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Some Lessons in Mathematics: a comparison of mathematics teaching in Japan and America

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Interest in the results of international surveys of education often focuses solely on the relative achievement of the countries taking part. Usually this is not good news as far as the performance of English pupils in mathematics is concerned. As David Reynolds and Shaun Farrell confirm in their comprehensive review of international surveys over the past thirty years, "performance in mathematics in England is relatively poor overall" and furthermore, they claim, it has "deteriorated relative to other countries" over that period of time [[1](#), page 52]. From their report it is possible to identify a range of factors that are characteristic of English mathematics teaching, compared with other countries. These factors include:

- a wider range of attainment in England with U.K. low achievers being much poorer
- greater use of grouping by ability
- less time devoted to mathematics
- a variation in access to essential mathematical knowledge
- particular weaknesses in number work
- ineffective use of calculators
- low proportion continuing mathematics into the sixth form
- less attention to homework

Such findings go some way towards indicating some of the issues that need to be tackled if the international achievement of English pupils in mathematics is to be improved. Yet we need to know much more about classroom teaching practices in different countries if we are to have an informed debate about the appropriate directions in which to develop mathematics teaching in the UK. Now we have some relevant data from one component of the largest international comparison yet undertaken; an innovative American study([1](#)) which reveals what actually goes on in typical mathematics classrooms in different countries. In this article I concentrate on some of the findings of this study by focusing on the similarities and differences between Japanese and U.S. mathematics classrooms (U.K. classrooms were not part of this particular study). The findings make interesting reading and should go some way towards dispelling the overly-simplistic myths often perpetuated in the media that the U.K. needs a so-called back-to-basics approach with "more tables, more drilling to emulate the Pacific Rim classrooms that topped the league" [[2](#)]. Given the proper attention, the results of the classroom study described below should provide some forceful lessons for mathematics teaching in the UK.

As part of the Third International Mathematics and Science Study (TIMSS)([2](#)), a representative sample of 131 year 9 classrooms in Japan and the United States were video-taped; 50 in Japan, and 81 in the United States. One complete lesson was videotaped in each classroom and examples of textbook pages or worksheets helpful for understanding the lesson were collected. The videotapes were encoded and stored digitally and analysed using multimedia database software developed especially for the project. All the lessons were also transcribed and analysed on a number of dimensions by teams of coders who were native speakers of the language used in the classroom. These analyses focused on the content and organisation of the lessons, as well as on the teaching practices used by teachers during the lessons.

The analyses of the data relating to typical Year 9 mathematics classrooms([3](#)) in the U.S. and Japan reveals the following [[3](#)]:

- U.S. students spend an average of *143 hours per year studying mathematics*. In Japan the figure is

117 hours.

- in the U.S., students of different abilities are typically divided into different teaching groups. In Japan, no ability grouping is practised
- textbooks were used during class in almost half of U.S. mathematics lessons, but in only 2 percent of Japanese lessons (in Japan textbooks are used for homework)
- learning a skill, such as being able to solve a certain type of problem, or using a standard formula, was listed as the goal by about 60 percent of the U.S. mathematics teachers, compared with 27 percent of the Japanese teachers
- mathematical thinking, such as exploring, developing, and understanding concepts, or discovering multiple solutions to the same problems, was described as the goal of the lesson by 71 percent of the Japanese mathematics teachers, compared with 24 percent of U.S. teachers
- in the U.S., 96 percent of working time in mathematics lessons was spent on routine procedures, in comparison to 41 percent in Japan.
- students were asked to invent new solutions, proofs, or procedures on their own which required them to think and reason in 44 percent of Japanese, and less than 1 percent of U.S. lessons.
- explicit links between concepts used in one part of the lesson to ideas or activities in another part of the lesson was a feature of 96 percent of Japanese mathematics lessons in comparison to about 40 percent of U.S. lessons.
- in the U.S., 86% of U.S. mathematics teachers set at least 90 minutes homework per week, compared to 21% of Japanese teachers (where, typically, set homework is less than one hour per week).
- nevertheless, both Japanese and U.S. students report spending about the same amount of time, between 30 and 60 minutes per night, studying mathematics (in Japan, one factor is that 64% of Japanese students in Year 9 report attending weekly extra tuition sessions in mathematics which tend to focus on the review and practice of mathematical skills).

This indicates that there are considerable differences⁽⁴⁾ between what goes on in U.S. mathematics classrooms, compared with Japanese ones. The video study found that, when comparing the steps typical of Year 9 mathematics lessons in Japan with that in the U.S., the emphasis on *understanding* was clearly evident in the Japanese lessons. The typical Year 9 mathematics lessons in Japan has the following format:

- Teacher poses a complex thought-provoking problem
 - Students struggle with the problem
 - Various students present ideas or solutions to the class
 - Class discusses the various solution methods
 - Teacher summarizes the class' conclusions
- Students practise similar problems

In contrast, an emphasis on skill acquisition was evident in the steps common to most U.S. mathematics lessons, where the following format was typical:

- Teacher instructs students in a concept or skill
 - Teacher solves example problems with the whole class
 - Students practise on their own while the teacher assists individual students

In U.S. mathematics classrooms, when a lesson included a mathematical concept, it was usually simply stated by the teacher, whereas it was developed in Japanese ones. For example, during a lesson on the Pythagorean theorem a U.S. teacher would typically state "we find the length of the hypotenuse of a right-angled triangle by using $a^2 + b^2 = c^2$." In contrast, in Japan the theorem would be proved, or derived, or explained in some detail.

An example of a sequence of Japanese Year 9 mathematics lessons is provided by Keiko Ito-Hino ^[4]. The

topic is introducing linear equations with two variables. At the start of the first lesson, the teacher shows the class two twelve-sided dice and asks the students:

"Suppose that you roll both dice at the same time. In what cases will the sum of two times A plus B equal fifteen."

During the lesson the students work on the problem, discuss it with their peers, and volunteer to present their solutions to the class. The following lesson begins with the teacher saying:

"Today I want you to solve the dice problem by using numerical expressions."

During this lesson the students are encouraged, again through discussion and student presentations, to examine the similarities and differences between the solutions of an equation with two variables and the solution of an equation with one variable. And so the work develops.

In contrast, the video study found that U.S. teachers rarely develop concepts during mathematics lessons. In the US it was often the teacher who did the mental work in developing the concept, while the students listened or answered short questions designed to add to the flow of the teacher's explanation. Japanese teachers, however, typically plan their lessons in such a way that the students *themselves* derived the concept from their own struggle with the problem.

Overall, the video study found that U.S. mathematics teachers were *more likely* to ask students to *practise computational skills*, in most or every class than were their Japanese colleagues. In contrast, Japanese teachers were *more likely* to ask students to *analyse relationships, write equations, explain their reasoning, and solve problems which have no obvious solution*, in most or every class than mathematics teachers in the U.S.

Other TIMSS data reveal that there are also striking differences between the working lives of teachers in Japan and America [2]. For example, with a 30 period teaching week, mathematics teachers in the U.S. most commonly reported teaching *26 periods per week*. *Japanese teachers reported teaching 16 periods*. In addition, Japanese schools are typically designed with one very large teachers' room, in which all teachers have their main desks, and the seating is arranged so that all teachers from a particular subject sit near one another. U.S. mathematics teachers also appear to lack the long and carefully mentored introduction to teaching that Japanese teachers enjoy. In Japan, newly qualified teachers receive intensive mentoring and training during their first year of teaching. Their first year includes at least 60 days of closely mentored teaching and 30 days of further training at resource centres run by local boards of education. Their teaching load is reduced to allow time for these activities. As is typical of Japanese society, mentoring and assistance between junior and senior teachers continues throughout teachers' working lives.

The findings of the TIMSS studies clearly point to significant differences between mathematics teaching in Japan and America. Now let us turn to the relative performance of the pupils(5). Out of 41 countries who took part in the latest TIMSS achievement tests for 13 year olds, *Japan came third, the U.S. came 28th. England came 25th, Scotland 29th*. The proportion of Japanese pupils in the top 10 per cent internationally was 34%; this compares to 7% in both the U.S. and England. In other words, the performance of U.S. and U.K. students is about the same. Japanese students do considerably better. This happens despite U.S. students spending *more school time on mathematics and being set more homework* than corresponding pupils in Japan. It suggests that *how mathematics is taught in school, the quality of professional life of the teachers, and what pupils do for homework (and how this is supported) are among the particularly crucial factors in influencing pupil attainment*. Of course there are other factors. Reynolds and Farrell [1], for example, point to the high status of teachers within Pacific Rim societies, and the high aspirations of parents for their children. However, they also claim that *"mathematics and science are the two areas of the curriculum where the effects of the educational system outweigh the effects of home*

background in determining achievement" (page 2, emphasis added).

The classroom study demonstrates what takes place in typical Year 9 mathematics classrooms in Japan, in contrast to typical U.S. classrooms. Differences between the working lives of mathematics teachers are highlighted by other components of the TIMSS study. The question we need to address now is what can we learn from this evidence in order to improve the situation in the UK. One problem is that without such comprehensive data on what happens in U.K. mathematics classrooms we are forced to rely on evidence from Ofsted inspections. Such inspections suggest that only about a third of mathematics lessons at Key Stage 3 can be considered, in Ofsted terms, to be good or very good [5,6,7,8] by providing, for example, opportunities to "investigate and solve problems" and develop "conceptual understanding" [5, page 21]. In the approximately one-third of mathematics lessons where Ofsted claim that classroom practice is poor, contributory factors are listed as including "knowledge taught in isolation, lacking appropriate contexts" with "little attempt to foster any understanding of underlying concepts" [6, page 25]. Such evidence appears to indicate that U.K. mathematics classrooms are likely to be *much closer* in style to U.S. classrooms than to Japanese ones.

A summary of the comparison of teaching approaches in mathematics between U.S. and Japan mathematics classrooms, based on the TIMSS classroom video study, is as follows:

Typical Year 9 U.S. mathematics lesson	Typical Year 9 Japanese mathematics lesson
is with a class set by ability	is with a mixed-ability class
relies on a textbook	begins with a complex problem
focuses on developing a mathematical skill	focuses on developing mathematical thinking
devotes most available time to practising routine procedures	devotes most time to mathematical reasoning and understanding
features isolated tasks	makes explicit links between concepts

Perhaps significantly, the Japanese mathematics teacher also has a substantially lighter teaching load with correspondingly more time to prepare lessons in well-provided conditions, *plus* they have more support in developing professionally, than does the typical U.S. mathematics teacher.

There is no doubting the relative performance of Japanese compared to both U.S. and U.K. students in mathematics. The research documented in this article helps us to identify what some of the issues are and indicates the sort of quality evidence required to properly inform decisions about the direction in which we need to develop mathematics teaching in the U.K.. It is now particularly clear that research needs to be funded to provide such quality data on what actually happens in U.K. mathematics classrooms. While international comparisons are fraught with difficulty and methods successful in some countries cannot easily be translated wholesale into other countries, there are lessons here for U.K. mathematics teaching that cannot be ignored.

References

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Footnotes

1 [The TIMSS Videotape Classroom Study](#) was directed by James W. Stigler, University of California, Los Angeles.

2 The Third International Mathematics and Science Study (TIMSS) is the largest and most comprehensive international study ever undertaken of educational achievement. Its focus is mathematics and science curricula "both as they are intended and as they are actually delivered in the classrooms, and as they are learned by students". The study is designed to compare and analyse curricula, teacher practices and student achievement in mathematics amongst the participating countries. Mathematics textbooks and curriculum documents have been analysed, the views of students and teachers gathered, classroom filmed, and more than half a million students aged 9, 13 and 16 in nearly 50 countries have been tested.

3 These findings are based on national averages. There will be many mathematics teachers in each country who will not be accurately characterised by the average.

4 In this article I have chosen to concentrate on the teaching approaches employed in the classroom but there also appear to be important differences between the U.S. and Japan in terms of the mathematics curriculum offered. What was "basic" in Year 9 mathematics in the U.S. differed greatly in Japan. For example, in the U.S. the basic content included arithmetic, fractions and a relatively small amount of algebra. In Japan, the basic content included intense coverage of algebra and geometry, much more than in the U.S.. The NCES report [3] suggests that the U.S. Year 9 curriculum attempts to cover more topics than the equivalent curriculum in Japan but may be as much as *one year behind* the Japanese curriculum in terms of depth.

5 Despite official TIMSS advice, reflected in the report on England [9], not to construct a league table, this, of course, is just what is being done. See, for example, the DfEE press release 394/96 dated 20 November 1996 which begins "An international survey of 13 year olds' achievements in science and maths shows that England is 6th out of 25 countries in science and 16th in maths".

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