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# Adaptive inertia tuning of an energy harvester for increasing its operational bandwidth

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#### Abstract

A rotational energy harvesting system comprises a sprung mass coupled to an electrical generator through a motion transmission system such as a ball screw. In this paper, the operational bandwidth of a rotational energy harvester is expanded by varying its moment of inertia and load resistance of the generator. It is shown that the resulting tuneable device produces significantly higher amounts of harvested power. In addition to mass and stiffness, the natural frequency of a rotational device is defined by its moment of inertia, an additional design parameter that enables implementing the approach presented here. This parameter also determines the apparent mass (inertance) of the device, an important factor that allows a small additional mass to increase the apparent mass hugely and hence increase the overall power density of the harvester.

It is shown that the system with variable load resistance shows a good performance at frequencies around the natural frequency of the device whereas away from resonance frequencies the system with variable moment of inertia produces more power. The approach described in this paper is a first step in the direction of having an autonomous energy harvester with a wide operational bandwidth. One of the advantages of the presented method is that, unlike some other methods, changing the adjustable parameters (i.e., moment of inertia and load resistance) can be conducted intermittently. In other words, this approach only consumes power during the tuning operations and does not use energy once the harvester is tuned at its optimum conditions.

These tuneable rotational systems should be used where the excitation frequency varies slowly (e.g., in marine environment) as any sudden changes in frequency would result in an instantaneous change in the apparent mass and the device may even stall. To implement the device effectively, some kind of predictive control may need to be used that can detect frequency variations fast enough for the inertia to change in a timely manner. This aspect that is outside the scope of this paper is currently under investigation.

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#### 1. Introduction

Electromechanical energy harvesters can be divided into two general groups of linear-reciprocating and rotational devices. In a linear device, the reciprocating nature of the environment is used directly to charge up an electrical energy storage device. This type of device is particularly suitable for high frequency-low amplitude excitations and not so efficient for the other extreme case of low frequency-high amplitude environment [1, 2]. In a rotational harvester, the reciprocating/oscillatory motion of the environment is translated to a rotational motion. It can be shown that the efficiency of a rotational system is higher than a linear system that has a maximum efficiency of 50%. Energy harvesters, either linear or rotational, are generally designed to produce maximum power at their resonant frequency, an aspect that has two major problems associated with. Firstly, the size of the device gets larger as the environmental frequency goes down and hence the power density of the device gets smaller. Secondly, in most applications the ambient vibration is spread over a wide range of frequencies and hence the harvesters would operate at sub-optimum conditions [3]. This limitation necessitates the design of a tuning mechanism for varying their resonance frequency and hence to increase their operational bandwidth. Besides changing the resonance frequency, widening the operational bandwidth is another strategy to improve the performance of energy harvesters in practical environments. Exploiting nonlinear springs [4], coupled oscillators [5], structures with multiple vibration modes [6] and bi-stable structures [7, 8] are some of the methods employed to widen the bandwidth of the harvesters. In this paper, we present a new method to broaden the operational bandwidth of a ball screw based harvester by tuning its moment of inertia. In addition, the effect of optimum control of the load resistance on the output power of the energy harvester and widening the operational bandwidth of the energy harvester is investigated.

### 2. System modelling

Fig.1 shows a ball screw based energy harvesting device in which a rotational rod with two moving masses, is perpendicularly attached to the coupling shaft between the generator and the ball screw. The free body diagram of the proposed harvester is shown in Fig.1, where l, m,  $c_m = c_{bg} \left(2\pi/l\right)^2$  and k respectively represent the ball screw lead, the oscillating mass, mechanical damping and the overall spring stiffness. Note that,  $c_{bg}$  represents the damping associated with all rotating parts and the input vibration is  $y(t) = Y \sin(\omega t)$ .

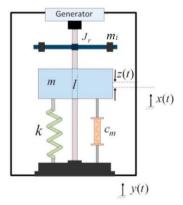


Fig. 1. An energy harvester consisting of a sprung mass coupled to a generator through a ball screw. It can be shown that the relative displacement (Z) of mass is given by

$$Z = \frac{mY \,\omega^2}{\sqrt{\left(k - \left(m + J_t \left(\frac{2\pi}{l}\right)^2\right)\omega^2\right)^2 + \left(\left(c_{bg} + \frac{T_i^2}{R_l + R_i}\right)\left(\frac{2\pi}{l}\right)^2\omega\right)^2}}$$
(1)

Where  $T_i$  refers to the generator's torque constant,  $R_i$  and  $R_l$  are the internal and load resistances, respectively.  $J_t$  refers to the total moment of inertia of the rotational components which is given by

$$J_t = J_{mi} + J_g + J_r + J_{bs} , (2)$$

where  $J_g$  and  $J_{bs}$  are the moment of inertia of the generator and the ball screw, respectively. Also,  $J_r$  is the moment of inertial of the rotational rod, given by

$$J_r = \frac{m_r L^2}{12}. (3)$$

where,  $m_r$  and L are the mass and length of the rotational rod. Also, if  $m_i$  is the moveable rotational mass and r is the distance between the center of moveable mass to the center of the rod, i.e. rotation axis, the moment of inertia due to the rotation of the two moveable masses  $J_{mi}$ , is defined as

$$J_{mi} = 2m_i r^2. (4)$$

Note that more than two moveable masses can be used. The natural frequency of the energy harvester is given by

$$\omega_n = \sqrt{\frac{k}{m + J_t \left(\frac{2\pi}{l}\right)^2}}.$$
 (5)

In addition, the maximum value of the relative displacement occurs when  $\partial Z/\partial\omega = 0$  which gives the resonance frequency of

$$\omega_r = \frac{2k}{\sqrt{4k\left(m + J_t\left(\frac{2\pi}{l}\right)^2\right) - 2\left(\left(c_{bg} + \frac{T_i^2}{R_l + R_i}\right)\left(\frac{2\pi}{l}\right)^2\right)^2}}.$$
(6)

The maximum relative displacement is then given by

$$Z_{resonance} = \frac{2mk^{2}Y}{\sqrt{4k^{3} \left( \left( c_{bg} + \frac{T_{i}^{2}}{R_{l} + R_{i}} \right) \left( \frac{2\pi}{l} \right)^{2} \right)^{2} \left( m + J_{t} \left( \frac{2\pi}{l} \right)^{2} \right) - k^{2} \left( \left( c_{bg} + \frac{T_{i}^{2}}{R_{l} + R_{i}} \right) \left( \frac{2\pi}{l} \right)^{2} \right)^{4}}}.$$
 (7)

The output power can be shown [2] to be given by

$$P_{l-out} = \frac{R_l}{2(R_l + R_i)^2} T_i^2 \left(\frac{2\pi}{l}\right)^2 \frac{\left(mY\omega^3\right)^2}{\left(k - \left(m + J_i \left(\frac{2\pi}{l}\right)^2\right)\omega^2\right)^2 + \left(\left(c_{bg} + \frac{T_i^2}{R_l + R_i}\right)\left(\frac{2\pi}{l}\right)^2\omega^2\right)^2},$$
(8)

where the optimum load resistance is obtained from  $\partial P_{l-out} / \partial R_l = 0$ , which results in

$$R_{l,\text{opt}} = \sqrt{R_i^2 + \frac{\left(\left(\frac{2\pi}{l}\right)^2 \omega T_i\right)^2 \left(2R_i c_m + T_i^2\right)}{\left(k - \left(m + J\left(\frac{2\pi}{l}\right)^2\right)\omega^2\right)^2 + \left(\left(\frac{2\pi}{l}\right)^2 c_m \omega\right)^2}}$$

$$(9)$$

From (8) it is seen that the output power of the system shown in Fig.1, is a function of its load resistance and the total

moment of inertia. This paper investigates the optimal control of these two parameters to maximize the output power when the system is subjected to a frequency varying excitation.

# 3. Tuneable moment of inertia in a constrained system

In this section, it is assumed that the amplitude of base displacement Y is 1 m. However, there is a constraint on the maximum allowable relative displacement of the mass which is  $Z_0 = 0.2$  m. In this section, the performance of four different systems are compared. These systems are named as: 1) Constant  $J_i$ . Constant  $J_i$  for system without any tuning parameter. We shall refer to this as the fixed system. 2) Variable  $J_i$ . Constant  $J_i$  for the system with moveable masses 3) Constant  $J_i$ . Variable  $J_i$  for system with tunable load resistance 4) Constant  $J_i$ . Variable  $J_i$  for system with both moveable masses and tunable load resistance. For the first system, the moment of inertia is selected so that for all excitation frequencies, the maximum displacement does not exceed the maximum allowable amplitude. For the system with adjustable moment of inertia, the optimum moment of inertia of the device is obtained by solving the following system of equations:

$$\begin{cases}
\max\left(P_{l-out}(J_{t},\omega)\right) \\
\text{subject to } : Z_{0} - Z(J_{t},\omega) \ge 0.
\end{cases}$$
(10)

For the system with variable resistance, the optimum load resistance is obtained by solving the following system of equations

$$\begin{cases}
\max\left(P_{l-out}\left(R_{l},\omega\right)\right) \\
\text{subject to}: Z_{0} - Z\left(R_{l},\omega\right) \ge 0.
\end{cases} \tag{11}$$

and for the system with variable moment of inertia and variable load resistance, the optimum values are obtained by solving the following system of equations

$$\begin{cases}
\max\left(P_{l-out}\left(J_{l},R_{l},\omega\right)\right) \\
subject \ to : Z_{0} - Z\left(J_{l},R_{l},\omega\right) \ge 0.
\end{cases} \tag{12}$$

A comparison between the relative mass displacements in these four configurations is shown in Fig.2. It is seen that, for the system with tuneable moment of inertia and load resistance, for all frequencies above 1.1 rad/sec, the load resistance and moment of inertia of the device can be tuned so that the mass oscillates at its maximum allowable amplitude of 0.2 m. None of the other three systems can achieve this.

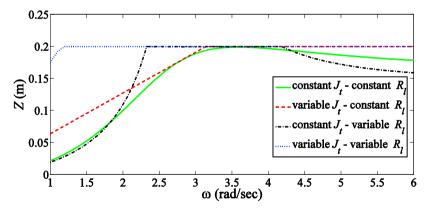


Fig. 2. Relative mass displacements of four systems in the constrained mode.

Fig. 3. shows the optimum moment of inertia of the harvester and the position of the moveable masses for different frequencies. It is seen that at low frequencies the moment of inertia of the system must be very large. It implies that the moveable masses are at the far ends of the rod. By increasing the frequency of oscillation, the moment of inertia should decrease which means that the two masses should be moved toward the centre of the rod. This trend is continued until the relative mass displacement is 0.2 m which occurs when the frequency of excitation is  $\omega = 3.12 \text{ rad/sec}$  (see

Fig 2). In the system with only variable moment of inertia, after this point and for a short range of excitation frequencies, i.e. up to  $\omega = 3.50 \, \text{rad/sec}$ , the moment of inertia of the device should increase in order to keep the relative displacement of the mass constant. However, by increasing the frequency of excitation, if the controller does not reduce the moment of inertia of the harvester, the relative displacement of the mass will decline. Therefore, to keep the mass displacement at its maximum allowable distance, the moment of inertia of the device should gradually decrease.

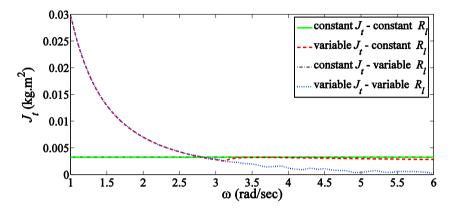


Fig. 3. The moment of inertia of four systems in the constrained mode.

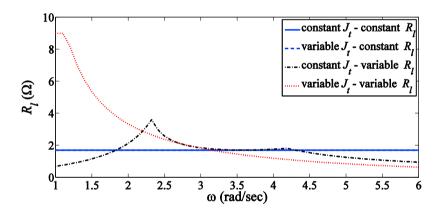


Fig. 4. The load resistance of four systems in the constrained mode.

Fig. 4 shows variations of the load resistance in these four systems. It is seen that in the system with only variable load resistance, i.e. constant moment of inertia, for frequencies below  $\omega = 2.25$  rad/sec, Where the relative displacement of oscillating mass is below 0.2 m (based on Fig.2), the tuned load resistance is selected according to the optimal load resistance obtained in (9). However, by increasing the excitation frequency, the load resistance is decreased to apply more electrical damping to the system and to keep the oscillating mass displacement in the constrained range. By increasing the excitation frequency, in systems with tuneable load resistance, the load resistance decreases to increase the damping of the system. However, the load resistance in the system with tuneable moment of inertia and load resistance obtained from (12) reduces faster, as in this system its natural frequency is much closer to the excitation frequency and hence more electrical damping is required to meet the constraint condition. The comparison between the output power of these four systems is shown in Fig. 5. The output power of the fixed system at frequencies of 1 rad/sec and 6 rad/sec, respectively, are 0.04 W and 92.15 W, whereas, the system with tuneable moment of inertia and load resistance can produce 0.61 W and 193.5 W output power at the same frequencies which are significantly higher than the power produced by the fixed system. It is seen that for all frequency ranges, the system with tunable moment

of inertia and load resistance can produce greater power in comparison with the other systems.

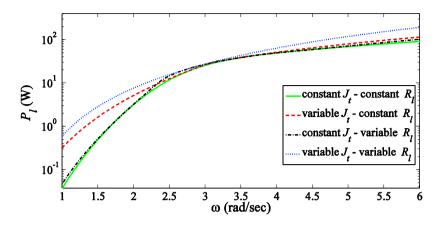


Fig. 5. Comparison between the output power of four harvesters in the constrained mode.

#### 4. Conclusion

It is shown that the combination of tuning the moment of inertia of a rotational harvesting device and adjusting its resistance load is a promising approach for broadening the operational bandwidth of an energy harvester with constrained range of motion. The approach described in this paper is a first step in the direction of having an autonomous energy harvester with a wide operational bandwidth. One of the advantages of the presented method is that unlike some other methods [9], changing the adjustable parameters, i.e., moment of inertia and load resistance, can be conducted intermittently. In other words, this approach only consumes power during the tuning operations and does not use energy once the harvester is tuned at its optimum conditions.

Finally, it is important to note that rotational systems should be used where the excitation frequency varies slowly (e.g., in marine environment) as any sudden changes in frequency would result in an instantaneous change in the apparent mass and the device may stall. To implement the device effectively, some kind of predictive control may need to be used that can detect frequency variations fast enough for the inertia to change in a timely manner. This aspect that is outside the scope of this paper is currently under investigation.

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