first one explains how the model calculates the optimal toll, and shows the evolution of social cost for different tolls and traffic levels. The second one analyzes the effect of the average VTT on the optimal toll. The third one studies the effect of the variance of VTT on the optimal price. The fourth analyses the combination of the three input variables of the model: average VTT (μ), variance of VTT (σ2) and TNPU.

Finally, the fifth subsection tests the model with a VTT distribution resembling the income characteristics of Spain.

Evolution of social cost and traffic

Before analysing figure 1, it is worth saying that three triangular and three rectangular distributions were also tested in the model with a similar performance (see Ortega, 2014). This is based on the demand and assignment for each group of users which are done in an "all or nothing" based on travel budget (equation 2) or on average flow costs (equation 3) respectively. Thus in order to simulate a realistic scenario we will just focus on lognormal distributions.

Figure 1 shows the evolution of SC as a function of T^H, for a particular VTT distribution. The lognormal VTT distribution chosen for this subsection has an average μ = 19 €/hour and a variance σ2 = 249.28 in the middle of the range previously selected. This distribution does not have any users pushed out.

FIGURE 1. Evolution of social cost (SC) for different toll prices (TH) and levels of potential users (TNPU)

(Figure 1. App. Here)

Figure 1 shows that the larger the potential traffic the greater the social cost and also the optimal toll. This figure also demonstrates that for higher levels of potential traffic if the toll is far from the optimum, the social cost will increase a lot. The shape of the different lines is explained by the share of the traffic between each road of the corridor. The straight lines on the left mean all users are travelling through the toll highway whereas the huge increase of SC on the right is due to a higher share of the road. In other words, at the left of the figure the toll is so low that travel cost through the road is more expensive than travel cost through the tolled highway, therefore all users choose the tolled highway. At the right of the figure, the effect is the opposite and the steps that can be seen on the right represent users changing from the tolled highway to the conventional road. In fact the continuous distribution of VTT has been implemented as 100 discrete values. Depending on the traffic level the minimum SC is always achieved with a toll between €3.7 and €4.7. The toll increases only modestly because in this type of network traffic demand is inelastic, so the maximum welfare can be achieved as an assignment problem between two roads. If the demand was elastic, some trips would be suppressed when the price rises, congestion would rise less quickly and tolls might rise more slowly too. However, the prices would be higher than with inelastic demand, because not all the potential users could internalize the externalities and therefore they would not travel. In other words, with inelastic demand users who cannot afford to pay the toll in the highway will always travel because there is available an untolled alternative.

In Figure 2 we can see how the larger the potential traffic the greater the road share stemming from the application of the optimal toll. For instance, the optimal toll for 3,000 potential traffic in the corridor is €3.7 and the share absorbed by the road is 17% of the total traffic. However, if the potential traffic is 7,000 then the optimal toll will be €4.7 and the traffic share of the road will rise to 35%. This result makes sense, given the fact that the more users in the corridor the greater the traffic and the congestion and therefore a higher cost of fuel, travel time, road maintenance and externalities to be internalized.

FIGURE 2. Road share and optimal toll for different levels of potential users, TNPU

(Figure 2. App. Here)

Effect of average VTT

In this section we analyze the effect of the average VTT on the optimal toll. Figure 3 shows the optimal toll corresponding to five different VTT distributions —all with the same variance (σ2=249.28) but with different averages μ— as a function of the potential users.

FIGURE 3. Optimal toll for different levels of potential users (TNPU) and different average VTT (μ)

(Figure 3. App. Here)

Before getting into Figure 3, it is necessary to clarify that for the distributions with μ=13€/hour and μ=16€/hour there is, respectively, a percentage of 32% and 8% of potential users who decide not to travel because this is too costly for them. However, the other three distributions —corresponding to wealthier people— do not have any group of potential users pushed out. From the preceding figure some conclusions can be drawn. First, the larger the potential traffic the higher the optimal toll. This conclusion is in line with previous findings identified in the literature review as well as with the previous subsection. Basically it proves the fact that users must internalize the externalities they produce and do not perceive, so this is done through a toll. If they cannot afford this toll, then they will have to shift to the conventional road and spend more time in their trip. Second, it is noteworthy that *ceteris paribus*, the higher the value of μ the higher the optimal toll. However, this increase in the toll is not proportional to μ, since the optimal toll rises at a lower rate than μ. In other words *ceteris paribus* the ratio μ/optimum toll is not constant, albeit for VTT distributions without users pushed out, an increase in μ of €1 always implies an increase of around €0.4 in the optimal toll. This rule does not apply to VTT distributions with users pushed out because the traffic volumes grow less than the regular steps of 500 vehicles. Third, when the traffic is near to the capacity of the corridor, the share of the road is also close to its relative capacity in the corridor although slightly above it. Later figure 6 will confirm this conclusion. And fourth, there is a discontinuity in the model for low level of potential users and low values of μ, where the optimal toll is zero. This result is related to the total number of potential users in the corridor. When the users of the corridor are below a certain threshold, the congestion is very low (i.e. level of service A or B) and the externalities of using the highway are lower than using the road, so it is better not to set any toll and all users will drive through the highway. In other words, the congestion is low enough that the marginal social cost of using the highway is lower than the marginal social cost of using the road. Optimal road usage is then zero, and with inelastic demand there is no point levying a toll because it will not affect any user's travel decisions. This threshold depends on both the number of potential users and the percentage of users who decide not to travel.

Effect of the VTT variance

In this subsection we examine the effect of the VTT variance on the optimal tolls. For this purpose, in Figure 4 we present the optimal toll corresponding to five different VTT distributions all with the same average (μ=19 €/hour), but with a different variance (see Table 1). For each VTT distribution, Figure 4 shows the evolution of the optimal toll as a function of the traffic level.

FIGURE 4. Optimal toll for different levels of potential users (TNPU) and different variances of VTT (σ2)

(Figure 4. App. Here)

Figure 4 proves that the higher the VTT variance (or dispersion), the lower the optimal toll. In other words, for the same average VTT (μ) and potential traffic (TNPU), the larger the Gini index the lower the optimal toll. This means that *ceteris paribus* tolls in regions with a widespread income distribution should be lower than in regions with a concentrated income distribution. With a higher variance there are more "wealthy" users in the distribution, but there are also more "poor" users, and as a result the toll must be lower because with a lower toll more people will travel on the road so the traffic share between the roads would reach a better proportion. This result can be generalized with continuous demand and when the capacity of the road is clearly lower than the capacity of the tolled highway, which is the case in many developed countries.

If the capacity of the tolled highway was lower than the capacity of the road, then the optimal toll might be higher since a lower toll could trigger more congestion on the tolled highway. In other words, the toll acts as thermostat that regulates the traffic share between both roads and in this relationship the relative capacity between both roads is quite important. As in the previous figures, the greater the potential traffic the higher the toll, which is again explained by the externalities. Finally, it is remarkable that along with the increase in traffic there is also an increase in the share of the conventional road. There are two potential explanations. First, as congestion becomes serious, it would be efficient to make the highway less congested than the road. By doing so, the cost of travel time of rich users (going through the highway) can be suppressed while the poor people, whose cost of travel time is low, can make a trip through the road.. Second, as congestion rises, the optimal highway toll increases, and this encourages travelers to use the toll-free road rather than the highway. What is somewhat surprising here is that the traffic share of the road is increased until it reaches a traffic level slightly above its relative capacity in the corridor. As in the previous results, this will be confirmed later in figure 6.

Combined effect of μ and σ2

In the previous two subsections, we have analyzed the effect of the average VTT (μ) and the variance of (σ2) separately. However the combined effect of both variables has not been considered yet. In order to study this effect, in Figure 5 we present the optimal toll corresponding to all VTT distributions (see Table 1). The Figure contains four graphs. In each graph, the y-axis indicates the optimal toll in euros and the x-axis represents different average VTT (μ). Each graph corresponds to a different potential traffic level.

FIGURE 5. Optimal toll for different levels of potential users (TNPU) and different VTT distributions (μ and σ2)

(Figure 5. App. Here)

The trends previously observed are confirmed in Figure 5. First, the greater the potential traffic the higher the toll. Second, the higher the average VTT (μ) the higher the optimal toll. Third, the higher the VTT dispersion (σ2) the lower the optimal toll.

The discontinuity observed in Figure 3 and in the upper left graph of Figure 5 can be better understood below in Figure 6, where the road share corresponding to the optimal toll is depicted for different VTT distributions. The horizontal black line in Figure 6 shows the relative capacity of the road in the corridor. This figure shows that the larger the potential traffic the greater the share of the conventional road. In fact, when the potential traffic reaches the capacity of the corridor, the traffic share in the conventional road is, in most cases, around 30/32%. Nevertheless, for distributions with low μ the road share lies between 24% and 27%. This is likely caused by the high percentage of potential users who do not travel when the average VTT of the distribution is low.

As can be observed in Figures 5 and 6, for low average VTT and low potential traffic levels, both the optimal toll and the road share are null. This result seems to be reasonable because with low traffic levels there are almost free flow conditions and there is no need to internalize the externalities through a toll. Albeit a very small externality would still exist the users are already paying gasoline taxes and as we previously said, in this type of network where there is an untolled alternative the traffic is the same for each TNPU (i.e. constant percentage of users who do not travel) so a toll in the highway would unnecessarily trigger some congestion in the conventional road. Finally the higher share of the road above its capacity is due to the fact that, according to Table 1, a higher β has been imposed on the toll highway.

FIGURE 6. Road share for different VTT distributions and different levels of potential users, TNPU

(Figure 6. App. Here)

These findings can be applied to traffic management. Under free flow congestion in the corridor the toll must be null; if there are less than 2,700 users the toll must be null. From that point, the toll must be increased and also the share of the road until the traffic in the road is slightly higher than the capacity of the road and the congestion in the corridor is unavoidable. In other words, when the level of service in the toll highway is B, it is necessary to set some toll, and as a consequence the traffic is shared between both roads. In societies with low μ and high $\sigma 2$, this limit is close to the level of service C. In the rest of societies, the limit is in the level of service B.

The Spanish Case

Finally, we have calculated the optimal toll for different levels of potential users, with a VTT distribution resembling the socioeconomic characteristics of Spain and considering average traffic flow (level of service B) in Spain. With these results, we intend to provide insight on the differences between the optimal toll according to our methodology and the current toll levels in a particular place. The income distribution has been obtained from the National Bureau of Statistics of Spain. The shape of the distribution is very similar to a lognormal. The main results of this analysis are summarized in Table 2.

TABLE 2. Optimal toll and road share for different levels of potential users (TNPU) considering the income distribution of Spain

(Table 2. App. Here)

The results for the case of Spain are similar to the ones obtained for low average (μ) and high variance ($\sigma2$) distributions. With free flow traffic conditions, the optimal tolls will be zero. By contrast, insofar as the potential traffic increases, the optimal toll will increase to efficiently distribute traffic to each road. When the potential traffic reaches the capacity of the corridor, the traffic share in the road will be around 26%. As previously explained, this is due to the high percentage of users who decide not to travel (27%). Finally, comparing the optimal tolls obtained through our methodology with the current tolls in Spain's highways with similar characteristics, we note that current tolls are around three times higher than the optimal ones. The main goal of the toll highway concessions in Spain has been to develop new infrastructure through different Public Private Partnership's schemes (Ortega, Baeza and Vassallo, 2015) and not to set a toll that maximizes welfare.

CONCLUSIONS AND POLICY LESSONS

This paper presents a model to obtain the toll that maximizes the social welfare in a corridor where a toll highway competes with a parallel conventional road. The model is devised for an interurban corridor with non-recurrent users who take their decisions assuming average traffic conditions. The results of the paper shed additional light on the topic. The most important contribution is that optimal tolls are not the same for different VTT distributions and consequently for users with different income distributions. Indeed, the results demonstrate that changes in VTT distribution affects toll in a non-linear manner. Under the assumptions made in the paper and for the type of corridor studied where the tolled highway has more capacity than the conventional road and there are no other transport modes available we obtained the following conclusions: 1) the higher the VTT average the higher the optimal price; 2) the higher the VTT dispersion the lower the optimal price; and 3) the larger the number of potential users the higher the optimal toll. In other words, the optimal toll is an increasing function of the VTT average and a decreasing function of the VTT dispersion. Consequently, tolls should be higher in regions with lower Gini indexes than in regions with higher Gini indexes.

This last result constitutes the main policy lesson added on the field. For instance, a country such as Brazil —with very unequally distributed income per capita— should have ceteris paribus optimal tolls lower than a country like Sweden —with high equally-distributed income per capita. The methodology presented in the paper can be easily applied to any other corridor of similar characteristics located in any other region where the same assumptions might apply.

The model shows that the optimal toll always triggers a traffic share according to the capacities of both roads. Under free flow conditions the optimal toll is the one that makes users travel only through the highway. As a general rule the greater the potential traffic the higher the traffic share in the conventional road. When the capacity of the corridor is reached, this share is slightly above its relative capacity. The results from the Spanish case suggest that current tolls in some developed regions are much higher than the optimal ones. This is particularly important for developed regions trying to impose a new tax to the vehicles since infrastructure manager could have the temptation of levying high tolls but this could trigger congestion and harm the economy.

This research may be extended with the use of a logit model for the users' decision making. Moreover, it would be interesting to design a new model for recurrent users (commuters) who perfectly know the traffic in the corridor, and consequently decide on the basis of both real travel time and monetary cost. It would also be necessary to adapt the criteria over which the users decide to travel and include in it a travel time budget as well.

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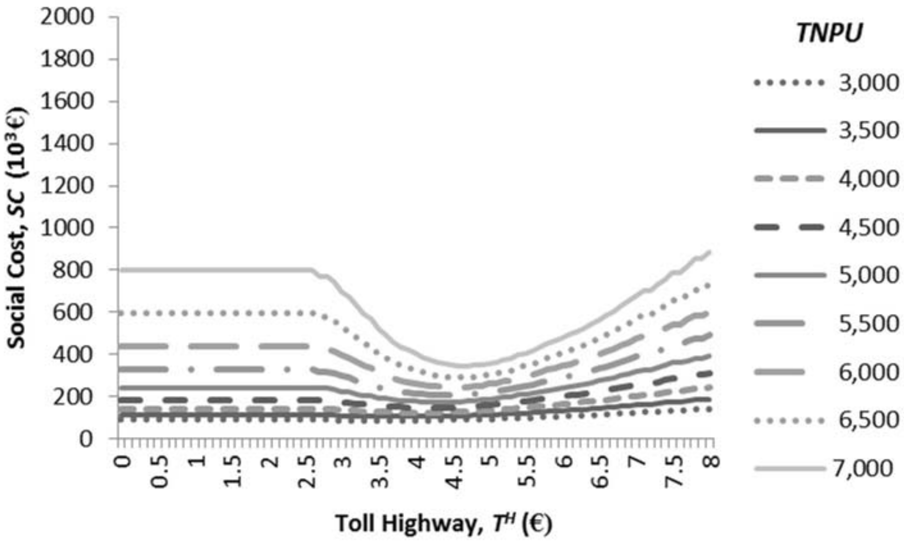
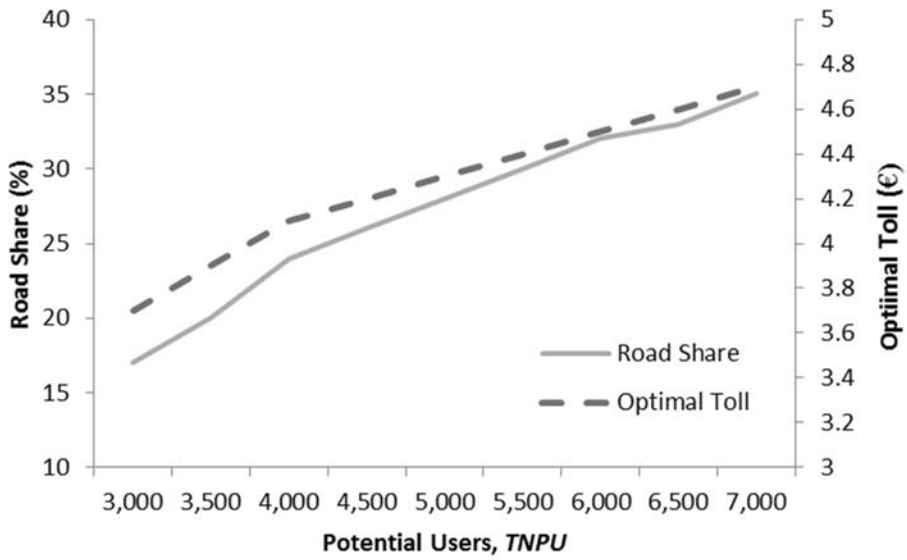
Table 1. Parameters and variables selected for the case study

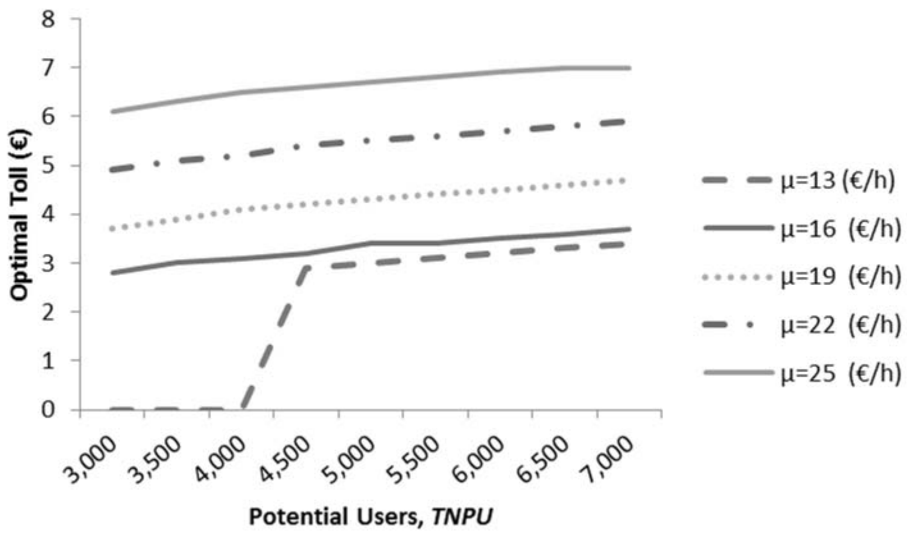
Variable	***Definition***	***Value***	***Source***
VTT_i	Distributions of the value of travel time	25 lognormal distributions with 5 different averages (μ= 13, 16, 19, 22 and 25€/hour) and 5 different variances (σ^2= 84.44, 171.28, 249.28, 324.8 and 391.95)	Small, Winston and Yan, (2005); Calvo, Cortiñas and Sánchez, (2012) ; De Rus et al. (2010); Fosgerau (2006); Brownstone and Small (2005); Abrantes and Wardman (2011); Shires and De Jong, (2009); Ortiz and Cummins (2012); Nie and Liu (2010)
T^H	Highway Toll	0 – 8.5 €	Ortega (2014)
$TNPU$	Potential Users in the corridor	3,000 to 7,000 vehicles	Kraemer et al. (2004)
Parameter	***Definition***	***Value***	***Source***
$DIST^H$	Distance in the highway	90 km	Vassallo, Ortega, and Baeza, (2012)
$DIST^R$	Distance in the road	100 km	Vassallo, Ortega, and Baeza, (2012)
ETT^H	Expected Travel Time in the highway	Level of Service C (51 minutes)	Own calculation
ETT^R	Expected Travel Time in the road	Level of Service C (75 minutes)	Own calculation
CAP^H	Highway capacity	2,400 users/lane/hour. 2 lanes	Kraemer et al. (2004)
CAP^R	Road capacity	1,700 users/lane/hour. 1 lane	Kraemer et al. (2004)
$ExternalityCost$	Kilometer cost of the externality due to the gasoline consumption, noise and so on	0.05€/veh – km for a fuel consumption of 11.4 liters/100Km	Vassallo, Lopez, and Perez-Martinez (2012)
$AccidentCost$	Cost of accidents	0.03 €/veh – km	Vassallo, Lopez, and Perez-Martinez (2012)
σ	Penalization of the exceeding travel time above free flow	0.3	Van den Berg and Verhoef (2011 a) and b)); Wardman and Ibañez (2012); Li, Hensher and Rose (2010); Carrion and Levinson (2012)
δ	Percentage of taxes paid due to fuel consumption	45 %	Ortega, Gómez and Vassallo (2012)
φ	User's perception with respect to the cost of gasoline	0.9	Matas, Raymond and Ruiz (2012); Huang and Burris (2013)
Θ	Limit of expenditure in transport	0.2	Litman (2007)
FP	Fuel price	1.5 €/liter	Ortega, Gómez and Vassallo (2012)

Table 2. Optimal toll and road share for different levels of potential users (*TNPU*) considering the income distribution of Spain.

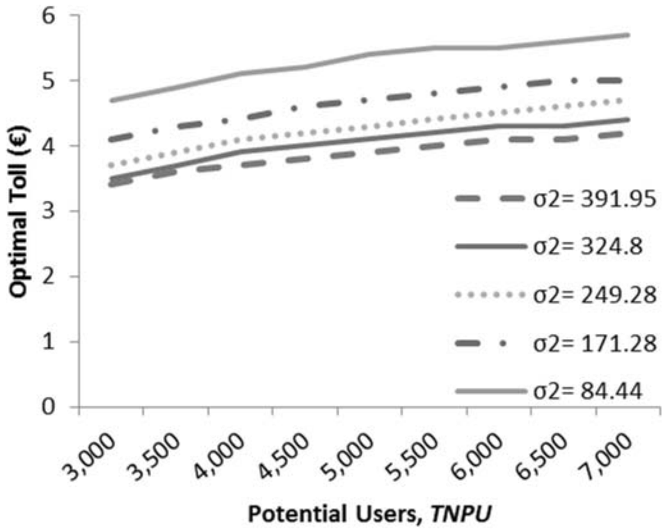
Kind of distribution	***Potential Traffic (Number of potential users)***	***Optimal price in the toll highway (€)/ (€/km)***	***Share of traffic in the conventional road (%)***
Proxylognormal (μ=13.09€/h; σ^2=469.92; GI=51.68; Exp=27%)	3,000	0	0
	4,000	2.5/(0.027)	17.80
	5,000	2.7/(0.03)	21.91
	6,000	2.9/(0.032)	26.02
	7,000	3.1/(0.034)	30.13

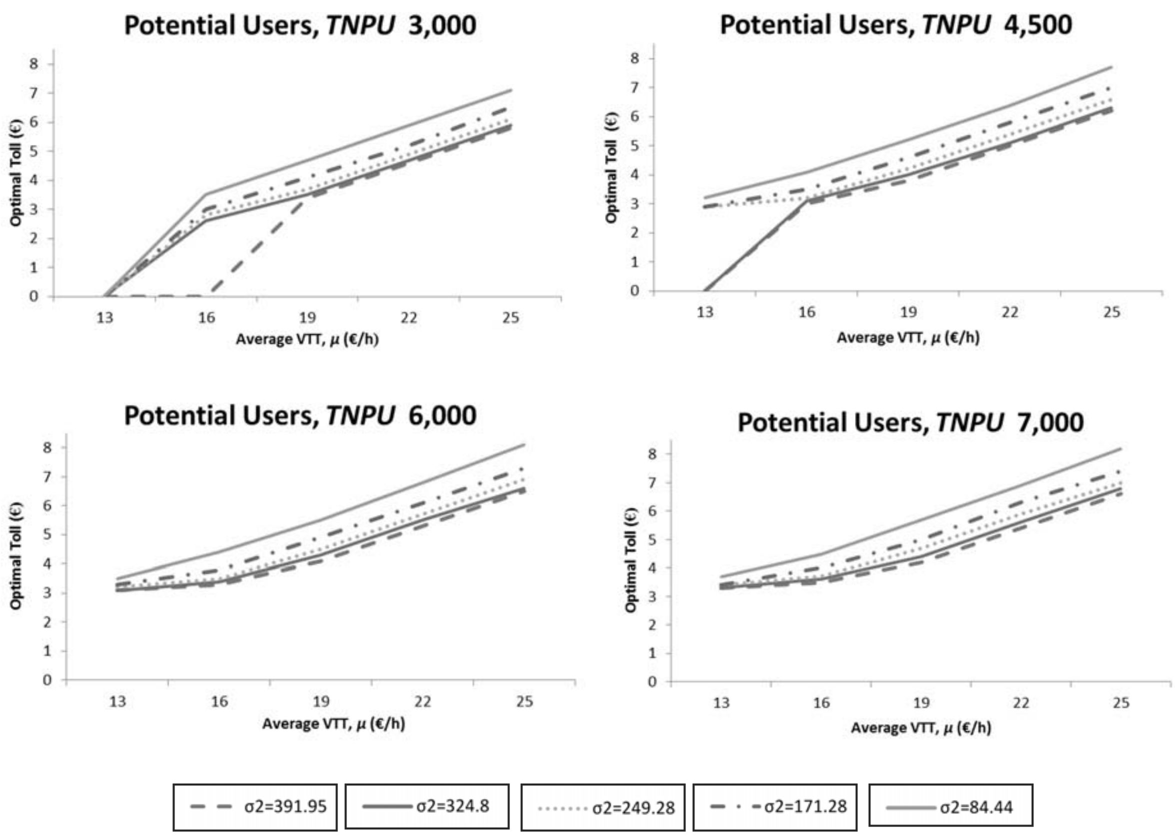

2000
1800
1600
1400
1200
1000
800
600
400
200
0
Social Cost, SC (10³€)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
Toll Highway, T^H (€)
TNPU
3,000
3,500
4,000
4,500
5,000
5,500
6,000
6,500
7,000


40
35
30
25
20
15
10
Road Share (%)
5
4.8
4.6
4.4
4.2
4
3.8
3.6
3.4
3.2
3
Optiimal Toll (€)
Road Share
Optimal Toll
3,000 3,500 4,000 4,500 5,000 5,500 6,000 6,500 7,000
Potential Users, TNPU

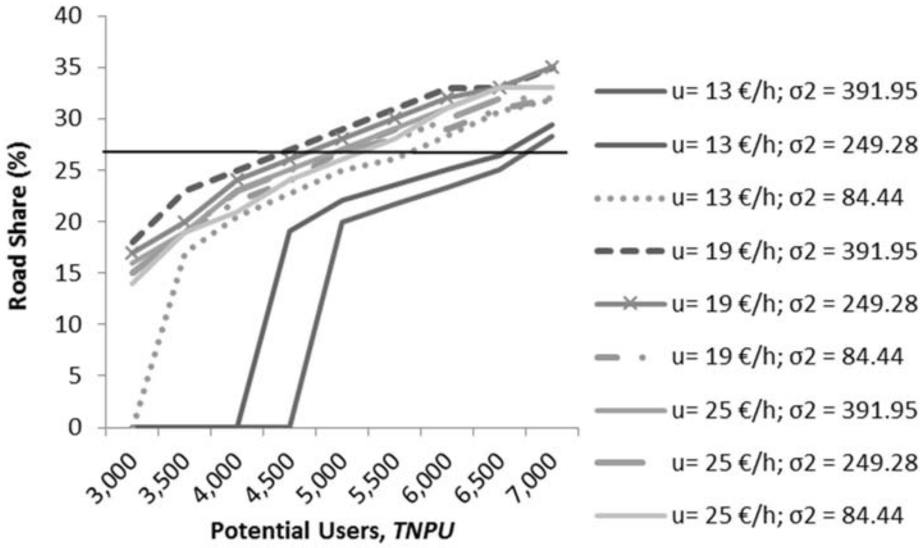


Optimal Toll (€)
8
7
6
5
4
3
2
1
0
3,000
3,500
4,000
4,500
5,000
5,500
6,000
6,500
7,000
Potential Users, TNPU
μ=13 (€/h)
μ=16 (€/h)
μ=19 (€/h)
μ=22 (€/h)
μ=25 (€/h)


6
5
4
3
2
1
0
Optimal Toll (€)
σ2= 391.95
σ2= 324.8
σ2= 249.28
σ2= 171.28
σ2= 84.44
3,000
3,500
4,000
4,500
5,000
5,500
6,000
6,500
7,000
Potential Users, TNPU



Potential Users, TNPU 3,000
Potential Users, TNPU 4,500
Potential Users, TNPU 6,000
Potential Users, TNPU 7,000
Optimal Toll (€)
Average VTT, μ (€/h)
σ2=391.95
σ2=324.8
σ2=249.28
σ2=171.28
σ2=84.44


40
35
30
25
20
15
10
5
0
Road Share (%)
3,000
3,500
4,000
4,500
5,000
5,500
6,000
6,500
7,000
Potential Users, *TNPU*
u= 13 €/h; σ2 = 391.95
u= 13 €/h; σ2 = 249.28
u= 13 €/h; σ2 = 84.44
u= 19 €/h; σ2 = 391.95
u= 19 €/h; σ2 = 249.28
u= 19 €/h; σ2 = 84.44
u= 25 €/h; σ2 = 391.95
u= 25 €/h; σ2 = 249.28
u= 25 €/h; σ2 = 84.44

LIST OF FIGURE CAPTIONS

FIGURE 1. Evolution of social cost (SC) for different toll prices (TH) and levels of potential users (TNPU)

FIGURE 2. Road share and optimal toll for different levels of potential users, TNPU

FIGURE 3. Optimal toll for different levels of potential users (TNPU) and different average VTT (μ)

FIGURE 4. Optimal toll for different levels of potential users (TNPU) and different variances of VTT ($\sigma2$)

FIGURE 5. Optimal toll for different levels of potential users (TNPU) and different VTT distributions (μ and $\sigma2$)

FIGURE 6. Road share for different VTT distributions and different levels of potential users, TNPU

Supplemental Data File

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