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Quadratic Time-Frequency Transforms based Brillouin Optical Time Domain Reflectometry

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*Abstract*— Linear and quadratic Time-Frequency (T-F) transforms are proposed for the signal processing of the Brillouin optical time domain reflectometry, to improve the system performance, in terms of the transition responsivity and frequency resolution. Various T-F transforms, including linear T-F transform (short time Fourier transform (STFT)) and quadratic transforms (Choi-Williams (CW), Zhao-Atlas-Marks (ZAM), Smoothed Pseudo Wigner-Ville (SPWV), S-method (SM), and Adaptive spectrogram (AS)) are applied to the experimental backscattered time-domain spontaneous Brillouin signals. Multiple time-frequency approaches can be jointly applied to efficiently improve the time and frequency resolutions simultaneously. Results show that the SWPV transform provides the best transition responsivity and frequency resolution simultaneously among the six T-F transforms because of its smoothing operations on frequency and time axis for reducing the cross-term interference.

*Index Terms*—Fiber optics sensors, backscattering, Brillouin, Time-frequency analysis.

# INTRODUCTION

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rillouin optical fiber sensors have the capability of a distributed sensing of strain and temperature along the fiber under test (FUT), which has attracted significant interest in the past few decades, as they can provide a convenient approach to health and safety monitoring for industrial applications [1]. Spontaneous Brillouin scattering (SpBS) has been utilized to measure the distributed temperature and strain along the fiber by applying proportionality between the strain and temperature in the FUT and the Brillouin Frequency Shift (BFS) [2]. Based on the principle, the Brillouin Optical Time Domain Reflectometry (BOTDR) is widely used in civil engineering applications [3], in which only one end of the fiber is required for detection. The BFS is obtained by measuring peak-power frequency of the Brillouin gain spectrum along the FUT. The distributed Brillouin Gain Spectrum (BGS) along the FUT is a nonstationary signal that depends on the distributed stain and temperature.

T-F representations map a one-dimensional time-domain signal into a two-dimensional function of time and frequency, which is time-varying spectral representation [4]. The values of the T-F surface give an indication as to which spectral components are present at which times. Therefore, T-F representations can analyze, modify, and synthesize the SpBS signal along the FUT, which is a fast time-varying frequency signal and is much weaker than the Rayleigh backscattering signal. The linewidth of SpBS is about 35-100 MHz, when the pulse width of the probe light is varying from continuous light wave to a 10ns square-wave pulse light [5]. In order to obtain the spontaneous Brillouin spectrum, the traditional frequency-sweeping method has been applied with a narrow bandpass filter before the signal receiver [1].

 In the previous studies, a discrete Fourier transform (DFT) based BOTDR is applied by adopting digital signal processing (DSP) and/or field programmable gate array (FPGA), so that the time-consuming frequency sweeping method can be replaced [6]. For the dynamic sensing, a distributed dynamic strain measurement has been demonstrated, which can detect 16.7 Hz of dynamic strain variation with a 4m spatial resolution and a 45 µɛ uncertainty on a 12 m section [7]. A study using the Choi-Williams transform was introduced and demonstrated a reduction in the trade-off between transition responsivity and frequency accuracy [8].

 In this study, a comprehensive analysis of the T-F transform based BOTDR is introduced and analyzed. T-F representations can be divided into two main classes: linear and quadratic [9]. Short time Fourier Transform (STFT) is the most widely used linear T-F method. The basic idea of STFT is to break up the signal into small segments, and Fourier analyzes each time segment in order to determine the frequencies that existed in that segment [10]. The motivation of applying the quadratic T-F frequency analysis is the crucial drawback of the linear T-F distribution, namely that the length of the window is related to the frequency resolution [11]. However, it does suffer an inherent cross-term contamination when analyzing multi-component signals, by utilizing a carefully chosen window function, the interference can be significantly mitigated, at the expense of the transition responsivity and frequency resolution [12, 13]. The definition of transition responsivity applied in this work is measured as having a 10%-90% rise in distance of a Brillouin central frequency [8]. In this work, six T-F analysis methods that possess various characteristics are introduced and applied in the BOTDR system, including linear T-F transform, STFT [4], and quadratic T-F transforms Choi-Williams (CW) [14], Zhao-Atlas-Marks (ZAM) [15,16], Smoothed Pseudo Wigner-Ville (SPWV) [4,14], S-method (SM) [17], and Adaptive spectrogram (AS)[18,19]. The BGS along the FUT is generated and processed by these linear and quadratic T-F transforms, from the time-domain backscattered Brillouin signal to the T-F domain Brillouin spectral profile, where the characteristics of each T-F distribution are analyzed and experienced in terms of BGS, frequency resolution and transition responsivity.

# Theoretical background of Time-Frequency (T-F) representations

A signal is classically represented in the time-domain and frequency-domain respectively, where the variables of time and frequency are retreated as mutually exclusive: to obtain a representation in terms of one variable, the other variable is integrated out [4]. This follows in some sense from the uncertainty relation that links time and frequency, with the consequence that the outcome of any transform depends not only on intrinsic characteristics of the analyzed signal, but also on the specific properties of the chosen transform [4]. T-F representations of signals map a one-dimensional function of time into a two-dimensional function of time and frequency. Most T-F representations are time-varying spectral representations, which gives an indication of the spectral components at each time location. The T-F representation and the corresponding equation are listed as Table 1.

TABLE 1. THE T-F REPRESENTATIONS AND THE CORRESPONDING EQUATIONS [4,10,12-19]

|  |  |
| --- | --- |
| Representation | Equation of the time-frequency representation |
| STFT | where *t* is the time, *ω* is the angular frequency, *τ* is the time lag, *x(t)* denotes the signal and *w(t)* denotes the window function |
| CWD | where *θ* is the angular frequency lag and *u* is the additional integral time variable |
| ZAMD | where *h(τ)* is the smoothing window |
| SPWVD | where *g(t)* is the smoothing window |
| AS | where *Ak* is the coefficient factor that indicates the signal projection on the elementary function |
| SM | where *P(τ)* is a narrow window that is used to adjust the transform from STFT to WVD |

Fourier-type linear T-F analysis, STFT, is highly dependent on the choice of the window function. Therefore, a major disadvantage of applying STFT in the BOTDR system is the trade-off between the transition responsivity and the frequency resolution [11]. The motivation of applying a quadratic T-F analysis is to conquer this crucial drawback of STFT. This inherent relationship between the time and frequency resolution becomes more important when one is dealing with a signal whose frequency content is varying rapidly. Therefore, a T-F characterization that is capable of overcoming the above drawback became a major goal for the quadratic T-F analysis methods. It is possible to obtain any quadratic T-F distribution by selecting a suitable kernel function according to Cohen’s generalization [20, 21]. The quadratic analysis means the analysis is bilinear where the signal involved twice in the calculation. Performing the transformation brings two dimensional planes which displays the changes of frequency components, named auto-terms. Unfortunately, the quadratic nature of the discussed transformations manifests itself in existing of undesirable components, known as cross-terms. The cross-terms are located between the auto-terms, which reduces the auto-components resolution, obscures the true signal features and makes an interpretation of the T-F distribution difficult [4,20]. The auto-terms (AT) are the analyzing result of the signal and noise result separately in both time and frequency domain, however, the cross-term (CT) will mix the signal and noise together to form an addition terms that the noise will be incorporated into the signals and sometimes the CT can be larger than the original signal. Therefore, one crucial matter of a kernel function is to smooth the cross-terms with preservation all useful properties of the distribution. CWD applies exponential kernel to suppress the cross-term [14]. For development of ZAMD, “cone kernel” was introduced to deliver finite time support and reduce cross terms [15, 22]. Influence of the function h(τ) is the smoothing window on the original T-F representation of Cohen’s class along the frequency axis. The cone shaped kernel function suppresses the cross terms away from the vertical axis and the origin of the ambiguity function plane [4, 12]. SPWVD is introduced for bringing additional operations, which can bear fruit with smoothing the cross terms [23]. Based on the smoothing window provided by h(τ), the second smoothing operation concerns a convolution of the pseudo-representation with the next additional smoothing window g(t) in the time domain. The chirplet-based adaptive spectrogram or AS is implemented using the matching pursuit algorithm, which searches for suitable elementary functions. This is regarded as a kind of parametric T-F analysis method. The advantage of AS is that it does not bring any cross terms. The parameter, Ak, is the coefficient factor, which indicates the signal projection on the elementary function derived from the input signal [18] and WVD x(t, ω) is the Wigner Ville distribution (WVD). However, this adaptive spectrogram is only suitable for certain types of signals that should be composed of chirplet signals. S-method (SM) for T-F signal analysis is proposed [17], and designed for reducing or completely eliminating cross-term interference without degrading the high resolution of the WVD. SM is a way of constructing quadratic distributions from STFT to WVD. Accordingly, SM is designed to combine the good properties of both; that is, a high resolution and significant reduction of the cross-term interference.

# Experimental setup

An experiment is performed to compare the performance of STFT, CWD, ZAMD, SPWVD, AS, and SM based BOTDR. Fig. 1 illustrates the schematic representation of the experimental BOTDR with a temperature calibration unit:



Fig. 1. Experimental setup of the BOTDR system

Branch A and B were split by a 90/10 coupler from a continuous-wave laser diode. Branch A was modulated by an Electro-Optical Modulator (EOM) with a 34 ns modulating pulse, amplified by an Erbium-Doped Fibre Amplifier (EDFA). The amplified pulse light went through an optical filter followed by a circulator, and then was launched into a 1.5 km FUT, which was heated by a temperature controlled water bath, with a temperature of around 75 °C. Branch B is regarded as the reference light connected by a polarization scrambler (PS) and utilized as an optical local oscillator (OLO). Branch B mixed with the backscattering light from the FUT via a coupler and then a 26 GHz photodetector (PD) is used for coherent detection. The down-converted signal was captured by a 5 GS/s digitizer. The water bath can offer a temperature control within 0.1 °C. There are multiple sections set from 10m to 0.3m, with a 10m length internal into the water bath, by setting the temperature to be 75°C, as shown in Fig. 2



Fig. 2. Experimental setup for the temperature-controlled unit of the BOTDR system

The plot of the Brillouin gain spectrum located at the same position of the FUT and processed by STFT, CWD, ZAMD, SPWVD, SM and AS, which are all filtered by a 128 points Kaiser filter (2.56m), are shown in Fig.3. Among these plots, the Brillouin gain spectrum produced by AS offers the most asymmetric Brillouin scattering distribution and the coarsest spectrum resolution compared with the other five. The values of Full Width at Half Maximum (FWHM) generated by bilinear T-F transforms (ZAM, CW and SPWV) are narrower than the value of STFT, which indicates that the concentration brought by ZAM, CW, SPWV and AS is higher than that of STFT [24,25].







Fig. 3. (A) Normalized Brillouin gain spectrum located at the same position, (B) BFS profile (from 50 m to 250m) and (C) Brillouin central frequency transition region for the 70 °C temperature measurement, generated by STFT, SM, SPWV, CW, ZAM, and AS.

The distributed BFS profiles, processed by STFT, SM, SPWV, CW, ZAM and AS, are shown in Fig. 3(B). It is observed that STFT, SM, SPWV, CW, and ZAM can properly measure the temperature change up to 1 m section. However, AS cannot successfully detect the sections with a length of less than 1 m. Here the results show that the STFT, SM, SPWV, CW and ZAM will give different spatial resolution because of their different ability in noise elimination. With the same data obtained by the digitizer and same filter for peak power detecting to observe the BFS along the fibre, the algorithm shows different analytical effective pulse width [26] which relates to the spatial resolution. In Fig 3 (B), ZAM gives the best spatial resolution, whereas STFT’s 1m section has less accuracy and AS method cannot detect 1m section at all. This shows that the ZAM has better time localization and the SNR in time domain can be improved to give a smaller effective pulse width.

It is also noticeable that in Fig 3 (B), at 0.5m section, the SM can detect more accurate than ZAM method while the transition response of SM is actually slower than ZAM. This is because SM shows rapid transition at beginning of the event change, however, the ZAM is rapid in the whole transition. Because 0.5m section is much smaller than spatial resolution. The power of large temperature change is averaged and therefore flattened to smaller value. SM method can better overcome this problem than ZAM so that the SM method is more reliable than ZAM when the event section is small.

The standard deviation and transition responsivity, which are measured as having a 10%-90% rise in time of a Brillouin central frequency transition for these six transforms calculated for the section from 60m to 100m, are listed in Table 2.

TABLE 2. STANDARD DEVIATION AND TRANSITION RESPONSIVITY OF THE BFS PROFILE

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| T-F Transform | STFT | SM | SPWV | CW | ZAM | AS |
| Standard Deviation (Hz) | 1.67E+06 | 1.48E+06 | 1.41E+06 | 1.45E+06 | 1.72E+06 | 1.58E+06 |
| Transition responsivity (m) | 0.40 | 0.52 | 0.32 | 0.37 | 0.03 | 0.8 |

 It is observed that the smallest frequency resolution is provided by SPWV and the largest standard deviation is produced by ZAM amongst the six T-F transforms. In the comparison between the STFT and the SM transforms, SM gains the advantage of a smaller frequency resolution, which is due to its bilinear T-F property: high concentration in the T-F distribution. The standard deviation generated by AS is between STFT and SM. The reason for the different BFS along the fiber compared with the other five is because the Brillouin scattering signal in this section is not composited by chirplet. In the CW, ZAM and SPWV transforms, SPWV can contribute the best frequency resolution, which is caused by the time and frequency domain smoothing. For ZAM, since the smoothing window is only applied to the frequency domain to reduce the cross-term interference, the resolution deteriorates in the time domain. In summary, the SPWV T-F transform can offer the best standard deviation among these six methods. It means that SPWV-BOTDR can offer a 0.26 MHz smaller frequency resolution than the STFT-BOTDR system in this temperature measurement.

 The transition responsivity, from the smallest to the largest, is offered by ZAM, SPWV, CW, STFT, SM and AS. The T-F transforms of Cohen’s class (SPWV, CW, and ZAM) can offer both smaller transition responsivity and frequency resolution compared with the linear T-F transform (STFT) simultaneously. For the comparison between STFT and SM, STFT can offer shorter transition responsivity than SM, but SM is capable of providing a smaller frequency resolution. AS performs moderately among the six T-F transforms and provides 0.8 m transition responsivity and 1.58 MHz frequency resolution. Unfortunately, there is no T-F transform that can offer both the shortest transition responsivity and the frequency resolution at the same time. If the application requires high transition responsivity, ZAM can provide 0.03 m transition responsivity. If the application needs low strain and temperature resolution, SPWV can be a good choice because it offers 1.41 MHz frequency resolution. In the overall evaluation, SPWV and CW can offer both better transition responsivity and frequency resolution, compared with STFT. For the comparison between SPWV and CW, SPWV provides a better transition responsivity and frequency resolution.

# Conclusion

In this study, linear and quadratic time-frequency (T-F) analyses are used to transform the received backscattered Brillouin signal in the time domain to the T-F domain. There are six T-F transforms, including linear T-F transform (STFT) and quadratic T-F transforms (CW, ZAM, SPWV, AS, and SM) and they were applied to generate the distributed Brillouin gain spectrum along the FUT in order to obtain the corresponding Brillouin central frequency. Among the six T-F transforms, ZAM can bring the shortest transition responsivity (0.03 m), and SPWV can offer the smallest frequency resolution (1.41 MHz). Multiple time-frequency approaches can be jointly applied to efficiently improve the time and frequency resolutions simultaneously. In the overall evaluation, SPWV can offer both better transition responsivity and frequency resolution at the same time, when it is compared to STFT.

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Disclosure

Following EPSRC policy on research data, the additional data will be accessible at https://www.repository.cam.ac.uk/handle/1810/247689

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