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CONTROLLING INCIDENT SOUND FIELDS WITH SOURCE ARRAYS IN FREE SPACE AND THROUGH APERTURES

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The active control of an incident sound field with an array of secondary sources is a fundamental problem in active control. In this paper the optimal performance of an infinite array of secondary sources in controlling a plane incident sound wave is first considered in free space. An analytic solution for normal incidence plane waves is presented, indicating a clear cut-off frequency for good performance, when the separation distance between the uniformly-spaced sources is equal to a wavelength. This result is then compared with numerical simulations of controlling the sound power radiated through an open aperture in a rigid wall, subject to an incident plane wave and using an array of secondary sources in the aperture. In this case the diffraction through the aperture becomes important when the size of the window is compatible with the acoustic wavelength, in which case only a few sources are necessary for good control. When the size of the window is large compared to the wavelength, and diffraction is less important but more secondary sources need to be used for good control, the results become similar to those for the free field problem.

Keywords: Active sound control, open windows

1. Introduction

The active control of sound using arrays of secondary sources through open windows is a topic of current concern [1]. The main merit of implementing such systems is its ability to control mainly low frequency noise while retaining natural ventilation. Currently, implementation has been limited to a small array of sources and thus, small aperture sizes, due to increasing computational complexities [2], [3].

Apart from the technological issues, it is of interest to consider the physical constraints on the performance of such systems. At low frequencies, where the size of the window is not large compared with the acoustic wavelength, the diffraction of the incident sound through the open window, acting as an aperture, will be important in determining the physical performance of such a system. At higher frequencies however, when the size window is large compared with wavelength, the arrangement begins to approximate a free field problem. In this paper we begin by considering the free field problem of controlling the transmission of an incident plane wave passed an infinite array of secondary sources. This turns out to have an analytic solution, due to its similarity with the analogous problem of controlling plane waves in ducts. The case of controlling a normally incident plane wave with a line array of secondary sources is considered in Section 2, since the problem is the simplest to visualise in 2D and can also be readily compared with numerical simulations of active control through

2D openings [4]. The 3D case is shown to be a simple extension of the 2-D case. The case of controlling a plane wave after transmission through a finite-sized aperture is then considered in Section 3 using numerical simulations.

2. Control of plane waves with arrays of monopole is in free space

It is convenient to begin with the 2D case, with a line array of secondary sources, since this is the simplest to visualise. Consider the case in which a plane wave is normally incident on an infinite array of controllable line sources in free space, as shown in Figure 1. By symmetry, all sources must have the same volume velocity per unit length for the optimal control of the plane wave. Consider a line normal to the line of secondary sources at a midpoint between any two secondary sources. Since there is an equal array of sources of the same strength at the same distances either side of this line, the particle velocity parallel to the array of secondary sources must be zero. The incident primary field also has no particle velocity in this direction. The acoustic situation is thus unchanged if there were two rigid walls midway between any one secondary source and the two adjacent secondary sources, as illustrated in Figure 1. These walls could be considered to be those of an imaginary duct, so that the control problem over the infinite domain is reduced to that of the active control of the plane wave in an infinite rigid-wall duct with a single secondary source at its centre.

Assuming axes as defined in Figure 1, the downstream pressure in this duct can be written as

$$p(x,z)=\sum_{n=0}^{\infty} A_n \psi_n(x) e^{-jk_{z_n} z}, \tag{1}$$

where A_n is the amplitude of the nth mode propagating in the duct, ψ_n is its mode shape across the duct and k_{z_n} is its wave number, where a tonal field proportional to $e^{j\omega t}$ has been assumed.

For a rigid wall deduct the mode shapes are given by

$$\psi_0 = 1 \qquad \psi_n(x)=\cos(n\pi x/d), \tag{2}$$

with associated cut-off frequencies

$$f_n = \frac{nc_0}{2d}, \tag{3}$$

where c_0 is the speed of sound and d is the separation distance between the secondary sources. The transverse wave numbers, in the x direction, associated with these modes are

$$k_{x_n} = \frac{n\pi}{d}. \tag{4}$$

The axial wave number for the nth mode is then given by

$$k_{z_n} = \sqrt{k_0^2 - k_{x_n}^2}, \tag{5}$$

where k_0 is equal ω/c_0. If k_0 is greater than k_{z_n}, at a given frequency, $k_{z_n}$is real and this mode will propagate along the duct. If k_0 is smaller than k_{z_n}, at a given frequency, $k_{z_n}$is imaginary and this mode is evanescent and will decay along the length of the duct.

The amplitude of the zeroth order mode in the duct is equal to the incident plane wave and the contribution due to the secondary source

$$A_0 = p_p + \rho c_0 v_s, \tag{6}$$

where ρ is the density of the medium and v_s is a suitably normalised secondary source strength. The amplitude of the higher order modes in the duct are due only to the secondary source and these are given by [5].

$$A_n = \frac{2\omega\rho v_s}{k_{z_n}}, \quad n = 2, 4, 6, \cdots, \tag{7}$$

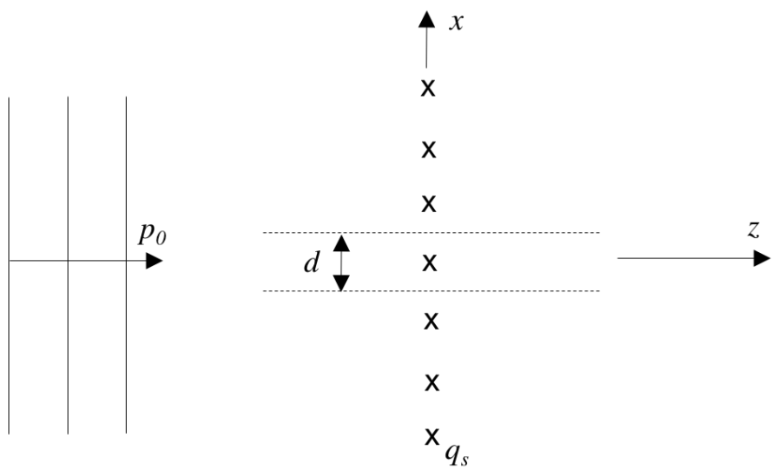


Figure 1: Active control of a normally incident plane wave by an infinite array of line sources of strength q_s in 2D. The dashed lines are the walls of an imaginary hard-walled duct with height equal to the source separation distance, d.

since only the even order modes are excited by the secondary source in this case, as it is exactly at the centre of the duct where the odd order mode shapes are zero. The pressure in the far field, i.e. as z tends to infinity, only has contributions from the modes that are propagating. The plane wave mode always propagates. At a given frequency, the number of higher-order propagating modes may be denoted as L, which varies with excitation frequency, as shown in Figure 2(a). Only the plane wave propagates up to a frequency of c_0/d and so $L = 0$. $L = 1$ for frequencies between c_0/d and $2c_0/d$, since only the $n = 2$ higher-order mode is excited and can propagate and $L = 2$ for frequencies from $2c_0/d$ to $3c_0/d$ since the $n = 4$ mode then also starts propagate.
The acoustic intensity in the duct can be written [5] in terms of the normalised secondary source strength and the complex pressure in the primary wave as

$$W = \frac{\rho c}{2}\left[\left|\frac{p_p}{\rho c} + v_s\right|^2 + 2\mathbb{R}\sum_{l=1}^{L}\frac{k_0}{k_{z_l}}\left|v_s\right|^2\right], \tag{8}$$

where $\mathbb{R}$ denotes the real part, to only include the propagating modes, and $l = 2n$ since since only the even order modes are excited by the secondary source at the centre of the duct. The dependence of the power on $1/k_{z_l}$, where k_{z_l} is zero at the cut-on frequency of the mode, would give rise to singularities in the power generated if v_s were finite, as illustrated in Fig 9.10 of Morse and Ingard [6] for example. A cost function, J, can be defined proportional to the acoustic intensity and written as

$$J = v_s^* A v_s + v_s^* b + b v_s + c, \tag{9}$$

where $A = 1 + 2\mathbb{R}\sum_{l=1}^{L} k_0/k_{z_l}$, $b = p_p/\rho c$, and $c = \left|p_p/\rho c\right|^2$.

The optimum source strength that minimises J is thus given by

$$v_{opt} = -A^{-1}b = \frac{-p_p/\rho c}{1 + 2\mathbb{R}\sum_{l=1}^{L}\frac{k_0}{k_{z_l}}}. \tag{10}$$

Figure 2(b) shows the variation of the real part of k_{z_n}, in Eq. (5), as a function of normalised frequency for n equal to 0, 2, 4, 6 and 8, and it can be seen from this that v_s, in Eq. (10), must take the form shown in Figure 2(c). The minimum value of the cost function is then given by

$$J(\min) = c - b^* A^{-1} b, \tag{11}$$

and the attenuation in dB is given by

$$Atten = 10\log_{10}\frac{J_{min}}{c}, \tag{12}$$

which is plotted in Figure 2(d).

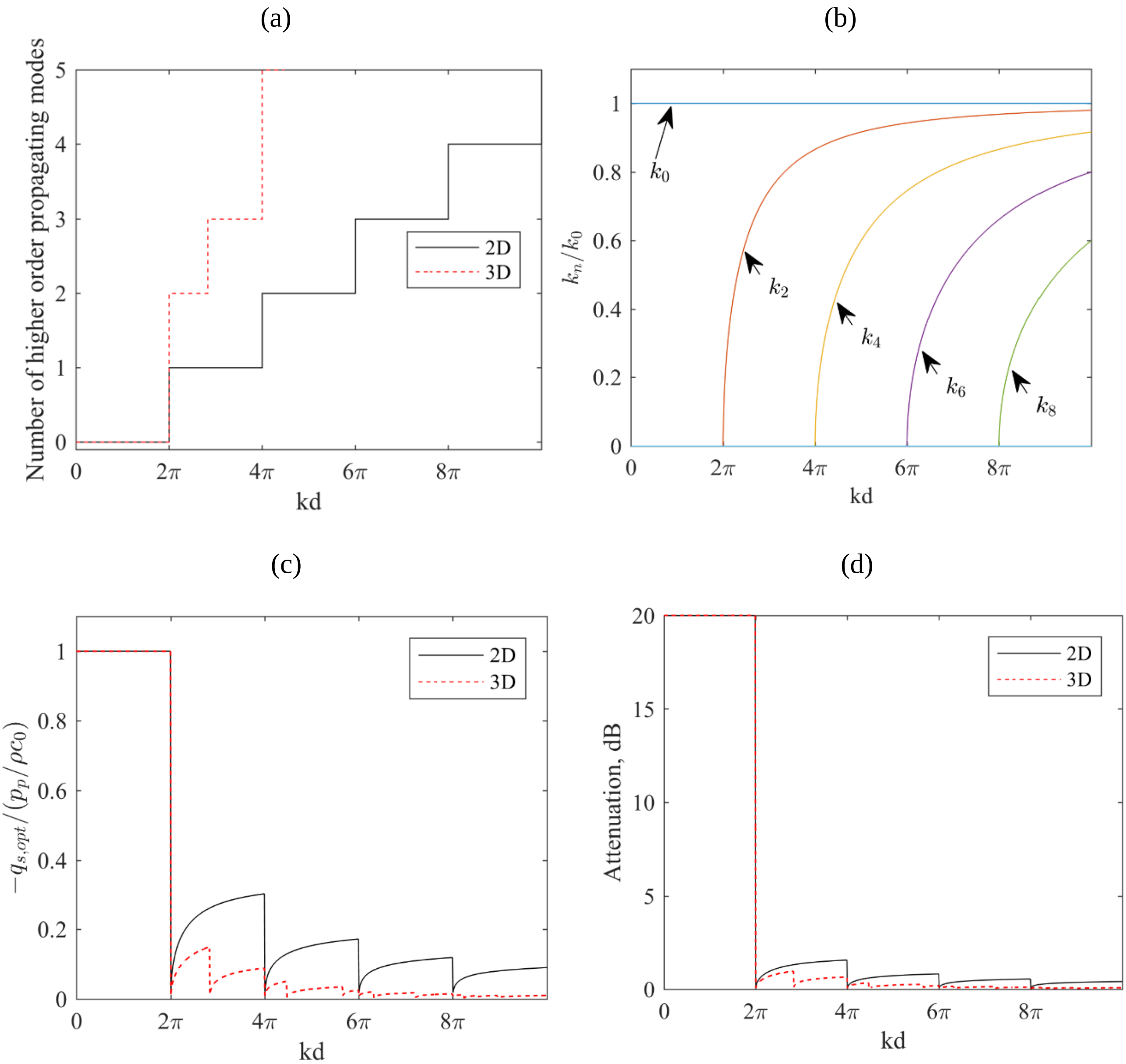


Figure 2: The number of higher-order propagating modes, L, (a) as a function of normalised frequency ($kd = \omega d / c_0$), together with the real parts of the wavenumber of the propagating modes (b), the optimum secondary source strengths (c) and the maximum attenuation in the far-field sound power (d). The solid line is for the 2D case considered in detail above and the dashed line is for the 3D case.

The attenuation is infinite, since the cancellation of the primary plane wave is perfect in this case, for frequencies below c_0/d . The secondary source strength goes to zero at the cut-on frequencies of each higher- order mode, however, since otherwise it would generate on infinite power output, as noted above and so the attenuation drops to zero at these frequencies. The attenuation rapidly falls off above the cut-off frequency $of\ c_0/d$.

A very similar analysis can be done in the case of a 2-D array of secondary sources controlling a normally incident wave in 3-D. In this case the imaginary duct used in the analysis has a square cross-section, with dimensions d by d. Instead of just the higher order modes in the x direction, in this case the corresponding modes in the y direction and the cross modes must also be taken into account, which significantly increases the number of higher-order modes that contribute to the sum over L in the expression for the optimum source strength, as shown by the dashed lines in Figure 2. In this case the wave number in the z direction for these modes is given by

$$k_{z_{nm}} = \sqrt{k_0^2 - k_{x_n}^2 - k_{y_m}^2}, \tag{13}$$

where k_{y_m} is equal to $m\pi/d$. The frequency above which perfect active control no longer is no longer achieved remains the same as in the 2-D case, however, at $f_c = c_0/d$, and the optimum source strength is given by a similar equation to (10), but taking the extra modes into account.

It is also possible to generalise this analysis to incident plane waves that are at an angle of θ to the secondary source array, in which case the frequency at which perfect far-field control is no longer achieved drops to

$$f_c = \frac{c_0}{d(1+\sin\theta)}. \tag{14}$$

3. Control of waves transmitted through a finite aperture

To analyse the physical limits of the open window active control, an analogous 2D representation depicted in Figure 3, is solved using 2D finite-element methods (FEM). The resolution of the simulation plane is set at one-sixth the wavelength of the highest frequency of interest, 4 kHz. Like the free-field cases, the primary noise to be controlled is a plane wave, but in this case it is transmitted through a finite aperture, of width w. It is travelling in the x-direction, at $\theta = 0°$ in this case. An array of N secondary line sources are symmetrically distributed $d = w/N$ apart, with the sources nearest to the edges being a distance of $d/2$ from the edge of the aperture.

The active control formulation in this case [7] can be written in terms of minimising a cost function equal to the sum of squared pressures at 1100 evaluation points on an arc 5 m away from the centre of the window, which is 2 m across, as shown in Figure 3. The cost function is now given by

$$J = \mathbf{e}^{\mathrm{H}}\mathbf{e} = \mathbf{q}_s^{\mathrm{H}}\mathbf{A}\mathbf{q}_s + \mathbf{q}_s^{\mathrm{H}}\mathbf{b} + \mathbf{b}^{\mathrm{H}}\mathbf{q}_s + \mathbf{d}^{\mathrm{H}}\mathbf{d}, \tag{15}$$

where $\mathbf{e} = \mathbf{d} + \mathbf{G}\mathbf{q}_s$, is the vector of complex pressures at the evaluation points, $\mathbf{d}$ representing the vector of disturbance signals due to the incident plane wave at these points, $\mathbf{G}$ is the matrix of plant responses, between the secondary sources and the pressures at the evaluation points, and $\mathbf{q}_s$ is the vector of secondary source strengths. Hence, after substitution, $\mathbf{A} = \mathbf{G}^{\mathrm{H}}\mathbf{G}$ and $\mathbf{b} = \mathbf{G}^{\mathrm{H}}\mathbf{d}$.

After equating the derivative of Eq. (15) to zero, the resulting optimal secondary source strengths are given by

$$\mathbf{q}_{opt} = -(\mathbf{G}^{\mathrm{H}}\mathbf{G} + \beta\mathbf{I})^{-1}\mathbf{G}^{\mathrm{H}}\mathbf{d}, \tag{16}$$

where β is a regularisation parameter [7]. The regularisation parameter is set at a suitable value to avoid the ill-conditioning of matrix $\mathbf{G}^{\mathrm{H}}\mathbf{G}$.

The resulting attenuations of the cost function, predicted from of the FEM simulations, are shown in Figure 4 for different numbers of secondary sources, N, in the plane of the aperture. The results are plotted as a function of the normalised frequency, both when it is normalised on the size of the window, kw in Figure 4(a), and when it is normalised on the separation distance between the sources, kd in Figure 4(b). The results in Figure 4(a) correspond to the practical case, in which the size of the window forming the aperture is fixed and the excitation frequency is increased. The performance clearly gets better as the number of secondary sources increases, as expected, but it is not clear whether this is entirely because these sources are closer together, or whether there is also an effect due to diffraction, which will be more important at low frequencies than at high frequencies. This question is resolved in Figure 4(b), which corresponds to the case in which the size of the aperture is increased as the number of sources gets larger, and is consistent with the representation used in Section 2. When $N = 1$, the source is placed in the centre of the aperture and d is set equal to $w/2$. It seen in Figure 4(b) that when the number of sources is large, the results approximate those for the infinite case, in Figure 2(d), with good levels of attenuation being achieved up to frequencies of $f = c_0/d$ ($kd = 2\pi$). With only a few sources the performance is not as great, indicating that diffraction has a significant effect, although there is still some sign of the cut-off frequency effect seen in the infinite case.

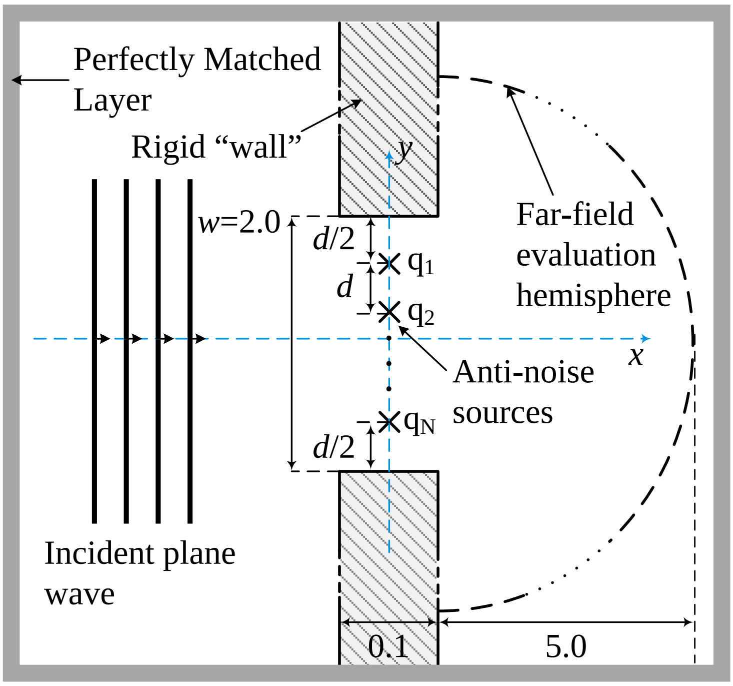
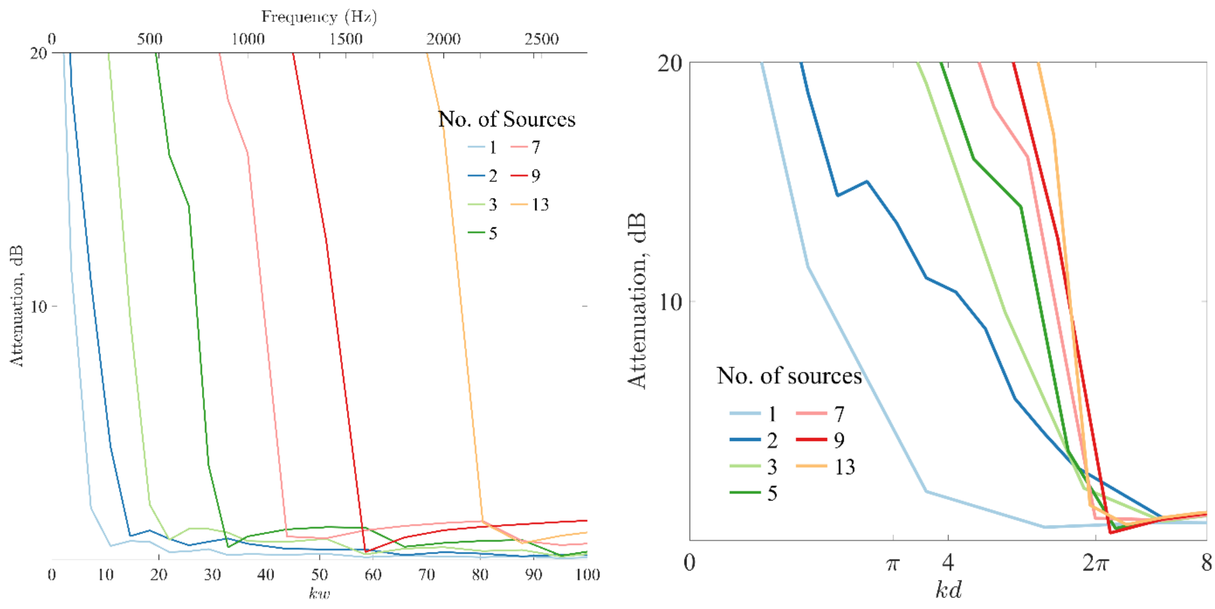


Figure 3: Finite element model geometry for the active control of a plane wave through a finite aperture with a finite linear array of secondary line sources in 2D.



(a) (b)

Figure 4: Attenuation of far-field pressure in the FE simulations with a normally-incident plane wave, plotted as a function of frequency, normalised both by the size of the aperture, kw in Figure 4(a), and when normalised by the separation between the sources, kd in Figure 4(b), for different numbers of secondary sources.

4. Discussion and conclusions

By comparison with an equivalent problem in a duct, a simple analysis can be performed of the active control of a normally-incident plane wave with an infinite array of secondary sources. It is found that perfect control of a normally-incident plane wave can be achieved in the far field, provided the separation between the uniformly-spaced secondary sources is less than the acoustic wavelength, i.e. at frequencies of less than $f = c_0/d$. This is found to be the case both for a 1D array of line sources in a 2D analysis, and also for a 2D array of monopoles in a 3D analysis.

A numerical simulation of active control is then performed for the active control of a plane wave after transmission through an aperture in a rigid wall, with an array of secondary sources in the aperture, which has practical applications for the reduction of sound through open windows. When there are many secondary sources, and the size of the aperture is large compared with the wavelength so that diffraction is not so important, significant attenuations in far field power are again found for frequencies up to $f = c_0/d$. For smaller numbers of sources, however, such that the size the aperture is not large compared with the wavelength and diffraction becomes more important, the attenuation is not as good as with a larger number of sources, even when plotted as a function of kd.

REFERENCES

1 B. Lam and W.-S. Gan, Active Acoustic Windows: Towards a Quieter Home, *IEEE Potentials*, **35** (1), 11–18, (2016).

2 T. Murao and M. Nishimura, Basic Study on Active Acoustic Shielding, *Journal of Environment and Engineering*, **7** (1), 76–91, (2012).

3 B. Kwon and Y. Park, Interior noise control with an active window system, *Applied Acoustics*, **74** (5), 647–652, (2013).

4 B. Lam, S. J. Elliott, J. Cheer, and W. Gan, The Physical Limits of Active Noise Control of Open Windows, in *12th Western Pacific Acoustics Conference*, (2015), 184–189.

5 P. E. Doak, Excitation, transmission and radiation of sound from source distributions in hard-walled ducts of finite length (I): The effects of duct cross-section geometry and source distribution space-time pattern, *Journal of Sound and Vibration*, **31** (1), 1–72, (1973).

6 P. M. Morse and K. U. Ingard, *Theoretical acoustics*. Princeton university press, (1968).

7 S. Elliott, *Signal Processing for Active Control*. Academic Press, (2001).