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Dynamics of two impacting beams with clearance nonlinearity

Larysa Dzyubak[a,*], Atul Bhaskar[b,*] [1]

[a]Department of Applied Mathematics, National Technical University "Kharkiv Polytechnic Institute", Kharkiv 61002, Ukraine
[b]Faculty of Engineering and the Environment, University of Southampton, Southampton SO16 7QF, United Kingdom

Abstract

Analytical solutions describing the transient dynamics of two Euler-Bernoulli beams with tips separated by clearance, are obtained. The tips of the beams impact when one of the beams is harmonically excited. Expressions of transient dynamics are presented as a superposition of particular solutions that satisfy to inhomogeneous boundary conditions, and eigenfunctions series with time dependent coefficients and homogeneous boundary conditions. The transition from impact phase to out-of-contact phase and vice versa is implemented using conditions that switch, involving construction of expressions for shear forces and relative position of beam tips. After each transition from one phase to another, the functions describing the time dependent coefficients in the eigenfunctions series are updated. This update involves the solution of ordinary differential equations with initial conditions corresponding to the end of the previous phase. The system of impacting beams reveals complex dynamics, including chaotic behaviour. Transient dynamics surfaces, time histories of beams deflections, impact forces, coefficients of restitution and phase planes are presented.

Keywords: Impacting beams; Clearance; Chaos

1. Introduction

Problems related to vibro-impact dynamics of structures are of great interest in mechanical or mechatronic engineering [1, 2]. The dynamics of two impacting beams with clearance nonlinearity can be associated with the study of vibrating bolted and riveted structures with loose fastening, noise generation and chaotic vibrations due to lose connections between structural elements as well as it can be applied to design of impact dampers employing attached flexible beam/beams. In numerous cases non-linear effects caused by clearance may lead to many undesirable effects, in particular to premature failure of structures.

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* Corresponding authors. Tel.: +38 057 707 6032, +44 2380593825; fax: +38 057 707 6601, +44 2380594813.
E-mail address: lpdzyubak@gmail.com, A.Bhaskar@soton.ac.uk

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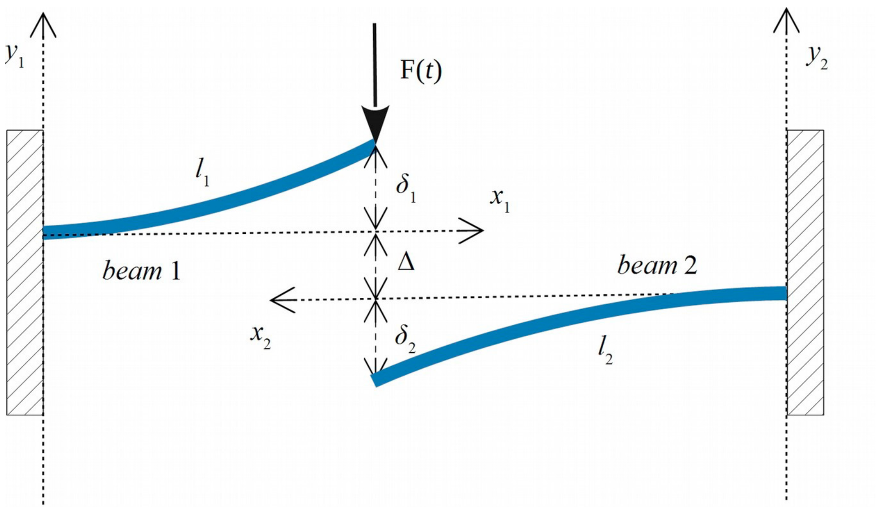


Fig. 1. Impacting beams under harmonic excitation.

2. Mathematical model

The transient dynamics of two impacting cantilever Euler-Bernoulli beams with tips separated by clearance Δ is depicted schematically in Fig. 1. The tips of the beams impact when beam 1 is harmonically excited by a force F(t). l_1 and l_2 are lengths of the beams, and δ_1, δ_2 are the initial uplifts at the free ends. The two beams are not allowed to cross each other at their tips, that is $y_1(l_1,t) \geq y_2(l_2,t)-\Delta$ always. After the impact-induced phase, they must separate in opposite directions. This interaction of the two beams is governed by the partial differential equations

$$a_1^2 \frac{\partial^4 y_1(x_1,t)}{\partial x_1^4} + \frac{\partial^2 y_1(x_1,t)}{\partial t^2} = 0 \quad , \quad a_2^2 \frac{\partial^4 y_2(x_2,t)}{\partial x_2^4} + \frac{\partial^2 y_2(x_2,t)}{\partial t^2} = 0 \tag{1}$$

with different set of boundary conditions for the impact phase and for the out-of-contact phase. For the impact phase the boundary conditions are

$$y_1(0,t)=0 \quad , \quad y_2(0,t)=0 \quad , \quad \frac{\partial y_1(0,t)}{\partial x_1}=0 \quad , \quad \frac{\partial y_2(0,t)}{\partial x_2}=0 \quad , \tag{2}$$

$$M_1(l_1,t)=E_1 I_1 \frac{\partial^2 y_1(l_1,t)}{\partial x_1^2}=0 \quad , \quad M_2(l_2,t)=E_2 I_2 \frac{\partial^2 y_2(l_2,t)}{\partial x_2^2}=0 \quad , \tag{3}$$

$$Q_1(l_1,t)=E_1 I_1 \frac{\partial^3 y_1(l_1,t)}{\partial x_1^3} \quad = \quad Q_2(l_2,t)+F(t)=E_2 I_2 \frac{\partial^3 y_2(l_2,t)}{\partial x_2^3}+F(t) \quad , \tag{4}$$

$$y_1(l_1,t)=y_2(l_2,t)-\Delta \quad . \tag{5}$$

Here E_i ,I_i,ρ_i,A_i, are the Young modulus, area moment of inertia, mass density and cross-section of i-th beam, $a_i=\sqrt{E_i I_i / \rho_i A_i}$, i=1,2. Boundary conditions (2), (3) are identical for the impact and out-of-contact phases. Conditions (2) correspond to fixed both the position and slope of the cantilever beams at points x_1=0 and x_2=0. Since no external bending moments are applied at the beam tips $x_1=l_1$ and $x_2=l_2$, the bending moments $M_1(x_1,t)$ and $M_2(x_2,t)$ at that locations are zero (conditions (3)). The shearing force $Q_1(x_1,t)$ at the tip $x_1=l_1$ is equal to the sum of the applied force $F(t)$ and the reaction force $Q_2(x_2,t)$ at the tip $x_2=l_2$ (condition (4)). Condition (5) is the necessary

(a) (b)

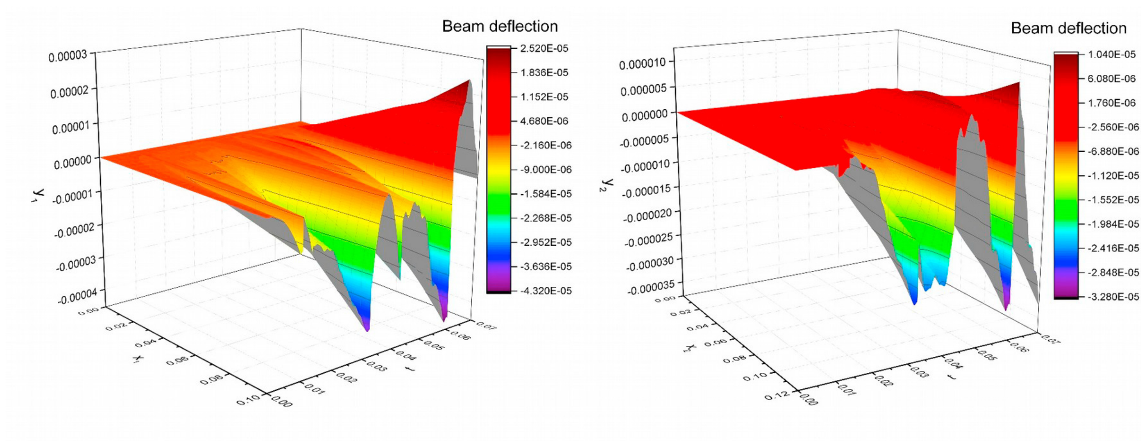


Fig. 2. Beam deflections surfaces depending on time and the lengths: (a) $y_1(x_1,t)$ for the first beam; (b) $y_2(x_2,t)$ for the second beam at A=0.0005, w=40, l_1=0.1, l_2=0.12, δ_1= δ_2=0.0, Δ=0.000011, Δt=0.0001, 0<t<0.07.

and sufficient condition for beam tips to be in contact. In the out-of-contact case boundary conditions (4) and (5) are replaced by boundary conditions

$$Q_1(l_1,t)=E_1I_1\frac{\partial^3 y_1(l_1,t)}{\partial {x_1}^3}=F(t), \qquad Q_2(l_2,t)=E_2I_2\frac{\partial^3 y_2(l_2,t)}{\partial {x_2}^3}=0. \tag{6}$$

The discretization of the system is based on the eigenvalue problem that gives infinite number of beam natural frequencies, as well as mode shapes for the cases of impact phase and out-of-contact phase.

2.1. Impact phase

Expressions of transient dynamics (functions $y_1(x_1,t)$ and $y_2(x_2,t)$) are presented as a superposition of particular solutions $y_{1s}(x_1,t)$ and $y_{2s}(x_2,t)$ that satisfy to inhomogeneous boundary conditions, and eigenfunctions series with time dependent coefficients and homogeneous boundary conditions:

$$y_1(x_1,t)=y_{1s}(x_1,t)+\sum_{m=1}^{\infty} Y_{1m}(x_1)q_m(t) \quad , \quad y_2(x_2,t)=y_{2s}(x_2,t)+\sum_{m=1}^{\infty} Y_{2m}(x_2)q_m(t) \quad . \tag{7}$$

Expressions for the particular solutions are obtained in the form:

$$y_{1s}(x_1,t)=\frac{1}{6}\frac{3E_2I_2\Delta-l_2^3F(t)}{E_2I_2l_1^3-E_1I_1l_2^3}\left(x_1^3-3l_1x_1^2\right) \quad , \quad y_{2s}(x_2,t)=\frac{1}{6}\frac{3E_1I_1\Delta-l_1^3F(t)}{E_2I_2l_1^3-E_1I_1l_2^3}\left(x_2^3-3l_2x_2^2\right) \quad . \tag{8}$$

The corresponding natural frequencies of vibration $\omega_{1m}=\omega_{2m}$, $\omega_{1m}=a_1k_{1m}^2$, $\omega_{2m}=a_2k_{2m}^2$, are defined from the equation that follows by conditions for the existence of non-trivial solutions

$$\begin{aligned}&E_1I_1k_{1m}^3\left(1+\cos k_{1m}l_1\cosh k_{1m}l_1\right)\left(\cosh k_{2m}l_2\sin k_{2m}l_2-\cos k_{2m}l_2\sinh k_{2m}l_2\right) \quad + \\ &E_2I_2k_{2m}^3\left(1+\cos k_{2m}l_2\cosh k_{2m}l_2\right)\left(\cosh k_{1m}l_1\sin k_{1m}l_1-\cos k_{1m}l_1\sinh k_{1m}l_1\right)=0.\end{aligned} \tag{9}$$

So, as it follows by the eigenvalue problem, the mode shapes in the impact phase are represented by the expressions:

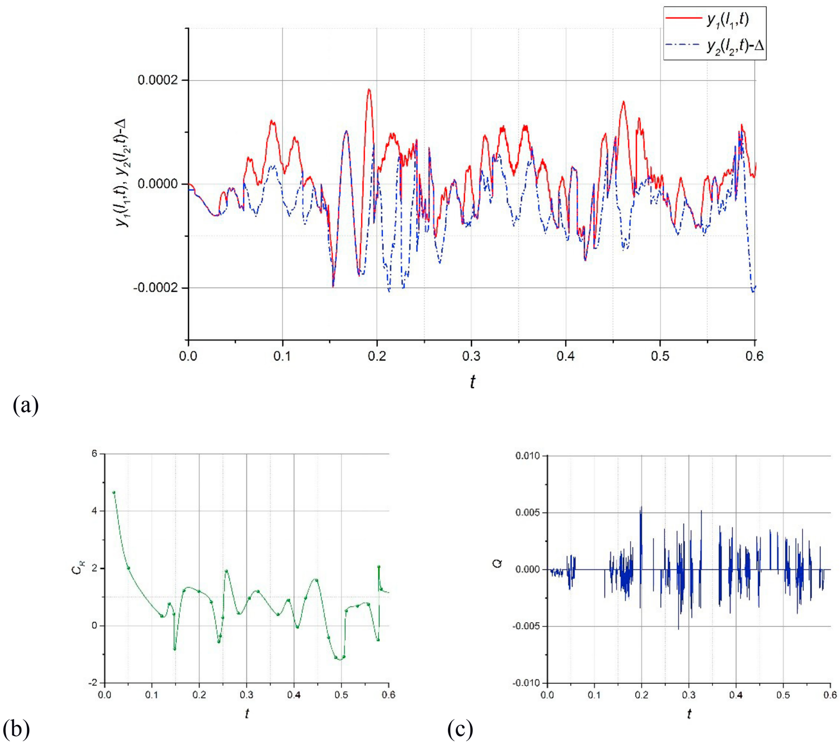


Fig. 3. (a) beams deflections $y_1(l_1,t)$, $y_2(l_2,t)$-Δ; (b) coefficient of restitution of impacting beams; (c) impact-induced force.

$$\begin{aligned}
&Y_{1m}=A_m\left[\sin k_{1m}x_1-\sinh k_{1m}x_1+\frac{\sin k_{1m}l_1+\sinh k_{1m}l_1}{\cos k_{1m}l_1+\cosh k_{1m}l_1}\left(\cosh k_{1m}x_1-\cos k_{1m}x_1\right)\right] \quad ,\\
&Y_{2m}=A_m\frac{E_1I_1k_{1m}^3\left(1+\cos k_{1m}l_1\cosh k_{1m}l_1\right)}{E_2I_2k_{2m}^3\left(\cos k_{1m}l_1+\cosh k_{1m}l_1\right)\left(1+\cos k_{2m}l_2\cosh k_{2m}l_2\right)}\times \\
&\left[\left(\cos k_{2m}l_2+\cosh k_{2m}l_2\right)\left(\sin k_{2m}x_2-\sinh k_{2m}x_2\right)+\left(\sin k_{2m}l_2+\sinh k_{2m}l_2\right)\left(\cosh k_{2m}x_2-\cos k_{2m}x_2\right)\right].
\end{aligned} \tag{10}$$

The time dependent coefficients are defined as follows

$$q_m(t)=q_m(0)\cos\omega_m t+\frac{1}{\omega_m}\dot{q}_m(0)\sin\omega_m t+\frac{1}{\omega_m}\int_0^t\ddot{\psi}_m(\tau)\sin(t-\tau)\,d\tau \quad . \tag{11}$$

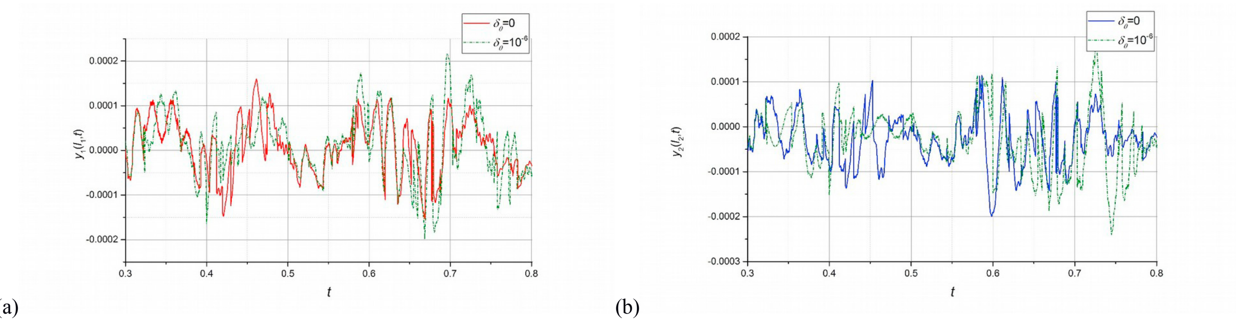
2.2. Out-of-contact phase

In this case the motions of the beams don't depend on each other. Natural frequencies $\omega_{1n}=a_1k_{1n}^2$, $\omega_{2n}=a_2k_{2n}^2$ are defined for both beams from equations: $\cos(k_{in}l_i)\cosh(k_{in}l_i)+1=0$, i=1,2.

Expressions for functions $y_1(x_1,t)$ and $y_2(x_2,t)$) are presented as a superposition of particular solutions $y_{1s}(x_1,t)=-F(t)\left(3l_1x_1^2-x_1^3\right)/6E_1I_1$ and $y_{2s}(x_2,t)$=0 that satisfy to inhomogeneous boundary conditions, and eigenfunctions series with time dependent coefficients and homogeneous boundary conditions:

$$y_1(x_1,t)=y_{1s}(x_1,t)+\sum_{n=1}^{\infty}Y_{1n}(x_1)q_n(t) \quad , \quad y_2(x_2,t)=\sum_{m=1}^{\infty}Y_{2n}(x_2)q_n(t) \quad . \tag{12}$$

the mode shapes in the out-of-contact phase are represented by the expressions:



(a) (b)

Fig. 4. Chaotic motion of the impacting beams (a) $y_1(l_1,t)$, (b) $y_2(l_2,t)$.

$$Y_{1i}=A_{1i}\left[\sin k_{in}x_1-\sinh k_{in}x_1+\frac{\sin k_{in}l_1+\sinh k_{in}l_1}{\cos k_{in}l_1+\cosh k_{in}l_1}\left(\cosh k_{in}x_1-\cos k_{in}x_1\right)\right] \quad , i=1,2. \tag{13}$$

The time dependent coefficients are defined as follows

$$q_{1n}(t)=q_{1n}(0)\cos\omega_{1n}t+\frac{1}{\omega_{1n}}\dot{q}_{1n}(0)\sin\omega_{1n}t+\frac{1}{\omega_{1n}}\int_0^t\ddot{\psi}_{1n}(\tau)\sin(t-\tau)\,d\tau,$$
$$q_{2n}(t)=q_{2n}(0)\cos\omega_{2n}t+\frac{1}{\omega_{2n}}\dot{q}_{2n}(0)\sin\omega_{2n}t \tag{14}$$

2.3 Switching between phases

The transition from impact phase to out-of-contact phase and vice versa is implemented using conditions that switch, involving construction of expressions for shear forces and relative position of beam tips:

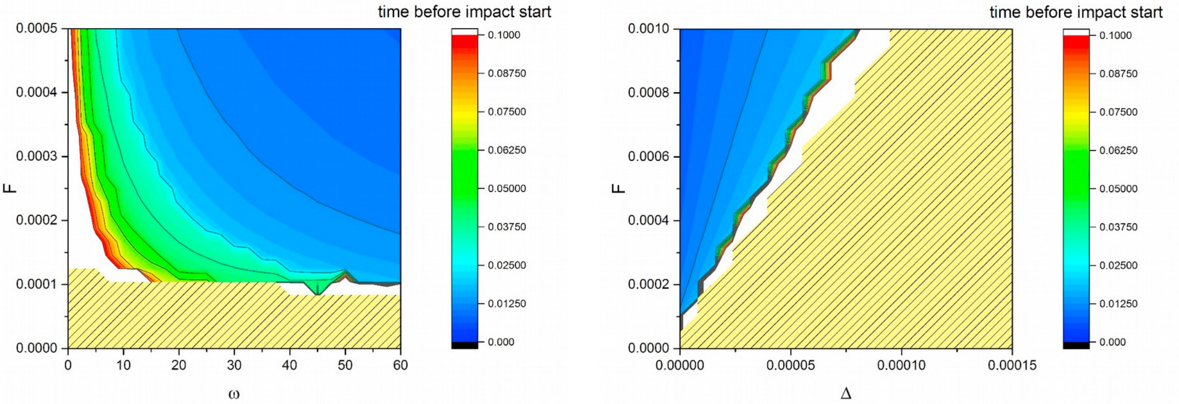
$$y_2(l_2,t)-y_1(l_1,t)=\Delta \qquad \frac{d}{dt}\left(y_2(l_2,t)-y_1(l_1,t)\right)\geqslant 0$$ out-of-contact phase → impact phase transition

$$P(t)=0, \qquad \frac{dP(t)}{dt}\leqslant 0, \quad \text{where} \quad P(t)=E_1I_1\frac{\partial^3 y_1(l_1,t)}{\partial x_1^3}-F(t)$$ impact phase → out-of-contact phase transition

After each transition from one phase to another, the functions describing the time dependent coefficients in the eigenfunctions series are updated. This update involves the solution of ordinary differential equations with initial conditions corresponding to the end of the previous phase.

3. Graphical representation of the analytical solutions obtained

The solutions obtained were tested on numerous examples with various set of parameters. Convergence of eigenfunction series as well as convergence of the solutions in time were investigated. Terms of eigenfunction series with time dependent coefficients evaluated at the tips in the case of out-of-contact phase $y_{1s}(l_1,t)$, $Y_{1n}(l_1)q_{1n}(t)$, $Y_{2n}(l_2)q_{2n}(t)$ (n=1,2,3,4,5) and in the case of impact phase $y_{1s}(l_1,t)$, $Y_{1m}(l_1)q_{1m}(t)$, $y_{2s}(l_2,t)$, $Y_{2m}(l_2)q_{2m}(t)$ (m=1,2,3,4,5) were seen to be fast decreasing. The solutions with truncated eigenfunction series at n=4, m=4 and at n=5, m=5 are almost identical. A necessary convergence of the solutions in time was reached by an appropriate choice of time steps. So, the time step is chosen correctly if the solutions obtained with this time step and with the doubled time step are identical. In Fig. 2 the beam deflections surfaces vs time and length for the first beam and for the second beam are shown. Fig. 3 represents beams deflections $y_1(l_1,t)$, $y_2(l_2,t)$-Δ, coefficient of restitution of impacting beams and the impact-induced force at amplitude and frequency of external harmonic excitation A=0.001 and ω=50,



(a) (b)

Fig. 5. Time before impact start (a) in (ω, *F*) control parameter plane; (b) in (Δ, F) control parameter plane.

initial uplifts $\delta_1=\delta_2=0.0$, clearance $\Delta=0.000011$, $a_1=0.7$, $a_2=0.7$, time instant is $0\le t\le 0.6$. Such time instant is chosen for convenience to observe the dynamics of the beams. During numerical experiment long term solutions were constructed for $0\le t\le 30$ and more. Fig. 4 shows exponential divergence of (a) nearby trajectories $y_1(l_1,t)$, (b) nearby trajectories $y_2(l_2,t)$ with very close initial uplifts $\delta_1=\delta_2=0.0$ and $\delta_1=10^{-6}$, $\delta_2=0.0$. It is sure sign of chaotic vibrations of the beams. In Fig. 5 contours describing time before impact start are plotted in control parameter planes (ω, *F*) and (Δ, *F*). Areas with hatch fill correspond to vibrations without any impact.

4. Conclusions

The analytical solutions, describing the transient dynamics of two impacting beams with clearance nonlinearity, were obtained in the form of eigenfunctions series with time dependent coefficients. Several examples were considered for various set of parameters. Transient dynamics surfaces, time histories of beams deflections, impact forces, coefficients of restitution as well as phase planes and Poincare sections were presented. Chaotic behavior of the beams was ascertained on the base of sensitive dependence of the trajectories of motion on the initial conditions using procedure that similar to one presented in [3]. Time before impact start level contours were obtained in various control parameter planes (ω, *F*), (Δ, *F*) and (ω, Δ). Solutions obtained allow to construct long term vibrations of the impacting beams.

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