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IDENTIFYING LOCAL PROOF 'MODULES' DURING PROVING

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In reviewing PME research, Stylianides et al. (2016) identified developing effective teaching interventions as one of the under-researched areas in argumentation and proofs. In developing such an intervention, we take the structure of deductive proofs as one of the essential elements of understanding (Miyazaki et al. 2017). In considering 'the structure of proof', there are at least two aspects. One is the logical structure consisting of hypothetical syllogism and universal instantiations; another, as Mejia-Ramos, et al. 2012, p. 12) explain, is that to understand a proof entails "breaking the proof into components or modules and then specifying the logical relationship between each of the modules". In this paper, we address the research question: What modules of proofs can be identified during the process of learning deductive proving? From our earlier research, we found that learners' understanding of the logical structure advances from elemental, via relational, to holistic level, and that the relational has two aspect distinguished by universal instantiation and hypothetical syllogism (Miyazaki et al., 2017). Through observations of grade 8 geometry lessons, and corresponding to these three level of understanding, we identified three structure 'modules': 1) vague 'chunks' of propositions, 2) small networks with universal and singular propositions by universal instantiations, and 3) series of small networks. We found, for example, that a learner at the elemental level recognizes elements of proofs such as assumptions or conclusion without their logical relationships, and, as such, this learner's proof 'modules' are no more than vague 'chunks' of singular propositions such as 'AB=DE' or '△ABC≡△DEF'. A learner who is at a relational level of understanding forms proof 'modules' in the form of small networked propositions such as '△ABC≡△DEF because SAS condition' or 'AB=DE because △ABC≡△DEF' when trying to prove a given statement. A learner who is at the holistic level utilises proof 'modules' formed by a series of small networks such as '△ABC≡△DEF because AB=DE, angles ABC=DEF & BC=EF', 'AB=DE because △ABC≡△DEF' and so on which is used to deduce a conclusion from given assumptions.

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