Development, implementation and evaluation of a tool for forecasting short term demand for beds in an intensive care unit.

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Abstract:
Variability in demand for staffed beds from existing patients and new referrals in intensive care units presents a substantial problem to managers. Short term fluctuations in the number of patients requiring a bed can result in demand for beds exceeding capacity, or alternatively, seemingly inefficient use of an expensive resource. While operational research methods can help in capacity planning, there are many barriers to implementing such methods in practice. In this paper we describe an entire operational research project cycle encompassing:

- deriving exact expressions for the probability distribution for the time-varying bed demand on an intensive care unit, taking account of occupancy at the point of forecast and future planned and emergency admissions;
- applying it to a specific hospital's intensive care unit using historical data;
- building easy-to-use software that the hospital staff can use daily to produce forecasts of short term bed demand;
- implementing the software within the hospital and
- an evaluation of this implementation from both a technical and non-technical perspective.

The major contribution of this paper is in describing the process of implementing an abstract mathematical model in a busy intensive care unit and the independent qualitative evaluation of the work about how potential barriers to implementation were addressed as part of a "modellers in residence" programme. In particular, we conclude by reflecting on how key factors in our "modellers in residence" approach resulted in building a software tool that is still being used by the hospital more than 4 years after initial implementation.
1. Introduction

Optimising bed use and staffing in intensive care is incredibly difficult. Utilisation of intensive care resources needs to be balanced with the need to accommodate emergency referrals and the need to maintain the flow of planned patients through, and the utilisation of, operating theatres. Short-term fluctuations in demand for beds, for instance from current patient with longer lengths of stay or a surge in emergency referrals, can result in the cancellation of elective surgeries or refusals of emergency referrals due to lack of capacity [1–4]. One major constraint on capacity is the availability of specialised staff; if managers could have an early warning of busy periods, there might be scope to plan ahead [2,4]. It is also reasonable to suppose that there may be other, less tangible, benefits associated with staff being ‘forewarned’ of busy periods (see also Littig and Isken [5] and Chow [6]).

Future demand for beds on an intensive care unit in the short term depends on (figure 1): the number of patients currently on the unit, the number of elective admissions planned, the number of emergency admission requests over the coming period, and the lengths of stay of patients currently on the unit and of those yet to be admitted.

Figure 1 - Representation of sources of demand for beds on an intensive care unit over a short period of time.

In this paper we describe the development, parameterisation, implementation and evaluation of a mathematical model and an accompanying software tool designed to provide clinical
teams with short-term forecasts of bed demand on an intensive care unit that admits emergency patients and planned, post-operative patients.

The mathematical model comprises the exact probability distribution for unfettered demand \([7,8]\) for intensive beds for a given time in the (near) future, building on the work of Utley and Gallivan \([9,10]\) by including the contribution to demand from current patients. We outlined the mathematical approach in conference proceedings \([11]\) but give here the full formulation of the model and details of its subsequent implementation and evaluation. The model complements the considerable existing literature on managing bed capacity that focuses on ‘steady-state’ demand \([2,4,10,12–22]\) or managing bed capacity in the short term but over a whole hospital \([5,23–27]\). Calculating the transient distributions permits use of the model for tactical/operational decision making related to staffing and theatre planning rather than the strategic capacity planning of the sort informed by steady state models.

Reviews of operational research methods applied to health care often highlight a lack of implementation and evaluation \([28–33]\). Implementing new information systems within hospitals is almost always harder than anticipated by decision makers, clinical teams and those designing the software \([34–37]\). Many of the barriers to success are in the details of how a system is implemented, whose work and goodwill are required to make it happen and how a system is incorporated into existing workflows \([38–42]\). Typical barriers are: lack of clarity about team roles \([43–49]\); resistance among senior clinicians and staff \([36,44–46,50–53]\); lack of time \([44,45,47,50]\); and low motivation and high perceived burden \([42,44,50,54–57]\). Understanding the ‘soft’ contextual factors that influence how new systems can actually be implemented cannot be ignored \([34,58–60]\).

Here we describe the full process from developing a mathematical solution to a real-life hospital capacity problem, to co-developing software to implement it, to evaluating its use in practice. While the mathematical analysis has novel components, we consider the main value of this paper lies in our documentation of how we worked to promote adoption and use of the tool and the independent qualitative evaluation of the work.

In the next section we present the mathematical model. In section 3 we introduce the case-study and discuss how we worked with the hospital and the factors that influenced the development and implementation of the software tool. In section 4 we present the statistical analysis performed in order to populate the model with length of stay distributions for different groups of patients. In section 5 we present the Excel based software tool that was co-produced with hospital staff and discuss how it was implemented and subsequently used.
In section 6 we present the OR team’s evaluation of technical and non-technical aspects of this project and an independent qualitative evaluation of the work of the OR team (CPa, MU) conducted by author Pope, to which we add some further reflections in section 7.

2. Mathematical model

2.1 Setting, intent, assumptions and notation

Our setting was an intensive care unit that has planned admissions as well as emergency admissions. In this context, a planned admission is an elective surgery patient for whom an intensive care bed is reserved. Not all such patients contribute to demand for intensive care beds as the planned elective surgery may not take place (for reasons other than intensive care capacity).

The intent of the model was to provide managers with a forecast of what demand for beds would be at some point in the future. To be reflective of true demand, we did not include in the model any effects of limited capacity such as cancellations of surgery due to there not being an intensive care bed, refusal of emergency referrals, or the discharge of patients being influenced by occupancy.

2.1.1 Length of stay

Our analysis is based on a discrete view of time and it is assumed that, once admitted, a patient will occupy a bed for a whole number of time units, in our case study, days. A key assumption in our analysis is that the lengths of stay of different patients can be treated as independently distributed, in line with our intent to not include effects of occupancy on discharge. There is scope for different groups of patients to have different length of stay distributions, which may depend on upon parameters known at booking or admission [11]. Our notation is summarised in Table 1.

Throughout our analysis, we make use of standard results concerning generating functions, which are reproduced in the appendix. In what follows, generating functions are denoted by the capitalised letter of their respective random variable.
<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>An integer number days into the future. In this case study, $0 \leq f \leq 7$</td>
</tr>
<tr>
<td>Length of stay</td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>Category of patient; $0 \leq c \leq C$</td>
</tr>
<tr>
<td>$d$</td>
<td>A counter variable for the integer number of days since admission for a given patient</td>
</tr>
<tr>
<td>$x_{d}^{c}$</td>
<td>the number of beds that a single patient of category $c$ occupies at time $d$ after being admitted to the unit</td>
</tr>
<tr>
<td>$p_{d}^{c}$</td>
<td>The probability that a patient of category $c$ still occupies a bed, $d$ time units after admission</td>
</tr>
<tr>
<td>Patients resident on the unit at time $f=0$</td>
<td></td>
</tr>
<tr>
<td>$j$</td>
<td>The $j$th patient resident on the unit at time 0, $0 \leq j \leq J$</td>
</tr>
<tr>
<td>$c_{j}$</td>
<td>The category of the $j$th patient</td>
</tr>
<tr>
<td>$u_{j}$</td>
<td>the number of time units the $j$th resident patient has already spent on the intensive care unit by time 0</td>
</tr>
<tr>
<td>$r_{j,f}$</td>
<td>the probability that the $j$th patient resident at time 0 will still require a bed at time $f &gt; 0$</td>
</tr>
<tr>
<td>$w_{j,f}$</td>
<td>the number of beds that the $j$th patient resident at time 0 will require at time $f &gt; 0$</td>
</tr>
<tr>
<td>Admissions to the unit</td>
<td></td>
</tr>
<tr>
<td>$n_{f}^{c}$</td>
<td>the number of planned admissions at time $f$ for patients of category $c$</td>
</tr>
<tr>
<td>$v_{c}$</td>
<td>the probability that a planned patient of category $c$ attends for admission</td>
</tr>
<tr>
<td>$q_{i,f}$</td>
<td>the probability that there are $i$ emergency admissions at time $f$</td>
</tr>
<tr>
<td>$a_{f}^{c}$</td>
<td>the number of admissions of patients of category $c$ at time $f$</td>
</tr>
<tr>
<td>Bed demand</td>
<td></td>
</tr>
<tr>
<td>$y_{f}$</td>
<td>the number of patients resident on the unit at time 0 who still require a bed at time $f &gt; 0$</td>
</tr>
<tr>
<td>$h_{b,f}^{c}$</td>
<td>the number of beds required at time $f$ by patients of category $c$ admitted at time $b$ where $0 &lt; b \leq f$</td>
</tr>
<tr>
<td>$t_{f}$</td>
<td>the total demand for beds at time $f &gt; 0$</td>
</tr>
</tbody>
</table>

Table 1 - A summary of notation used

2.2. Analysis

We formulate a mathematical expression that can be used to calculate the exact probability distribution for bed demand in the (short-term) future. Our analysis extends previous work by Utley et al [9] by taking into account the patients currently resident on the unit in addition to new arrivals that stay up to the time of interest.

A key building block in our analysis is the binary random variable $x_{d}^{c}$, the number of beds occupied by a patient of category $c$, $d$ time units after admission. The occupation or otherwise of a bed by a patient of type $c$, a time $d$ after admission, is a single Bernoulli trial with probability $p_{d}^{c}$. That is to say, the generating function for $x_{d}^{c}$ is given by:

$$X_{d}^{c}(s) = (1 - p_{d}^{c}) + p_{d}^{c}s$$

where we note $p_{0}^{c} = 1$, $0 \leq c \leq C$.  

(1)
2.2.1 Patients currently on the unit

Similarly, the occupancy or otherwise of a bed at time \( f \geq 0 \) by the \( j^{th} \) patient resident on the unit at time 0 is a Bernoulli trial with probability \( r_{j,f} \). The generating function for the random variable \( w_{j,f} \), the number of beds occupied by the \( j^{th} \) patient at time \( f \) is given by

\[
W_{j,f}(s) = (1 - r_{j,f}) + r_{j,f}s
\]  

(2)

where

\[
r_{j,f} = \frac{p_{j+1}^e}{p_j^e}
\]  

(3)

If there are \( J \) patients resident on the unit at time 0, then the number of beds required by these patients at time \( f \) is given by the random variable

\[
y_f = \sum_{j=1}^{J} w_{j,f}
\]  

(4)

which has generating function

\[
Y_f(s) = \prod_{j=1}^{J} W_{j,f} = \prod_{j=1}^{J} \left( (1 - r_{j,f}) + r_{j,f}s \right).
\]  

(5)

2.2.2 Demand due to future arrivals

The generating function \( A^0_f(s) \) for the number of emergency (\( c=0 \)) admissions at time \( f \) is, by definition, given by

\[
A^0_f(s) = \sum_{i=0}^{\infty} q_{i,f}s^i
\]  

(6)
The admission or otherwise of a planned patient of category \( c \) is a single Bernoulli trial with probability \( v_c \). The number of planned patients of class \( c \) admitted at time \( f \), \( a^c_f \), is the sum of \( n^c_f \) such independent trials. The generating function for \( a^c_f \) is hence given by

\[
A^c_f(s) = [(1 - v_c) + v_c s]^n^c^f \quad 1 \leq c \leq C\tag{7}
\]

The number of beds required at time \( f \) by patients of category \( c \) admitted at time \( 0 < b \leq f \), \( h^c_{b,f} \), is the sum of a random number \( a^c_f \) of independent random variables \( x^c_{f,b} \). Using the standard result given in equation (A2) of the appendix, the generating function for \( h^c_{b,f} \) is given by

\[
H^c_{b,f}(s) = A^c_b(X^c_{f-b}(s))\tag{8}
\]

Equations (1), (6) and (7) in (8) give

\[
H^0_{b,f}(s) = \sum_{i=0}^{\infty} q_i b \left( (1 - p^0_{f-b}) + p^0_{f-b} s \right)^i
\]

\[
H^c_{b,f}(s) = \left[ (1 - v_c) + v_c \left( (1 - p^c_{f-b}) + p^c_{f-b} s \right) \right]^n^c^b 1 \leq c \leq C\tag{9}
\]

2.2.3 Exact solution for total bed demand

The total number of beds, \( t_f \), required at time \( f \) is the sum the number of beds still occupied by patients resident at time 0 and contributions from the admissions up to an including time \( f \). That is to say:

\[
t_f = y_f + \sum_{c=0}^{C} \sum_{b=1}^{f} h^c_{b,f} . \tag{10}
\]

The generating function for \( t_f \) is given by

\[
T_f(s) = Y_f(s) \prod_{c=0}^{C} \prod_{b=1}^{f} H^c_{b,f}(s) . \tag{11}
\]

Using (5) and (9) in (11) and standard properties of generating functions, the probability of \( m \) beds being required at time \( f \) is given by the coefficient of \( s^m \) in the polynomial
\[ T_j(s) = \prod_{j=1}^{f} \left[ (1 - r_j,f) + r_j,f s \right] \]
\[ \times \prod_{b=1}^{f} \sum_{i=0}^{\infty} q_{i,b} \left( (1 - p_{f-b}^0) + p_{f-b}^0 s \right)^i \times \prod_{c=1}^{C} \prod_{b=1}^{f} \left( (1 - v_c) + v_c \left( (1 - p_{f-b}^c) + p_{f-b}^c s \right) \right)^{c_b} \]  

(12)

where we recall from (3) that \( r_{j,f} = \frac{p_{c,j}^{\text{CICU}}}{p_{c,j}} \).

The expression given in equation (12) is complex and, although calculable, could be difficult to enumerate computationally in the general case. However, the current context is that of a short-term forecasting tool for a relatively small pool of beds, where we need consider only small values of \( f \) (<8). Additionally, although these equations give the probability for any possible level of demand, we need only calculate a moderate number (<30) of coefficients in equation (12) given that in practice a unit will have a maximum number of beds it could possibly staff. In the software tool described in section 5, independent forecasts are calculated and presented for successive time points.

3. The setting and context of the implementation and our ways of working

Great Ormond Street Hospital for Sick Children (GOSH) is a tertiary paediatric hospital in London, UK. The cardiac intensive care unit (CICU) at the hospital admits patients from the hospital's paediatric cardiac surgery programme, emergency cardio-respiratory admissions from the North Thames area of the UK (approximately 5-6 million population) and patients from other clinical environments within the hospital. In addition to the CICU, the hospital has other intensive care facilities dedicated to neonates and other patients not under the care of the cardiothoracic team.

The Clinical Operational Research Unit (CORU) at University College London has a 15 year working relationship with the cardiothoracic team at Great Ormond Street. The OR team on this project (CPa, MU) have worked with GOSH on several collaborative research projects, mainly related to clinical outcomes. The work reported here was part of a "Modellers in Residence" programme at the hospital launched by MU and CPa with the support of the clinical lead for cardiac services (AG) and UK based charity The Health Foundation [59] with the aim of strengthening the implementation of OR models. The project arose through discussions between MU and AG, who at that time was clinical lead of the unit and very concerned with problems of patient flow.
Our collaborative work involved the OR team observing multi-disciplinary team meetings and case conferences at which discussions of theatre planning and capacity took place, CICU nursing meetings where workload and staffing for coming shifts were discussed and a series of meetings with the lead analyst for the unit (VB) the unit manager (PW) as well as the project sponsor (AG) and one of the senior intensive care doctors (KB). At two points in the project, the OR team presented their progress on the project to the multi-disciplinary team and invited feedback. Once the software tool was developed, the OR team sat with the data manager in CICU twice a week for 6 weeks and collated the information required as input for the model and ran the model – not divulging predictions.

Once the software was implemented, the OR team maintained contact with the staff using the tool and those receiving the model output, asking for feedback and encouraging requests for changes that would make the software easier to use or more valuable.

4. Statistical analysis of referrals and length of stay

The model described in section 2 is based on knowing the daily rates of emergency referrals for intensive care, the likelihood that a planned surgery is cancelled for reasons unrelated to capacity (and so an ICU bed is not required after all) and the length of stay distributions among different groups of patients. In this section we discuss the data sets and statistical analyses used to parameterise the model for use in the context of the Cardiac Intensive Care Unit at Great Ormond Street Hospital.

4.1 Data Sets

We used three local data sets to estimate the parameters used in the model: data collected by GOSH for the UK paediatric intensive care audit data set (PICAnet), data collected by GOSH for the UK congenital heart audit data set (NCHDA) and a local in-house data set used within CICU. We used data on all admissions to CICU from 1 April 2007 to 31 March 2011, comprising 2833 patient admissions. The datasets were linked using hospital ID number and admission date.

4.2 Analysis of emergency referrals

Our first step was to use historical data to determine the probability distribution for the number of beds required for emergency patients on any given day \( n^0_i \). After discussion with the data manager (VB), emergency referrals were identified using “Unplanned” status from the PICAnet dataset. Since emergency referrals had increased since 2007, we only used referrals since 2010 to estimate mean current demand for emergency referrals. Daily
The demand for referrals was calculated using the mean observed emergency admissions by day of week added to the mean daily emergency referrals that were refused for capacity reasons as genuine demand. Refusals are only recorded weekly and from 2010 to 2011 there were on average 0.88 refusals a week. We distributed these refusals evenly throughout the week by adding $0.88/7=0.126$ to the mean number of emergency admissions for each day of the week. We then fitted emergency referrals as a Poisson distribution with the mean given by the calculated emergency referral rate by day of week shown in Table 1. We note that, unlike a general ICU, there is no seasonal dependence for cardiac emergency referrals. While the mathematical model does not rely on distributional assumptions about emergency referrals, the equal mean and variance of emergency referrals supported the use of the Poisson distribution, our choice of which flowed from a reasonable assumption of independent inter-arrival times.

**Table 1 - mean emergency demand by day of week for CICU.**

<table>
<thead>
<tr>
<th>Mon</th>
<th>Tues</th>
<th>Weds</th>
<th>Thurs</th>
<th>Fri</th>
<th>Sat</th>
<th>Sun</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.64</td>
<td>0.68</td>
<td>0.84</td>
<td>0.70</td>
<td>0.72</td>
<td>0.66</td>
<td>0.72</td>
</tr>
</tbody>
</table>

### 4.3 Analysis of cancellations for planned surgical patients

The number of beds required for planned surgical patients depends on the surgical lists for that week (known in real time by the intensive care unit) and the likelihood that a planned patient actually requires an intensive care bed ($\nu_c$). To determine this latter parameter, we considered only cancellations due to the family cancelling (e.g. through patient illness) or problems with theatre or ward capacity. After discussion with the clinical team, we also counted planned surgeries which were cancelled on the day of surgery due to a more urgent emergency case (accounted for under emergency referrals) as a cancellation. All of these factors represent reasons that a CICU bed might be available and are not affected by changes in CICU capacity. We used hospital collected data on planned surgeries that were cancelled between 3 August 2008 and 31 March 2011. Over this period there were 143 cancellations out of 1306 planned surgeries, giving a cancellation rate of 10.9%. We model the number of planned surgical patients of each category being admitted to the ICU on a given day as a Binomial distribution with the probability of success set at 89.1%.

### 4.4 Grouping patients based on length of stay distributions

We used retrospective data to develop the patient categories $c$, $0 \leq c \leq C$, based on length of stay characteristics, so that we could use the empirical length of stay distributions to
determine $x_d^c$. Patients already on the ward were considered separately from elective patient before admission who were in turn considered separately from emergency patients ($c=0$).

To determine the $C$ groups, we first separated the dataset into a (randomly assigned) development set (1957 records, 70% of the data) for the analysis to define length of stay groups and a validation set (876 records) in which to test the final allocation.

We worked with the data manager (VB) and clinical team (PW, KB, AG) to identify which information in the datasets was known in real-time for patients on admission to the ward and prior to surgery for planned patients. This was important since any allocation to length of stay groupings needed to be done while the child was still on CICU or (for future planned arrivals) before they arrived on CICU.

The subset of factors considered are given in Table 2.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planned admission</td>
<td>Binary</td>
</tr>
<tr>
<td>Referring group (proxy for diagnosis)</td>
<td>Categorical</td>
</tr>
<tr>
<td>Comorbidity present</td>
<td>Binary</td>
</tr>
<tr>
<td>Had cardiac surgery?</td>
<td>Binary</td>
</tr>
<tr>
<td>Patient had had a cardio-pulmonary bypass</td>
<td>Binary</td>
</tr>
<tr>
<td>Had a major event in theatre?</td>
<td>Binary</td>
</tr>
<tr>
<td>Neonate (less than one month old)</td>
<td>Binary</td>
</tr>
<tr>
<td>Admitted from? (e.g. home, clinic, hospital)</td>
<td>Categorical</td>
</tr>
<tr>
<td>Sex</td>
<td>Binary</td>
</tr>
<tr>
<td>On Extra-coporeal life support</td>
<td>Binary</td>
</tr>
<tr>
<td>Had had cardiac arrest</td>
<td>Binary</td>
</tr>
<tr>
<td>Age</td>
<td>Continuous</td>
</tr>
<tr>
<td>Weight</td>
<td>Continuous</td>
</tr>
</tbody>
</table>

Table 2 - Factors considered for developing length of stay groupings

From this subset of factors we then tested for univariate association with patient length of stay (LoS) in the following way:

- for binary factors, we used a t-test for difference in mean LoS and the log rank test for difference in LoS persistence distributions to determine association with increased LoS
• for categorical variables with more than two categories, we used analysis of variance (ANOVA) to test for significant differences in mean length of stay between categories and the log rank test to test for difference in LoS persistence distributions
• for continuous variables (e.g. weight and age), we used linear regression to test for significant association with length of stay.

In categorical variables with many categories that showed significant association with LoS, we further collapsed categories that were either pairwise not significantly different (using a t-test for difference in means) or were categories with very few patients.

4.4.1 Patients currently on the unit
Together with our clinical authors (KB, AG), we chose the final variables from Table 2 considered for inclusion in LoS groupings for patients on the unit based on univariate association with length of stay and clinically relevant criteria (e.g. “neonate”).

Possible length of stay groupings which discriminated in length of stay based on these factors were then generated using Classification and Regression Tree (CART) analysis. This resulted in eight length of stay of groupings. The CART analysis was performed using IBM SPSS Statistics for Windows, Version 20.0. The maximum number of levels of the tree was set to 5, the minimum parent size was 100, the minimum child size was 50. Category splits were determined by Gini improvement (minimum improvement set to 0.0001). Generated groups were then discussed with the clinical teams and manually adjusted and pruned.

The final factors determining these groupings were:
• Referring group (proxy for diagnosis: reduced to 4 categories)
• Patient had another health condition (comorbidity)
• Patient had had a cardio-pulmonary bypass
• Patient had had major event in surgery
• Patient age

4.4.2 Planned patients pre-arrival to the unit
The only both useful and available factors in this case were age (continuous) and referring service. CART analysis (performed as in 4.4.1) produced five groups based on referring service and various age bands.

4.4.3 Final length of stay groups
We presented the resulting length of stay groups to the whole CICU clinical team to check for face validity and to address any concerns or questions. The team agreed that the developed groups had clinical validity. After discussion with clinical team, the age thresholds identified from the analysis were adjusted manually to correspond to clinically meaningful age bands (for instance 5.2 months became a 6 month threshold and a 32 day threshold became a 28 day threshold to match the existing threshold between “neonates” and “infants”). We then successfully tested the performance of the groupings in the validation set.

The final grouping algorithms for current CICU patients and pre-arrival planned patients are shown in figures 2 and 3.

**Figure 2 - Length of stay groupings for patients currently on the unit. Yellow boxes show the final groups.**
Figure 3 - Length of stay groupings for planned patients before admission to the unit. Yellow boxes show the final groups.

The length of stay distributions in the validation set for each set of groupings are shown in figures 4 and 5 respectively.

Figure 4 - length of stay distributions for the eight groupings for patients already on the unit in the validation data set.
Figure 5 - length of stay distributions for the five groupings for planned patients yet to arrive on the unit in the validation data set.

The length of stay distributions for each group between the development and validation and data sets were not significantly different for all groups except group 3 in figure 2 and group C in figure 3 (using the Mann-Whitney U test).

4.4.4 Emergency patients

Finally, we note that while nothing is known prospectively about emergency patients, historical analysis shows that they tend to stay longer on the unit than other patient groups. We thus used the historical length of stay distribution for unplanned patients when considering the contribution to future demand of future emergency arrivals. Note that we made no attempt to distinguish among different groups of emergency admissions as the length of stay distribution for emergency patients is only used to model the bed demand associated with unplanned patients not yet admitted and, by their very nature, no information other than emergency referral rate is available prospectively for this cohort.

5. Development, implementation and use of software tool

We wrote computer code using Microsoft VBA for Excel 2010 to implement the mathematical model described in section 2 as a spreadsheet tool with the parameters from section 4. The tool takes as user-input the information necessary to categorise each patient currently on
CICU to one of the 8 relevant “length of stay groups”, identified through the CART analysis described in section 4.4 and the date and time of each admission, and planned theatre activity in terms of the number of patients from each length of stay group planned for admission on each day of the forthcoming period (figure 6). Note that the empirical historical length of stay distribution (in hours) over the whole of the dataset (development + validation) was hard-coded into the tool for each group.

The historical rates of emergency referrals discussed in section 4 were hard-coded into the tool, as were the proportions of planned patients that are not admitted for reasons other than CICU capacity. We assume that future arrivals are admitted at 12pm and the output shows bed occupancy distribution at 5pm each day.

Based on the analysis presented in section 2, the probability distribution for the demand for CICU beds is calculated for a user-defined number of future days. Since exact distributions are calculated explicitly, processing time required us to look ahead in a whole number of days and no further than seven days in advance.

These distributions are then used automatically to produce a graphical display indicating the probability that unconstrained demand for beds will be at or above a certain level at each respective time (see figure 7). While there is in reality fluctuation in demand within a day, the tool was intended to inform planning decisions such as staffing and theatre listing where forecasting at daily intervals was felt by the hospital team to be sufficient.

Figure 6 – Example input screen for the short-term bed demand tool. In this example the forecast is run on a Friday afternoon. Note that the hospital numbers and dates of birth have been changed to avoid identification.
Figure 7 - Example output screen for the short-term bed demand tool. The colours indicate the likelihood that that number of beds will be filled for that day. In this example, Friday, and Tuesday to Friday of the following week are likely to be busy (normally 16-18 beds on the CICU were staffed).

5.2 Implementation

Following the 6 week period in which the tool was run by the OR team without the output being shared, the team at Great Ormond Street have been using the software tool daily since October 2012, with a snapshot of the output screen emailed by the data manager to all intensive care doctors and the nursing bed manager each morning. After their early experience of using the tool, the hospital made a few requests for modifications in 2013, namely

- to add a function to highlight those patients on the ward with an estimated chance of 50% or higher of still being on the unit in a week’s time
- to reduce the number of coloured bands in the model output from 7 to 5
- to extend the model to consider 21 beds rather than the original 18

The first two of these requests were trivial to implement. The third request was challenging as it potentially involved implementing over 45 million computations as opposed to 5 million
computations. Meeting the request involved two agreed compromises: switching from (12 hour) shift to (24 hour) day forecast intervals; and combining demand for at least 20 or at least 21 beds into a single output (>19 beds needed).

6. Evaluation

6.1 OR team’s reflections on technical and non-technical aspects of evaluation

6.1.1 Technical challenges

At the outset, we identified three key questions for the technical evaluation of this work: whether the CART analysis of length of stay among resident and forthcoming patients would yield useful information, whether the run time of the model would be acceptable, and whether the predictions of demand would be sufficiently accurate.

The 8 distinct groups among patients already resident on the unit and the 5 groups among planned patients (for example see figure 4) showed considerable variation in length of stay with 8 fold and 5 fold differences in median length of stay across groups. The validity of these groupings was confirmed in data set aside for testing (see section 4).

After several initiatives to improve the efficiency of the computer code and some compromise on the number of time points at which predictions are made, a run time of 15-30 seconds was achieved.

The model predicts demand in the absence of any cancellations of planned theatre cases or refusal of emergency admissions due to shortage of beds on CICU. Once there is a cancellation or refusal, the predictions are null and void. To verify the model in these circumstances, we isolated the predicted demand associated with patients already resident on the unit and checked the series of predicted distributions against the series of observations. Given that each observation of demand corresponds to a potentially unique predicted distribution of demand, we compared the proportion of observations that fell at or below a series of threshold values on the cumulative distribution function of their respective predicted distribution to what would be expected if the predicted distributions were correct. Since the distributions are discrete, each observation corresponded to a range on the relevant cumulate distribution function (CDF) so we did separate assessments using the lower limits of these ranges, the mid points and the upper limits.
Figure 8 shows the results of this validation exercise, with the observations of demand among resident patients 3 days after the point of forecast compared to the model predictions. The three comparisons shown correspond to using the lower limits, midpoints and upper-limits of the range on the CDF corresponding to each observation. The good concurrence between observations and the predicted distributions confirmed that our underlying assumptions were sound and the computer code a correct implementation of the mathematical model.

![Diagram showing the comparison of observed demand with model predictions.](image)

**Figure 8** - Comparing the observed demand 3 days after the point of forecast among patients resident on the ward model to predictions. Essentially this chart shows good agreement between observations and the predicted distributions generated using the model and observations.

During the 6 weeks when we ran the model without sharing output with the hospital team, we prospectively logged the occasions on which there was predicted to be either a greater than 60% or a less than 60% chance of demand being at least the (then) notional capacity of 16 beds over each 7-day forecast period. Results of this exercise are shown in table 3 below. Note that not all forecasts could be used due to system cancellations or refusals. Although numbers in table 3 are small, there is nothing to suggest that the demand model is not fit for purpose.
Table 3 - simple validation of forecasting tool from the 6 week blind trial period.

<table>
<thead>
<tr>
<th>Number of days in the future</th>
<th>Proportion of time at least 16 beds were filled or there was a cancellation/refusal when forecast suggested &gt;60% chance that at least 16 beds would be needed.</th>
<th>Proportion of time fewer than 16 beds were filled (and there was no cancellation/refusal) when forecast suggested &lt;60% chance that at least 16 beds would be needed.</th>
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<td>1 day</td>
<td>5/6 (83%)</td>
<td>4/7 (57%)</td>
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<td>2 days</td>
<td>3/5 (60%)</td>
<td>7/9 (78%)</td>
</tr>
<tr>
<td>3 days</td>
<td>1/3 (33%)</td>
<td>8/10 (80%)</td>
</tr>
<tr>
<td>5 days</td>
<td>3/4 (75%)</td>
<td>6/8 (75%)</td>
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</tbody>
</table>

6.1.2 Non-technical challenges

One non-technical challenge was the differing perceptions of what data were readily available for use in real time. To provide data on arrivals, refusals, cancellations and clinical features of admitted patients, the data manager needed to interrogate 4 separate databases whereas we’d been given the impression by the project sponsor that these data were available from a single source.

More problematic was that some fields in a key data source were often only completed or validated after discharge. We had not anticipated this since all the fields originally considered for use in the length of stay CART analysis concerned features of the patient known on admission. It had not occurred to us that features known, or at least knowable at admission would not necessarily be recorded until after discharge and this restricted the data we could use for the length of stay analysis. Additionally, one important field used for the length of stay analysis proved difficult to establish without additional work. Another, related observation is that, despite there being talk of “the theatre list”, it became apparent that, at any one time, there might be different versions circulating and it is subject to many changes.

6.2 Independent qualitative evaluation

A researcher with 25 years’ experience of evaluating health care and expertise in qualitative methods (Pope) evaluated the use of the model. She conducted interviews with 7 individuals: members of the CORU team (2), a GOSH data manager (1), consultant medical doctors (3), and a nurse manager (1). The GOSH staff interviewed were all closely involved in the project and were familiar with the demand forecasting model. Interviews were conducted face to face or by telephone and took between 40-90 minutes. Hand written notes were taken to
provide a near verbatim record. (The decision not to audio-record these interviews helped establish rapport and encouraged the interviewees to make negative as well as positive comments about the project).

An interview topic guide was iteratively developed from discussions with the CORU team, informed by a review of documents and by early interviews. Broadly it covered the history of the individual’s engagement with the project, their role and responsibilities, the development and use of the modelling software, and views about the utility and design of the software. The GOSH respondents were asked to comment on their experiences of working with the CORU team, how process of development and implementation of the software could be improved and the possibilities for transfer beyond their team – i.e. to other teams in the NHS Hospital and beyond this.

6.2.1 Thematic analysis

Given the small scale nature of this work, we have simply presented the data and commentary under three broad thematic headings to capture the important features associated with the initial adoption and use of the demand forecasting software at GOSH. We have used some near verbatim quotes where these articulate respondents’ views well.

6.2.2 Achieving engagement

The interviews provided evidence of strong clinical engagement with the project. This had been enabled by a clinical champion, a senior clinical manager, who was highly influential in initiating the programme at GOSH. This individual had a strong desire to improve services and this motivation was echoed in comments by other respondents. As the project progressed, another clinician took on the role of ‘implementer’ taking direct responsibility for ensuring that the modelling work was used by the clinical team. While this person had not been heavily involved in the initiation they proved key to making sure that the software was brought into sustained everyday use.

Personal characteristics of CORU team played an important part in establishing a good working relationship. It is clear from the interviews that the CORU team worked hard to engage the GOSH team and to respond to requests to adapt the software. There was a sense that CORU ‘did their homework’; the CORU director described this as ‘just listening’. The clinicians reported that the CORU team spent considerable time, unobtrusively, on the ward and with staff and this was key to reducing resistance - such as the nurse manager and
data manager - who could have felt threatened by software which potentially deskillled them (both appeared to view the software as a resource not a replacement for their expertise). Close working with the GOSH team, particularly those involved in data creation and collection was crucial in developing the model: for example the CORU team discovered that expected or promised data was often not available and adapted the model accordingly. In thinking about why the demand forecasting modelling was successfully implemented this effort by the team to enrol staff should not be overlooked. Indeed some interviewees noted that this effort had been absent in previous external consultancy projects which had not been successful.

6.2.3 Using the Demand Forecasting software

The CORU team expressed a strong desire to create models and software that would actually be used in everyday practice. This motivated their efforts to build positive relationships with the GOSH team. A clinician described the CORU approach as one which ‘maps things out and relates it to patients in a way that makes sense’; the CORU team were seen as providing ways of thinking about problems ‘in a way that goes beyond what the clinician is capable of seeing on their own’. Knowledge of the underpinning mathematical models did not appear to be essential to using the software - but it may be worth noting that in cardiac care, especially paediatric cardiac care, clinicians may be more familiar with predictive modelling and risk measures, perhaps sharing more of an affinity with the kinds of operational research and statistical approaches which CORU deploy, than staff in other specialties. Nonetheless the GOSH respondents felt that the ‘software can be understood without needing to understand the whole modelling process’.

The software was intuitive, quick and easy to use and crucially highly meaningful for all the GOSH respondents. The use of coloured rectangles to display the probability scores (figure 6) meant that this resembled a ward full of beds and this was significant in its successful adoption. An earlier version of the software displayed a graph which had not been received so well. The current version of the display appeared ‘congruent with how we see things’. One clinician explained that it captured the tacit and experiential knowledge they used on the ward. The nurse manager explained that it ‘gives a flavour of what is in the unit’ such that if ‘I see a lot of red, say up to 16/17 ‘beds’ coloured in red that means we are going to have lots of cancellations.’

The GOSH interviewees appreciated the responsiveness of the CORU team to requests to adapt the visual display - for example changing the colour of the display and adding or
removing items. The CORU team exploited the functionality of MS Excel to keep the main information required and provided by the software relatively simple; hiding unnecessary formulae and providing on screen instructions and memos to help explain how the software worked. This functionality, combined with careful adaptation and refinement of the software in collaboration with the users, secured buy-in by the GOSH team and appeared to give them a sense of co-ownership and therefore interest in the continued use of the software.

As part of each interview, the GOSH respondents were asked to run the software and ‘think aloud’ to explain how they used it. From this it was clear that they were all comfortable with using the software, and using the language of probability and risk to interpret what the model ‘told them’. The software is used in different ways and at different times by members of the GOSH team. The nurse manager and data manager ran the software each morning to prepare for the day ahead - for example to inform liaison with nursing staff. They used the information provided by the software in conjunction with other information – operating lists, Trust targets for waiting lists and the types of cases admitted and planned to inform decision making. The software was also used regularly in the daily clinical planning meeting at noon to plan and respond to admissions. The work done by the CORU team in validating the model and software, notably the shadow modelling and continual review of the forecasts meant that the GOSH team were positive about the software and trusted its ability to accurately predict demand. While some clinicians felt that the software simply confirmed what they instinctively knew about the work flows, they all felt the system had merits in providing a formal and numerical model which could be shared with a wider team to legitimate decisions.

7. Operational research “modeller in residence” approach: reflections

In this section we reflect on some of the findings from the independent qualitative evaluation for the purpose of identifying generalizable learning from this work.

7.1 Understanding context

The interviews with staff highlighted that the model output was not viewed by the hospital team as entirely new information on forthcoming demand for beds, but rather as formal confirmation of their “gut feel”. This chimes with our observations from planning meetings, where heuristic rules of thumb for planning theatre activity were being promoted by some members of the team, such as “we really shouldn’t list more than 12 cases a week because we have never managed to get more than 12 through theatres” without these being
universally accepted in the room. It became clear to the OR team that some of the hospital team felt that decisions regarding who to list when for surgery led to unduly optimistic planning. In small group discussions later in the project, this impression was reinforced. Another, related issue that arose was the possibility that not all staff groups are exposed directly to the impact of short term cancellations on patients and their carers. This led us to conclude that model output might be co-opted by some staff as an explicit form of evidence in support of their position, and this understanding of context influenced how we talked about the tool with different staff groups.

7.2 The value of co-design

The interviews with staff found that including them in the design of the model output did not only lead to more useful output, but also gave the hospital staff a sense of ownership over the tool. Without this, it is not certain that they would have felt able to ask for the later modifications discussed in section 5, or that the model would have been used in practice.

The OR team had initially hesitated before simplifying the output from the model from the precise probability distributions to show just 7 bands of probability. The later request for an even less refined level of output suggested that the OR team had misjudged the level of detail that would be useful to the hospital. That said, we feel the process sharing the full detail offered by the analytical model but being willing to simplify had considerable benefits for the project as a whole.

7.3 The importance of trust

The staff interviews display a good level of trust in the model output. We consider that a key factor in establishing this trust was getting the data right and this is where close liaison between the OR team and the data manager and lead intensivists was crucial, particularly during the length of stay analysis described in section 4. In addition to the issue of some data not being recorded electronically until patient discharge (see section 6), it was only through our frequent discussions with the lead intensivist that we avoided missing a key piece of dataset, which was stored in a dataset the OR team didn’t know about. The data manager had not mentioned the other data set since we had not asked for it and so in this way it would have been very easy to finish the length of stay analysis without this extra information. Without this additional data set, we suspect that the length of stay groupings would have had less clinical face validity, regardless of their statistical performance. Although this would likely
have been picked up when presenting the analysis to the MDT, we would have lost credibility
and trust

The 6 week pilot discussed in section 6.1 also helped to build trust. The clinical lead had
wanted to start using the model output immediately the tool was ready. We think our
insistence on this extra step of validation helped convince the wider team (and especially the
data manager and bed manager) that we were genuinely committed to the tool being useful
and genuinely interested in their feedback.

It is worth noting that while the hospital team took reassurance from the pilot data, they were
not particularly interested in the technical validation we conducted – part of their trust in the
tool seemed to flow from a general trust in the OR team built up over a decade. Right or
wrong, it is not certain that an identical tool developed by a team without that history of
collaboration would have been trusted and used. This has implications for how academic OR
groups work in health, and points towards the value of embedded teams or residency models
[59,61].

7.4 Little things make a big difference

The other key factor influencing the use of the tool identified in the interviews was the ease of
use of the software, and the commitment of the OR team to making the tool easy to use.
The pilot was incredibly valuable here. The OR team using the tool as it would be used by
the data manager routinely enabled small modifications to be identified that were simple but
very effective in streamlining data entry. These included things like: adding the patient
hospital ID to the input screen, allowing patients to be entered in any order, sorting patients
by admission date once entered (very useful for iterative data entry on subsequent days),
and adding various consistency checks to look for inconsistent dates, patient IDs etc.

These last steps of making the tool easy to use did not feel like “academic research” and nor
was it particularly interesting. If these last small steps are crucial for successful
implementation, and we believe they are, this raises the issue of how academic operational
researchers can be incentivised to take them.

8. Conclusion

This paper describes the complete project cycle of implementing a mathematical model to
help manage short-term demand for beds within a paediatric intensive care unit. The project
was successful, with sustained daily, unsupported, use of the tool in the unit 3.5 years after implementation (at time of writing). Leading on from this project, one of the authors (CPa) is now working with the critical care units at GOSH 2 days a week as part of a "modellers in residence" programme. As part of this work, she is currently updating the parameters used in the forecasting tool (e.g. length of stay distributions) and exploring the opportunity for expanding this work to the general paediatric intensive care unit.

Acknowledgements
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Conflicts of interest
The authors declare that they have no conflicts of interest.

References


Appendix

Generating functions

Here we give some standard results concerning generating functions.

Let $y$ be a positive integer valued random variable where $P(y = i) = r_i$. The generating function that describes the probability distribution of $y$, $Y(s)$, is defined as

$$Y(s) = r_0 s^0 + r_1 s^1 + r_2 s^2 + \ldots = \sum_{i=0}^{\infty} r_i s^i, \quad 0 < s \leq 1. \tag{A1}$$

The parameter $s$ is a dummy variable used only to define the generating function and has no physical significance.

The generating function for the probability distribution of the sum, $z$, of a random number $k$ of independent random variables $y$ is given by

$$Z(s) = K(Y(s)) \tag{A2}$$

where $K(s)$ is the generating function for the probability distribution of $k$.

The generating function $C(s)$ for $c$, the sum of two independent random variables $a$ and $b$ that have generating functions $A(s)$ and $B(s)$ is given by:

$$C(s) = A(s)B(s). \tag{A3}$$

For proofs of these standard results see, for example, Grimmett and Stirzaker (1992).

**Figure Captions**

Figure 1 - Representation of sources of demand for beds on an intensive care unit over a short period of time.

Figure 2 - Length of stay groupings for patients currently on the unit. Yellow boxes show the final groups.

Figure 3 - Length of stay groupings for planned patients before admission to the unit. Yellow boxes show the final groups.

Figure 4 - Length of stay distributions for the eight groupings for patients already on the unit in the validation data set.

Figure 5 - Length of stay distributions for the five groupings for planned patients yet to arrive on the unit in the validation data set.

Figure 6 – Example input screen for the short-term bed demand tool. In this example the forecast is run on a Friday afternoon. Note that the hospital numbers and dates of birth have been changed to avoid identification.

Figure 7 - Example output screen for the short-term bed demand tool. The colours indicate the likelihood that that number of beds will be filled for that day. In this example, Friday, and Tuesday to Friday of the following week are likely to be busy (normally 16-18 beds on the CICU were staffed).

Figure 8 - Comparing the observed demand 3 days after the point of forecast among patients resident on the ward model to predictions. Essentially this chart shows good agreement between observations and the predicted distributions generated using the model and observations.

**Table Captions**

Table 1 - A summary of notation used
Table 2 - mean emergency demand by day of week for CICU.
Table 3 - simple validation of forecasting tool from the 6 week blind trial period.
Tomorrow's demand for beds

Current patients discharged

Current patients staying another day

ICU occupancy today

Planned elective surgery patients

Emergency referrals

Tomorrow’s demand for beds
1. Referring service

Referring Group: Cardiac Surgery or Thoracic

2a. Major event in theatre?

- No Major event in theatre

Referring Group: Cardiology, Tracheal, Other

2b. Had cardio-pulmonary bypass?

- No CPB

Referring Group: ECMO or Transplant

3a. Patient age?

- >6 months
  - GROUP 1
  - Median LOS: 1.2

- >28 days and < 6 months
  - GROUP 3
  - Median LOS: 2.9

- <= 28 days
  - GROUP 6
  - Median LOS: 4.9

3b. Comorbidity?

- No comorbidity
  - GROUP 2
  - Median LOS: 2.4

- Comorbidity
  - GROUP 4
  - Median LOS: 3.1

- Had CPB
  - GROUP 8
  - Median LOS: 9.6

- No CPB
  - GROUP 7
  - Median LOS: 6.3
1. Referring service

Referring Group:
Cardiac or Thoracic

Referring Group: Not Cardiac or Thoracic
GROUP E
Median LOS: 5

2. Patient age?

> 1 year
GROUP A
Median LOS: 1.1

> 6 mths & <= 1 year
GROUP B
Median LOS: 2.2

> 28 days & <= 6 mths
GROUP C
Median LOS: 3.0

<= 28 days
GROUP D
Median LOS: 4.8
Probability patient still resident on CICU

N=876 patients
Probability patient still resident on CICU

N=604 patients

Number of days (log scale)
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<th>Number of patient on ward</th>
<th>Name</th>
<th>Hospital Number</th>
<th>Date of Birth</th>
<th>Date and Time of Admission to CICU</th>
<th>Referring Service</th>
<th>Had Cardiopulmonary bypass (CPB)?</th>
<th>Major Event in Theatre? (Yes/No)</th>
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**KEY**
- More than 90% likely to be needed
- 70-90% likely to be needed
- 40-70% likely to be needed
- 10-40% likely to be needed
- Less than 10% likely to be needed
Proportion of instances $\leq X$ on cdf

darker dots – model
lighter dots - observations