Price-based Online Mechanisms for Settings with Uncertain Future Procurement Costs and Multi-unit Demand

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ABSTRACT

We examine the use of online mechanism design in settings where consumers have multi-unit demand, goods are procured and allocated over time, and future procurement costs are uncertain and only become known at the time of allocation. An important application with such characteristics is demand response, where electricity wholesale prices depend on overall demand and the availability of renewables. We formulate this as a mechanism design problem and focus specifically on the property that the mechanism does not revoke any allocated items. In this setting, we characterise a class of price-based mechanisms that guarantee dominant-strategy incentive compatibility, individual rationality, and no cancellation. We present three specific such mechanisms in this domain and evaluate them in an electric vehicle charging setting. By using extensive numerical simulations, we show that a mechanism based on the first-come first-served principle performs well in settings where future procurement costs can be estimated reliably or supply is very tight, while a responsive mechanism performs very well when the estimated procurement costs are highly uncertain and supply is not as tight. We moreover show that a well-defined price-based mechanism can lead to high profits for the operator of the mechanism in many real-world situations.

KEYWORDS

online mechanism design; demand response; EV charging

ACM Reference Format:
Keiichiro Hayakawa, Enrico H. Gerding, Sebastian Stein, and Takahiro Shiga. 2018. Price-based Online Mechanisms for Settings with Uncertain Future Procurement Costs and Multi-unit Demand. In *Proc. of the 17th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2018), Stockholm, Sweden, July 10–15, 2018,* IFAAMAS, 9 pages.

1 INTRODUCTION

In many settings, consumers wish to acquire resources at specific times in the future, such as when booking hotels or airline tickets, participating in demand response schemes in the Smart Grid, or reserving cloud computing space. Because of uncertain future demand and fluctuating wholesale prices, current systems for booking such resources typically employ dynamic pricing schemes. For example, revenue management systems continuously adjust prices and capacities to maximise the seller's revenue [2, 3, 14], while day-ahead, hour-ahead, and real-time electricity markets allow the trading of electricity at varying timescales in the future. However, a key shortcoming of these systems is that they require consumers to speculate about the best time to buy resources.

To address this shortcoming, we propose using online mechanism design, which aims to devise mechanisms that are dominant strategy incentive compatible (DSIC). Under such mechanisms, consumers maximise their utility by truthfully revealing their preferences. Their advantage is twofold: they both negate the need for speculation and enable the more efficient procurement and allocation of resources, as consumers are incentivised to promptly and accurately reveal their private preferences. In addition to DSIC, we are interested in ensuring individual rationality (IR), which holds when consumers do not make a loss by participating in the mechanism. Depending on the application, we may also require that allocations made cannot be revoked. In this study, we define this property as *no cancellation*, or NC for short.

A considerable strand of the literature has proposed auction-based systems to ensure DSIC [4, 9], including for settings with complex preferences [1]. However, these studies take place in a one-shot setting. By contrast, Porter [12], Hajiaghayi [7], and Parkes [10] considered online settings in which consumers arrive dynamically over time. They proposed an online DSIC mechanism that does not need any prior knowledge of future arrivals, which is the so-called 'model-free' property. In addition to DSIC and IR, this mechanism considers weak budget balance (WBB), which ensures that the mechanism does not run a deficit. To keep DSIC, IR, and WBB, it first defines a (weakly) monotonic allocation and then uses a critical-value payment. In this study, we call this type of common mechanism a *critical-value-based mechanism*.

Generally, in model-free online settings, no allocation can be optimal; hence, existing work is typically interested in the *efficiency* of the mechanism, defined as the proportion of social welfare achieved compared with the offline optimal. For example, in settings with single-minded bidders, Parkes [10] proposed a critical-value-based online mechanism that is DSIC, IR, and WBB and has an efficiency of $1/2$. By contrast, Gerding et al. [5] proposed a critical-value-based online mechanism by considering the multidimensional preferences of agents, assuming non-increasing marginal valuations for agents.

Proc. of the 17th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2018), M. Dastani, G. Sukthankar, E. André, S. Koenig (eds.), July 10–15, 2018, Stockholm, Sweden.

Owing to the nature of critical-value-based mechanisms, their proposed mechanism is DSIC, IR, and WBB. However, in settings with multidimensional preferences, such mechanisms require the cancellation of certain allocations that have already been made in order to achieve good efficiency. Indeed, Robu et al. [13] showed that the efficiency of an NC variant of this mechanism is $1/C$, while a variant with cancellation achieves an efficiency of $1/2$.

By contrast, Hayakawa et al. [8] proposed a price-based NC mechanism that first defines the price and then determines the allocation. Unlike critical-value-based mechanisms, it is not WBB. Moreover, simple price-based online mechanisms are not completely general because certain allocations are not possible [10]. However, Hayakawa et al. [8] empirically showed that a well-defined price-based mechanism satisfies both good *efficiency* and high profit in certain real-world situations (e.g. when procurement costs vary).

However, a wide range of options are available to determine the allocation and payment policy of price-based mechanisms and the performance of the mechanism greatly depends on the choice of these policies. To design mechanisms in complicated procurement settings in the real world, we consider settings with uncertain future costs and unlimited supply (but increasing marginal costs). We assume that resources are perishable (i.e. once procured, they have to be allocated immediately). We also assume that consumers arrive over time and wish to procure one or more units of a good, with arbitrary marginal valuations of the quantity of this good (including complementarities). Further, consumers have constraints on how the resource can be consumed, including a consumption interval, consumption rate, and deadline by when an allocation needs to be confirmed by the mechanism. No existing work in the field of mechanism design has thus far simultaneously considered these challenges.

A key motivating example of this setting is an electric vehicle (EV) charging mechanism operated by a demand response aggregator [6]. Considering this, in this study, we make the following novel contributions:

- We characterise a class of price-based online mechanisms that are DSIC, IR, and NC in settings with uncertain procurement costs and multi-unit demand.
- We introduce three novel mechanisms within this class, two of which also satisfy WBB.
- In real-world situations in which price-based mechanisms perform much better than common critical-value-based mechanisms, we empirically evaluate the performance of the proposed mechanisms in various settings of supply tightness and the accuracy of estimated future costs.

2 SYSTEM OVERVIEW

In this section, we introduce a general model of consumers and of the procurement process as well as the theoretical properties we would like the mechanism to satisfy. We assume that decisions are made at discrete time steps $t \in T$, where T is the set of all time steps.

2.1 Consumer Model

We refer to consumers as agents and use I to denote the set of all agents. Agents arrive over time and, for each agent, we distinguish between the time when they become aware of their demand and when they can actually receive the resources. Formally, $a_i \in T$ denotes the time when agent $i \in I$ realises its demand. Furthermore, we consider a deadline $d_i \geq a_i \in T$ by which the agent must know the total number of units to be allocated and the payment. In addition, i has a time interval $[\alpha_i, \beta_i] \subset T$ in which the resources can actually be allocated (see Figure 1). Note that existing online mechanisms (e.g. [5, 10, 11]) assume that $a_i = \alpha_i$ and $d_i = \beta_i$, meaning that this model is more general. In particular, when $d_i < \alpha_i$, it captures settings with advance reservations. For example, in the EV charging case, although agents can only be charged when they are plugged in, they may know their demand and book the use of a charging station in advance.

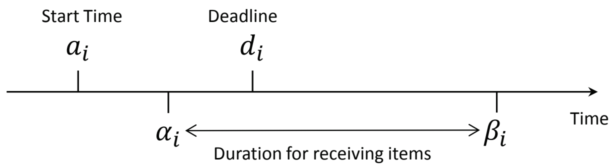


Figure 1: Agents' time preferences.

Agents are interested in multiple units of the same resource, and we use $\boldsymbol{v_i} = \{v_{i,0}, v_{i,1}, \ldots\}$ to denote the valuations of agent i, where $v_{i,k}$ is the *total* value when k units are allocated between α_i and β_i. A mild assumption is that preferences are monotonic (a.k.a. free disposal), namely $\forall k' \leq k : v_{i,k'} \leq v_{i,k}$. Note that unlike Gerding et al. [5], this approach allows for increasing marginal valuations. W.l.o.g. we set $\forall i : v_{i,0} = 0$. Agents may have restrictions on the maximum number of resources that can be allocated in each time step, which we denote by r_i. We assume that this is bounded by r_{max}, i.e. $\forall i : r_i \leq r_{max}$, and that r_{max} is known to the mechanism.

Given the above, the *type* of agent i is denoted by

$$\theta_i = \{a_i, d_i, \alpha_i, \beta_i, r_i, \boldsymbol{v_i}\}.$$

We consider agents to be strategic and able to misreport their type if this increases their utility. We use $\hat{\theta}_i = \{\hat{a}_i, \hat{d}_i, \hat{\alpha}_i, \hat{\beta}_i, \hat{r}_i, \hat{\boldsymbol{v}}_{\boldsymbol{i}}\}$ to denote the report of agent i. Note that the mechanism only becomes aware of agent i at time $\hat{a}_i$. We use θ to denote the types of all agents and θ_{-i} to denote the types of all agents except i. Similarly, $\hat{\theta}$ is the reported types of all agents and $\hat{\theta}^{\langle t \rangle}$ is the reported types of all agents $i \in I : \hat{a}_i \leq t$.

As is standard in online mechanism design and natural in many applications, we assume limited misreports [10], namely $\hat{a}_i \geq a_i$, $\hat{d}_i \leq d_i$, $\hat{\alpha}_i \geq \alpha_i$, $\hat{\beta}_i \leq \beta_i$, and $\hat{r}_i \leq r_i$. That is, an agent can only report more restrictive preferences (except for the valuations). For example, an agent cannot report a_i before it is aware of its own preferences; however, it can easily delay its report. In addition, we make the natural assumption that $\hat{\alpha}_i \geq \hat{a}_i$ and $\hat{\beta}_i \geq \hat{d}_i$. Finally, we assume that each agent has a maximum number of units that it can receive, Q_i, which is known to the mechanism. For example, in the EV charging case, Q_i is the remaining capacity of the battery, and the mechanism can detect this when it is plugged into the charging equipment.

2.2 Procurement and Assignment Model

The mechanism has to procure resources from an external market. We use $c(t,k)$ to denote the *marginal procurement cost* of the kth unit at time t. We assume that the supply of the external market is infinite, although the marginal costs can be arbitrarily high. We also assume that the mechanism knows the actual cost $c(t,k)$ right before it allocates items to agents at time t, but does not know these in advance. However, it knows the maximum marginal cost $c_{max}(t,k)$ and expected cost $\tilde{c}(t,k)$ in advance.

The mechanism computes, for each agent $i \in I$, a price, $P_{i,k} = P_i(k, \hat{\theta}^{\langle \hat{d}_i \rangle})$, which depends on the allocation (i.e. the number of units received) k, based on the information available at the reported deadline $\hat{d}_i$.

At each time point, the mechanism decides and renews the allocation schedule for all agents. Let $y^{\langle t \rangle}_{i,t'} = y^{\langle t \rangle}_{i,t'}(\hat{\theta}^{\langle t \rangle})$ denote the scheduling function, which specifies the total number of units allocated to agent i by time t' when the current time is t. This function makes (potentially temporary) allocation decisions for future time points $t' > t$, given the information available at time t. In addition, we denote by $y_{i,t} = y^{\langle t \rangle}_{i,t}$ the number of units actually allocated to agent i by time t. Obviously, the scheduling algorithm cannot change past allocations, and thus

$$\forall t, t' > t : y^{\langle t' \rangle}_{i,t} = y^{\langle t \rangle}_{i,t} = y_{i,t}. \tag{1}$$

The total number of units allocated to agent i is fixed by the reported deadline $\hat{d}_i$, meaning we have

$$\forall t \geq \hat{d}_i : y^{\langle t \rangle}_{i,\hat{\beta}_i} = y^{\langle \hat{d}_i \rangle}_{i,\hat{\beta}_i} = y_{i,\hat{\beta}_i}. \tag{2}$$

However, the allocation schedule (i.e. the time at which the mechanism will allocate the items to each agent) can be flexible because agents are only interested in the final allocation at the end of the reported time interval $\hat{\beta}_i$. We denote by Y_i the final allocation of agent i, such that

$$Y_i = y_{i,\hat{\beta}_i}. \tag{3}$$

Given the foregoing, we use $\Delta y_{i,t}$ to denote the number of units actually allocated to i at time t, such that

$$\Delta y_{i,t} = \begin{cases} y_{i,t} - y_{i,t-1} & \text{if} \quad t > \alpha_i \\ y_{i,\alpha_i} & \text{if} \quad t = \alpha_i. \end{cases} \tag{4}$$

The number of units m_t allocated to all agents at time t is

$$m_t = \sum_{i \in I} \Delta y_{i,t}. \tag{5}$$

Thus, the procurement cost at time t is $\sum_{k=1}^{m_t} c(t,k)$.

2.3 Mechanism Properties

Given the models above, the utility u_i of agent i is $u_i(\hat{\theta}^{\langle \hat{d}_i \rangle}) = v_{i,Y_i} - P_{i,Y_i}$. Furthermore, the revenue R of the mechanism is $R(\hat{\theta}) = \sum_{i \in I} P_{i,Y_i} - \sum_{t \in T} \sum_{k=1}^{m_t} c(t,k)$. We define the total social welfare $SW(\hat{\theta})$ as the sum of the utilities of all agents and the revenue of the mechanism:

$$SW(\hat{\theta}) = \sum_{i \in I} u_i(\hat{\theta}^{\langle \hat{d}_i \rangle}) + R(\hat{\theta}) = \sum_{i \in I} v_{i,Y_i} - \sum_{t \in T} \sum_{k=1}^{m_t} c(t,k).$$

Now, we are interested in the following properties. A mechanism is said to be DSIC if $\forall \theta_i, \hat{\theta}_i, \hat{\theta}_{-i} : u_i(\{\theta_i, \hat{\theta}_{-i}\}) \geq u_i(\{\hat{\theta}_i, \hat{\theta}_{-i}\})$. Furthermore, the mechanism is IR if $\forall \theta_i : u_i(\{\theta_i, \hat{\theta}_{-i}\}) \geq 0$. The mechanism is *ex-post* WBB if $\forall \hat{\theta} : R(\hat{\theta}) \geq 0$, and it is *ex-interim* WBB if R is computed using the expected costs. Finally, we state that the mechanism satisfies the property of no cancellation (NC) if

$$\forall i, \forall t; \hat{\alpha}_i < t \leq \hat{\beta}_i : y_{i,t} \geq y_{i,t-1}. \tag{6}$$

This is important because in most real-world scenarios, cancelling (i.e. removing) previously allocated units is impossible or impractical.

3 MECHANISM CHARACTERISATION

Here, we characterise a general class of mechanisms that satisfy DSIC, IR, and NC in the given setting. As shown in the previous section, the mechanism consists of a pricing function $P_{i,k}$ and scheduling function $y^{\langle t \rangle}_{i,t'}$. In this section, we define a price-based DSIC (and IR) mechanism and discuss the properties that the pricing function should satisfy. We then show a set of constraints for the scheduling function through which the mechanism is guaranteed to satisfy NC.

3.1 Price-Based Online DSIC Mechanism

To extend the simple price-based online mechanism for single-minded bidders [10] to our setting, we first define the *monotonicity* of pricing functions as follows.

Definition 3.1. A pricing function is monotonic if it is (weakly) monotonically increasing over $\hat{a}_i$ and $\hat{\alpha}_i$, and (weakly) monotonically decreasing over $\hat{d}_i$, $\hat{\beta}_i$ and $\hat{r}_i$.

We define the price-based online DSIC mechanism for the multi-minded domain as follows.

Definition 3.2. A mechanism is a price-based DSIC online mechanism for multi-minded bidders if

1. The mechanism decides the price $P_{i,k}$ for every possible allocation k to agent i by adopting a pricing function that is monotonic and independent of $\hat{\boldsymbol{v}}_i$.
2. The final allocation Y_i for agent i coincides with k such that the value of $\hat{v}_{i,k} - P_{i,k}$ is maximised (over all k that can be allocated to i for any choice of $\hat{\boldsymbol{v}}_i$).

Then, we have the following theorem.

THEOREM 1. *A price-based online mechanism defined by Definition 3.2 is DSIC.*

PROOF. The proof is shown in Appendix A. □

In addition to being DSIC, assuming $P_{i,0} = 0$, the mechanism is IR because the payment of i cannot exceed its valuation. This definition is quite general and common critical-value-based mechanisms are also included in this domain.

3.2 Constraints of the Scheduling Function for NC Mechanisms

Definition 3.2 only considers the prices at the (reported) deadline and corresponding final allocations. However, allocations are generally made over time. To this end, we describe a framework for designing mechanisms that neither under- nor overallocate by the end of the reported time interval, where the allocations already made are never revoked.

First, we define an NCF (no cancellation and feasible) schedule as follows.

Definition 3.3 (NCF schedule). A schedule for i, determined at time t, satisfies NCF if

$$\forall t'; \hat{\alpha}_i < t' \leq \hat{\beta}_i : 0 \leq y^{\langle t \rangle}_{i,t'} - y^{\langle t \rangle}_{i,t'-1} \leq \hat{r}_i. \tag{7}$$

LEMMA 1. *If the schedules for all agents at every time step satisfy NCF, the mechanism with such a scheduling function satisfies NC.*

PROOF. By definition, at the deadline d_i of agent i, this ensures that $\forall t'; \hat{\alpha}_i < t' \leq \hat{\beta}_i : 0 \leq y^{\langle \hat{d}_i \rangle}_{i,t'} - y^{\langle \hat{d}_i \rangle}_{i,t'-1}$, and thus $\forall i, \forall t; \hat{\alpha}_i < t \leq \hat{\beta}_i : y_{i,t} \geq y_{i,t-1}$ holds. □

To ensure this, we discuss a set of constraints for the scheduling function $y^{\langle t \rangle}_{i,t}$. First, we introduce an upper-bound function $h^{\langle t \rangle}_i$, which satisfies

$$y^{\langle t \rangle}_{i,t} \leq h^{\langle t \rangle}_i, \tag{8}$$

for all i and t. Given this, we focus on the *upper limit number* of units $h^{\langle t \rangle}_i$ and *temporarily assigned number* of units $y^{\langle t \rangle}_{i,\hat{\beta}_i}$, and call these *u-number* and *t-number*, respectively. Here, *u-number* $h^{\langle t \rangle}_i$ expresses the maximum number of units assigned to agent i until time t, whereas *t-number* $y^{\langle t \rangle}_{i,\hat{\beta}_i}$ expresses the number of units (temporarily) assigned to agent i during its available time interval, determined by the schedule at time t. Given these numbers, the allocation schedule $y^{\langle t \rangle}_{i,t}$ to agent i is determined. To avoid revoking the allocation, *u-number* $h^{\langle t \rangle}_i$ should be (weakly) increasing over time, that is,

$$\forall t : h^{\langle t+1 \rangle}_i \geq h^{\langle t \rangle}_i, \tag{9}$$

because otherwise the allocation at time t is cancelled if $y^{\langle t \rangle}_{i,t} = h^{\langle t \rangle}_i$. Then, to avoid under- or overallocation,

$$\forall t : 0 \leq y^{\langle t \rangle}_{i,\hat{\beta}_i} - h^{\langle t \rangle}_i \leq (\hat{\beta}_i - t)\hat{r}_i \tag{10}$$

should hold. Here, the difference $y^{\langle t \rangle}_{i,\hat{\beta}_i} - h^{\langle t \rangle}_i$ between *t-number* and *u-number* represents the minimum number of units that the mechanism should allocate between t and $\hat{\beta}_i$ based on the schedule (temporarily) decided at time t. Hence, it can neither be negative nor exceed the number achievable by the maximum rate $\hat{r}_i$. Then, we use Π_i to denote the maximum number of units that agent i could receive, such that, $\Pi_i = \min\left\{Q_i, (\hat{\beta}_i - \hat{\alpha}_i + 1)\hat{r}_i\right\}$. Given this,

$$\forall t : y^{\langle t \rangle}_{i,\hat{\beta}_i} \leq \Pi_i, \tag{11}$$

holds. We then define a cut-off time δ_i such that

$$\delta_i = \beta_i - \lceil \frac{\Pi_i}{\hat{r}_i} \rceil + 1. \tag{12}$$

If $t < \delta_i$, the mechanism can allocate a maximum of Π_i units to the agent before the end of the reported time interval, $\hat{\beta}_i$, because $\hat{r}_i(\hat{\beta}_i - t + 1) \geq \Pi_i$. However, if $t \geq \delta_i$, the mechanism may be unable to allocate Π_i, and thus we introduce the following constraint:

$$\forall t \geq \delta_i : y^{\langle t+1 \rangle}_{i,\hat{\beta}_i} \leq y^{\langle t \rangle}_{i,\hat{\beta}_i}. \tag{13}$$

This means that *t-number* $y^{\langle t \rangle}_{i,\hat{\beta}_i}$ is (weakly) decreasing over time after the cut-off time δ_i.

Summarising the above constraints, we define the term *YH constraints* as follows.

Definition 3.4 (YH constraints). A scheduling function $y^{\langle t \rangle}_{i,t'}$ satisfies the YH constraints if it satisfies Eqs. 8, 9, 10, 11, and 13 with an upper-bound function $h^{\langle t \rangle}_i$.

We then have the following result.

LEMMA 2. *Given unlimited supply, if a schedule that is NCF at time $t-1$ satisfies the YH constraints with an upper-bound function $h^{\langle t \rangle}_i$, there always exists an NCF schedule at time t.*

PROOF. The proof is shown in Appendix B. □

LEMMA 3. *Given unlimited supply, there always exists a scheduling function that is NCF at time $\hat{a}_i$ and that satisfies the YH constraints with an upper-bound function $h^{\langle t \rangle}_i$.*

PROOF. This can be achieved by setting $y^{\langle \hat{a}_i \rangle}_{i,\hat{\beta}_i} = h^{\langle \hat{\beta}_i \rangle}_i$ and $y^{\langle \hat{a}_i \rangle}_{i,\hat{a}_i} = \min(\hat{r}_i, h^{\langle \hat{a}_i \rangle}_i)$, with an arbitrary upper-bound function $h^{\langle t \rangle}_i$ that satisfies Eq. 9 and $h^{\langle \hat{\beta}_i \rangle}_i \leq \Pi_i$. □

We now have the following result.

THEOREM 2. *Given unlimited supply, an NCF schedule that satisfies the YH constraints with an upper-bound function always exists and an allocation mechanism with such a scheduling function satisfies NC.*

PROOF. This theorem is proven by Lemmas 1, 2, and 3. □

Thus, it is possible to design an NC mechanism by using the *YH constraints*.

3.3 Price-Based Online DSIC and NC Mechanism

We now have a price-based online DSIC and NC mechanism for our setting, as defined below.

Definition 3.5. A mechanism is a price-based online DSIC and NC mechanism for our setting if

1. The mechanism decides the price $P_{i,k}$ for every possible allocation k to agent i by adopting a pricing function that is monotonic and independent of $\hat{\boldsymbol{v}}_i$.

2 At each time, the mechanism makes an allocation to agents by adopting a scheduling function $y^{\langle t \rangle}_{i,t'}$ that satisfies the YH constraints with an upper-bound function $h^{\langle t \rangle}_i$.

3. The final allocation Y_i for agent i coincides with k such that the value of $\hat{v}_{i,k} - P_{i,k}$ is maximised (over all k that can be allocated to i for any choice of $\hat{\boldsymbol{v}}_i$).

Then, we have the following result.

THEOREM 3. *A price-based online mechanism defined by Definition 3.5 satisfies DSIC and NC.*

PROOF. The theorem is proven by Theorems 1 and 2. □

In our setting with uncertain marginal procurement costs and multidimensional preferences, there is no model-free allocation mechanism with a bounded competitive ratio (see Appendix C). In the following part of this paper, we therefore consider a real-world situation in which our proposed price-based mechanism is used effectively and show its properties as well as the results of quantitative evaluations.

4 MECHANISMS

We now consider several specific mechanisms within the general class of price-based online DSIC and NC mechanisms discussed in the previous section. These offer various trade-offs between achieving WBB and high efficiency in practice (as we explore empirically in the next section).

4.1 First-Come First-Served Mechanism

Our first mechanism is based on the well-known first-come first-served (*FCFS*) principle. This mechanism produces an optimal allocation schedule for each agent in the order of their arrival while keeping existing schedules fixed, and the prices are based on the marginal costs given existing commitments. We consider two variants: one using the maximum cost, $c_{max}(t, m)$, and the other using the expected cost, $\tilde{c}(t, m)$, referred to as *FCFS(Max)* and *FCFS(Est)*, respectively. In more detail, suppose that, when i arrives, $k_{-i,t}^{\langle \hat{a}_i \rangle}$ units have already been allocated to preceding agents at some future point t. Then, the price for i of the μth unit at time t is given by $\rho_1 \cdot c_{max}(t, k_{-i,t}^{\langle \hat{a}_i \rangle} + \mu)$ and $\rho_1 \cdot \tilde{c}(t, k_{-i,t}^{\langle \hat{a}_i \rangle} + \mu)$, respectively for all μ such that $1 \leq \mu \leq \hat{r}_i$. Here, ρ_1 is a parameter that can be set by the mechanism.

The marginal price $p_{i,l}^{\langle \hat{a}_i \rangle}$ is then obtained by sorting all the prices during the period $t \in [\hat{\alpha}_i, \hat{\beta}_i]$ in ascending order. The prices (and hence the allocation schedule) are set on arrival and remain unchanged. Note that the prices obtained in this way do not depend on $\hat{\boldsymbol{v}}_{\boldsymbol{i}}$ and are *monotonic*. Given these prices, the scheduling function $y_{i,t}$ is set to maximise the value of $\hat{v}_{i,k} - P_{i,k}$. Defining the upper-bound function that coincides with the scheduling function, namely $h_i^{\langle t \rangle} = y_{i,t}$, the FCFS mechanism satisfies the condition of Theorem 3, and thus it is DSIC and NC. In addition, if $\rho_1 \geq 1$, *FCFS(Max)* is *ex-post* WBB and *FCFS(Est)* is *ex-interim* WBB.

4.2 Responsive Mechanisms

We now introduce mechanisms that unlike *FCFS* respond to the actual costs $c(t, m)$ and agents arriving by adapting to future prices. At each time t, the mechanism determines a price $p_{i,l}^{\langle t \rangle}$ for all agents using the information given by $c(t, m)$ at the current time and $\tilde{c}(t, m)$ or $c_{max}(t, m)$ in the future. Here, $p_{i,l}^{\langle t \rangle}$ denotes the provisional marginal price of the lth unit for agent i at time t, and $p_{i,0}^{\langle t \rangle} = 0$. The provisional price at time t for agent i for receiving k units until the end of the reported time interval, β_i, is described as $P_{i,k}^{\langle t \rangle} = \sum_{l=0}^{k} p_{i,l}^{\langle t \rangle}$. We describe two specific ways of determining the marginal prices in Sections 4.2.1 and 4.2.2.

By using these provisional prices, the upper-bound function $h_i^{\langle t \rangle}$ is defined as follows:

$$h_i^{\langle t \rangle} = \min\{\underset{k \leq \Pi_i}{\operatorname{argmax}}\{\hat{v}_{i,k} - P_{i,k}^{\langle \min(t, \hat{d}_i) \rangle}\}\}. \tag{14}$$

Given this upper-bound function $h_i^{\langle t \rangle}$, at each time, the mechanism myopically decides the allocation schedule that maximises total social welfare (including future allocations), while satisfying the *YH constraints* with $h_i^{\langle t \rangle}$. To keep Eq. 9, which is a part of the *YH constraints*, the price $P_{i,k}^{\langle t \rangle}$ has to be (monotonically) decreasing over time. Hence, if the provisional marginal price $p_{i,l}^{\langle t \rangle}$ is set independent of $\hat{\boldsymbol{v}}_{\boldsymbol{i}}$, is *monotonic*, and is (monotonically) decreasing over time, the mechanism is DSIC, IR, and NC because of Theorem 3. In the following subsection, we show two pricing algorithms that satisfy these conditions, namely *Count* and *PayEX*.

4.2.1 Count All Agents (Count).

At each time t, this pricing algorithm calculates the maximum number of units $n_{t'}^{\langle t \rangle}$ that can be allocated at time t', given the information available at time t not considering the report of i, as follows: $n_{i,t'}^{\langle t \rangle} = \sum_{j \in I_{-i,t'}^{\langle t \rangle}} \hat{r}_j + r_{max}$. Here, $I_{-i,t'}^{\langle t \rangle}$ denotes the set of agents available at time t' based on the information available at time t, excluding agent i. Namely,

$$\forall j \in I_{-i,t'}^{\langle t \rangle} : j \neq i, \hat{\alpha}_j \leq t', \hat{\beta}_j \geq t', \hat{a}_i \leq t. \tag{15}$$

Using this, the virtual cost ${c'}_{i,t'}^{\langle t \rangle}$ is defined as follows:

$${c'}_{i,t'}^{\langle t \rangle} = \begin{cases} c(t', \lfloor \rho_2 \cdot n_{i,t'}^{\langle t \rangle} \rfloor) & (t' \leq t) \\ c_{max}(t', \lfloor \rho_2 \cdot n_{i,t'}^{\langle t \rangle} \rfloor). & (t' > t) \end{cases} \tag{16}$$

Here, parameter $\rho_2 \leq 1$ can be set by the mechanism. Then, the cost $c_{i,t'}^{\langle t \rangle}$ is set for each agent as follows:

$$c_{i,t'}^{\langle t \rangle} = \begin{cases} {c'}_{i,t'}^{\langle t \rangle} & (t = \hat{a}_i) \\ \min\{c_{i,t'}^{\langle t-1 \rangle}, {c'}_{i,t'}^{\langle t \rangle}\}. & (otherwise) \end{cases} \tag{17}$$

If $t \leq \delta_i$, the marginal price $p_{i,l}^{\langle t \rangle}$ is obtained by sorting all $c_{i,t'}^{\langle t \rangle}$ during the period $t \in [\hat{\alpha}_i, \hat{\beta}_i]$ in ascending order, and if $t > \delta_i$, $p_{i,l}^{\langle t \rangle} = p_{i,l}^{\langle t-1 \rangle}$.

This pricing function is (monotonically) decreasing over time. It also does not depend on $\hat{\boldsymbol{v}}_{\boldsymbol{i}}$ and is *monotonic*. Hence, a responsive mechanism using this pricing algorithm is DSIC, IR, and NC. It also satisfies ex-post WBB if $\rho_2 = 1$.

4.2.2 Pay Externality of the Virtual Market (PayEX).

This pricing algorithm calculates agent i's price based on a virtual market M_{-i}, which considers only the reports $\hat{\theta}_{-i}^{\langle \hat{d}_i \rangle}$ of all other agents except i. The virtual market is rerun from the beginning of T. Considering M_{-i}, we myopically find the schedule that optimises the total social welfare of M_{-i} and allocate units to the virtual agents in M_{-i}. This is repeated at each time step, using information about new arrivals; however, allocations before the current time t remain fixed. We use $SW(t, M_{-i})$ to denote the achieved social welfare at time t. Then, to compute the price of agent i, we recalculate the optimisation with the constraint that agent i obtains k units during $[\hat{\alpha}_i, \hat{\beta}_i]$. Let $SW_k(t, M_{-i})$ denote the achieved social welfare, excluding agent i's value. Given this, denote the externality $EX_i(t, k)$

of agent i as $EX_i(t, k) = SW(t, M_{-i}) - SW_k(t, M_{-i})$. Then, the provisional marginal price $p_{i,l}^{\langle t \rangle}$ of agent i is determined as follows:

$$p_{i,l}^{\langle t \rangle} = \begin{cases} \rho_3 \cdot \{EX_i(t,l) - EX_i(t,l-1)\} & (t = \hat{a}_i) \\ p_{i,l}^{\langle t-1 \rangle} & (t > \delta_i) \\ \min\{\rho_3 \cdot \{EX_i(t,l) - EX_i(t,l-1)\}, p_{i,l}^{\langle t-1 \rangle}\} & (otherwise) \end{cases}$$

Here, ρ_3 can again be set by the mechanism.

This pricing algorithm is independent of $\hat{\boldsymbol{v}}_i$, is *monotonic*, and is (monotonically) decreasing over time. Therefore, a responsive mechanism using this pricing algorithm is also DSIC, IR, and NC. However, unlike the two previous mechanisms, *FCFS* and *Count*, it does not guarantee *ex-interim* WBB (i.e. the revenue of the mechanism can become negative).

4.3 Theoretical Bounds on Social Welfare

As Theorem 5 in Appendix C states, none of the presented mechanisms guarantees a bounded competitive ratio. However, the total social welfare of *FCFS* and *Count* is guaranteed to be non-negative when $\rho_1 \geq 1$ or $\rho_2 = 1$ because of the IR and BB properties. On the contrary, the total social welfare of *PayEX* can be negative.

5 NUMERICAL ANALYSIS

While there are no theoretical performance bounds, we now consider the empirical performance of the mechanisms in real-world settings, where EV charging is coordinated by a demand response aggregator [6], which acts as a broker between EV agents and the electricity market by procuring electricity from a mixture of local renewable generators.

5.1 Experimental Setup

Agents (EV drivers) submit their requirements (i.e. type) at time $\hat{a}_i$. EVs are plugged into the charging equipment during $t \in [\hat{\alpha}_i, \hat{\beta}_i]$. To obtain the distribution of $\hat{\alpha}_i$ and $\hat{\beta}_i$, we use the results of a questionnaire about daily travel patterns answered by 340 citizens in Nagoya City, Japan. The booking time $\hat{a}_i$ is uniformly drawn from $\hat{\alpha}_i - 12$ to $\hat{\alpha}_i$ and the deadline $\hat{d}_i$ is uniformly drawn from $\hat{a}_i$ to $\hat{\beta}_i$. We assume that the charging speed of all agents is 3 kW. Furthermore, a single time step is one hour and a single unit of electricity is 3 kWh; $\forall i \in I : r_i = 1$. The capacity Q_i of each agent is uniformly drawn from 1 to 6.

The model of procurement costs is described below. First, given the cost $c_{t,1}$ of the first unit at time t, the marginal cost $c(t, m)$ of the mth unit is obtained from Eq. 18:

$$c(t, m) = \gamma^{m-1} c_{t,1}. \qquad (18)$$

Here, $\gamma \geq 1$ is a parameter that indicates supply tightness. If $\gamma = 1$, the marginal costs are constant and they increase otherwise. The aggregator estimates $c_{t,1}$ with a certain error band ε. We use $\tilde{c}_t$ to denote the estimated value:

$$(1 - \varepsilon)\tilde{c}_t \leq c_{t,1} \leq (1 + \varepsilon)\tilde{c}_t. \qquad (19)$$

We assume that the aggregator knows γ and ε accurately in advance, and thus the maximum cost $c_{max}(t, m)$ is obtained as $c_{max}(t, m) = (1+\varepsilon)\gamma^{m-1}\tilde{c}_t$. In this analysis, we set $\tilde{c}_t$ as shown in Figure 2, using the real market price of a typical fine June day in Japan[1]. The actual

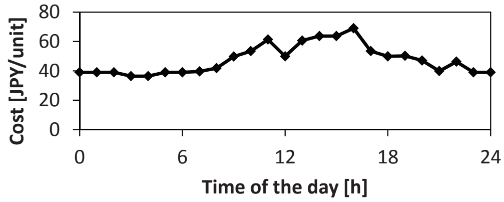


Figure 2: Estimated cost.

$c_{t,1}$ is uniformly drawn from the range shown by Eq. 19 at each time. The analysis is run for a 48-hour period, and the estimated cost is the same for these two days, while the actual cost is different.

The optimisation in all mechanisms is carried out by solving the corresponding mixed integer program by using the Gurobi[2] solver without any tolerance.

As benchmarks, common critical-value-based online DSIC mechanisms may be considered. However, such mechanisms [5, 10] are based on the greedy algorithm and thus unable to take predictions about future procurement cost into account. Indeed, Hayakawa et al. [8] showed that *FCFS* outperforms the existing state-of-the-art mechanisms in procurement settings. Thus, we adopt *FCFS* as a baseline of the analysis.

5.2 Empirical Results

We now numerically evaluate our proposed mechanisms. In all the figures, each point shows the average over 30 trials and the error bar shows the 95% confidence intervals.

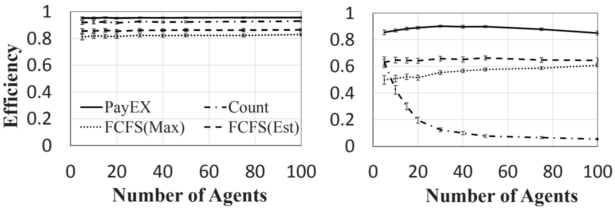
5.2.1 Efficiency. First, we compare *efficiency*, which is defined as the obtained social welfare as a proportion of that of the offline optimal. We evaluate two cases. In case 1, $\gamma = 1.0$ and $\varepsilon = 0.2$; in case 2, $\gamma = 1.1$ and $\varepsilon = 0.5$. These represent constant/rising marginal costs and low/high prediction errors, respectively. The parameters of the mechanisms are set as $\rho_1 = \rho_2 = \rho_3 = 1$. Figure 3 shows the results where the number of agents varies from 5 to 100. These results show that *PayEX* outperforms the other two mechanisms and achieves over 80% efficiency in these settings.

Then, we fix the number of agents to 100 and vary ε, which regulates the error in the cost prediction, from 0.0 to 1.0. The results in Figure 4 show that the efficiency of *FCFS* is sensitive to ε regardless of the adopted pricing function. Responsive mechanisms, *Count* and *PayEX*, are less sensitive to ε because they adapt their schedule after observing the actual cost, as opposed to *FCFS*, which does not adapt.

Finally, we fix the number of agents to 100 and vary γ, which regulates the supply tightness, from 1.0 to 1.5. The results in Figure 5 show that the responsive mechanisms perform well if the supply is not too tight because they have the option to accept late-arriving high-valued agents, while *FCFS* always gives priority to the early-arriving agents. However, the responsive mechanisms result in poor performance if supply is very tight. The efficiency of *Count* decreases rapidly over γ but is bounded to be positive, while the efficiency of *PayEX* decreases more slowly but is unbounded.

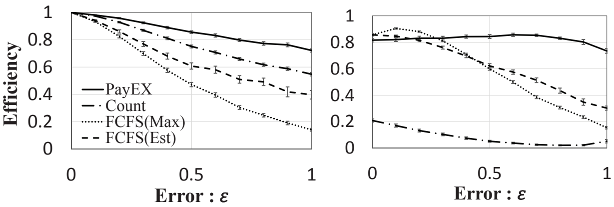
[1]http://www.jepx.org/english/index.html.

[2]http://www.gurobi.com.



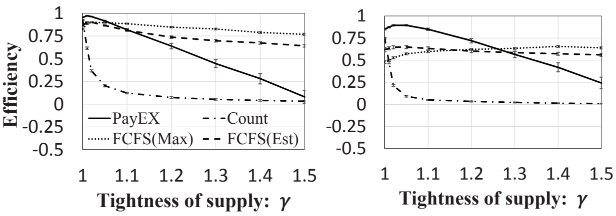
Case 1: $\gamma = 1.0, \varepsilon = 0.2$ Case 2: $\gamma = 1.1, \varepsilon = 0.5$

Figure 3: Efficiency of the mechanisms.



Case 1: $\gamma = 1.0$ Case 2: $\gamma = 1.1$

Figure 4: Influence of the error band ε.



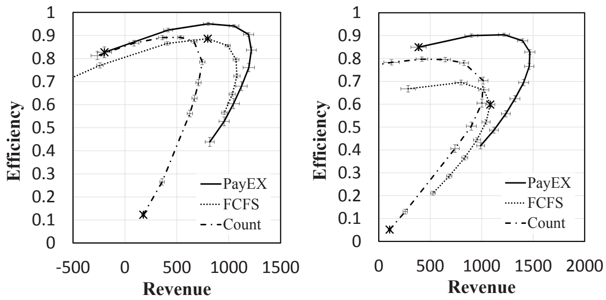
Case 1: $\varepsilon = 0.2$ Case 2: $\varepsilon = 0.5$

Figure 5: Influence of supply tightness γ.

5.2.2 Trade-off between Efficiency and Revenue. In all three mechanisms, there is a trade-off between revenue and efficiency. Specifically, we fix γ to 1.1 and consider the two cases where ε is 0.2 or 0.5. The number of agents is set to 100. Figure 6 shows the trade-off achieved by these mechanisms by setting various parameters, $0.7 \leq \rho_1 \leq 1.5$, $0.06 \leq \rho_2 \leq 1.0$, $1.0 \leq \rho_3 \leq 2.0$. In this setting, *FCFS(Est)* is a variant of *FCFS(Max)*, where ρ_1 is set to $\frac{1}{\varepsilon+1}$. Hence, we show the performance of *FCFS* based on *FCFS(Max)*. For all mechanisms, the point $\rho_1 = \rho_2 = \rho_3 = 1.0$ is shown by an asterisk.

In both cases, *PayEX* outperforms the other two mechanisms. In particular, when $\varepsilon = 0.5$, the aggregator can obtain about 32% more revenue by using *PayEX* compared with *FCFS*, while it can achieve about 14% higher social welfare.

Note that for a wide range of ρ parameters, namely for all the plots in Figure 6, the mechanism satisfies DSIC, IR, and NC. Unlike critical-value-based mechanisms that require unique scheduling and pricing algorithms, a wide range of algorithms can thus be adopted in our proposed framework, considering the trade-off between efficiency and revenue.



($\varepsilon = 0.2$) ($\varepsilon = 0.5$)

Figure 6: Trade-off between efficiency and revenue.

6 CONCLUSIONS

In this study, we addressed how to design online mechanisms in settings with uncertain future procurement costs and multi-unit demand, focusing on the no cancellation (NC) property. In these settings, no mechanism has a bounded competitive ratio in terms of efficiency. We characterised the price-based DSIC, IR, and NC mechanism for such settings. We focus on the domain of multi-unit demand, in which agents are only interested in the finally assigned number of units and do not care when they are assigned, and propose a novel algorithm that uses this flexibility to improve performance. The proposed mechanisms are model-free w.r.t. future agents, but can consider stochastic information about future procurement cost. We presented three specific mechanisms within the framework, each of which has some flexibility to consider the trade-off between efficiency and revenue.

As shown in our numerical analysis, first-come first-served mechanism performs well in settings where the aggregator can estimate future procurement costs reliably or supply is very tight. However, when the estimated procurement costs are highly uncertain and supply is not as tight, *PayEX* performs very well, although it is not WBB. These conditions are typically true in the EV charging setting, where procurement costs depend more on the time of day rather than the number of customers, and there can be high uncertainty about the costs because of the increasing reliance on the generation of renewable energy. As shown in the presented results, in certain real-world situations, a price-based mechanism that is not WBB not only achieves better *efficiency* but also makes more profit for the operator than common WBB mechanisms.

In the future, we plan to explore algorithms in various fields, such as booking hotels, airline tickets, and the Smart Grid, considering the specific constraints of each field.

A PROOF OF THEOREM 1

Here, we show the proof of Theorem 1. We first use the characterisation of the DSIC mechanisms for offline settings presented by Bartal et al. [1], namely where the misreports of $\hat{a}_i$, $\hat{d}_i$, $\hat{\alpha}_i$, $\hat{\beta}_i$, and $\hat{r}_i$ are not considered.

LEMMA 4. *Assuming truthful reports of $\hat{a}_i$, $\hat{d}_i$, $\hat{\alpha}_i$, $\hat{\beta}_i$, and $\hat{r}_i$, a direct revelation mechanism for our setting is DSIC if and only if*

1. The pricing function $P_i(k, \hat{\theta}^{\langle \hat{d}_i \rangle})$ for every possible allocation k to agent i does not depend on $\hat{\boldsymbol{v}}_{\boldsymbol{i}}$.

2. The allocation function $Y_i(\hat{\theta}^{\langle \hat{d}_i \rangle})$ allocates k items to i such that the value of $\hat{v}_{i,k} - P_i(k, \hat{\theta}^{\langle \hat{d}_i \rangle})$ is maximised (over all k that can be allocated to i for any choice of $\hat{\boldsymbol{v}}_{\boldsymbol{i}}$).

Lemma 4 is proven by Theorem 1 in Bartal et al. [1]. Next, we consider the situation where agents can misreport $\hat{a}_i$, $\hat{d}_i$, $\hat{\alpha}_i$, $\hat{\beta}_i$, and $\hat{r}_i$ in settings with *limited misreports*. We then have the following result.

THEOREM 4. *A direct revelation mechanism for our setting is DSIC if and only if*

1. The pricing function $P_i(k, \hat{\theta}^{\langle \hat{d}_i \rangle})$ for every possible allocation k to agent i is monotonic and independent of $\hat{\boldsymbol{v}}_{\boldsymbol{i}}$.

2. The allocation function $Y_i(\hat{\theta}^{\langle \hat{d}_i \rangle})$ allocates k items to i such that the value of $\hat{v}_{i,k} - P_i(k, \hat{\theta}^{\langle \hat{d}_i \rangle})$ is maximised (over all k that can be allocated to i for any choice of $\hat{\boldsymbol{v}}_{\boldsymbol{i}}$).

PROOF. First, we show that this is a sufficient condition. An agent cannot manipulate its own price by misreporting $\hat{\boldsymbol{v}}_{\boldsymbol{i}}$, because this is determined independent of $\hat{\boldsymbol{v}}_{\boldsymbol{i}}$. Therefore, due to the second condition, it can always maximise its own utility by reporting $\hat{\boldsymbol{v}}_{\boldsymbol{i}} = \boldsymbol{v}_{\boldsymbol{i}}$. If the agent reports the true $\hat{\boldsymbol{v}}_{\boldsymbol{i}}$, it can only increase its utility by decreasing prices. However, owing to the monotonicity of the pricing function, only misreporting $\hat{a}_i < a_i$, $\hat{d}_i > d_i$, $\hat{\alpha}_i < \alpha_i$, $\hat{\beta}_i > \beta_i$, or $\hat{r}_i > r_i$ can reduce prices, and this is not possible because of the assumption of limited misreports. Thus, the agent always maximises its own utility by truthfully reporting a_i, d_i, α_i, β_i, and r_i.

Second, we show that this is necessary. Assume to the contrary that the first condition does not hold. If the pricing function depends on $\hat{\boldsymbol{v}}_{\boldsymbol{i}}$, then the mechanism is not DSIC because of Lemma 4. Now, suppose that the pricing function is not *monotonic*, namely there is some $a'_i < a''_i$ such that $P_i(k, a'_i) > P_i(k, a''_i)$, while $\hat{d}_i$, $\hat{\alpha}_i$, $\hat{\beta}_i$, and $\hat{r}_i$ are the same. In this case, an agent whose true type is a'_i and who is allocated k units when reporting truthfully can increase its utility by misreporting a''_i even when the allocation remains the same. Thus, the first condition is necessary.

Now, assume that the first condition holds but the second condition does not. Then, for some $\hat{\boldsymbol{v}}'_{\boldsymbol{i}}$, the mechanism allocates k' such that $\hat{v}_{i,k'} - P_i(k', \hat{\theta}^{\langle \hat{d}_i \rangle}) < \hat{v}_{i,k''} - P_i(k'', \hat{\theta}^{\langle \hat{d}_i \rangle})$, where k'' can be allocated by reporting a certain $\hat{\boldsymbol{v}}''_{\boldsymbol{i}}$. In this case, if its true valuation is $\boldsymbol{v}_{\boldsymbol{i}} = \hat{\boldsymbol{v}}'_{\boldsymbol{i}}$, its utility can increase by misreporting $\hat{\boldsymbol{v}}''_{\boldsymbol{i}}$. Thus, the second condition is necessary. □

Hence, Theorem 1 is proven. The conditions stated in Definition 3.2 are necessary and sufficient to be DSIC.

B PROOF OF LEMMA 2

Due to the assumption that a schedule $y_{i,t'}^{\langle t-1 \rangle}$ at time $t-1$ satisfies NCF,

$$y_{i,\hat{\beta}_i}^{\langle t-1 \rangle} - y_{i,t-1}^{\langle t-1 \rangle} \leq (\hat{\beta}_i - t + 1)\hat{r}_i. \tag{20}$$

In this case, if the schedule at time t is set by

$$y_{i,t}^{\langle t \rangle} = \min(y_{i,t-1}^{\langle t-1 \rangle} + \hat{r}_i, h_i^{\langle t \rangle}), \tag{21}$$

with the upper-bound function $h_i^{\langle t \rangle}$ that, with the scheduling function $y_{i,t}^{\langle t \rangle}$, satisfies the *YH constraints*, the schedule $y_{i,t}^{\langle t \rangle}$ at time t is NCF for the following reason.

First, because of Eqs. 1, 8, and 9, $y_{i,t-1}^{\langle t \rangle} = y_{i,t-1}^{\langle t-1 \rangle}$ and $h_i^{\langle t \rangle} \geq h_i^{\langle t-1 \rangle} \geq y_{i,t-1}^{\langle t-1 \rangle}$ hold; thus, from Eq. 21,

$$0 \leq y_{i,t}^{\langle t \rangle} - y_{i,t-1}^{\langle t \rangle} = \min(\hat{r}_i, h_i^{\langle t \rangle} - y_{i,t-1}^{\langle t-1 \rangle}) \leq \hat{r}_i, \tag{22}$$

holds. Then, we consider two cases for the value of the right-hand side of Eq.21.

(1) Case 1, where $y_{i,t-1}^{\langle t-1 \rangle} + \hat{r}_i \leq h_i^{\langle t \rangle}$

In this case, $y_{i,t}^{\langle t \rangle} = y_{i,t-1}^{\langle t-1 \rangle} + \hat{r}_i$. If $t < \delta_i$, according to Eqs.11 and 12, $y_{i,\hat{\beta}_i}^{\langle t \rangle} - y_{i,t}^{\langle t \rangle} \leq \Pi_i \leq (\hat{\beta}_i - t)\hat{r}_i$. Moreover, if $t \geq \delta_i$, according to Eqs.13 and 20,

$$y_{i,\hat{\beta}_i}^{\langle t \rangle} - y_{i,t}^{\langle t \rangle} \leq y_{i,\hat{\beta}_i}^{\langle t-1 \rangle} - y_{i,t}^{\langle t \rangle} = y_{i,\hat{\beta}_i}^{\langle t-1 \rangle} - (y_{i,t-1}^{\langle t-1 \rangle} + \hat{r}_i) \leq (\hat{\beta}_i - t)\hat{r}_i.$$

(2) Case 2, where $y_{i,t-1}^{\langle t-1 \rangle} + \hat{r}_i \geq h_i^{\langle t \rangle}$. In this case, $y_{i,t}^{\langle t \rangle} = h_i^{\langle t \rangle}$. Then, according to Eq.10, $y_{i,\hat{\beta}_i}^{\langle t \rangle} - y_{i,t}^{\langle t \rangle} = y_{i,\hat{\beta}_i}^{\langle t \rangle} - h_i^{\langle t \rangle} \leq (\hat{\beta}_i - t)\hat{r}_i$.

This means that in both cases, there exists a schedule that satisfies $\forall t'; \hat{\alpha}_i < t' \leq \hat{\beta}_i : 0 \leq y_{i,t'}^{\langle t \rangle} - y_{i,t'-1}^{\langle t \rangle} \leq \hat{r}_i$, and thus an NCF schedule can be obtained.

C COMPETITIVE ANALYSIS

We say that an allocation mechanism Y has a bounded competitive ratio p if $\forall \hat{\theta} : pSW_Y(\hat{\theta}) \geq SW_{OPT}(\hat{\theta})$, where $SW_Y(\hat{\theta})$ is the social welfare achieved by Y and $SW_{OPT}(\hat{\theta})$ is the social welfare of the offline optimal allocation given all information about future arrivals and procurement costs.

THEOREM 5. *No allocation mechanism has a bounded competitive ratio.*

PROOF. Assume there is a single agent $\hat{\theta}_1 = \{\hat{a}_1 = 1, \hat{d}_1 = 2, \hat{\alpha}_1 = 1, \hat{\beta}_1 = 2, \hat{r}_1 = 1, \hat{v}_1 = 0, \hat{v}_2 = c_{max}(2,1)\}$; in other words, this agent derives a value of $c_{max}(2,1)$ if it receives two units and 0 otherwise. Furthermore, assume that the (known) cost at time 1 is $c(1,1) < c_{max}(2,1)$.

Now, any mechanism Y needs to decide whether to allocate a unit to agent 1 at time 1 or not. If $y_{1,1} = 0$, then assume that $c(2,1) < c_{max}(2,1) - c(1,1)$. In this case, $SW_Y(\hat{\theta}) \leq 0$ (regardless of $y_{1,2}$), as the agent can no longer be allocated two units. However, with perfect foresight, the optimal solution allocates two units, so $SW_{OPT}(\hat{\theta}) = c_{max}(2,1) - c(1,1) - c(2,1) > 0$. Hence, there is no bounded competitive ratio $p > 0$. On the contrary, if $y_{1,1} = 1$, then assume that $c(2,1) > c_{max}(2,1) - c(1,1)$. In this case, $SW_Y(\hat{\theta}) < 0$ (regardless of $y_{1,2}$). The optimal solution allocates no units, so $SW_{OPT}(\hat{\theta}) = 0$. Hence, there is no bounded ratio either. □

REFERENCES

[1] Yair Bartal, Rica Gonen, and Noam Nisan. 2003. Incentive compatible multi unit combinatorial auctions. In *Proceedings of the 9th conference on Theoretical aspects of rationality and knowledge*. ACM, 72–87.

[2] Gabriel Bitran and René Caldentey. 2003. An overview of pricing models for revenue management. *Manufacturing & Service Operations Management* 5, 3 (2003), 203–229.

[3] René Caldentey and Gustavo Vulcano. 2007. Online auction and list price revenue management. *Management Science* 53, 5 (2007), 795–813.

[4] Shahar Dobzinski, Ron Lavi, and Noam Nisan. 2012. Multi-unit auctions with budget limits. *Games and Economic Behavior* 74, 2 (2012), 486–503.

[5] Enrico H Gerding, Valentin Robu, Sebastian Stein, David C Parkes, Alex Rogers, and Nicholas R Jennings. 2011. Online mechanism design for electric vehicle

charging. In *The 10th International Conference on Autonomous Agents and Multiagent Systems-Volume 2*. International Foundation for Autonomous Agents and Multiagent Systems, 811–818.

[6] Lazaros Gkatzikis, Iordanis Koutsopoulos, and Theodoros Salonidis. 2013. The role of aggregators in smart grid demand response markets. *Selected Areas in Communications, IEEE Journal on* 31, 7 (2013), 1247–1257.

[7] Mohammad T Hajiaghayi. 2005. Online auctions with re-usable goods. In *Proceedings of the 6th ACM conference on Electronic commerce*. ACM, 165–174.

[8] Keiichiro Hayakawa, Enrico H Gerding, Sebastian Stein, and Takahiro Shiga. 2015. Online Mechanisms for Charging Electric Vehicles in Settings with Varying Marginal Electricity Costs. In *Proceedings of the 24th International Joint Conference on Artificial Intelligence (IJCAI)*. 2610–2616.

[9] Noam Nisan, Jason Bayer, Deepak Chandra, Tal Franji, Robert Gardner, Yossi Matias, Neil Rhodes, Misha Seltzer, Danny Tom, Hal Varian, and Dan Zigmond. 2009. Google's auction for tv ads. In *Automata, Languages and Programming*. Springer, 309–327.

[10] David C Parkes. 2007. Online Mechanisms. In *Nisan, N., Roughgarden, T., Tardos, E., and Vazirani, V. (Eds.) Algorithmic Game Theory*. 411–439.

[11] David C Parkes and Satinder P Singh. 2004. An MDP-based approach to online mechanism design. In *Advances in neural information processing systems*. 791–798.

[12] Ryan Porter. 2004. Mechanism design for online real-time scheduling. In *Proceedings of the 5th ACM conference on Electronic commerce*. ACM, 61–70.

[13] Valentin Robu, Enrico H Gerding, Sebastian Stein, David C Parkes, Alex Rogers, and Nick R Jennings. 2013. An online mechanism for multi-unit demand and its application to plug-in hybrid electric vehicle charging. *Journal of Artificial Intelligence Research* 48 (2013), 175–230.

[14] Kalyan T Talluri and Garrett J Van Ryzin. 2006. *The theory and practice of revenue management*. Vol. 68. Springer Science & Business Media.