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# High energy seesaw models, GUTs and Leptogenesis

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**Abstract.** I review high energy (type I) seesaw models and in particular how they can be nicely embedded within grand-unified models and reproduce the observed matter-antimatter asymmetry with leptogenesis. I also discuss how high energy (type I) seesaw models can provide a candidate for very heavy cold dark matter, within the TeV-EeV range, whose decays might explain part of the IceCube high energy neutrino events in addition to an astrophysical component.

## 1. Introduction

The  $\Lambda$ CDM cosmological model provides a minimal model able to describe all cosmological observations [1]. However, within the Standard Model of particle physics and fundamental interactions (SM), we cannot explain the intimate nature and origin of some of the features of the  $\Lambda$ CDM, in particular the observed matter-antimatter asymmetry of the Universe and the necessity of a non-baryonic dark matter component. For this reason these *cosmological puzzles* have to be regarded as strong motivations for new physics beyond the SM. Neutrino masses and mixing also call for an extension of the SM and it is then reasonable that the same new physics model should also address the cosmological puzzles. I will mainly focus on a simple extension based on the introduction of right-handed (RH) neutrinos with Yukawa couplings, generating a Dirac mass term for neutrinos as for the other massive fermions, and an additional (left-left) Majorana mass term. This simple extension leads to the seesaw formula for the light neutrino masses and mixing and opens the opportunity to solve some of the cosmological puzzles in quite a minimal way, in any case without *ad hoc* additions beyond this minimal one.

The matter-antimatter asymmetry of the Universe. is precisely and accurately measured by CMB temperature anisotropies in terms of baryon-to-photon number ratio. The *Planck* satellite collaboration finds [1]  $\eta_{B,0}^{(CMB)} = (6.05 \pm 0.06) \times 10^{-10}$ .

Neutrino mixing experiments are well explained, barring anomalies hinting at possible light sterile neutrino states, by neutrino mixing among three active neutrino mass eigenstates with masses  $m_1 < m_2 < m_3$  and with mass squared differences given in the case of normal (inverted) ordering by  $m_3^2 - m_1^2 \equiv m_{\text{atm}}^2 \simeq 0.05 \text{ eV}^2$  and  $m_2^2 - m_1^2 (m_3^2 - m_2^2) \equiv m_{\text{sol}}^2 \simeq 0.009 \text{ eV}^2$  [2]. Flavour neutrinos are in general given by an admixture of neutrino mass eigenstates described by a leptonic mixing matrix  $U$  such that  $\nu_\alpha = \sum_i U_{\alpha i} \nu_i$ . Latest neutrino oscillation experiments global analyses find for the mixing angles and the Dirac phase  $\delta$ , in the case of NO, the following best fit values and  $1\sigma$  errors [2]:  $\theta_{13} = 8.4^\circ \pm 0.2^\circ$ ,  $\theta_{12} = 33^\circ \pm 1^\circ$ ,  $\theta_{23} = 41^\circ \pm 1^\circ$ ,  $\delta = -108^\circ \pm 36^\circ$ . It is interesting that there is already a  $3\sigma$  exclusion interval  $\delta \ni [27^\circ, 153^\circ]$  and that  $\sin \delta > 0$  is excluded at more than  $2\sigma$  clearly favouring  $\sin \delta < 0$  (a lower statistical significance is found in [3]). There is no signal from  $00\nu\beta$  experiments that, therefore, place an upper bound on the  $00\nu\beta$  effective neutrino mass  $m_{ee}$ . Currently, the most stringent reported upper bound comes from the KamLAND-Zen collaboration finding,



at 90% C.L.,  $m_{ee} \leq (61\text{--}165)\text{ meV}$  [4], where the range accounts for nuclear matrix element uncertainties. Cosmological observations place an upper bound on the sum of the neutrino masses,  $\sum_i m_i \lesssim 230\text{ meV}$  at 95% C.L. [1] that, taking into account the measured values of the solar and atmospheric neutrino mass scales, translates into an upper bound on the lightest neutrino mass  $m_1 \lesssim 70\text{ meV}$ . Neutrinos only carry lepton number as a global charge and, therefore, having introduced RH neutrinos and without modifying the SM Higgs sector, one can also have, in addition to the Dirac mass term  $m_D$ , a right-right Majorana mass term  $M$  without conflicting with any experimental bound. In the *seesaw limit*,  $M \gg m_D$ , the mass spectrum splits into 2 sets: a set of three light neutrinos (dominantly LH) with masses given by the seesaw formula  $\text{diag}(m_1, m_2, m_3) = -U^\dagger m_D M^{-1} m_D^T U^*$ , and a set of  $N$  very heavy RH neutrinos with masses  $M_1 \leq M_2 \leq \dots \leq M_N$ . For definiteness and since we will be interested in  $SO(10)$ -inspired models, we will refer to the case  $N = 3$ . These new very heavy RH neutrinos can now play a cosmological role.

## 2. Minimal scenario of leptogenesis

The minimal scenario of leptogenesis [5] relies on two main assumptions:

i) A (type I) seesaw extension of the SM discussed above. In the flavour basis, where both charged lepton and Majorana mass matrices are diagonal,  $h$ , that is in general complex, encodes all source of  $CP$  violation that can translate into a macroscopic  $B - L$  asymmetry, injected in the form of a lepton asymmetry, thanks to the out-of-equilibrium decays of the very heavy RH neutrinos. A RH neutrino can decay either into lepton and higgs doublets with rate  $\Gamma_i$  or into anti-leptons and (h.c.) higgs doublets with rate  $\bar{\Gamma}_i$ . The two rates are in general because of  $CP$  violation and one can define the total  $CP$  asymmetries  $\varepsilon_i \equiv -\Gamma_i - \bar{\Gamma}_i / (\bar{\Gamma}_i + \Gamma_i)$ . Each decay of a RH neutrino  $N_i$  will then generate on average a  $B - L$  asymmetry, in the form of lepton asymmetry, given by  $\varepsilon_i$ . In general the final  $B - L$  asymmetry will be the sum of the contributions from each RH neutrino species, so that  $N_{B-L}^f = \sum_i N_{B-L}^{(i)f}$ .

ii) *Thermal production* of the RH neutrinos in the early Universe. This implies a reheat temperature at the end of inflation  $T_{RH} \gtrsim T_{\text{lep}} = M_i/z_B$ , where  $M_i$  is the mass of the RH neutrino whose decays dominantly produce the asymmetry and  $z_B = 2\text{--}10$  is a factor taking into account that the surviving asymmetry is generated in a relatively sharp range of temperatures below  $M_i$ , *about the value corresponding to a departure of equilibrium*, while at higher temperature all asymmetry is quite efficiently washed-out. This is true for the *a production in a (mildly) strong wash-out regime that however is strongly favoured (and desirable) by the measured values of the solar and atmospheric neutrino mass scales*. A necessary condition for successful leptogenesis is  $T_{\text{lep}} \gtrsim T_{\text{sph}}^{\text{off}}$ , where  $T_{\text{sph}}^{\text{off}} \simeq 140\text{ GeV}$  is the temperature below which sphaleron processes switch off and go out-of-equilibrium (i.e. when  $\Gamma_{\text{sph}} \lesssim H$  where  $H$  is the expansion rate). In this way sphalerons can convert part of the lepton asymmetry into a baryon asymmetry conserving the  $B - L$  asymmetry. In this way one has from leptogenesis  $\eta_{B,0}^{\text{lep}} = a_{\text{sph}} N_{B-L}^f / N_{\gamma,0}$ , where  $a_{\text{sph}} \simeq 1/3$  is the fraction of  $B - L$  asymmetry that ends up into a baryon asymmetry. Successful leptogenesis of course requires  $\eta_{B,0}^{\text{lep}} = \eta_{B,0}^{(CMB)}$ .

### 2.1. A problem with too many parameters

The seesaw parameter space contains 18 additional parameters: 3 RH neutrino masses and 15 additional parameters in the Dirac mass matrix. Thanks to the seesaw formula, the 15 parameters in the Dirac mass matrix can be re-expressed through the 9 low energy neutrino parameters, 3 light neutrino masses and 6 parameters in  $U$ , the 3  $M_i$  and 6 parameters in a orthogonal matrix  $\Omega$ , explicitly  $m_D = U D_m^{1/2} \Omega D_M^{1/2}$ . The orthogonal matrix  $\Omega$  encodes information on the 3 lifetimes and the 3 total  $CP$  asymmetries of the RH neutrinos. Therefore, low energy neutrino experiments by themselves cannot test the seesaw mechanism. The baryon-

to-photon number ratio calculated from leptogenesis,  $\eta_B^{\text{lep}}$ , depends on all 18 seesaw parameters in general. The successful leptogenesis condition is conceptually very important since introduces a constraint on the RH neutrino parameters: it is like if with leptogenesis we are able to read the result of a very special experiment that occurred in the very early Universe, the origin of matter, getting information on the physics at those very high energies. However, by itself in general it is clearly insufficient to over-constrain the seesaw parameter space providing a conclusive phenomenological test. On the other hand a few things might help in this direction: i) Successful leptogenesis might be satisfied only about ‘peaks’; ii) Some of the parameters might cancel out in the calculation of  $\eta_B^{\text{lep}}$ ; iii) Imposing some cosmologically motivated condition to be respected such as the *strong thermal leptogenesis* (independence of the initial conditions) or, even stronger, that one of the heavy RH neutrino species is the dark matter candidate; iv) Adding particle physics phenomenological constraints; v) Embedding the seesaw within a model leading to conditions on  $m_D$  and  $M_i$ .

## 2.2. Vanilla leptogenesis

A particular successful scenario that illustrates the possible strategies to reduce the number of parameters listed above in order to obtain testable constraints or predictions on observables, is represented by so called *vanilla leptogenesis*. It relies on the following set of assumptions: i) the flavour composition of the final leptons does not influence the calculation of the final asymmetry; ii) a hierarchical RH neutrino spectrum ( $M_2 \gtrsim M_1$ ); iii) the asymmetry produced by the heavier RH neutrino is negligible. iv) (momentum integrated) Boltzmann equations fairly describes the kinetic evolution.

Under these four assumptions the predicted baryon-to-photon number ratio gets a contribution only by the lightest RH neutrino decays and has a very simple expression,  $\eta_B^{\text{lep}} \simeq 0.01 \varepsilon_1 \kappa^f(K_1)$ , where the final *efficiency factor*  $\kappa^f$  depends only on the lightest RH neutrino decay parameter  $K_1 \equiv \tilde{\Gamma}_1/H(T = M_1)$  and is basically corresponding to the number of RH neutrinos decaying out-of-equilibrium and from iv) one can find a simple analytical expression that show that it is exponentially suppressed at masses  $M_1 \gtrsim 10^{13} \text{ GeV}$  for  $m_1 \gtrsim m_{\text{atm}}$ . In addition, v) barring fine-tuned cancellation in the see-saw formula, one also obtains an upper bound  $\varepsilon_1 \lesssim 10^{-6} (M_1/10^{10} \text{ GeV}) m_{\text{atm}}/(m_1 + m_3)$  [6]. When these results are combined, from the successful leptogenesis condition one finds a lower bound  $M_1 \gtrsim 10^9 \text{ GeV}$  [6, 7] and an upper bound  $m_1 \lesssim 0.1 \text{ eV}$  [8, 9] that is now interestingly confirmed by the cosmological upper bound  $m_1 \lesssim 0.07 \text{ eV}$  placed by the *Planck* collaboration. This upper bound is also very interesting, since it provides an example of how, despite one starts from 18 parameters, the successful leptogenesis condition can indeed produce testable constraints. The reason is that the final asymmetry in vanilla leptogenesis does not depend on the 6 parameters in  $U$ , since this cancels out in  $\varepsilon_1$ , and on the 6 parameters associated to the two heavier RH neutrinos. There are only 6 parameters left ( $m_1, m_{\text{atm}}, m_{\text{sol}}, M_1, \Omega_{11}^2$ ) of which two are measured leaving only 4 free parameters. The asymmetry moreover has a ‘peak’ strongly suppressed by the value of  $m_1$  and that is where the upper bound on  $m_1$  comes from. Another interesting feature of *vanilla leptogenesis* is that the value of the decay parameter  $K_1$  is not too large to prevent successful leptogenesis but strong enough to wash-out any pre-existing asymmetry (including an asymmetry generated by the heavier RH neutrinos). There is however a corner in the parameter space where  $K_1 \lesssim 1$  and in this case the asymmetry can be generated by the  $N_2$ ’s this is the  $N_2$ -dominated scenario of leptogenesis [12]. An unpleasant feature of vanilla leptogenesis is that imposing so called  $SO(10)$ -inspired conditions and barring very fine-tuned crossing level solutions, one has  $M_1 \sim 10^5 \text{ GeV}$ , well below the lower bound on  $M_1$  for successful  $N_1$ -dominated leptogenesis.  $N_2$ -dominated leptogenesis also cannot be realised since in  $SO(10)$ -inspired leptogenesis one has strictly  $K_1 \gg 1$ . We will see how this problem can be circumvented.

### 3. Flavour effects

The stringent lower bound on  $M_1$  has been one of the main motivations to investigate scenarios of leptogenesis beyond vanilla leptogenesis, that is strictly a  $N_1$ -dominated scenario. There have been different main directions of investigation but certainly that one with the most far-reaching implications, certainly in connection with models and low energy neutrino experiments, are those from flavour effects.

#### 3.1. Charged lepton flavour effects

If  $5 \times 10^8 \text{ GeV} \lesssim M_1 \lesssim 5 \times 10^{11} \text{ GeV}$  then the flavour composition of the leptons (and anti-leptons) produced in the  $N_1$ -decays influence the value of the final asymmetry since leptons have to be described as an incoherent mixture of a tauon component and, of a coherent superposition of the electron and muon components due to the fast  $\tau$ -interactions [10, 11]. In this situation a *two-flavoured regime* is realised and the final asymmetry has to be calculated as the sum of a tauon component and of a electron+muon component, since the two in general experience a difference wash-out. If  $M_1 \lesssim 5 \times 10^8 \text{ GeV}$  then also muon interactions are fast enough to break the coherence of the electron-muon component and one has to consider a three flavoured regime where the final asymmetry has to be calculated as the sum of three different contributions from each charged lepton flavour.

#### 3.2. Heavy neutrino flavour effects

In general one should also consider the asymmetry produced by the heavier RH neutrinos. In an unflavoured approximation, one would obtain that this is efficiently washed-out and can be neglected except for a special region in parameter space [12]. However, when flavour effects are considered, the wash-out has to be considered along different flavour directions and is in general reduced. Even when all three masses are above  $10^{12} \text{ GeV}$  and charged lepton effects are absent, one still has to consider that a lighter RH neutrino  $N_i$  can only wash-out the asymmetry along the  $\ell_i$  flavour direction but not along the orthogonal direction in flavour space (heavy neutrino flavour effects) [10]. When both flavour effects are considered, one has 10 different RH neutrino hierarchical scenarios giving rise to different expressions for the calculation of the final asymmetry from Boltzmann equations.

### 4. $N_2$ -dominated scenario and $SO(10)$ -inspired leptogenesis

An important scenario is obtained for  $M_1 \ll 10^9 \text{ GeV}$  since in this case necessarily the asymmetry has to be generated by the two heavier RH neutrinos and typically the one generated by the heaviest is negligible so that one obtains a  $N_2$ -dominated scenario. This scenario has two interesting features: it emerges naturally when  $SO(10)$ -inspired conditions are imposed and it is the only one that can realise independence of the initial conditions, quite an interesting combination of completely independent features. In the unflavoured case we have seen that imposing  $SO(10)$ -inspired conditions and barring fine-tuned crossing level solutions successful leptogenesis cannot be attained. However, when flavour effects are considered then there is a set solutions satisfying successful leptogenesis. Typically the final asymmetry is in the tauon flavour. Interestingly the set of solutions requires certain constraints on the low energy neutrino parameters. For example the lightest neutrino mass cannot be below  $\simeq 1 \text{ meV}$ , i.e. one expects some deviation from the hierarchical limit though right now we do not know how to fully test this lower bound. However it should be said that also  $SO(10)$ -inspired leptogenesis strongly favours normally ordered neutrino masses. Imposing  $SO(10)$ -inspired conditions and barring crossing level solutions, it is possible to find quite accurate expressions for all important quantities necessary to calculate the asymmetry. We refer the reader to [13] for a detailed discussion. In this way one arrives to a fully analytical expression for  $\eta_B^{\text{lep}}$  in terms of the low energy neutrino

parameters and of the three RH neutrino Dirac mass eigenvalues. It is possible also to consider a supersymmetric framework for  $SO(10)$ -inspired leptogenesis [15].

#### 4.1. Strong thermal $SO(10)$ -inspired leptogenesis

When flavour effects are taken into account there is only one scenario of (successful) leptogenesis allowing for independence of the initial conditions: the tauon  $N_2$ -dominated scenario, where the asymmetry is produced by  $N_2$  decays in the tauon flavour [14]. The conditions are quite special since it is required that a large pre-existing asymmetry is washed-out by the lightest RH neutrino in the electron and muon flavour. The next-to-lightest RH neutrinos both wash-out a large pre-existing tauon asymmetry and also produce the observed asymmetry in the same tauon flavour escaping the lightest RH neutrino wash-out. It is then highly non trivial that this quite special set of conditions can be realised by a subset of the  $SO(10)$ -inspired solutions satisfying successful leptogenesis. For this subset the constraints are quite stringent and they pin down a quite well defined solution: the strong  $SO(10)$ -inspired solution, characterised by non-vanishing reactor mixing angle, normally ordered neutrino masses, atmospheric mixing angle in the first octant and  $\delta$  in the forth quadrant ( $\sin\delta < 0$  and  $\cos\delta > 0$ ). In addition the lightest neutrino mass has to be within quite a narrow range of values about  $m_1 \simeq 20$  meV corresponding to a sum of neutrino masses, the quantity tested by cosmological observations,  $\sum_i m_i \simeq 95$  meV, implying a deviation from the normal hierarchical limit predicting  $\sum_i m_i \simeq 60$  meV detectable during next years. At the same time the solution also predicts a  $00\beta\nu$  signal with  $m_{ee} \simeq 0.8 m_1 \simeq 16$  meV. In light of the latest experimental results discussed in the introduction, this solution is quite intriguing since it has already corrected predicted a non-vanishing reactor mixing angle and it is currently in very good agreement with the best fit parameters from neutrino mixing experiments (to our knowledge is the only model that has truly *pre*-dicted  $\sin\delta < 0$ ).

#### 4.2. Realistic models

An example of realistic models satisfying  $SO(10)$ -inspired conditions and able to fit all lepton and quark mass and mixing parameters are of course  $SO(10)$  models. A specific example is given by renormalizable  $SO(10)$ -models for which the Higgs fields belong to 10-, 120-, 126-dim representations yielding specific mass relations among the various fermion mass matrices [16]. Recently reasonable fits have been obtained typically pointed to compact RH neutrino spectrum with all RH neutrino masses falling in the two-flavour regime. However, also fits realising the  $N_2$ -dominated scenario have been obtained [17]. Note however that  $SO(10)$ -inspired conditions can be also satisfied not necessarily within  $SO(10)$ -models. For example recently a Pati-Salam model combined with  $A_4$  and  $Z_5$  discrete symmetries has been proposed satisfying  $SO(10)$ -inspired conditions [18] and also successful  $SO(10)$ -inspired leptogenesis [19]. A realistic model realising strong thermal  $SO(10)$ -inspired leptogenesis has not yet been found.

### 5. Two RH neutrino scenario and dark matter

Another popular scenario of leptogenesis is realised within a 2 RH neutrino model [20]. In this case the heaviest RH neutrino has a mass  $M_3 \gg 10^5$  GeV and it effectively decouples from the seesaw formula. In this case there is a lower bound  $M_1 \gtrsim 2 \times 10^{10}$  GeV from leptogenesis [21]. Also in this case there are regions in the parameters space, though more special, where the  $N_2$ -production is essential to get successful leptogenesis. Recently a realistic 2 RH neutrino scenario of leptogenesis has been shown to emerge within a  $A_4 \times SU(5)$  SUSY GUT model [22] and also within a  $\Delta(27) \times SO(10)$  model [23].

Intriguingly within a 2 RH neutrino seesaw scenario one can also consider the case when the third RH neutrino decouples from the seesaw formula not because it is very heavy but because its Yukawa coupling is very small. In the limit when this vanishes the RH neutrino becomes stable and can play the role of dark matter [24]. The difficulty is to find a plausible production



mechanism. A minimal way is to introduce a non-renormalizable Higgs portal-like operator  $(\lambda_{ij}/\Lambda) \phi \phi^\dagger N_i \bar{N}_j^c$ , where  $\Lambda$  is the scale of new physics (or a combination of more scales). The very interesting feature of this operator is that it can at the same time be responsible for the RH neutrino production through Landau-Zener non adiabatic resonant conversion, enhancing medium effects, and at the same time make the RH neutrino unstable. Interestingly a an allowed region exists such that both requirements of production and stability on cosmological scales can be satisfied and this region is for a mass of the DM RH neutrino in the range  $\text{EeV} > M_{DM} > \text{TeV}$ . At the same time the model necessarily predicts some contribution to the flux of very high energy neutrinos at IceCube [25]. The scenario is also compatible with leptogenesis in a two RH neutrino model, realising in this way a unified picture of leptogenesis and dark matter that will be tested in next years at IceCube.

## 6. Conclusions

Despite the absence of new physics at colliders, with neutrino physics and cosmology (and hopefully high energy neutrinos at Neutrino Telescopes) we might still have a chance in the next years to disclose the nature of the SM extension that is necessary in order to explain neutrino masses and the cosmological puzzles. High energy seesaw models embedded within GUT theories provide a very simple and attractive way in this respect.

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