

# Constraint on the branching ratio of $B_c^- \rightarrow \tau \bar{\nu}$ from LEP1 and consequences for $R(D^{(*)})$ anomaly

A.G. Akeroyd<sup>1,\*</sup> and Chuan-Hung Chen<sup>2,†</sup>

<sup>1</sup>*School of Physics and Astronomy,  
University of Southampton, Highfield,  
Southampton SO17 1BJ, United Kingdom*

<sup>2</sup>*Department of Physics, National Cheng-Kung University, Tainan 70101, Taiwan*

(Dated: September 22, 2017)

## Abstract

Recently there has been interest in the correlation between  $R(D^*)$  and the branching ratio (BR) of  $B_c^- \rightarrow \tau \bar{\nu}$  in models with a charged scalar  $H^\pm$ . Any enhancement of  $R(D^*)$  by  $H^\pm$  alone (in order to agree with current data) also enhances  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$ , for which there has been no direct search at hadron colliders. We show that LEP data taken at the  $Z$  peak requires  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu}) \lesssim 10\%$ , and this constraint is significantly stronger than the recent constraint  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu}) \lesssim 30\%$  from considering the lifetime of  $B_c$ . In order to respect this new constraint, any explanation of the  $R(D)$  and  $R(D^*)$  anomaly in terms of  $H^\pm$  alone would require the future measurements of  $R(D^*)$  to be even closer to the Standard Model prediction. A stronger limit on  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$  (or its first measurement) would be obtained if the L3 collaboration used all its data taken at the  $Z$  peak.

---

\*Electronic address: [a.g.akeroyd@soton.ac.uk](mailto:a.g.akeroyd@soton.ac.uk)

†Electronic address: [physchen@mail.ncku.edu.tw](mailto:physchen@mail.ncku.edu.tw)

## I. INTRODUCTION

The  $B_c$  meson is the ground state of a quarkonium system that is composed of a  $c$  and a  $b$  quark. Prior to the operation of the LHC there were only a few measurements of its properties from Tevatron data [1–4]. The LHC experiments (in particular LHCb) promise the first detailed study of  $B_c$ . More precise measurements of its mass and lifetime are now available, and several decay channels have been observed for the first time. It is well known that precise measurements of the branching ratios (BRs) of hadrons play an important role in constraining the properties of new physics particles. The measured BRs of decays such as  $b \rightarrow s\gamma$ ,  $B_u^- \rightarrow \tau\bar{\nu}$  and  $B_u^- \rightarrow D^{(*)}\tau\bar{\nu}$  all provide constraints on the coupling constants and the masses of new physics particles, and often such constraints are stronger than those that are derived from direct searches at the LHC. There have been a few works on the potential of the  $B_c$  meson to probe the presence of new physics particles. In particular the BR of the leptonic decay  $B_c^- \rightarrow \tau\bar{\nu}$  could be significantly enhanced by a charged Higgs boson ( $H^\pm$ ) [5–7] or by supersymmetric particles with specific R-parity violating couplings [8, 9].

The potential of the  $B_c$  meson to constrain the properties of new physics particles has attracted renewed attention recently. It was shown in [10] that the measured value of the lifetime of  $B_c$  disfavors an explanation of the  $R(D)$  and  $R(D^*)$  anomaly (in  $B_u^- \rightarrow D^{(*)}\tau\bar{\nu}$  decays) in terms of an  $H^\pm$  alone\*. This is because any enhancement of  $R(D^*)$  by an  $H^\pm$  would also cause an enhancement of the BR of the unobserved decay  $B_c^- \rightarrow \tau\bar{\nu}$ . In order to comply with the current world average of the  $B_c$  lifetime it was shown that  $\text{BR}(B_c^- \rightarrow \tau\bar{\nu}) \lesssim 30\%$  is necessary, but accommodating the measured values of  $R(D)$  and  $R(D^*)$  by  $H^\pm$  alone would require  $\text{BR}(B_c^- \rightarrow \tau\bar{\nu}) > 30\%$ .

In this paper we derive a stronger bound on  $\text{BR}(B_c^- \rightarrow \tau\bar{\nu})$  than that obtained from the lifetime of  $B_c$ . LEP data taken at the  $Z$  peak constrained a combination of  $B_u^- \rightarrow \tau\bar{\nu}$  and  $B_c^- \rightarrow \tau\bar{\nu}$  [12–14]. This was first pointed out in [6], and in an earlier work [7] we showed that a signal for the sum of the processes  $B_u^- \rightarrow \tau\bar{\nu}$  and  $B_c^- \rightarrow \tau\bar{\nu}$  might be observed if the L3 collaboration (which had the strongest limits [12] from the LEP collaborations) performed the search with their full data sample. A crucial input parameter for the detection prospects of  $B_c^- \rightarrow \tau\bar{\nu}$  is the transition probability (denoted by  $f_c$ ) of a  $b$  quark hadronising to a  $B_c$ .

---

\* For a study of the impact of the  $B_c$  lifetime on a leptoquark explanation of the  $R(D)$  and  $R(D^*)$  anomaly see [11].

In [7] the value of  $f_c$  was obtained (with sizeable errors) from early Tevatron measurements.

Building on the analysis of [7], we first obtain a much more precise evaluation of  $f_c$  from measurements of  $B_c$  production/decay with the full Tevatron data [15] and from LHC measurements [16–19]. We then derive a formula for the bound on  $\text{BR}(B_c^- \rightarrow \tau\bar{\nu})$  from LEP data, which was not obtained in [7]. The bound can be expressed in terms of experimentally determined quantities and just one theoretical input parameter, which is the BR of  $B_c^- \rightarrow J/\psi\ell\bar{\nu}$ . Guided by recent lattice QCD calculations of the form factors for  $B_c^- \rightarrow J/\psi\ell\bar{\nu}$ , we present the preferred range for its theoretical BR. We then obtain a bound on  $\text{BR}(B_c^- \rightarrow \tau\bar{\nu})$  that is considerably stronger than the bound in [10] from considering the lifetime of  $B_c$ . Finally we discuss the consequences of this stronger bound on  $\text{BR}(B_c^- \rightarrow \tau\bar{\nu})$  for an interpretation of the  $R(D)$  and  $R(D^*)$  anomaly in terms of an  $H^\pm$  alone.

## II. THE DECAY $B_c^- \rightarrow \tau\bar{\nu}$ AND SEARCHES AT LEP

The LEP searches for  $B_u^- \rightarrow \tau\bar{\nu}$  with data taken at  $\sqrt{s} \sim 91$  GeV (the “ $Z$  peak”) [12–14] were sensitive to  $\tau\bar{\nu}$  events originating from both  $B_u^- \rightarrow \tau\bar{\nu}$  and  $B_c^- \rightarrow \tau\bar{\nu}$  [6]. Hence the published limits constrain an “effective branching ratio” defined by:

$$\text{BR}_{\text{eff}} = \text{BR}(B_u^- \rightarrow \tau\bar{\nu}) \left( 1 + \frac{N_c}{N_u} \right). \quad (1)$$

This expression applies to all searches for  $B_u^- \rightarrow \tau\bar{\nu}$  at  $e^+e^-$  colliders with data taken at the  $Z$  peak. For searches at the  $\Upsilon(4S)$  (i.e. the BABAR and BELLE experiments operating with  $\sqrt{s} \sim 10.6$  GeV) the  $B_c$  meson cannot be produced. Thus in those experiments  $N_c = 0$  and  $\text{BR}_{\text{eff}} = \text{BR}(B_u^- \rightarrow \tau\bar{\nu})$ . At the  $Z$  peak one has the following expression for  $N_c/N_u$ :

$$\frac{N_c}{N_u} = \frac{f_c}{f_u} \frac{\text{BR}(B_c^- \rightarrow \tau\bar{\nu})}{\text{BR}(B_u^- \rightarrow \tau\bar{\nu})}. \quad (2)$$

Substituting for  $N_c/N_u$  in eq. (1) gives rise to following expression for  $\text{BR}(B_c^- \rightarrow \tau\bar{\nu})$  in terms of  $\text{BR}_{\text{eff}}$ :

$$\text{BR}(B_c^- \rightarrow \tau\bar{\nu}) = \frac{f_u}{f_c} [\text{BR}_{\text{eff}} - \text{BR}(B_u^- \rightarrow \tau\bar{\nu})]. \quad (3)$$

Here  $\text{BR}(B_u^- \rightarrow \tau\bar{\nu}) = (1.06 \pm 0.19) \times 10^{-4}$ , which is the world average [20] of BABAR and BELLE measurements. The L3 collaboration obtained the bound  $\text{BR}_{\text{eff}} < 5.7 \times 10^{-4}$  [12]. If  $f_c/f_u$  is known then a bound on  $\text{BR}(B_c^- \rightarrow \tau\bar{\nu})$  can be derived from eq. (3). The value of  $f_c/f_u$  can be obtained from Tevatron and LHC data (see later).

In the Tevatron Run I and II the following ratio was measured:

$$\mathcal{R}_\ell = \frac{\sigma(B_c) \cdot \text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu})}{\sigma(B_u) \cdot \text{BR}(B_u^- \rightarrow J/\psi K^-)}. \quad (4)$$

Tevatron Run I data with  $0.11 \text{ fb}^{-1}$  gave the result  $\mathcal{R}_\ell = 0.13 \pm 0.06$  [1]. Tevatron Run II data with  $0.36 \text{ fb}^{-1}$  gave  $\mathcal{R}_\ell = 0.28 \pm 0.07$  [3], and this measurement was used in the analysis of [7] when extracting  $f_c/f_u$ . Recently, using the full CDF Run II data ( $8.7 \text{ fb}^{-1}$ ) the result  $\mathcal{R}_\ell = 0.211 \pm 0.012 \pm 0.021$  was obtained [15]. The transition probability  $f_c$  determines  $\sigma(B_c)$  and several theoretical calculations are available for  $\text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu})$  [21–32].

The LHC collaborations have not yet measured  $R_\ell$  directly. However, two related ratios have been measured, from which a measurement of  $R_\ell$  can be obtained. The ratio  $\mathcal{R}_{\pi/K}$  is defined as:

$$\mathcal{R}_{\pi/K} = \frac{\sigma(B_c) \cdot \text{BR}(B_c^- \rightarrow J/\psi \pi^-)}{\sigma(B_u) \cdot \text{BR}(B_u^- \rightarrow J/\psi K^-)}. \quad (5)$$

The measurements at CMS with  $\sqrt{s} = 7 \text{ TeV}$  and  $5 \text{ fb}^{-1}$  [16], LHCb collaboration with  $\sqrt{s} = 7 \text{ TeV}$  and  $0.37 \text{ fb}^{-1}$  [17], and LHCb collaboration with  $\sqrt{s} = 8 \text{ TeV}$  and  $2 \text{ fb}^{-1}$  [18] have been averaged in [20], with the result  $\mathcal{R}_{\pi/K} = (6.72 \pm 0.19) \times 10^{-3}$ . The ratio  $\mathcal{R}_{\pi/\mu}$  is defined as:

$$\mathcal{R}_{\pi/\mu} = \frac{\text{BR}(B_c^- \rightarrow J/\psi \pi^-)}{\text{BR}(B_c^- \rightarrow J/\psi \mu \bar{\nu})}. \quad (6)$$

The measured value at LHCb with  $\sqrt{s} = 7 \text{ TeV}$  and  $1 \text{ fb}^{-1}$  is  $\mathcal{R}_{\pi/\mu} = 0.0469 \pm 0.0054$  [19]. Now the ratio  $\mathcal{R}_{\pi/K}$  in eq. (5) can be written as:

$$\mathcal{R}_{\pi/K} = \mathcal{R}_\ell \cdot \mathcal{R}_{\pi/\mu}. \quad (7)$$

Hence  $\mathcal{R}_\ell$  can be extracted from the LHCb measurements of  $\mathcal{R}_{\pi/\mu}$  and  $\mathcal{R}_{\pi/K}$ . One obtains

$$\mathcal{R}_\ell = \frac{\mathcal{R}_{\pi/K}}{\mathcal{R}_{\pi/\mu}} = 0.143 \pm 0.017. \quad (8)$$

All the above measurements of  $\mathcal{R}_\ell$  are summarised in Table I. We note that there is some tension between the Tevatron Run II and LHC results, but the average of the Tevatron Run I and II measurements agrees well with the LHC measurement. Since  $\sigma(B_c)/\sigma(B_u) = f_c/f_u$  then from the definition of  $\mathcal{R}_\ell$  one has:

$$\frac{f_c}{f_u} = \frac{\text{BR}(B_u^- \rightarrow J/\psi K^-)}{\text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu})} \mathcal{R}_\ell. \quad (9)$$

	Tevatron Run I	Tevatron Run II	Average I+II	LHC
$\mathcal{R}_\ell$	$0.13 \pm 0.06$	$0.211 \pm 0.024$	$0.171 \pm 0.032$	$0.143 \pm 0.017$

TABLE I: Measured values of  $\mathcal{R}_\ell$  at Tevatron Run I and II, average of Run I+II, and LHC.

Here  $\text{BR}(B_c^- \rightarrow J/\psi K^-) = (1.028 \pm 0.04) \times 10^{-3}$ . Using the measured values of  $\mathcal{R}_\ell$  from the Tevatron and LHC gives the following expression:

$$\frac{f_c}{f_u} = \frac{10^{-4}}{\text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu})} \begin{cases} 1.758 \pm 0.336 & (\text{Tevatron data}), \\ 1.470 \pm 0.184 & (\text{LHC data}). \end{cases} \quad (10)$$

In Fig. 1 we display contours of  $\mathcal{R}_\ell$  as a function of  $\text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu})$  and  $f_c/f_u$ , and the band denotes the prediction of the various theoretical calculations for  $\text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu})$  whose values lie in the range  $(1.5 \sim 2.5)\%$  [21–32].

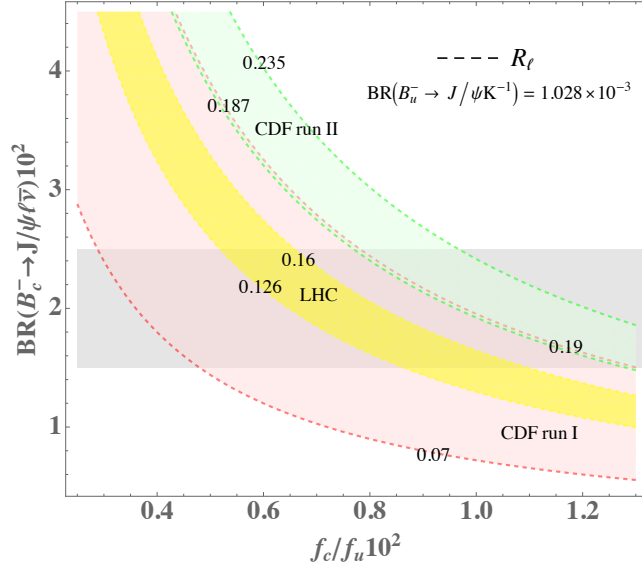


FIG. 1:  $R_\ell$  as a function of  $\text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu})$  and  $f_c/f_u$ , where the different bands denote the results from the CDF Run I (red), Run II (green), and LHC (yellow) with  $1\sigma$  errors.

We now substitute the expression for  $f_c/f_u$  in the expression for  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$  in eq.(3). Using  $\text{BR}_{\text{eff}} < 5.7 \times 10^{-4}$  [12], and the Tevatron/LHC data for  $f_c/f_u$  in eq. (10) one obtains the expression:

$$\text{BR}(B_c^- \rightarrow \tau \bar{\nu}) < \text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu}) \begin{cases} 2.64 \pm 0.52 & (\text{Tevatron data}), \\ 3.16 \pm 0.42 & (\text{LHC data}). \end{cases} \quad (11)$$

Here the error is from  $B_u$ ,  $\text{BR}(B_u^- \rightarrow J/\psi K^-)_{exp}$  and  $\mathcal{R}_\ell$ , which can be seen from the explicit formula:

$$\text{BR}(B_c^- \rightarrow \tau \bar{\nu}) = \text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu}) \frac{1}{\mathcal{R}_\ell} \frac{\text{BR}_{\text{eff}} - B_u^{\text{exp}}}{\text{BR}(B_u^- \rightarrow J/\psi K^-)_{exp}}. \quad (12)$$

Various theoretical calculations for  $\text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu})$  are available [21–32]. In Table II we present the bounds on the ratio  $R$  defined by

$$R = \frac{\text{BR}(B_c^- \rightarrow \tau \bar{\nu})}{\text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu})}. \quad (13)$$

In Fig. (2) we show the bounds on  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$  as a function of values of  $\text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu})$  that span the range of the theoretical predictions [21–32]. The four bands are obtained with the  $1\sigma$  ranges of the measured values of the input parameter  $\mathcal{R}_\ell$  from i) CDF (Run I), ii) CDF Run II, iii) LHC, and iv) the average of all three measurements. One can see that the weakest bounds (which are obtained for  $\text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu}) = 2.5\%$ ) are still stronger than the bound of  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu}) \lesssim 30\%$  [10] from considering the lifetime of  $B_c$  e.g. with the LHC data alone one has  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu}) \lesssim 10\%$ . The strongest bounds (which are obtained for  $\text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu}) = 1.5\%$ ) are very close to the SM prediction of  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu}) \approx 2\%$  e.g. with the CDF run II data alone one has  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu}) \lesssim 3\%$ .

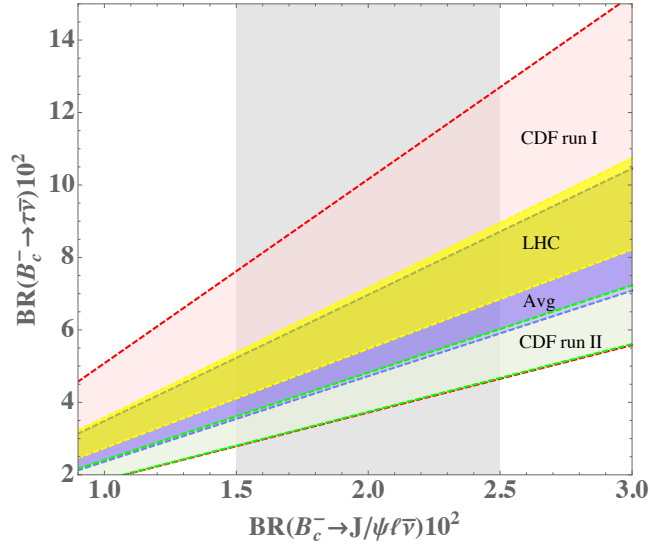


FIG. 2: The limit on  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$  as a function of  $\text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu})$  where the different bands correspond to  $1\sigma$  ranges of the measured values of the input parameter  $\mathcal{R}_\ell$  from CDF Run I (red), Run II (green), LHC (yellow), and the average of all three (blue).

	Tevatron Run I	Tevatron Run II	LHC	Avg
$R$	$3.47 \pm 1.61$	$2.14 \pm 0.27$	$3.16 \pm 0.42$	$2.92 \pm 0.56$

TABLE II: Bound on  $R = \text{BR}(B_c^- \rightarrow \tau \bar{\nu})/\text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu})$ .

A sizeable uncertainty in the extraction of the bound on  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$  is the theoretical prediction for  $\text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu})$ , of which there are several calculations. The estimated values for  $\text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu})$  mostly fall within the range 1.50 – 2.50 % [21–32], with the exception being a value of 6.7% that was obtained in [33]. Without further information from experimental measurements or from first-principle QCD calculations, it is not clear which value of  $\text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu})$  to select from the widespread values when evaluating the constraint on  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$ . Recently, the HPQCD collaboration has made progress in the calculations of the form factors for the decays  $B_c^- \rightarrow J/\psi$  [34], and the obtained (preliminary) results are as follows:

$$A_1 = [0.49, 0.79], \quad V = [0.77, \text{None}]. \quad (14)$$

Here  $F = [F(q^2 = 0), F(q_{max}^2)]$  denotes the values of a form factor at  $q^2 = 0$  and  $q_{max}^2$ . We note that all the errors have not been fully determined, but the total error in the form factors is expected to be of the order of 10% or less. Taking the HPQCD results as a theoretical guidance, we select the QCD model results from [21–33] for which the predicted form factors at  $q^2 = 0$  are within 15% of the values of the HPQCD calculation. Accordingly, the results of the selected QCD approaches are shown in Table III, where the last column is the predicted  $\text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu})$ . It can be clearly seen that values of  $\text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu})$  in range  $\approx (2.0 \pm 0.5)\%$  are favoured when using the values of the form factors from lattice QCD as a guide.

### III. IMPACT ON $H^\pm$ INTERPRETATION OF $R(D)$ , $R(D^*)$ ANOMALY

The following ratios  $R(D)$  and  $R(D^*)$  are defined as follows:

$$R(D) = \frac{\text{BR}(B \rightarrow D \tau \nu)}{\text{BR}(B \rightarrow D \ell \nu)}; \quad R(D^*) = \frac{\text{BR}(B \rightarrow D^* \tau \nu)}{\text{BR}(B \rightarrow D^* \ell \nu)}. \quad (15)$$

The current world averages [35] of their measurements at BABAR [36, 37], BELLE [38–40] and LHCb [41] are:

$$R(D) = 0.407 \pm 0.039 \pm 0.024; \quad R(D^*) = 0.304 \pm 0.013 \pm 0.007. \quad (16)$$

TABLE III: Form factors for  $B_c^- \rightarrow J/\psi$  at  $q^2 = 0$  and  $q_{max}^2$ .

$(F(0), F(q_{max}^2))$	$A_1$	$V$	$\text{BR}(B_c \rightarrow J/\Psi \ell \bar{\nu})$
HPQCD[34]	( 0.49, 0.79)	(0.77, None)	None
NW[26]	(0.53, 0.76 <sup>a</sup> )	(0.73, 1.29 <sup>a</sup> )	1.47%
IKS[28]	(0.55, 0.85)	(0.83, 1.53)	2.17%
WSL[31]	(0.50, 0.80)	(0.74, 1.45)	1.49%

<sup>a</sup> We follow the formulae in [26] to estimate the form factor values.

The predictions in the SM for  $R(D)$  [42, 43] and  $R(D^*)$  [44] are given by:

$$R(D) = 0.300 \pm 0.008; \quad R(D^*) = 0.252 \pm 0.003. \quad (17)$$

The above measurements of  $R(D)$  and  $R(D^*)$  exceed the SM predictions by  $2.3\sigma$  and  $3.4\sigma$  respectively. Taking into account the  $R(D)$ - $R(D^*)$  correlation, the deviation with respect to the SM prediction is  $4.1\sigma$ . Consequently, there have been many works that explain this deviation by invoking the contribution of new physics particles. One such candidate particle is  $H^\pm$ , which is predicted in many well-motivated extensions of the SM e.g. models that contain two or more  $SU(2) \otimes U(1)$  scalar doublets (which includes supersymmetric models).

It has been shown that an  $H^\pm$  from a Two Higgs Doublet Model (2HDM) with type II couplings and natural flavour conservation cannot accommodate the above data for  $R(D)$  and  $R(D^*)$ . However, an  $H^\pm$  in a 2HDM without natural flavour conservation (called the “generic 2HDM” or “Type III 2HDM”, in which both Higgs doublets couple to each fermion type) can give rise to the measured values of  $R(D)$  and  $R(D^*)$  [45–51].

However, recently it has been shown that there is a correlation between  $R(D^*)$  and  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$ , and any enhancement of the former by  $H^\pm$  gives rise to an enhancement of the latter [10]. In [10] the direct limit on  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$  (that is derived in section II) is not considered. Instead, an indirect limit of  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu}) \lesssim 30\%$  was derived by considering the current measurement of the lifetime of  $B_c$  i.e. the partial decay width of  $B_c^- \rightarrow \tau \bar{\nu}$  is bounded from the knowledge of the total decay width (inverse of lifetime) of  $B_c$ . The bound  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu}) \lesssim 30\%$  restricts  $R(D^*)$  to values  $\lesssim 0.275$ , which at the moment slightly disfavors an explanation of the  $R(D)$  and  $R(D^*)$  anomaly from  $H^\pm$  alone. The bound



$\text{BR}(B_c^- \rightarrow \tau \bar{\nu}) \lesssim 30\%$  has been implemented in subsequent studies that consider  $H^\pm$  as a candidate for explaining the  $R(D)$  and  $R(D^*)$  anomaly e.g. [49].

We now study the effect of  $H^\pm$  on  $R(D)$ ,  $R(D^*)$  and  $B_c^- \rightarrow \tau \bar{\nu}$ . In  $R(D)$  and  $R(D^*)$  the underlying quark decay is  $b \rightarrow c \tau \bar{\nu}$ , while  $B_c^- \rightarrow \tau \bar{\nu}$  proceeds via annihilation of the meson to a  $W^\pm$  or  $H^\pm$ . The effective Lagrangian for the contribution of  $W^\pm$  and  $H^\pm$  bosons to all three decays is given by:

$$\mathcal{H}_{\text{eff}} = \frac{G_F V_{cb}}{\sqrt{2}} [(\bar{c}b)_{V-A}(\bar{\tau}\nu_\tau)_{V-A} + (C_R^\tau(\bar{c}b)_{S+P} + C_L^\tau(\bar{c}b)_{S-P})(\bar{\tau}\nu_\tau)_{S-P}], \quad (18)$$

where  $(\bar{f}'f)_{V-A} = \bar{f}'\gamma_\mu(1-\gamma_5)f$ ,  $(\bar{f}'f)_{S\pm P} = \bar{f}'(1\pm\gamma_5)f$ , and  $C_{L,R}^\tau$  are the effective couplings which combine the quark and tau-lepton Yukawa couplings. In general the neutrino can be any flavour, but since the enhancement of  $R(D^{(*)})$  is mainly from the constructive interference of  $H^\pm$  with the SM contribution, we only consider  $\nu_\tau$  in the effective Lagrangian. The couplings  $C_L^\tau$  and  $C_R^\tau$  are functions of  $\tan\beta$  and  $m_{H^\pm}$  in a 2HDM with natural flavour conservation. In a generic 2HDM,  $C_L^\tau$  and  $C_R^\tau$  have an additional dependence on parameters that lead to flavour changing neutral currents see e.g. [50].

To demonstrate the impact of  $B_c^- \rightarrow \tau \bar{\nu}$  on  $R(D^{(*)})$ , we show contours of the 2HDM prediction for  $R(D)$  (band),  $R(D^*)$  (dashed), and  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$  (dash-dotted) as a function of  $C_R^\tau$  and  $C_L^\tau$  in Fig. (3), where the estimations for  $R(D)$  and  $R(D^*)$  are based on the formulae in [44]; the ranges of  $R(D) = [0.3, 0.45]$  and  $R(D^*) = [0.25, 0.35]$  (i.e. the upper values correspond to  $1\sigma$  and  $3\sigma$  above the respective central values of the experimental measurements), and  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu}) < 30\%, 10\%$  are used. It can be seen that the bound  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu}) < 10\%$  reduces the maximum allowed value of  $R(D^*)$  to  $\sim 0.26$ . Hence in context of an enhancement of  $R(D)$  by  $H^\pm$  alone in a 2HDM, the maximum allowed value of  $R(D^*)$  is reduced from  $R(D^*) \sim 0.275$  (for  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu}) \lesssim 30\%$  and see e.g. [49]) to  $R(D^*) \sim 0.26$  i.e. to within  $3\sigma$  of the SM prediction for  $R(D^*)$  in eq. (17). We note that other models with new physics particles (e.g. leptoquarks) can give rise to other terms in the effective Hamiltonian for the  $cb\tau\nu$  vertex. These models are not strongly constrained by  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$ , as discussed in [10].

Prospects for more precise measurements of  $R(D)$  and  $R(D^*)$  are good. Although LHCb has currently only measured  $R(D^*)$  (for two separate decay modes of the  $\tau$ , and with the data taken at  $\sqrt{s} = 7$  TeV and  $\sqrt{s} = 8$  TeV) it is capable of measuring  $R(D)$  [52]. Measurements with data taken at  $\sqrt{s} = 13$  TeV data will further reduce the error in the world averages

of both observables. The BELLE-II experiment will eventually have roughly fifty times as much integrated luminosity as the final integrated luminosities from the  $B$  factories (BABAR and BELLE), and hence significantly more precise measurements of  $R(D)$  and  $R(D^*)$  will become available. In contrast, it is challenging for the LHC experiments to directly measure (or set direct limits on)  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$ . As discussed in [7], the best prospect for observing the decay  $B_c \rightarrow \tau \bar{\nu}$  is a period of operation of an  $e^+e^-$  linear collider at  $\sqrt{s} \sim 91$  GeV. We note that the L3 limit [12] only used 40% of the available data taken at  $\sqrt{s} \sim 91$  GeV. If the full L3 data sample were used, the limit  $\text{BR}_{\text{eff}} < 5.7 \times 10^{-4}$  could be improved, or even evidence for first observation of  $B_c^- \rightarrow \tau \bar{\nu}$  could be obtained. As shown in Fig. (2), the strongest bound on  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$  is  $\lesssim 3\%$ , which is just above the SM prediction of  $\sim 2\%$ . In Fig. (4) contours for  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$  are shown as a function of  $\text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu})$  and

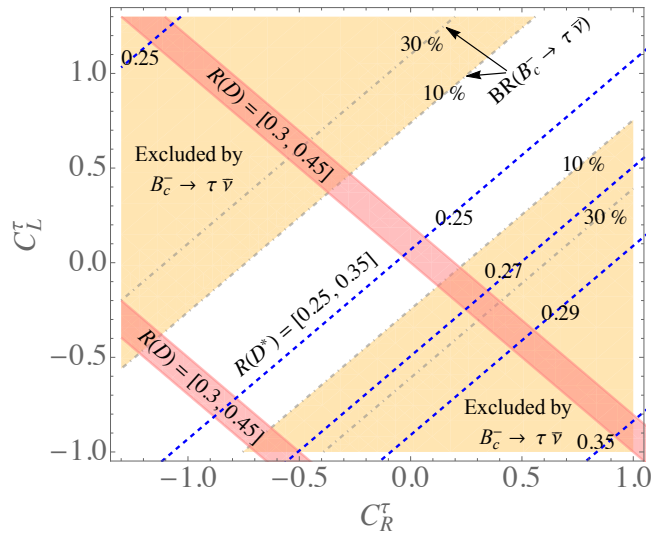


FIG. 3: Contours of the 2HDM prediction for  $R(D)$  (band),  $R(D^*)$ , and  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$  as a function of  $C_{R,L}^\tau$ , where the ranges of  $R(D) = [0.30, 0.45]$  and  $R(D^*) = [0.25, 0.35]$ , and  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu}) \lesssim 30\%, 10\%$  are taken.

$\text{BR}_{\text{eff}}$ . We take  $\mathcal{R}_\ell = 0.161$ , which is the central value of the average of the CDF Run I, CDF Run II and LHC measurements. The shaded region corresponds to the range of theoretical predictions of  $\text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu})$ . It was suggested in [7] that sensitivity to  $\text{BR}_{\text{eff}} \sim 4 \times 10^{-4}$  might be reached if L3 used all the data taken at  $\sqrt{s} \sim 91$  GeV. From Fig. (4) it can be seen that this limit is close to the value of  $\text{BR}_{\text{eff}}$  that is obtained for a SM-like value ( $\approx 2\%$ ) for  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$ .

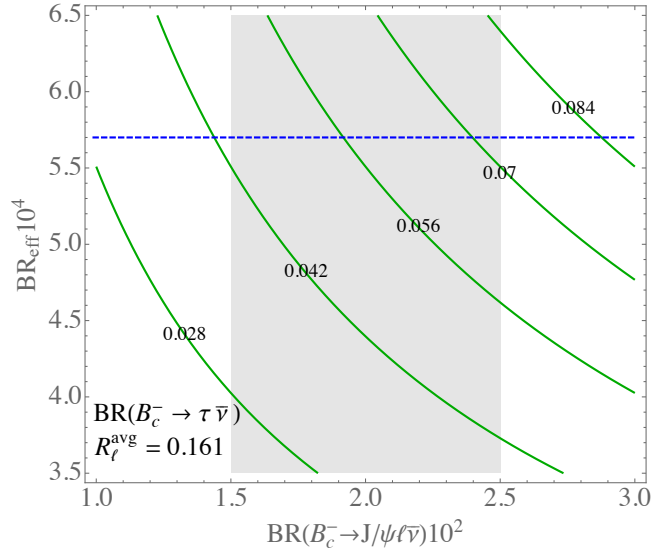


FIG. 4: Contours for  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$  as a function of  $\text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu})$  and  $\text{BR}_{\text{eff}}$ , for  $\mathcal{R}_\ell = 0.161$ . The current limit  $\text{BR}_{\text{eff}} < 5.7 \times 10^{-4}$  is shown.

#### IV. CONCLUSIONS

As discussed in [6, 7], LEP data taken at the  $Z$  peak constrained a combination of the decays  $B_u^- \rightarrow \tau \bar{\nu}$  and  $B_c^- \rightarrow \tau \bar{\nu}$ . This is the only data that directly constrains the magnitude of  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$ . From the L3 limit [12] we derived for the first time an explicit bound on  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$ . The bound can be conveniently written in terms of experimentally determined quantities and just one theoretical input parameter, which is the branching ratio of  $B_c^- \rightarrow J/\psi \ell \bar{\nu}$ . Using the theoretically preferred range for  $\text{BR}(B_c^- \rightarrow J/\psi \ell \bar{\nu})$  we showed that  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu}) \lesssim 10\%$ , which is considerably stronger than the bound from considering the lifetime of  $B_c^-$  [10].

It is known that any bound on  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$  has consequences for an explanation of the  $R(D)$  and  $R(D^*)$  anomaly in terms of an  $H^\pm$  alone. In such scenarios, any enhancement of  $R(D^*)$  leads to an enhancement of  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$ . Our new bound on  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$  further reduces the maximum enhancement of  $R(D^*)$  from an  $H^\pm$ . Thus if future values of  $R(D)$  stay significantly higher than the SM predictions, any explanation that uses  $H^\pm$  alone would require the measured value of  $R(D^*)$  to approach values that are closer to the SM prediction.

The observables  $R(D)$ ,  $R(D^*)$  and  $B_c^- \rightarrow \tau \bar{\nu}$  all proceed via the same effective Lagrangian,

and thus measurement of  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$  would provide independent information on the relevant couplings. Direct searches for  $B_c^- \rightarrow \tau \bar{\nu}$  at the LHC are challenging. However, as stressed in [7], a stronger limit on  $\text{BR}(B_c^- \rightarrow \tau \bar{\nu})$  (or even first observation of this decay) could be obtained if the L3 collaboration used all their data to update the limit in [12] (which used  $\sim 40\%$  of the available data). Operation of an  $e^+e^-$  linear collider at the  $Z$  peak would have sensitivity to the SM branching ratio of  $B_c^- \rightarrow \tau \bar{\nu}$ .

### Acknowledgements

This work was partially supported by the Ministry of Science and Technology of Taiwan, under grant MOST-106-2112-M-006-010-MY2 (CHC). We thank A. Lytle for useful comments.

- 
- [1] F. Abe *et al.* [CDF Collaboration], Phys. Rev. D **58**, 112004 (1998).
  - [2] A. Abulencia *et al.* [CDF Collaboration], Phys. Rev. Lett. **96**, 082002 (2006).
  - [3] A. Abulencia *et al.* [CDF Collaboration], Phys. Rev. Lett. **97**, 012002 (2006).
  - [4] T. Aaltonen *et al.* [CDF Collaboration], arXiv:0712.1506 [hep-ex].
  - [5] D. S. Du, H. Y. Jin and Y. D. Yang, Phys. Lett. B **414**, 130 (1997) [hep-ph/9705261].
  - [6] M. L. Mangano and S. R. Slabospitsky, Phys. Lett. B **410**, 299 (1997) [arXiv:hep-ph/9707248].
  - [7] A. G. Akeroyd, C. H. Chen and S. Recksiegel, Phys. Rev. D **77**, 115018 (2008) [arXiv:0803.3517 [hep-ph]].
  - [8] S. Baek and Y. G. Kim, Phys. Rev. D **60**, 077701 (1999) [hep-ph/9906385].
  - [9] A. G. Akeroyd and S. Recksiegel, Phys. Lett. B **541**, 121 (2002) [hep-ph/0205176].
  - [10] R. Alonso, B. Grinstein and J. Martin Camalich, Phys. Rev. Lett. **118**, no. 8, 081802 (2017) [arXiv:1611.06676 [hep-ph]].
  - [11] X. Q. Li, Y. D. Yang and X. Zhang, JHEP **1608**, 054 (2016) [arXiv:1605.09308 [hep-ph]].
  - [12] M. Acciarri *et al.* [L3 Collaboration], Phys. Lett. B **396**, 327 (1997).
  - [13] P. Abreu *et al.* [DELPHI Collaboration], Phys. Lett. B **496**, 43 (2000).
  - [14] R. Barate *et al.* [ALEPH Collaboration], Eur. Phys. J. C **19**, 213 (2001).
  - [15] T. A. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. D **93**, no. 5, 052001 (2016)

- [arXiv:1601.03819 [hep-ex]].
- [16] V. Khachatryan *et al.* [CMS Collaboration], *JHEP* **1501**, 063 (2015) [arXiv:1410.5729 [hep-ex]].
- [17] R. Aaij *et al.* [LHCb Collaboration], *Phys. Rev. Lett.* **109**, 232001 (2012) [arXiv:1209.5634 [hep-ex]].
- [18] R. Aaij *et al.* [LHCb Collaboration], *Phys. Rev. Lett.* **114**, 132001 (2015) [arXiv:1411.2943 [hep-ex]].
- [19] R. Aaij *et al.* [LHCb Collaboration], *Phys. Rev. D* **90**, no. 3, 032009 (2014) [arXiv:1407.2126 [hep-ex]].
- [20] Y. Amhis *et al.*, arXiv:1612.07233 [hep-ex].
- [21] C. H. Chang and Y. Q. Chen, *Phys. Rev. D* **49**, 3399 (1994).
- [22] A. Y. Anisimov, I. M. Narodetsky, C. Semay and B. Silvestre-Brac, *Phys. Lett. B* **452**, 129 (1999) [hep-ph/9812514].
- [23] V. V. Kiselev, A. K. Likhoded and A. I. Onishchenko, *Nucl. Phys. B* **569**, 473 (2000) [hep-ph/9905359].
- [24] A. Abd El-Hady, J. H. Munoz and J. P. Vary, *Phys. Rev. D* **62**, 014019 (2000) [hep-ph/9909406].
- [25] P. Colangelo and F. De Fazio, *Phys. Rev. D* **61**, 034012 (2000) [hep-ph/9909423].
- [26] M. A. Nobes and R. M. Woloshyn, *J. Phys. G* **26**, 1079 (2000) [hep-ph/0005056].
- [27] D. Ebert, R. N. Faustov and V. O. Galkin, *Phys. Rev. D* **68**, 094020 (2003) [hep-ph/0306306].
- [28] M. A. Ivanov, J. G. Korner and P. Santorelli, *Phys. Rev. D* **71**, 094006 (2005) Erratum: [*Phys. Rev. D* **75**, 019901 (2007)] [hep-ph/0501051].
- [29] M. A. Ivanov, J. G. Korner and P. Santorelli, *Phys. Rev. D* **73**, 054024 (2006) [hep-ph/0602050].
- [30] E. Hernandez, J. Nieves and J. M. Verde-Velasco, *Phys. Rev. D* **74**, 074008 (2006) [hep-ph/0607150].
- [31] W. Wang, Y. L. Shen and C. D. Lu, *Phys. Rev. D* **79**, 054012 (2009) [arXiv:0811.3748 [hep-ph]].
- [32] H. W. Ke, T. Liu and X. Q. Li, *Phys. Rev. D* **89**, no. 1, 017501 (2014) [arXiv:1307.5925 [hep-ph]].
- [33] C. F. Qiao and R. L. Zhu, *Phys. Rev. D* **87**, no. 1, 014009 (2013) [arXiv:1208.5916 [hep-ph]].

- [34] B. Colquhoun *et al.* [HPQCD Collaboration], PoS LATTICE **2016**, 281 (2016) [arXiv:1611.01987 [hep-lat]].
- [35] Heavy Flavour Averaging Group: <http://www.slac.stanford.edu/xorg/hfag/semi/fpcp17/RDRDs.html>
- [36] J. P. Lees *et al.* [BaBar Collaboration], Phys. Rev. Lett. **109**, 101802 (2012) [arXiv:1205.5442 [hep-ex]].
- [37] J. P. Lees *et al.* [BaBar Collaboration], Phys. Rev. D **88**, no. 7, 072012 (2013) [arXiv:1303.0571 [hep-ex]].
- [38] M. Huschle *et al.* [Belle Collaboration], Phys. Rev. D **92**, no. 7, 072014 (2015) [arXiv:1507.03233 [hep-ex]].
- [39] Y. Sato *et al.* [Belle Collaboration], Phys. Rev. D **94**, no. 7, 072007 (2016) [arXiv:1607.07923 [hep-ex]].
- [40] S. Hirose *et al.* [Belle Collaboration], Phys. Rev. Lett. **118**, no. 21, 211801 (2017) [arXiv:1612.00529 [hep-ex]].
- [41] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **115**, no. 11, 111803 (2015) Erratum: [Phys. Rev. Lett. **115**, no. 15, 159901 (2015)] [arXiv:1506.08614 [hep-ex]]; Talk by G. Wormser at FPCP Conference 05 June, 2017
- [42] J. A. Bailey *et al.* [MILC Collaboration], Phys. Rev. D **92**, no. 3, 034506 (2015) [arXiv:1503.07237 [hep-lat]].
- [43] H. Na *et al.* [HPQCD Collaboration], Phys. Rev. D **92**, no. 5, 054510 (2015) Erratum: [Phys. Rev. D **93**, no. 11, 119906 (2016)] [arXiv:1505.03925 [hep-lat]].
- [44] S. Fajfer, J. F. Kamenik and I. Nisandzic, Phys. Rev. D **85**, 094025 (2012) [arXiv:1203.2654 [hep-ph]].
- [45] A. Crivellin, C. Greub and A. Kokulu, Phys. Rev. D **86**, 054014 (2012) [arXiv:1206.2634 [hep-ph]].
- [46] A. Celis, M. Jung, X. Q. Li and A. Pich, JHEP **1301**, 054 (2013) [arXiv:1210.8443 [hep-ph]].
- [47] A. Crivellin, A. Kokulu and C. Greub, Phys. Rev. D **87**, no. 9, 094031 (2013) [arXiv:1303.5877 [hep-ph]].
- [48] A. Crivellin, G. D'Ambrosio and J. Heeck, Phys. Rev. Lett. **114**, 151801 (2015) [arXiv:1501.00993 [hep-ph]].
- [49] A. Celis, M. Jung, X. Q. Li and A. Pich, Phys. Lett. B **771**, 168 (2017) [arXiv:1612.07757 [hep-ph]].

- [50] C. H. Chen and T. Nomura, arXiv:1703.03646 [hep-ph].
- [51] A. Arbey, F. Mahmoudi, O. Stal and T. Stefaniak, arXiv:1706.07414 [hep-ph].
- [52] C. Bozzi [LHCb Collaboration], PoS CKM **2016**, 049 (2017).