ICT and Mathematics: a guide to learning and teaching mathematics 11-19

The original guide was produced for the Teacher Training Agency by The Mathematical Association in July 2002. This updated version has been produced as part of the DfES `*KS3 ICT Offer to Schools'* in September 2004.

The guide has been written to support the continued professional development of serving mathematics teachers, and the preparation of teachers in training, by the following group:

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Context:
The guide has been designed to be of particular relevance and usefulness to groups such as the following:

- Those involved with organising programmes of continued professional development (CPD) for mathematics teachers in schools and colleges.
- Those responsible for organising and providing `Hands On Support'.
- Those responsible for initial teacher training courses for new mathematics teachers.
- Teachers in schools and colleges in receipt of laptop computers under the DfES Laptops for Teachers scheme, those who have received computers under previous schemes and those who have purchased their own equipment.
- LEA mathematics staff advising schools, colleges and teachers including KS3 strategy consultants, advisory teachers, advisers and inspectors.
- Those planning the deployment of ICT resources in school and college mathematics departments including senior managers, middle managers, heads of departments and mathematics teachers.
- Those schools and colleges with specialist status, such as SS Trust members, which includes targets for mathematics (technology, science, engineering, business & enterprise, mathematics & computing) and those making applications for such status.
- Those charged with developing future policy, materials and support involving ICT use in mathematics 11-19.

Structure:
The guide is written in a direct style addressing you, the reader. We, the authors, have tried to give you enough concrete information, illustrated by examples, to help you extend the use of ICT in supporting learning and teaching. We are conscious that we can only include a limited amount of information in such a guide, so we have tried to provide as many references to further useful sources of information as we can. Similarly we have tried to be as-up-to-date as we can with regard to the resources covered in the guide, but we are aware that this will change over time and that new technologies will be developed which may well impact on mathematics.

The guide is structured in three parts.

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Introduction

The government has made a huge investment in ICT in schools over the past few years and is committed to maintain it. One aspect of this investment was the Continued Professional Development opportunities provided for all teachers in ICT through the training groups approved by the New Opportunities Fund (NOF). The rate at which secondary school subjects, especially mathematics, have taken up the use of ICT to support teaching and learning has not been as rapid as might have been expected. This could be for a number of reasons, and we hope that this guidance document might make a contribution to improving the situation. Two new forms of CPD in the use of ICT are now available. The first, called Enhancing Subject Teaching Using ICT (ESTUICT), is a mixture of on-line resources, face-to-face meetings, and e-mail support. The ESTUICT(CPD) programme for mathematics is provided by the Mathematics Consortium – a partnership between the Mathematical Association, University College Chichester and New Media plc – http://www.cpd4teachers.co.uk . At the time of writing the ESTUICT(CPD) programme is under government review and is likely to be provided in the future in a different form than at present. The Standards Fund contains a section *`31 ICT in Schools Grants'*. This includes an element (not ring-fenced) for Hands On Support (HOS) which provides funding for schools to buy-in external help. http://www.dfes.gov.uk/ictinschools/funding/ .

The most recent Ofsted report *`Mathematics in secondary schools'* (HMI 1978 Feb 2004) reports on schools visited in 2002/3. It can be downloaded from:
http://www.ofsted.gov.uk/publications/index.cfm?fuseaction=pubs.summary&id=3541
It summarises the present situation and does not find the overall picture very bright! One if its main findings is that:

> "*Despite significant government funding, the use of information and communication technology (ICT) to promote progress in mathematics remains a relatively weak aspect of provision. Its use is less effective than in many other subjects and is unsatisfactory in one school in three*."

It continues:

> "*The use of ICT to support teaching and learning within mathematics remains underdeveloped. While there are examples of good practice, there are significant inconsistencies between schools as well as within mathematics departments. A minority of teachers are still not confident in the use of ICT and require further training. In some schools and colleges, access to ICT facilities, including graphing calculators, is too limited and an appropriate range of software has not been made available. In other places, where resources are adequate, they are often not used frequently enough or to promote better teaching and learning*"

So don't be downhearted if your department is not making as full use of ICT as it might – you are far from alone! Once you have read this document we trust that you will be in a much better position to plan a successful strategy for improving the situation for the benefit of your students and your mathematics staff alike.

Section 1: **Planning the effective use of ICT in teaching and learning mathematics**:

ICT is just another, albeit very powerful, resource which you, as a mathematics teacher, will need to consider when planning work for your students inside and outside mathematics lessons. Of course finance for resources is finite. But there has been a huge public investment in ICT hardware, software, support and training for education which is ongoing. Parents, students and teachers have a right to expect that teaching of all subjects in the curriculum should make the best possible use of ICT resources, and that means that the selection and deployment of ICT resources should be made to fit the needs of the subject – and not the reverse. The test of whether it makes sense to deploy ICT is a simple one: "Does it benefit the students' effective learning of mathematics?"

Developments in ICT have been very rapid, and in general costs have fallen considerably, so it is now the case that many homes contain more powerful ICT resources than are currently available to many teachers of mathematics in schools and colleges. Many common ICT tools have been developed for personal use, and their deployment in a teaching environment requires careful consideration. In business it is a common rule of thumb that hardware accounts for less than half the investment in ICT, and that investment in hardware is of very little use without investment in appropriate software, as well as training and support.

Unfortunately a culture has arisen in education in which the provision of resources to support the acquisition of appropriate curriculum software lags far behind that of hardware, which often comes with a range of so-called `generic' software bundled-in. In section 2 we give a review of the many software tools which have already been developed to support teaching and learning mathematics. Most of these have been widely available for some considerable time at a relatively modest cost. Inspection evidence consistently shows very little use of appropriate ICT tools in mathematics teaching at all levels. Reasons for this may include: (i) lack of support and/or training, (ii) lack of funding devolved to mathematics departments, (iii) scepticism about benefits of using ICT and (iv) lack of knowledge of what is available. We hope this guidance will help enable you to articulate your needs in a way which may improve the situation with respect to (i) and (ii). If you feel you don't know what ICT resources are available, nor the benefits of their use to teachers and learners, then we hope that this guidance will contribute to improving your knowledge. For a guide to the funding sources currently available to all maintained schools in England see the article `*What would it take to get ICT established in the maths department*?' by Adrian Oldknow in ATM's Micromath journal Vol 20/1, Spring 2004, posted at: http://www.adrianoldknow.org.uk/

The TTA's principal aim for the NOF-funded ICT training was to equip teachers with the necessary knowledge, skills and understanding to make sound decisions about **when, when not, and how to use ICT effectively in teaching particular subjects**. Our experience is that the effective use of ICT in mathematics teaching gives a powerful context which contributes to, and promotes, learning. The key to this is **how** you plan to integrate the use of appropriate ICT tools. When used well, such tools make your students the active participants in the learning. In the next section we will review a number of models of classroom use of ICT. In each case you need to be able to answer the question: "What does the ICT in this particular model of use bring to the learning?".

The recent Ofsted (2004) Report - *`ICT in schools – the impact of government initiatives Secondary mathematics'* HMI 2185 states that:

> *"the most effective use of ICT occurs where mandatory activities with clear mathematical aims and objectives are identified and planned for in schemes of work. Many schemes go no further than offering possible activities, and the extent and frequency of use with different groups depend very much on the interest and inclination of their teacher. Too often, pupils' ICT experience is not monitored to establish the impact on standards. It is therefore not surprising that departments are rarely able to say which ICT resources and types of teaching strategies were having the most impact. In order to allow them to do so, monitoring needs to be more rigorous and to focus on the impact of ICT on achievement, not just on quantity or frequency of use. A related issue is the degree to which heads of department recognised the opportunity for developing ICT in mathematics as part of their response to the Key Stage 3 Strategy. Many did not, because they did not believe it to be a priority, initially, within the mathematics strand of the National Strategy."*

The earlier Ofsted report - *`ICT in Schools'* (2002) – contains the following useful statement in Paragraph 90 about some of the options for ICT deployment in secondary school subject teaching:

> *"The characteristics of good general secondary provision include:*
> - *availability of different groupings of resources to match the needs of departments, for example computer rooms, clusters of machines and individual workstations around the site*
> - *computers networked and well maintained with good Internet access from all workstations*
> - *well-lit, comfortable computer rooms with sufficient space for pupils to work away from computers and for teachers to circulate and talk to individual pupils*
> - *effective communication with the whole class using digital projectors or the capacity to control all the computers*
> - *an efficient and equitable booking system for computer rooms.*
>
> *Increasingly schools are exploring the possibility of greater flexibility in serving subject needs through the use of wireless links between banks of laptops, managed centrally and linked to the school network."*

Of course different subjects will also have further styles of ICT use to consider – for example in mathematics we have hand-held devices such as graphical calculators and data-loggers, and whole-class teaching tools such as interactive whiteboards to consider. Whatever style of hardware use we choose, the key issue will be the choice of software to support the learning and teaching of mathematics. Paragraph 41 of Ofsted's earlier report, `*ICT in Schools*' (2001), advised that:

"Less successful use of ICT in core subject teaching typically stems from weak links between the computer task and the lesson objectives. Too many teachers select software packages for their visual appeal rather than their relevance to lessons. In one case, a primary

science lesson revolved around a science program, which, although quite stimulating, placed very few demands on pupils, who watched a simulation of materials dissolving, rather than carrying out experiments of their own."

The Ofsted ICT Secondary Mathematics report (2004) has the following summary, which suggests that while some improvements have been seen there is still a long way to go:

- *"There is an unacceptably wide variation between schools in the use of ICT to enhance learning in mathematics. The best practice is excellent but it is not shared widely enough.*
- *The majority of mathematics teachers use ICT effectively outside the classroom in the preparation of teaching materials and in the analysis of data. A minority are still not confident even in this use of ICT and require further training.*
- *There are now many more good teaching ideas and applications from a range of sources. However, there needs to be better co-ordination and distribution of materials, ideas and resources if teachers are not to waste time searching for appropriate materials.*
- *Similarly, there is a good range of software available to support the teaching of mathematics but some schools need better guidance on selecting and utilising the software that best meets their needs.*
- *The most significant impact of ICT is when it is used to enable pupils to model, explore, analyse and refine mathematical ideas and reasoning.*
- *In the best practice, schools have made great strides in integrating the use of ICT into their lessons in a way that enhances learning and progress for pupils. In many schools, however, ICT activities are not written into the scheme of work and hence fail to occur consistently.*
- *In general, many schools make effective use of calculators but this is not always the case, even in sixth form lessons.*
- *Some New Opportunities Fund (NOF) training has been very successful but, more commonly, it has been ineffective in helping mathematics teachers make good use of ICT in classrooms.*
- *The leadership and management of ICT are very variable. Departments that are furthest forward reflect the vision and direction provided by the head of department or the teacher responsible for ICT. Many others have not had ICT as a priority while implementing the Key Stage 3 Strategy because it was not a priority within the mathematics strand of the National Strategy.*
- *National Grid for Learning (NGfL) funding has had a significant effect on the availability of resources in a number of schools. Despite significant improvement in the provision of hardware resources, access to ICT facilities for mathematics departments in some schools and colleges remains inadequate. 'Mobile' technology, including graphing calculators and wireless laptops, is not used sufficiently to address such problems."*

While we have done our best to juggle our desire to give you as much helpful advice and information as possible within a document of tolerable length you will probably find it useful to have some other resources to hand when wanting to go rather further into aspects we can only touch on.

Such resources include:

- the book `*Teaching Mathematics using ICT – 2nd edition'* by Adrian Oldknow and Ron Taylor, Continuum 2004 ISBN 0-8264-7059-9
- recent editions of the ATM's `*Micromath*' journal (say from Vol 17 onwards)
- articles collected from previous editions of `*Micromath*' in the book `*More Teaching, Learning & Mathematics with ICT*', ATM 2001 ISBN 1-8986-1112-2
- TTA's free booklet `*Identification of Training Needs: Secondary Mathematics'*, TTA publication 10/3-99.

1.1 **Models of ICT deployment in schools and colleges**

In recent years mathematics teaching has seen a renaissance of whole class teaching approaches. So clearly you need to know what sorts of ICT resources are available to support this. In many cases, though, you will also want the students themselves to have access to ICT resources. This might be through individual access, or working in pairs, or working in groups. Nearly all schools and colleges have special purpose ICT suites where students can have individual or paired access to computer resources. These may be available to subject departments through a booking system – but there are issues about such use which you will need to consider carefully. An alternative is to make ICT resources available within mathematics teaching rooms – and here, too, there are issues you need to think about. Of course, learning is not restricted to timetabled lessons and need not take place solely in the school or college. So you need also to think about the access which students might have to resources away from lessons which can support their learning and your teaching.

We are fortunate that mathematics was one of the three subjects, along with Latin and Japanese, chosen to be the subject of a DfES funded trial project in 2000/1 which has been extensively evaluated. Lessons from the mathematics project, based on 20 Year 7 classes and their teachers, should help inform us about each of the deployment issues above. In this project it was assumed that all the teachers and many of the pupils would have Internet access outside lessons. A central tenet of the project was that the classroom should provide for as flexible use of ICT as possible. In the photograph below you can see at the front, scanning from left to right, (a) an overhead projector on which there is a liquid crystal display panel for projecting a teacher's model of a graphical calculator, (b) an interactive whiteboard (IWB) mounted at a height suitable for pupil use and connected to the PC and printer at the front, as well as the ceiling-mounted data projector and (c) one of a further two PCs in the corner of the room. Not shown are (d) the teacher's own laptop and printer, (e) the teacher's graphical calculator for whole-class display with either OHP or the data projector, (f) the compatible data-logger (motion detector) and (g) the half-class set of a further 15 graphical calculators for pupil use.

Whole class work with ICT

The images may be **generated** by ICT hardware such as:
- a single PC or laptop
- hand-held technology such as a teacher's graphical calculator
- another ICT device such as a digital camera, DVD player or cam-corder.

They may be **projected** by display technology such as:
- a data-projector – either portable or permanently sited, possibly wireless
- an overhead projector (OHP) with a liquid crystal display (LCD) pad
- an interface to convert the input signal to a standard output such as PAL video
- a document camera.

It may be **displayed** on devices such as:
- a fabric screen
- an ordinary whiteboard suitable for use with marker pens
- an interactive whiteboard (IWB) designed for use with special styli and pens
- one or more large screens such as display monitors, TV sets, plasma displays…

Paragraph 69 of the 2001 Ofsted ICT report referred to PCs for whole class displays:

*" Many schools have invested in **interactive whiteboards and computer projection equipment**. Generally these have led to more effective whole-class teaching, with all pupils able to see a large, clear display. Pupils too have made good use of this new equipment, for example in illustrating a prepared talk with the aid of presentational software. In particular, the use of interactive whiteboards is bringing benefits to pupils with SEN, for example by making teachers' presentations clearer and more interesting, and by providing instant notes via*

printouts of the text and examples used in presentations. In an effort to provide more flexible access to ICT resources for pupils, some schools have increased the availability of ***laptop computers****."*

The 2004 Ofsted report on ICT and secondary mathematics finds:

> *"A number of schools have used a range of funding sources to purchase interactive whiteboards, in some cases together with associated software. Over half of the schools visited had at least one such board within the mathematics department. One of the best examples of such resource provision is in a specialist school where, in addition to reasonable access to the school's ICT suites, all eight mathematics classrooms are equipped with an interactive whiteboard, integral software, ceiling mounted data projector and networked PC. This equipment came with dedicated training for all mathematics staff. As a consequence, the staff are confident and enthusiastic users of the full range of facilities. They share worksheets and presentations freely across the team, providing a good exchange of ideas and commonality of activities for parallel teaching groups."*

Paragraph 70 of the 2001 Ofsted ICT report refers to hand-held devices such as graphical calculators:

"In specific subjects in secondary schools, NGfL funding has been used to purchase specialised ICT equipment. … There is increasing availability of hand-held devices in ***mathematics****, with more powerful calculators that can be used to display graphs, capture and analyse data, manipulate algebraic expressions and solve equations. These devices can significantly enhance mathematics teaching, especially where teachers can provide demonstrations via an overhead projector."*

However the 2004 Ofsted ICT and secondary mathematics report finds:

> *"In mathematics work, calculators are an obvious ICT tool. Many schools make effective use of calculators, but relatively few use graphing calculators to overcome the difficulties of access to school computer suites."*

As we have seen, the DfES Y7 project made use both of PC displays projected via a ceiling-mounted data-projector onto an interactive whiteboard and graphical calculator (GC) displays projected via a conventional OHP onto any flat clear surface. If possible it is preferable to display either PC or GC displays onto a surface on which you can write, so that you can easily annotate diagrams etc. You can see further examples of the use of whole class displays with mathematics classes in each of the four case studies in the Secondary Mathematics part of disk 3 of the TTA's ICT CD-ROM needs identification materials, obtainable from the Publications Line on 0845 606 0323. The DfES produced further video case studies for BETT 2004 called `*Embedding ICT@Secondary'* where the mathematics class shown uses both an interactive whiteboard and group work with laptops for KS3 Shape, space and measures. Further video case studies are currently being prepared for the DfES to release at BETT 2005. In Section 3 below we give more details about whole class displays such as interactive whiteboards.

ICT resources for whole class work can be used in a number of ways including mental and oral work, demonstration, interactive whole class teaching, class discussion and plenary activities. If they are PC-based then, at a minimum, single user licences would be required to give access to dynamic geometry software (such as *Cabri* or *Geometer's Sketchpad*), a programming language (such as *MSW Logo*) , a graph plotter (such as *Omnigraph*, *Autograph* or *Coypu*) and/or other powerful integrated mathematics software (such as *TI InterActive!*). Most of this software is registered for Curriculum Online (http://www.curriculumonline.gov.uk), and can be bought using the e-Learning Credits (e-LCs) available to schools up to August 2005. This would allow teachers and pupils to make interactive use of ICT in a variety of ways, although pupils would only have limited opportunities for hands-on work. It could also be possible to access resources from the Internet (either 'live' or downloaded), such as games, simulations, data and animations as well to access other `generic' software (such as *Excel* or *Powerpoint*). If the resources include hand-held devices then much of this software is already built in to graphical calculators. With so-called `flash ROM' (such as in the TI-84 Plus) further applications can be downloaded and run, including spreadsheet, small software (such as *SMILE* programs), computer algebra and dynamic geometry.

Individual, or paired, use of ICT

In a minority of schools there is a mathematics room equipped with a suite of PCs on a network which supports a good base of curriculum software. In such a case the allocation of mathematics rooms can usually be adjusted so that each class has reasonable access to hands-on use at an appropriate time. More commonly, though, there will be one or more ICT suites in the school available for sharing between subject and ICT teaching using a booking system. In either case there are a number of issues concerning the layout of the room. Ideally it should provide:

- a whole-class display visible to all students
- scope for visual and oral interaction between teacher and pupils in both one-to-one and whole class situations
- space for pupils to work away from the computers
- a range of software appropriate to the mathematics curriculum.

An alternative approach to giving all students hands-on access to ICT in their normal classroom is provided by having a set of portable battery-driven devices which could range, in descending order of cost, from a set of laptops (possibly linked wirelessly to a network), through palm-top computers (such as the Palm Pilot, Casio Classpad and TI Voyage 200) to graphical calculators. There are advocates both of having enough for all pupils to use individually and for making pairs of students share. The model taken following the DfES project uses 20 TI-83 Plus graphical calculators – enough so that students can work in pairs, but also to support a few individual students who may, for example, be working at a different pace. The Y10 algebra case study on the TTA's CD-ROMS illustrates a lesson including graphical calculators both for whole class work and for working in pairs. A major advantage of the use of portable ICT is that students can use it in their normal work-space, and make their own decisions about when, and when not, to use it.

Whatever means of access you have at your disposal you will still need to plan carefully for collaborative activities. Thought also needs to be given to practical issues such as:

- how students' work may be saved for future access
- whether hard-copy devices should be available
- the possibility of students displaying their own work to the whole class.

Group work with ICT

It may be possible to have just a few PCs in the mathematics classroom which can be used for work with small groups of students – say 3 or 4 to a machine. If there are not enough for the whole class then some kind of `circus-style' system of management will be required with some students working on tasks away from PCs while others work with them – maybe changing over during the lesson or for the next lesson. The kind of activity using the PC will require careful planning – ideally it should involve the students working collaboratively as a group to plan the next move involving the PC. For example this might be participating in an adventure game or simulation, or in a problem-solving or modelling task where the PC is just one of the available tools. While primary school teachers often adopt such circus-style of activity, with or without ICT, it is less common in secondary school mathematics and needs careful planning.

Mixed models

Of course you may not be restricted to any one of the above models, in which case you will be able to make flexible decisions about the best way to deploy ICT to meet a particular curriculum objective. This was the resourcing model followed in the DfES Y7 project – which extends well beyond just Y7 to the whole 11-19 age-range.

In planning ICT use for a department you need to bear in mind how you can best provide an initial deployment of ICT resources which maintains flexibility and which allows you to continually improve the types of access as finances allow. For example if you already have access to a number of laptops for teachers' use then you may want to aim to build up a stock of portable data-projectors so that they can be used for whole class work. Or you might want to plan a phased equipping of a number of mathematics classrooms with interactive whiteboards. In either case you might also want to build up a stock of graphical calculators which can be taken into any lesson. You need to be able to make a reasoned case for what you reckon is a realistic base from which to meet the ICT demands for organising the teaching and learning of your subject in the 21st Century. Remember that hardware is only part of the story – you will need to plan also for acquiring appropriate software licences – for which the e-Learning Credits are usually available. Later in this section we give advice on management issues, and extensive advice on ICT products and software comes in Section 2. A useful summary of issues is given in Ronnie Goldstein's article `*Organise yourself*' in *Micromath* 17/2, page 48.

In order to help you achieve what you would like to do, you will probably need some form of CPD or other support. The Standards Fund provides for this, so you should explore routes such as the ESTUICT programme from the Mathematics Consortium (www.cpd4maths.co.uk), Hands On Support or the T-cubed programme.

Summary

We hope that you have been able to gain an understanding of the main types of organisation of ICT resources for mathematics classes, and that you are aware of the range of issues associated with each. We conclude this section with some comments from the recent Ofsted reports on ICT in schools.

1.2 Changes in teaching approaches

It is quite possible to use ICT in mathematics lessons in a way which does not throw into question your own teaching approaches. However the power of the medium is such that to restrict its use in this way is to deprive the students of access to a resource which could revolutionise their learning. It was to address this issue that the DfES' mathematics curriculum IT support group produced its `*Mathematics and IT – a pupil's entitlement'* leaflet in 1995, now republished on the Becta ICT Advice website: http://www.ictadvice.org.uk/downloads/entitlement_doc/entitle_maths_sec.pdf . It lists six major ways in which ICT can provide opportunities for students learning mathematics:

Learning from feedback*: The computer often provides fast and reliable feedback which is non-judgemental and impartial. This can encourage students to make their own conjectures and to test out and modify their ideas.*

Observing patterns*: The speed of computers and calculators enables students to produce many examples when exploring mathematical problems. This supports their observation of patterns and the making and justifying of generalisations.*

Seeing connections*: The computer enables formulae, tables of numbers and graphs to be linked readily. Changing one representation and seeing changes in the others helps students to understand connections between them.*

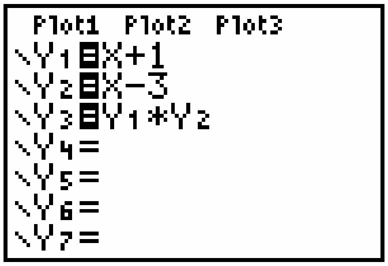
Working with dynamic images*: Students can use computers to manipulate diagrams dynamically. This encourages them to visualise the geometry as they generate their own mental images.*

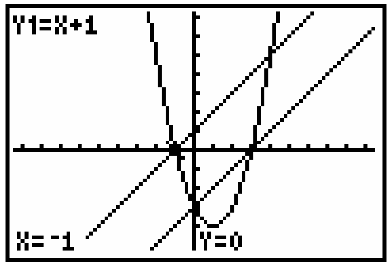
Exploring data*: Computers enable students to work with real data which can be represented in a variety of ways. This supports interpretation and analysis.*

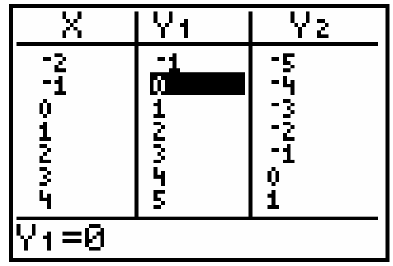
`Teaching' the computer*: When students design an algorithm (a set of instructions) to make a computer achieve a particular result, they are compelled to express their commands unambiguously and in the correct order; they make their thinking explicit as they refine their ideas.*

These opportunities are equally valid now, but we have more widely available tools to help us provide them for our students. QCA have established the National Curriculum in Action website http://www.ncaction.org.uk/subects/maths/ to help teachers map these opportunities onto the main types of ICT resources such as graph-plotters, spreadsheets, dynamic geometry software, computer algebra, small software, *Logo*, statistics packages, graphical calculators, data-loggers and web-based resources. The DfES have refocused their overall ICT strategy for 2003-6 to aim to see ICT embedded in the teaching of all subjects. They are currently funding groups, such as the Mathematical Association, to produce a variety of lesson plans, resource materials, case studies and guidance as part of what will be known as the DfES Key Stage 3 ICT Offer to schools to be launched at BETT 2005.

As an example we can look a different approach to quadratic functions, their roots and factorisation which illustrates the "Seeing connections" opportunity above. This could be approached using graphical calculators, graph-plotters or integrated software. The idea is taken from page 33 of the `*Secondary Mathematics with a Graphic Calculator'* book from the NCET's `*The I.T. Maths Pack'* published in 1994.







Can you explain why the curve cuts the axes at the same points as each of the lines? Why is the curve negative between these two `roots', and positive on either side? Can you enter a quadratic expression for X in Y4 which gives an identical curve to the one in Y3?

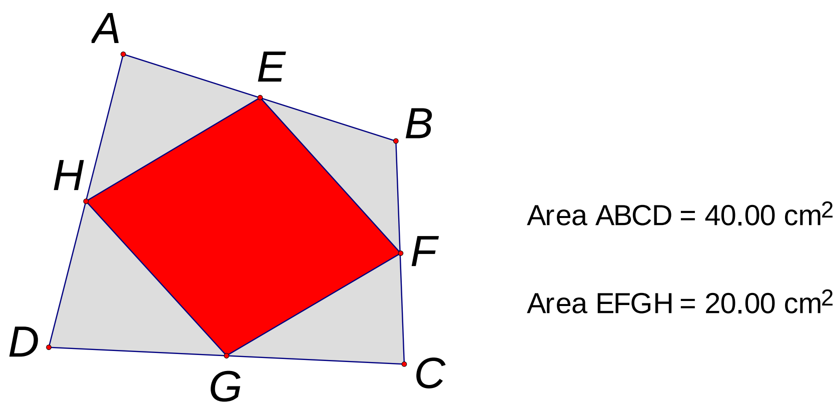
A specific example of the kinds of changes in teaching approach which may be needed is provided by the Royal Society/JMC report `*Teaching and learning geometry 11-19*'. (http://www.royalsoc.ac.uk policy document 16/01) The following extract is taken from section 5: the development of the curriculum:

"The revision of the National Curriculum by QCA in 1999 gave the opportunity for greater exemplification of the ways in which ICT impacts on many subjects and their teaching. Yet there is very little specific reference to the use of ICT in the mathematics National Curriculum in general, and in geometry in particular. Geometrical software is now widely used in, for example, engineering and design. Through government and commercial initiatives many secondary schools and colleges have acquired powerful CADCAM packages for use in teaching Design and Technology. By contrast relatively few schools have access to software for teaching geometry in mathematics. Yet by using such software in appropriate ways, pupils can apply their ICT skills to increase their knowledge and understanding of geometry. The software also provides them with the opportunity to acquire and practice geometric skills. Opportunities occur when pupils create, analyse and interpret dynamic spatial images; make and test conjectures about geometrical relationships that they can manipulate; and record and present the results of their investigations.

As with any approach to teaching, the educational use of ICT needs to be well thought through and carefully planned. The TTA has produced documentation to accompany the current programme of lottery funded ICT training for all teachers in which it emphasises the importance of a critical approach to the use of ICT. This expects teachers to know where, when and how to apply ICT to enhance the teaching and learning of their subjects. This advice is particularly important in geometry where a variety of approaches are needed including mental, practical, and ICT enhanced work. Increasingly powerful software is becoming available in education, such as for simulations in science and geography, much of which relies on sophisticated mathematical algorithms. Pupils and teachers in all subjects need to be cautious about accepting computer produced results without question, and mathematics is probably the subject best placed in the curriculum to engender a critical approach. In teaching geometry, caution is particularly needed to avoid making assertions based solely on computational illustrations.

Thus the working group would like to see further development of the curriculum with respect to … the use of ICT. .. We recognise that this will have implications for resources, materials, assessment and teachers' professional development, as will the effective teaching of proof, modelling, problem-solving and other aspects of geometry. In many respects we need develop a completely new pedagogy in geometry."

Here is an example of dynamic geometry software used to explore the relationship between the areas of quadrilaterals and the areas of the figures formed by joining their midpoints.



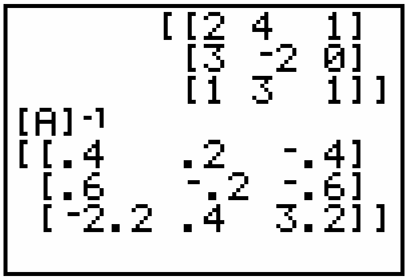
The dynamic geometry software tells us that the two areas appear to be equal, however we drag *A*,*B*,*C* or *D* about. So we can formulate a conjecture that the area of *EFGH* is always half that of *ABCD*. But lots of pictures drawn with ICT do not, of course, constitute a proof – only very strong circumstantial evidence! Noting relationships, like *EFGH* always appears to be a parallelogram, or adding constructs to the diagram, such as a diagonal like *BD*, might suggest geometric relationships from which we could arrive at a convincing proof.

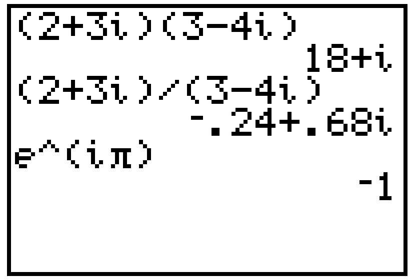
1.3 Impact on the curriculum

As we know, the mathematics curriculum can take a very long time to respond to technological changes, such as the development of the electronic calculator. Yet now ICT is so well established in all branches of industry, commerce and research as a mathematical and statistical modelling and problem-solving tool there are few people who still use pencil-and-paper techniques. So while the impact may be slow, we should at least be prepared to re-examine our own ideas of what are the fundamentally important elements of the curriculum. To quote again from `*Mathematics and IT – a pupil's entitlement*':

"As the technology progresses and becomes more prevalent, teachers will also need to be continually reconsidering the mathematical content of their teaching. Having software which can, for example, solve systems of equations at the touch of a button has strong implications for the way particular topics are approached."

Certain topics, such as matrices and complex numbers, have all but disappeared from the mathematics curriculum. This is ironic since, for example, most graphical calculators will perform the standard arithmetic operations such as matrix inversion or complex multiplication. Using these facilities we could now concentrate on deepening an understanding of what matrices and complex numbers are, and can be used for, without the need to perform routine and unenlightening manipulations. Given the widespread use of matrices and complex numbers in engineering, science and other numerate disciplines it would be good if we could harness ICT to help students understand their significance.





Another example is in data-handling where it a very simple matter to transform data-sets and to display either *x*- or *y*-axes, or both, scaled to logarithms.

One important aspect of the widespread availability of mathematical ICT tools, and easy access to sources of mathematical information via the Internet, is that they can enable you to keep in touch with developments in your own subject. While it is the norm for teachers of many subjects to keep themselves up-to-date with their subject, this does not appear always to be the case in mathematics. So maybe you could help lead the way by creating a mathematics learning community and ethos in your own school – and hopefully inspire more students to want to study, and perhaps teach, the subject?

1.4 Using ICT to provide enrichment activities to support differentiation

There are three terms that are closely associated with enrichment and, whilst they are not entirely mutually exclusive, they do represent different agendas.

- <u>Acceleration</u>: takes pupils into areas of the curriculum normally covered by older children often resulting in early entry to public examinations. This is sometimes achieved by moving pupils into higher year groups for all subjects or just the subject in which they excel. Alternatively, children can be accelerated within their own class working independently, often with some additional support.
- <u>Extension</u>: involves moving outside the syllabus and looking at aspects of mathematics not normally covered within the `normal' curriculum. The mathematics can give opportunities for problem solving which requires pupils to use their mathematical skills in non-standard contexts.
- <u>Enrichment</u>: is about extending pupils' understanding of the mathematical ideas they have already met by applying them to other situations and problems, often requiring decisions on what area or areas of mathematics to employ. The aim is to develop higher level problem solving and communication skills. It extends the notion of using and applying. The aim is to produce a thinking mathematician who can look beyond the standard `test' type questions. Enrichment also implies a teaching approach which encourages mathematical discussion and communication.

More able pupils need to be stretched and motivated but this is often very difficult in a classroom with 30 or more pupils all needing individual attention. One of the most powerful resources available to you is the computer. More able pupils will no more benefit from the computer than other students but in the same way as ICT can support basic literacy or numeracy skills through targeted, skills focussed, software or the use of generic software (such as word processors and spreadsheets) or access to resources via the Internet, it can support the needs of the more able mathematicians. Effective use of ICT will enable you to:

- find appropriate online resources that can be used off-line or converted to a paper based resource – an example of this are the resources to be found on the NRICH website (offering enrichment materials for mathematics to pupils of all ages)
- use generic software and open ended tasks that will engage pupils who are struggling but offer real opportunities for stretching the most able. Software such as *Logo*, dynamic geometry software and spreadsheets all have this potential but closed, skills based, software does not give the flexibility to encourage enrichment.

This can be achieved by:

- Planning tasks for the whole group that will involve all pupils but stretch the most able (differentiation by outcome). So, for example the whole group might be using *Logo* to find their way through a maze – working on right and left and estimating distance. At the same time in the same room more able pupils may be creating a maze of their own. What are the properties of mazes? How can you ensure that the width of passages remain constant? This will involve using the same mathematical ideas but extending them to more demanding contexts. What they are not doing is `more of the same'.
- Sending individual or small groups of pupils to work at a computer on tasks particular to their ability but related to the activities being undertaken by the rest of the class. So if the group are working on angle properties of 2D shapes a small group might be investigating the angle properties by creating tiling patterns in *Logo*.
- Using the Internet to locate resources, such as those from NRICH to provide enrichment material that supports the lesson objectives and the more able pupils.

So where might you find ideas for appropriate activities? Some useful sites that might provide some starting points for enrichment activities:

NRICH	http://www.nrich.maths.org
Games by Conway	http://www.cs.uidaho.edu/~casey931/conway/games.html
Komal	http://komal.elte.hu/info/bemutatkozas.e.shtml
Abacus	http://www.gcschool.org/pages/program/Abacus.html
Mazes	http://www.logicmazes.com/index.html
Fibonacci	http://www.ee.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html
Mathsnet	http://www.mathsnet.net

1.5 Implications for management

Inevitably you will become involved with management – not only of deploying ICT in your own teaching. You may be responsible for managing others, and you may need to persuade your own managers to support you with your own, or the department's, development plans.

The role of school management at any level is to develop, support, and direct the learning community. For the purposes of this document we will focus on the management aim of developing, and supporting, a culture of creative use of ICT within the learning and teaching of mathematics, where classroom mathematics, at its best, is understood as a social activity fostering a community of enquiry.

Two general features of ICT applications that enhance the learning experience are software that is **responsive** and techniques that provide **leverage**, making easy something that was otherwise a considerable effort. We will try and give a flavour of what we mean.

<u>Responsive software</u> – examples include where a change to one value on a spreadsheet automatically changes all calculations that depend on that as a source value, and where movement of one point on a dynamic geometry display causes movement in all constructions that depend upon that point.
It is useful to make explicit the value of `interactivity', i.e. responsiveness, in software.

- Interactive software encourages learning by experiment and discovery – the "try it and see what it does" and "why did that happen?" approach.
- A facility to make changes, with instant effect, invites generalisation.

<u>Leverage</u> - making easy something that was otherwise a considerable effort. Examples of leverage include moving text around within a word processor document, replication of formulae on a spreadsheet and the automatic production of graphs on a graphical calculator. Any of these would have been tedious, or even worthless, without ICT.

There is a proposed ICT Development Cycle with phases which managers need to support.

The <u>familiarisation</u> phase applies both to students and teachers – this is a period of orientation and uncertainty. During the familiarisation period an important initial perspective forms about what the new object is, and what it might be worth, together with an assessment about what is being threatened. The familiarisation phase is important and needs to be handled by managers with awareness, imagination, and clear purpose. The familiarisation phase is passed through many times as a teacher or pupil moves on to encounter each major new environment. The teacher who, for example, has used graphical calculators in the classroom for some time, but is coming to a dynamic geometry package for the first time, will experience again the excitement, uncertainty, and perhaps frustration, of the familiarisation phase.

In managing the familiarisation phase, the context within which the initial encounter occurs matters greatly. The initiation experience needs not only to reveal just what the tool does, and give enough practice to sow the seeds of confident use, but also to convince the new user that the tool has value. For example, the first use of a spreadsheet should be based on something mathematically interesting, such as a number puzzle like the magic square.

This is followed by a phase known as `Routine Specific Activity', which is needed before a general culture of creative use establishes itself. In this period of routine but specific use, particular lessons will involve particular activities which make use of particular techniques. The management of routine specific activity includes the provision of schemes of work, the production of supporting resources, the provision of classroom support, and, of course, provision of access to hardware.

Barbara Ball, the ATM's Professional Officer, contributed to the review *`Developing IT Across the Mathematics Department'* published in 1996. There she compared her recent experiences with ICT tools to those some 6 years previously.

"I think I have found both Geometer's Sketchpad and Cabri relatively easy to learn because essentially what each of them does is to simulate ruler-and-compass constructions in a straightforward way. Of course, I needed to learn which menus to look in and which buttons to press. But once I had found my way round that I could do almost anything with the software and I could easily switch between the two programs. This is because I know about ruler-and-compass constructions. Of course, if someone does not know about ruler-and-compass constructions it might be difficult for them to do very much at all.

My experience with the Pinpoint data-base software was different. In order to use Pinpoint I had to learn, not only which buttons to press, but also how the program author was choosing to interpret and organise data and how I was expected to present the data to the parts of the program that draw graphs and charts and how the program author interpreted the meaning of different graphs and charts. I suppose this could be summarised by saying that different people organise information, and interpret the meaning of words such as 'histogram', in slightly different ways. There are no right answers about this; it is largely a matter of convention. In contrast, there is only one ruler and one pair of compasses and either we know how to use them or we don't."

In order to establish the mathematics classroom as a community of enquiry there are issues for management. These include the provision of resources and CPD for teachers in order to ensure all students have equal opportunity to experience mathematics using ICT. At a senior level this could involve a clear development plan to meet national targets for the proportion of lessons using ICT. It also involves ensuring that the knowledge base is kept up-to-date in order, for example, to make informed decisions about the relative strengths of different ICT provision, such as computers with interactive white boards as against hand-held technology.

There are management issues for the development of ICT in mathematics at all level. Appraisal by middle managers can be used to identify needs in a supportive way and also to relate these to training needs. Middle managers have a pivotal position in facilitating teachers' development and in ensuring students have a good experience of ICT in mathematics.

Appraisal can provide a useful means to identify needs, provide support and monitor the professional development of teachers in the use of ICT. In the best management practice this supports the whole school and departmental development plan. Our experience is that teachers respond positively and find these aspects of the appraisal and target setting process constructive.

One examples of good practice is to identify key modules in schemes of work which will require all mathematics teachers to use ICT in an aspect, say, of the geometry curriculum. Planning this, and finding department time for training, means that teachers who need support can get it, but at the same time they are challenged to deliver lessons with ICT. Another successful strategy has been to hold mathematics enrichment days for a particular year group which involve the use of ICT.

1.6 Evaluating the impact of the use of ICT

Evaluation occurs at many levels – e.g. of the use of some ICT as part of a lesson, or of the lesson as a whole, or maybe of your own, or the department's, facility with ICT. You will also need to be able to evaluate particular ICT tools in considering their usefulness for mathematics teaching. Let's start with how you might evaluate a lesson which used ICT.

<u>Evaluating a lesson using ICT</u>

Useful references here might be the `Review and Evaluation' questions on page 210 and the `Planning a lesson' on page 238 of Oldknow & Taylor's `*Teaching Mathematics using ICT*' . Also the list of bullet points on page 9 of the TTA's `*Identification of Training Needs – Secondary Mathematics'*.

In order to set the context we will need to know:

- What are you trying to do?
- How are you trying to do it?
- Who are you going to work with?

Then we will need to pose appropriate questions such as:

- Did you meet the learning objectives?
- Were the students actively engaged and were all abilities catered for?
- Were the students confident?
- Did the plenary work – could they transfer knowledge to other scenarios?
- What mathematics did they use?
- What progress did they make?
- Can it be extended further?

The Oldknow & Taylor book includes a sample activity review sheet. Below is a reproduction of a teacher's evaluation of a lesson exploring number sequences with graphical calculators:

1. *What did you expect to get from the task, i.e. purpose and learning intentions?*
   These are defined in the Teaching/learning objectives in the lesson plan.

2. *What additional knowledge and skills did* ***you*** *need :*

   about the technology
   Using the data handling facilities associated with the "stat" key and the calculator's ability to manipulate "lists". For the follow up lesson: how to plot data and superimpose functions.

   about the mathematics
   Fairly confident with the mathematics, but talked to mentor about the expectations and level of challenge appropriate for this Year 9 group.

   about teaching strategies and approaches?
   How to engage all pupils in the whole class interaction, use of the growing design and matchsticks as an initial task accessible to all pupils. The use of more open-ended questioning at the start of the lesson to establish what pupils already know. Getting the pupils to discuss in pairs to ensure everyone is involved was a good idea. I like collaborative work and often use this approach in my lessons. Pupils learn so much when they have to explain their thinking to others. It also improves their listening skills.

3. *What additional knowledge and skills did the* ***pupils*** *need:*

   about the technology
   How to enter, edit and manipulate data in lists.

   about the mathematics
   An ability to analyse growing patterns and convert observations in words into conventional mathematical symbols

   about learning strategies and approaches?
   Be prepared to explore, look for patterns and try things out, develop their use of visual imagery to explain their thinking. Work and co-operate with other pupils in their group. Explain and communicate ideas to other pupils in the class

4. *Was the focus of your teaching on developing skills or understanding?*
   Both; I wanted pupils to develop pupils reasoning and communication skills through an accessible context. I also wanted to develop pupils understanding of equivalent algebraic expressions and introduce them to algebraic manipulation and give purpose for further work on this topic. Although I think it is much more important that pupils can use algebra to formulate/model situations.

5. *Did the pupils focus on understanding or pressing buttons?*
   At the beginning of the first lesson some time was spent getting used to the data handling facilities on the GC. However, pupils were quick to come to terms with these and after an initial hiatus were able to use the GC with confidence to explore, check and validate their algebraic rules.

6. *In what ways were your answers to questions 4 and 5 affected by the use of the technology for the topic?*
   The technology acted as an extra "teacher". Pupils were able to try out their ideas on the calculator and it provided them with instant feedback on the validity of their conjectures and formulae.

7. *Would the use of the technology for this topic change the order in which concepts were taught?*
   Might try to introduce simple modelling and formal algebraic conventions earlier. Pupils coped well with the symbolism in the context of growing geometric patterns and had little difficulty using the GC.

8. *What were the benefits/disadvantages of using the technology?*
   Benefits; see 5,6 and 7. Disadvantages; some initial problems getting to know the GC. I had considered using "mouseplotter" and/or a spreadsheet in the school's computer room but chose the GC because of its easy use in the classroom and the similarity of the GC's notation with the standard algebraic convention.

9. *What would you do differently next time?*
   As pupils become familiar with the GC I will get the pupils to plot the results and link the graphs to the rules such as the kind of "rule that produces a straight line and the connection with the gradient and the intercept.

Evaluating the ICT use of the mathematics department

Part of the work of the DfES' mathematics curriculum IT support group was to produce materials to help departments evaluate their use of ICT and their future needs. A booklet giving case studies of schools called `*Developing IT Across the Mathematics Department'* is available on-line at: http://curriculum.becta.org.uk/docserver.php?docid=1892

"This booklet … is designed to help schools monitor their progress and form their action plans. It is based on case studies of six schools. These are all very different and there is no single clear route which they all advocate, but the practices in each school are outlined and policies for the use of IT in mathematics are pursued. In addition to the descriptions of the schools, this book contains discussions of some important issues common to the process of review:

- *Staff development*
- *Deployment of resources*
- *Linking practice with Entitlement opportunities*
- *Departmental policy."*

A 4-page form `*Review of IT in Mathematics – a departmental self-audit'* to help departments with the process of review is also available on-line at:
http://curriculum.becta.org.uk/docserver.php?docid=1844

The review is divided into 7 sections: (this has been updated slightly, for instance where the original review form referred to IT we have updated this to ICT)

1. ICT in learning mathematics

This section establishes the extent to which ICT is built into the teaching of mathematics in all classes. In which years are there modules of work including ICT use, support materials and teaching notes? How do we use the technology as a teaching and learning resource?

Year 7

Year 8

Year 9

Year 10

Year 11

Years 12,13

What aspects of ICT do we emphasise?

2. Current ICT skills of mathematics teaching staff

This section establishes the types of software, and devices, with which teachers are familiar and how widespread ICT skills are across the department. Since the development of all these skills takes time, the first part is about continuing professional development (CPD).

Recent ICT/mathematics CPD activities for the department or for individuals:

Number of teachers who teach mathematics: [ ]

Number of full-time equivalents in mathematics department: [ ]

Number of teachers who have developed specific ICT skills e.g.:

Data-handling	used database or statistical software:	[ ]
Dynamic geometry	used DGS for constructions and transformations:	[ ]
	extended their use e.g. for coordinates, measures etc.:	[ ]
Graph-plotters	used graph-plotting software in their teaching:	[ ]
Graphical calculators	used numeric and graphing aspects of graphical calculators:	[ ]
	used data-handling and other aspects of graphical calculators:	[ ]
	used graphical calculators with data-loggers:	[ ]
Logo	used Logo to manipulate a screen turtle and write procedures:	[ ]
	extended their use of Logo e.g. recursion, microworlds…:	[ ]
Small software	used curriculum focused software e.g. Smile, Maths Alive..	[ ]
Spreadsheets	set up a basic spreadsheet e.g. entered and replicated formulae:	[ ]
	used extended facilities of a spreadsheet e.g. statistical graphing:	[ ]
Symbol manipulation	used computer algebra software:	[ ]
Web-sites	explored web-sites relevant to mathematics teaching:	[ ]
Whiteboards	used electronic whiteboards for whole-class work:	[ ]

Other relevant ICT skills:

3. Curriculum resources
This section establishes whether there are appropriate mathematical activities and other support materials (published, or produced within school) for the different categories of ICT:
Departmental resources/starting points for working with:

Data-handling

Data-loggers

Dynamic geometry

Graph-plotters

Graphical calculators

Logo

Small software

Spreadsheets

Symbol manipulation

Web-sites

Whiteboards

Other relevant ICT skills:

4. ICT resources
This section is about the hardware available in the school. How many machines (desktop PCs, laptops, palmtops, graphical calculators etc.) are available in mathematics classrooms? Comment on suitability, sufficiency, accessibility and reliability.

How many purpose equipped computer suites are there; how many are bookable for maths?

Comment on the adequacy, or otherwise, of the bookable resources:

Time available for maths classes:

Availability and suitability of generic software (e.g. spreadsheets):

Availability and suitability of mathematics software:

5. Entitlement opportunities:
`Mathematics and IT – a Pupil's Entitlement' highlights six major opportunities for exploiting the power of the computer. This section establishes which opportunities are offered in your school. Indicate on a scale from 1 (minimal) to 5 (extensive) the extent to which pupils in your school have access to the following opportunities:

Learning from feedback:

Observing patterns:

Seeing connections:

Working with dynamic images:

Exploring data:

Teaching the computer:

What do you believe are the most important benefits which using ICT brings to mathematics teaching?

6. ICT development
This section establishes how you arrived at your current stage of development.
Can you identify helpful factors? Have you had contact with other schools through an LEA group, subject association, project etc.? Are ICT resources available for teachers to take home? Can you identify helpful sources of ICT support and advice inside the department, school or elsewhere?

7. Action plan
Completing the previous sections may have suggested directions for development. Your action plan could be written under the following headings, giving some indication of time scales.
Embedding ICT (entitlement opportunities and curriculum objectives):

Staff development, both within and outside school:

Updating and improving ICT resources:

What is needed in order to achieve this plan within the stated times?

During 2002 Becta developed materials to support secondary school subject departments evaluate their work. The mathematics materials are published on Becta's ICT advice site under the title: `*Timesaver: Carrying out an ICT audit of your Mathematics department'* **:** http://www.ictadvice.org.uk/index.php?section=ap&catcode=as_pl_aud_03&rid=1957

Evaluating resources such as software and web-sites.

The Becta Educational Software Database http://besd.becta.org.uk/ contains some useful information about ICT resources, which can be searched by subject, age-range, or both.

It gives details about, and links to, suppliers including costs of licences etc. In some cases it links to evaluations provided by teachers contributed as part of a group called TEEM, standing for Teachers Evaluating Educational Multimedia http://www.teem.org.uk/. Other evaluations are available at the Schoolzone web-site: http://www.schoolzone.co.uk

The TEEM evaluations are conducted to a common framework and use headings such as:

Overview of teaching with this title:

Installation:

Content

Curriculum relevance

Design and navigation

Ease of use

Special needs

Courseware

Conclusion

When evaluating software or web-sites we recommend that you consider the philosophy behind their design as well as any implicit or explicit teaching and learning styles when you use them.

Section 2: **Review of ICT tools of particular relevance to mathematics education**

Here we aim to provide practical advice about specific resources for you to consider when planning ICT developments in your own teaching, or for your department. This includes both software and hardware. As with many aspects of ICT, it is a sensible starting point to see what advice is already available and then to bring it as up-to-date as possible.

During 1999 Becta held a number of seminars to review the software base in relation to subjects in the secondary school curriculum. The mathematics report identifies a `toolkit' approach structured around a number of categories of software. This also forms the basis of ICT guidance given in the Key Stage 3 Mathematics Strategy's *`Framework for teaching mathematics: Years 7,8 and 9'*, page 25. A different approach is adopted in the TTA's *`The Use of ICT in Subject Teaching: Identification of Training Needs: Secondary Mathematics'*. On page 9 there is a very useful analysis of aspects of KS3/4 content where teaching and learning can be enhanced with ICT, as well as examples of ICT use to support the teacher.

We have taken this opportunity to update this useful information base, and to restructure the categories of both software and other ICT products. In order to make this as useful as possible we will refer to specific products which the group has experience of using in a mathematical context. Inevitably this cannot be comprehensive, nor stay up-to-date, but we hope that it will give you a clear idea of the extensive, and not particularly expensive (particularly if you use e-Learning credits!), range of ICT tools already `out there' waiting for you to experiment with.

The DfES's Year 7 RM MathsALIVE! project aimed to ensure that its pilot schools were equipped with just such a `toolkit'. This project has since been developed into the full Key Stage 3 *Maths Alive Framework Edition* service available from Research Machines.

One of the main lessons from that project is the benefit which comes from selecting a range of tools which are (a) not too many in number and (b) have widespread uses across the whole mathematics curriculum 11-19. This way the time you spend learning to use a tool is amply repaid when it can be used throughout the classes which you teach. It allows the department to share out responsibility for taking a lead role with a particular tool.

The main tools used in the project were:

- **Whole class displays**: Smartboard interactive whiteboards with data projectors and PCs using RM's *Easiteach Maths* software - and also Texas Instruments' graphical calculators with LCD pads projected using OHPs.
- **Hand-held technology**: TI graphical calculators and data-loggers.
- **`Small programs'**: addressing particular aspects of the curriculum – mostly in the form of games and simulations written by RM's 3T production company.
- **Programming languages**: *MSW Logo*
- **General purpose software**: Microsoft *Office* products such as *Excel*, *Word*, *Internet Explorer* and *Powerpoint*
- **Mathematics teaching software**: *The Geometer's Sketchpad, V.4* dynamic geometry software and the *TI InterActive!* integrated mathematics package – including graph-plotting, data-handling, symbolic algebra etc.

In this section we review a range of such ICT tools. An important maxim to bear in mind when considering the hardware and software combinations you might use is to **try to keep as much flexibility as possible in the deployment of ICT**. The ICT should be adapted to the needs of mathematics, and not the reverse! It is also worth bearing in mind that it is possible to use ICT to teach mathematics in a way which merely reinforces pre-ICT practice – e.g. `drill-and-practice' dressed-up as computer games, or `notes and exposition' dressed-up as *MS Powerpoint* presentations.

2.1 Whole class displays

The TTA's Needs Identification CD-ROM pack for secondary school has four case studies on the mathematics disk. These each illustrate a different means of obtaining a whole class display. Technology has moved so fast that when these were produced, just five years ago, it was considered too `leading edge' to illustrate the use of an interactive whiteboard! For an extensive review of whole class display technology see the Becta www.ictadvice.org.uk web-site. This also includes an extensive list of suppliers. As the information goes out of date very quickly you will need to check what products are currently available, as well as the prices!

Digital data projectors

These projectors usually accept a range of signals from external devices such as:

- computers, using a standard graphical output format such as SVGA, XVGA etc.;
- video recorders and players, cassette and DVD, using a standard format such as PAL;
- other electronic display and capture devices, such as digital cameras and camcorders.

They can be permanently mounted e.g. from the ceiling (as in the photograph on page 9), or put on a stand or table. While still relatively expensive items, their price has fallen sharply in the last couple of years and portable versions are available at under £1000. The major part of the cost is the lamp which has to produce a very bright image indeed. Because of the heat generated, the unit includes a cooling system. This makes it imperative not to move the unit until the lamp has been sufficiently cooled after it is switched off. The increasing home use of data projectors, e.g. for showing movies from DVDs, means that prices are likely to continue to fall, and that a good range of units should stay available on the market. It also means that projectors have become more attractive to thieves and that security is an issue to be considered. Wireless data projectors capable of receiving images without needing a cable are beginning to appear on the market. At the time of writing the transfer rate is not yet fast enough to do much more than browse slides, such in a *Powerpoint* presentation, but as the technology develops these should make a considerable impact in classrooms.

The image can be projected on any flat, light surface – but preferably one which is not reflective – so an ordinary classroom whiteboard may not be ideal. The Geometry case study on the TTA CD-ROM illustrates the use of a data-projector.

To aid readability over a wider range, it is usually preferable to set the computer to display at a resolution of 800x600 pixels. This can be selected by right-clicking the mouse over the desktop and selecting `Properties' and then `Settings' where there is a slide-bar which you can use to change the screen resolution.

Interactive whiteboards

In just a couple of years these have come from being an expensive rarity in education, to now being commonplace - which is not to say that they are always put to the best use! The huge investment being made in them in English schools fits well with the return to more whole-class mathematics teaching within the primary and KS3 strategies. An example of use with dynamic geometry software is shown in the mathematics video case study contained in the DfES' `*Embedding ICT@Secondary'* CD pack distributed at BETT 2004. The national strategies are producing a range of *Flash* animations (called *Interactive Teaching Programs* or *IT*Ps) for use with IWBs to support the mathematics strand. These can be downloaded from http://www.standards.dfes.gov.uk/numeracy/publications/

Teachers report that having an IWB enables better lesson preparation – where screens, texts, diagrams etc. can be brought up at the touch of a button. Before exploring what constitutes an IWB we should offer a note of caution. The IWB is one possible ICT solution to enable interactive whole class teaching – but not necessarily the most convenient one in all circumstances, and certainly not the cheapest. We will consider some alternatives – which also rely on a data projector or large class display – later in this section.

An Interactive WhiteBoard (IWB) is one part of a system which includes a computer and a data projector. It is both an input, and an output, device which allows information to be sent to, and received from, the computer. There are two main types of IWB. The digital version uses a special stylus or electronic `pen'. The analog version responds to any touch, such as with a finger. In either case the IWB sends back information to the computer about exactly where the pointing tool is placed. A matter of cognitive importance is that you click at the point of action on the display screen rather than via a mouse alongside a PC's keyboard or with a tracker-ball, toggle-stick or touch pad built-in to a laptop.

An IWB comes with supporting utility software which can allow screens which have been created to be saved and printed. It also allows the pointing tool to be used to annotate the output from other software. Increasingly manufacturers are improving their software to support teaching and learning in subjects such as mathematics – so if you are considering the purchase of an IWB you need to consider what range of useful mathematics tools and templates come with it.

The Becta ICT Advice web-site contains much helpful information e.g. at:
http://www.ictadvice.org.uk/index.php?section=ae&page=questions&theme=46
http://www.ictadvice.org.uk/index.php?section=te&catcode=as-pres_02&rid=4925&wn=1

An extensive review of IWBs was carried out by the NESTA-funded REVIEW project at the University of Hull and a CD called `The Good Guide to Interactive Whiteboards' has been widely distributed to schools and ITTEs in 2004. http://www.thereviewproject.org

The use of an IWB can be considered at several levels:

- low interactivity: for example you move from one slide to the next in a *Powerpoint* presentation by clicking on an on-screen icon;
- medium interactivity: for example where teachers and students are able to control relevant software from the front of the class for whole class teaching and discussion;
- high interactivity: for example where students are able to interact with geometrical constructions produced using dynamic geometry software by dragging diagrams into different configurations – the evaluation of the DfES' Y7 project referred to this as "pupils touching the mathematics".

In addition to the main display screen – usually wall-mounted or stand-mounted – and its pointing device(s), you can purchase other external input devices called `tablets' (not to be confused with `tablet PCs'), which are modern plastic and electronic versions of the Victorian school slate which can be passed around the class with an accompanying stylus. There are some routine maintenance procedures you need to adopt to ensure that the PC keeps track reliably of the position of the pointing device. IWBs come with software to enable you to recalibrate the board by clicking on a number of crosses displayed on the board. This needs to be carried out periodically even if you are using a fixed projector.

Sources of IWBs include:

Promethean Activ Board	http://www.promethean.co.uk/index.html
Hitachi Starboard	http://www.hitachi-soft.com
Smartboard	http://www.smartboard.co.uk/

Most manufacturers supply a range of software tools which have curriculum specific facilities – such as producing a range of grids and coordinate axes in mathematics.

A review of IWBs can be found in `*Teaching ICT*' Vol 1, issue 2, Autumn 2001 - the journal of ACITT: the association for ICT in Education. A typical large board (60" to 75" diagonal) costs around £1200, but you will also need to budget for additional items such as a data projector, stands and/or fixing costs.

One of our contributors provided the following comments:

"These comments follow my personal full-time use of an IWB in my classroom for the last six months. There is no doubt that the system represents a considerable advantage over just a plain whiteboard, or over a digital projector and static screen (which I was using before). The learning curve for the user is not steep: at the lowest level of competence you can just use the system as a conventional whiteboard, but with the advantage of being able to review material that would otherwise have had to be wiped off due to lack of space. Graphics and web links can be prepared beforehand and a lesson's `scribbling' can be saved and/or printed out.

One major use is the ability to annotate over other packages. So, if you're working with a dynamic geometry document or a spreadsheet, you can simply switch to annotation mode and write over the top of the document window. The ability, with a single pen, to quickly switch between different colours, highlighters, and specialist drawing tools, adds significantly to the quality of presentations."

Other means of supporting interactive whole class teaching

Most uses of IWBs require someone – teacher and/or student – to stand at the front of the class and to take control of the board so that the ICT interaction is made by physical contact with the board. Whether or not you use an IWB as the display screen with a data projector, you can interact with it and the computer by using the familiar input devices of the mouse and/or keyboard. This is where wireless technology really has come into its own. There is a wide range of wireless mice and keyboards on the market from about £30 to £150. A receiver/transmitter unit is plugged into the computer (usually via a USB port) so that the mouse and/or keyboard act as standalone devices which can passed around the class. The cheaper models usually have a range of about 2m, but there are others, such as those from Gyration, which have ranges up to 25m. So when planning a classroom for ICT supported interactive whole class work you will need to consider whether, in addition to the computer and projector, you need either wireless mouse/keyboard, or an IWB, or both.

Another possibility is the use of a `tablet PC' as the computer. This is essentially a laptop computer with a built-in screen which can sense the presence of a special stylus (like a larger version of the electronic `personal organiser'). Linked to a projector it can be passed around so that the teacher or a pupil can use the tablet like a `laptop IWB'. Classroom use of tablet PCs are shown in the new DfES video case studies to be released at BETT 2005. Most tablet PCs have high specifications and so are fast, have good battery life, will pick up wireless networks etc. The current drawback is the cable to connect them to the projector. When wireless projectors are fully developed then this will enable a tablet PC to passed anywhere in the classroom.

TV/video output

A cheaper solution than a projector (with or without IWB) is to echo the computer display onto one or more large screen TV sets, monitors or plasma screens. This usually requires a different form of output (PAL video) than that sent to an external monitor (usually a variety of VGA, such as SVGA or XGVA). Some computers and laptops have such video output built-in. Otherwise you will need a `scan-converter' which takes a VGA input and produces a video output. The Data-handling case study on the TTA CD-ROM illustrated the use of a VGA/TV converter connecting a laptop and a TV display.

OHP pads

There are a number of devices using Liquid Crystal Displays (LCDs) designed for use with an overhead projector. These include:

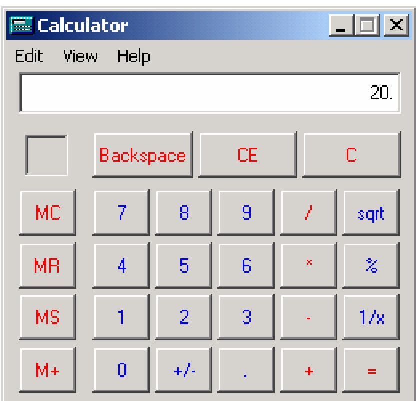
- transparent four-function and scientific calculators which sit on the OHP
- monochrome LCD displays which echo the output from graphical calculators
- colour LCD displays which echo the output from a PC or laptop.

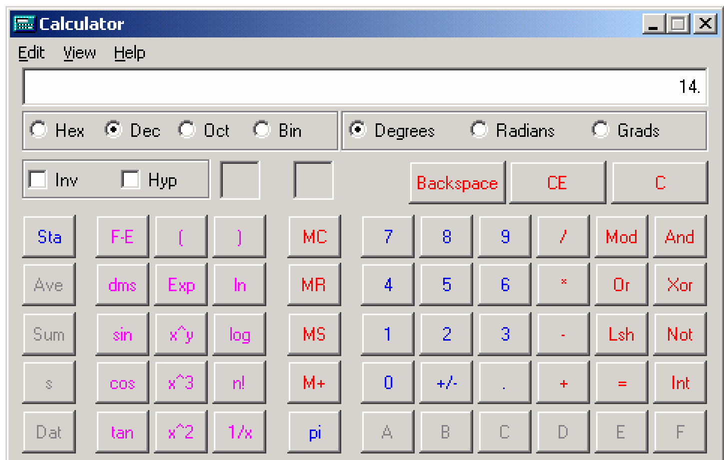
While the first two of these devices work quite well in normal lighting conditions, the third option usually requires some form of black-out. On the TTA CD-ROM, the Algebra case study illustrates the second option and the Number case study illustrates the third.

2.2 Hand-held technology

Calculators

While these are far from leading edge technology, there are still several issues concerning their use which need to be borne in mind. Clearly it is important that pupils know how to use them, and also that they recognise when it is sensible to use them. The use of calculators is required in Key Stage 2 mathematics. They are used in many subjects of the curriculum, and advice about their use is provided in KS3 Mathematics Strategy's *`Numeracy Across the Curriculum'* materials. OHP versions of calculators can be useful in supporting whole class teaching. Another approach is to display the simulations of basic and scientific calculators provided in the *MS Windows* accessories! You do need to know how your calculator works.





There are two major types of calculator operating systems, arithmetic and algebraic, the latter tending to be slightly more expensive. Note that they can give different answers e.g. to 2 + 3 x 4 . What would you need to do to ensure you get the same answers? Where would you predict other differences? Algebraic OS calculators will provide correct solutions to calculations without need for rearrangement. They also have a useful constant memory function which can be useful for routine conversions e.g. £ to euros.

Graphical Calculators

It is convenient to use the label `graphical calculator', or the abbreviation GC, to refer to a hand-held device which has all the features of a calculator together with an LCD display screen with built-in graph-plotting software. But in the 15 years or so since these first came on the market they have developed greatly and there are considerable differences between models, just as there are between PCs! There is very useful advice, together with good links and references, on the Becta ICT advice web-site at:
http://www.ictadvice.org.uk/index.php?section=tl&rid=3802&pagenum=1&NextStart=1&country=sco

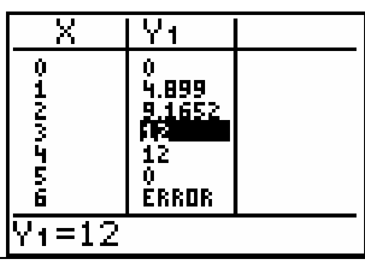
In this country the use of GCs at A-level was investigated and developed by the MESU project *`Graphical Calculators at A-level'* directed by Ken Ruthven and at KS3/4 through the NCET `

Portable Computers Pilot Evaluation'. There are many opportunities for their use to support teaching and learning in KS3/4 mathematics and which have been considerably reinforced by the large numbers of references in the KS3 strategy's `*Framework for mathematics*'. HMI report: "*Despite significant improvement in the provision of hardware resources, access to ICT facilities for mathematics departments in some schools and colleges remains inadequate. 'Mobile' technology, including graphing calculators and wireless laptops, is not used sufficiently to address such problems.*"

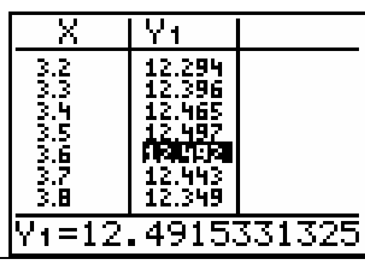
At A/AS-level there is a specific assessment objective concerned with their use, and many of the Free Standing Mathematics Qualifications have a specific requirement to use them. In many other countries (e.g. Netherlands, Denmark, Portugal) it is compulsory to teach the use of GCs in secondary schools. Their use is strongly encouraged in national examinations at 16-19, such as the US Advanced Placement examinations and the Scottish Highers. They are compulsory in 3 of the 4 mathematical strands in the International Baccalaureate, and strongly recommended in the fourth.

An example of their use is provided in Section 3: `*Using ICT in secondary mathematics – a case study'* (pp 34-47) in the TTA's `*Identification of Training Needs: Secondary Mathematics*' from which the following is extracted:

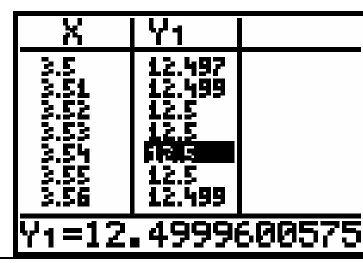
Specific teaching points
Now that the pupils were familiar with the problem the teacher reminded them how to enter a function such as "Y1 = X√(5^2-X^2)" on the graphic calculators and show the class how to use them to produce automatically a table of values of the areas Y1 for the values as before for the heights X, as in Fig. 4a. (She would expect pupils to explain why the word "ERROR" appeared against the value X=6.)



[Fig. 4a]



[Fig. 4b]



[Fig. 4c]

Step 5 - drawing graphs of functions
The teacher now discussed with the class how the data might be represented differently and more accurately using a graph. She then checked whether the pupils remembered how to use the graphic calculators to draw the graph of the function stored in Y1.

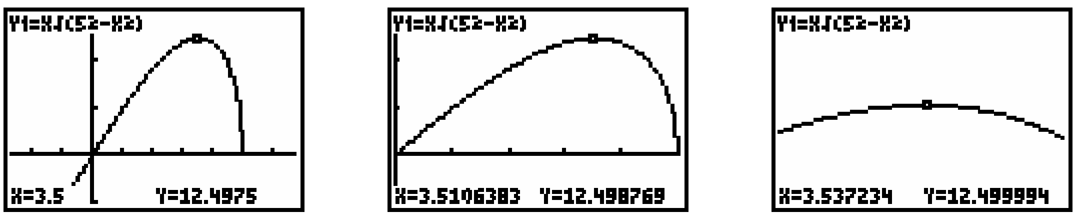
The teacher worked through a complete example with the class using the values in Fig. 4a as co-ordinates for the graph.

Commentary on teacher's decisions about whether or not to use ICT in meeting the teaching and learning objectives and justifying and explaining any use of ICT

The teacher judged that graphical calculators were the best resource for this as pupils could produce graphs quickly using the data. The split screen facility which allowed pupils to view the graph with the table of results alongside it, gave the teacher the opportunity to draw out in discussion with pupils the connections between the two representations.

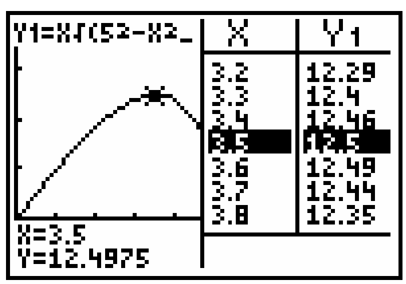
Specific teaching points

The teacher checked that pupils knew how to decide upon suitable values for the scales on the X- and Y-axes based on the data in their tables: "*What's the smallest value X can be? What's the largest....*" She checked that pupils remembered how to trace along the graph. She also reminded them how to "zoom in" on places of interest on the graph, and also how to arrange the scale on the x-axis so that each sideways movement of the cursor corresponded to a "round number" for X (as in Figs 5 a, b, c).



[Fig. 5 a, b, c Tracing the graph of the area function on a graphic calculator]

Finally she showed pupils how to arrange the screens of the graphic calculators so that the table and the graph were displayed alongside each other. Now when the graph was traced the pupils could see (a) the point on the graph, (b) the algebraic expression for the function which generates it, (c) the co-ordinates of the point, and (d) the corresponding entry in the table (Fig. 6).



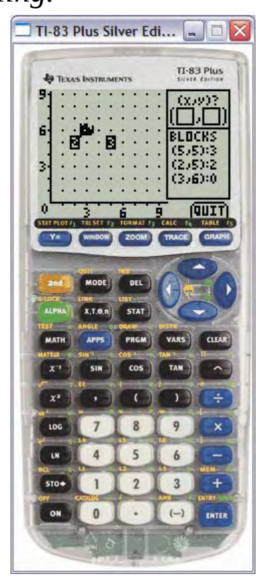
[Fig. 6. Split screen display]

The same case study was used as the basis for the Year 10 Algebra example on the accompanying TTA CD-ROM. This shows both pupils working in pairs at the problem using GCs, and a teacher's GC being used by both teacher and students for whole class displays.

At a basic level a graphical calculator is a small computer running built-in software specifically designed for mathematical, statistical and financial applications and displayed on a low-resolution monochrome screen. In that respect there is very little difference between the various models from the main manufacturers: Casio, Sharp and Texas Instruments. As with the purchase of any ICT, the maxim is: **you get what you pay for!** For around 50% more than the cost of a basic model you can purchase models which are designed for easy transfer of data, screens and programs between calculators, PCs and compatible devices such as data-loggers and even robots, as well as for projection using an OHP or a video input. A new range of models now have USB ports which enable both faster data transfer, and for pupil units to be able to connect to whole class displays.

The large amounts of so-called flash-ROM now available in such models mean that you now have the equivalent of a mini hard-drive on which to store application programs. Such applications can be pre-loaded, or obtained via the Internet, and cover a wide range of aspects from mathematics (e.g. dynamic geometry), ICT (e.g. spreadsheet), science (e.g. periodic table) etc. A recent Becta project explored the use of versions of software, such as from SMILE, on graphical calculators. It concluded that:

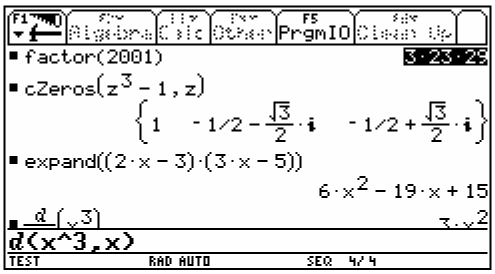
"This short project demonstrated the potential of graphic calculators to enhance the teaching and learning of mathematics throughout the 11-16 curriculum. Both teachers and students were positive about the use of the calculators. Given the difficulty of accessing computer suites in school, graphic calculators offer mathematics teachers a dedicated resource that can be used to target particular areas of the curriculum in a way which encourages mathematical thinking. The outcomes show the flexibility of usage that graphic calculators allow: whole class with one demonstration machine, whole class with individual or paired use, small group or individual use when other activities are also going on in the classroom. There is also tremendous potential for covering a range of mathematics content: number, algebra and data handling."

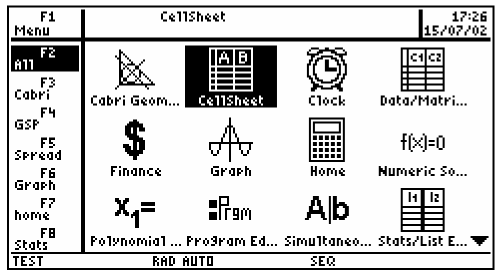


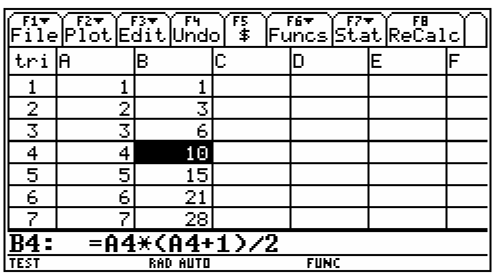
Teachers' models for whole class display can be projected using an OHP or data projector. Another interesting possibility is the use of on-screen emulator provided by a manufacturer such as Casio or TI, especially with an IWB. The illustration shows the SMILE *Rhino* game being played on the on-screen software called the *TI Flash Debugger* – downloaded with the free *Software Developer's Kit* (SDK) from the TI website http://education.ti.com . It currently provides a full emulation of the TI-83 Plus and TI-83 Plus Silver Edition graphical calculators which you can use for a whole class display – particularly with an Interactive Whiteboard. Just press the keys as you would do on the real thing! You can also download and install free `Apps' such as *SmileMath.*

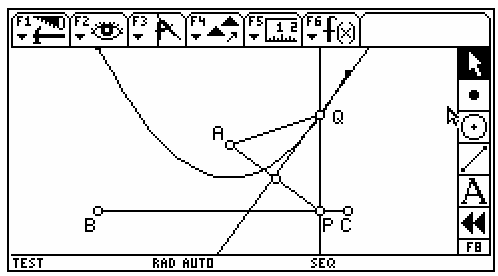
Other hand-held devices

The range of these is constantly increasing. Personal data assistants (PDAs), palm-tops and new generation mobile phones all have educational potential, although they were not designed for that purpose. Their facilities vary widely, and their use in schools is not yet well established, so it will not be possible to give sound pointers to their use at this stage. However there is one specifically educational product, based on a `game-boy' design, which has been in use in education for a number of years, although mostly outside the UK. This is the Texas Instruments TI-92 Plus which has the features of an advanced graphical calculator, but whose mathematical software includes symbolic manipulation (written by one of the authors of the popular *Derive* software). The built-in software includes dynamic geometry (either *Cabri* or *Sketchpad*). The French government supported a successful trial of this technology used by Lycée teachers with an OHP pad as an electronic blackboard for teaching mathematics 16-19. Information on the newer Voyage 200, using similar technology, but with a large flash-ROM memory can be found at: http://education.ti.com/us/product/tech/89/features/features.html . The Casio Classpad has similar features.









One educational use of PDAs and new mobile phones which we can expect to see grow is to access the Internet. For example there is now a range of free software resources for KS3 mathematics and science made available by Intel at http://Skoool.co.uk predicated on the use by students including such hand-helds.

2.3 `Small programs' which address particular aspects of the curriculum

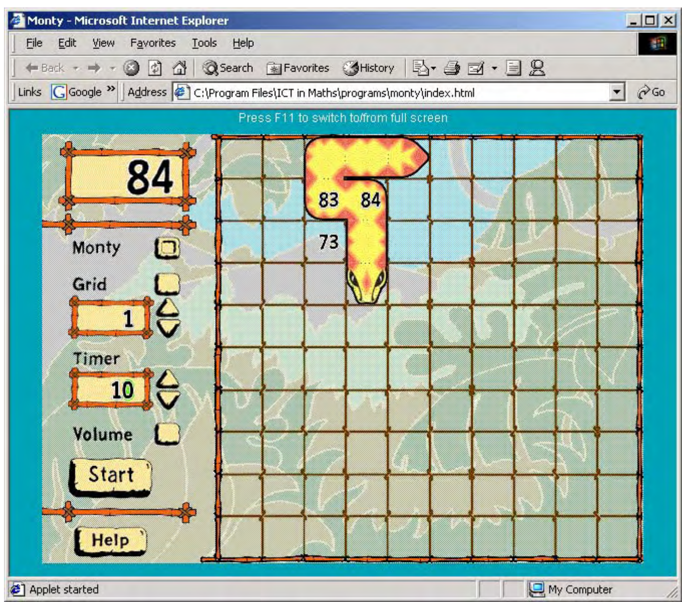
There are many programs currently available which are designed to exemplify particular mathematical topics. Some are games, others are more didactic. They vary widely in quality. A number of titles have been developed by educational groups and trialled in classrooms. These include those produced by SMILE, the ATM and NRICH. Some are available at no cost via the Internet, such as NRICH. Others are commercial products. An interesting recent development is the availability of versions of such software to run on graphical calculators.

The criteria for using such software should start from the mathematical learning objectives. The programs can be very motivating for pupils, but it is important to consider the quality of the interaction between the pupil and the software. For example, feedback can be crucial. Sometimes these resources work best with a number of pupils, as a stimulus for discussion and thinking. Some will also work well as resources for working with a whole class

Examples of such software are included in Becta's `*Using web-based resources in the daily mathematics lesson*', designed for the National Numeracy Strategy in primary schools: http://www.ictadvice.org.uk/index.php?section=tl&catcode=as_cu_pr_sub_08&rid=3669

One well-known example is *Monty* which is a program (originally developed by the ATM) that is based on the hundred square which can be arranged in various patterns. These patterns can be partially obscured by a snake (Monty Python!) and the pupils have to predict which numbers have been hidden. Some of the patterns can be very challenging. *Monty* can be used by individual pupils, small groups or by a teacher working with a whole class.

The software is freely available on CD-ROM as part of the National Numeracy Strategy's `*Using ICT to support mathematics*' pack, available from DfES publications dfee@prolog.uk.com , and can be downloaded from the Standards site: http://www.standards.dfes.gov.uk/numeracy/publications/ict_resources/12890/



2.4 Programming languages

Algorithms, sets of instructions to carry out procedures, play a key role in mathematics. Students encounter a number of algorithms, from long multiplication to Newton-Raphson, which are amongst the major achievements of mathematics. All too often learning such algorithms is a chore, partly because learners are not given the opportunity to experience the power of creating algorithms. `Teaching the computer' is an opportunity for students to gain some insight into what it means to design an algorithm and hence to begin to appreciate the importance of algorithms.

Algorithms can be explored within modelling software such as spreadsheets, and by writing `macros' to add functionality to a particular package, such as *Excel* or *Cabri Geometry*. An implementation of *Basic*, called MS *Visual Basic* is available both as a stand-alone programming package and as the macro language for tools such as *Excel*. There are also both free and low cost versions of *True Basic* available to download from http://www.truebasic.com Two other accessible vehicles for programming are *Logo* and graphical calculators.

Logo

Logo has a graphical interface which students use to instruct an on-screen `turtle'. This is a good place to start. Giving pupils a few turtle instructions, such as Forward: FD, Right: RT, Clear Screen: CS, Left: LT, means that a simple question like "Can you draw your initials?" has enormous potential for mathematical discussion and discovery (linked to areas such as estimation of angles and distances and properties of shapes). Alternatively, providing students with the instructions to draw a rectangle can allow students to draw a square by changing some of the instructions. The instructions to draw a square can be saved as a *Logo* procedure and this procedure, in turn, can be modified to take inputs so that the turtle can draw any regular polygon. In this way the turtle has been taught a general algorithm.

```
to Rectangle
Repeat 2 [FD 100 RT 90 FD 50 RT 90]
end
```

```
to Square :edge
Repeat 4 [FD :edge RT 90]
end
```

Logo draws right and left turns dynamically and emphasises the true nature of angle being an amount of turn. *Logo* is not just a geometrical tool but can be used to investigate number patterns and sequences. There is potential to use *Logo* as a tool for developing an understanding of variables and encouraging mathematical communication. A key feature is that any *Logo* procedure that is created can be used again within more complex procedures. By building such `super-procedures', students can move on from turtle graphics to computational modelling of probability, iterative processes, etc. Some versions of *Logo* are available free, such as *MSW Logo* (which can be found at http://www.softronix.com). Other useful versions of *Logo* include *StarLogo* and *LogoWriter*.

Graphical calculators

All models of graphical calculators are programmable, so that facilities which are not readily available on a particular model can usually be added by writing a small program for the purpose. Their programming language is a variation on BASIC. An interesting account `*Algebra by stealth – cartoons on graphic calculators'* of programming work with a class of Y8 pupils is given by Alison Clark-Jeavons in the report `*Good Practice in the Use of ICT in Mathematics, Science and Geography at Key Stage 3*'edited by Joyce Wood and obtainable from RM plc: http://www.rm.com/_RMVirtual/Media/Downloads/good_practice_ICT.pdf

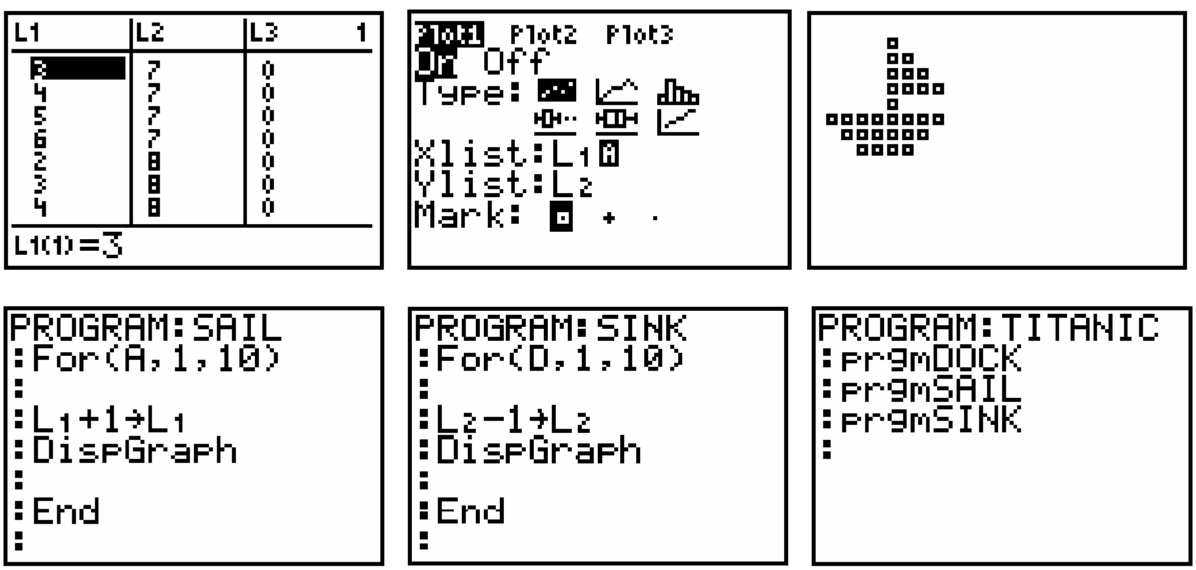
"My Year 8 students learn to produce animated pictures on graphic calculators. The purposes of this activity are to develop students' understanding of co-ordinates, provide a real application of algebra and develop their problem solving skills, by answering the question, "Here is a co-ordinate picture; now how can we make it move?"

The activity uses relatively low-level, inexpensive technology and can be carried out in the classroom. Each student has a hand-held TI-83 graphical calculator and the teacher has a calculator Viewscreen placed on an overhead projector. The Viewscreen facilitates the projection of the teacher's calculator screen for the whole class to see. It is possible to link a student's calculator easily with the teacher's to share an individual student's work with the whole class.

The first task is to create a co-ordinate picture. To do this, the students input a set of co-ordinates into the statistical list facility on the calculator and set up a statistical plot. The level of difficulty can be varied here, with students entering positive/negative integers or decimals to create a picture in one, two or four quadrants. I use a planning sheet to enable less able students to plan their pictures initially and note the co-ordinates on paper. The feedback given by the calculator display allows students to spot and amend any errors quickly and easily when producing the static picture.

The next stage of the activity is to introduce the question, "How can you make your picture move?" The first consideration for the students is that the effect of adding a constant to (or subtracting a constant from) all the numbers in the list will alter the image in some way. The teacher is able to test and verify a student's conjectures using the Viewscreen, observed by the whole class. The whole list is manipulated by using its label. For example, L1 + 1 $\rightarrow$ L1 would increase all of the X coordinates by one and translate the image one unit to the right.

To achieve a succession of translations, the students need to write a short program with a loop to repeat an instruction. Students are able to build several animations to produce a short film. Samantha's animation showed the Titanic sailing towards the iceberg, hitting it and sinking."



2.5 General purpose software:

We do rather take it for granted now that any PC, laptop or network station will give us access to a good range of general purpose software – and by default this often consists of elements of the Microsoft *MS Office* suite of software. This contains tools such as a word-processor, spreadsheet, database, slide-presenter, web-browser and e-mailer. Many teachers make use of such software for their own preparation of materials outside the classroom. HMI report: "*The majority of mathematics teachers use ICT effectively outside the classroom in the preparation of teaching materials and in the analysis of data. A minority are still not confident even in this use of ICT and require further training.*"

For teaching it is quite possible to prepare an *MS Powerpoint* slide-show, or to use the web-browser to access pages on-line via the Internet, or locally via an Intranet. The one general purpose piece of software which gets most use in mathematics teaching, though, is the spreadsheet. This has its fair share of both adherents and critics – many of whom are quite vocal! So we have taken care to debate the position of spreadsheets within mathematics teaching amongst ourselves and hope that the result gives a useful and balanced view!

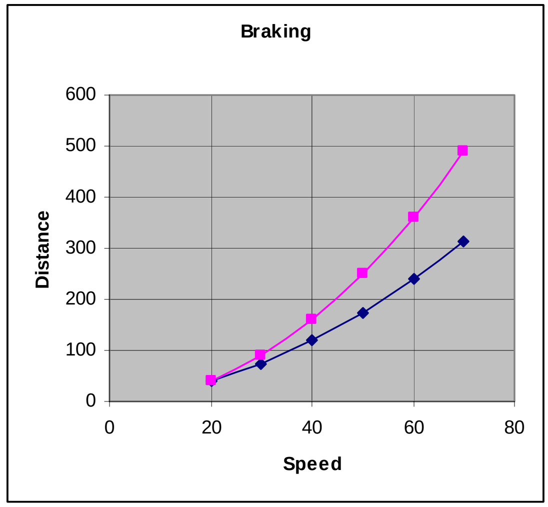
<u>Spreadsheets</u>

As the name suggests, these were originally developed as accountancy tools and are very widely used in all aspects of finance as well as many other situations involving mathematical and/or statistical modelling. Of course there is no agreed meaningful definition of just what comprises a spreadsheet. But a key feature is that parts of the sheet (usually called `cells') can be defined in terms of previous ones in such a way that any change to any cell automatically causes the values in every dependent cell to change instantly.

For mathematics we frequently want to be able to set up tables of data, which may use formulae to generate other data, and from which we can draw graphs.

For example the figure alongside shows a table of the minimum braking distances in feet corresponding to speeds in mph, taken from the `*Highway Code*'. The entries `Speed' and `Distance' in cells A3 and B3 are just labels for the columns. The value 20 has been entered into cell A4. The formula `=A4+10' has been entered into cell A5. This has been `replicated down' from cell A5 to A9. This is an example of the so-called `battleships' notation where each cell has a column letter and a row number. The data for distances has been entered into cells B4:B9.

	A	B	C	D
1	*Braking distances*			
2				
3	Speed	Distance	k	Model
4	20	40	0.1	40
5	30	75		90
6	40	120		160
7	50	175		250
8	60	240		360
9	70	315		490



With the range of cells A3:B9 highlighted, a graph (or chart) can be drawn. In this case it is a scattergram with the data-points joined by line-segments. The range of available graph types usually also reflects the major usage of spreadsheet graphics – such as the display of sales results! Hence it is usually much easier to draw a pie-chart than a box-plot.

In order to try to fit a function to the data we can define some function of the speed from a formula. Here we might try a quadratic function of the form $d = ks^2$. So the parameter k has been entered into cell C4.

Now we can define a `model' in column D by defining the cell D4 by the formula: `=C$4*A4^2' and replicating it to the block D4:D9. Note that here we have had to use a mixture of `relative' and `absolute' references so that D5 refers to A5, D6 refers to A6 etc., but that all the formulae use C4. The *MS Excel* spreadsheet allows you to define names for cells, and so you can use `Insert Name' to define `s' as the name for the block A4:A9 and `k' as the name for the cell C4. Then the formula for cell D4 can be entered as `=k*s^2' which looks rather more like algebraic notation. Since the name `k' refers to a cell, while `s' refers to a range, we will not have to worry about absolute and relative references when replicating the formula down column D.

Now we can just change the value of `k' in the cell D4 to see the effect on the graph. This gives a powerful way of exploring problems involving data-handling and/or algebra – but some forethought is needed into the best way of laying out the sheet's design.

So in what ways might you use a spreadsheet in teaching mathematics? You might start with a blank sheet and develop an activity `from scratch'. An example of this is shown in the Y8 case study on the TTA CD-ROM where the class enter their wrist and neck sizes as the data. They can sort the data, for example by wrist size, to find the median and quartiles. They also plot the neck against wrist size as a scatterplot and start to look for a linear model which gives a good `fit' to the data. Materials to support this lesson have been developed by the Mathematical Association and will be made freely available to schools as part of the DfES' ` *KS3 ICT Offer'* at BETT 2005.

Another approach is to have a pre-designed sheet, known as a `template', into which students enter their data without having to worry about issues such as formatting. This allows them to spend more time concentrating on the mathematics rather than on the mechanics of the spreadsheet.

An alternative approach is to obtain ready-to-use sheets designed for a specific task. Examples of these are the materials produced from the Royal Statistical Society's Centre for Statistical Education such as for census data: http://www.censusatschool.ntu.ac.uk/SpreadGraph.asp and for experimental data: http://experimentsatschool.lsz.ntu.ac.uk/ . Other good materials are available from DISCUS: `*Discovering Important Statistical Concepts Using Spreadsheets'*: http://www.mis.coventry.ac.uk/research/discus/discus_home.html .

Below are some further ideas for ways of using spreadsheets, based on example in *MS Excel*, provided by one of the contributors:

Characteristics of the spreadsheet which make it a useful tool in the teaching and learning of mathematics include:

- A spreadsheet is responsive.
  Calculated values within a sheet respond instantly to changes in input values, which enables the effect of variables within a context to be explored.
- A spreadsheet amplifies intellectual effort.
  Replication of formulae, by Copy-Paste or Fill, enables the generation of a large number of new calculated values with minimum effort.
- A spreadsheet offers a content-free modelling space.
  Dynamic representation of the context under investigation allows connections between the elements within that context to be explored.

The spreadsheet can also be valuable as an interactive presentation environment :

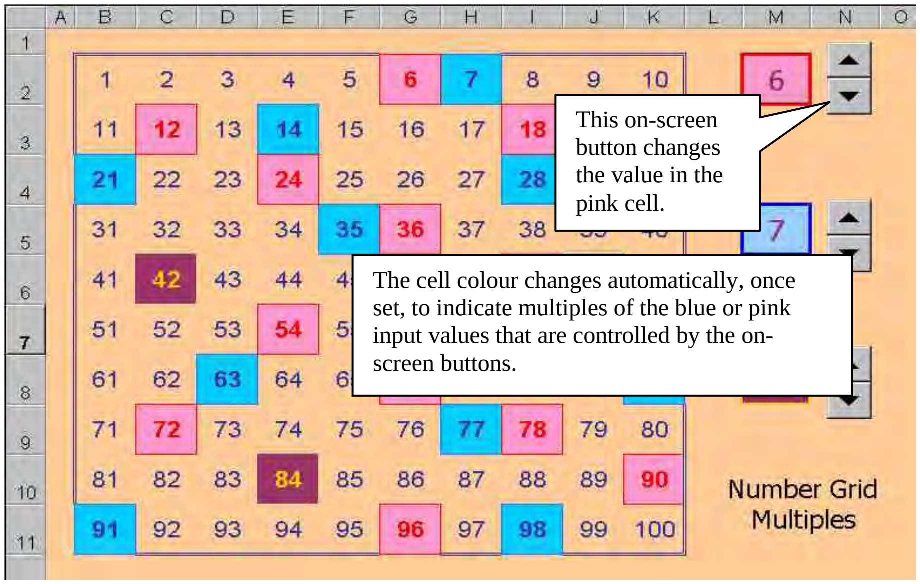
- Linked values are a defining feature of the spreadsheet. The capability to link and respond enables the spreadsheet to offer interactivity.
- On screen buttons enable incremental changes to cell values without using the keyboard.
- Value sensitive formatting allows the appearance of text, values, and background, to change according to the intentions of the designer, and so give high value feedback, or other information, to the user.

The very wide range of contexts that can be presented in this way allows individual pupils, or even the whole class, to explore mathematical relationships without the repetitive calculations which were previously necessary but added little value for the learner.

There are a number of valuable functions or operations, beyond basic calculation, which offer pupils, at both KS 3 and KS 4, access to interesting ideas.

- Maximum and minimum values (and other statistical calculations) can be automatically obtained.
- The Modulo function can be of great value when investigating factors, multiples, or remainders.
- The Integer function, which returns only the integer part of a value and discards the decimal or fraction component, has many uses, including splitting a number into its component digits.
- Charts, or graphs, on a spreadsheet are dynamically linked to the source data values so that graphical representations are automatically re-drawn as source data values change.

- The appearance of individual cells can be made to vary in response to values falling within `trigger' intervals (`conditional formatting').
- On-screen buttons can be defined which make the keyboard unnecessary for entering / changing cell values (an *Excel* feature on the Forms toolbar) – these are particularly useful on an interactive whiteboard. See the illustration screen shot below. *Number Grid Multiples* is a spreadsheet-based resource for use on an interactive whiteboard.



- Producing a table for a function of two independent variables can be a very valuable technique. Many contexts of interest even at KS3 involve more than one independent variable. The illustration alongside shows part of a table used to discover Pythagorean Triples, each cell in the table contains the square root of the sum of the squares of values in the first row and first column. The basic formula is entered in the first cell only, then copied across the complete range of the table. Additionally, in this illustration, conditional formatting has been applied to drawn attention to cells that hold integer values and indicate triples.

	1	2	3	4	5	6	7	8	9	10	11	12
1	1.414	2.236	3.162	4.123	5.099	6.083	7.071	8.062	9.055	10.05	11.05	12.04
2	2.236	2.828	3.606	4.472	5.385	6.325	7.28	8.246	9.22	10.2	11.18	12.17
3	3.162	3.606	4.243	5	5.831	6.708	7.616	8.544	9.487	10.44	11.4	12.37
4	4.123	4.472	5	5.657	6.403	7.211	8.062	8.944	9.849	10.77	11.7	12.65
5	5.099	5.385	5.831	6.403	7.071	7.81	8.602	9.434	10.3	11.18	12.08	13
6	6.083	6.325	6.708	7.211	7.81	8.485	9.22	10	10.82	11.66	12.53	13.42
7	7.071	7.28	7.616	8.062	8.602	9.22	9.899	10.63	11.4	12.21	13.04	13.89
8	8.062	8.246	8.544	8.944	9.434	10	10.63	11.31	12.04	12.81	13.6	14.42
9	9.055	9.22	9.487	9.849	10.3	10.82	11.4	12.04	12.73	13.45	14.21	15
10	10.05	10.2	10.44	10.77	11.18	11.66	12.21	12.81	13.45	14.14	14.87	15.62
11	11.05	11.18	11.4	11.7	12.08	12.53	13.04	13.6	14.21	14.87	15.56	16.28
12	12.04	12.17	12.37	12.65	13	13.42	13.89	14.42	15	15.62	16.28	16.97

The value of the spreadsheet as a tool, or environment with which to support the teaching and learning of mathematics, is much reduced if use is limited to a series of isolated experiences. The spreadsheet, like other ICT, facilitates the most significant learning gains in mathematics where establishing a `culture of use' is a curriculum development goal. Teachers and students need to be free to give their attention to the ideas being explored - uncertainty with the technology, due to infrequent use, seriously inhibits

Another contributor comments: "*One powerful aspect is the ability to apply a repeated sequence of operations a large number of times. This is particularly relevant to the generation of sequences, and to iterative methods. The 'suck it and see' aspect is very relevant to trial and improvement methods. In some circumstances spreadsheets can be very good simulators of random processes with a large number of trials; RND() can be copied just as easily as any other function*." See e.g. the article `*Probability Simulation with a Spreadsheet'* by Richard Bridges in *Mathematics in Schools*, September 1999.

Databases

There are a number of packages designed to aid sorting and classifying data in a variety of ways, as well as their representation in graphs and charts. These are not specific to mathematics and can be used in any subject where classification according to different criteria is important. They differ from spreadsheets as they can be used to classify and represent non-numeric data. They can be used to support the `PCAI' data handling cycle. Some packages start from questionnaire design, then automatically sort and classify responses. This shifts the focus from collecting and classifying data to deciding on appropriate representations, interpreting data and evaluating findings. These are the important skills of handling data, often not given sufficient attention. The use by other subject departments of databases, particularly where they prompt discussions about the appropriate forms of representing different data sets, should be part of each school's agenda for Numeracy Across the Curriculum. Databases linked to surveys can successfully form the focus of group work in mathematics lessons. Using a database for questionnaires and surveys in KS3 will give pupils opportunities to consider appropriate ways of representing data and will equip them for coping with statistical elements of the current GCSE requirements for coursework.

Wordprocessors

MS Office includes a tool called *Equation Editor* for type-setting mathematical notation. You use the Insert Object menu to open its editor. *MathType* 5 is the professional version of the *Equation Editor*. It claims to include new technology for converting *Word* documents to web pages that use high-quality mathematical notation. Another useful addition is SPA's *FxDraw* software to enable you to include mathematical diagrams, symbols etc.

2.6 Mathematics specific software:

There is a range of tools specifically designed to help solve mathematical problems, some of which have an explicitly educational intent, while others may be used to make useful contributions to teaching and learning mathematics.

<u>Graph plotters</u>

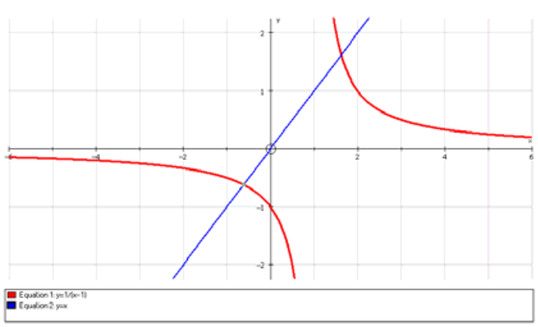
These are probably the most frequently used tools in mathematics teaching. There are several software packages whose primary function is to plot graphs. These include *Autograph*, *Coypu*, *Omnigraph* and *Winplot*. The built-in software in graphical calculators includes a graph-plotter. Other packages also include facilities to plot graphs. These include dynamic geometry such as *Geometer's Sketchpad 4* and *Cabri II Plus*, computer algebra systems such as *Derive* and integrated mathematics software such as *TI InterActive!*

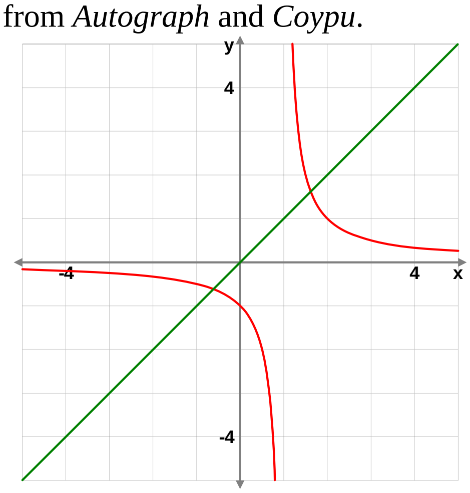
Common features include the ability to:

- use a wide variety of mathematical functions (such as sin(x), e^x etc.);
- show several graphs on the same axes;
- vary the colour, style and weight of graphs of functions;
- label graphs, axes, points etc.;
- rescale the axes easily, including zooming in and out;
- control the way axes and grids are displayed;
- mark points of interest, such as maxima or intersections;
- enter data and display statistical plots.

Different packages offer additional facilities, often with a certain age-range or syllabus in mind. These include tools for calculus, numerical mathematics, statistical distributions, regression and transformation geometry. Most plotters allow function definitions in standard $y = \mathrm{f}(x)$, parametric $x = \mathrm{f}(t)$, $y = \mathrm{g}(t)$ and polar $r = \mathrm{f}(\theta)$ formats. Some plotters allow alternative formats such as $x = \mathrm{f}(y)$ and implicit definitions: $\mathrm{f}(x,y) = k$. Some also have tools for the exploration and solution of differential equations.

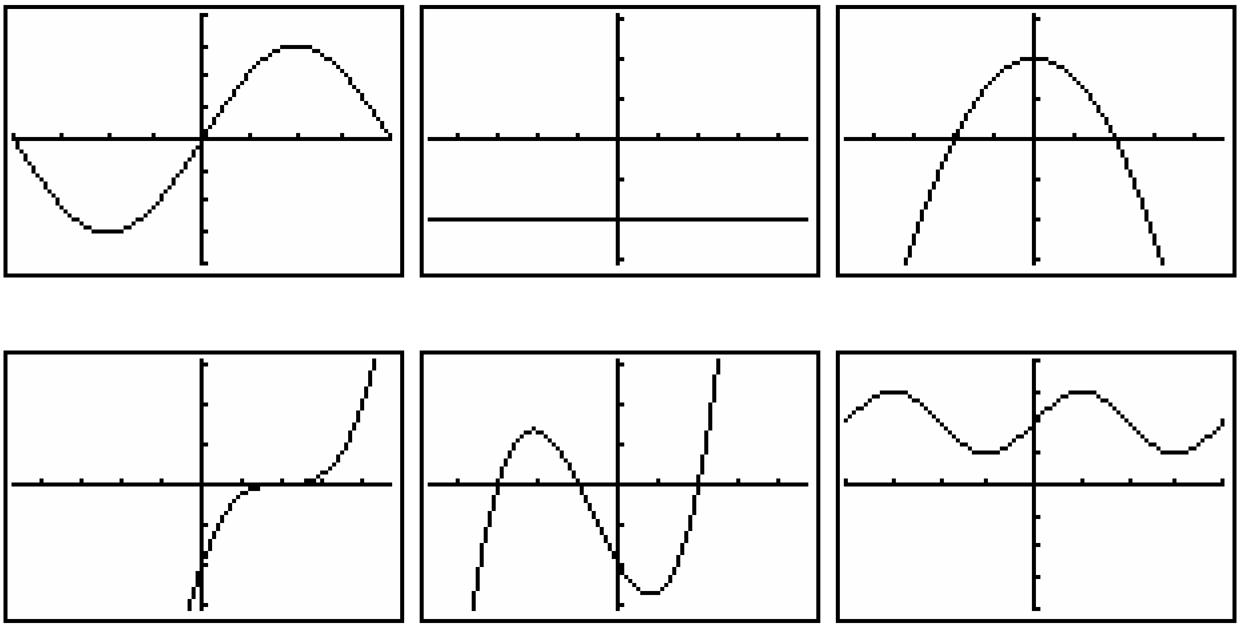
Below are example of plots of $y = 1/(x\text{-}1)$ and $y = x$ from *Autograph* and *Coypu*.





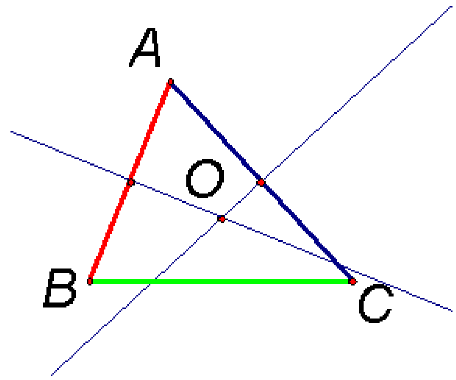
If you plan to use graph-plotters for whole class displays then it will be important to be able to select line styles and colours which are very visible. Of course graph-plotters are also very useful outside the classroom so that you can copy and paste graphs into documents such as task-sheets and examination papers!

An activity that has validity at almost any level is to plot the graph of a function whose equation is hidden from view and to try to find an equation whose graph matches it. (If you use this activity with graphical calculators it is useful to have graphs pre-printed on pieces of acetate, such as OHTs, which can be overlaid on the screen.) Below are some examples extracted from pp 90-1 of `*Teaching Mathematics using ICT*'.

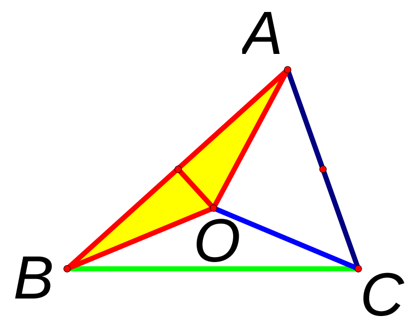


Dynamic geometry software

This software has evolved from a package written for the Apple II computer over 20 years ago called the *Geometric Supposer*, written by Judah Schwarz and others at MIT. Essentially it provides a range of tools for you to construct geometric objects from a range of `primitive' objects (points, segments, lines, circles etc.) using both `classical' constructions (midpoint, perpendicular, parallel etc.) and transformations (reflect, rotate, translate etc.) Once drawn, measurements can be taken from objects (length, angle, area etc.). The `dynamic' aspect comes from the ability to drag defining objects, such as points, round the screen to deform the resulting shape – and hence to be able to look for aspects that always stay the same (`invariants'). The dragging is usually done with some analog control device. Originally this was the `game paddle' of the Apple II. Now it is usually the mouse, or tracker-ball, of the PC or laptop. Of course dynamic geometry software (DGS) is ideally suited for use with an interactive whiteboard where the dragging can be done with a stylus or even the finger. The two most common DGS packages for schools are *Cabri Geometry* and the *Geometer's Sketchpad* (GSP). There is also a free DGS package called *Wingeom* in the Peanuts software series obtainable from Richard Parris at: http://math.exeter.edu/rparris . Another package called *Cinderella* has been particularly designed with the production of interactive web-pages in mind. (Most DGS packages now have facilities to export `live' diagrams in *Javascript*.) *MathsNet* maintains a page detailing these packages at http://www.mathsnet.net/dynamic/ and also has many examples of interactive geometry web-pages.



As an example we will show the construction for the circumcentre *O* (and hence the circumcircle) of a triangle *ABC* using *Sketchpad.* The points *A,B,C* have been constructed and segments drawn between them. With segment *AB* highlighted the Midpoint tool is selected from the Construction menu. With both the midpoint and the segment *AB* selected the Perpendicular tool is selected to construct the perpendicular bisector of *AB*. A similar sequence is used to construct the perpendicular bisector of *AC*. With both perpendicular bisectors highlighted *O* is constructed with the Intersection tool from the Construction menu.



The display of the figure can be adjusted by selecting differing line styles, weights and colours for segments and lines, and by labelling some or all of the points in a variety of fonts, sizes and styles. Using the mouse you can drag any of the points *A*, *B* or *C* around independently to deform the triangle *ABC*. This shows some of the similarities and differences between drawing by hand and with DGS software.

With the dynamic image we can test hypotheses such as: if we drag *A* around

- does *O* always lie within the triangle;
- for what sort of triangle does *O* lie below *BC*;
- when will *O* lie on *BC*;
- what is the locus of *O*…?

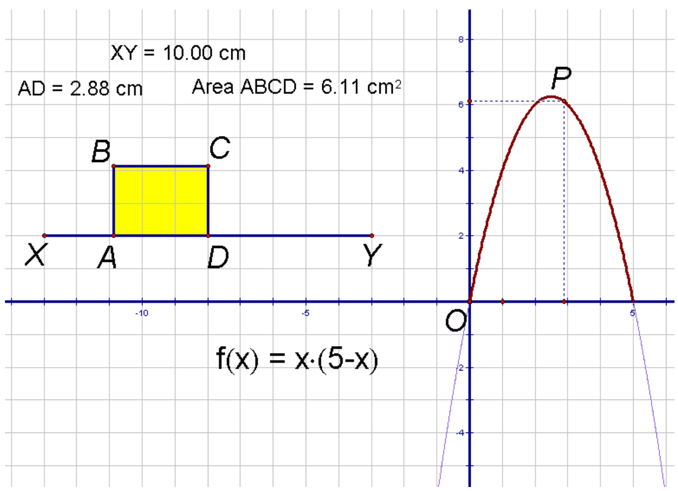
Here we start to arrive at a real issue in the use of DGS – the software can't prove anything! It can give us lots of clues, but we have to be alert to the geometric reasoning that is needed to establish results like *O* is the point equidistant from *A*,*B* and *C*. In the second of the pair of diagrams above we have highlighted the triangle *AOB*. If we can assume that we already know that the locus of points equidistant from *A* and *B* is the perpendicular bisector of *AB*, then we can deduce that *OA* = *OB* and hence that *AOB* is **isosceles**. Similarly we can deduce that *BOC* is isosceles, and hence that *OB* = *OA* = *OC*. Which explains why *BOC* must also be isosceles and why *O* must lie on the perpendicular bisector of *BC*.

The ATM publish a useful set of resources for teaching geometry with both *Cabri Geometry* and *The Geometer's Sketchpad* called *Active Geometry*. These comprise 17 files and over 100 associated worksheets offering various activities on exploring mainstream topics in shape and space.

The report `*Teaching and Learning Geometry 11-19*' by the Royal Society/JMC working group recommends the increased use of ICT in teaching geometry and says:

"Geometry teaching outside primary schools can be, and has been, conducted with a minimal amount of equipment - such as a stick of chalk and a piece of string (echoing images of ancient Greeks drawing in the sand). Our view is that teachers should now have at their disposal an appropriate variety of equipment from which to select, depending on fitness for purpose. In particular we wish to see the potential of ICT realised in supporting the teaching and learning of geometry. There is already software available, such as for Dynamic Geometry, but its use is not widespread. Many schools do not have licences for the software. There is also a need for the development of additional software, such as to support work in 3 dimensions. Increasing numbers of schools and colleges are now being equipped with `interactive whiteboards' - where a computer image is projected onto a touch sensitive screen. This medium has considerable potential for interactive whole-class teaching of geometry. We would like to see the funding to schools for ICT being used more effectively to support the geometry curriculum."

As well as tools for constructions, transformations and measurement, DGS usually include the means of drawing loci, performing animations and working with coordinates. Both *Cabri* and *Sketchpad* include a Calculator tool which enables measurements to be combined. Measurements, and functions of them, can thus be used to define the coordinates of a point plotted in an axis system – and the locus of such a point will represent the graph of function. Hence DGS are not restricted solely to geometric applications but can bridge neatly into algebra. As DGS evolves it will embrace more tools already associated with other forms of mathematical software. For example, new features in version 4 of *Sketchpad* and in *Cabri Geometry II Plus* included some graph-plotting tools.



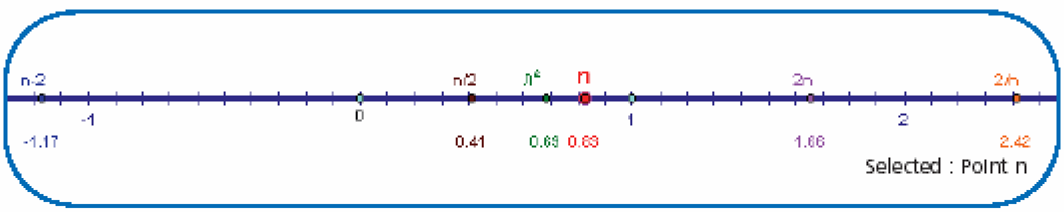
The illustration below shows a modelling situation where a segment *XY* has been set up to represent the length of a piece of string which is used to enclose a rectangle *ABCD*, whose area and side have been measured. The point *A* can slide on the segment formed by *X* and the midpoint *D* of *XY*. Thus *A* is the independent variable with segment *XD* as its domain. The values of the distance *AD* and area *ABCD* are used to define the coordinates of the point *P* plotted on the axis system. As *A* slides on *XD* so *P* traces out the graph of area against side – which is shown as the locus of the dependent variable *P* against *A*. The analytic form of this curve can be derived, entered as a function f(x), and graphed to compare with the locus.

Another very powerful feature is the ability to import digital images either from files or using the clipboard. An illustration is shown in the section 2.10 below on digital image technology.

These extensions to the power of dynamic geometry software have been explored through a small project `*Using ICT as a bridge between algebra and geometry*' supported by QCA in 2003 as part of its Algebra & Geometry project. The project was co-directed by Ken Ruthven and Adrian Oldknow and involved 4 schools in Hampshire. The QCA publication `*Developing reasoning through algebra and geometry*' (£6, QCA/04/1289 from QCA publication 01787 884444 or download from: http://www.qca.org.uk/ages14-19/subjects/7421.html) pp23-27 gives examples of the sorts of activities trialled in the schools. One modelling activity, known as `*Beachwatch'* is the basis of a video case study to be disseminated in January 2005 by DfES. Another activity copied below, `*Dynamic number lines'*, is the basis of a lesson, and supporting materials, in the Mathematics Consortium ESTUICT on-line CPD programme (www.cpd4maths.co.uk).

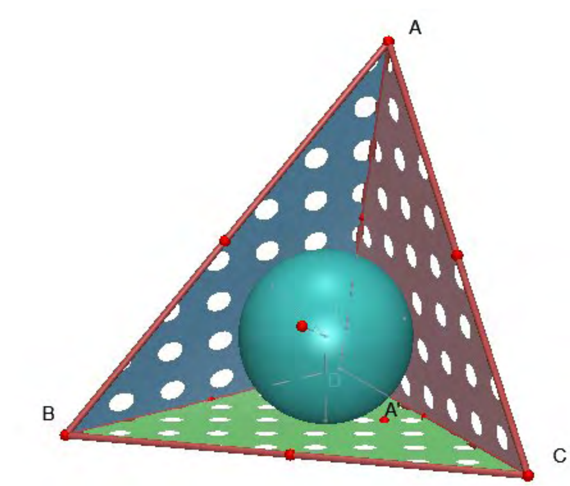
Example – Dynamic number lines

A *dynamic number line* is a construction using dynamic geometry which gives a way of visualising the relationship of a variable to various functions of it. In the diagram below, as the point representing the variable n is dragged along the line the points representing $n - 2$, $n/2$, n^2, $2n$ and $2/n$ move accordingly.



From the starting position shown, as n increases – moves to the right – all the other points do likewise with the exception of $2/n$ which decreases – moves to the left. Likewise, as n decreases, so initially do all the other points except $2/n$. However, as n passes through 0, the direction of movement of n^2 reverses; it seems to 'bounce off' the 0 position; in addition, n^2 'overtakes' n as they pass together through 1. Similarly, as $n/2$, n, and $2n$ pass through 0, their relative positions reverse; and these three variables move along the number line at different, but clearly related, 'speeds'. By contrast, $n - 2$ never exchanges position with n; rather, both move at identical 'speed', and maintain a fixed 'distance'.

By the time this guidance is published, the latest version of *Cabri Geometry* will have been launched, called *Cabri 3D*. This contains an extensive set of tools for investigating 3D geometry using primitives such as planes, cylinders, cones etc. together with their intersections. Modern operating systems such as Windows XP contain tools developed to support the writers of games and simulations to manipulate and display 3D objects. This means that once a 3D object has been built-up then you can easily rotate it and zoom in or out.



Data-handling tools

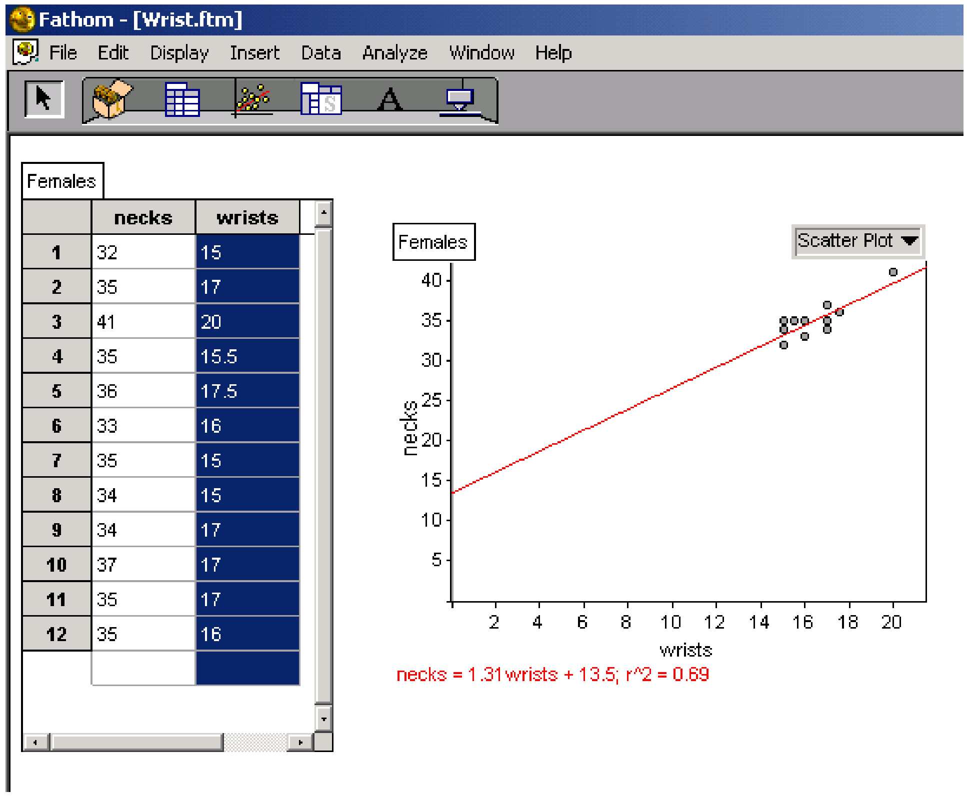
A wide range of ICT tools has been developed for handling data. These tools facilitate the collection, communication, analysis and interpretation of data. Tools include data loggers, graphical calculators, the Internet and specific software for computers. ICT tools enable students to:

- formulate questions;
- undertake purposeful enquiries;
- collect and represent data;
- engage in practical and experimental work;
- appreciate randomness;
- question the representation of data;
- draw and criticise conclusions.

ICT tools provide the opportunity to work with data that has meaning to students so that they have a measure of ownership that is real to them. They aid students in specifying problems, in testing hypotheses, in dispelling myths, and collecting data. Large data sets can easily be gathered, e.g. from the Internet or the KS3 Mathematics Strategy CD-ROM. They can be differentially sectioned within the software. The resulting data sets can then be explored by students using whatever is the most convenient ICT tool. For example they can be distributed to graphical calculators for analysis individually by students. This exploratory analysis is supplemented through built in analytical procedures and tests which can lead to interpretation and discussion, before returning results to the PC in order to write up a report.

Fundamental concepts of statistics are made accessible through the use of ICT. For example the nature of data and the distinction between discrete and continuous data can be brought to life using a radiation monitor to collect times of random events. The resulting data could then be modelled by a Poisson distribution or an exponential distribution. The distinction between the data and the scale used to record the data becomes a reality for students. Bivariate data, and the concepts of correlation and regression, can be easily explored through the use of data logging equipment. The use of one distribution to approximate probabilities for another can be explored graphically and numerically. ICT provides many opportunities for students to consider issues surrounding sampling, such as methods, stratification, repeated measures, random samples, convenience samples, sample sizes and bias.

Many of the ICT tools listed separately here can be readily employed for data-handling work, such as spreadsheets like *Excel*, graphical calculators like TI-83 Plus, data-loggers like the CBR, graph plotters like *Autograph* and integrated mathematics packages like *TI InterActive!*. There are also special purpose tools designed for educational data-handling. Perhaps the most interesting of these is the `*Fathom*' package from Key Curriculum Press which can be obtained from both Chartwell-Yorke and QED. Another shareware package from Peanut software http://math.exeter.edu/rparris/ is *WinStat*. The figure below shows the fitting of a linear regression model to wrist and neck measurements using *Fathom*.

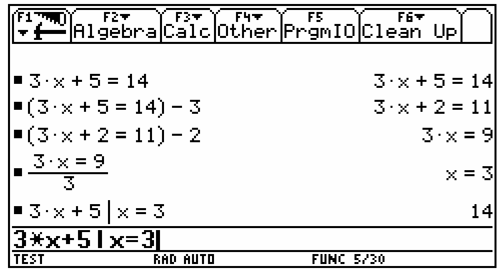


The use of online questionnaires such as those offered by the RSS' CensusAtSchool Project are very beneficial. They collect and disseminate real data from and about the children giving the pupils personal involvement in the data thus encouraging meaningful and realistic analysis and inference.

Computer algebra systems

Originally computers were designed for civil and military tasks involving the manipulation and computation of numeric data. But as computer tools were developed for working with strings of symbols it became apparent that many of the rules we use for symbol manipulation in algebra and calculus could also be turned into computer algorithms – an obvious example is the rule for differentiating x^n where x is real and n is an integer. Such tools were developed for mainframe computers to handle the vast amount of routine manipulation involved in applications such as in theoretical physics – where their complexity meant any attempt to carry these out by hand would lead to many errors. From these a variety of packages were developed to help researchers in mathematics and physics, onto which a variety of facilities were added to make them useful for teaching and learning as well. Examples of these Computer Algebra System (CAS) packages include *Mathematica*, *Maple* and *Reduce*. Of considerable educational importance was the development of the *Derive* package with many of the features of its predecessors, but designed to run on common PCs at a price education could afford – it used to be advertised as "*a college education for $200*". Another educational version of CAS is called *LiveMath*. CAS is also available on a number of hand-held platforms, including some graphical calculators. As with ordinary calculators, which throw into question just how much arithmetic facility we now need, so CAS raises similar issues with respect to symbolic facility in algebra and calculus. The RS/JMC report `*Teaching and Learning Algebra pre-19'* raises some of these issues. The Mathematical Association's report `*Symbolic Manipulation by Computers and Calculators'* addresses these further. The ATM's `*Computer Algebra at A-level'* focuses more closely on issues in 16-19 mathematics.

One problem with symbol manipulation is that there can be many equivalent forms of output and the software usually has to `guess' which form the user expects. But in teaching this may be something which can be exploited to advantage. As an example the figure shows some manipulation to solve an equation and to check the result.



While most examination syllabuses currently prohibit the use of CAS there are exceptions. Both the Scottish Highers and the International Baccalaureate are planning to introduce their use in the near future. Whether or not students are allowed to use CAS in examinations they can still be of benefit both to teachers and to learners. Since they contain many features of a more advanced nature, such as for calculus, complex numbers, matrices etc their main use may extend beyond the KS3/4 curriculum.

Integrated mathematics packages

We have already seen how different types of mathematical software can have very different roots – for example DGS was explicitly developed for an educational context while CAS has been developed mainly for academic research. Many working engineers and applied mathematicians needed rather different tools which would allow them to set up problems for numerical solution, produce results, test them and write a report explaining the process. Mathsoft's *MathCad* was, and is, the major package for such `computer-assisted mathematical modelling'.

By contrast Texas Instrument's *TI InterActive!* was developed explicitly for education and contains a blend of functions found in (a) *MathCad*, (b) *Derive*, (c) the TI-83 graphical calculator and (d) *MS Office*! It was this range of functions that led to *TI InterActive!* (TII) being adopted as the main mathematics tool, along with graphical calculators and the *Geometer's Sketchpad*, for the DfES Year 7 pilot project: MathsAlive! Its features include:

- a graph-plotter for functions and data;
- a spreadsheet – a subset of *MS Excel*;
- a symbol manipulator – very similar to *Derive*;
- a data editor with facilities for data-handling, probability and statistics with lists;
- a word-processor – a subset of *MS Word*;
- a web-browser with easy means of capturing data into lists – a subset of *MS Internet Explorer*;
- an e-mailer – a subset of *MS Outlook Express*;

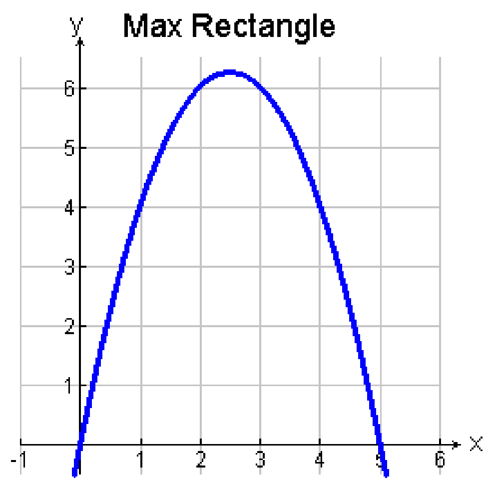
- live mathematics boxes where values and functions may be defined and altered so that any dependent boxes, graphs etc. are recomputed when changes occur;
- an easy interface with data-loggers (CBR, CBL) and graphical calculators (TI-83) for capture and exchange of data;
- easy ways to transporting data and documents using a variety of compatible file formats.

You can click on any live part of a TII document to edit it – and used with large fonts, thick lines etc. it is an ideal medium for whole class displays of mathematics such as with an interactive whiteboard. Its use with a CBR data-logger is shown on one of the video case studies to be launched by DfES at BETT 2005. It is also a very useful tool for students to use to communicate their mathematical results, such as in undertaking the mandatory Handling Data coursework task at GCSE. The software qualifies for e-Learning Credits. An extensive review of *TI InterActive!* is available from the ATM's Micromath site:
http://www.atm.org.uk/journals/micromath/articles/index.html

A rectangle has a perimeter of 10 and a side length of x units.
So its breadth is given by 5-x units and its area by x(5-x) square units.
We can explore both the table and the graph of this function:

x	x*(5-x)
0	0
1	4
2	6
3	6
4	4

x	x*(5-x)
2	6
2.1	6.09
2.2	6.16
2.3	6.21
2.4	6.24



And if we knew some calculus we could do some manipulation!

$\frac{d}{dx}(x\cdot(5-x))$ $\quad$ $\text{solve}(ans = 0, x)$ $\quad$ $x\cdot(5-x) \mid ans$

$5 - 2\cdot x$ $\quad$ $x = \frac{5}{2}$ $\quad$ $\frac{25}{4}$

2.7 Integrated Learning Systems

An Integrated Learning System (ILS) consists of curriculum software with a built-in management system. This means that the software records all the work that each individual student does. Student reports can be produced when required. These reports can then be used to identify individuals' strengths and weaknesses. Two of the most common ILS systems in UK secondary schools are *Successmaker* and *Headstart*. *Successmaker* includes two main mathematics courses – `Concepts and Skills' and `Investigations'. `Concepts and Skills' can give students practice in basic work. The `Investigations' course is a suite of modules providing challenges and open work – probably for more able students. However you will need to be selective with these modules. *Successmaker* is not only for mathematics but includes English (and science) curriculum software as well. *Headstart* began as an ILS to support GCSE Mathematics. The original product claimed to cover the National Curriculum programmes of study at Key Stage 4 for level 6 and above. It now includes a Foundation version covering Levels 2-5 of the National Curriculum.

Schools need to have a clear rationale for using this type of software - aims and objectives need to have a clear focus. The implementation model is critical to the success of the system and needs to be monitored and perhaps adapted over time. Some schools encourage their students to use ILS at breakfast clubs, lunchtime, or sometimes after school. Use during lesson time poses a number of management issues. Some schools withdraw students from normal lessons to use ILS. However this has the disadvantage of students missing some of their timetabled lessons. The model of use needs to be considered in the light of the current National Strategies. The cost of ILS is quite considerable and schools will need to consider carefully whether better value for money could be achieved by using other ICT approaches.

Teachers need appropriate training on the use of these systems. This is normally provided by the supplier as part of the purchase price. Students should have an induction into the courses and not just be left to start alone. The courses should have appropriate regular use by particular students. For example, daily use is recommended for the basic `Concepts and Skills' course of *Successmaker*. It seems to work better if the students can have some form of supervision. It is recommended that reports should be printed off regularly to inform staff of student progress. These should then be analysed and used to help inform future teaching and learning.

Independent evaluation and advice reports can be obtained from Becta. The suppliers of the software should be able to give information about successful implementation sites. Information about *Successmaker* can be obtained from RM Learning Systems, New Mill House, 183, Milton Park, Abingdon, Oxon, OX14 4SE. Tel. 0870 908 6700. E-mail address: rmls@rm.com. Information about *Headstart* can be obtained from Headstart Software Marketing Ltd, Vauxhall Industrial Estate, Ruabon, Wrexham, LL14 6AH. Tel. 01978 810000. E-mail address: post@headstartsoftware.co.uk.

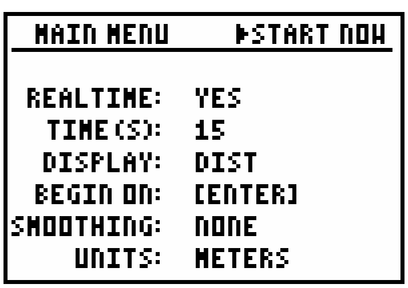
2.8 Data-loggers

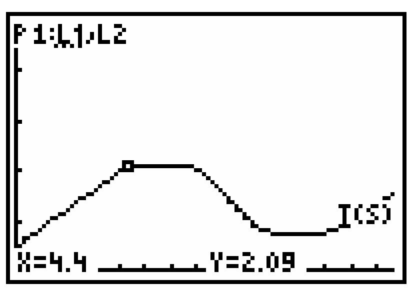
Perhaps it is surprising to see these included when their more obvious curriculum `homes' would seem to be science, D&T and ICT. Of course the forging of cross-curricular links using

ICT as a catalyst cannot be a bad thing. But there are many aspects of the mathematics curriculum whose origins lie in modelling scientific phenomena and which can be brought to life through practical contexts. The most obvious of these is the interpretation of distance-time graphs which can be greatly facilitated with the use of a motion detector (i.e. a distance measurer). Distance data may be captured from a variety of activities which include someone walking to and fro, or a pendulum swinging, or a model car running down a ramp, or a ball bouncing, or a spring-mass system in oscillation. Once the data have been captured using a data-logger then they can be manipulated, displayed, analysed etc. using any of the normal ICT tools for data-handling. Thus an experiment can be performed in front of the whole class, and the data then distributed for group, paired or individual work.

There is a wide range of data-capture systems and accompanying probes in use in other subjects, but we will concentrate on two particular devices designed for use both with graphical calculators and PCs. These are the Texas Instruments Calculator Based Ranger (CBR) and Calculator Based Laboratory (CBL). The first of these is a device about the size of a pack of three golf-balls which emits and collects an ultra-sound signal (like the way a bat navigates in flight). The second is a more versatile control unit which can be connected to a number of probes simultaneously – such as for temperature, light intensity and voltage. The Key Stage 3 Framework for mathematics includes examples of the use of both devices, e.g. for the successive bounces of a ball, and for the cooling of a liquid. They can be used throughout the 11-19 curriculum providing data for modelling using linear, quadratic, inverse, sine and exponential functions. One of the DfES video case studies for BETT 2005 shows Y8 pupils working in groups with laptops, CBRs and *TI InterActive!* for activities with distance-time graphs. The corresponding lesson materials will also be freely available to schools as part of the DfES' KS3 Offer to schools from BETT 2005.

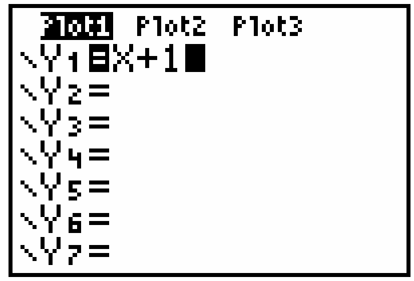
An account of their use by mathematics and science teachers at KS3 can be found in the Becta publication `*Data-capture and modelling in mathematics and science*'. This can be downloaded from: http://vtc.ngfl.gov.uk/uploads/application/datacapture-16796.pdf



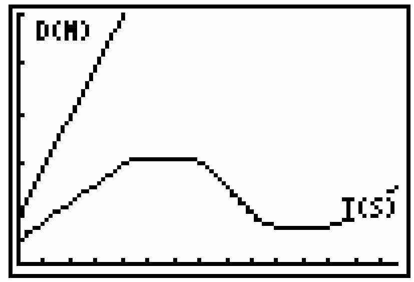


The TI-83 Plus graphical calculator, as used in RM Maths Alive, has a built-in application `CBL/CBR' to facilitate the use of data-loggers. In this example we capture distances walked in a straight line.

You can use the left and right cursor keys to trace and read off from the graph. You can also make `qualitative' observations, such as `walking away', `standing still', `walking toward', `constant speed', `faster', `slower', `longer', `shorter' etc. You can take measurements from the graph and thus can make `quantitative' remarks such as "in the first 4.4 seconds you moved from 0.46m to 2.09m away" or "your average speed was 0.37 m/s". A popular activity is to draw a target distance-time graph on the whiteboard, or OHT overlay, and to ask students to try to match it. When you choose to "QUIT" the application, the time data (in seconds) are stored



in list L_1 and displacement data (in metres) are in list L_2. Pressing [GRAPH] redisplays the last graph shown, and by pressing [TRACE] you can take readings from it. So you can now try to `fit' a model to any part of the motion. Enter a function in Y_1 which you think might match the first part of the distance-time graph. The screens above show a first attempt - can you improve on it?



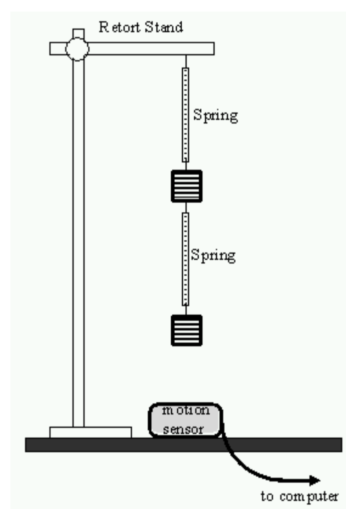
Now think hard about the various terms in the definition of Y_1 above. $Y_1(X)$ is a measurement in metres of the distance from the CBR at any time X in seconds. So when you use a function like $Y_1(X) = 0.37X+0.46$ it is clear that 0.46 must also be measured in metres, and that $Y_1(0)=0.46$, so that it is the distance away from the CBR when the time is 0, i.e. the starting, or `initial', displacement. Similarly 0.37X must be in metres, but X is in seconds, so 0.37 must be in metres/sec (or $ms^{-1}$) and so is a speed (or, since it can be negative, strictly a `velocity'). Hence the gradient of the line represents the average, or constant, velocity.

An alternative to hand-held technology is given by Tim Morland:

"A lot of emphasis has been given to the place of 'handheld technology' in school Mathematics, and for good reasons. We have chosen another route, largely because we have reasonable access for our students to desktop PCs and also because it seems unnecessary to duplicate resources when the school's science department are equipped with data-logging equipment. To show the sort of thing that we have done, I would refer enquirers to my article "Modelling a Simple Mechanical System" in the IMA journal "Teaching Mathematics and its Applications" (OUP), Volume 18, No.2.

The necessary mathematical background required for students is

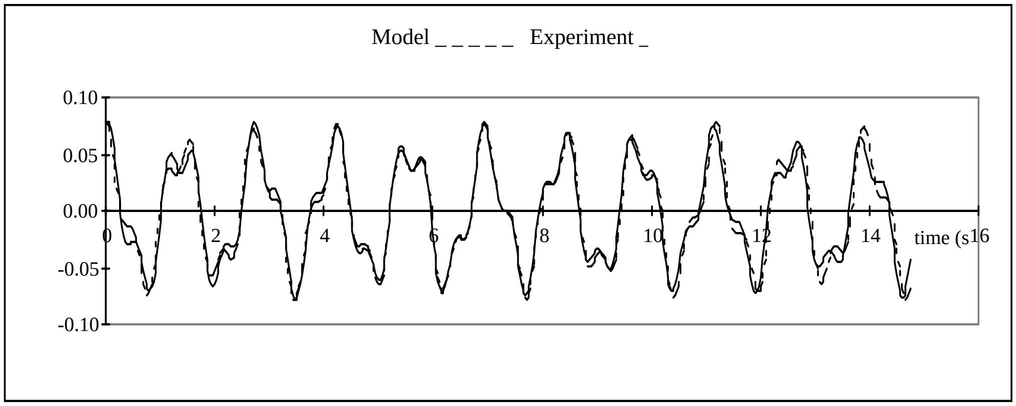
- *Hooke's Law*
- *Newton's Laws*
- *Solution of linear differential equations*



This would be covered by most students preparing for a Further Mathematics qualification.

One experiment uses the physical apparatus shown opposite, and the displacement of the lower body is recorded after the system is set in motion according to a variety of initial conditions.

The actual motion of the body is compared to the predictions of a mathematical model constructed by students. After some refinement of the model, satisfying agreement can be reached."



2.9 Sources of data and mathematical stimulus: the Internet and CD-ROMS

The Internet

There are some technological changes which revolutionise education. It is difficult to imagine how we coped before the advent of the photo-copier! The same is true of the Internet. It is an unparalleled mechanism for making available to anyone, almost instantly, and at near-zero cost, anything that is expressible on a PC. Fortunately there is already a good deal of both advice and resources to support ICT use in mathematics already there waiting to be found! In the next section we give a selective list, such as those available from Becta and the Virtual Teacher Centre (VTC). One of the most significant sites for UK mathematics is MathsNet www.mathsnet.net designed, maintained and funded by a full-time teacher: Bryan Dye. Everyone will have their own personal favourites, and web-sites come and go surprisingly quickly, but we hope the list below forms a good starting point.

- ATM - The Association of Teachers of Mathematics http://www.atm.org.uk
- BBC http://www.bbc.co.uk/learning/index.shtml
- CensusAtSchool International project http://www.censusatschool.ntu.ac.uk
- Count On – the follow up from MathsYear 2000 http://www.counton.org
- Explore Math http://www.bbc.co.uk/learning/index.shtml
- MA - The Mathematical Association http://www.m-a.org.uk
- MathAid http://www.mathaid.com
- MathsNet http://www.mathsnet.net
- Maths Online http://www.univie.ac.at/future.media/moe/
- NC Online - choose mathematics http://www.nc.uk.net
- NRICH http://www.nrich.maths.org
- Oundle School links http://users.argonet.co.uk/oundlesch/mlink.html
- Sidney Tyrrell's Statistics http://www.mis.coventry.ac.uk/~styrrell/resource.php
- Standards Site http://www.standards.dfes.gov.uk/keystage3/subjects/maths/
- TI graphical calculator support http://education.ti.com/uk/teacher/teacher.html
- US library of Virtual Manipulatives http://matti.usu.edu/nlvm/nav/vlibrary.html
- Virtual Teachers Centre - choose subjects, then mathematics http://vtc.ngfl.gov.uk
- World of Mathematics http://mathworld.wolfram.com/

Our general advice to you is to begin with a fairly clear sense of what learning experience the resources are to support, before casting about for helpful resource-providing websites. But you also need to allow yourself some undirected browsing to stimulate your imagination and to widen your awareness of new possibilities. You will find that many sites change quite frequently so that you may find it difficult to retrace your steps to useful pages. It might be a good idea to copy sets of useful pages to a recordable CD Rom every so often.

As with reviewing software, some criteria are needed when reviewing mathematics websites. We have used the following criteria in drawing up our recommendations:

- ease of navigation around the site;
- transparency of design;
- emphasis of content over design;
- a large body of interactive material available;
- high relevance of content to school use.

The emphasis above is on `interactive': meaning pages that require action by the user, often through varied and continual use of the mouse. This may involve Macromedia *Flash* and *Shockwave*, *Java* or *JavaScript*, though these technicalities should not be apparent to the user - apart from issues of download time. Some sites dress material up with `attractive' graphics, or provide sophisticated but cumbersome methods of getting at the material. Unfortunately some of the BBC's material (*BiteSize*) suffers from this.

There are sites catering for specific areas of Mathematics at Advanced Level, particularly Statistics. Four excellent examples are:

- Inferential Stats at http://faculty.vassar.edu/lowry/webtext.html
- Rice Virtual Lab in Statistics at http://www.ruf.rice.edu/~lane/rvls.html
- CAST at http://cast.massey.ac.nz/
- Calc101 at http://www.calc101.com

Other sites offer excellent research and background material, such as:

- the History of Maths site at St Andrews http://www-groups.dcs.st-and.ac.uk/~history/
- Ron Knott's Fibonacci numbers site at http://www.ee.surrey.ac.uk/Personal/R.Knott/Fibonacci/fib.html
- The Interactive Mathematics Miscellany and Puzzles site at http://www.cut-the-knot.org/

There are also excellent online strategy puzzles scattered about the web, such as chess, go, reversi, draughts, Chinese checkers, though no one site has a comprehensive selection. MathsNet has made a small collection of them at www.mathsnet.net/puzzles.html .

MathsNet includes a huge collection of interactive material for use in schools using online versions of dynamic geometry packages at all Key Stages at www.mathsnet.net/geometry/ . The site covers Shape, Transformations and Geometric construction in detail, along with smaller topics, such as Euclid's elements, construction without a ruler, properties of triangles and circles.

Jennifer Piggott, NRICH Project Director, brings a slightly different perspective to the question "what use is the Internet?"

"In order to make effective use of an online resource it is essential to know what the site is trying to achieve and why it will suit your purpose.

These resources can take a number of forms from lesson plans to subject specific software through to pure information sources, such as sites containing population statistics. The first requirement is for you to be clear about the type of resource you are looking for so that you can begin to look in the right places. Sites will offer support in a variety of forms and it is useful to be aware of what a site is trying to achieve so that you know what to expect. The rest of this section looks at types of sites and why you might find them useful.

The range of places to start searching for mathematics resources is growing. Sites like the BBC http://bbc.co.uk/learning *offer opportunities for revision and some, such as the NGfL* http://ngfl.gov.uk *and National Curriculum Online* http://www.nc.uk.net/home.html *, simply refer to further sites and resources (rather like a compendium). There are also sites that have specific, curriculum linked, resources such as Counton* http://www.counton.org *and the NRICH online mathematics club* http://nrich.maths.org *, which offers enrichment materials. This site contains a wealth of problems, games and articles and offers a search facility for locating material related to particular aspects of mathematics. There is also a mathematical thesaurus, which is being continually updated, and gives access to definitions and explanations of mathematical term and makes links to related terms. Access to online discussion groups, like the ones found on the NRICH site enable pupils, teachers and parents to discuss mathematics with peers. Finally, simply searching using a search engine such as Google can also be useful.*

One problem is the vast number of sites and finding time to assess the potential of each one. The NGFL `flags' sites they feel meet their assessment criteria and TEEM http://www.teem.org.uk *is beginning to evaluate web based resources as well as multimedia software. These evaluations are carried out by teachers who use the resource(s) with their pupils over several lessons and therefore they have the advantage of being evaluated in the `real world'.*

This is all beginning to show how complicated the task of identifying and using resources can be. The answer is to start small. Get to know one resource well and share it with colleagues. The site you pick will reflect your approach to teaching mathematics so your choice may be quite different from someone else's, simply because you are a different teacher. As your confidence grows you can move on or use a network of colleagues to develop supporting materials for pupils to use with online resources. Many hands make light work."

Ron Knott adds these comments:

"The advantages of the Internet over other software are that:

- *browsers are free*
- *pages are available at home as well as school and everywhere else*
- *students' work and teachers' ideas can be shared world-wide*
- *platform independent*
- *useful for student revision later*
- *students who missed class can easily catch up*
- *keen students can carry on working on their own using the resource*
- *web-generated interactive test pages give instant feedback to students and allow plenty of practice."*

CD ROM

CD ROMs provide an alternative storage and delivery mechanism in the same sense as the Internet, but with some significant differences.

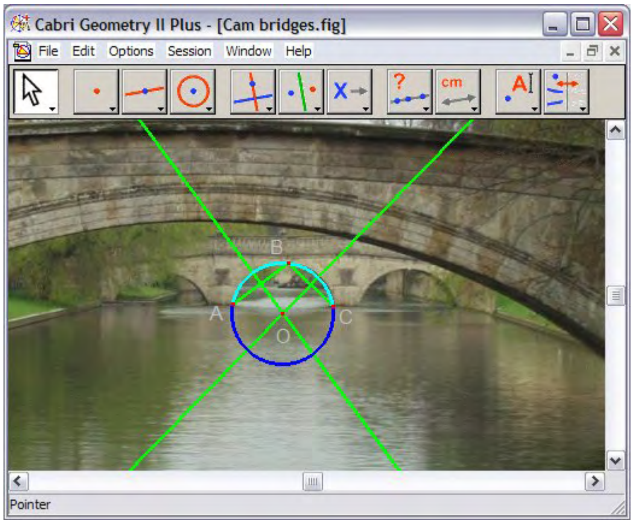
- They may be more reliable, and load faster, across a multi-user school network.
- They may make pupils' learning more manageable by offering scope within boundaries. The structure of a CD-ROM may prevent students following web-trails in all directions, which can quickly lead to their becoming `off-task'.
- They do not have the same quality of *currency* when compared to Internet-based resources, which may not be highly significant in all cases but needs consideration.

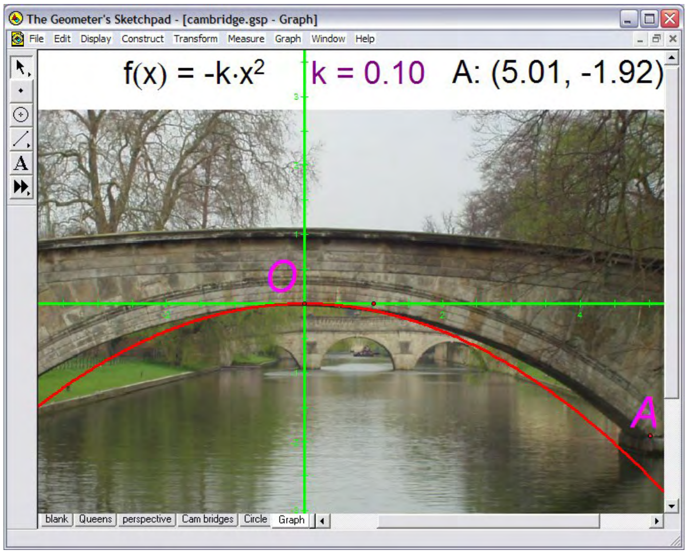
One large scale use of CD-ROMs in mathematics is for item-banking past test and examination questions, often with tools to enable you to construct your own test papers.

2.10 Digital image technology

The increased availability of digital cameras and video camcorders provides a valuable source of geometric and other data which can be used in a number of ways. For example a photograph can be captured from a digital camera, as a still from a video camcorder or scanned from an original or copy. The image can be manipulated in standard imaging software and the result saved in a standard image file format or pasted to the clipboard. The resulting image or file can then be opened in software such as the *Geometer's Sketchpad* or *Cabri II Plus* and constructions, measurements etc. made using the image. See for example the Micromath article: http://www.atm.org.uk/journals/micromath/articles/mmarchivepdfs/mm192oldknow.pdf

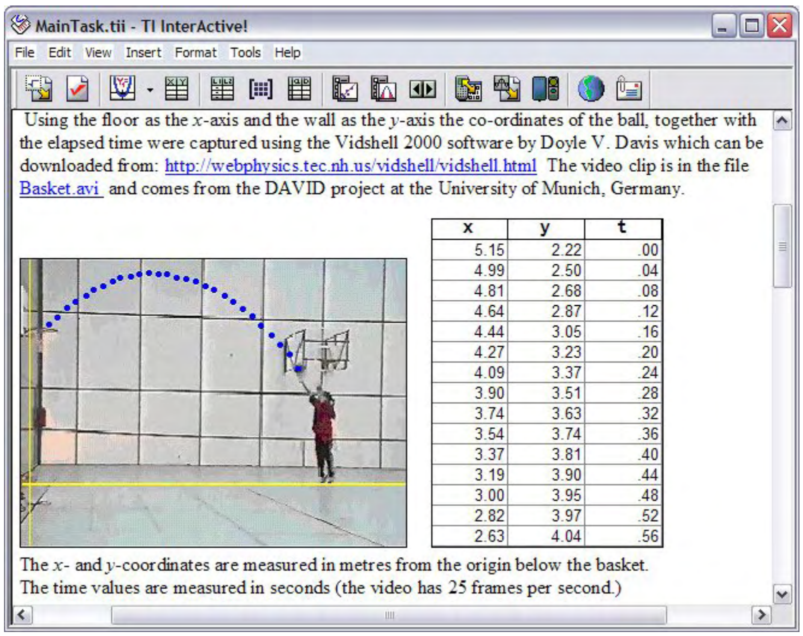
One of the figures below shows *Cabri II Plus* with three points *A,B,C* superimposed on a curved span of the further bridge, and used to fit a circular arc by constructing its centre *O*. The other shows *Geometer's Sketchpad* with origin and axes superimposed on the nearer bridge. A quadratic function is being explored by varying a parameter *k* – what value should it take to pass through the superimposed point *A*?





Still images and video clips are also available on CD-ROMs and via the Internet. For example the *Problem Pictures* CD-ROM http://www.problempictures.co.uk/ produced by Richard Phillips provides a massive base of digital images together with notes suitable for use in teaching and learning aspect such as ratio and proportion, symmetry, 3D visualisation etc.

The *Multimedia Motion II* CD-ROM http://www.csmedia.demon.co.uk from Cambridge Science Media provides a large number of video clips of actions such as the striking of a golf ball, the path of a squash ball and the collision of a vehicle together with tools for recording coordinates of objects from successive frames so that distance-time graphs can be drawn, velocities and accelerations estimated etc. There is also free software on the Internet, such as *Vidshell*, which can be used to export data taken from frames of a digital video clip. See the Micromath article: http://www.atm.org.uk/journals/micromath/articles/mmarchivepdfs/mm192oldknowa.pdf
The image below shows data taken from a clip of a basketball throw and imported into *TI InterActive!*



Using the floor as the *x*-axis and the wall as the *y*-axis the co-ordinates of the ball, together with the elapsed time were captured using the Vidshell 2000 software by Doyle V. Davis which can be downloaded from: http://webphysics.tec.nh.us/vidshell/vidshell.html The video clip is in the file Basket.avi and comes from the DAVID project at the University of Munich, Germany.

x	y	t
5.15	2.22	.00
4.99	2.50	.04
4.81	2.68	.08
4.64	2.87	.12
4.44	3.05	.16
4.27	3.23	.20
4.09	3.37	.24
3.90	3.51	.28
3.74	3.63	.32
3.54	3.74	.36
3.37	3.81	.40
3.19	3.90	.44
3.00	3.95	.48
2.82	3.97	.52
2.63	4.04	.56

The *x*- and *y*-coordinates are measured in metres from the origin below the basket. The time values are measured in seconds (the video has 25 frames per second.)

Images from a digital camera (or scanned images from photographs, books etc) can be imported into a variety of graphic software, such as *MS Paint*, from which cursor positions can be read off as co-ordinates and used as data sets for modelling. Frames from a digital video camcorder (or stored input from a VCR or TV) can be similarly digitised manually with the frame number and film speed giving the time data. For examples of each of these see `*Teaching Mathematics using ICT*', Oldknow & Taylor, Continuum 2004.

2.11 Video conferencing

This consists of much more than just making a telephone call with pictures!

Features of the technology include the following.

- At any location, the image and sound can be projected to be available to a group rather than just an individual.
- The connected activity can involve more than two parties/locations simultaneously.
- The technology supports shared sight and control of applications, simulating the experience of working together at the same machine.

In order for it to be an effective means of communication, participants need to be able to:

- see the other people person to feel connected;
- see the objects of discussion;
- have shared control/manipulation of the objects of discussion.

Three of the major models of use for mathematics work are:

- Conference – this is `presenter centred' (such as in the *Motivate* activities of the NRICH Project);
- Workshop – which may be a significant model for teachers' professional development as well as for activities with students;
- Peer to peer – where teachers and/or pupils work in a style of collaboration closer to the familiar work/learning style in school.

A successful case study of video-conferencing in mathematics contained the following elements:

- 12 students prepared in advance for a conference using activities located on the web;
- they introduced themselves and shared findings with other schools in the conference;
- the presenter picked up points and provided an input, after which the students presented results to others during the conference;
- a final task was set for pairs of schools to work together on an aspect of the topic of the conference;
- notes and support were provided on the web for teachers and students;
- in a second session schools reported back their findings and reviewed the work – which led to extensions and generalisations.

2.12 Courseware

A number of published schemes, such as the `*Folens Maths Programme'* and `*Key Maths'*, are now including materials making use of ICT such as graphical calculators and spreadsheets. Similarly there are supporting materials to accompany A/AS schemes such as MEI and SMP16-19. *Absorb Mathematics* software from *Crocodile-Clips* is their new courseware for GCSE Mathematics. It combines Crocodile Mathematics models with animations and interactive investigations: http://www.crocodile-clips.com .

The current moves towards a `digital curriculum' mean that organisations, principally the BBC, will provide large amounts of downloadable materials to accompany the National Curriculum and GCSE courses.

At the moment, though, there is a unique ICT based mathematics course in Research Machine's *Maths Alive Framework Edition* (MAFE) which has been developed from the DfES Y7 project to cover the whole of the objectives in the KS3 Framework for Mathematics. This is based on the use of the mixed model of hardware described in section 1 above, which can include:

- Interactive whiteboard, projector, PC, printer and Internet connection
- Further PCs to support group work
- Teacher's graphical calculator displayed via video adaptor and/or OHP pad
- Graphical calculators and data-loggers for student's paired or individual use.

The software can include:
- RM *Easiteach Maths* for use with the interactive whiteboard
- MS *Office* elements; *Word, Powerpoint, Excel, Internet Explorer*..
- Small software, videos and stimulus materials written by RM's 3T production company
- *MSW Logo*
- *Geometer's Sketchpad V4*
- *TI InterActive!*

The service includes a large number of supporting files containing materials to support whole-class work, student activities, mental-and-oral starters, task sheets, homework sheets, assessment sheets, teachers' notes, students' notes etc. See http://www.mathsalive.co.uk
Of course the implementation of such an extensive scheme needs careful planning, such as in the provision of professional development for staff.

2.13 Overview

We hope that we have convinced you that there is no shortage of ICT resources for teaching mathematics at all levels! While we have tried to give a fair coverage of the range of materials available we are aware that we cannot be exhaustive. A question often asked though is "what should be the absolute baseline for ICT resources in mathematics department?". So, with a certain amount of trepidation, here are some pointers we hope you will find useful.

Hardware:
1. Some means of supporting whole-class work with ICT to include any of the following:
- PC/laptops displayed with interactive whiteboards, data projectors and/or large screens;
- Graphical calculators/data-loggers displayed with OHP pad and/or video adaptor.

2. Some means of supporting paired and/or individual access, preferably in a normal classroom including any of the following:
- networked PCs;
- laptops;
- hand-held devices such as graphical calculators, data-loggers and palm-tops.

Software – with site-licences:
3. A version of *Logo*, such as the free *MSW Logo*.
4. Dynamic Geometry Software, such as the *Geometer's Sketchpad* or *Cabri Geometry*.
5. Other mathematics teaching software for graphing, data-handling etc, such as the integrated maths package *TI InterActive!* or individual titles such as *Omnigraph*, *Coypu*, *Autograph* etc.
6. Software and cables to support the easy transfer of data, programs, applications etc. between PCs, graphical calculators, data-loggers and the Internet.
7. A library of resource materials such as small software from *SMILE*, stimulus materials on CD-ROM such as *Problem Pictures* etc.
8. Resources to support the use of general purpose tools such as those in *MS Office*.

Materials:

9. Publications to support mathematics teachers in their use of ICT, including those from TTA and Becta, journals such as ATM's `*Micromath'* and books such as Oldknow and Taylor's `*Teaching Mathematics using ICT'*.

In selecting mathematical ICT tools such as software and graphical calculators bear in mind the ratio of time taken to become familiar with the tool and curriculum opportunities for its use! **Tools such as dynamic geometry software, integrated mathematics software, graphical calculators and data-loggers are deployable at all levels from Y7 through to Further Maths.**

Maybe now would be a good time to take a look at what the 2000 National Curriculum includes under Breadth of study for Mathematics key stage 4 higher:

"During the key stage, pupils should be taught the knowledge, skills and understanding through…choosing appropriate ICT tools and using these to solve numerical and graphical problems, to represent and manipulate geometrical configurations and to present and analyse data."

So, do you think you've got that sorted now?

Section 3: **Supplementary information**

We conclude with a collection of references to supporting materials, information about ICT support for teachers, references to past trials of ICT in mathematics and on-going projects, as well as a glossary of the acronyms which go with computers and government initiatives!

3.1 Current sources of information and materials

The following documents are valuable primary sources of information:
Becta *Mathematics and IT - A pupils entitlement*
http://curriculum.becta.org.uk/docserver.php?temid=290
DfES (2003), *Fulfilling the Potential: Transforming teaching and learning through ICT in schools*, DfES/0265/2003 ISBN 1-84185-982-6
http://www.dfes.gov.uk/ictinschools/publications/publication.cfm?publicationid=45
Jones, K. (2004), `Celebrating 20 Years of Computers in Mathematics Education: a research bibliography', *Micromath* V20/1 Spring 2004
NGfL *Transforming the Way we Learn* http://www.dfes.gov.uk/ictfutures
Ofsted reports on *ICT in Schools – Secondary mathematics* 2004 and 2002
http://www.ofsted.gov.uk/publications/index.cfm?fuseaction=pubs.summary&id=3652
Oldknow, A.& Taylor, R. (2004), *Teaching Mathematics using ICT* , Continuum ISBN 0-8264-7509-9 http://www.amazon.com/, http://www.continuum-books.com/
RM *Good Practice in the Use of ICT* http://www.rm.com/reports/
Ruthven, K. & Hennessy, S. (2003), `Successful ICT use in Secondary Mathematics', *Micromath* 19/2 Summer 2003
TTA *Identification of Training Needs*
http://www.tta.gov.uk/php/read.php?sectionid=94&articleid=500

The following web-sites contain valuable information and links, and are described below:

The Virtual Teacher Centre (VTC)	http://www.vtc.ngfl.gov.uk/
The Teacher Resource Exchange (TRE)	http://tre.ngfl.gov.uk/
The Becta Educational Software Database (BESD)	http://besd.ngfl.gov.uk/
The Becta ICT Advice Site	http://www.ictadvice.org.uk/
The Becta Website	http://www.becta.org.uk/
The Further Education Resources for Learning (FERL)	http://ferl.becta.org.uk/

The Virtual Teacher Centre (VTC)
The mathematics area on the VTC is divided into the following sections:

- Subject resources: containing links to web sites that have been approved as having useful curriculum content, as well as the Teacher Resource Exchange and other curriculum resources.
- The curriculum: providing direct links to the National Curriculum documents, schemes of work and frameworks for teaching for mathematics.
- ICT support: here you will find information, advice, case studies and resources for using ICT in mathematics. (See below for more detail)
- Tests and exams: this section provides links to the on-line examination syllabuses for Mathematics and links to revision materials. There is information and support for SATs.
- Organisations: here you will find links to subject associations, national projects and other mathematics related bodies.
- Discussion groups: take the opportunity to communicate, discuss and share ideas with other mathematics teachers.

The ICT support pages for mathematics
http://curriculum.becta.org.uk/docserver.php?temid=89
These contain the following sections:

- **ICT and Mathematics**
  A simple guide to ways in which ICT can support pupils learning mathematics in both primary and secondary phases
- **ICT materials**
  A catalogue of software and equipment for mathematics together with some suggestions and advice about their use.
- **ICT in Practice**
  This section contains classroom ideas and activities that focus on integrating ICT with the teaching and learning of mathematics. It contains a large number of resources and ideas for the use of graphical calculators here.
- **Managing ICT**
  This provides case studies of six schools developing their use of ICT and a departmental audit of ICT use in mathematics
- **Internet resources**
  Some links to mathematics resources on the internet

The Teacher Resource Exchange (TRE)

The Teacher Resource Exchange is designed to help teachers develop and share ideas for activities and resources.

Contributions are many and varied from simple ideas and questions, to complete contributions/lesson plans or schemes of work, which will enable other teachers to use these resources within their own lessons.

Registering with the TRE is not compulsory. You can browse and download resources without registering. You will, however, need to register if you would like to submit new resources and add comments or materials to existing resources.

Registration also provides you with a personal profile based on the specific interests selected. For example, a mathematics teacher logging on to the TRE will see a home page drawing attention to all new mathematics resources on the site (over 300 and rising). It is possible to amend the registration profile at any time. Your personal homepage also provides easy access to any resources submitted by you.

The Becta Educational Software Database (BESD)

BESD contains information about software from pre-school to further education. Software included in the BESD is educational in nature and available in the UK.

The basic search will allow you to search by subject area, phrase, platform, format or keyword. A `Help' facility is available.

BESD information has been provided directly by software publishers. Each of the product records may contain an outline of system requirements, formats, networking options, a description of the software and references to reviews. Where available, there are also links to reviews of the product on the TEEM (Teachers Evaluating Educational Multimedia) Web site. Publisher contact details are included for most records with links to publisher and suppliers web sites.

The ICT Advice Site

The ICT advice web site offers a new concept in ICT support and advice to education professionals working in schools. Aimed at classroom teachers, leadership teams, heads of departments, subject co-ordinators, ICT co-ordinators and special needs co-ordinators, this new site is a described as `one-stop shop for all ICT education needs'.

As a practitioner you will find a dedicated set of tools to provide access to advice and support that is relevant to your specific professional needs, including classroom and management applications, purchasing, sourcing and planning, technology and inclusion, and teaching and learning.

The site caters for a wide range of experience and skill, so you don't need to be a computer wizard to take advantage of the advice and support available on the ICT advice site. Your school might, for example, be still in the early days of its ICT development, in which case a visit to the A-Z Guide will provide you with a glossary of concise definitions and terms to help you get to grips with new ideas and applications.

The Timesaver feature is specifically designed to help you find solutions to ICT questions, and has been designed with teacher workloads very much in mind. This contains specific advice to help in evaluating the ICT provision of a mathematics department.

For more detailed descriptions of technology or ICT-related subjects, click on the 'What is' button. This will provide answers to a wide range of questions – for example, `What is a digital camera?' or `What is a computer virus?' If you want support to achieve a task which involves using technology, the 'How to' guides give clear and concise instructions.

This website will be the main repository of advice and materials for schools generated through the DfES funded "KS3 ICT Offer to Schools".

The Becta Website

The BECTa web site provides information, advice and dialogue relating to ICT in education for the schools and FE sectors. It includes information from case studies and examples of good practice to practical guidance on using ICT within the curriculum and for administration.

Further Education Resources for Learning (FERL)

The FERL web site contains a wealth of information for lecturers, trainers, managers, ILT Champions and support staff; in fact anyone involved in post compulsory education. Despite the `FE' reference in the title there is much here, particularly in the 'Teaching and Learning' section, which will be of value to teachers in schools.

There are over 100 resources for supporting mathematics teaching and learning.

The report of a seminar, March 2002, to disseminate the findings of various Becta projects is at: http://ferl.becta.org.uk/display.cfm?page=25&catID=36&resID=2720
The reports and resources, including downloadable software for both Casio and TI technology can be found at: http://curriculum.becta.org.uk/docserver.php?docid=4445
Resources to support this application of technology are available from: http://curriculum.becta.org.uk/docserver.php?docid=1760
Further ideas including a report from a project on the use of graphical calculators for data capture and modelling in mathematics and science are available at: http://curriculum.becta.org.uk/docserver.php?temid=292

The *CensusAtSchool* Project

The Royal Statistical Society (RSS) Centre for Statistical Education, based at Nottingham Trent University, started the ***CensusAtSchool*** project in 2000 in conjunction with National Statistics. In addition it was funded by the Maths Year 2000 initiative and the RSS itself. The project is on going and, by continuing it, the Centre has three main aims:

1) to provide real data for data-handling activities across the National Curriculum;
2) to increase awareness of what a national census is, and what it is for;
3) to show how Information and Communications Technology (ICT) can be used effectively to enhance learning and teaching resources for good practice in data handling.

Between Autumn 2000 and March 2001, across England, Wales and Northern Ireland thousands of young people between the ages of 7 to 16 took part in the project through their schools, using the Internet site www.censusatschool.ntu.ac.uk. The ***CensusAtSchool*** questionnaire itself comprises a single A4 sheet with simple questions covering information about pupils, their households and their school life. While some of the questions were identical to those in the UK adult census that took place in April 2001, others were designed to appeal to the pupil's own interests and enthusiasms. Over 2000 primary, secondary and special schools registered for the project and there is now a database of over 60,000 responses with excellent coverage of all regions in England, Wales and Northern Ireland. The project can be run equally well using paper methods, computer technology or a combination of the two.

The questions posed can be asked of school children across the globe with minor adjustments to reflect local culture and traditions. The project has, or is being run, in South Africa, Queensland, South Australia, New Zealand, Canada plus involvement to a smaller extent in Italy and Norway. Links to all our International colleagues and data from their projects is available on the website.

A further phase of the project is run each year to add to the portfolio of online questionnaires available. Phase 2 ran during 2001/2, Phase 3 2002/3, Phase 4 2003/4 etc. All questionnaires are available in the classroom material section of the website at:
http://www.censusatschool.ntu.ac.uk/keystage.asp

There is a vast selection of resources for educational purposes that can be downloaded from the website at http://www.censusatschool.ntu.ac.uk. These currently comprise:

1) A large, and expanding portfolio of worksheets for Key Stages 2, 3 and 4 which focus on the use of ICT and data handling skills across the school curriculum. Subject areas covered include Mathematics and Statistics, Geography, History, Science, ICT and Key Skills. These documents can be viewed and freely downloaded after browsing the web site at http://www.censusatschool.ntu.ac.uk/curriculum.asp. All these worksheets are in both *Acrobat .pdf* and *Word .doc* formats, the former being visually appealing, while you can choose to alter the latter to suit individual or local needs;
2) More ideas, lesson plans and help sheets in the teachers notes section;
3) Hundreds of tables of summary data that can be easily copied and pasted into *Word* or *Excel* for use in the classroom;
4) A variety of *Excel* Spreadsheets containing sections of data for use in school projects;
5) A random data selector where random samples of raw data can be requested for use in the classroom from a number of databases from the UK, South Africa, Queensland,

South Australia and New Zealand. These data are returned as a CSV file attachment or downloaded directly for importation into spreadsheets.

The worldwide database, from which random samples may be taken for use in creating learning and teaching materials, is available from the web site and, as more countries participate, it will be expanded accordingly. This will enable the enhancement of teaching and learning data handling in many countries. The involvement of these countries vastly increases the potential for exchange of information between school-aged children and is a unique way to assess global, social and other changes. It has the added bonus of providing ICT, education and motivational resources for both teachers and children.

Additional sources of information

Journals

Mathematics in School (MA), http://www.m-a.org.uk/resources/periodicals/mathematics_in_school/

Micromath (ATM), http://www.atm.org.uk/journals/micromath.html

Teaching Mathematics and Its Applications (IMA), http://www3.oup.co.uk/teamat/

Calculator support

A+B books (Alan Graham and Barrie Galpin) http://www.fineshade.u-net.com/a+b/

Jones, K. (2003), `Research bibliographies: Calculators', *Micromath* 19/1 Spring 2003

Computer Algebra

Goldstein, R. at al *Computer Algebra at A Level* ATM http://www.atm.org.uk/buyonline/products/books/dis001.html

Oldknow, A. & Flower, J. (1996), *Symbolic Manipulation by Computers and Calculators* MA, ISBN 0 906588 37 5 http://www.m-a.org.uk/resources/publications/books/

Small programs

SMILE Mathematics http://www.smilemathematics.co.uk

TI web-store – for Flash Applications http://epsstore.ti.com/webs/Home.asp

Programming languages

Miller, D. (1996), *Logo for Windows*. Keele Mathematical Education Publications http://www.cthree.freeserve.co.uk/education/kmep/webcat1temp.htm

MSW *Logo* web-site http://www.softronix.com/logo.html

Programs for TI graphical calculators http://www.ticalc.org/

TrueBASIC http://www.truebasic.com/

Spreadsheets

Green, D., Armstrong, A. & Bridges, R.(1993), *Spreadsheets: Exploring their Potential in Secondary Mathematics* , MA, ISBN 0 906588 29 4 http://www.m-a.org.uk/resources/publications/books/

Graph plotters

Autograph web-site http://www.autograph-math.com/

Omnigraph web-site http://www.spasoft.co.uk/omnigraph.html

Peanuts software http://math.exeter.edu/rparris/default.html

TI InterActive! http://education.ti.com/product/software/tii/features/features.html

Dynamic Geometry Software

Becta, *Dynamic geometry*: http://curriculum.becta.org.uk/docserver.php?docid=1937
Cabri web-site http://www-cabri.imag.fr/index-e.html
Clark G., Redden, E., *Geometrical investigations : a Companion to Cabri* II , AAMT http://www.aamt.edu.au
Geometer's Sketchpad web-site http://www.keypress.com/sketchpad/index.html
Jones, K. (2002), `Research Bibliographies : Dynamic Geometry Software', *Micromath* 18/3 Autumn 2002
Michael De Villiers web-site http://mzone.mweb.co.za/residents/profmd/homepage4.html
QCA (2004), *Developing Reasoning through algebra and geometry*. QCA/04/1289 http://www.qca.org.uk/ages14-19/subjects/mathematics_7420.html
Royal Society /JMC (2001), *Teaching and Learning Geometry 11-19* http://www.royalsoc.ac.uk/policy/index.html
Schumann, H. & Green, D. (1994), *Discovering geometry with a computer - using Cabri Géomètre* , London, Chartwell-Bratt.

Data-handling

Fathom web-site http://www.keypress.com/fathom/

Graphical Calculators and Data Loggers

Burill, G. (2002) *Handheld Graphing Technology in Secondary Mathematics*, Dallas: TI
French, D. (2000) *Press ENTER to get the point*, TES Curriculum Special May 19 2000
Galpin, B., Graham, A. (eds.) (2001) *30 Calculator Lessons for Key Stage 3,* A+B Books ISBN 0 9541020 0 2
Galpin, B., Graham, A. (2000) *Calculator Maths – series of 5 topic books*, A+B Books
Green, D., Pope, S. (eds) (1993) *Graphic Calculators in the Mathematics Classroom*, Leicester: MA, ISBN 0 906588 31 6
Oldknow, A. (2001) *Networked Data Logging*, Coventry: BECTa. http://www.becta.org.uk/page_documents/corporate/events/wirelessclass.pdf
Oldknow, A. (2001) `The graphic calculator is dead, long live hand held technology', *Micromath* 17/3 Autumn 2001
Oldknow, A. & Taylor, R. (1998) *Data-capture and modelling in mathematics and science*, Coventry: BECTa, ISBN 1853794295 http://vtc.ngfl.gov.uk/uploads/application/datacapture-16796.pdf
Secondary Maths with a graphic calculator (NCET) – in the IT Maths Pack (MA/ATM)
TI-Time journal http://education.ti.com/uk

Integrated Mathematics Software

Drijvers, P (ed.), (2004) TI InterActive! in the Classroom, T-cubed Europe, ISBN 3-934064-51-5 2004 http://www.t3ww.org

Integrated Learning Systems

BECTa (1996) *Integrated Learning Systems: A report of Phaes 2 of the pilot evaluation of ILS in the UK*, Coventry: BECTa, ISBN 1 85379 358 2
McFarlane, A. (1999) *ILS: A guide to good practice* Coventry: Becta ISBN 0 43509 690 7
Underwood, J, Brown, J (eds) (1997) *Integrated Learning Systems: Potential into practice* London: Heinemann ISBN 0 43509 690 7

Interactive Whiteboards
Jones, K. (2004), `Using Interactive Whiteboards in the Teaching and Learning of Mathematics: a research bibliography', *Micromath* 20/2 Summer 2004

Internet
Becta (2004), `Getting the most from your interactive whiteboard'
http://ferl.becta.org.uk/display.cfm?resID=7674
Jones, K. (2003), `Using the Internet in the Teaching and Learning of Mathematics: a research bibliography', *Micromath* 19/2 Summer 2003

General books and reports
Becta (2002) *Using web-based resources in the daily mathematics lesson*
http://www.ictadvice.org.uk
Bloomfield, A., Harries, A. (1995) *Teaching, Learning and Mathematics with IT*, Derby: ATM, ISBN 0 900095 98 9
Cobden, D., Gibbon, J. (eds) (2001) *More Teaching, Learning & Mathematics with ICT*, Derby: ATM, ISBN 1 898611 12 2
French, D. (2002) *Teaching and Learning Algebra*, London: Continuum, ISBN 0826452221
Mann, W., Tall, D. (eds.) (1992) *Computers in the Mathematics Curriculum*, Leicester: MA. ISBN 0 906588 28 6
Ofsted (2002) *ICT in Schools*, HMI 423 http://www.ofsted.gov.uk
Oldknow, A. & Taylor, R. (1999). *Engaging Mathematics*, London: TCTrust, ISBN 873882459.. http://www.schoolsnetwork.org.uk/

Some suppliers of software and other ICT resources for mathematics

Chartwell-Yorke http://www.chartwellyorke.com
114 High Street, Belmont, Bolton BL7 8AL
info@chartwellyorke.com (tel) 01204 811001 (fax) 01204 811008

Oxford Educational Supplies http://www.oxford-educational.co.uk/
Unit 19, Weston Business Park, Weston on the Green, Bicester, OX6 8SY
info@exford-educational.co.uk (tel): 01869 344 500

Mathsite (formerly QED/Griffin) http://www.mathsite.co.uk/

Science Studio Limited http://www.sciencestudio.co.uk/acatalog/index.html
Hanborough Park, Long Hanborough, Witney OX29 8SF
calculators@sciencestudio.co.uk (tel) 01993 883 598 (fax) 01993 883 317

SMILE Mathematics http://www.smilemathematics.co.uk/
Isaac Newton Centre, 108A Lancaster Road, London W11 1QS
info@smilemathematics.co.uk (tel) 020 7598 4841 (fax) 020 7598 4838

Finally, a reminder of information contained in Section 1 of the KS3 Strategy's Framework for Mathematics, page 25.

Information and communication technology (ICT)

ICT includes calculators and extends to the whole range of audiovisual aids, including educational broadcasts and video film. The National Curriculum suggests points at which ICT can be used in mathematics. For example:

> *Generate functions from plots of data, for example, from a science experiment, using simple curve fitting techniques on graphic calculators, or with graphics software to explore the transformation of graphs.* The National Curriculum 1999

The main uses of ICT in mathematics in Key Stage 3 stem from:

- the use of calculators for calculating purposes (see page 12);
- small programs, such as number games or investigations in a particular context;
- programming languages, such as Logo or Basic, and the programming capabilities of graphical calculators;
- general-purpose software, particularly spreadsheets, but also databases;
- content-free mathematics software, such as graph plotters, dynamic geometry software and data-handling packages;
- ILS (Independent Learning Systems), which provide and manage practice in mathematical techniques tailored to the needs of individual pupils;
- graphical calculators and data-loggers;
- CD-ROMs and the Internet.

The supplement of examples includes some references to ICT. Small programs are not mentioned by name, since the focus in the supplement is on mathematical outcomes, not the resources that can be used to achieve them. Where appropriate, you could add references to the small programs and other ICT resources that you have access to.

ICT training and support The New Opportunities Fund, which will continue to 2003, will provide most teachers with about 30 hours of subject training on the use of ICT, tailored to their individual needs. To help identify training needs, the Teacher Training Agency (TTA) has produced a CD-ROM of four case studies of different mathematics lessons in Years 7 to 10, in which dynamic geometry software, a number game, a spreadsheet and graphical calculators are used. The case studies also show different ways of projecting screen images to whole classes.

The National Grid for Learning (NGfL) aims to connect all schools to the Internet. A major element of the NGfL is the Virtual Teachers' Centre, which includes useful information, resources and advice for mathematics teachers. There is more useful information, including reviews of software, on the websites of The Mathematical Association at www.m-a.org.uk and The Association of Teachers of Mathematics at www.atm.org.uk.

Funding for hardware and software The Standards Fund can be used to buy hardware and software. It also supports teachers' use of ICT for their professional development and for administrative work associated with teaching. You can buy:

- ICT equipment, including class sets of portable computing devices such as graphical calculators, whole-class teaching aids such as projection equipment, or equipment to help create teaching materials;
- technical training which complements but does not replicate the pedagogical training supported by the New Opportunities Fund (see above);
- software related to mathematics, including online resources and site licences.

In addition, the pages of Y789 Examples in the Framework contain very many references to the use of graphical calculators, dynamic geometry software, data-loggers and other ICT tools in mathematics throughout the Key Stage.

3.2 Support for teachers

The National Lottery funded (NOF) programme of ICT training for teachers finished in 2003. Here we give information about some alternative models. There are other opportunities provided locally e.g. by LEAs, as well as by commercial companies.

The T-cubed Programme

http://education.ti.com/uk/teacher/training/training.html

T^3 (Teachers Teaching with Technology) is a programme of ICT professional development mainly focused on hand-held technology for secondary school mathematics and science teachers. Jo Gough is the UK co-ordinator: t3@ti.com . Its coordinating group includes members of a number of leading mathematics subject associations. T^3 , through the Mathematical Association, supported the pilot schools on the 2000/1 DfEE Year 7 mathematics/ICT *MathsAlive*! project, managed by Research Machines. Groups of teachers, consultants, advisers etc. can apply to mount a T^3 course or workshop. Provided these meet certain criteria of numbers and duration, T^3 provides master copies of course materials free of charge for course organisers to duplicate for participants. It also provides an experienced tutor, whose fee and expenses will be normally met by the T^3 programme. Equipment for use by participants in the course/workshop can be provided on loan free from the TI Workshop Loan Programme (see below) - which also covers the delivery and collection charges. Participants in courses/workshops, including the organiser(s), can access a range of free PC software and GC Applications.

The TI Workshop Loan Programme

http://education.ti.com/uk/support/workshop/workshoploan.html

Loan requests can be made via the web-site, or via ti-loan@ti.com , (tel) 020 8230 3184, (fax) 020 8230 3132. All the equipment for a workshop or course can be borrowed, usually for a period of up to two weeks. Delivery is made two or three days beforehand, and instructions are provided on how to return the equipment by pre-paid courier after the loan period. For additional information about TI's educational service contact Melanie Horsburgh, Marketing Manager, UK and Ireland, Texas Instruments, 800 Pavilion Drive, Northampton, NN4 7XL Tel: 01604 663059 Fax: 01604 663004 mhorsburgh@ti.com. Other useful contacts are: education.ti.com/uk
Customer Service Centre: 020 8230 3184 **,** and e-mail help from ti-cares@ti.com

Online CPD and support materials in the use of ICT in KS3 mathematics teaching

The DfES set up a competition to provide this service, following on from a pilot which involved the development of on-line support for KS1/2 numeracy and literacy, as well as KS3 Science. The DfES announcement in July 2002 included the following:

2.1 "ICT occupies an important position as a subject of the National Curriculum. As schools' provision of ICT improves, it is becoming increasingly important as a teaching and learning tool to enhance pupils' learning experience and raise levels of attainment in all subjects. Teachers' ICT capability is of central importance to the effective delivery of the National Curriculum. This was recognised several years ago, and was the reason for the introduction in 1998 of the Initial Teacher Training National Curriculum in the use of ICT in subject teaching. To address the training needs of the force of serving teachers, the New Opportunities Fund is drawing on

£230 million of proceeds from the National Lottery to provide all serving teachers in the maintained sector with the opportunity to receive ICT training comparable to the training received by teachers now entering the profession.

2.2 *Whilst important, this initial ICT training is only the first stage in the development of teachers' ICT capability and further steps need to be taken to raise teachers' levels of competence and confidence in the use of ICT in subject teaching and learning. The development of higher order ICT skills, building on initial teacher and NOF training, must be regarded as a key part of teachers' continuing professional development. The Department has a responsibility to address these further training and development needs. During the 2001/02 academic year pilots were run for a number of subject areas and the Department for Education and Skills are now looking to 'roll-out' the pilots nationally. It is in the context of this national roll-out that the present procurement of online CPD is being carried out.*

2.3 *This competition covers the development and piloting of materials in English and mathematics at Key Stage 3. …The contractor(s) will be expected to consider making available an integrated package of support materials, which must include online web based material, and may include paper-based materials, CD-ROMs, DVDs, or videos. Resources need to be complementary to classroom lessons, and their aim is to support interactive teaching. It is envisaged that the online material will also support the strengthening of teachers' subject knowledge. The contractor(s) should develop this as an integral part of the package by developing high quality online teacher training material using presentations with text, sound, graphics, video and animations.*

2.5 *In the case of the projects in English and mathematics at key stage 3 the strategies' network of consultants will support the development of materials and their rollout to selected schools.*

2.6 *The use, by teachers, of the materials developed by this project will not be compulsory. It will therefore be essential that the materials are engaging, easily accessible and understood, and wholly relevant to teachers' individual needs. The proposals should, therefore, include innovative and exciting ways of delivering these materials, and should include a range of strategies to deliver the aims of the project.*

2.7 *The materials and associated support models should:*

- *Build on the outcomes of the New Opportunities Fund (NOF) ICT teacher training, whilst recognising that teachers will have varied levels of expertise;*
- *Deliver training online and meet the popular demand by teachers to have face to face/'human' support;*
- *Draw on the lessons learnt through NOF training for the most preferred and effective methods of teaching;*
- *Involve the Key Stage 3 English and mathematics consultants in the dissemination and evaluation of the training materials, whilst also developing a model which can be employed in the delivery of training in subjects which do not have such support;*
- *Link directly to the teaching objectives and classroom approaches of the Key Stage 3 English and mathematics strategies and the strategies of any other subjects covered where relevant;*
- *Include plans, using existing delivery mechanisms, for teachers to exchange information regarding the use of ICT in their particular subject area, to share good practice and to have the facility to engage in discussions with colleagues online; and*
- *Make use of existing online resources already available.*

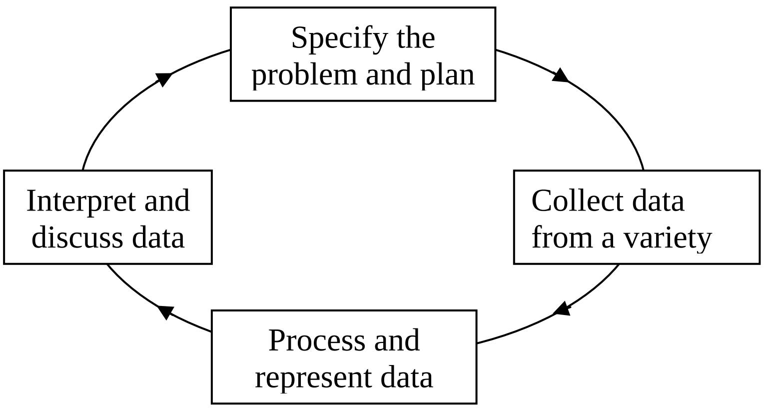
2.8 The successful contractor(s) should work closely with the strategies and the subject associations in meeting these aims."

The final products were launched by DfES at the BETT show, Olympia in January 2004 with the publication of `*Key Stage 3: Enhancing Subject Teaching Using ICT (CPD)*' – including samples from the various web-sites on CD-ROMS. The Mathematics programme is provided by the Mathematics Consortium. This is a partnership between the Mathematical Association, the Mathematics Centre at University College Chichester and New Media plc (now a branch of Plato Learning) http://www.cpd4maths.co.uk
As mentioned in the Introduction, this scheme is currently under review by the DfES and an announcement on making it more widely available to teachers is expected soon.

Using ICT and Web Based Materials for Learning & Teaching and Data Handling Across the Curriculum in Schools

This course developed by the Royal Statistical Society (RSS) Centre for Statistical Education wraps ICT training around data handling specifications across the national curriculum. It uses the international web-based teaching resource the ***CensusAtSchool*** project to show how to integrate the use of software, the Internet and web-based resources into the learning and teaching environment. There is significant hands-on experience with computer-based tools such as word processing, spreadsheets and use of the Internet, using them to develop existing or new web-based teaching and learning resources. The course is based upon a two-stage approach: stage 1 is an interactive teacher course and stage 2 involves self-learning and home activities. Teachers use their own specialist curriculum knowledge to develop web-based teaching resources specific to their own disciplines. The length of the course was originally designed to last two days but is very flexible and arranged into a number of sessions that can be adapted to suit most time constraints.

The underpinning for material in this course is data handling, since it is a topic that is common to many core curriculum subjects. As stated in the National Numeracy Strategy, data handling is best taught in the context of real statistical enquiries, in a coherent way so that teaching objectives arise naturally from the whole cycle, as represented in the following diagram:



The revised GCSE Mathematics syllabus for first examination in 2003 requires candidates to produce one piece of extended work that demonstrates an ability to handle and compare data. This is worth 10% of the total marks. This data handling project also offers opportunities for candidates to produce a piece of work that will satisfy the vast majority of requirements for

Key Skills in both Application of Number and Information Technology (provided ICT methods are utilised) at level 1 or 2 and indeed much of level 3 of Application of Number.

The aims of the course are:

- training teachers in both data handling and ICT skills, linking these together;
- empowering teachers with confidence and transferable skills in these areas;
- using ***CensusAtSchool*** as a way to involve pupils fully in the data handling cycle;
- helping the professional and personal development of teachers through interactive training, with special emphasis on web-based resources;
- appreciating the need to collect data using samples or censuses, in order to make informed decisions;
- providing a framework for the creation of information technology-based and traditional learning and teaching material across a broad range of curriculum subjects, and the actual creation of web-based and other technology-based teaching resources for sharing.

For further information contact Doreen Connor at The Royal Statistical Society Centre for Statistical Education, Nottingham Trent University, Nottingham UK
Phone + 44 115 848 4471 Fax: +44 115 848 4771 http://science.ntu.ac.uk/rsscse

3.3 Past, and ongoing, trials of ICT and where to find about them

This introduction is a brief summary of some of the range of projects, initiatives and reports on the use of ICT to support the teaching and learning of mathematics from the past decade, where there has been an outcome which is still accessible and relevant.

A resource which is still of interest is the set of video clips of teachers using ICT in mathematics available on the `*Identification of Needs*' CD ROM from the TTA. This is still available from the TTA publishers. The TTA website is also a source for the reports by teachers who have received research grants for the use of ICT in mathematics.

Other projects which have produced resources which are still available are the:

- GEST funded projects in 1993-1999. Reports and advice from schools involved in these are available on the ICT support pages of the Virtual Teacher Centre (VTC) http://curriculum.becta.org.uk/docserver.php?temid=89 eg:
  - A simple guide to the variety of experiences which ICT can give pupils in mathematics
  - An account of the development of some mathematics departments who received GEST funding and a departmental audit
  - A guide to data capture and modelling in mathematics and science
  - A report on software available for teaching and learning mathematics
  - Resources for teaching advanced mathematics with graphical calculators
- More recent projects (also available through the ICT support pages on the VTC) by the British Educational Communications and Technology Agency (BECTa) have produced
  - reports and resources for the use of handheld technology (graphical calculators) running small software for KS3 and KS4
  - resources for the use of four function calculators for whole class activities (primary focused but with some relevance for KS3)

- resources for the use of web based resources for whole class activities (primary focused but with some relevance for KS3)

Some additional useful reports:

- *A summary of research* (including the NCET portables project); http://www.becta.org.uk/page_documents/research/conf2003/Conference2003Report.pdf
- *Effective pedagogy using ICT for Literacy and Numeracy* (in Primary but still relevant) http://www.ncl.ac.uk/education/ttaict
- BECTa *ILS (Integrated Learning System) evaluation*: from BECTa publications: BECTa, Milburn Hill Rd, Science Park, Coventry CV4 7JJ
- Examples of effective use of ICT in subjects: http://rm.com/_RMVirtual/media/Downloads/good_practice_ICT.pdf
- *Advanced Calculators in Mathematics Education* (Scottish CCC) http://www.svtc.org.uk/sccc/acme
- Royal Society reports on Algebra and Geometry http://www.royalsoc.ac.uk/policy/index.html
- ImpaCT2 report: *Emerging Findings from the Evaluation of the Impact of Information and Communications Technologies on Pupil Attainment* (NGfL) http://www.becta.org.uk/impact2
- Ofsted report: *ICT in Schools – Effect of government initiatives* (HMI 423) http://www.ofsted.gov.uk/publications/index.cfm?fuseaction=pubs.summary&id=19
- Ofsted report: *supplement on Secondary Mathematics* 2002 (HMI 705) http://www.ofsted.gov.uk/publications/index.cfm?fuseaction=pubs.summary&id=2593
- Ofsted report*: ICT in Schools 2004* (HMI 2050) http://www.ofsted.gov.uk/publications/index.cfm?fuseaction=pubs.summary&id=3652
- Ofsted report: *supplement on Secondary Mathematics* 2004 (HMI 2185) http://www.ofsted.gov.uk/publications/index.cfm?fuseaction=pubs.summary&id=3646
- NGfL *Transforming the Way we Learn – a vision for the future of ICT in schools* http://www.dfes.gov.uk/ictfutures

3.4 Glossary

AAMT	The Australian Association of Mathematics Teachers
ACITT	The Association for IT in Education – subject association for ICT teachers
AS	Advanced Supplementary – 1year `half A-level’ qualifications
ASE	Association of Science Education
ATM	The Association of Teachers of Mathematics
BASIC	Simple programming language developed for education in the 1960s
BBC	British Broadcasting Corporation – developing a `digital curriculum’
Becta	British Educational and Communication Technology Agency
BESD	Becta Educational Software Database
BETT	Educational Technology show held in London Olympia each January
BSRLM	British Society for Research in Learning Mathematics
CADCAM	Computer Aided Design and Computer Aided Manufacture
CAS	Computer Algebra System – capable of symbolic manipulation
CBL	Calculator Based Laboratory – multi-purpose data-logger from TI
CBR	Calculator Based Ranger – motion detector from TI
CPD	Continued Professional Development (formerly known as INSET)
CSV	Comma Separated Variables – a form of file structure
D&T	Design and Technology
DES	Department of Education and Science – later became DFEE, then DfES
DFEE	Department for Education and Employment (now DfES)
DfES	Department for Education and Skills
DGS	Dynamic Geometry Software
DISCUS	Discovering Important Statistical Concepts Using Spreadsheets
doc	The file format used by MS *Word*
DVD	Digital Video Disk
FERL	Further Education Resources for Learning
FSMQ	Free Standing Mathematics Qualifications – post-16 (formerly FSMUnits)
GC	Graphical Calculators
GCSE	General Certificate for Secondary Education
GEST	Grants for Educational Support and Training – replaced by Standards fund
Google	An Internet `search-engine’
html	Hypertext Mark-up Language – file format used by MS *Internet Explorer*
ICT	Information and Communications Technology
IB	The International Baccalaureate – an integrated qualification 16-19
ILS	Integrated Learning System – also Individualised LS or Independent LS
IMA	The Institute of Mathematics and its Applications
INSET	In-service Education of Teachers – now usually called CPD
IT	Information Technology
ITT	Initial Teacher Training
IWB	Interactive White Board
Java	A file format used to run applications over the Internet
JMC	The Joint Mathematical Council of the United Kingdom
KS3	Key Stage 3 – i.e. the first 3 years of secondary school = Y7,Y8,Y9
LCD	Liquid Crystal Display
LEA	Local Education Authority
Logo	Simple programming language developed for education in the 1960s
LSP	Learning Schools Programme – a NOF ICT partnership of RM and the OU
LTSN	Learning and Teaching Support Network

MA	The Mathematical Association
MAFE	Maths Alive Framework Edition – a KS3 scheme developed by RM
MEI	Mathematics in Education and Industry
MESU	Microelectronics Support Unit – precursor to NCET
MIT	Massachusetts Institute of Technology
MS	Microsoft
NAMA	National Association of Mathematics Advisers
NANAMIC	National Association for Numeracy and Mathematics in Colleges
NC	National Curriculum
NCET	National Council for Educational Technology – replaced by Becta
NGfL	National Grid for Learning
NOF	New Opportunities Fund – for distributing national lottery proceeds
NRICH	A mathematics enrichment web-site from Cambridge University
Ofsted	The Office for Standards in Education
OHP	Overhead Projector
OHT	Overhead (projector) Transparency – also called a `foil'
OU	The Open University
OUP	The Oxford University Press
PAL	The standard used in the UK for colour television signals
PC	Personal Computer
PCAI	The Pose, Collect, Analyse & Interpret cycle for data-handling
PDA	Personal Digital Assistant – new breed of hand-held technology
pdf	A file format for documents used by Adobe *Acrobat Reader*
PE	Physical Education
PHSE	Personal Health and Social Education
QCA	The Qualifications and Curriculum Authority
RM	Research Machines – a supplier of educational ICT
RSS	The Royal Statistical Society
SAT	Standardised Assessment Task –such as those set at KS3
SEN	Special Educational Needs
SMILE	A mathematics curriculum resource and support provider in London
SMP	The School Mathematics Project
SST	The Specialist Schools Trust – formerly TCT
SVGA	Super VGA – a standard display signal for PCs, monitors and projectors
T-cubed	Teachers Teaching with Technology – an ICT CPD programme
TCT	The Technology Colleges Trust – now the SST
TEEM	Teachers Evaluating Educational Multimedia
TI	Texas Instruments – a supplier of educational ICT
TII	TI InterActive! - integrated mathematics software
TRE	Teacher Resource Exchange
TTA	The Teacher Training Agency
TV	Television
UK	The United Kingdom
VGA	A standard format of display signal for PCs, monitors and projectors
VTC	The Virtual Teachers Centre
XVGA	Extra VGA - a standard display signal for PCs, monitors and projectors