MEO DYNAMICS AND GNSS DISPOSAL STRATEGIES

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ABSTRACT

In the last years significant understandings in the knowledge of the dynamics of the MEO satellites were achieved. Much work was done in the analysis of viable disposal strategies and technologies for the spacecraft belonging to the Global Navigation Satellite Systems (GNSS), with particular emphasis on the European Galileo system. In the framework of an ESA-ESOC Contract an extensive numerical simulations of different long term evolution scenarios, implementing different disposal strategies were performed. A detailed analysis of the collision risk and manoeuvres need, related to the different scenarios, was performed. In terms of the long term evolution, the scenarios where the orbital instabilities are exploited to remove the objects from the operational regions seems favourite. That is, if the focus is on the long term sustainability of the space environment, the possibility to dilute the collision risk and to aim at the re-entry in the atmosphere of a subset of the disposed GNSS spacecraft is the most attractive. The most "problematic" constellations are Glonass and Beidou. This conclusion is driven by the future launch traffic hypothesized for these constellations and by the past practices that left already a significant number of large uncontrolled spacecraft in the constellation orbital zone, in the case of Glonass. On the other hand, the Galileo constellation is well detached from the others and faces the lowest collision risks. The Stable scenarios seems to minimize the interactions (crossings) with the operational constellations and, therefore, might be preferred for operational reasons. In particular, in the Stable scenarios the inter-constellations interaction is negligible. Particular care should be devoted to the efficiency and reliability of the disposal manoeuvers. A significant share of the collision risk faced by the operational satellites in every simulated scenario can be traced back to the ``failed" satellites (the success rate of the disposal manoeuvers was assumed to be 90 % for all the constellations).

1 INTRODUCTION

The Medium Earth Orbit (MEO) region, home of the operational Global Navigation Satellite Systems (GNSS) GPS and Glonass, is becoming more and more exploited with the advent of the European Galileo and the Chinese Beidou constellations, both in their build-up phase. The sensitive applications of the navigation satellites and the absence of any natural sink mechanism, such as the atmospheric drag, call for a careful debris prevention policy able to preserve the MEO environment, avoiding in the future the problems now already faced by the LEO and the Geostationary Orbit (GEO) environments.

Previous works on the MEO region simulations can be found in [5][6][8][13].

The analysis of different disposal strategies for the spacecraft belonging to the GNSS, with particular emphasis on the European Galileo system, is the aim of this study. The possibility to store the disposed spacecraft in stable circular orbits above the operational orbits is the currently adopted strategy and seems, at first sight, the most viable one. Nonetheless, this apparently straightforward procedure is hindered by a few drawbacks. First, the accumulation, in the next decades, of a significant number of spent uncontrolled spacecraft in a limited region of space can give rise to a local collisional activity, with no possibility to control it from the ground with space surveillance means and avoidance manoeuvres. Moreover the noted instability of the GNSS disposal orbits can lead the disposed uncontrolled spacecraft back to dangerous crossings with the operational orbits in a not too distant future (see [10] and the references therein).

To tackle these issues, in the framework of and ESA-ESOC Contract we performed an extensive study to explore the benefits and drawbacks of the possible disposal strategies for the GNSS spacecraft. Within the study a detailed overview of the current configurations of the GNSSs, along with their operational, maintenance and disposal procedures was given. Then, an analysis of the dynamics of the orbits in the MEO region was performed, with the aim of looking for stable and unstable orbits able to meet the requirements of the proposed disposal strategies. Then, with an eye on future applications, alternative methods for de-orbiting of GNSS satellites at end-of-life, exploiting low-thrust propulsion and non-gravitational perturbations, was studied too [1]. Finally, a large number of numerical simulations of different long term evolution scenarios, implementing different disposal strategies, were performed using the SDM model [9], along with a detailed analysis of the collision risk and manoeuvres need related to the different scenarios.

The detailed results of the whole study can be found in [11]. In Sec. 6 a short summary of the main findings of all the study will be given. In the following we will instead show a summary of the main results concerning the long term evolution of the MEO environment.

2 MEO DYNAMICS

The highly-inclined, medium-Earth orbits (MEOs) of the Global Navigation Satellite Systems (GNSS) lies in a region populated by orbital resonances. A clear picture of their physical significance is of interest for the design of disposal strategies for the four constellations. This concerns particularly the question as to whether suitable stable graveyard orbits exist such that satellites in the disposal orbit will not interfere with the GNSSs, or whether strong instabilities exist that can be exploited to permanently clear this region of space from any future collision hazard.

For this reason we performed an extensive numerical investigation of the MEO phase space to identify the values for the argument of perigee ω corresponding to any given combination of initial epoch, longitude of ascending node and inclination (t_0, Ω, i), which ensure in 200 years stability or instability of the orbital evolution of a disposed satellite in MEO. The results of this analysis are reported in [2]. Here we show some of the obtained results and highlight the main conclusions.

We addressed the possibility of disposing the satellites to an almost circular graveyard orbit located about 500 km above the operational one, or to an eccentric orbit whose eccentricity and semi-major axis depend on the available $\Delta v-$budget, which was about 100 m/s in this work. In both cases, we analysed three different configurations for the initial inclination, namely, the nominal one and a decrease/increase of 1°.

It turns out that we are almost always able to provide initial conditions which can be considered safe over 200 years (except for GLONASS), but we cannot always define initial conditions which lead to an Earth re-entry. This is due to the fact that in practice we do not have the freedom to change (Ω, i) to get a more favourable positioning, but more importantly to the intrinsic dynamics where these satellites live. It turns out that the MEO region displays a significant chaoticity related to the interaction of two or more orbital resonances, according to the Chirikov criterion. This might imply a non-predictability of the long-term behaviour of the bodies. The chaotic zones defined by the regions of overlapping resonances do not preclude the existence of regular trajectories embedded within it. Indeed, the character of the motion depends sensitively upon the initial orientation angles of the satellite and the initial lunar node. This is revealed by the fact that we found large stable regions in the graveyard case for each constellation, even though the constellations exist in such precarious states in the *e*-*i* phase space, always perched on the threshold of instability [3,7]. The work described in [2] revealed that the harmonics $2\omega + \Omega$ for Galileo, BeiDou and GPS and ω for GLONASS are not the only ones responsible for the eccentricity evolution, though to stay far enough from the critical value of inclination associated with the variation of such harmonics seems to ensure a longer stability. Instead, the periodicity associated with the initial epoch shows that the relative Earth–Moon–Sun initial configuration must not be ignored.

As an example of the work performed, in search of proper initial conditions for the simulations described in the next Sections, Figs. 1 and 2 show, as a function of the initial epoch t_0 and longitude of ascending node Ω, for the graveyard orbit scenarios, the values of argument of pericenter, ω, which ensures that the eccentricity will not exceed 0.02 in 200 years, for the four GNSSs. On the other hand, Figs. 3 and 4 show, on the same quantities for the unstable (eccentricity growth) scenarios for BeiDou, GPS an Galileo.

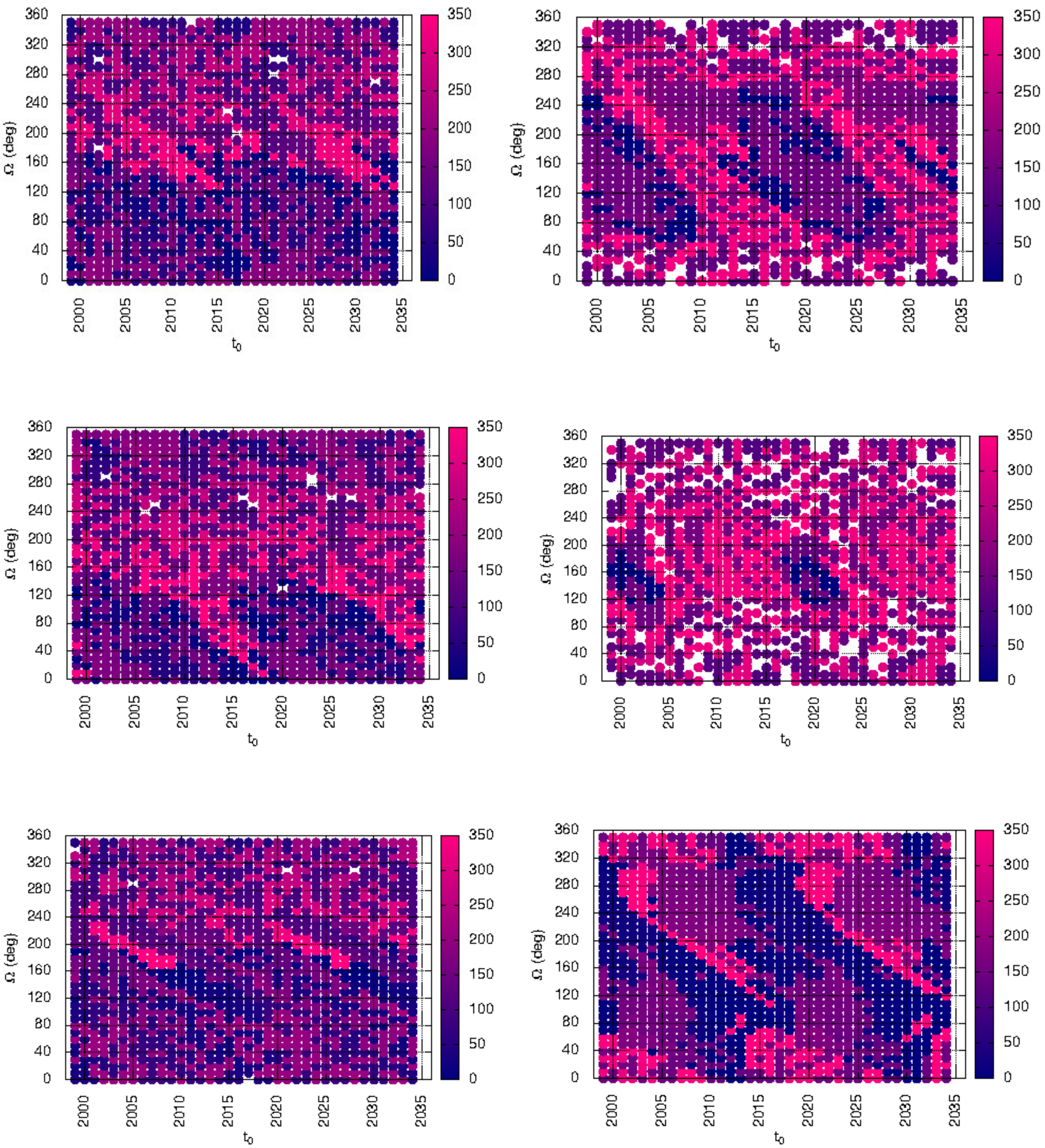

Figure 1. As a function of the initial epoch t_0 and longitude of ascending node Ω we show, in the graveyard orbit scenario for Galileo (left) and GLONASS (right), the value of ω in degrees (color bar) which ensures that the eccentricity will not exceed 0.02 in 200 years. Whenever there exist two values, we represent the one corresponding to the minimum eccentricity growth. Top nominal initial inclination; middle initial inclination decreased by 1°; bottom: initial inclination increased by 1°.

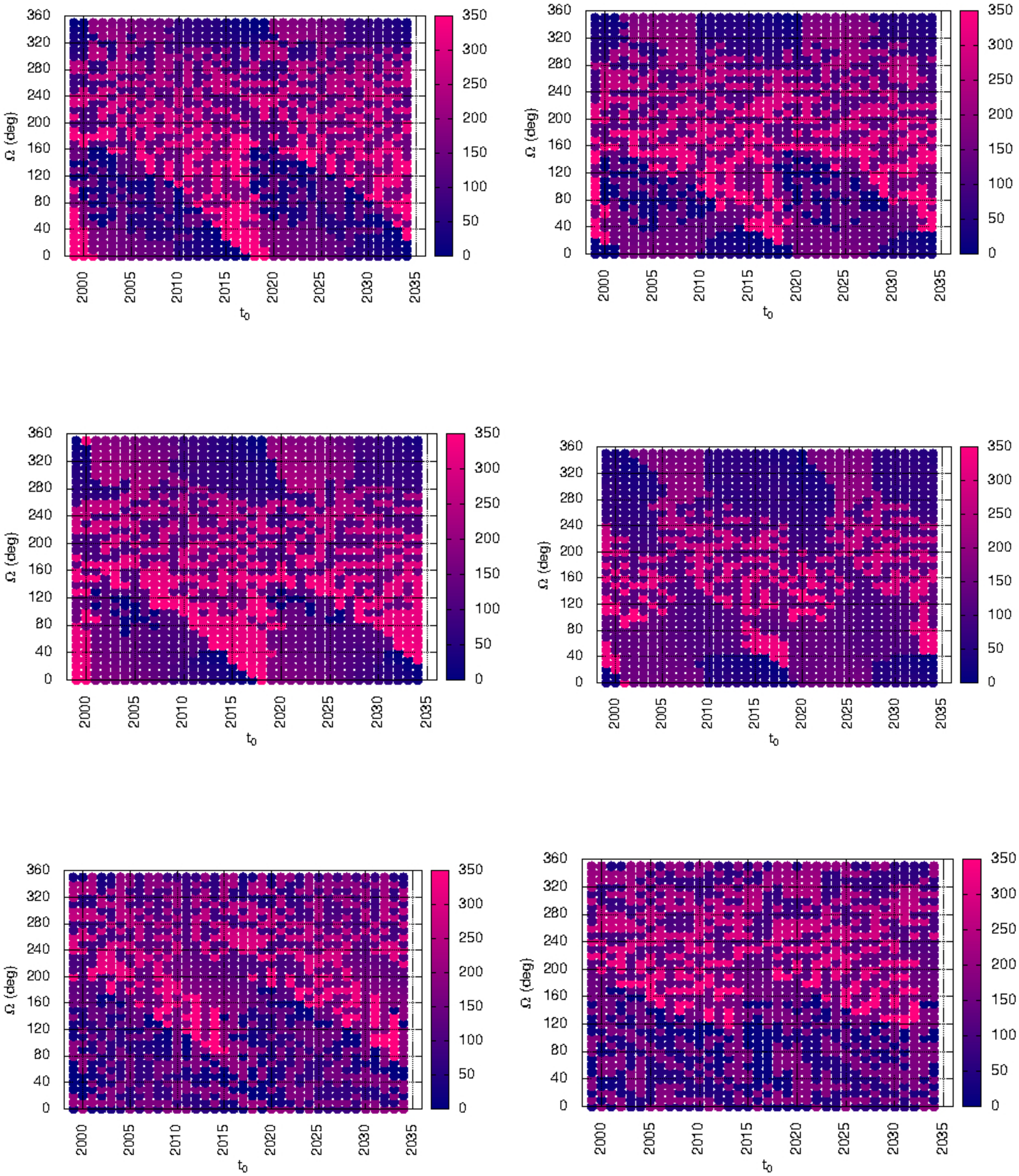

Figure 2. The same as in Fig. 1, for the BeiDou (left) and GPS (right) constellations.

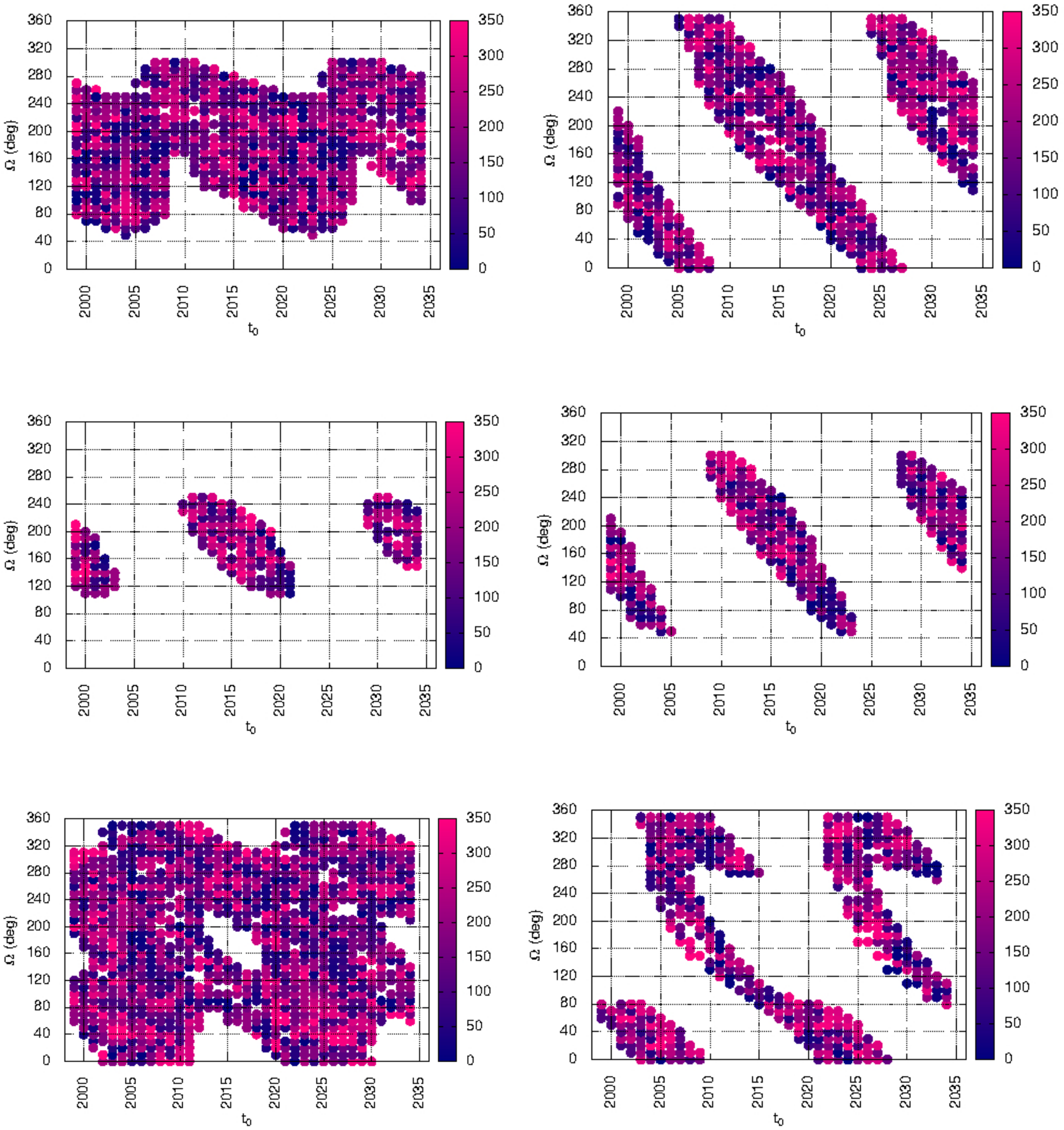

Figure 3. As a function of the initial epoch t 0 and longitude of ascending node Ω we show, for the eccentricity growth scenario for BeiDou (left) and GPS (right) (see Table 2), the value of argument of pericenter in degrees (color bar) which ensures a re-entry. Top: nominal initial inclination; middle initial inclination decreased by 1˚; Bottom initial inclination increased by 1˚.

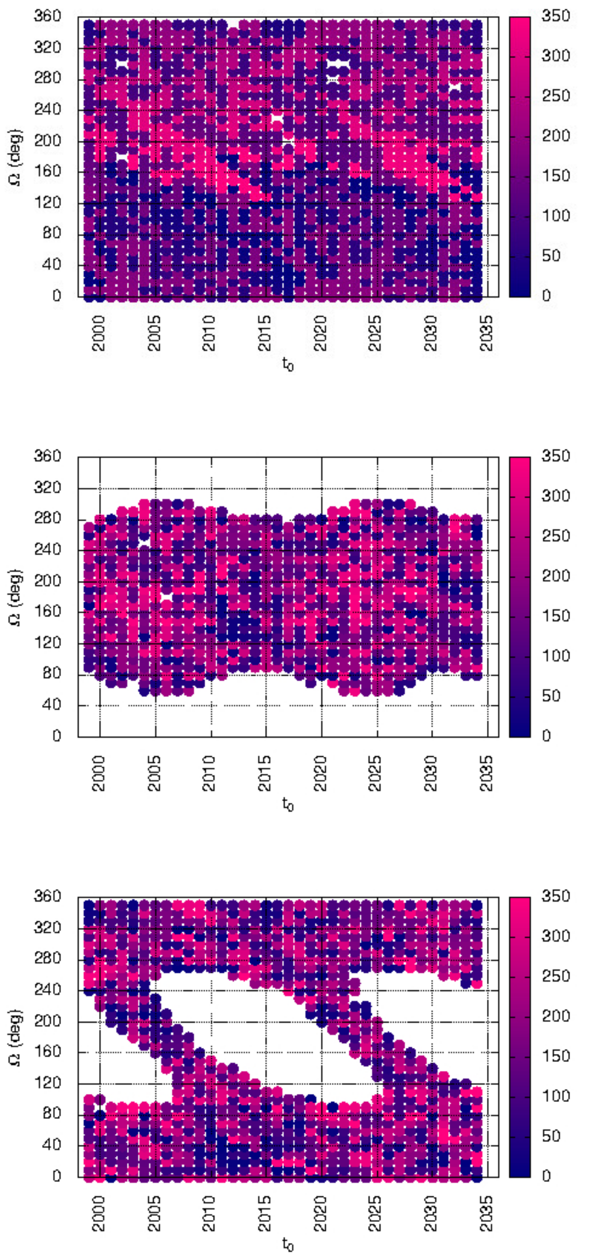

Figure 4. As a function of the initial epoch t_0 and longitude of ascending node Ω we show, in the eccentricity growth scenario for Galileo, the value of argument of pericenter in degrees (color bar) which ensures a re-entry. Top: nominal initial inclination; middle initial inclination decreased by 1°; Bottom initial inclination increased by 1°.

Further work on the subject of the MEO dynamics can be found in the paper by Skoulidou *et al.* in this volume.

3 LONG TERM SIMULATION SCENARIOS

A detailed simulations setup was defined for the various scenarios envisaged in the study.

Beyond the GNSS management procedures detailed here, all the scenarios share the same main assumptions and a common simulation plan for the LEO and GEO traffic. In particular:

- the initial population consists of the objects *larger than 5 cm*, taken from MASTER;
- the simulations consider the whole circum-terrestrial space, from LEO to GEO;
- the launch traffic cyclically mimics the activity of the past decade;
- an 8-year mission lifetime for future spacecraft is assumed;
- the post-mission disposals measures are applied to upper stages and spacecraft with *60 % success rate*;
- the future explosions are set to zero;
- no station-keeping and no collision avoidance manoeuvre are allowed;
- the NASA Breakup model is used;
- 200 years projections are performed.
- 50 Monte Carlo runs are performed for each scenario.

3.1 Reference scenario

The Reference Scenario is the main scenario against which all the others will be compared. It is basically a revised “business-as-usual” scenario where most of the maintenance practices currently adopted by the different constellations are simulated. In some cases, the simulated procedures might not be exactly the same adopted by a particular constellations in the recent past, but instead what are deemed to be the most probable procedures for the near future operations of that system.

In every constellation we assume that the success rate of the end-of-life disposal will be 90% (at difference to what is simulated for the LEO spacecraft, for which a success rate of 60 % is assumed).

Note that, irrespective of the simulated scenario, unless explicitly mentioned, the spacecraft are always disposed to orbits having the same inclination of the operational orbit *at the epoch of disposal* (i.e., the inclination is routinely propagated during the simulation time span and no inclination change manoeuvre is done at the disposal epoch). Similarly, the right ascension of the node

(RAAN) of each object is evolved in time and, at end-of-life, the disposal is done on the same orbital plane of the operational satellite *at the epoch of disposal* (i.e., no change of RAAN in performed with the disposal manoeuvre).

The details of the configuration, launch, maintenance and disposal strategies adopted for the four GNSSs simulated are given in Table 1 and Table 2. Note that, when an interval is indicated, the actual value is randomly drawn from a flat rectangular distribution within the interval.

Table 1. Configuration of the four simulated GNSSs.

	GPS	**Glonass**	**Galileo**	**Beidou**
a [km]	26560	25508	29600	27906
e	<0.023	<0.004	<0.001	<0.001
e [deg]	55°±2°	65° ± 2°	56°±2°.	55°± 2°
Orbit planes	6	3	3	3
Total number of satellites (including spares)	30	27	30	27
Total number of satellites per plane	5	9	10	9
Satellite average lifetime [years]	10	8	10	10
Satellite mass [kg]	1630	1480	665	800
Satellite random tumbling average area [m^2]	16.7	20.2	9.3	20
Upper stage mass [kg]	2850	920	1480	3062
Upper stage random tumbling average area [m^2]	33.5	8.4	8.5	28

3.2 Stable and unstable scenarios

Before detailing the other simulation scenarios, an introduction to the computational method used in

SDM for the end-of-life disposal on stable or unstable orbits is given.

3.2.1 The matrix method

Based on the results of MEO dynamics analysis described in [2], the spacecraft of all four GNSS constellations in MEO will be disposed at the end-of-life in orbits where either the minimal or maximal eccentricity growth is foreseen.
When a spacecraft reaches the end-of-life some of its orbital elements can be changed with a series of impulsive maneuvers, taking into account the available propellant. Disregarding the mean anomaly, these elements are the semimajor axis (a) the eccentricity (e) and the argument of perigee (ω). In some cases it would be beneficial to change also the inclination and the Right Ascension of the Ascending Node (RAAN), but it is well known that the change in the orbital plane implied by a change of these elements would require a very expensive maneuver, almost always incompatible with the available Delta V. Therefore, the inclination and the RAAN of the disposal orbit are kept equal to the ones of the specific satellite operational orbit at the epoch of disposal.
An analytical expression able to catch the whole complexity of the long term behaviour of the eccentricity in the MEO orbital region over the 200-year time span is not currently available.
In the end, what is needed for the purpose of identifying the best disposal orbit is an algorithm that allows to choose the proper values of a, e and ω, given i and RAAN at the disposal epoch, that guarantee the desired long term behaviour of the eccentricity. From the practical point of view of the long term simulations, the values of the disposal semimajor axis and eccentricity are dictated by the available Delta V, and can therefore be considered fixed (i.e., given in input).
Therefore we are left with the choice of the argument of perigee given a set of 4 orbital elements.
As mentioned in Sec. 2, given the nominal disposal semimajor axis and eccentricity, a large number of numerical integrations were performed, sampling the ω-RAAN space (from 0 to 360 degrees, at steps of 10 degrees). For all the cases the time history of all the orbital elements is available. Looking at the growth of the eccentricity, maps of the phase space were produced, for every GNSSs, both for the case of circular disposal orbits and for the case of eccentric disposal orbits (in this case, the disposed satellite is assigned an initial eccentricity

equal to the value used for the computation of the matrix, i.e., e_{disp}=0.0539±0.001).

In the matrix method, implemented in SDM, these plots are translated in matricial form and stored in ASCII files for each navigation constellation. Every time a spacecraft, belonging to a GNSS, has to be de-orbited, according to the scenario simulated (e.g., stable or unstable), the deorbiting algorithm searches within the matrices, for the given epoch and for the RAAN of the epoch, the (single) value of ω that minimizes or maximizes the eccentricity growth.

Note that the matrices were computed considering the nominal GNSSs inclinations. During the SDM runs, as a default, the slight difference between the inclination of the satellites to be disposed and the nominal inclination for whom the matrix was computed is neglected. A further set of simulations, with matrices computed for different inclinations close to the nominal one, were performed too.

3.2.2 Stable scenario: minimal eccentricity growth

The spacecraft of all four GNSS constellations in MEO will be disposed at the end-of-life in orbits where the minimal eccentricity growth is expected, taking into account the proper angular arguments obtained with the matrix method. That is, the elements of the disposal orbits are the same as those used in the Reference case, except for the argument of perigee which is selected with the matrix method.

3.2.3 Unstable scenario: maximal eccentricity growth

In this case the spacecraft will be put in disposal orbits as unstable as possible, again by properly targeting the argument of perigee using the matrix method. In particular:

- an initial disposal manoeuvre with a ΔV ~ 100 m/sec is performed to increase the eccentricity. The value of the initial disposal eccentricity reached for all the four navigation constellations is about 0.05 ± 0.001. The disposal semimajor axis is such that the initial apogee of the disposal orbit is at the altitude of the operational orbit, plus (or minus) the spread.
- the optimal value of the argument of perigee, ω, leading to a fast eccentricity growth is selected from the proper matrix.

3.2.4 Stable and Unstable Galileo scenarios

The simulation scenario is similar to the Stable scenario. The difference here is that, in this case, only the Galileo spacecraft will be disposed with a targeting of the optimal argument of perigee. The satellites of the other constellations will be disposed as in the Reference Case. The purpose of these simulations is to highlight the potential benefits, if any, of a “proper” disposal management of the Galileo constellation alone.

3.2.5 Stable and Unstable scenarios with inclination change

The actual inclination of the constellation orbits can vary with respect to the nominal values by about ± 1 degree, due to launch dispersions and orbital perturbations. As detailed in [5], the stability/instability zones in the phase space are sensitive to small inclination changes. Since orbital maneuvers to change the inclination are very expensive in terms of ΔV, it is not very realistic to simulate orbital plane changes to target the preferred regions of phase space. Therefore, there is the risk to miss the right argument of perigee with the matrix method.

For this reason it was decided to repeat the stable and unstable simulation scenarios, described above, taking into account the actual orbital inclination at the epoch of the disposal. For this purpose, different matrices were computed for disposal orbits having inclination ± 1 degree around the nominal orbits. At the moment of the disposal maneuver, the algorithm is using the matrix that refers to the inclination closer to the actual orbital inclination of the epoch.

4 RESULTS

Although all the simulations were performed considering the whole circumterrestrial space, the focus of the following analysis will be on the MEO region and, in particular, on the GNSSs related spacecraft. The LEO environment is not the goal of this study. Nonetheless, it is worth stressing that the LEO population evolution is fully evolved and is duly considered to properly account for the collision risk of MEO spacecraft on eccentric orbits that might be crossing LEO at perigee. All the simulations consider objects larger than 5 cm.

4.1 Main scenarios results

Figure 5 (top panel) shows the time evolution of the effective number of objects, larger than 5 cm, in the region between 15000 and 35000 km. The thick lines are the average over 50 MC runs in the three scenarios (as detailed in the figure caption), while the thin red lines are the ± 1 sigma uncertainty interval, coming from the MC averaging process of the Reference case (the 1 sigma lines for the other two cases are not shown to avoid cluttering). It is immediately clear that, looking in terms of the number of objects, the three scenarios are basically statistically indistinguishable. At variance from the LEO region, the environment evolution in MEO is driven mostly by the deterministic pace of the launch and removal actions and by a very limited number of collisional fragmentation

Table 2: Details of the launch, maintenance and disposal strategies for the simulated GNSSs.

	GPS	Glonass	Galileo	Beidou
Launches (build-up)	N/A	N/A	2 double launches per year with Soyuz-STB/ Fregat-MT	3 double launches per year with CZ-3B
Launches (constellation maintenance)	1-2 single launches per year with Delta-4	2-3 single launches per year with Soyuz-2-b/Fregat-M	1 double launch per year with Soyuz-STB/Fregat-MT	2 double launches per year with CZ-3B
Spacecraft orbit keeping	No control of RAAN and inclination is foreseen	No control of RAAN and inclination is foreseen	The RAAN of the planes will be kept within a 2° window, i.e. ± 1° around the nominal precessing value. The satellites are launched at one extreme of the control window, so that the slow precession of the nodes will remain within the desired boundaries without the need of a control manoeuvre during the spacecraft lifetime. In in the simulations the satellites will be placed not in the centre of the window, but at the "right" extreme value.	No control of RAAN and inclination is foreseen.
Spacecraft disposal	Re-orbiting 500 ± 10 km above the operational altitude. No targeting of a stable resonant angle will be carried out. Initial ecc. < 0.01.	Re-orbiting 500 ± 10 km above the operational altitude. No targeting of a stable resonant angle will be carried out. Initial ecc. < 0.01.	For the Galileo satellites at the end-of-life, a re-orbiting ΔV budget of approximately 100 ± 10 m/s will be considered, which is translated into a re-orbiting altitude of about 800 km above the constellation altitude. An upper limit of about 800 km above the operational altitude will be considered for the circular disposal orbits, even if the available ΔV would allow a higher disposal. The value applied to each satellite will be extracted between 750 and 800 km. No targeting of a stable resonant angle will be carried out. Initial e < 0.001.	Re-orbiting 500 ± 10 km above the operational altitude. No targeting of a stable resonant angle will be carried out. Initial ecc.< 0.01.
Upper stages disposal	Delta-4 second stages left 950 ± 100 km above the constellation altitude, with eccentricity ≤ 0.01. No targeting of a stable resonant angle.	Fregat-M stages left 300 ± 100 km above the constellation altitude with eccentricity ≤ 0.01. No targeting of stable resonant angle.	Upper stages (Fregat-MT) left 310 ± 10 km above the constellation altitude, with eccentricity ≤ 0.0006.	CZ-3B third stages left in 150 × 21,500 km elliptical transfer orbit.

It is worth stressing that the large width of the 1 sigma bars in Fig. 5 (top panel) is due to the fact that the pace of the growth is due to a very small number of collisions changing significantly the number of objects from one MC occurrence from the others.

The bottom panel of Fig. 5 shows the time evolution of the number of objects larger than 5 cm, divided by object types. Note the linear pace of the intact spacecraft in contrast with the more than linear pace of the fragments. Table 3 lists all the collisional fragmentations recorded in all the MC runs, involving a spacecraft belonging to one of the GNSSs in the three scenarios. It can be noticed how, on average, we can expect less than one collision in the 200-year time span. It is worth noting that only the first entry in the table involved an operational spacecraft, so that one might assume that "in reality" this collision could have been avoided with a proper maneuver triggered by the space surveillance systems. On the other hand, all the other fragmentations involve only disposed, non maneuverable, spacecraft. It can also be noticed that the majority of the collisions are recorded in the Stable cases. Despite the small number of events, this might be a first indication that the accumulation of uncontrolled spacecraft in the disposal regions above the operational orbits can be the source of a future collision activity.

Note that, no feedback collisions, i.e., no collisions between fragments generated in the events of Table 3 and other GNSS related objects, are recorded in any MC run. As indicated in Table 3, all the collisions involving GNSSs objects happen on circular orbits in the MEO region. A few collisional fragmentations are happening also in highly elliptical orbits, mostly of Molniya type, during their LEO crossings at perigee.

4.1.1 Eccentricity evolution

Due to the assumptions in the three main scenarios, we expect a different long term evolution of the orbits of the disposed objects, i.e., mainly in the growth of the eccentricities.

Some statistical measures of the eccentricity distribution, at the end of the 200-year time span, for all the disposed GNSS satellites, in the three scenarios are listed in Table 4, while Table 5 shows the values of the different statistical measures for each constellation.

The global eccentricity values show how the proper choice of the initial disposal angles, performed with the matrix method, allows a better stability or instability of the disposal orbits.

Considering the eccentricities that the uncontrolled disposed satellites must keep in order not to interfere with the operational GNSSs it can be noticed how, while in the Reference case the mean eccentricity for all the constellations is above the allowed eccentricity

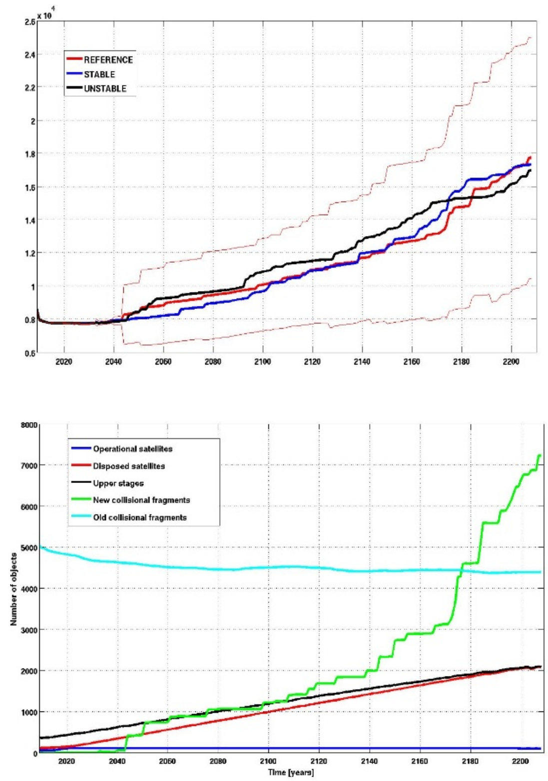

Figure 5. Top panel: Effective number of objects between 15000 and 35000 km, larger than 5 cm in the three scenarios: Reference (thick red line), Stable (blue line) and Unstable (black line). The thin red lines show the ± 1 sigma of the MC averaging in the Reference case. Bottom panel: Effective number of objects between 15000 and 35000 km, larger than 5 cm divided by type: operational satellites (blue), disposed satellites (red), upper stages (black), old collisional fragments (cyan) and new collisional fragments (green).

values, in the Stable case the limiting values are not exceeded, with the exception of the Galileo disposed spacecraft for which the mean value of the eccentricity is exceeding the maximum allowed value (whereas the median is below the maximum allowed value).

The maximum allowed value (whereas the median is below the maximum allowed value).This is related to the noted larger instability of the Galileo orbits and to the presence, in the examined sample, of about 10 % of satellite for whom the disposal manoeuvre did not succeed and that are therefore allowed to reach higher values of the eccentricity.

In any case it can be stated that, on average, the disposed spacecraft are not interfering with the operational ones, both within one GNSS and with the neighbouring ones.

On the other hand, the eccentricity reached in the unstable case while clearly larger than in the other two cases, is still, on average, too small to guarantee a significant number of atmospheric re-entry for the disposed satellites (e.g., only about the the 4.6 % of all the disposed Galileo satellites in the investigated time frame). Figure 6 shows the perigee altitude distribution of the disposed satellites of the four constellations in the three simulation scenarios in year 2209.

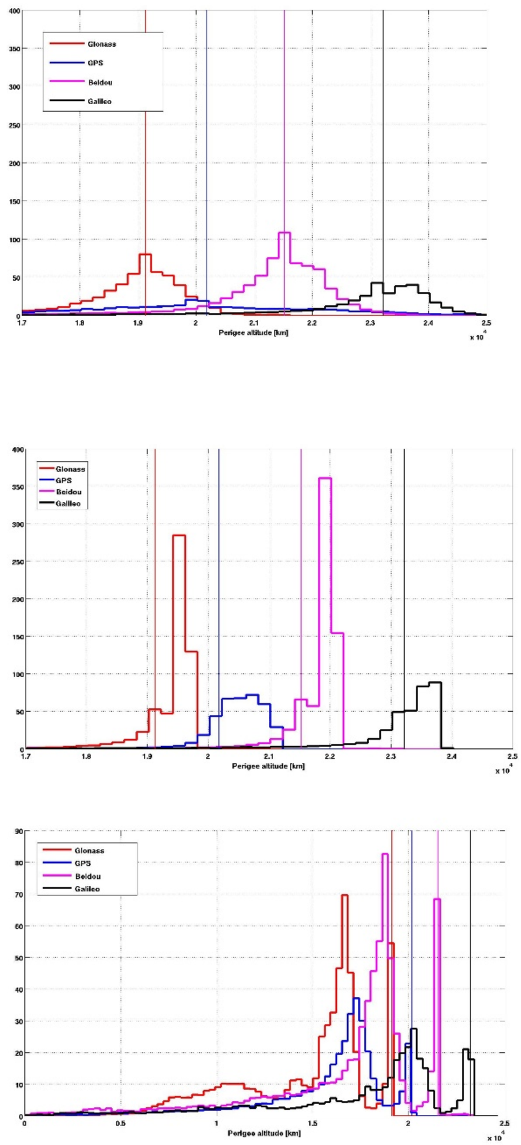

Figure 6. Perigee altitude distribution, in the year 2209, of the disposed satellites in the Reference (top panel), Stable (middle) and unstable (bottom) scenarios. The thin vertical lines mark the altitude of the four GNSSs operational orbits.

Note the differences between the peaks in Fig. 6, mainly related to the difference in traffic (hence number of satellites) between the constellations. The minimal interaction between the disposed GNSS spacecraft and the LEO protected region is noticeable. Checking also the apogee distribution, it can be noticed how, even in the Unstable case, where the maximal eccentricity growth is sought for, the interaction with the GEO protected zone is *de-facto* negligible.

4.1.2 Collision probability evolution

The situation described in the previous sections translates into a picture of the collision probability depicted by Fig. 7. In the plot the overall collision expectancy for all the satellites (operational and disposed) of the four constellations is computed by cumulating over time all the collision probabilities stemming from orbital crossings, involving at least one GNSS object, as recorded by the CUBE algorithm.

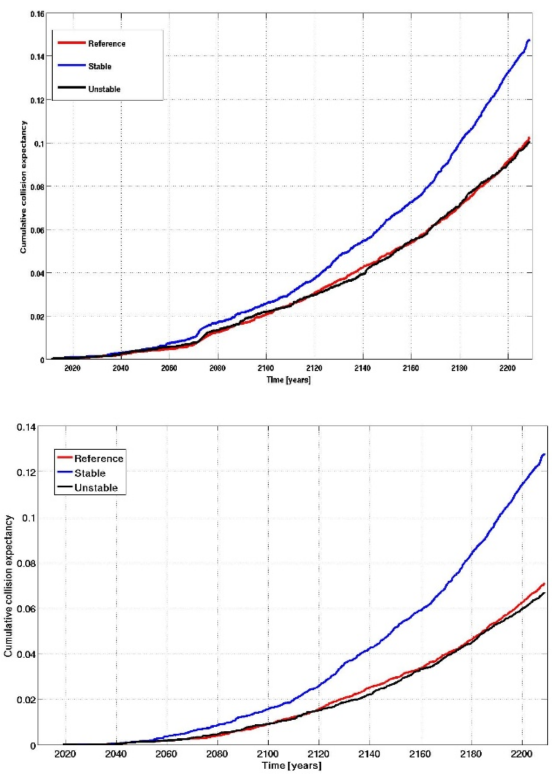

Figure 7. Top panel: cumulative collision expectancy for objects belonging to the GNSSs (both operational and disposed) in the three main scenarios: Reference (red), Stable (blue) and Unstable (black). Bottom panel: cumulative collision expectancy for non-operational objects belonging to the GNSSs against every other non-controlled object (i.e., other GNSSs non-operational, upper stages, fragments, MRO,....). See text for details

Table 3. List of all the collisional fragmentations recorded in the whole set of MC runs performed in the three scenarios, involving, at least, one object belonging to one of GNSSs. The columns list: the year of the event, the semimajor axis, eccentricity, inclination and object type of the target (T) and of the projectile (P).

Scenario	Year of event	T. a [km]	T. e	T. i [deg]	T. Type	P. a [km]	P. e	P. i[deg]	P. Type
Reference	2075	26560	0.0051	55.48	Operational GPS	26577	0.0061	64.75	Disposed Glonass
Reference	2173	27358	0.0028	53.88	Disposed Beidou	27411	0.0010	57.72	Disposed Beidou
Stable	2109	25932	0.0004	66.15	Disposed Glonass	25810	0.0051	63.05	Upper stage
Stable	2167	27060	0.0004	53.08	Disposed GPS	27060	0.0008	56.74	Disposed GPS
Stable	2171	25988	0.0026	65.02	Disposed Glonass	25990	0.0014	63.69	Disposed Glonass
Stable	2179	26036	0.0027	65.75	Disposed Glonass	25810	0.0131	64.27	Upper stage
Stable	2181	28374	0.0007	52.86	Disposed Beidou	28403	0.0005	52.12	Disposed Beidou
Unstable	2056	26580	0.0951	55.96	Disposed Beidou	26567	0.0512	54.80	Disposed Beidou
Unstable	2206	24258	0.0764	65.80	Disposed Glonass	24322	0.0511	62.95	Disposed Glonass

Table 4. Statistical measures of the eccentricity distribution, in the year 2209, for the three main simulation scenarios.

	Mean	Standard Deviation	50th percentile (median)	75th percentile	90th percentile
Reference	0.0404	0.0735	0.0122	0.0394	0.1119
Stable	0.0110	0.0272	0.0024	0.0086	0.0258
Unstable	0.1338	0.1455	0.0752	0.1839	0.3540

Table 5. Statistical measures of the eccentricities of the disposed spacecraft in the years 2209, for each constellation in the three scenarios.

	Mean	Standard deviation	50th percentile (median)	75th percentile	90th percentile
Glonass (Reference)	0.0484	0.0879	0.0126	0.0449	0.1410
Glonass (Stable)	0.0098	0.0247	0.0018	0.0067	0.0250
Glonass (Unstable)	0.1414	0.1328	0.0816	0.2381	0.3543
GPS (Reference)	0.0523	0.0851	0.0159	0.0570	0.1578
GPS (Stable)	0.0089	0.0279	0.0020	0.0063	0.0175
GPS (Unstable)	0.1345	0.1305	0.0859	0.1715	0.3169
Beidou (Reference)	0.0294	0.0522	0.0108	0.0299	0.0756
Beidou (Stable)	0.0069	0.0155	0.0019	0.0056	0.0166
Beidou (Unstable)	0.1355	0.1516	0.0746	0.1730	0.3548
Galileo (Reference)	0.0487	0.0751	0.0206	0.0542	0.1281
Galileo (Stable)	0.0257	0.0425	0.0110	0.0249	0.0667
Galileo (Unstable)	0.1595	0.1682	0.0875	0.2252	0.4230

Keeping in mind the Fig. 6, it can be seen that the concentration of objects in the disposal zones, obtained in the Stable scenarios, while possibly advantageous in terms of operations for the GNSSs, is actually slightly increasing the probability of collision between uncontrolled object, in the long run (as also testified by the higher number of collisions recorded in the SDM runs for the Stable scenario, as detailed in Table 3).

It is worth stressing that operational satellites will be able to perform collision avoidance manoeuver, therefore it is reasonable to assume that most (if not all) the collision risk between operational satellites and other large, trackable intact objects can be reduced to negligible levels (see later for further discussions on the expected rate of avoidance manoeuvers). The bottom panel of Fig. 7 shows the cumulated collision expectancy computed considering only cases where an uncontrolled satellite, from one of the GNSSs, is involved against any other uncontrolled object, i.e., other uncontrolled GNSS satellites, upper stages, MRO, fragments. In particular, in the specific figure, at least an uncontrolled satellites launched after the beginning of the simulation has to be involved, in order to highlight the effects of the scenarios (i.e., a crossing between two uncontrolled GNSS satellites, both launched before the year 2009, is not included in the computation). Again it can be noted how the accumulation of uncontrolled objects in the disposal zones leads to higher values for the Stable scenario, with the Unstable slightly below the Reference one. This plot somehow summarizes the potential environmental effects of the simulated scenarios, since the potential collisions stemming from this collision expectancy cannot be avoided.

It is worth remembering that the actual value of the collision probability computed by CUBE depends from the geometry of an orbital crossing (i.e., trivially, the two objects must be in the same cube at the epoch of the time sampling) and from the velocity of the crossing. Since, as a matter of fact we are recording very few collisional fragmentations and a limited number of crossings in our simulations, a few *deep encounters* can actually unbalance the statistical computation of the cumulated collision expectancy.

Studying the actual number of crossings, in all the 50 MC runs, involving MEO objects for the three main scenarios some preliminary conclusions can be drawn.

The largest number of crossings clearly involve disposed spacecraft. The most affected constellations are Glonass and Beidou and this is strictly related to their launch and traffic characteristics. The largest number of crossings, involving mainly disposed Glonass and Beidou spacecraft, is recorded in the Reference scenario. On the other hand the spreading of the disposal orbits in the Unstable scenario significantly decreases the total number of crossings, according to the so-called "dilution of collision risk". In particular, the crossing between *new objects* is strongly reduced whereas the number of crossings with *historical objects* is increased since the disposed objects in eccentric orbits tend to interact with other populations of objects in the MEO and upper LEO regions.

As was stated above, no feedback collisions are happening in all the MC runs. On the other hand, as seen in Fig. 5 some collisional fragmentations are happening and therefore there are fragments spread around the MEO region. In all the scenarios the new fragments play a minor role, with about 100 crossings in all the 50 MC runs, i.e., about 2 crossings per MC run. The cumulative collision expectancy for all the GNSSs objects against fragments generated in the 200-year investigated time span (i.e., excluding fragments already present in space before the beginning of the simulations) remains below 10^{-2}, even after 200 years.

The plots in Fig. 8 show the breakdown of the collision expectancy for the operational satellites within each constellation, coming from any other object. Whereas the collision risk for the operational satellites can be prevented, if a space surveillance system is in place, these plots can give an initial idea of the relative need for avoidance manoeuvers within the single GNSSs in the different scenarios.

Looking at the Fig. 8 it can be noted how Galileo is facing systematically the lowest risk, due to its detachment in altitude from the other constellations. On the other hand, Glonass is always on top of the others, also due to larger number of objects (past and future) present in its altitude range.

Thanks to the higher statistics, the Glonass results in the three scenarios appear more separated showing how the so-called dilution of the collision probability, caused by the increased eccentricities of the disposed satellites, is indeed minimizing the cumulative collision expectancy for the Unstable scenario, which is about 30% lower with respect to the other two. Then, the Reference and the Stable scenario show very similar cumulative collision expectancies. Note, however, that the number of orbital crossings follow a different pattern: summed over all the 50 MC runs, in the Reference scenario there are 289 crossings, in the Stable scenario there are 244 crossings and in the Unstable scenario there are 306 crossings. This means that, on average, the fewer encounters recorded in the stable case are indeed much deeper (lower relative velocity) than in the other scenarios and their weight in the low numbers statistics we are dealing with is more important.

It is important to note that, in the Stable case, the 34% of the orbital crossing involving operational Glonass satellites are against disposed Glonass satellites. Nevertheless, at a closer look, it can also be noticed how the majority of the disposed satellites involved in these crossings are actually *failed satellites*, i.e., satellites for which the disposal maneuver did not take place and that are left stranded at the operational altitude.

Whereas the encounters in the Unstable scenarios, due to the higher eccentricities, happens with geometric conditions leading to smaller values of the collision probability.

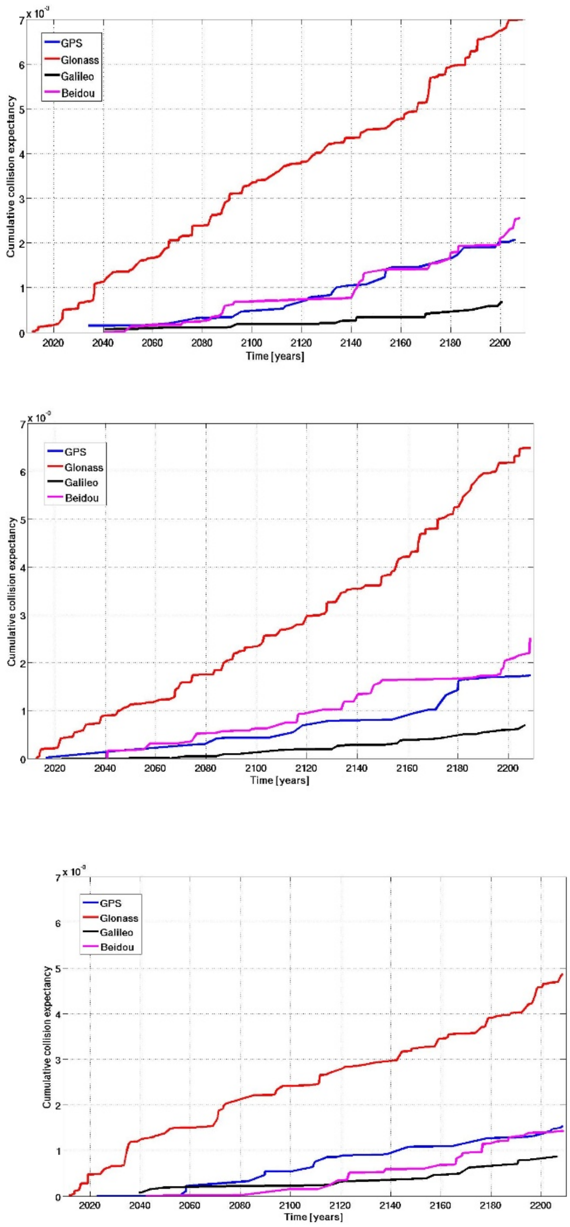

Figure 8. Cumulative collision expectancy for operational satellites belonging to the four GNSSs in the Reference (Top panel), Stable (middle panel) and Unstable (bottom panel) scenarios. The red line refers to Glonass, the blue line to GPS, the magenta line to Beidou and the black line to Galileo

A similar behaviour is found in the other three constellations, with the results for the three scenarios less separated due to the lower statistics. For the GPS and the Galileo constellations, the values are much lower than those obtained for Glonass and Beidou. In particular, for Galileo it remains at the negligible level of 10^{-4} for the first 100 years, barely reaching 10^{-3} only after 200 years. This is due to a significantly lower number of orbital crossings recorded for the operational Galileo and GPS. Looking in detail to the overall number of crossings recorded for the Galileo operational spacecraft, in the 50 MC runs, it can be stated that the reference and unstable scenarios are more prone to orbital crossings of uncontrolled objects with the operational satellites.

The cumulative collision expectancy for operational Galileo satellites against disposed satellites from the same constellations was computed. As expected, in the Unstable scenarios the interaction between operational and disposed spacecraft is increased. The number of crossings is as follows: 17 in the Reference scenario, 12 in the Stable one and 24 in the Unstable one. It is, again, important to stress that, e.g., in the Stable case, out of the 12 crossings, 11 involve *failed disposed satellites*. Only one of the crossings pertains to a disposed satellites for which the disposal manoeuver was actually performed. The number of crossings between operational spacecraft and upper stages is even lower and is basically equal in the three scenarios.

In essence, we are dealing with very low numbers both in terms of overall number of crossings and in terms of collisions expectancies (below 10^{-3} even after 200 years). This makes it difficult to clearly discriminate the three scenarios and to draw firm conclusions. Nonetheless it appears reasonable to state that the Stable scenario minimizes the interaction between operational Galileo satellites and disposed Galileo satellites. Moreover, the importance of the reliability of the disposal manoeuver is once again highlighted.

Conversely, Figure 9 shows the cumulative collision expectancy for non-operational Galileo satellites against all other GNSS operational satellites in the three scenarios. The situation looks similar since the higher eccentricity reached by the disposed satellites in the Unstable scenarios brings them to an increased interaction with the other constellations. In particular, note that, in the Stable scenario, all the orbital crossings happen with disposed Galileo satellites, a part from a single crossing with a disposed Beidou satellite. On the other hand, in the Unstable case, the interaction of the disposed Galileo with the other constellations become apparent with the following orbital crossings. Whereas, the situation is clearly different between the two scenarios and is a clear indication of the expected trend

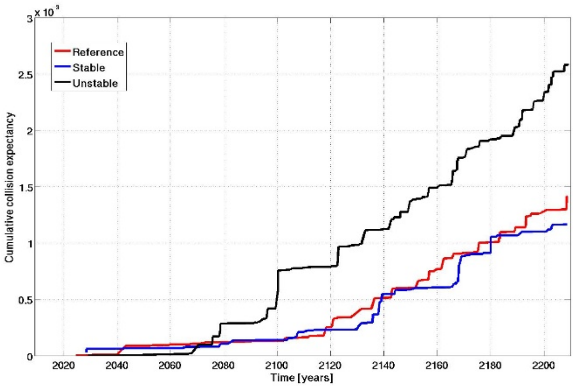

Figure 9. Cumulative collision expectancy for operational satellites belonging to the four GNSSs in the Reference (Top panel), Stable (middle panel) and Unstable (bottom panel) scenarios. The red line refers to Glonass, the blue line to GPS, the magenta line to Beidou and the black line to Galileo.

4.2 Results of the scenarios with inclination change

The Stable and Unstable scenarios were simulated again choosing, for every disposal, the matrix computed for the inclination closest to one of the spacecraft at the disposal epoch. These two new scenarios are dubbed Stable Inc. and Unstable Inc.
As a matter of fact, the results of these new scenarios are very similar to those described in Sec. 4.1.
The statistical measures of the eccentricity, at the end of the 200-year time span, for all the disposed GNSS satellites, in the two new scenarios are almost identical to those listed in Sec. 4.1 for the standard scenarios, telling us that the effect of the slight inclination difference in the choice of the disposal orbits is not significant. Slightly higher values for the Unstable inclined scenario are found and this might be an indication of a possible improvement attainable with an optimized disposal strategy, taking into account the actual inclination of the satellites at the end-of-life. On the other hand, it is clear that, on average, over all the cases treated in the long term simulation described in this note, these small differences cannot play any significant role in the global picture.

In fact, comparing the cumulative collision expectancy in the new scenarios (not shown here for lack of space, see [11]), with respect to three main cases shown in Fig. 9, it can be clearly noticed how the new scenarios give substantially the same results, in terms of collision risks for the GNSSs. The only noticeable difference, with respect to the trend already seen in Fig. 9, is that the Unstable case leads to a slightly reduced cumulative collision expectancy, which is not altering the general conclusions drawn in Sec. 4.1.

As a general comments, it can be stated that the overall long-term statistical behaviour of the MEO environment in not significantly affected by possible inclination inaccuracies

4.3 Results of the scenarios with Galileo only targeted disposal

The results of Sections 4.1-4.2 confirm the conclusions already reached in previous works [8][12], calling for a global management of all the GNSSs, where the mitigation measures are harmonized between all the constellations. On the other hand, political, economical and practical reasons will most probably prevent the realization of this idealized scenario. Therefore we devised two new simulation scenarios where only the Galileo spacecraft are disposed targeting stable or unstable orbits. The other three constellations were instead managed following the Reference scenario. These two new scenarios were dubbed Stable Galileo and Unstable Galileo.

For the disposal of the Galileo spacecraft the method described in Sec. 2 (i.e., considering the actual inclinations of the spacecraft at end-of-life) was adopted. Note that, at variance from the Unstable Inc. scenario, in the Unstable Galileo one *only the Galileo spacecraft are moved to an elliptic unstable orbit*.

The purpose of the two “Galileo only” scenarios is to check whether the application of targeted disposal policies, only for the Galileo constellations, are still useful for minimizing the collision risk and the avoidance manoeuver rate on the constellation itself.

Comparing the cumulative collision expectancy for operational Galileo satellites as expected, only small differences can be noticed between the three main scenarios of Sec. 4.1 and the *Galileo-only* scenarios (see [11] for details). Looking at the results of the Stable-Galileo case, it can be noticed how the Stable-Galileo is very similar to the Reference case and shows an increased collision expectancy with respect to the Stable Inclined case due, to the fact that the disposed spacecraft of the other constellations are actually placed in non-targeted orbits with possibly growing eccentricities.

In any case, it can be concluded that the detached orbit of the Galileo satellites makes them only marginally sensible to the management policies of the other three GNSSs.

On the other hand, a comment on the interaction between the disposed Galileo satellites and the other constellations, in this scenarios where only Galileo is performing disposal manoeuver. The conclusions from this study is that a minimal interaction is recorded for the Stable Galileo scenario, whereas an increased interaction is seen in the Unstable Galileo scenario. It must be stressed that the level of this interaction is very limited and thus it does not appear as a strong argument to prevent the adoption of a “dilution of collision risk” strategy.

5. Collision risk and expected maneuver rate

Following the approach described in [1] and [4], a detailed analysis the collision risk against selected targets was performed. The output of the SDM simulations described in Sec. 4 is used as the background environment and a specific method detailed in the following section is used to identify the most relevant features of the collision risk over short periods of time. The possible need of avoidance manoeuvres is the different simulated scenarios is investigated too.

The simulation setup is as follows:

- The overall debris environment obtained as output of the SDM simulations described in Sec. 4 is used as the “background” population against which a selected “target” object is flown. That is, in a post-processing phase the orbit of a target object is propagated, along with the orbits of all the background population and the orbital crossings are recorded.
- Each object has its own (diagonal) covariance matrix according to the orbital regime (LEO-MEO-GEO).
- The CUBE algorithm (implements in SDM [9]) is used as a filter to identify orbital crossings. For this purpose, CUBE is evaluated with a much shorter time-step of 10^{-4} days (i.e., 8.64 sec). The time step is chosen to be short enough to catch most of the orbital crossings, while keeping the computational burden to an acceptable level. It is worth remembering that the standard CUBE evaluation time step for an SDM run is 5 days.
- To cumulate statistics, at each evaluation time step, the anomalies of the population objects (projectiles) are randomized and the CUBE evaluation is performed for the 500 randomized anomalies (resulting in a local Monte Carlo experiment).
- Every time, in anyone of the 500 MC occurrences, two objects are found within an enlarged cube ($30 \times 30 \times 30$ km^3) the collision probability is evaluated with the Foster algorithm [4].
- Due to the heavy computational burden, related to the short time steps and the large number of MC evaluations, 1-month snapshots are evaluated at different epochs (e.g. in the years 2009, 2029, 2059, 2109, etc.).

This analysis was performed on a significant number of test objects orbiting in different regions of space. In particular the interactions of the disposed GNSS spacecraft with objects in LEO, GEO and HEO was explored. Moreover, the collision risk faced by the operational constellation satellites against all the other GNSS related objects was checked. In these latter cases, for each constellation, a satellite on the operational orbit was selected as the test target. The focus of the analysis was on the interaction only against objects related to the GNSSs, to highlight the effects of the different simulated scenarios. The interaction against the background objects transiting in MEO is considered similar for all the tested target and is therefore not shown and discussed.
For all the cases the three main scenarios (Reference, Stable and Unstable), were simulated, in one-month snapshots at 5 different epochs: 2009, 2059, 2109, 2159 and 2209.

Furthermore, for the Galileo constellation, the existence of possible asymmetries in the collision risk on the constellation planes was tested by computing the collision risk on an operational satellite located on a different constellation plane (i.e., having the same semimajor axis, eccentricity and inclination, but a different RAAN, separated by 120 degrees).

Looking in detail to short time spans the main features, advantages and disadvantages, of the scenarios described in Sec. 2 can still be noticed. For a complete report on the collision risk analysis the reader can refer to [11]. Here we only summarize the main outcome.

As expected, due to orbital characteristics, the interaction of the disposed GNSS satellites with the LEO and GEO protected zones is negligible, both in absolute terms and, even more, compared to the background risk in those regions of space.

For the operational GNSS spacecraft, the highest interaction with other MEO objects is generally recorded in the Unstable scenarios. In none of the scenarios considered a collision risk higher than the thresholds commonly adopted for collision avoidance was ever recorded, Only a few crossings with probability higher than 10^{-6} were found. This is related to the very low spatial density of objects in the MEO region, which makes orbital crossings statistically rare events.

The low interaction of the Galileo constellation with the other three GNSS is confirmed. The majority of the risk for operational Galileo comes from disposed Galileo spacecraft and from GNSS related upper stages.

In all the constellations, and in particular for Galileo, the role of the “failed” satellites appears of paramount importance. Most of the intra-constellation collision risk is due to satellites that were not able to manoeuvre out of the operational zone, thus somehow nullifying the efforts made in devising optimal complex mitigation strategies.

The analysis of the collision risk on two different Galileo constellation planes (with RAAN separated by 120 degrees) did not show the evidence of any effect related to the considered plane. That is, there is no notable angular asymmetry in the distribution of the collision risk for the Galileo planes.

5 CONCLUSIONS

The main results of the long term simulation campaign can be summarized as follows:

- In terms of the long term environment evolution, the Unstable scenario seems favourite. That is, if the focus is on the long term sustainability of the space environment, the possibility to dilute the collision risk and to aim at the re-entry in the atmosphere of a subset of the disposed GNSS spacecraft is the most attractive.
- The most "problematic" constellations are Glonass and Beidou. This conclusion is driven by the future launch traffic hypothesized for these constellations and by the past practices that left already a significant number of large uncontrolled spacecraft in the constellation orbital zone, in the case of Glonass.
- The Stable scenarios seems to minimize the interactions (crossings) with the operational constellations and, therefore, might be preferred for operational reasons. In particular, in the Stable scenarios the inter-constellations interaction is negligible.
- The Galileo constellation is well detached from the others and faces the lowest collision risks. This relates both to the interaction of the operational Galileo satellites with the disposed satellites from the other GNSSs and to the interaction between disposed Galileo satellites and the satellites belonging to the other GNSSs.
- Particular care should be devoted to the efficiency and reliability of the disposal manoeuvers. A significant share of the collision risk faced by the operational satellites in every simulated scenario can be traced back to the "failed" satellites (the success rate of the disposal manoeuvers was assumed to be 90 % for all the constellations).

Concerning the collision risk analysis the main conclusions are as follows:

- As expected, due to orbital characteristics, the interaction of the disposed GNSS satellites with the LEO and GEO protected zones is negligible, both in absolute terms and, even more, compared to the background risk in those regions of space.
- For the operational GNSS spacecraft, the highest interaction with other MEO objects is recorded in the Unstable scenarios.
- The low interaction of the Galileo constellation with the other three GNSS is clearly confirmed. The majority of the risk for operational Galileo comes from disposed Galileo spacecraft and from GNSS related upper stages.
- In all the constellations, and in particular for Galileo, the role of the "failed" satellites appears of paramount importance. Most of the intra-constellation collision risk is due to satellites that were not able to manoeuvre out of the operational zone, thus somehow nullifying the efforts made in devising optimal complex mitigation strategies.
- The analysis of the collision risk on two different Galileo constellation planes (with RAAN separated by 120 degrees) did not show the evidence of any effect related to the considered plane. That is, there is no notable angular asymmetry in the distribution of the collision risk for the Galileo planes.
- Concerning the manoeuvre rate for each of the simulated scenarios, as a matter of fact, none of the orbital crossings actually triggered a manoeuvre, even considering a very low threshold of 10^{-5}. This is related to the very low spatial density of objects in the MEO region, which makes orbital crossings statistically rare events.

6 ACKNOWLEDGMENTS

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