Advanced Open Rotor Far-Field Tone Noise

by

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ABSTRACT
Doctor of Philosophy
ADVANCED OPEN ROTOR FAR-FIELD TONE NOISE
by Celia Maud Ekoule

An extensive analysis of the far-field tonal noise produced by an advanced open rotor is presented. The basis of the work is analytical methods which have been developed to predict the tonal noise generated by several different mechanisms. Also to provide further insight and validation, an in-depth analysis of rig-scale wind tunnel measurements and Computational Fluid Dynamics (CFD) data has been carried out.

The loading noise produced by a propeller operating at angle of attack is formulated modeling the blades as a cascade of a two-dimensional flat plates. The simplified geometry enables the prediction of rotor-alone tones with a reduced computer processing time. The method shows a good ability to predict the relative change of the sound pressure level with changes of the angle of attack.

The radiated sound field caused by aerodynamic interactions between the two blade rows is investigated. A hybrid analytical/computational method is developed to predict the tonal noise produced by the interaction between the front rotor viscous wakes and the rear rotor. The method assumes a Gaussian wake profile whose characteristics are determined numerically in the inter-rotor region and propagated analytically to the downstream blade row. A cross validation of the prediction method against wind tunnel measurements and CFD data shows reasonable agreement between the three techniques for a wide number of interaction tones.

Additionally a theoretical formulation is derived for expressions for the tonal noise generated by the interaction between the bound potential field of one rotor with the adjacent rotor. The bound potential field produced by the thickness and bound circulation of each blade is determined using a distributed source model. The analytical model is compared with an existing model for a point source and is validated using advanced CFD computations. Results show that the potential field mostly affects the first interaction tone. The addition of the potential field component provides an improvement in the agreement with wind tunnel measurements.

Overall this work provides new tools for the rapid assessment of the tonal noise produced by an advance open rotor at various operating conditions, and the analytical result can provide further physical insight into the sources of noise.
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Declaration of Authorship

I, Celia Maud Ekoule, declare that the thesis entitled *Advanced Open Rotor Far-Field Tone Noise* and the work presented in the thesis are both my own, and have been generated by me as the result of my own original research. I confirm that:

- this work was done wholly or mainly while in candidature for a research degree at this University;

- where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated;

- where I have consulted the published work of others, this is always clearly attributed;

- where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work;

- I have acknowledged all main sources of help;

- where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself;

- parts of this work have been published as: (Kingan et al., 2014) and (Ekoule et al., 2015)

Signed:

Date: April 21st, 2017
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Nomenclature

List of Symbols

Roman Letters

\( B \) = Number of propeller blades
\( c \) = Blade chord
\( c_0 \) = Speed of sound in ambient fluid
\( C_L \) = lift coefficient
\( e_{ij} \) = viscous stress tensor
\( f_i \) = ith vector component of the surface force exerted by the air on the blade per unit area
\( G \) = time dependent Green’s function
\( G_m \) = harmonic Green’s function
\( k_m \) = harmonic wave number, \( mB/c_0 \)
\( L \) = blade section lift
\( M \) = flight Mach number
\( M_t \) = tip rotational Mach number, \( (\Omega r_t)/c_0 \)
\( M_r \) = Mach number of the blade relative to the air
\( p \) = pressure
\( p_0 \) = ambient pressure
\( p' \) = pressure disturbance, \( p' = p - p_0 \)
\( r \) = distance from propeller centre to observer
\( r_h \) = blade radius at hub
\( r_t \) = blade radius at tip
\( r_{y} \) = source radius on propeller
\( s \) = blade sweep
\( t \) = observer time
\( T \) = time limit in acoustic analogy integrals
NOMENCLATURE

\( T'_{ij} \) = Lighthills tensor in a moving frame
\( U_r \) = relative flow velocity
\( v_i, v'_i \) = velocity disturbance component in stationary fluid, in a moving frame
\( V_n, V'_n \) = source normal velocity to the blade surface in the stationary, moving medium
\( \bar{X} \) = chordwise coordinate
\( (x_1, x_2, x_3) \) = observer (field point) coordinates
\( (y_1, y_2, y_3) \) = source coordinates in the flight system
\( (y'_1, y'_2, y'_3) \) = source coordinates in the propeller system

Greek Letters

\( \alpha \) = stagger angle
\( \alpha_p \) = pitch angle of propeller shaft with respect to flight coordinate system
\( \alpha_y \) = yaw angle of propeller shaft with respect to flight coordinate system
\( \alpha_r \) = roll angle of propeller shaft with respect to flight coordinate system
\( \beta \) = \( \sqrt{1 - M^2} \)
\( \gamma \) = angle of attack
\( \theta, \theta' \) = radiation angle to observer from flight direction, propeller axis
\( \Omega \) = rotational speed of propeller shaft
\( \phi, \phi' \) = azimuthal angle to observer in retarded system in flight, propeller system
\( \Psi_L \) = noncompactness factor defined in equation 3.50
\( \rho \) = density of the fluid
\( \rho_0 \) = ambient density in the undisturbed fluid
\( \rho' \) = density disturbance, \( \rho - \rho_0 \)
\( \sigma \) = phase radius

Acronyms

AOA = Angle Of Attack
AOR = Advanced Open Rotor
BPF = Blade Passing Frequency
CFD = Computational Fluid Dynamics
DNW (LLF) = German-Dutch Wind Tunnels (Large Low-speed Facility)
DREAM = valiDation of Radical Engine Architecture SysteMs
ICAO = International Civil Aviation Organisation
ISVR = Institute of Sound and Vibration Research
RPM = Revolutions Per Minute
SPL = Sound Pressure Level
SROR = Single Rotating Open Rotor
URANS = Unsteady Reynolds-Averaged Navier-Stokes
Chapter 1

Introduction

1.1 Background

Improvement of the thermal and propulsive efficiency of aircraft engines has always been a major objective of the aviation industry. Increasingly, the economic benefit of increased global air traffic has to be balanced against the environmental impact, driving the need for improved fuel efficiency and reduced emissions. A recent study published in the Airbus Global Market Forecast (Airbus, 2013) showed that worldwide annual traffic (measured in Revenue Passenger Kilometres (RPK)) increased by 61% between 2000 and 2013 (see figure 1.1). With the continuous increase of global air traffic, and the environmental concerns relating to global warming, air quality and noise emissions, there is a strong incentive for aeronautical companies to develop new aircraft engines that will reduce pollutant and noise emissions as well as a requirement to meet the increasingly strict certification standards set by the International Civil Aviation Organisation (ICAO).

Open rotor technology is considered as a possible alternative to turbofan engines due to its significant improvement in fuel efficiency in comparison to conventional high bypass ratio turbofans. Conventional propellers are designed with a relatively small number of blades (two to six) which are usually long and thin, so that the propeller can displace a large mass of air for small changes in flow velocity and efficiently produce high thrust at low forward speeds. Although conventional propellers provide high performance at moderate flight speeds (below a Mach number of approximately 0.6), their efficiency decreases significantly at higher flight speeds due to the development of shock waves around the rotating blades as their relative Mach number reaches the supersonic regime (Peake and Parry,
Current commercial aircraft are designed to cruise at a Mach number of approximately 0.8. An alternative to the conventional propeller is the propfan or single-rotation open rotor (SROR) which is designed with a higher number of blades whose aerodynamic characteristics provide better performance at high speed. One of the contributors to the loss of efficiency of propellers is the swirl in the wake. Adding a contra-rotating rear row increases efficiency by significantly reducing the swirl created by the front row, providing a potential additional 8% reduction in fuel burn compared to a single-rotating propeller (Parry, 1988). A cutaway view of an advanced open rotor (AOR) is shown in figure 1.2.

Because of advances in propeller design, it is now possible to design an “advanced open rotor” which will operate at a reasonable cruise Mach number ($M \approx 0.7$) at good efficiency. The advanced open rotor allows a higher effective bypass ratio and provides a 30% reduction in fuel burn compared to modern turbofans (Parker 2011). However, unlike the turbofan whose nacelle contains the noise, the rotors
of the AOR operate in the free field and produce relatively high levels of noise. Figure 1.3 shows a comparison of specific fuel consumption (SFC) against Effective Perceived Noise Level (EPNL) reduction between a year 2000 turbofan, an advanced open rotor and a future turbofan engine. It is believed that the advanced open rotor design will comply with the current noise requirements. Clearly, the specific fuel consumption of an open rotor is 20% to 30% less than that of both the year 2000 and the advanced turbofan. However, this technology will compete with future turbofans, which are expected to continue to become quieter, therefore the minimization of its noise remains a critical issue if the AOR happens to be viable.

![Figure 1.3: Relative SFC and EPNL noise reduction for Turbofan of the year 2000, Open Rotor and Advanced Turbofan (picture courtesy of Rolls-Royce plc.)](image)

In the 1960s the number of aircraft powered by turbofan engines was greater than those powered by conventional propellers due to their relatively good efficiency at high subsonic Mach number. An increase in fuel prices in the 1970s brought back interest in open rotors owing to their fuel efficiency. Many studies were conducted in the 1980s, for example the development of the General Electric Unducted Fan (UDF) GE36 and the Pratt & Whitney/Allison 578DX (Peake and Parry, 2012). However, the concept was abandoned by the manufacturers because of relatively
high noise emissions and a significant decrease in fuel prices in the 1990s. For
the past fifteen years, interest in Open Rotors has been renewed because of the
recent emphasis on reducing carbon emissions and the increase in fuel costs over
the years 2000-2010 (Airbus (2013)).

The noise produced by an AOR is characterized by its high tonal content which is
produced by a number of different sources. Predictions of sound radiated from a
propeller can be performed using either time domain or frequency domain methods.
Lowson (1965) developed a time domain model for predicting the noise produced
by an acoustic source in arbitrary motion, which can be used to predict the noise
radiated from a propeller. Most of the current time domain methods are based
on the work of Farassat (Farassat (1981), Farassat (1985), Farassat (1986), Faras-
sat and Brentner (1998)) who extended Lowson’s work and developed prediction
methods for calculating the sound of both a subsonic and a supersonic propeller.
A comparison between time domain and frequency domain methods is given by
Carley (1996), who developed a time domain method to calculate the sound pro-
duced by a propeller in a moving medium based on the work of Farassat. The
analytical models developed in the present work are formulated in the frequency
domain as it gives direct access to the frequency spectrum and is thus suitable
for tonal analysis. Consequently, the following sections are focused on frequency
domain methods.

1.2 Single-rotating open rotor

A single-rotating open rotor generates rotor-alone tones which are produced by
thickness, loading and quadrupole sources. Such tones occur at integer multiples
of the rotor’s blade passing frequency (BPF). Thickness noise is produced by
the periodic volume displacement of air by the rotating blades. Its amplitude is
dependent on the geometry of the blades. This source is equivalent to a surface
distribution of monopole sources. Steady loading noise is produced by the steady
surface stresses acting on the surface of the blades. Whereas thickness noise is
generally important at high velocities, the loading noise component is dominant
at low and moderate speeds (Magliozzi and Hanson, 1991). Both steady loading
and thickness noise sources constitute the linear content of the sound field that
can be modelled by the linearized equations of motion for an inviscid fluid. The
quadrupole component accounts for non-linear effects that can be significant when
the flow over the blade becomes transonic.
Chapter 1 Introduction

The first model for predicting propeller noise was published by Gutin (1936) who modelled the propeller blades as rotating point forces (the thrust and torque components of the loading were accounted for) in a stationary fluid. As this method was only valid in the far-field, Hubbard and Regier (1950) extended Gutin’s formulation and developed a method which could predict noise in the near-field, close to the tip of the propeller. They investigated the effect of various parameters such as the tip clearance, power coefficient and the blade tip shape on the measured free-field pressure; and compared their analytical results with measurements which gave quite satisfactory agreement with the theory. However this method was limited to a propeller in a static fluid.

The effects of subsonic forward speed was first investigated by Garrick and Watkins (1954). They expressed the noise produced by rotating concentrated forces, which were defined following Gutin’s formulation, in a rectilinear uniform motion. The forces exerted by the propeller on the medium are assumed to be fixed periodic forces acting at the propeller disk and normal to the blade surface. The model can be used to calculate both the near field and far field sound pressure. The formulation was also extended to a radial distribution of forces. The application of the model to a two-blade propeller showed a general dependency of the sound pressure levels on the flight Mach number, with indications that at high subsonic Mach numbers ($\approx 0.9$) the noise produced near the propeller blade tips can be significant. The effect of non-uniform chordwise blade loading was investigated by Watkins and Durling (1956) who showed that a non-uniform distribution of forces could significantly alter the sound amplitude. They modeled any chordwise distribution of loading such that its Fourier decomposition can be expressed as a superimposition of rectangular and triangular shaped force distribution elements.

A more realistic model was developed by Hanson (1980) who formulated a far-field frequency domain model for a single rotating propeller using a helicoidal surface representation of the blades: the motion of the sources follows a helicoidal path corresponding to the rotation of the rotor blade chordline as the blade moves in the fluid. In this formulation, the blade design parameters are explicitly taken into account so that the effect of blade sweep and offset (lean) can be investigated. In view of modern blade designs (which have significant sweep and lean), this provides a more realistic model to predict the tonal noise produced by a propeller.
1.3 Contra-rotating open rotor or Advanced Open Rotor (AOR)

When adding a second rotor, acoustic and aerodynamic interference occurs between the two blade rows. In addition to rotor-alone tones produced by each single rotor, a number of interaction tones are produced by the interaction of the distorted flow field of one rotor with the adjacent rotor. This causes the level of noise emitted from an AOR to be much higher than an equivalent single-rotating open rotor (Magliozzi and Hanson, 1991). Interaction tones occur at frequencies corresponding to the sum or difference of integer multiples of the BPF of the two rotors. The generation of interaction tones can be due to various sources, which are illustrated in figure 1.4, such as:

1. The impingement of the upstream rotor’s viscous wakes onto the downstream rotor.
2. The impingement of the upstream rotor’s tip vortices onto the downstream rotor.
3. The interaction of the bound potential field of each rotor with the adjacent rotor.

When installed on an aircraft, additional sources of tonal noise are produced. For example, extra tonal noise is produced by the periodic loading on the rotor blades due to rotor interaction with the flow distortion produced by nearby structures such as a pylon (see the source labeled 4 in figure 1.4) or the change in the inflow angle with respect to the engine shaft axis (i.e. the angle of attack). They can significantly alter the noise produced by the AOR (Ricouard et al. (2010), Paquet et al. (2014)).

The earliest study on AOR noise was published by Hubbard (1948), based on Gutin’s theory. He highlighted the presence of the acoustic interference mechanism between the two blade rows: the phase difference between the propellers blades could result in destructive or constructive interference patterns if tones were produced by both rotors at the same frequencies. Hubbard did not include the effect of an axial flow or the forward speed of the rotor in this model.

Hanson conducted numerous studies of open rotor tonal noise, and developed the first comprehensive prediction model for AOR tonal noise. In addition to his helicoidal model, Hanson (1985b) developed a far-field analytical model to account
for acoustic and aerodynamic interferences between the two rotors, including a wake prediction model which gave satisfactory agreement with hot wire measurements. Parry (1988) developed a model to predict the AOR tonal noise using a flat plate approximation to calculate the loading on the blades, and he applied the method to an acoustic formulation identical to that of Hanson (1980, 1985). The model includes a method for predicting interaction tones produced by the unsteady loading on the rear rotor blades due to interactions with front rotor wakes and the unsteady loading on both rotors due to the interaction with the bound potential fields of the adjacent rotor. This comprises the calculation of the front rotor wake’s velocity profile, the representation of the wakes as a sum of harmonic gusts impinging on the rear rotor blades and the associated blade response, the potential flow around each blade row and the blade response to each harmonic of the unsteady velocity field (both for incompressible and compressible flow). Comparison of three different wake models showed that, for small wake wavelengths, the sound pressure levels due to the wake interaction is relatively independent of the inter-rotor spacing (see Parry (1988) p.121), however for small spacing the potential field may be dominant at low frequencies. The model did not include the noise due to the front rotor tip vortex which is an important source of the AOR tonal noise.

A method for predicting tones produced by the interaction of the tip vortex from the front rotor with the rear rotor was presented by Majjigi et al. (1989) who used a 2-D representation of the physical phenomenon. The size, strength, growth or decay, and tangential locations of the tip vortex were determined using empirical models derived from experimental data. More recently, new efficient analytical methods were developed in order to predict the tip vortex interaction noise (Kinguan and Self (2009), Roger et al. (2012)). Owing to progress in computational techniques, recent studies on AOR tonal noise allowed the acquisition of aerodynamic
data via various numerical schemes. Strip theory methods calculate the unsteady field along several ‘strips’: the blade is split into several radial sections and the incident flow field is assumed to be uniform on each of them. The aerodynamic properties of the flow are then calculated for each blade section. CFD simulations provide information on the flow behaviour around the blade geometry and may deliver more accurate aerodynamic data. Carazo et al. (2011) used both methods to predict the wake interaction noise of an AOR. The blade segments were modeled as trapezoids in unwrapped coordinates. The unsteady loading on each section was calculated based on Amiet’s gust-airfoil response models (Amiet et al., 1989) with inclusion of the spanwise variation of blade chord and sweep. Some coefficients were determined using semi-empirical wake models. Comparison with CFD simulations and wind tunnel measurements showed some disparities between the methods. The analytical model seemed to over-predict the sound field whereas better agreement with the measurements was found using the numerical computation. Some of the discrepancies between the analytical and the numerical results were later on found to be influenced by the existence of a leading edge vortex (Jaouani et al., 2016). A recent comparison between another strip theory code and CFD by Kingan et al. (2014) showed that the aerodynamic inputs obtained with CFD simulations gave better agreement with measurements than the data obtained with the strip theory code. However obtaining high fidelity CFD data is critical to accurately predict noise levels.

Due to the advances in research and efforts to reduce the tonal noise of the AOR, the broadband noise has become an important contributor to the total sound field (Parry et al., 2011). Although the present work focuses on tonal noise, it is worth mentioning where the broadband noise originates from. Broadband noise is thought to be produced by the unsteady stresses on the rotor blades due to turbulence interacting with the rotor blades. Possible sources of broadband noise which were identified in the literature (Blandeau, 2011) are:

1. The impingement of the turbulent component of the upstream rotor’s wakes onto the leading edge of the downstream rotor’s blades.

2. The impingement of the upstream rotor’s turbulent tip vortices onto the leading edge of the downstream rotor’s blades.

3. Turbulent ‘self-noise’, produced by the interaction of the turbulent boundary layer on the surface of each rotor blade with the trailing edge.

4. Atmospheric turbulence ingestion.
5. The interaction of the turbulent component of a pylon’s wake with the front rotor’s blades.

6. Hub boundary layer ingestion noise.

7. Secondary flow between the two rotors.

1.4 Installation effects

Most of the studies cited above do not take into account installation effects (angle of attack, interactions with the flow field produced by structures such as the pylon, fuselage, and the aircraft wings, tail sections), which cause significant changes in the sound field produced by the propeller. This section illustrates the different analyses that have been conducted to describe the sound field produced by an installed AOR.

1.4.1 Angle of attack

The angle of attack (AOA) corresponds to the angle between the engine axis and the flow (see figure 1.5). The noise produced by an AOR when operating at angle of attack will be significantly different to that produced when the AOR is at 0° AOA. The investigation of this effect started in the 1980s with experimental studies of propeller blade passing frequency noise with angular inflow conducted by Block (1986) and Woodward (1987). Block tested a lightly loaded propeller with a relatively small number of blades and noticed an increase and a decrease in noise levels respectively under and above a single rotating propeller at angle of attack, in comparison with a propeller at zero angle of attack, and small changes in sound pressure levels for a contra-rotating propeller at AOA. The results of Woodward for a contra-rotating propeller with higher loading and number of blades (11 and 9) showed some important variations in the noise levels and directivity patterns of an AOR at angle of attack compared to an AOR at zero angle of attack, notably for the azimuthal variation of aerodynamic loading.

Stuff (1988) attempted to quantify the noise produced by a propeller at angle of attack in a uniform flow using an analytical method. He showed that the angular inflow significantly affects the directivity of the sound field with an increase in the sound pressure level in the forward and downward directions. However, this was applicable only for a point source.
Mani (1990) developed a model for predicting the sound field due to the unsteady loading caused by the AOA. The model was compact in the chordwise direction. The results were compared with the experiments of Block and Woodward and showed that the sound pressure level produced by a propeller at angle of attack was underestimated for the case of a highly loaded propeller (Woodward). Mani found that, in addition to the unsteady loading, the AOA caused azimuthal variations in the radiation efficiency of the propeller. Including this effect in his model, the results were in a better agreement with Woodward’s measured data for an AOR. However, the blade passing frequency of the rear rotor was still underestimated. This may have been due to difficulties in accurately calculating the flow incident on the rear rotor.

Krejsa (1990) developed an analytical model to predict the noise produced by a single rotation propeller at angle of attack, based on the derivation of Parry (1988) for a propeller at zero angle of attack. The model was applicable to a larger range of angles of attack in comparison to the study of Mani. The study confirmed that in addition to the unsteady loading due to the inflow incidence, the inclusion of the effects of angle of attack on the radiative efficiency of the loading and thickness noise components improves agreement with the experiments. Unlike Parry’s model (which was limited to the case of zero angle of attack), the location of the source distribution was limited to the blade midchord i.e. the loading was modelled as chordwise compact.

Other prediction methods for propeller and propfan noise at angle of attack have been formulated, for example Envia (1991) who did not formulate a near or far-field approximation but used a large blade count limit. The blade surface was segmented into a large number of elements and the sound field of a propeller at
angle of attack was expressed in terms of Airy functions restricted to small AOA (under 5 degrees). Blade design parameters were not explicitly specified in the formulation. The work of Hanson and Parzych (1993) is the most complete and comprehensive model available for modelling the far-field noise due to a propeller at angle of attack. The model is based on Goldstein’s acoustic analogy (Goldstein, 1974). All sources of noise are considered including the radial contribution of the loading (which is often ignored in prediction models). The monopole and dipole sources are located on the blades surface and the quadrupole sources in the volume around the blades. The previous simplifications for blade geometry have been removed so the derivation provides an exact formulation of the propeller tonal noise at angle of attack. Hanson (1995) extended this analysis to show that the increase of radiation efficiency due to the crossflow was due to the increase of the source Mach number relative to the observer caused by the propeller tilt. The work presented in this thesis is based on the former model (Hanson and Parzych (1993)) with simplification of the blade geometry.

The effects of the angle of attack on the noise produced by an AOR was investigated by Brandvik et al. (2012). The interest was to describe the altered velocity profile and loading distribution in between the two rotors. They conducted CFD simulations for an AOR at take-off and observed significant changes in the flow field due to the angle of attack, notably for angles of attack greater than 3° (the AOA varied from 0° to 12°). They investigated the effect of the rotor AOA on the front rotor viscous wake and tip vortex interaction sources. It was clearly shown that the angle of attack induces circumferential variations in the front rotor’s wakes and in the strength and size of the tip vortex. The acquisition of high fidelity aerodynamic data at angle of attack would thus be of interest for the present study in order to predict the tip vortex and wake profiles between the two rotors.

1.4.2 Fuselage scattering

When the AOR is mounted on the aircraft, the interaction of the incident sound field with the fuselage can have a significant effect on the radiated noise. This effect must be taken into consideration when predicting the total sound field produced by an installed AOR.

To the author’s knowledge, the first consideration of the effect of the aircraft fuselage was made by Hubbard and Regier (1950) who investigated the fuselage
response to the oscillating pressures produced by an adjacent propeller. They conducted experiments modelling the fuselage as a flat vertical wall and a circular wall. The tests showed a considerable increase in the sound pressure levels produced by the propeller due to the presence of the walls, with a larger increase for the vertical wall which is not a realistic representation of the fuselage geometry. The analysis provides calculations of the vibration amplitude of the panel as a function of its mass, natural frequency and structural damping.

The presence of the fuselage can engender shielding effect. The latter was identified during flight tests of a propfan mounted on a Jetstar business aircraft, see Hanson (1984) who made the first attempt to model the shielding effect of the fuselage boundary layer. His study treated plane incident waves on a flat plate, which is a crude approximation for a fuselage but was suitable for a preliminary study. The results showed that the boundary layer provided shielding upstream of the rotor and that the shielding effect increased with the boundary layer thickness.

Hanson and Magliozzi (1985) developed a more advanced frequency-domain method to assess the effect of the refraction and scattering of propeller tones by the aircraft fuselage and boundary layer. The fuselage was modelled as an infinite rigid cylinder to simplify the analysis. The boundary layer profile was assumed to be constant around the circumference of the cylinder and also axially. The incident sound field was calculated using Hanson’s near-field theory for propellers (Hanson (1985a)), the acoustic field inside the fuselage boundary layer was calculated by solving a form of the Pridmore-Brown equation, and the solutions inside and outside the boundary layer were matched. The method can be utilised to calculate the noise levels at the fuselage surface, which can then be used to evaluate cabin noise. Fuller (1989) derived a simple formulation considering the scattering effects of an infinite rigid cylinder (with no boundary layer) for stationary monopole and dipole acoustic sources. Lu (1990) extended Hanson and Magliozzi’s model to determine the noise emission both on the fuselage surface and in the far field using an asymptotic solution. More recently McAlpine and Kingan (2012) studied the effect of fuselage scattering on the noise emitted by a rotating point source and an open rotor (modeled as distributed sources). They applied Hanson and Magliozzi’s formulation neglecting the fuselage boundary layer. They showed evidence that the far-field directivity of an open rotor is affected by the source rotational direction, location and distance from the fuselage. More recently, Brouwer (2016) presented a method to estimate the scattering of open rotor tones by a rigid cylindrical cylinder including its boundary layer. He developed a simplified model to reproduce the directivity and propagation properties of an AOR. He showed that
the presence of the boundary layer causes reductions of acoustic pressure levels on
the fuselage upstream of the AOR as well as variations of SPL with circumferential
angle in both the near-field and the far-field. Although the source modelling is
not truly representative of the noise radiated from an installed AOR, the method
provides quick estimates of the scattered sound field.

1.4.3 Wing and pylon

The presence of an upstream pylon or a downstream wing can significantly influ-
ence the noise produced by an AOR. The pylon generates a potential field and
viscous wakes upstream of the rotors, which produces additional sources of noise.
The presence of a wing would produce supplementary flow distortions and scat-
tering.

Only a few studies on pylon and wing installation effects were identified by the
author. Tanna et al. (1981) conducted experiments on an installed single rotation
propeller and highlighted an increase in tonal noise as a result of the inflow dis-
tortions due to the upwash induced by a downstream wing. Amiet (1986, 1991)
studied the diffraction of sound radiated from a point source by a half plane in a
uniform flow and derived analytical solutions for both swept and unswept wings.
The latter was used by Kingan and Self (2012) to describe the scattering of the
sound from a rotating point source by a rigid half plane in a stationary medium,
and extended to the case of distributed sources in a moving medium. They ap-
plied this method to assess the scattered sound field of an open rotor. For an open
rotor mounted above the wing, the sound pressure levels are significantly reduced
by the presence of the half-plane, which is a promising solution for current and
future noise reduction schemes.

Shivashankara et al. (1990) conducted tests of a scale model of the UDF engine
and investigated the influence of the presence of an upstream pylon on the noise
field of this AOR. They noticed an increase of up to 10-12 dB of the blade pass-
ing frequency tones but negligible variations on the interaction tones. Recent
experiments by Ricouard et al. (2010) lead to similar observations: the pylon sig-
nificantly affects the rotor-alone tones (especially the front rotor BPF) but has a
negligible effect on rotor-rotor interaction tones. Furthermore, results showed that
the polar and azimuthal directivities of the AOR sound field were greatly affected
by the pylon, and that the velocity profile of the pylon wake, which appeared to
significantly influence the sound field, could be reduced by the application of pylon trailing edge blowing. This benefit was highlighted in by Paquet et al. (2014) whose experimental data analysis showed that the pylon blowing was more effective at low thrust than at high thrust. More recently Sinnige et al. (2015) analysed the effects of pylon blowing on the noise emissions and the propulsive performance of a propeller in a pusher configuration using Particle Image Velocimetry (PIV) measurements. Again, the application of pylon blowing proved to be beneficial, using appropriate blowing coefficients. The influence of the pylon design is thus an interesting subject of research to reduce the noise produced by a AOR. In their study of the scattered field of an open rotor by a fuselage McAlpine and Kingan (2012) included a representation of a pylon wake in terms of an impulsive loading component. They pointed out that the noise could be reduced at certain observer locations depending on the position of the pylon. They also found that the noise radiated by the open rotor was affected by the pylon length, as it is directly related to the distance between the source and the fuselage. More recently, Jaouani et al. (2015) developed a semi-analytical model to predict the effect of the pylon wake on the front rotor-alone tones radiated by an AOR in a pusher configuration. The pylon wake in the vicinity of the front rotor leading edge and the unsteady-loading on the front rotor blades were determined numerically, and the radiated sound field was calculated analytically. The model included the computation of the steady-loading and thickness noise, which appeared to interfere significantly with the unsteady-loading noise caused by the pylon wake.
1.5 Aims, objectives and original contributions

A thorough assessment of the noise produced by an advanced open rotor (AOR) would require precise theoretical and computational models supplemented with truly representative wind-tunnel tests. As the implementation of such tests is a long and expensive process, the development of efficient analytical and numerical prediction methods enables fast analysis of the AOR’s noise, and thus is a useful asset for aircraft manufacturers. An existing prediction model for tonal noise is currently used by Rolls-Royce plc. The aim of the present project is to develop a new prediction method which can be used to calculate the far-field tonal noise produced by an advanced open rotor, as it remains a major contributor to the total noise, and which can be implemented into the existing tools.

The first objective is to predict the unsteady loading noise produced by an isolated advanced open rotor at angle of attack relative to the aircraft flight path. The second objective is to use Computational Fluid Dynamics (CFD) data to validate the unsteady loading calculation methods and to predict interaction tones produced by an isolated AOR. Of particular interest is to quantify the differences between current and new prediction methods using various aerodynamic input data.

The third objective is to develop methods to analyse experimental data and to validate the analytical models against test measurements.

Accordingly, in the thesis the material is ordered as follows. The data analysis is presented in Chapter 2. Next, the detailed derivation of the loading noise produced by a propeller at angle of attack is presented in Chapter 3. The derivation of the interaction tone noise radiation expressions is detailed in Chapter 4. In Chapter 5 a hybrid analytical/numerical method for predicting wake interaction noise is described. The method utilizes predictions from a three-dimensional CFD simulation of an advanced open rotor rig to acquire the wake characteristics at a particular location in the inter-rotor region. An analytical model is used to propagate the wake profiles to the leading edge of the downstream rotor blades. The blade response is then calculated used in an analytical method to predict the far-field noise. In Chapter 6 a model is derived to predict the noise produced by the interaction between the potential field of one rotor and the adjacent rotor, considering
both the thickness and the loading problems. The model is compared with existing methods and the relative contribution of wake and potential field interaction tones is assessed.

Finally, it is noted that the experimental results are compared with predictions in both Chapters 5 and 6. Concluding remarks are then given in Chapter 7.

The original contributions in the thesis are highlighted below:

1. Development of a method to post-process wind tunnel experimental data. Creation of algorithms specific to the format of the data collected during Rolls-Royce’s latest test campaigns. Extensive analysis of the data, of which one part is presented in this thesis and the second part was provided to Rolls-Royce via a proprietary deliverable.

2. Development of a simple and fast prediction method for predicting the far-field tonal noise produced by a propeller at angle of attack.

3. Development of a numerical routine to post-process CFD data specifically to the delivered format. This includes the extraction of the flow disturbances associated with each rotor in moving and stationary frames, around the blades and in the inter-rotor region.

4. Development of a hybrid analytical/computational method for predicting wake interaction noise. The novel contribution of this method is the analytic propagation of CFD wakes in order to calculate the downstream blade response and the radiated noise. The assessment of the analytic development of the wake characteristics in the inter-rotor region relative to that given by the CFD.

5. Extension of existing methods for predicting the noise produced by potential field interactions in an AOR. The method models the potential field of each rotor blade using a distribution of sources along the rotor blade chordline.
Chapter 2

Model scale open rotor data analysis

A number of experimental test campaigns using model scale advanced open rotors have recently been undertaken in the German-Dutch Wind Tunnels (DNW) Large Low-speed Facility (LLF), the Netherlands. The first of the recent experimental campaigns was undertaken in 2008 as a part of the EU project DREAM ("valiDation of Radical Engine Architecture SysteMs"), and used Rolls-Royce’s one-sixth scale advanced open rotor rig (known as Rig 145 Build 1). A second experimental campaign using the same rig but with aeroacoustically optimised blades (known as Rig 145 Build 2) was undertaken in 2010. More recently, tests were run as a part of the Clean Sky Joint Technology Initiative (JTI) EU programme in 2012-2013 using a one-seventh scale advanced open rotor rig designed by Airbus (known as the Z08 rig) at the DNW wind tunnel. The tests of interest in this Chapter were conducted on Airbus’ Z08 rig mounted with blades designed by Rolls-Royce. Extensive analysis was performed for different pitch and thrust settings. The author’s contribution to this study is described in this chapter. The principal tasks involved the post-processing and the analysis of experimental data from the Rig145 Build 2 and Z08 experimental test programmes.

2.1 Description of the wind tunnel facility

The DNW LLF wind tunnel is located in Marknesse, the Netherlands. It is a closed circuit, continuous low-speed wind tunnel in which flight operating conditions such as take-off, approach and cutback can be tested. The forward flight simulation
is produced by an open jet of variable cross-section. It is capable of providing continuous jet velocities up to 152 m/s for the smallest jet section (6 m x 6 m).

The instrumentation used for both the Rig 145 Build 2 and Z08 experiments comprises far-field out-of-flow microphone arrays located in the roof, walls, floor and the door of the facility, as well as inflow microphones mounted on a traversing system specific to each test campaign. A description of the set-up for the Rig 145 Build 2 tests is shown in figure 2.1.

![Figure 2.1: Schematic showing the experimental test set-up during testing of Rig 145 Build 2 in the DNW wind tunnel. Published in Parry et al. (2011).](image)

### 2.1.1 Rig 145 Build 2 test campaign

The objective of this campaign was to measure and analyse the noise produced by a contra-rotating open rotor and to assess the quality of the acoustic measurement method. The experiments were conducted using Rolls-Royce’s one-sixth scale advanced open rotor with respectively 12 and 9 front and rear rotor blades. The series of tests was run with an open jet nozzle measuring 8 m x 6 m with a Mach number of approximately 0.2. A set of 1/2” and 1/4” inflow microphones was mounted on a C-shaped traversing frame located at a fixed sideline distance from the open rotor rig. The data collected from the inflow microphones was recorded as the traverser translated upstream and downstream of the open rotor rig. This enabled coverage at a wide range of polar and azimuthal locations. Similarly, a
series of out-of-flow microphones were mounted on the traverser. A photograph of the experimental setup is shown in figure 2.2, and the traversing system is described in figure 2.3.

![Figure 2.2: Rig145 Build 2 installed in the DNW wind tunnel. Published in Parry et al. (2011).](image)

The experiments were conducted for the following configurations:

- Isolated rig (uninstalled): The effect of varying parameters such as the flow Mach number, the propeller rotational speeds, the blade pitch angles was investigated.

- Isolated rig with variation of the angle of attack: The same parameters as the uninstalled case were investigated.

Note that the previous Rig145 Build 1 experiments also tested an isolated rig with a pylon. The same parameters as the uninstalled case could be modified as well as the location of the pylon. A set up including an Airbus designed blowing system was also added. Its effect was investigated and described in details by Ricouard et al. (2010), see section 2.3.3.
2.1.2 Z08 rig test campaign

The aim of the Z08 experimental campaign was to investigate the effects of installation on the sound emitted from an advanced open rotor, and to measure the noise produced by a number of different AORs operating at different conditions. This campaign was, with the support of the EU funding Clean Sky, a collaboration between the manufacturers Airbus, Rolls-Royce and Snecma. The engine used for the experiments was Airbus Z08 air turbine driven rig, for which different sets of blades were provided by Airbus, Rolls-Royce and Snecma. The present Chapter focuses on the Rolls-Royce/Airbus phase. The rotor geometry was taken from open rotor one-seventh scale 12x9 blades designed by Rolls-Royce to fit onto the Z08 rig. The data from the Rolls-Royce/Airbus phase is jointly owned by Airbus and Rolls-Royce, who have given the author the permission to use them in the present thesis. Note that the Airbus phase (utilizing Airbus’ generic 11x9 blades called AI-PX7) is described further in details by Paquet et al. (2014).

The inflow microphones used to measure the pressure levels around the engine were mounted on a wing-shaped horizontal traverser (see an example of a test configuration in figure 2.6). This instrumentation comprised a set of inflow wire-mesh and flush-mounted microphones. The wire-mesh microphones are mounted in a cavity with the upper surface closed by a porous mesh (see figure 2.4). The presence of the mesh suppresses the noise due to the turbulent flow at the panel surface. The mesh is relatively impermeable to the flow passing over the traverser but is effectively acoustically transparent. Pressure perturbations due to the turbulent...
hydrodynamic flow decay evanescently within the cavity. Thus the microphone at the bottom of the cavity measures little flow noise. However, because the microphone is located within the cavity, the signal measured by these microphones will be dependent on the angle of incidence of the acoustic waves. This effect is not taken into account in the analysis presented here and may account for some of the differences observed between the measurements of the wire-mesh and the flush mounted microphones in Section 2.4.2.

The flush-mounted microphones have their diaphragm aligned with the upper surface of the traverser airfoil and so these microphones measure the pressure perturbation due to the turbulent boundary layer in addition to the incident acoustic pressure (see figure 2.5). They can capture the tonal noise in all directions, provided that the level of the tones is greater than the aerodynamic noise.

The test data was collected for the following configurations:

- **Isolated rig (uninstalled):** Test parameters such as the flow Mach number, the propeller rotational speeds, the blade pitch angles, the inter-rotor spacing, and number of blades could be modified. Additional sets of blades were provided by Rolls-Royce and Snecma in order to assess the engine’s acoustic sensitivity to the blade design.

- **Isolated rig with variation of the angle of attack:** The same parameters as the uninstalled case could be modified.

- **Rig installed with a downstream wing (puller configuration):** The wing sweep, angle of attack, distance from the propellers and the parameters previously cited could be modified.
• Rig attached to an upstream pylon (pusher configuration): The position of the pylon and the same parameters as the uninstalled case could be modified.

• Full aircraft tests. In addition to the parameters previously cited, the presence of landing gear, the slat and flap settings and parameters for pylon blowing could be modified.

Investigation of the effect of the last two configurations was recently published by Paquet et al. (2014) who confirmed that the pylon blowing system can be used to significantly reduce the noise caused by the interaction of the pylon wakes with the front rotor blades.

Figure 2.6: Z08 rig installed with a pylon in the DNW LLF wind tunnel. Note that in this picture the rig is mounted with Airbus’ blades. Published in Paquet et al. (2014).
2.2 Acoustic data processing

The data analysis was conducted using a post-processing routine developed by the author. This routine is suitable for analysing either Rig 145 or Z08 data, as the data format is common for both test campaigns. Note that a preliminary correction for propagation distance was made by Rolls-Royce so that each microphone is calibrated to measure the noise of the contra-rotating propeller at a reference radiation distance (emission radius) of 16.6 meters. The correction was performed using the inverse-square law based on the principle that the acoustic intensity is inversely proportional to the square of the radiation distance. To be applicable, the method assumes a spherical spreading of the sound waves and no reflections or reverberation effects in the wind tunnel. Note that as far-field results for an AOR are generally expressed in terms of emission coordinates, all the measurements are presented in the emission coordinate system. A conversion from reception to emission coordinates is described in the following section.

2.2.1 Emission and reception coordinate systems

When the aircraft is flying, the noise emission time differs from the reception time at the observer position. In fact, when the observer receives the noise at time \( t \), it corresponds to a position of the aircraft at time \( t - \Delta t \) where \( \Delta t = r_e/c_0 \), \( r_e \) is the distance at emission time between the observer and the aircraft, and \( c_0 \) the sound propagation speed. In propeller noise studies, we often use the emission (or retarded) coordinate system. Figures 2.7(a) and 2.7(b) describe the emission and reception coordinate systems for the flyover case and in a wind tunnel. In a wind tunnel, as the propeller is fixed in a moving flow, the emission position will correspond to a virtual source position.

Let \((x_r, r, \theta)\) and \((x_e, r_e, \theta')\) be the axial, radial and polar coordinates at reception and emission time respectively. The origin of both coordinate systems is defined on the engine axis and half way between the two rotors. The coordinate systems are represented in figure 3.3. Let \( M \) be the flow Mach number in the positive axial direction, and \( R \) the sideline radius. As the sound wave propagates during the time \( \Delta t \), the acoustic wave front is convected by the flow by a distance \( \Delta r = Mr_e \).
(a) Flyover experiments, the source and observer are moving at flight Mach number $M$ relatively to the medium which is at rest.

(b) Wind tunnel experiments, the source and observer are fixed relatively to the uniformly moving medium at flow Mach number $M$.

Figure 2.7: Emission and reception coordinates for the flyover (a) and wind tunnel (b) testing cases.
The reception coordinates $x_r$ and $r$ can be defined in terms of emission coordinates as

$$x_r = -r_e \cos \theta' + \Delta_r = r_e (\cos \theta - M),$$  \hspace{1cm} (2.1)

$$R = r_e \sin \theta'.$$  \hspace{1cm} (2.2)

To convert from reception to emission coordinates we consider the relationships

$$x_e = -x_r + \Delta_r = -R \cos \theta',$$  \hspace{1cm} (2.3)

$$r_e^2 = (-x_r + \Delta_r)^2 + R^2 = (Mr_e - x_r)^2 + R^2,$$  \hspace{1cm} (2.4)

$$\theta' = \cos^{-1} \left( \frac{Mr_e - x_r}{r_e} \right).$$  \hspace{1cm} (2.5)

Equation 2.4 leads to the following quadratic equation

$$(M^2 - 1)r_e^2 - 2x_rMr_e + (x_r^2 + R^2) = 0.$$  \hspace{1cm} (2.6)

The solutions of equation 2.6 are given by

$$r_e = \frac{x_rM \pm \sqrt{x_r^2M^2 - (M^2 - 1)(x_r^2 + R^2)}}{M^2 - 1}.$$  \hspace{1cm} (2.7)

Only the solution which gives positive values of $r_e$ is used in the present case since a negative emission radius would be nonphysical.

### 2.2.2 Post-processing methods

The post-processing method developed by the author consists in five steps. These are described in the following and illustrated in figures 2.8 to 2.12.

**Correction for background noise**

The data collected during the tests comprise a set of 'raw' measured noise data of the operating AOR and a set of background noise levels. For each test, the corresponding background noise levels were measured running a 'blank' test at identical Mach number during which the engine was driven
without any blades attached. These background noise levels were thus subtracted from the given measured or ‘raw’ data using the equation

\[ SPL_{\text{corrected}} = 10\log \left( \frac{10^{SPL_{\text{measured}}/10} - 10^{SPL_{\text{background}}/10}}{10^{SPL_{\text{background}}/10}} \right). \]  

(2.8)

Note that the two peaks observed at the lowest frequencies are believed to be associated with background noise and do not contribute to the tonal noise emitted by the AOR. However, this contribution remains apparent on the spectrum after the removal of the background noise as its levels were close but not equal to the ‘raw’ measured noise levels.

![Figure 2.8: Background noise correction.](image)

**Calculation of broadband noise levels**

Broadband noise levels were deduced from the background corrected data using a moving median method. This consisted of calculating the median for a sample of frequencies and incrementing over the whole frequency range of the noise spectra. As the tones are expected to have distinctly higher sound pressure levels than the broadband noise, their contribution would not affect the median value of each sample. This calculation was performed for the whole available range of polar angles.

![Figure 2.9: Broadband noise calculation.](image)
Identification of the tones

In order to distinguish the tones from the broadband noise in the measured spectra using an automated procedure, a filter was applied to select only the background corrected data whose levels were 3 dB above the broadband noise levels.

Correction for broadband noise

The contribution of the broadband noise was removed from the filtered data by subtracting the broadband noise levels from the tones data in a similar manner as equation 2.8 (yielding to slightly different sound pressure levels close to the broadband noise levels, as shown in figure 2.11).
Unbroadening of the tones

The data obtained from the filtering tended to be spread amongst the frequencies neighbouring the relevant tone frequency. The tones’ levels were calculated by summing the energies contained within the adjacent frequency bands, thereby deducing the level at each discrete frequency tone.

![Figure 2.12: Tones unbroadening.](image)

The post-processing method determines the polar directivities of any tone, provided that their level is high enough relatively to the background noise level. In addition, the method provides broadband noise directivity patterns at any frequency, and noise spectra at any polar angle within the measured range. The noise spectra were initially measured at successive frequency bandwidths of 6.25Hz. A conversion to one-third octave band spectra was implemented in order to have a quick overview of the tones and broadband noise contribution to the total noise.
2.3 Measurement issues

2.3.1 Haystacking

The present analysis is restricted to inflow data as the measurement of tones by out-of-flow microphones could be altered by reflections from nearby structures, background noise, atmospheric absorption, refraction and most importantly haystacking. Haystacking, or the spectral broadening of tones, occurs when the sound waves propagate through the jet shear layer. The tones amplitudes are reduced and, centered on each tone, a broadband bump or "haystack" is measured in the narrow band spectrum. The haystacking is considered 'weak' when the proportion of scattered energy is small compared to the energy remaining in the tone and 'strong' when the tone becomes undetectable and is replaced by a broadband hump. In wind tunnel experiments, the measurements from out-of-flow microphones can be significantly affected by haystacking. Figure 2.14 shows a comparison of 'raw' measurements captured from an inflow and an out-of-flow microphone during the Rig 145 tests. The spectral broadening of the tones captured by the out-of-flow microphones is clearly visible and tends to become greater at high frequencies. Two effects can be observed:

![Figure 2.13: Scattering effects in a wind tunnel.](image-url)
1. A loss in the tones protrusion.

2. An increase in the broadband noise levels.

These effects are a specific issue for out-of-flow measurements, which can lead to errors in the evaluation of the tones and broadband noise. As the correction of such data is not of concern here, the analysis is focused on data measured by inflow microphones.

![SPL spectrum comparison](image)

Figure 2.14: Narrow band SPL spectrum at 90° of an inflow and an out-of-flow microphone at the same take-off case - Rig 145 tests.

The prediction of tones haystacking has been of concern in recent studies. McAlpine et al. (2013) developed an analytical model to predict and quantify the spectral broadening of the tones radiating from the exhaust nozzle of turbofan engines. This method was developed using a weak-scattering formulation (Powles et al., 2010) and provided encouraging results for which the features of the tones broadening were well reproduced. This method can be applied to predict the tones detected by the out-of-flow microphones provided that the scattering is ‘weak’. In figure 2.14 weak scattering is observed at low to medium frequencies. However once we move to the high frequency range, the tones energy collapse into the haystacks (strong scattering) and numerous tones cannot be detected. Numerical approaches were developed by Ewert et al. (2008) and more recently Clair and Gabard (2015) who, based on the work of Ewert, analysed the scattering of a
harmonic monopole by a synthesized turbulent layer and investigated the effect of
the mean flow velocity and source frequency on the scattering. It was highlighted
that an increase in the source frequency produces stronger scattering effects. Al-
though an interesting study would be the modelling of strong scattering of tones
in order to predict the phenomenon occurring at high frequencies in figure 2.14,
the recovery of the tones by unbroadening would be challenging if several tones
are scattered in the same haystack.

2.3.2 Measurements uncertainties

The data collected from the inflow microphones were exempt from haystacking
effects. However, the microphones used for the Rig 145 experiments suffered from
scattering effects from the vertical bar of the traversing system on which the
microphones were mounted. The microphone locations are shown in figure 2.15.
The scattering of the tones manifested itself upstream of the advanced open rotor,
at polar angles below 60°. The changes in the polar directivity pattern of the
first rotor-alone tone of the front rotor can be distinctly seen in figure 2.16 for
the microphones A, B, and C (which are labelled in figure 2.15). Note that polar
angles were measured from the AOR axis, 0° and 180° corresponding respectively
to the locations upstream and downstream of the engine.

![Microphone set-up on the Rig 145 inflow traversing system](source: Parry et al. (2011)).

Despite the correction for radiation distance applied to the inflow microphones,
figure 2.16 shows evidence that the variation measured in the sound pressure levels
is seemingly quite large (typically ± 5 dB). Thus one should be careful when using
the data owing to the large variance in the measurements.
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Figure 2.16: Polar directivity pattern of the first front rotor alone tone measured at a typical takeoff case by a selection of inflow traversing microphones and at a given emission radius - Rig 145.

The measurement disparities between the microphones are shown in terms of errorbars, see the polar directivity pattern in figure 2.17. Note that this excludes the obviously faulty microphones. In this figure, the continuous line corresponds to the median of the values of the sound pressure level measured by the different

Figure 2.17: Polar directivity of the first rotor alone tone measured by the inflow microphones at a typical take-off case - Rig 145 build 2.
microphones, and the errorbars show the maximum and minimum sound pressure levels. The results obtained from Z08 rig experiments showed similar variations in levels between each microphone’s measurement but did not exhibit evidence of scattering of the tones by the traverser.

2.4 Results

2.4.1 Rig 145 tests

This section presents some illustrative examples of results obtained using the post-processing method described in section 2.2.2. As a starting point, only the uninstalled data i.e. measurements from the isolated rig at zero angle of attack have been treated. Note that the results presented in the present section were from a single inflow microphone. As tones occur at the sum or difference of the BPF of each rotor, the tones will be labelled \( \{n_1, n_2\} \) where \( n_1 \) and \( n_2 \) are respectively integers associated with the front and rear rotors. For example, the label \( \{1, 0\} \) denotes the first front rotor alone tone and the notation \( \{1, 2\} \) corresponds to an interaction tone that occurs at a frequency which is the sum of the front rotor BPF and twice the rear rotor BPF. A typical SPL spectrum in which the tones are identified is shown in figure 2.18.

![Figure 2.18: A typical spectrum with tones annotations for a particular take-off case](image-url)
Influence of rotor speed

Test data was collected for several “pitch-lines”. One pitch-line corresponds to one specified blade setting. Each pitch-line is denoted by a label \( P_1 \times P_2 \) corresponding to the increase in rotational speed (RPM) of the rotor at take-off relative to its design cruise rotational speed. We denote \( F \) the speed-up factor. A series of take-off, approach and cutback cases were simulated for each pitch line with different rotational speeds in order to reach the required full-scale thrust and a specific torque-split between the two rotors. Although the main objective of the data analysis is to be able to validate the current and future prediction methods against experimental data, it is interesting to assess the influence of varying the rotors’ rotational speed on the noise. Indeed, as the same thrust can be reached using different rotor blade pitch settings and rotational speeds, these parameters may be of interest to optimise flight operating conditions.

Figures 2.19 to 2.21 show the polar directivity of the first rotor-alone tone produced by the front rotor, \( \{1, 0\} \), and also some interaction tones at take-off, approach and cutback at a constant emission radius. As expected, the sound pressure level is higher as the thrust is larger, the take-off case being the loudest. Note that the tones at approach condition could not be captured properly as the measurements were too close to the background noise levels. The directivity pattern of the rotor-alone tone observed in figure 2.19 is that of a single “lobe” whose pressure peaks

\[
\begin{array}{c}
\text{SPL} \\
20 40 60 80 100 120 140 \\
\text{Polar emission angle (°)}
\end{array}
\]

Figure 2.19: Polar directivity of the tone \( \{1, 0\} \) at typical take-off, approach and cut-back conditions - Rig 145.
around 80° and decays strongly away from the peak pressure. The directivity pattern of interaction tones shown in the following figures can be seen either as a single lobe or a succession of lobes.

Figure 2.20: Polar directivity of the tone \{1,1\} at typical take-off, approach and cut-back conditions - Rig 145.

Figure 2.21: Polar directivity of the tone \{1,2\} at typical take-off, approach and cut-back conditions - Rig 145.
The effect of varying the rotors rotational speed (keeping the thrust constant) is shown in figure 2.22 for the front rotor fundamental blade passing frequency \{1, 0\}, and in figure 2.23 for the first rotor-rotor interaction tone \{1, 1\}.

Figure 2.22: Polar directivity of the \{1, 0\} tone at take-off at different rotational speed settings - Rig 145.

Figure 2.23: Polar directivity of the \{1, 1\} tone at take-off at different rotational speed settings - Rig 145.
Figure 2.24: Polar directivity of the \( \{1, 2\} \) tone at take-off at different rotational speed settings - Rig 145.

Clearly, the sound pressure level of the front rotor rotor-alone tone increases with the rotational speed. Indeed, as the blades’ velocity relative to the flow increases, the lift forces acting on the blades become greater. As the mechanisms inducing interaction tones are more intricate, figures 2.23 and 2.24 don’t exhibit any obvious effect related to the increase of the rotors’ rotational speed. It is interesting to note that at the same given thrust setting the polar directivity of the interaction tones can significantly vary at some frequencies and observer positions depending on which pitch-line is considered. Due to the complexity of the physics behind this phenomenon the results presented hereby are for illustrative purpose only. The behaviour of the flow and different sources of interaction tones will be investigated in greater details in the following chapters.

**Relative contribution of broadband and tone noise**

In many studies of AOR noise the contribution of broadband noise was neglected due to the spectra dominated by the tones. A typical spectrum is plotted in figure 2.25 and shows a significant number of tones. However, as efforts have been concentrated on reducing the tonal noise emissions of AORs, the broadband noise may have a greater impact on the total sound pressure levels (Parry et al., 2011).
To investigate the relative contribution of tones and broadband noise to the total sound pressure level, one third octave band spectra are plotted for typical take-off, cutback and approach cases at $\theta = 90^\circ$ for a given emission radius. The results are shown in figures 2.26 to 2.28. The results show that the broadband noise is significant on a One-Third Octave Band (OTOB) basis over a significant portion of the spectra.

In the take-off case shown in figure 2.26, the first tone (the tone with the lowest frequency) corresponds to the first BPF of the rear rotor, i.e. the \{0, 1\} tone. Its amplitude is relatively close to the broadband noise level. The first tone that appears in the cutback case shown in figure 2.27 has a level is 3dB higher than the broadband noise level and corresponds to the \{1, 0\} tone. The \{0, 1\} tone was not visible above the background noise for this particular case.

At approach, the first rotor-alone tone of the front and the rear rotor was not distinguishable from the background noise, and the first tonal contribution is due to the first rotor-rotor interaction tone, whose frequency lies in 1250Hz OTOB. The results shown in figure 2.28 demonstrate that at approach the broadband noise level is generally higher than the tonal component.
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Figure 2.26: One-third octave band spectra at 90 degrees (take-off).

Figure 2.27: One-third octave band spectrum at 90 degrees (cutback).

Next, the effect of varying polar angle on the resulting noise spectrum is investigated. The results are presented in figure 2.29 for three different angles.

It is noticeable that the tones are highly directional with variations in the sound pressure levels of up to 22 dB being observed. Although some more sizeable
variations are noticeable at low frequencies, the broadband noise levels tend to be relatively constant with a maximum change in amplitude of about 2 dB.
Comparison with tonal noise predictions

This section provides some examples of comparisons between Rig 145 measurements and predictions calculated using currently available methods. The predictions were performed using the current prediction tool used by Rolls-Royce. The code was developed by M. Kingan (ISVR) and is based on the Landahl method described by Parry. The prediction tool provides the tonal noise predictions for all types of AOR noise sources, as well as calculations of the broadband noise components. The tool also includes a model developed by Kingan and Sureshkumar (Kingan et al. (2010), Kingan and Sureshkumar (2014)) which accounts for the acoustic scattering of the tones from the centerbody of the AOR engine. Illustrative results for a typical take-off case are shown in figures 2.30 and 2.31 for the first rotor alone tone of the front rotor and the first interaction tone respectively. These results were published by Kingan et al. (2014).

![Figure 2.30: Comparison with predictions - rig145.](image)

The predictions show a reasonable agreement with the experimental data. As expected, the loading noise is the principal source of rotor-alone tone noise, especially at take-off when the engine is highly loaded. The \{1, 0\} tone is essentially dominated by the viscous wake interaction, although the forward potential (interaction of the front rotor blades with the rear rotor bound potential field) has a significant contribution at some observer angles, as shown in figure 2.31.
2.4.2 Z08 rig tests

The analysis performed on the Z08 data is similar to that performed on the Rig 145 data. The polar directivity of tones and broadband noise, narrow band and one-third octave band spectra for different rotational speeds, pitch and thrust settings can be provided. The polar directivity patterns of rotor-alone and interaction tones appear reasonable. Consequently, only a few illustrative examples are shown in this section. An important feature of this analysis is that, whereas the Rig 145 measurements suffered from scattering by the C-shaped traverser, there was no evidence of scattering effects in the measurements taken using the horizontal traversing system (see figure 2.6) during the Z08 tests. Thus, it is believed that the Z08 data will provide a high quality assessment of the noise field of an advanced open rotor.

However, as for the Rig 145 data, the results for a particular tone displayed differences in the sound pressure levels measured by each microphone. Figures 2.32 and 2.34 show an illustration of the discrepancies between microphones for the polar directivity patterns of a selection of tones. The continuous line and the errorbars respectively show the median and the maximum and minimum values of sound pressure level measured by all the microphones. The measurements from both wire-mesh and flush mounted microphones are plotted separately.
The results show a satisfactory agreement between the noise levels and directivity patterns measured by the wire-mesh and the flush mounted microphones. The median values of the range of data measured by the flush-mounted microphones are over most of the frequency range within the measurement range of the wire-mesh microphones.
Similar representation can be applied to compare the relative contributions of broadband noise and tones for Z08 measurements. A one-third octave band spectrum for a typical take-off case, measured by the wire-mesh microphones, is shown in figure 2.35. Differences in sound pressure levels up to 5 dB can be observed within a one-third octave frequency band. It can be seen that the tones can be up to 10 dB higher than the broadband noise levels.

Figure 2.34: Polar directivity of the \{1, 2\} interaction tone at take-off measured by wiremesh microphones - Z08.

Figure 2.35: One-third octave band spectra at 90° measured by wire-mesh microphones (take-off).
2.5 Concluding remarks

The analysis of data collected during the Rig 145 and Z08 experimental test programmes provides polar directivity patterns of tones and broadband noise, as well as narrow band and one-third octave band spectra at typical take-off, approach and cutback cases. Preliminary comparisons show a good agreement with the current analytical prediction model. The experimental results shown in this chapter have been published by Kingan et al. (2014). The post-processing enables further analysis of Rig 145 and Z08 data, to show the contribution from the different sources of noise from the AORs. The analytical models presented in the following sections will be compared with wind tunnel measurement data processed using the methods described in this chapter.
Chapter 3

Loading noise produced by a single rotor at angle of attack

3.1 Steady loading

The steady loading noise is caused by the component of the aerodynamic steady forces exerted by the blades on the air. The prediction method presented here is based on Hanson’s derivation of far-field loading noise produced by a propeller at angle of attack (Hanson and Parzych, 1993) with a simplified blade geometry. The starting point of Hanson and Parzych’s model is the acoustic analogy of Goldstein (1974) which states that the acoustic density disturbance $\rho'$ at a location $x$ at time $t$ in a moving medium with constant velocity in any volume $\nu$ bounded by surfaces $S$ in arbitrary motion is expressed as

$$c_0^2 \rho(x,t) = \int_{-T}^{T} \int_{S(\tau)} \rho_0 V'_n \frac{D_0 G}{D_\tau} dS(y) d\tau$$

$$+ \int_{-T}^{T} \int_{S(\tau)} f_i \frac{\partial G}{\partial y_i} dS(y) d\tau + \int_{-T}^{T} \int_{\nu(\tau)} T'_{ij} \frac{\partial^2 G}{\partial y_i \partial y_j} dy d\tau,$$

where $G$ is a Green’s function which satisfies the convected wave equation. The source coordinates and time are respectively $y = (y_1, y_2, y_3)$ and $\tau$. The first term of this equation corresponds to the thickness noise where $V'_n$ is given by $V'_n = V_n - n_i U_i$ with $U_i = [U_1, U_2 = 0, U_3 = 0]$. The terms $V_n$ and $U_i$ are
respectively the velocity of the blade surface in a direction normal to the blade surface and the velocity of the fluid (which is constant in time and space). The unit normal \( n_i \) is pointing outward (away from \( \nu \)) into the surface \( S \). The second and third terms of equation (4.1.1) are respectively the loading noise and a quadrupole noise term where \( f_i \) is the force per unit area exerted by \( S \) on the air and \( T'_{ij} \) the Lighthill’s stress tensor in a moving reference frame. These are defined by the following equations

\[
T'_{ij} = \rho v'_i v'_j + \delta_{ij}[(p - p_0) - c_0^2(p - p_0)] - e_{ij},
\]

\[
f_i \equiv n_i (p - p_0) + n_j e_{ij},
\]

in which \( \rho = \rho_0 + \rho' \) is the density (\( \rho_0 \) is the density in the undisturbed fluid), \( v_i \) is the fluid velocity, \( e_{ij} \) is the viscous stress tensor, \( \delta_{ij} \) is the Kronecker delta and \( p' = p - p_0 \) is the pressure perturbation (\( p_0 \) is the pressure in the undisturbed fluid). The velocity in the moving frame is:

\[
v_i = v_i - \delta_{i1} U_1.
\]

The convective derivative is given by:

\[
\frac{D_0}{D_\tau} \equiv \frac{\partial}{\partial \tau} + U_i \frac{\partial}{\partial y_i}.
\]

The observer coordinates are \( x = (x_1, x_2, x_3) \). The aircraft is fixed with respect to the coordinate system and the flow is moving at a uniform Mach number \( M_x \) in the \( x_1 \) direction. The Cartesian source coordinates in the flight system are \( y = (y_1, y_2, y_3) \) and \( y' = (y'_1, y'_2, y'_3) \) in the propeller system, see Appendix B. The rotation of a point on a blade can be specified in cylindrical coordinates as a function of the radius \( r'_y \), and axial and tangential positions \( y'_1 \) and \( \phi'_y \). The configuration is shown in figure 3.1.

Consider the loading noise term of equation (3.1) in which \( S(\tau) \) represents the surface of the moving blade. It is convenient to perform the integration in blade-fixed coordinates, where the source position on the blade is specified by the coordinates \( r'_y s, \phi'_y s \) and \( y'_1 s \). Consider unwrapped blade sections at a radius \( r_y = r'_y s \) onto a plane \( (r_y \phi'_y, y'_1) \), as shown in figure 3.2. In this system, the motion of the blade at rotational speed \( \Omega \) involves only the coordinate \( \phi'_y \) which is defined by
\[ \phi_y' = \phi_{ys}' + \Omega \tau. \]

The force \( f_i \) is a function of \( r_y, \phi_y' \) and \( y_1' \) only, and the Green’s function derivative is evaluated in terms of \( r_y = r_{ys}', \phi_y' \) and \( y_1' = y_{1s}' \). The surface \( S \) is no longer dependent on \( \tau \) and the loading component can be written

\[
p_L(x, t) = \int_S \int_{-T}^{T} \left[ f_i(\Omega \tau) \frac{\partial G}{\partial y_i} \right]_{\phi_y' = \phi_{ys}' + \Omega \tau} d\tau dS(y). \tag{3.6}
\]

Whereas Hanson expresses the locations of source area elements in terms of radial, axial and tangential coordinates along the blade geometry, the present model uses a flat plate approximation. The blade is assumed to be infinitely thin so the loading forces are represented as net lift and drag forces per unit area acting normal to the blade chordline and the integration is performed along the blade span and chordwise coordinates. The blade is also assumed to be aligned with the local flow direction. Note \( \bar{X} \) the non dimensional chordwise coordinate normalized against the half-chord \( \left( \bar{X} = \frac{2X}{c} \right) \). According to this approximation \( p_L(x, t) \) is expressed as

\[
p_L(x, t) \approx \int_{r_h}^{r_l} \int_{-1}^{1} \int_{-T}^{T} \left[ F_i(\Omega \tau) \frac{\partial G}{\partial y_i} \right]_{\phi_y' = \phi_{ys}' + \Omega \tau} d\tau \frac{c}{2} d\bar{X} dr_y, \tag{3.7}
\]

where \( F_i \) is the lift and drag force per unit area acting on the blade chordline.
Chapter 3 Loading noise produced by a single rotor at angle of attack

The pressure \( p_L(x, t) \) is periodic with period \( \frac{2\pi}{\Omega} \), therefore it can be expressed as a Fourier series:

\[
p_L(x, t) = \sum_{n=-\infty}^{+\infty} P_{Ln}(x) \exp \{ -in\Omega t \},
\]

with the Fourier coefficients \( P_{Ln} \) given by:

\[
P_{Ln}(x) = \frac{\Omega}{2\pi} \int_{0}^{\frac{2\pi}{\Omega}} p_L(x, t) \exp \{ in\Omega t \} \, dt.
\]

The time dependant Green’s function in the free space in a moving medium at Mach number \( M_x \) is given by Garrick and Watkins (1954) by

\[
G = \frac{\delta(t - \tau - \sigma/c_0)}{4\pi S},
\]

where \( S \) is the amplitude radius

\[
S = \sqrt{(x_1 - y_1)^2 + \beta^2[(x_2 - y_2)^2 + (x_3 - y_3)^2]},
\]

and \( \beta \) and \( \sigma \) are respectively the compressibility factor and the phase radius given by

---

**Figure 3.2: Flat blade approximation.**

\( f_i \): force exerted by the blades on the fluid
\[ f_i \rightarrow F_i = L_i + D_i \]

\( L_i \): Lift force exerted by the blades on the fluid
\( D_i \): Drag force exerted by the blades on the fluid

---
\[
\beta = \sqrt{1 - M_x^2}, \quad (3.12)
\]
\[
\sigma = \frac{M_x(x_1 - y_1) + S}{\beta^2}. \quad (3.13)
\]

Thus inserting the time dependant Green’s function from equation (3.10) into equation (3.7) and using equation (3.9) yields

\[
P_{Ln}(x) = \frac{\Omega}{2\pi} \int_{r_h}^{r_{t1}} \int_{-1}^{1} \int_{-T}^{T} \left[ F_i(\Omega\tau) \frac{\partial}{\partial y_i} \left( \frac{1}{4\pi S} \exp \left\{ \frac{i\Omega}{c_0} \sigma \right\} \right) \right] \phi'_{\gamma = \phi'_{\gamma_s} + \Omega \tau} \frac{c}{2} d\tau d\bar{X} dr_y. \quad (3.14)
\]

However, the limits of the integration over \( \tau \) are indeterminate. To avoid this one revolution of the rotor is considered, for example take values of \( \tau \) between 0 and \( 2\pi/\Omega \), which corresponds to one period of the signal received from a time \( t_1 \) to \( t_1 + 2\pi/\Omega \). So the noise pulse will be equal to \( p_L(x,t) \) for \( t_1 \leq t < 2\pi/\Omega \) and will be zero outside this interval. Thus the following can be written

\[
P_{Ln}(x) = \frac{\Omega}{2\pi} \int_{-\infty}^{+\infty} P_L(x,t) \exp \{i\Omega t\} dt. \quad (3.15)
\]

Applying this in equations (3.14) gives

\[
P_{Ln}(x) = \frac{\Omega}{2\pi} \int_{r_h}^{r_{t1}} \int_{-1}^{1} \int_{0}^{2\pi/\Omega} \left[ F_i(\Omega\tau) \frac{\partial}{\partial y_i} \left( \frac{1}{4\pi S} \exp \left\{ i\Omega \frac{\sigma}{c_0} \right\} \right) \right] \phi'_{\gamma = \phi'_{\gamma_s} + \Omega \tau} \frac{c}{2} d\tau d\bar{X} dr_y. \quad (3.16)
\]

The harmonic Green’s function is denoted by

\[
G_n = \frac{\exp \{i\Omega \sigma/c_0\}}{4\pi S}. \quad (3.17)
\]

This leads to the following expression:
\[ P_{Ln}(x) = \frac{\Omega}{2\pi} \int_{r_h}^{r_t} \int_{-1}^{1} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} F_i(\Omega\tau) \frac{\partial G_n}{\partial y_i} \left[ \phi_y - \phi_{ys} + \Omega \tau \right] \exp \left\{ i n \Omega \tau \right\} \frac{c}{2} d\tau d\bar{X}d\bar{y}, \quad (3.18) \]

The development of \( P_L(x, t) \) deals with only one blade. For a propeller with \( B \) blades it becomes

\[ p_{LB}(x) = p_L(x, t) \sum_{q=0}^{B-1} \exp \left\{ i 2\pi nq/B \right\}, \quad (3.19) \]

where the sum over \( m \) equals the constant \( B \) if \( n/B \in \mathbb{Z} \) and is equal to zero elsewhere. This means that the sound will radiate only if \( n \) is a multiple of \( B \). Denote \( n = mB \). Thus for a propeller with \( B \) equally spaced blades, each blade encounters identical loading and radiates only at the blade passing frequencies \( (n = mB) \). Equation (3.18) becomes

\[ P_{LmB}(x) = \frac{B\Omega}{2\pi} \int_{r_h}^{r_t} \int_{-1}^{1} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} F_i(\Omega\tau) \frac{\partial G_{mB}}{\partial y_i} \left[ \phi_y - \phi_{ys} + \Omega \tau \right] \exp \left\{ imB\Omega \tau \right\} c d\tau d\bar{X}d\bar{y}. \quad (3.20) \]

The lift force per unit area can be expressed as a Fourier series of the form

\[ F_i(\Omega\tau) = \sum_{k=-\infty}^{+\infty} F_k \exp \left\{ -ik\Omega \tau \right\}, \quad (3.21) \]

\[ F_k = \frac{\Omega}{2\pi} \int_{0}^{2\pi} F_i(\Omega\tau) \exp \left\{ ik\Omega \tau \right\} d\tau. \quad (3.22) \]

Inserting equation (3.21) into equation (3.20) and changing the variable of integration in the inner integral using \( \phi'_y = \phi_{ys} + \Omega \tau \) gives
\[ P_{lmB}(x) = \frac{B}{2\pi} \int_{r_h}^{r_i} \int_{-1}^{1} \exp \left\{ -i(mB - k)\phi'_{ys} \right\} \]
\[ \times \int_{0}^{2\pi} F_k \frac{\partial G_{mB}}{\partial y_i} \exp \left\{ i(mB - k)\phi'_{y} \right\} \frac{c}{2} d\phi'_{y} dX dy \]. \quad (3.23)

**Far-field Green’s function in the emission coordinate system**

In propeller noise studies, the emission (or retarded) coordinate system is often used, cf. the conversion from reception to emission coordinates in Chapter 2. From now, the far field Green’s function \( G_{mB} \) will be evaluated in terms of retarded coordinates. According to Hanson (1993) \( G_{mB} \) can be written in this system

\[ G_{mB} = \frac{\exp \left\{ ik_m(r_e - S_0y'_1) \right\}}{4\pi S_0} \exp \left\{ -ik_m S_s r_y \cos(\pm\phi' - \phi_y) \right\}, \quad (3.24) \]

where \( r_e \) is the radiation distance to the observer from the emission point, and the terms \( S_0, S_s \) and \( S_c \) are given by

\[ S_0 = r_e (1 - M_x \cos \theta), \quad (3.25) \]
\[ S_c = \frac{\cos \theta'}{1 - M_x \cos \theta}, \quad (3.26) \]
\[ S_s = \frac{\sin \theta'}{1 - M_x \cos \theta}, \quad (3.27) \]

and \( k_m = mB\Omega/c_0 \) is the harmonic wavenumber. The notation \( \theta' \) corresponds to the angle to the observer in retarded coordinates aligned with the propeller axis and \( \theta \) corresponds to the angle to the observer from the flight direction (see figure 3.3).

The partial derivatives \( f_i \frac{\partial G_{mB}}{\partial y_i} \) in vector notation correspond to \( \nabla \cdot \mathbf{G} \) which can be evaluated in cylindrical coordinates as
Chapter 3 Loading noise produced by a single rotor at angle of attack

\[
f \cdot \nabla G = \frac{\partial G_{mB}}{\partial r_y} + \frac{1}{r_y} \frac{\partial G_{mB}}{\partial \phi_y} + \frac{\partial G_{mB}}{\partial y'_1}, \tag{3.28}
\]

with

\[
\frac{\partial G_{mB}}{\partial r_y} = -ik_m S_s \cos(\pm \phi' - \phi_y) G_{mB}, \tag{3.29}
\]

\[
\frac{1}{r_y} \frac{\partial G_{mB}}{\partial \phi_y} = -ik_m S_s \sin(\pm \phi' - \phi_y) G_{mB}, \tag{3.30}
\]

\[
\frac{\partial G_{mB}}{\partial y'_1} = -ik_m S_c G_{mB}. \tag{3.31}
\]

Consider the loading in a two dimensional model and ignore drag components. The radial component of the loading force is then suppressed. Note \(F_{y'_1}^{(k)}\) and \(F_{\phi_y}^{(k)}\) the Fourier harmonics of the axial and tangential loading components respectively. With \(\phi'_y = \phi_y\) and inserting equations (3.30) and (3.31) into equation (3.23) yields
\[ P_{LmB}(x) = -ik_mB \int_{r_h}^{r_t} \int_{-1}^{1} \exp \left\{ -i(mB - k)\phi'_{ys} \right\} \exp \left\{ ik_m(r_e - Scy_1') \right\} \]

\[ \times \left[ \frac{1}{2\pi} \int_{0}^{2\pi} \sin(\pm \phi' - \phi_y) \exp \{i(mB - k)\phi_y\} \exp \{-ik_mScr_y \cos(\pm \phi' - \phi_y)\} \, d\phi_y \right] \frac{4\pi S_0}{S_s(F_{\phi_y}^{(k)})} \]

\[ + \frac{1}{2\pi} \int_{0}^{2\pi} \exp \{imB\phi_y\} \exp \{-ik_mScr_y \cos(\pm \phi' - \phi_y)\} \, d\phi_y \]

\[ \times \left( S_sI_{\phi_y}F_{\phi_y}^{(k)} + ScI_{y_1}F_{y_1}^{(k)} \right) \frac{c}{2} \bar{X} \, d\bar{X} \, dr_y. \]

This expression can be written as

\[ P_{LmB}(x) = -ik_mB \frac{\exp \{ik_mr_e\}}{4\pi S_0} \int_{r_h}^{r_t} \int_{-1}^{1} \exp \left\{ -i(mB - k)\phi'_{ys} \right\} \exp \left\{ -ik_mScy_1' \right\} \]

\[ \times \left( S_sI_{\phi_y}F_{\phi_y}^{(k)} + ScI_{y_1}F_{y_1}^{(k)} \right) \frac{c}{2} \bar{X} \, d\bar{X} \, dr_y, \]

with

\[ y_1' = \frac{c}{2} \bar{X} \cos \alpha, \]  

\[ \phi'_{ys} = \frac{c}{2} \bar{X} \frac{\sin \alpha}{r_y}, \]

and

\[ I_{\phi_y} = \frac{1}{2\pi} \int_{0}^{2\pi} \sin(\pm \phi' - \phi'_{ys}) \exp \{i(mB - k)\phi_y\} \exp \{-ik_mScr_y \cos(\pm \phi' - \phi_y)\} \, d\phi_y, \]

\[ I_{y_1} = \frac{1}{2\pi} \int_{0}^{2\pi} \exp \{i(mB - k)\phi_y\} \exp \{-ik_mScr'_{ys} \cos(\pm \phi' - \phi_y)\} \, d\phi_y. \]
The two components $I_{\phi_y}$ and $I_{y_1}$ can be expressed as follows. Details for their derivation are supplied in appendix A.

$$I_{\phi_y} = \frac{mB - k}{k_mS_y} \exp\left\{i(mB - k)(\pm \phi' - \frac{\pi}{2})\right\} J_{mB-k}(k_mS_y r_y), \quad (3.38)$$

$$I_{y_1} = \exp\left\{i(mB - k)(\pm \phi' - \frac{\pi}{2})\right\} J_{mB-k}(k_mS_y r_y). \quad (3.39)$$

The components of the loading force are given by

$$F_{\phi_y}^{(k)} = L \cos \alpha, \quad (3.40)$$

$$F_{y_1}^{(k)} = -L \cos \alpha, \quad (3.41)$$

where $L$ is the amplitude of the lift force acting on the blade per unit area defined as a sum of a steady ($L_0$) and an unsteady ($L_k$) component. Consider only the steady component of the loading

$$L_0 = \frac{1}{2} \rho V (r_y')^2 C_L(r_y') f(\bar{X}), \quad (3.42)$$

where $f(\bar{X})$ is a function describing the distribution of the lift forces along the chord wise coordinate, $C_L$ is the lift coefficient and $V$ the axial flow velocity. The far-field steady loading noise of a propeller at angle of attack in a uniform flow is finally given by:

$$P_L(x, t) = \sum_{m=-\infty}^{+\infty} \frac{-iB}{8\pi S_0} \exp\left\{imB \left(\Omega t - \frac{r_x}{c_0}\right) + i(mB - k) \left(\pm \phi' - \frac{\pi}{2}\right)\right\}
\times \int_{r_h}^{r_1} \int_{-1}^{1} \exp\left\{-i \left[(mB - k) \frac{\sin \alpha}{r_y} + k_mS_c \cos \alpha\right] \frac{c\bar{X}}{2}\right\}
\times \frac{c}{2} \rho V (r_y') C_L(r_y') f(\bar{X}) \left(\frac{mB - k}{r_y} \cos \alpha - k_mS_c \sin \alpha\right) J_{mB-k}(k_mS_y r_y) d\bar{X} dr_y. \quad (3.43)$$
3.2 Unsteady loading due to angle of attack

The present section comprises derivations of expressions to calculate the unsteady lift on the airfoil produced by the periodic change in the angle of the airflow onto each rotor blade as it rotates. In this formulation the blade sweep $s$ is accounted for by setting

$$y'_{1} = \left( s + \frac{cX}{2} \right) \cos \alpha, \quad (3.44)$$

$$\phi'_{ys} = \left( s + \frac{cX}{2} \right) \frac{\sin \alpha}{r_y}. \quad (3.45)$$

The far-field pressure perturbation is thus

$$P_L(x, t) = \sum_{k=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \frac{-iB}{4\pi S_0} \exp \left\{ \frac{imB}{\rho_c} \left( \Omega t - \frac{r_c}{c_0} \right) + i(mB - k)(\pm \phi' - \frac{\pi}{2}) \right\}$$

$$\times \int_{r_t}^{r_h} \Psi_L \exp \left\{ -i\Phi_s \right\} k_y J_{mB - k}(k_m S_s r_y) dr_y. \quad (3.46)$$

with

$$k_c = \left( \frac{mB \Omega \cos \theta'}{c_0(1 - M_x \cos \theta')} \cos \alpha + (mB - k) \frac{\sin \alpha}{r_y} \right) \frac{c}{2}, \quad (3.47)$$

$$k_y = \left( \frac{mB - k}{r_y} \cos \alpha - \frac{mB \Omega \cos \theta'}{c_0(1 - M_x \cos \theta')} \sin \alpha \right) \frac{c}{2}, \quad (3.48)$$

$$\Phi_s = \left( \frac{mB \Omega \cos \theta'}{c_0(1 - M_x \cos \theta')} \cos \alpha + (mB - k) \frac{\sin \alpha}{r_y} \right) s, \quad (3.49)$$

$$\Psi_L = \int_{-1}^{1} L_k \exp \left\{ -ik_c \bar{X} \right\} d\bar{X}. \quad (3.50)$$

**Blade lift calculation**

The blade loading is calculated by estimating the upwash incident on each blade. Note $U_i$ and $U'_i$ the axial flow velocity in the flight and propeller system respectively.
The velocity in the propeller system is given by

\[
\begin{align*}
U_1' &= U_1 \cos \gamma, \\
U_2' &= -U_1 \sin \gamma, \\
U_3' &= 0.
\end{align*}
\] (3.51) (3.52) (3.53)

The blade velocity and the air velocity relative to the observer in the propeller system are respectively

\[
\begin{align*}
V_{BO} &= \Omega r_y [- \sin \phi_y \hat{y}_2 + \cos \phi_y \hat{y}_3], \\
V_{AO} &= U_1 \cos \gamma \hat{y}_1 - U_1 \sin \gamma \hat{y}_2.
\end{align*}
\] (3.54)
Thus the air velocity relative to the blade is

\[
V_{AB} = U_1 \cos \gamma \hat{y}_1 + [\Omega r_y \sin \phi' y - U_1 \sin \gamma] \hat{y}_2 - \Omega r_y \cos \phi' y \hat{y}_3.
\] (3.55)

![Figure 3.6: Coordinates system for velocity components.](image)

The air velocity relative to the \( q = 0 \) blade in the axial, tangential and radial directions are respectively

\[
\begin{align*}
V'_{1} &= -U_1 \cos \gamma, \\
V_{\phi' y} &= -\sin \phi' y V'_2 + \cos \phi' y V'_3 = -\Omega r_y + U_1 \sin \gamma \sin(\Omega \tau + \phi_{ys}), \\
V'_r &= 0.
\end{align*}
\] (3.56)

Therefore the flow velocities parallel and normal to the blade are given by the following expressions (see figure 3.6).

\[
\begin{align*}
V_X &= V'_1 \cos \alpha - V_{\phi' y} \sin \alpha, \\
V_Y &= -V'_1 \sin \alpha - V_{\phi' y} \cos \alpha.
\end{align*}
\] (3.57)

As the blade is assumed to lie parallel to the time average flow, the stagger angle \( \alpha \) can be expressed (recall that \( \phi' y = \Omega \tau + \phi'_{ys} \))
\[ \alpha = \text{atan} \left( \frac{\Omega r_y}{U_1 \cos \gamma} \right) \]  
(3.58)

The chordwise distribution of the steady component of lift force per unit area acting on the rotor blade is calculated according to the expression of the pressure distribution on a flat plate inclined at angle of incidence relatively to the airflow given by the flat plate theory (Ashley and Landahl, 1965):

\[ L_0 = \frac{1}{2} \rho \bar{V}_x^2 C_L \frac{2}{\pi} \sqrt{\frac{1 - \tilde{X}}{1 + \tilde{X}}}. \]  
(3.59)

Therefore,

\[ \Psi_L = \frac{1}{2} \rho \bar{V}_x^2 C_L \left[ J_0(k_c) + i J_1(k_c) \right], k = 0, \]  
(3.60)

where \( \bar{V}_x \) is the steady component of flow parallel to the blade chordline. The unsteady component of flow normal to the blade chordline is given by

\[ \nu_y = -U_1 \sin \gamma \cos \alpha \sin \phi'_y. \]  
(3.61)

The upwash is defined as the sum of two gusts impinging on the flat airfoil such that

\[ \nu_y = \sum_{k' = \pm 1} \nu'_y \exp \{ i k_X (V_X \tau - X) \}, \]  
(3.62)

where \( k_X = k' \Omega / V_X \) and

\[ \nu'_y = \frac{i}{2} \text{sgn}(k') U_1 \sin \gamma \cos \alpha \exp \{ -i k_X s \}. \]  
(3.63)

At low frequency, the blade response to a gust of the form \( \nu'_y \exp \{ i k_X (V_X \tau - X) \} \) can be expressed using the compressible Sears’ response function as

\[ \Delta p = 2 \rho V_X \nu'_y S_c(\sigma_X) \sqrt{\frac{1 - \tilde{X}}{1 + \tilde{X}}} \exp \left\{ i k_X V_X \tau + i \frac{\sigma_X \tilde{M}_X^2}{\beta^2} \tilde{X} \right\}, \]  
(3.64)

where \( \sigma_X = k_X \frac{c}{2} \) and \( S_c(\sigma_X) \) is given by
\[ S_c(\sigma_X) = \frac{S(\frac{\sigma_X}{\beta})}{\beta} \left[ J_0 \left( \frac{M_X^2 \sigma_X}{\beta^2} \right) - i J_1 \left( \frac{M_X^2 \sigma_X}{\beta^2} \right) \right] \exp \left\{ i \frac{f(M_X)}{\beta^2} \right\}, \]  

(3.65)

with

\[
\begin{align*}
S\left( \frac{\sigma_X}{\beta^2} \right) &= \frac{\beta^2}{i \sigma_X} \left[ K_0 \left( \frac{i \sigma_X}{\beta^2} \right) + K_1 \left( \frac{i \sigma_X}{\beta^2} \right) \right], \\
f(M_X) &= (1 - \beta) \ln(M_X) + \beta \ln(1 + \beta) - \ln(2), \\
M_X &= \frac{V_X}{c_0}.
\end{align*}
\]  

(3.66)

The above formulation is only valid in the low frequency regime, provided that \( \sigma_X M_X / \beta^2 < 0.4 \). In the present application \( \sigma_X M_X / \beta^2 < 0.1 \) for the first rotor alone tone, hence the low frequency assumption is justified for the range of interest. Assume that the unsteady component of the lift force per unit area \( L_1 \) can be expressed as a Fourier series of the form

\[ L_k(r_y, X, \tau) = \sum_{k=-\infty}^{+\infty} l_k(r_y, X, k) \exp \{ ik \Omega \tau \}, \]  

(3.67)

\[ l_k(r_y, X, k) = \frac{\Omega}{2\pi} \int_0^{2\pi} L_k(r_y, X, \tau) \exp \{ -ik \Omega \tau \} d\tau. \]  

(3.68)

According to equation (3.64) the unsteady loading per unit area is thus given by

\[ L_k(r_y, X, \tau) = \sum_{k'=\pm1} 2\rho V_X \nu_y S_c(\sigma_X) \sqrt{\frac{1 - X}{1 + X}} \exp \left\{ ik_X V_X \tau + i \frac{\sigma_X M_X^2}{\beta^2} \bar{X} \right\} \]  

(3.69)

Substituting into equation (3.50) and making use of the change of variables \( \bar{X} = \cos \theta/2 \), the chordwise distribution of the unsteady component of lift force acting on the blade is

\[ \Psi_L = \pi \rho V_X \nu_y S_c(\sigma_X) \left[ J_0 \left( k_c - \frac{\sigma_X M_X^2}{\beta^2} \right) + i J_1 \left( k_c - \frac{\sigma_X M_X^2}{\beta^2} \right) \right], k = \pm1. \]  

(3.70)
3.3 Results

The model described in this Chapter has been validated internally against another code developed by M. Kingan, based on the same method but written independently (there is no available publication for this code). Comparisons with wind tunnel measurements issued from Rig 145 test campaign are shown in figure 3.7 for the first rotor-alone tone of the upstream rotor at $\phi' = 0^\circ$. Predicted levels were calculated for angles of attack in the range $\pm 12^\circ$ at a given emission radius. The relative changes in sound pressure level produced by the changes in angle of attack are well captured by the prediction method. Notice an increase in sound pressure level with increase of the angle of attack, compared to the steady loading case (black line), and vice versa. Those results were published and discussed in great details in Kingan et al. (2014).

![Figure 3.7: Polar directivity of the first rotor-alone tone for Rig 145 operating at a condition representative of take-off ($\phi' = 0^\circ$). Measured SPL (left) predicted SPL (right).](image)

3.4 Concluding remarks

An analytical model for predicting the loading noise produced by a propeller at angle of attack was developed based on an existing method using a simplified blade geometry. This model constitutes an improvement to the currently used model. An extension to this work will be the derivation of a method to calculate the thickness noise produced by a propeller at angle of attack.
Chapter 4

Interaction tone prediction

As mentioned in section 1.3, the rotor-rotor interaction tones can be caused by: (1) the interaction of the front rotor’s wakes and tip vortices with the rear rotor’s blades; (2) the interaction between the bound potential field of the front (or rear) rotor with the adjacent blade row. Those tones are produced by the unsteady forces caused by each of these flow distortions, and occur at frequencies corresponding to the sum or difference of integer multiples of each rotor’s BPF, i.e. if $\Omega$ and $B$ denote the rotor’s rotational speed and blade number (with the subscripts 1 and 2 corresponding to the front and rear rotor respectively), the interaction tones would manifest at frequencies equal to

$$n_1 B_1 \Omega_1 + n_2 B_2 \Omega_2, \quad n_1 \in \mathbb{N}, n_2 \in \mathbb{N}. \tag{4.1}$$

The circumferential mode order of the tones is

$$\nu = n_1 B_1 - n_2 B_2. \tag{4.2}$$

In the present chapter far-field expressions to predict the rotor-rotor interaction tones for an isolated AOR are derived, following the derivations of Hanson (1985b) and Parry (1988). The blades are modelled as flat plates and are assumed to be aligned with the flow direction. It is assumed that the noise is only produced by the loading forces exerted by the blades on the fluid and the thickness and quadrupole sources are neglected. For convenience the notations $\{X_i, Y_i\}$ and $\{x_i, y_i\}$ are used for the coordinate systems attached to each airfoil, where $i = 1, 2$. The configuration of the two blade rows is shown in figure 4.1.
4.1 Downstream interactions

This section covers the noise produced by the interaction of the unsteady flow field of the front rotor with the downstream rotor. The pressure perturbation at location $\mathbf{x}$ and time $t$ due to a rotor with $B_2$ identical, evenly spaced blades is given by

$$P_L(\mathbf{x}, t) = \sum_{q=0}^{B_2-1} \int_{-T}^{T} \int_{S(q)} F_q \nabla G(\tau) dS^{(q)} d\tau,$$  \hspace{1cm} (4.3)

where $F_q(\tau)$ and $G(\tau)$ are respectively the force per unit area exerted by the fluid on the $q^{th}$ blade’s surface $S^{(q)}$ and the free field Green’s function that can be written as

$$G(\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{G}_{\omega''} \exp \{i\omega''(t - \tau)\} d\omega'',$$  \hspace{1cm} (4.4)

and

Figure 4.1: Cascade representation of the front and rear rotor blades at constant radius $r_y$. 
\[
F_q(\tau) = \int_{-\infty}^{+\infty} \tilde{F}_q(\omega') \exp \{i\omega'\tau\} d\omega', \quad (4.5)
\]
\[
\tilde{F}_q(\omega') = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F_q(\tau) \exp \{-i\omega'\tau\} d\tau.
\]

The times \(t\) and \(\tau\) are respectively the observer time and the source time. Inserting equations (4.4) and (4.6) in (4.3) yields

\[
P_L(x, t) = \frac{1}{2\pi} \sum_{q=0}^{B_2-1} \int_{-T}^{T} \int \int_{S(n)} \int_{-\infty}^{+\infty} \tilde{F}_q(\omega') \tilde{G}_{\omega''} \exp \{i\omega''(t-\tau) + i\omega'\tau\} d\omega'' d\omega' dX_2 dr_y d\tau.
\]

The free-field Green’s function is given by (see Hanson)

\[
\tilde{G}_{\omega''} = \exp \{-i\omega''/c_0\} \frac{4\pi S_0}{4\pi S_0}, \quad (4.7)
\]

where \(S_0\) and \(\sigma\) are the amplitude and phase radius defined, in the far-field, by

\[
S_0 = r_e(1 - M_x \cos \theta), \quad (4.8)
\]
\[
\sigma = r_e + \frac{x_s \cos \theta}{1 - M_x \cos \theta} - \frac{r_y \sin \theta}{1 - M_x \cos \theta} \cos(\phi' - \phi_y) - \Omega_2 \tau + \frac{2\pi}{B_2} q. \quad (4.9)
\]

The quantity \(r_e\) corresponds to the distance from the source to the observer and the coordinates \((x_s, r_y \phi'_y)\) correspond to the source coordinates with

\[
\phi_y = \phi'_y + \Omega_2 \tau + \frac{2\pi}{B_2} q. \quad (4.10)
\]

The quantity \(\phi'_y\) is the source’s azimuthal coordinate. Note that the axial coordinate \(x_s\) is defined in the opposite direction to Hanson’s notations. The force \(\tilde{F}_q(\omega')\) can be decomposed in a sum of axial and tangential components relatively to the blade stagger angle \(\alpha_2\) as
\[ \tilde{F}_q(\omega') = \tilde{x}\tilde{F}_x + \tilde{\phi}\tilde{F}_\phi = \tilde{x}\sin \alpha_2 \tilde{F}_q + \tilde{\phi}\cos \alpha_2 \tilde{F}_q, \]  
\[ (4.11) \]

The product \( \tilde{F}_q(\omega') \cdot \nabla \tilde{G}_{\omega''} \) can be evaluated in cylindrical coordinates as

\[ \tilde{F}_q(\omega') \cdot \nabla \tilde{G}_{\omega''} = \tilde{F}_q(\omega') \left[ \sin \alpha_2 \frac{\partial \tilde{G}_{\omega''}}{\partial x_s} + \cos \alpha_2 \frac{\partial \tilde{G}_{\omega''}}{\partial \phi_y} \right], \]
\[ (4.12) \]

which gives the following result

\[ \tilde{F}_q(\omega') \cdot \nabla \tilde{G}_{\omega''} = \frac{i \tilde{F}_q(\omega') \omega'' \exp \left\{ -i\omega'' \frac{\sigma}{c_0} \right\}}{4\pi S_0 c_0 (1 - M_x \cos \theta)} \left[ -\cos \theta \sin \alpha_2 + \sin \theta \cos \alpha_2 \sin(\phi' - \phi_y) \right]. \]
\[ (4.13) \]

Thus the pressure \( P_L \) can be written

\[ P_L(x, t) = \sum_{q=0}^{B_2-1} \int_{-T}^{T} \int_{-\infty}^{+\infty} \int_{S(\theta)} \frac{i \tilde{F}_q(\omega') \omega''}{4\pi S_0 c_0} \exp \left\{ -i\frac{\omega''}{c_0} (r_e + S_c x_s) \right\} \times \left[ -\sin \alpha_2 S_c + \cos \alpha_2 S_s \sin(\phi' - \phi_y) \right] \exp \left\{ i\omega''(t - \tau) + i\omega' \tau \right\} \times \exp \left\{ i\frac{\omega''}{c_0} r_y S_s \cos(\phi - \phi_0) \right\} \ d\omega'' d\omega' dX_2 dY d\tau, \]
\[ (4.14) \]

with

\[ S_c = \frac{\cos \theta'}{1 - M_x \cos \theta'}, \]
\[ (4.15) \]

\[ S_s = \frac{\sin \theta'}{1 - M_x \cos \theta'}. \]
\[ (4.16) \]

Making use of the Jacobi-Anger formula yields
\[
P_L(x, t) = \sum_{q=0}^{B_2-1} \int_{-T}^{T} \int_{S(v)} \int_{-\infty}^{+\infty} \frac{i\tilde{F}_q(\omega')}{8\pi^2 S_0 c_0} \exp\left\{ -\frac{i\omega''}{c_0} (r_c + S_c x_s) \right\} \times \left[ -\sin \alpha_2 S_c + \cos \alpha_2 S_s \sin(\phi' - \phi_y) \right] \sum_{n=-\infty}^{+\infty} J_n \left( \frac{\omega'' r_y S_s}{c_0} \right) \times \exp \left\{ in \left( \phi' - \phi_y + \frac{\pi}{2} \right) + i\omega''(t - \tau) + i\omega'\tau \right\} d\omega'' d\omega' dX d\tau.
\]

(4.17)

The following product

\[
\sin(\phi' - \phi_y) \sum_{n=-\infty}^{+\infty} J_n(z) \exp \left\{ in \left( \phi' - \phi_y + \frac{\pi}{2} \right) \right\} = \sum_{n=-\infty}^{+\infty} \sum_{j=\pm 1} \frac{\text{sgn}(j)}{2}
\]

\[
\times J_n(z) \exp \left\{ i(n+j)(\phi' - \phi_y) + i(n-1)\frac{\pi}{2} \right\},
\]

is equal to

\[
\frac{-n}{z} \exp \left\{ in(\phi' - \phi_y + \frac{\pi}{2}) \right\} J_n(z).
\]

Thus, substituting \( \phi_y \) for its definition in equation (4.10), performing the \( \tau \) integration over infinity*,

\[
\int_{-\infty}^{+\infty} \exp \left\{ -i(\omega'' - \omega' + n\Omega_2)\tau \right\} d\tau = 2\pi\delta(\omega' - \omega'' - n\Omega_2),
\]

(4.19)

and integrating over \( \omega'' \), equation (4.17) can be written

\[
P_L(x, t) = \sum_{q=0}^{B_2-1} \int_{S(v)} \int_{-\infty}^{+\infty} \frac{i\tilde{F}_q(\omega')}{4\pi S_0} \sum_{n=-\infty}^{+\infty} \exp \left\{ -\frac{i\omega' - n\Omega_2}{c_0} (r_c + S_c x_s) \right\} \times \left[ -\sin \alpha_2 S_c - \frac{nc_0 \cos \alpha_2}{r_y (\omega' - n\Omega_2)} \right] \omega' - n\Omega_2 \frac{\omega' - n\Omega_2 r_y S_s}{c_0} \times \exp \left\{ i(\omega' - n\Omega_2)t + in \left( \phi' - \phi_y + \frac{2\pi q}{B_2} + \frac{\pi}{2} \right) \right\} d\omega'dX d\tau.
\]

(4.20)
Setting $n = -n$ yields

$$P_L(x, t) = \sum_{q=0}^{B_2-1} \int \int_{S} \int_{-\infty}^{+\infty} \frac{i \tilde{F}_q(\omega')}{4\pi S_0} \sum_{n=-\infty}^{+\infty} \exp \left\{ -i \frac{\omega' + n\Omega_2}{c_0} (r_e + S_c x_s) \right\}$$

$$\times \left[ -\sin \alpha_2 S_c + \frac{nc_0 \cos \alpha_2}{r_y (\omega' + n\Omega_2)} \right] \frac{\omega' + n\Omega_2}{c_0} (-1)^n J_n \left( \frac{(\omega' + n\Omega_2)r_y S_s}{c_0} \right)$$

$$\times \exp \left\{ i(\omega' + n\Omega_2)t - in \left( \phi' - \phi'_{ys} - \frac{2\pi q}{B_2} + \frac{\pi}{2} \right) \right\} d\omega'dX_2dr_y. \quad (4.21)$$

The force acting on the rear rotor blade is periodic with period $T = \frac{2\pi}{B_1(\Omega_1 + \Omega_2)}$ thus $\tilde{F}_q(\omega')$ can be expressed as a sum of impulses as

$$\tilde{F}_q(\omega') = \sum_{k=-\infty}^{+\infty} F_k \delta (\omega' - kB_1(\Omega_1 + \Omega_2)), \quad (4.22)$$

where $F_k$ is the harmonic of the unsteady lift force per unit area. Thus substituting equation (4.22) into equation (4.21) and integrating over $\omega'$ yields

$$P_L(x, t) = \sum_{q=0}^{B_2-1} \int \int_{S} \sum_{k=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} (-1)^n \frac{i F_k}{4\pi S_0} \exp \left\{ -i (kB_1(\Omega_1 + \Omega_2) + n\Omega_2)(r_e + S_c x_s) \right\}$$

$$\times \frac{k B_1(\Omega_1 + \Omega_2) + n\Omega_2}{c_0} \left[ -\sin \alpha_2 S_c + \frac{nc_0 \cos \alpha_2}{r_y (kB_1(\Omega_1 + \Omega_2) + n\Omega_2)} \right]$$

$$\times J_n \left( \frac{(kB_1(\Omega_1 + \Omega_2) + n\Omega_2)r_y S_s}{c_0} \right) \exp \left\{ i (kB_1(\Omega_1 + \Omega_2) + n\Omega_2)t \right\}$$

$$\times \exp \left\{ -in \left( \phi' - \phi'_{ys} - \frac{2\pi q}{B_2} + \frac{\pi}{2} \right) \right\} dX_2dr_y. \quad (4.23)$$

Setting $n = n - k B_1$ yields
\[ P_L(x, t) = \sum_{q=0}^{B_2-1} \int_{\Sigma(\phi)} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{i F_k}{4\pi S_0} \exp \left\{ -i(kB_1\Omega_1 + n\Omega_2) \frac{r_e + S_c x_s}{c_0} \right\} \frac{kB_1\Omega_1 + n\Omega_2}{c_0} \times (-1)^{n-kB_1} \left[ -\sin \alpha_2 S_c + \frac{(n - kB_1)c_0 \cos \alpha_2}{ry(kB_1\Omega_1 + n\Omega_2)} \right] J_{n-kB_1} \left( \frac{(kB_1\Omega_1 + n\Omega_2)ryS_s}{c_0} \right) \times \exp \left\{ i(kB_1\Omega_1 + n\Omega_2)t - i(n - kB_1) \left( \phi' - \phi'_{ys} - \frac{2\pi q}{B_2} + \frac{\pi}{2} \right) \right\} dX_2dy_y. \] 

The unsteady lift harmonic \( F_k \) can be written

\[ F_k = F_0 \exp \left\{ ikB_1 \frac{2\pi q}{B_2} \right\}, \] 

Thus

\[ P_L(x, t) = \sum_{q=0}^{B_2-1} \int_{\Sigma(\phi)} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{i B_2 F_0}{4\pi S_0} \exp \left\{ -i\omega \frac{r_e + S_c x_s}{c_0} \right\} \frac{\omega}{c_0} (-1)^\nu \times \left[ -\sin \alpha_2 S_c + \frac{\nu c_0 \cos \alpha_2}{ry\omega} \right] J_\nu \left( \frac{\omega ryS_s}{c_0} \right) \times \exp \left\{ i\omega t - i(mB_2 - kB_1) \left( \phi' - \phi'_{ys} + \frac{\pi}{2} \right) \right\} dX_2dy_y, \] 

where \( \omega = kB_1\Omega_1 + mB_2\Omega_2 \) and \( \nu = mB_2 - kB_1 \). The above equation is equal to
\[ P_L(x, t) = \int_{S(x)} \sum_{k=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} iB_2 F_0 \exp \left\{ i\omega \left( t - \frac{r_e + S_c x_s}{c_0} \right) \right\} \exp \left\{ -i\nu \left( \phi' - \phi'_{ys} \right) \right\} \times \exp \left\{ -i\nu \frac{\pi}{2} \right\} \left[ -\omega \sin \alpha_2 \frac{S_c}{c_0} + \frac{\nu \cos \alpha_2}{r_y} \right] J_{-\nu} \left( \frac{\omega r_y S_s}{c_0} \right) dX_2 dr_y. \] (4.28)

The axial and tangential source coordinates are given by

\[ x_s = (s_2 + X_2) \cos \alpha_2 + (l_2 + Y_2) \sin \alpha_2, \] (4.29)
\[ \phi'_{ys} = -(s_2 + X_2) \frac{\sin \alpha_2}{r_y} + (l_2 + Y_2) \frac{\cos \alpha_2}{r_y}, \]

where \( s_2 \) and \( l_2 \) are respectively the sweep and lean of the downstream airfoil.

Figure 4.2: Blade sweep and lean definition.

Note that on the blade surface, \( Y_2 = 0 \). We denote \( m = n_2 \) and \( k = n_1 \). Inserting equation (4.30) into equation (4.28) and using the non-dimensional chordwise coordinate \( \tilde{X}_2 = \frac{2X_2}{c_2} \) yields

\[ P_L(x, t) = \frac{iB_2}{4\pi S_0} \int_{r_2}^{r_2} \sum_{n_1=-\infty}^{+\infty} \sum_{n_2=-\infty}^{+\infty} \exp \left\{ i(n_2 \Omega_2 + n_1 \Omega_1) \left( t - \frac{r_e}{c_0} \right) \right\} \times \exp \left\{ -i(n_2 B_2 - n_1 B_1) \left( \phi' + \frac{\pi}{2} \right) \right\} \exp \left\{ -i\Phi_s - ik\gamma \right\} \times k\gamma J_{n_1 B_1 - n_2 B_2} \left( \frac{\omega r_y S_s}{c_0} \right) \Psi_L \, dr_y, \] (4.30)

where \( \Psi_L \) is the "acoustically weighted" lift per unit span which is defined as
\[ \Psi_L = \int_{-1}^{1} F_0 \exp \{-i k_{Xs} \bar{X}_2\} \, d\bar{X}_2, \quad (4.31) \]

and the wave numbers \( k_{Xs} \) and \( k_{Yl} \) are given by

\[
  k_{Xs} = \frac{c_2}{2} \left[ \omega \frac{S_c}{c_0} \cos \alpha_2 + (n_2 B_2 - n_1 B_1) \frac{\sin \alpha_2}{r_y} \right], \quad (4.32)
\]

\[
  k_{Yl} = \frac{c_2}{2} \left[ \omega \frac{S_c}{c_0} \sin \alpha_2 - (n_2 B_2 - n_1 B_1) \frac{\cos \alpha_2}{r_y} \right]. \quad (4.33)
\]

\[
  \Phi_s = s_2 \left[ \omega \frac{S_c}{c_0} \cos \alpha_2 + (n_2 B_2 - n_1 B_1) \frac{\sin \alpha_2}{r_y} \right]. \quad (4.34)
\]

### 4.2 Upstream interactions

Now consider the noise produced by the interaction between the disturbed flow field of the downstream rotor and the upstream blade row. The derivation is identical to that of section 4.1, expressing the force exerted by the upstream blades on the flow as follows.

\[
  \bar{F}_q(\omega') = -\hat{\mathbf{e}} \bar{F}_q \sin \alpha_1 + \hat{\phi} \cos \alpha_1 \bar{F}_q. \quad (4.35)
\]

Hence, the acoustic pressure perturbation is given by

\[
  P_L(x, t) = \frac{i B_1}{4 \pi S_0} \int_{r_{t1}}^{r_{t1} + \infty} \sum_{n_1 = -\infty}^{+\infty} \sum_{n_2 = -\infty}^{+\infty} \exp \left\{ i (n_2 \Omega_2 + n_1 \Omega_1) \left( t - \frac{r_y}{c_0} \right) \right\} \times \exp \left\{ -i (n_1 B_1 - n_2 B_2) \left( \phi' + \frac{\pi}{2} \right) \right\} \exp \{-i \Phi_s - i k_{Yl} l\} \times k_{Yl} J_{n_2 B_2 - n_1 B_1} \left( \frac{\omega r_y S_s}{c_0} \right) \Psi_L \, dr_y, \quad (4.36)
\]

with the wave numbers \( k_{Xs} \) and \( k_{Yl} \) given by
\[ k_{Xs} = \frac{c_1}{2} \left[ \frac{\omega S_c}{c_0} \cos \alpha_1 + (n_2 B_2 - n_1 B_1) \frac{\sin \alpha_1}{r_y} \right] \]  \hspace{1cm} (4.37) \\
\[ k_{Yl} = \frac{c_1}{2} \left[ \frac{\omega S_c}{c_0} \sin \alpha_1 + (n_2 B_2 - n_1 B_1) \frac{\cos \alpha_1}{r_y} \right] \]  \hspace{1cm} (4.38)

and

\[ \Phi_s = s_1 \left[ \frac{\omega S_c}{c_0} \cos \alpha_1 + (n_2 B_2 - n_1 B_1) \frac{\sin \alpha_1}{r_y} \right] \],  \hspace{1cm} (4.39)

\[ \Psi_L = \int_{-1}^{1} F_0 \exp \left\{ -i k_{Xs} \tilde{X}_1 \right\} d\tilde{X}_1. \]  \hspace{1cm} (4.40)

### 4.3 Concluding remarks

Far-field expressions for predicting the rotor-rotor interaction tones produced by an isolated advanced open rotor were derived based on existing methods. Those expressions will be used in the following chapter to calculate the far-field. The formulation of the downstream interactions given by equation (4.30) will be used to calculate the noise produced by: (1) the impingement of the front rotor viscous wakes onto the downstream rotor in Chapter 5; (2) the interaction of the potential field of the upstream rotor with the downstream rotor in Chapter 6. Formulation (4.36) will be used to calculate the noise produced by the interaction between the potential field of the downstream rotor with the upstream blade row.
Chapter 5

Wake interaction noise

5.1 Introduction

The wake interaction is a dominant source of interaction noise for an advanced open rotor. Predicting wake interaction noise requires the knowledge of the aerodynamic properties of the flow around the blades. These parameters can significantly influence the resulting sound pressure levels, thus the input of high fidelity aerodynamic data into the analytical models is essential. Calculating such accurate data is critical as the flow field around the blades is unsteady and complex, however some meticulous unsteady CFD simulations can improve the agreement between predictions and experimental data (Kingan et al. (2014), Carazo et al. (2011)). Another alternative is the use of experimental methods to capture the features of the distorted flow field around the blades (Santana et al. (2013), Sinnige et al. (2015)). Previous work by Kingan et al. (2014) investigated the prediction of the far field noise produced by an advanced open rotor using a method in which the steady aerodynamics of the rotor were predicted using both steady CFD and a strip method. The steady aerodynamics of the front rotor were input into a purely analytical model which predicted the unsteady flow field produced by the front rotor, the unsteady response of the downstream blade, and the radiated far-field tonal noise. It was suggested that the accuracy of the method relied on an accurate representation of the steady aerodynamics of the rotor namely, that obtained from high fidelity three-dimensional CFD. However, the accuracy of the method also relied on the ability of the analytical models to accurately predict the rotor wake development using this steady aerodynamic data. In this chapter, the ability of such analytical methods to accurately model the development of the
front rotor wakes is assessed using a hybrid CFD/analytical method. This work was published by Ekoule et al. (2015).

5.2 Hybrid CFD/analytical model

5.2.1 Method

The present method is inspired from the work of Carazo et al. (2011) who used URANS CFD data to model the incident flow field at the rear rotor leading edge. The numerical data were then input into analytical models to calculate the blade response and the far-field radiated noise. Because the development of the wakes provided by such simulations can be subject to numerical dissipation, analytical methods for modelling the evolution of the viscous wakes are still of interest. In the present model, the wake characteristics such as wake width and wake centerline velocity are determined from high fidelity unsteady three-dimensional CFD data collected at a particular axial plane and an analytical model is used to convect the wakes to downstream locations. A similar approach was followed by Jaron et al. (2014), who used another analytical model to determine the front rotor wake and potential field upstream of the rear rotor from RANS measurements taken at upstream locations.

The CFD simulations used in this method were supplied by N.Sohoni from the University of Cambridge’s Rolls-Royce University Technology Centre. Subsequently the author appraised the simulation data and post-processed ‘raw’ instantaneous velocity data. The simulations were performed using Turbostream (Brandvik and Pullan, 2011), which is a fast, three-dimensional, Unsteady Reynolds-Averaged Navier-Stokes (URANS) solver running on structured multi-block meshes. Turbostream uses an algorithm based on the TBLOCK solver, a code originally developed by Denton (1983). The numerical scheme is a finite-volume method, second-order accurate in time and space. The CFD simulations used structured multiblock meshes with an O-mesh surrounding each blade. The domains of each rotor were connected by a sliding surface using polynomial interpolation in the circumferential direction. The mesh topology is shown in figure 5.1 where $\frac{N_{\theta F}}{Z_F}$ and $\frac{N_{\theta R}}{Z_R}$ correspond to the number of circumferential grid points per blade pitch at the upstream and downstream blocks respectively. The data analyzed in this Chapter were captured at various locations upstream and downstream of the sliding surface. The rotor geometry used in the simulations were taken from open rotor
blades designed by Rolls-Royce and tested on Airbus Z08 test rig (note that the Z08 test rig is described in Chapter 2).

Figure 5.1: Blade-to-blade mesh topology, originally published by Sohoni et al. (2015). $S$ and $\epsilon$ correspond to pitch and thickness quantities.

The model assumes that the stream tube contraction is small downstream of the location at which the data is collected such that the wake profile measured at a particular radial location convects downstream of this point at a constant radius. The validity of this assumption may be assessed by inspection of figure 5.2 which plots the instantaneous velocity components and instantaneous velocity magnitude in a meridional plane. It is observed that, apart from very close to the front rotor blades, the radial component of velocity is significantly less than the axial component of velocity in the region between the two rotors. This should ensure that fluid particles propagate along a path which has an approximately constant radius.

Figure 5.2: Contours of constant instantaneous velocity in a meridional plane: (a) axial velocity, (b) radial velocity. The color scaling limit $V_{\text{max}}$ corresponds to the maximum absolute velocity magnitude across the meridional plane and $V_{\text{min}} \approx -0.7V_{\text{max}}$. 

\[
\frac{N_{\theta F}}{Z_F} = 152 \quad \frac{N_{\theta R}}{Z_R} = 256
\]
5.2.2 Coordinate system

Consider the coordinate system \( \{ X_1, Y_1 \} \) which rotates with the ‘reference wake’ of the front rotor ‘reference blade’, for which the coordinates \( X_1 \) and \( Y_1 \) are, respectively, tangent and normal to the mean flow direction in a frame of reference rotating with the front rotor at a particular radius. The \( \{ x_1, y_1 \} \) coordinate system also rotates with the front rotor with \( x_1 \) parallel to the rotor axis and \( y_1 = r\phi_1 \) parallel to the tangential direction. The deficit velocity due to the viscous wake from each blade on the front rotor is assumed to be identical and aligned with the mean flow direction at the measurement plane. The origin of the \( \{ X_1, Y_1 \} \) and \( \{ x_1, y_1 \} \) coordinate systems is selected to be located at the same axial position as the trailing edge of the front rotor blades. Note that in the method described by Parry (1988), the origin of this coordinate system was placed at the mid chord of the front rotor blade. The \( \{ X_1, Y_1 \} \) and \( \{ x_1, y_1 \} \) coordinate systems are related by

\[
\begin{align*}
X_1 &= x_1 \cos \alpha_1 + y_1 \sin \alpha_1, \\
Y_1 &= -x_1 \sin \alpha_1 + y_1 \cos \alpha_1,
\end{align*}
\]

(5.1)  
(5.2)

where \( \alpha_1 \) is the local flow angle. The coordinates are shown in figure 5.3.

![Figure 5.3: Cascade coordinate system.](image)
5.2.3 Wake velocity profile

The analytical model assumes that, at each radius, the wake can be modelled as two-dimensional and propagates in the mean flow direction which is modelled as constant in the inter-rotor region. Finally, the wakes of the front rotor are assumed to be far enough from each other to be considered as isolated wakes. In fact, in the present configuration the solidity is low enough for the cascade effects to be ignored \((s/c_1 \gtrsim 1\), where \(s/c_1\) is the pitch/chord ratio), see Blandeau et al. (2011).

The wake velocity deficit is modeled by a Gaussian function whose variables were determined from data at a measurement plane, \(P_1\), which is located at a constant axial location \(0.0902D_1\) (where \(D_1\) is the diameter of the front rotor) downstream of the front rotor blade pitch change axis, see figure 5.4. Figure 5.5 shows contours of constant tangential velocity calculated from the CFD simulation for one front rotor blade passage at this particular plane. It is assumed that the velocity field at the measurement plane can be modeled as a linear sum of the velocity produced by the front and rear rotors.

Thus, in the frame of reference of the front rotor, the flow disturbances due to the front rotor are steady in time and the perturbations due to the rear rotor are fluctuating with time. As described by Carazo et al. (2011), a time average of the velocity at the measurement plane in a frame of reference moving with the front rotor, will remove the unsteady portion of the flow due to the rear rotor leaving only the steady component of the flow and the unsteady components associated with the front rotor. The instantaneous and time-averaged tangential velocity at 80% of the tip radius are plotted as a function of \(\phi_1\), an azimuthal angle which rotates with the front rotor, in Figure 5.6.
Chapter 5 Wake interaction noise

Figure 5.5: Contours of constant tangential velocity (normalized by the maximum tangential velocity) at $x/D = 0.0902$ downstream of the front rotor blade pitch change axis for one blade passage.

Figure 5.6: Plot of normalized tangential velocity versus $\phi_1$ at $r = 0.8R_1$, $x/D_1 = 0.0902$ downstream of the front rotor blade pitch change axis.

The velocity deficit of the reference wake, $u'$ is modeled using a Gaussian function of the form

$$u' = u_c \exp\left\{-\beta y_1^2\right\},$$

(5.3)
where the wake centerline deficit velocity $u_c$ and the parameter $\beta$ are determined from the CFD data on the measurement plane by matching the centerline velocity and momentum deficit of the analytic and CFD velocity profiles. The method is described in details in Appendix C. As a result, the relationship between $\beta$ and $u_c$ is given by

$$
\beta = \pi \left( \frac{u_c}{2 + \frac{u_c \sqrt{2}}{U_r} \frac{\rho_0 U_r}{dD/dr \cos \alpha_1}} \right)^2,
$$

(5.4)

where $\rho_0$ is the air density and $dD/dr$ is the drag force per unit span. Figure 5.7 shows the wake velocity deficit normalized against the mean velocity $U_r$ at radius $r = 0.8R_{t_1}$ (where $R_{t_1}$ is the tip radius of the front rotor) for both the analytic model and the CFD data at the measurement plane. The principal features of the wake are reproduced with a reasonably good agreement with the CFD wake profile. The additional velocity components at the edges of the wake will be subject to further investigations. It is assumed that the fluctuations in the velocity deficit outside the viscous wake region are primarily due to the bound potential field of the front rotor. This contribution is investigated in Chapter 6.

![Normalized wake velocity deficit at radius $r = 0.8R_{t_1}$](image)

**Figure 5.7:** Normalized wake velocity deficit at radius $r = 0.8R_{t_1}$. Comparison between the analytical model and the CFD data at the measurement plane.

In figure 5.8, the tangential velocity profiles given by the CFD and those calculated using the analytical method over the entire $(r, \theta)$ measurement plane are compared.
In this plane, there is clearly good qualitative agreement between the CFD and the Gaussian models of the viscous wake. Nevertheless one source of discrepancy is the tip vortex which is noticeable in the full CFD solution but which is not included in the analytical model.

![Figure 5.8: Tangential velocity at the plane $P_1$ normalized by the front rotor tip speed given by the CFD data (a) and the analytical model (b).](image)

To assess the development of the wake along the axial coordinate and to calculate the upwash incident on the surface of the downstream blade, the wakes are convected downstream following a scaling law deduced from the analytical expressions of $u_c$ and $\beta$ given by Eq. (5.2.31) and (5.2.40) in Parry (1988)

$$\beta \propto \frac{1}{X_1},$$

(5.5)
$u_c \propto \frac{1}{\sqrt{X_1'}}. \quad (5.6)$

The results for the analytically-propagated wakes are shown in Figures 5.9 to 5.13. In Figure 5.9, the wake decay in the inter-rotor region is assessed in terms of the velocity deficit $u'_c$. The wake velocity deficit measured on a cylindrical CFD plane at $r = 60\% R_{t_1}$ is compared with the wake velocity deficit of the analytically-propagated wake (calculated using the scaling law (5.6) and the data on the measurement plane $P_1$). There is a very good agreement between the two results close to the measurement plane $P_1$. As we move further downstream of the measurement plane the analytical and numerical results slightly differ, however follow a similar trend. The large discrepancies at the upstream airfoil’s trailing edge axial location ($x_1 = 0$) are due to the fact that the scaling law arises from far-wake assumptions and thus is not valid close to the wake’s formation point. Note that in the far-wake, the decay rate of the wake centerline velocity and width (which is inversely proportional to $\sqrt{\beta}$) is the same as the correlations made by Reynolds (1979) for highly loaded fans and compressors.

![Graph](image)

Figure 5.9: Evolution of the wake velocity deficit $u_c$ between the two rotors at $r = 60\% R_{t_1}$.

Figure 5.10 shows a similar comparison for the scaling law in equation (5.5). There is a reasonable agreement between the CFD data and the convected-analytical results at most of the axial locations in terms of decay rate except in the near-wake (approximately up to 20% of the chordlength downstream of the upstream airfoil’s trailing edge). However, the CFD results present a discontinuity, which is in fact located at the axial position of the sliding plane. This will lead to differences
in the wake width of the CFD and the analytical profiles close to the leading edge of the downstream airfoil.

Figure 5.10: Evolution of $\beta$ (m$^{-2}$) between the two rotors at $r = 60\% R_t$.

Consider now a plane $P_2$ located at a constant axial position at a distance 0.144$D_1$ downstream of the pitch change axis of the front rotor, as illustrated in figure 5.11. The wake deficit velocity across one passage on the plane $P_2$ is plotted in figure 5.12. Figure 5.12(a) shows the velocity deficit calculated by matching a Gaussian profile on the CFD wake at the plane $P_2$ using equation (5.3). Figure 5.12(b) shows the velocity deficit calculated by matching the wake at the upstream measurement plane $P_1$, and propagating it downstream to the plane $P_2$ using equations (5.5) and (5.6). The results show a good agreement between the two profiles over most of the blade span except close to the hub and tip.

Figure 5.11: Location of the plane $P_2$. 

Figure 5.12: Evolution of $\beta$ (m$^{-2}$) between the two rotors at $r = 60\% R_t$. 

Consider now a plane $P_2$ located at a constant axial position at a distance 0.144$D_1$ downstream of the pitch change axis of the front rotor, as illustrated in figure 5.11. The wake deficit velocity across one passage on the plane $P_2$ is plotted in figure 5.12. Figure 5.12(a) shows the velocity deficit calculated by matching a Gaussian profile on the CFD wake at the plane $P_2$ using equation (5.3). Figure 5.12(b) shows the velocity deficit calculated by matching the wake at the upstream measurement plane $P_1$, and propagating it downstream to the plane $P_2$ using equations (5.5) and (5.6). The results show a good agreement between the two profiles over most of the blade span except close to the hub and tip.

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For this particular case, there is very good agreement between the profiles at radial locations between 75% and 85% of the front rotor tip radius. Figure 5.13 shows a comparison between the data from figures 5.12(a) and 5.12(b) at approximately a radius of 80% of the front rotor tip radius. There is remarkably good agreement between the two, particularly being cognizant of the fact that no streamtube contraction effects or radial flow effects have been included in the model.

Figure 5.12: Normalized wake deficit, at a distance $0.144D_1$ downstream of the front rotor pitch change axis calculated by (a) fitting a Gaussian profile to the CFD data in this plane, (b) using the data collected at the original measurement plane and propagating the wakes downstream using the scaling laws (5.5) and (5.6).

However, some of the discrepancies between the analytical and the numerical result show some important features. In accordance with figure 5.10, slightly larger wakes are observed up to around 60% of the front rotor tip radius. This is in part due to
the presence of acceleration regions at the edges of the CFD wake as well as the discontinuity previously observed at the sliding plane.

Figure 5.13: Normalized wake velocity deficit at the plane $P_2$ at $r \approx 0.8R_{t_1}$. The term $U_{r_1}$ is the mean velocity magnitude at this location.

At some radii, a difference is observed in the angular position of the wake centerline, which engenders a difference in the relative phases between the sources of both models at each spanwise location. This would result in a different treatment of interferences in the calculation of the sound radiation. A preliminary investigation of those disparities showed that those angular shifts correspond to a variation of distance to the leading edge in the chordwise direction of up to 6% of the chordlength (on average, the difference is approximately 1.4%) and differences in phase of up to 2.6% (1% on average). As a result, this contributed to a difference of 1-3dB in the radiated noise levels at particular angles and frequencies, but it had little effect on the directivity pattern of the considered tones. Yet, further investigation of those disparities is required to assess their importance in greater details.

We now wish to propagate the reference wake velocity deficit from $P_1$ to a curved surface at the location of the leading edge of the downstream rotor and to calculate the upwash incident on it. It is assumed that in the vicinity of this location the wake profile develops only very slowly in the axial direction (thus $\beta$ and $u_c$ can be regarded as being constant at axial locations close to the axial position of the leading edge of the downstream rotor blades). Due to the periodicity of the front
rotor wake profiles in the tangential direction, the velocity deficit incident on the rear rotor blades can be decomposed as a Fourier series:

\[
    u(r, \phi_1) = \sum_{n_1=-\infty}^{+\infty} u_{n_1} \exp \{in_1 B_1 \phi_1 \},
\]

(5.7)

with

\[
    u_{n_1} = \frac{B_1}{2\pi} \int_{\phi_c-\frac{\pi}{B_1}}^{\phi_c+\frac{\pi}{B_1}} u(r, \phi_1) \exp \{-in_1 B_1 \phi_1 \} d\phi_1
\]

(5.8)

and \( \phi_c \) is the value of \( \phi_1 \) at the wake centerline at the axial location of the rear rotor blades. Substituting equations (5.2) and (5.3) into equation (5.8) and evaluating the integral yields

\[
    u_{n_1} = (\cos \alpha_1, \sin \alpha_1) \frac{u_c B_1}{2r \cos \alpha_1 \sqrt{\pi \beta}} \exp \left\{ -in_1 B_1 x_1 \frac{\tan \alpha_1}{r} \right\} \exp \left\{ - \left( \frac{n_1 B_1}{2r \cos \alpha_1 \sqrt{\beta}} \right)^2 \right\}.
\]

(5.9)

The upwash on the \( q^{\text{th}} \) blade of the downstream rotor can be rewritten in the \( \{x_2(q), y_2(q)\} \) coordinate system which rotates with the downstream blade row and which is related to the \( \{x_1, y_1\} \) coordinate system by the following equations.

\[
    x_1 = x_2(q) + g + \left( s_2 - \frac{c_2}{2} \cos \alpha_2 + l_2 \sin \alpha_2 \right),
\]

(5.10)

\[
    y_1 = y_2(q) + \frac{2\pi rq}{B_2} + (\Omega_1 + \Omega_2) rt - \left( s_2 - \frac{c_2}{2} \sin \alpha_2 + l_2 \cos \alpha_2 \right),
\]

(5.11)

where \( g \) is the axial distance between \( x_1 = 0 \) and the pitch change axis of the downstream rotor, \( \alpha_2 \) is the stagger angle of the rear rotor blades and \( s_2 \) and \( l_2 \) are, respectively, the rear rotor blade sweep and lean. It is also convenient to define a downstream blade coordinate system, \( \{X_2(q), Y_2(q)\} \) which is related to the coordinate system \( \{x_2(q), y_2(q)\} \) by

\[
    x_2(q) = X_2(q) \cos \alpha_2 + Y_2(q) \sin \alpha_2,
\]

(5.12)

\[
    y_2(q) = -X_2(q) \sin \alpha_2 + Y_2(q) \cos \alpha_2.
\]
The incident upwash normal to the \( q^{th} \) downstream blade, \( v' \), is thus given by

\[
v' = \sum_{n_1=-\infty}^{+\infty} V_{n_1} \exp \left\{ i k_X (U_{r_2} t - X_2^{(q)}) - i k_Y (X_2^{(q)} + in_1 B_1 \frac{2\pi q}{B_2}) \right\},
\]

(5.13)

where

\[
V_{n_1} = \frac{\sin(\alpha_1 + \alpha_2) u_c B_1}{2r \cos\alpha_1 \sqrt{\beta \pi}} \exp \left\{ - \left( \frac{n_1 B_1}{2r \cos\alpha_1 \sqrt{\beta}} \right)^2 \right\} \times \exp \left\{ -i k_X \left( s_2 - \frac{c_2}{2} \right) - i k_Y l_2 + in_1 B_1 \tan\alpha_1 \frac{r}{g} \right\},
\]

(5.14)

\[
k_X = \frac{n_1 B_1}{r \cos\alpha_1} \sin(\alpha_1 + \alpha_2),
\]

(5.15)

\[
k_Y = -\frac{n_1 B_1}{r \cos\alpha_1} \cos(\alpha_1 + \alpha_2),
\]

(5.16)

Figure 5.14 shows the magnitude of the upwash harmonics calculated at one particular radial location at the rear rotor inlet. The upwash velocity for one blade passage is shown in figure 5.14 (a) and the Fourier harmonics are shown in figure 5.14 (b) (the case \( n_1 = 0 \) is not considered here since it does not affect the noise field). It is of particular interest to note that, although the spatial velocity fields from the CFD are well-replicated by the model, the Fourier components show some disagreements for a number of harmonics. These disagreements are clearly influenced by the flow regions at the edge of the wake and these regions contain contributions from velocity fields other than just that of the viscous wake. In fact, the averaging process used in this method may not perfectly isolate the viscous wake thus additional flow components may have been preserved. Consequently, there are likely to be associated errors in the predictions of the interaction tones which are, of course, driven by all the unsteady velocity fields.
Figure 5.14: Normalized upwash velocity at the rear rotor inlet plane ($r \approx 0.8R_t$): (a) velocity magnitude, (b) Fourier harmonics.
5.2.4 Airfoil response

In the present case, only the high frequency regime is considered since the reduced frequency is relatively high for the interaction tones of interest. Thus the acoustic wavelength is sufficiently smaller than the chord so that leading edge noise sources can be decoupled from the trailing edge. Since in the case of wake-interaction most of the loading is concentrated at the downstream blade leading edge, only the latter is considered, the trailing edge contribution being small at high frequency. The downstream airfoil is thus modeled as a semi-infinite plate extending to downstream infinity. The validity of this assumption is discussed at the end of this section. The response of the downstream rotor blade to the incident upwash is calculated using standard two-dimensional unsteady airfoil theory, as described in Goldstein (1974) and Parry (1988), for the high frequency response of an aerofoil to a harmonic convected gust in compressible flow. The total unsteady lift per unit area acting on the planform of the $q^{th}$ downstream blade is equal to the sum of the components of the unsteady lift acting on the blade due to its interaction with each upwash harmonic i.e.

\[
\Delta p(q) (r, X_2(q)) = \sum_{n_1=-\infty}^{+\infty} \Delta p_{n_1} (r, X_2(q)) \exp \left\{ i n_1 B_1 (\Omega_1 + \Omega_2) t + i 2\pi n_1 B_1 B_2 \right\},
\]

(5.17)

where

\[
\Delta p_{n_1} (r, X_2(q)) \exp \left\{ i n_1 B_1 (\Omega_1 + \Omega_2) t \right\}
\]

(5.18)

is the response of the $q^{th}$ blade to a normal incident gust of the form

\[
V_{n_1} \exp \left\{ ik_X (U_r t - X_2(q)) - ik_Y Y_2(q) \right\}.
\]

(5.19)

The pressure jump on the $q^{th}$ rear rotor blade, due to its interaction with each upwash harmonic, is given by

\[
\Delta p_{n_1} (r, X_2(q)) = \frac{2 \rho_0 U_r V_{n_1}}{\pi \sigma_2 (1 + M_{r_2}) \frac{2X_2}{c_2}^{0.5}} \exp \left\{ -i \frac{\pi}{4} - i \frac{\sigma_2 M_{r_2} 2X_2}{1 + M_{r_2}^2} \right\},
\]

(5.20)
where $\sigma_2 = k_X c_2^2$. The lift per unit span is thus given by

$$\frac{dL}{dr} = c_2 \int_0^{c_2} \Delta p_{n_1} \left( r, X_s^{(q)} \right) \exp \left\{ -i \frac{k_{X_s}}{c_2} \left( \frac{2X_s^{(q)}}{c_2} - 1 \right) \right\} dX_s,$$

(5.21)

where $k_{X_s}$ is defined in equation (4.33), and is equal to

$$\frac{dL}{dr} = \sqrt{2\rho_0 U_r c_2 V_{n_1}} \exp \left\{ i\omega t + ik_{X_s} - i\frac{\pi}{4} \right\} E^* \left[ \frac{2}{\sqrt{\pi}} \left( k_{X_s} + \frac{\sigma_2 M_r}{1 + M_r} \right)^{0.5} \right].$$

(5.22)

The function $E^*$ represents the conjugate of the complex Fresnel integral. The first three harmonics of the pressure jump on one rear rotor blade are plotted in figures 5.15, 5.16, and 5.17 for the hybrid analytical/numerical and the strictly numerical methods. With the latter, data were acquired at very-finely spaced radial locations and a linear interpolation was performed over the blade surface. At each radius, the pressure jump on the plate was obtained by taking the orthogonal projection of the difference of pressure between the pressure and suction sides onto the chordwise coordinate. Although the analytical method can capture the general behaviour of the flow field around the blades quite well in comparison to the CFD results, there are few discrepancies in terms of amplitude and spatial distribution of pressure. While the higher harmonics provide better agreement with the CFD data, more disparities are visible for the first loading harmonic from the leading edge up to the blade midchord.

Differences in amplitudes could be due to the fact that the analytically convected wakes are larger at the hub and the tip regions than those simulated by the CFD data (see figure 5.12); and that at some radial locations the amplitudes of the first upwash harmonics provided by the CFD were lower than those calculated analytically, see for example figure 5.14(b). Recall that the potential flow effects, hub and tip vortices, are not included in the model. Besides, the blade is modelled as a flat plate and the shape of the leading edge would influence the distribution of pressure across the blade. Note also that the wake skew and the sweep of the downstream airfoil are not included in the formulation of the 2D airfoil response. In addition, the swirl is not taken into account in the analytical model.
Goldstein (1974) stated that the above formulation was accurate within 10% wherever the reduced frequency $\sigma_2 M_r^2 / \beta_2^2 > 1$, which is the case everywhere except, for the first interaction tone only, from around 98% of the tip radius. This could explain some of the differences between the CFD and the analytical results, as well as the larger differences at low frequency. By inspection of figure 5.15, it is observed that the Kutta condition is satisfied by the CFD whereas the analytical response presents some fluctuations of pressure close to the tip region. Hence, for the first interaction tone and at the tip region, the trailing edge term may not be negligible anymore therefore its contribution shall be investigated in the future, in particular by the addition of a trailing edge back-scattering correction.

Finally, possible reasons of the discrepancies could be numerical dissipation in the wake, errors induced by the linear interpolation, or difficulties of the numerical scheme capturing the acoustics features of the problem.

In figure 5.18 we compare the first three harmonics of the acoustically weighted lift per unit span calculated using the analytical (black lines) and the numerical (blue lines) blade response. Clearly, the lift per unit span calculated analytically is higher than the one given by the CFD, as the amplitudes of the corresponding pressure jump harmonics are also higher. For the first harmonic, the maximum amplitude occurs at a similar radius for both the CFD and the analytical model, however, for higher harmonics the peak is shifted further inboard of the blade. Again, this may be caused by a difference in the wake velocity profiles provided by the analytical model and the CFD at the blade tip region, and the reasons cited above.
Figure 5.15: Airfoil response for \( k = 1 \). (a) predicted levels, (b) numerical results.
Figure 5.16: Airfoil response for $k = 2$. (a) predicted levels, (b) numerical results.
Figure 5.17: Airfoil response for $k = 3$. (a) predicted levels, (b) numerical results.
Figure 5.18: Lift per unit span for (a) $k = 1$, (b) $k = 2$, (c) $k = 3$. 
5.3 Wake interaction noise prediction

The interaction noise due to the impingement of the front rotor viscous wakes onto the rear rotor blades can be calculated using the radiation formula given by equation (4.30) from section 4 giving

\[
P_L(x, t) = \sum_{k=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \int_{r_{h2}}^{r_{t2}} \frac{iB_2}{4\pi S} \exp \left\{ i(n_2\Omega_2 + n_1\Omega_1) \left( t - \frac{r}{c_0} \right) \right\} \\
\times \exp \left\{ -i(n_2B_2 - n_1B_1) \left( \phi' + \frac{\pi}{2} \right) \right\} \exp \left\{ -i\Phi_s - ikYl \right\} \\
\times kYl J_{n_1B_1-b_2B_2} \left( \frac{\omega r_y S_s}{c_0} \right) \Psi_L \, dr_y,
\]

(5.23)

where \( \Psi_L = \frac{dL}{dr} \) is the acoustically-weighted unsteady lift per unit span.

Noise predictions made using the method described in the previous sections are compared with noise measurements made during testing of Airbus’s Z08 rig with Rolls-Royce’s blades during the Z08 test campaign conducted in the DNW large, low speed wind tunnel (see Chapter 2), as well as with noise predictions made using the blade response given by the CFD. Figure 5.19 shows the noise spectrum at \( \theta' = 90^\circ \) for a particular experiment conducted with an isolated rig running at a condition representative of take-off at zero angle of attack. All the results presented in the present section are representative of this particular operating condition.

Each tone emanates from an integration of ‘sources’ over the radial coordinate \( r_y \). Parry (1988) introduced that the radiation efficiency of the interaction tones is dependent on the non-dimensional sonic radius \( z^* \), the radius at which the source would be moving toward the observer at sonic velocity, defined by

\[
z^* = \frac{\nu(1 - M_x \cos \theta')c_0}{(n_1B_1\Omega_1 + n_2B_2\Omega_2)R_{t3} \sin \theta'}.
\]

(5.24)

A detailed discussion about the sonic radius given by Kingan et al. (2014) suggests that if the sonic radius lies inboard of the tip \( (0 < |z^*| < 1) \), the tones will radiate efficiently in the vicinity of \( z^* \). In the present case, the absolute value of the sonic radius related to the frequency for each interaction tone is plotted in figure 5.20. Note that tones that do not appear in the spectrum are labelled using red font, whereas tones which do appear are labelled in black. It is clearly observed that tones for which the sonic radius lies somewhat outboard of the tip (i.e. tones for
which $|z^*|$ is not much greater than approximately 1) do not appear in the noise spectrum plotted in figure 5.19.

![Figure 5.19: SPL spectrum for an isolated open rotor rig operating at a take-off type thrust measured at a polar emission angle $\theta' = 90^\circ$ and normalized to a constant emission radius.](image)

Figure 5.20: Sonic radius plotted against frequency at a polar emission angle $\theta' = 90^\circ$ for each of the interaction tones expected to occur at frequencies $\omega$ for the given take-off case. Tones are labelled $\{n_1, n_2\}$. 

![Figure 5.20: Sonic radius plotted against frequency at a polar emission angle $\theta' = 90^\circ$ for each of the interaction tones expected to occur at frequencies $\omega$ for the given take-off case. Tones are labelled $\{n_1, n_2\}$.](image)
Chapter 5 Wake interaction noise

The predicted polar directivities of a selection of interaction tones are compared with the wind tunnel measurements and the CFD-based calculations in figures 5.21 to 5.26. It is important to note that, once the wake profiles have been obtained (on this occasion, via unsteady CFD) the additional wake decay, unsteady blade response and acoustic radiation can be performed immediately, and very quickly, using the analytical formulae (5.23). The predictions show good agreement with the experimental data for both the low frequency and the high frequency tones in terms of absolute level and in terms of the envelope shape of the directivity characteristics, except at some particular observer locations (note that some of the discrepancies could be due to measurements uncertainties). There is also a good agreement between the CFD-based calculations and the analytical model in terms of overall trend. In terms of noise levels, the numerical and analytical results agree within 1dB to 10dB margin at a wide range of angles and frequencies. However, larger differences are observed at distinct locations and, at few frequencies, over a wide range of angles, e.g. between 100° and 150° for the tone \{3, 4\}, see figure 5.24. Some phase shifts are also observed between the directivity lobes of the tones predicted by each method.

Note here that the unsteady velocities produced by the rotors’ bound potential fields, and by the front rotor tip vortex (which are not included in these results) affect both the total unsteady flow field and the levels of the radiated interaction tones at some frequencies and some observer positions; particularly as their directivity characteristics differ from those of convected wake interaction tones. However, it is most likely that the low frequency results will be affected as the potential fields decay rapidly with frequency, and because the tip vortex contribution is likely to be small for the cropped rear row used in our example. Indeed, the results in section 5.2.3 above, showed that the harmonics of the velocity upwash onto the rear rotor are affected by the potential field component of the flow, particularly at the lower harmonics. The potential field effects are investigated further in section 6.

Other possible sources of discrepancies between the analytical and the numerical results could be attributed to numerical errors (e.g. the discontinuity at the sliding plane or numerical dissipation), or the assumptions of the method, for example the neglect of the wake skew or sweep effects in the analytical blade response (which would affect both the phase and levels). Since the differences in noise levels are directly linked to the differences in the lift distributions, future efforts will be concentrated on the improvement of the modelling of the source and the blade response.
Figure 5.21: Polar directivity of the tone \{1, 1\} for a given take-off case compared with wind tunnel measurements and CFD data.

Figure 5.22: Polar directivity of the tone \{1, 2\} for a given take-off case compared with wind tunnel measurements and CFD data.
Figure 5.23: Polar directivity of the tone \( \{2, 2\} \) for a given take-off case compared with wind tunnel measurements and CFD data.

Figure 5.24: Polar directivity of the tone \( \{3, 4\} \) for a given take-off case compared with wind tunnel measurements and CFD data.
Figure 5.25: Polar directivity of the tone \( \{3, 6\} \) for a given take-off case compared with wind tunnel measurements and CFD data.

Figure 5.26: Polar directivity of the tone \( \{4, 4\} \) for a given take-off case compared with wind tunnel measurements and CFD data.
5.4 Concluding remarks

A hybrid method which can be used to predict the development of the front rotor viscous wakes and to calculate the far-field tonal noise caused by the impingement of such wakes on the advanced open rotor’s downstream blades has been developed. The method uses a simple two-dimensional analytical model for which the wake start-up characteristics are taken from data on a CFD plane between the two blade rows, and convected to the leading edge of the downstream rotor. Comparisons at other wake propagation distances show that the method gives good agreement with the CFD for most radii. However, possible inaccuracies in the numerical scheme or some phenomena such as the bound potential fields of both blade rows, the tip vortex of the front rotor, and stream tube contraction are not included and account for some of the discrepancies in the total noise predictions.

The analytical method for predicting the unsteady flow field associated with the upstream rotor wakes and the radiated far-field tonal noise was assessed by comparison with wind tunnel measurements and CFD data. Comparisons of the unsteady loading on the downstream blade calculated using both the CFD simulation and the hybrid method showed promising results in the ability to predict the general behaviour of the flow field around the blades. The noise predictions show satisfactory agreement between the wind tunnel measurements, the analytical model, and the CFD-based method in a wide range of observer locations and frequencies. Yet, the origins of the discrepancies observed in the analysis need further investigation, in particular the the source and blade response modelling, as well as the reasons cited above.
Chapter 6

Potential field interactions

6.1 Introduction

Along with the viscous wakes, the bound potential field of each rotor blade produces an 'incident' unsteady (periodic) flow field on the blades of the adjacent rotor. The potential field is produced by the blade thickness and loading (due to the airfoil’s camber and angle of attack). For a single rotor blade, the bound potential field decays away from the rotor relatively quickly whereas the viscous wake convects with the local flow. Because the blade rows can be separated by a sufficient distance to ensure that the potential field interactions are much less significant than the viscous wake source, the potential field effects have, to the author’s knowledge, received little attention in the literature. For smaller distance between the two blade rows, the potential field effects can become significant. Kingan et al. (2014) showed that for some cases the interaction between the potential field of a blade row with the adjacent rotor can be a dominant source of tonal noise.

Kemp and Sears (1953) first derived an expression for the potential field produced by a cascade of blades. They only accounted for the loading component, modelling each blade as a point vortex of strength equal to the steady blade circulation. Parry (1988) extended the analysis to incorporate the compressibility effects and derived an expression for the potential field due to blade thickness. In the latter the blades were modelled as a cascade of point sources whose strength was determined from the potential field of an ellipse. In the present section, the loading and thickness components of the potential field produced by each blade are modeled by a distribution of sources along the blade chordline. The upwash on the adjacent
blade and the corresponding blade response are then formulated. The effect of varying the airfoil geometry is also investigated. In section 6.4 the potential field interaction noise is calculated for a given take-off case. Results are compared with Parry’s model for point sources, and with wind tunnel measurements and URANS CFD data. For convenience, the theory presented in this chapter will be termed ‘model 1’ and Parry’s formulation will be termed ‘model 2’. The latter is presented in Appendix D.

The velocity produced by the bound potential field of a rotor blade is equal to the gradient of the velocity potential \( \phi \)

\[
\mathbf{u} = \nabla \phi.
\]  

(6.1)

The velocity field \( \mathbf{u} \) is irrotational and the potential satisfies the inhomogeneous Laplace equation

\[
\nabla^2 \phi = S,
\]  

(6.2)

where \( S \) represents a source term. Since the governing equations are linear, solutions may be superimposed linearly. The solution is thus written as the sum of the free-stream potential and a disturbance potential. The latter is decomposed into two terms, one giving the field due to thickness, and the other the potential due to loading, see Ashley and Landahl (1965). Thus, the thickness and loading problems will be solved separately. For each problem, the potential field produced by the compressible flow over a thin airfoil may be regarded as the potential field produced by a distribution of elementary sources of strength \( f \) along the blade chord line.

\[
\phi(X,Y) = \int_{-c}^{c} f(X_s) \vartheta((X,Y)|(X_s,0)) \, dX_s,
\]  

(6.3)

where \( c \) is the blade chordlength and \( \vartheta((X,Y)|(X_s,0)) \) is the potential produced by one elementary source of unit strength, located at \( X = X_s \) and \( Y = 0 \), and satisfying the governing equations. The value of \( f \) is to be determined for the thickness problem and the loading problem independently. The blade geometry and the coordinates \( \{X,Y\} \) are shown in figure 6.1.

The location of the upper (+) and lower (−) surface of the airfoil is
\( Y^\pm = \pm \frac{t_{\text{max}}}{2} G(X) + \theta h(X) - \gamma X, \)  

\[(6.4)\]

where the functions \( G(X) \) and \( h(X) \) represent dimensionless thickness and camber distributions along the blade chord respectively, \( t_{\text{max}} \) is the maximum thickness, \( \gamma \) is the angle of attack, and \( \theta \) represents the maximum camber. The free-stream potential is related to the free-stream velocity \( U_r \) by

\[ \phi_\infty = U_r x. \]  

\[(6.5)\]

### 6.2 Upstream interactions

In this section we derive expressions for the unsteady potential field induced by the downstream blade row, the upwash incident on the upstream airfoil and the airfoil’s response. The blade rows are modeled by infinite cascades of equally spaced flat airfoils. We note \( s = \frac{2\pi r}{B} \) is the distance between two airfoils. The configuration is shown in figure 6.2, where \( g \) is the gap between the two rotors, \( \alpha \) is the blade stagger angle, \( \{X,Y\} \) are the coordinates parallel and normal to the chordline, and the \( \{x,y\} \) coordinate system is related to the cylindrical polar coordinate system \( \{x,r,\varphi\} \), via \( y = r\varphi \). The subscripts 1 and 2 denote values associated with the upstream and downstream rotor respectively.
6.2.1 Potential field due to blade thickness

6.2.1.1 Velocity potential of the downstream airfoil

Isolated Airfoil. We consider a thin symmetric airfoil at zero angle of attack. We denote $U_{r_2}$ the velocity incident on the airfoil and $M_{r_2}$ the corresponding Mach number. The potential due to blade thickness $\phi^t$ satisfies the governing equations and boundary conditions which are given by Ashley and Landahl (1965) [5-29 to 5-30]

$$\beta_2^2 \frac{\partial^2 \phi^t}{\partial X_2^2} + \frac{\partial^2 \phi^t}{\partial Y_2^2} = 0,$$

(6.6)

$$\lim_{Y_2 \to 0^\pm} \frac{\partial \phi^t}{\partial Y_2} \left( -\frac{c_2}{2} < X_2 < \frac{c_2}{2}, Y_2 \right) = \pm U_{r_2} \frac{\partial y^\pm}{\partial X_2},$$

(6.7)

where $c_2$ is the local chordlength of the downstream airfoil, $\beta_2 = \sqrt{1 - M_{r_2}^2}$, and $y^\pm$ is defined in equation (6.4). The boundary condition associated with the thickness component is deduced from equations (6.7) and (6.4) giving

$$\lim_{Y_2 \to 0^\pm} \frac{\partial \phi^t}{\partial Y_2} \left( -\frac{c_2}{2} < X_2 < \frac{c_2}{2}, Y_2 \right) = \pm U_{r_2} \frac{t_{\text{max}}}{2} \frac{\partial G}{\partial X_2}.$$

(6.8)
Introducing the stretched Prandtl-Glauert coordinates \( \{ \chi_2, \zeta_2 \} = \{ X_2/\beta_2, Y_2 \} \), and substituting \( \chi_2 \) into equation (6.6) yields a two dimensional form of the Laplace equation. The potential \( \vartheta^t \) of a point source located at \( \chi_2 = X_s/\beta_2 \) and \( \zeta_2 = 0 \) actually satisfies

\[
\frac{\partial^2 \vartheta^t}{\partial \chi^2} + \frac{\partial^2 \vartheta^t}{\partial \zeta^2} = \delta \left( \chi_2 - \frac{X_s}{\beta_2} \right) \delta(\zeta_2). \tag{6.9}
\]

The above equation can be written in the \( \{ X_2, Y_2 \} \) coordinate system as

\[
\beta_2^2 \frac{\partial^2 \vartheta^t}{\partial X^2} + \frac{\partial^2 \vartheta^t}{\partial Y^2} = \beta_2 \delta (X - X_s) \delta(Y). \tag{6.10}
\]

An elementary singular solution to equation (6.10) is

\[
\vartheta^t((X_2, Y_2)|(X_s, 0)) = \frac{1}{2\pi} \log \left( \frac{1}{\beta_2^2 (X_2 - X_s)^2 + \beta_2^2 Y^2} \right). \tag{6.11}
\]

The airfoil thickness effect is modeled as a distribution of two dimensional elementary sources and sinks of volume flow rate per unit area \( f^t \ (m.s^{-1}) \) located at points \( X_s = \{ X_s, Y_s = 0 \} \) along the blade chordline, see figure 6.3. The potential can be obtained by integrating the contribution of each point source over the airfoil chord as follows

\[
\varphi^t(X_2, Y_2) = \frac{1}{2\pi} \int_{-c^2}^{c^2} f^t(X_s) \log \left( \frac{1}{\beta_2^2 (X_2 - X_s)^2 + \beta_2^2 Y^2} \right) dX_s. \tag{6.12}
\]

Hence

\[
\frac{\partial \varphi^t}{\partial Y_2} = \frac{1}{2\pi} \int_{-c^2}^{c^2} \frac{\beta_2^2 Y}{(X_2 - X_s)^2 + \beta_2^2 Y^2} dX_s. \tag{6.13}
\]

Taking the limit of equation (6.13) as \( Y_2 \to 0^\pm \), noting that

\[
\lim_{Y_2 \to 0^\pm} \frac{\beta_2^2 Y_2}{(X_2 - X_s)^2 + \beta_2^2 Y_2^2} = \pm \beta_2 \pi \delta(X_2 - X_s), \tag{6.14}
\]
and equating the above result with the boundary condition defined in equation 6.7 gives the following expression for the source strength.

\[
f^t(X_2) = \frac{U_{r_2} t_{max}}{\beta_2} \frac{\partial \mathcal{G}(X_2)}{\partial X_2}.
\]  

(6.15)

Figure 6.3: Thickness source distribution.

To allow fast and simple calculations of the velocity potential, we chose to model the airfoil’s thickness distribution using two sine functions whose coefficients are deduced from a given airfoil’s geometry. The non-dimensional thickness function \( \mathcal{G}(X_2) \) is thus expressed as

\[
\begin{align*}
\mathcal{G}(X_2) &= \sin \left( \frac{\pi}{2X_m} \left( X_2 + \frac{c_2}{2} \right) \right) \quad \text{if } -\frac{c_2}{2} < X_2 < -\frac{c_2}{2} + X_m, \\
\mathcal{G}(X_2) &= \sin \left( \frac{\pi}{2} + \frac{\pi}{2} \frac{X_2 + \frac{c_2}{2} - X_m}{c_2 - X_m} \right) \quad \text{if } -\frac{c_2}{2} + X_m < X_2 < \frac{c_2}{2}. 
\end{align*}
\]  

(6.16)

From Equations (6.15) and (6.16), the source strength is

\[
\begin{align*}
f^t(X_2) &= \frac{U_{r_2} t_{max} \pi}{2\beta_2 X_m} \cos \left( \frac{\pi}{2X_m} \left( X_2 + \frac{c_2}{2} \right) \right) \quad \text{if } -\frac{c_2}{2} < X_2 < -\frac{c_2}{2} + X_m, \\
f^t(X_2) &= -\frac{U_{r_2} t_{max} \pi}{2\beta_2 (c_2 - X_m)} \sin \left( \frac{\pi}{2} \frac{X_2 + \frac{c_2}{2} - X_m}{c_2 - X_m} \right) \quad \text{if } -\frac{c_2}{2} + X_m < X_2 < \frac{c_2}{2}. 
\end{align*}
\]  

(6.17)

Hence the potential due to blade thickness of a single airfoil is given by
\begin{equation}
\phi' = \frac{U_r t_{\text{max}}}{4\beta_2} \left[ \frac{1}{X_m} \int_{-\frac{c_2}{2}+X_m}^{\frac{c_2}{2}+X_m} \cos \left( \frac{\pi}{2X_m} \left( X_2 + \frac{c_2}{2} \right) \right) \log \left( \frac{1}{\beta_2} \sqrt{(X_2 - X_s)^2 + \beta_2^2 Y_2^2} \right) dX_s 
- \frac{1}{c_2 - X_m} \int_{-\frac{c_2}{2}+X_m}^{\frac{c_2}{2}} \sin \left( \frac{\pi X_2 + \frac{c_2}{2} - X_m}{2 \frac{c_2}{2} - X_m} \right) \log \left( \frac{1}{\beta_2} \sqrt{(X_2 - X_s)^2 + \beta_2^2 Y_2^2} \right) dX_s \right].
\end{equation}

The velocity potential around an isolated airfoil in incompressible flow ($\beta_2 = 1$) is illustrated in figures 6.4 calculated using equation (6.18). In figure 6.5 the influence of the downstream airfoil’s geometry on the upwash incident on the upstream airfoil is investigated at $r = 60\% R_{t_1}$ at various axial locations. The upwash velocity calculated using the thickness distribution defined in equation (6.16) (geometry A) is compared to that calculated using the thickness distribution function for an elliptic airfoil (geometry B)

\begin{equation}
G_{\text{ellipse}}(X_2) = \sqrt{1 - \left( \frac{2X_2}{c_2} \right)^2}.
\end{equation}

Figure 6.4: Velocity potential calculated with model 1.

It is important to note that there is a good agreement between model 1 and model 2 when using the same geometry. There is a notable difference in values between the
upwash velocities calculated using geometries A and B as input in the distributed model at a wide range of axial locations. This shows that the type of profile inputed in the model greatly influences the results. The differences between the two models are assessed further in figure 6.6 in terms of percentage difference in the amplitude of the upwash velocity incident on the upstream airfoil at a fixed radius and various azimuthal locations. Close to the downstream airfoil’s leading edge, differences up to 60% are notable. Further away from the airfoil the differences decrease but remain around 35%. When using the geometry of an ellipse in the distributed model, which was the geometry used in Parry’s model, the differences in upwash velocity amplitude are much lower close to the downstream airfoil (up to $\approx 30\%$ one chordlength away and up to $\approx 10\%$ two chordlength away) and the two models converge in the far field within an error of about 5 %, see figure 6.7.

![Figure 6.5: Upwash velocity (normalized against the free stream velocity) calculated using models 1 and 2 at $r = 60\%R_1$. Geometry A: sum of sine functions; Geometry B: ellipse.](image)

Indeed, in Appendix D it is shown that Parry’s expression for a very thin ellipse ($t_{\text{max}} \ll c_2$) (D.7) can be recovered from the formulation of the distributed source model and the error between the two models can be determined analytically. Taking into account the distribution of sources along the airfoil chordline can thus have a non negligible influence on the resulting potential field when the gap between the two rotors is less than two chordlengths.
Figure 6.6: Percentage difference in upwash velocity between model 1 (with geometry A) and model 2.

Figure 6.7: Percentage difference in upwash velocity between model 1 (using an elliptic geometry) and model 2.
Cascade of airfoils. Consider now the potential produced by a cascade of equally spaced airfoils of the downstream row at locations \( y_2 = n_2 s_2 \), as described by figure 6.2. The \( \{ x_2, y_2 \} \) and \( \{ X_2, Y_2 \} \) coordinate systems are related by

\[
\begin{align*}
X_2 &= \cos \alpha_2 x_2 - \sin \alpha_2 y_2, \\
y_2 &= \sin \alpha_2 x_2 + \cos \alpha_2 y_2,
\end{align*}
\]

(6.20)

and

\[
\begin{align*}
x_2 &= \cos \alpha_2 X_2 + \sin \alpha_2 Y_2, \\
y_2 &= -\sin \alpha_2 X_2 + \cos \alpha_2 Y_2,
\end{align*}
\]

(6.21)

The governing equation (6.10) becomes

\[
\frac{\partial^2 \vartheta}{\partial x_2^2} + \frac{\partial^2 \vartheta}{\partial y_2^2} = \beta_2 \sum_{n_2 = -\infty}^{+\infty} \delta (X_2 - X_s + n_2 s_2 \sin \alpha_2) \delta (Y_2 - n_2 s_2 \cos \alpha_2). \tag{6.22}
\]

The above equation can be written in terms of \( \{ x_2, y_2 \} \) coordinates making use of the following theorem (see Izzo (2005))

\[
\delta \left[ \mathbf{x} - \hat{x}(\mathbf{y}) \right] = \sum_i \frac{\delta [\mathbf{y} - \mathbf{y}_i]}{|J|_{y=y_1}}, \tag{6.23}
\]

where \( J = \frac{\partial \hat{x}_i}{\partial y_i} \) is the Jacobian of the transformation \( \hat{x}(\mathbf{y}_i) \) and \( \mathbf{y}_i \) are the solutions to the equation \( \mathbf{x} = \hat{x}(\mathbf{y}_i) \). In the present case the Jacobian determinant related to the change from the \( \{ X_2, Y_2 \} \) to the \( \{ x_2, y_2 \} \) coordinates is

\[
\det \left[ \frac{\partial \{ x_2, y_2 \}}{\partial \{ X_2, Y_2 \}} \right] = 1. \tag{6.24}
\]

Note that the arguments of the Dirac delta functions in equation (6.22) are related to the \( \{ x_2, y_2 \} \) coordinates by

\[
\begin{align*}
X_2 - X_s + n_2 s_2 \sin \alpha_2 &= (x_2 - X_s \cos \alpha_2) \cos \alpha_2 - (y_2 - n_2 s_2 + X_s \sin \alpha_2) \sin \alpha_2, \\
Y_2 - n_2 s_2 \cos \alpha_2 &= (x_2 - X_s \cos \alpha_2) \sin \alpha_2 + (y_2 - n_2 s_2 + X_s \sin \alpha_2) \cos \alpha_2,
\end{align*}
\]

(6.25)
and that they equal zero if and only if \( x_2 = X_s \cos \alpha_2 \) and \( y_2 = n_2 s_2 - X_s \sin \alpha_2 \). Therefore

\[
\delta (X_2 - X_s + n_2 s_2 \sin \alpha_2) \delta (Y_2 - n_2 s_2 \cos \alpha_2) = \delta (x_2 - X_s \cos \alpha_2) \delta (y_2 - n_2 s_2 + X_s \sin \alpha_2).
\]

Equation (6.22) can be rewritten by making use of equation (6.26) giving

\[
\nabla_c^2 \vartheta^t = \beta_2 \sum_{n_2=-\infty}^{+\infty} \delta (x_2 - X_s \cos \alpha_2) \delta (y_2 + X_s \sin \alpha_2 - n_2 s_2),
\]

with

\[
\nabla_c^2 = \left( \sin^2 \alpha_2 + \beta_2^2 \cos^2 \alpha_2 \right) \frac{\partial^2}{\partial x_2^2} + 2 \sin \alpha_2 \cos \alpha_2 \left( 1 - \beta_2^2 \right) \frac{\partial^2}{\partial x_2 \partial y_2} \\
+ \left( \cos^2 \alpha_2 + \beta_2^2 \sin^2 \alpha_2 \right) \frac{\partial^2}{\partial y_2^2}.
\]

Applying Poisson’s summation formula to equation (6.27) leads to

\[
\nabla_c^2 \vartheta^t = \frac{\beta_2}{s_2} \delta (x_2 - X_s \cos \alpha_2) \sum_{n_2=-\infty}^{+\infty} \exp \left\{ -2i\pi \frac{n_2}{s_2} (y_2 + X_s \sin \alpha_2) \right\}.
\]

To calculate the velocity potential \( \vartheta^t \), it is convenient to use the Fourier transform pair

\[
\widetilde{\vartheta^t}(k_x, k_y) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{+\infty} \vartheta^t(x_2, y_2) \exp \{ ik_x x_2 + i k_y y_2 \} \, dx_2 \, dy_2, \quad (6.30)
\]

\[
\vartheta^t(x_2, y_2) = \iint_{-\infty}^{+\infty} \widetilde{\vartheta^t}(k_x, k_y) \exp \{ -ik_x x_2 - i k_y y_2 \} \, dk_x \, dk_y. \quad (6.31)
\]

Hence, performing the Fourier transform of equation (6.29) yields
\[ \vartheta_t(x_2, y_2) = -\frac{\beta_2}{2\pi s_2} \sum_{n_2 = -\infty}^{+\infty} \frac{\delta(k_y - K_y)}{D} \exp \left\{ -iK_{y_2}X_s \sin \alpha_2 + ik_xX_s \cos \alpha_2 \right\}, \]

\[ \quad \text{(6.32)} \]

where \( K_{y_2} = \frac{2\pi n_2}{s_2} \) and

\[ D = k_x^2 \left( \sin^2 \alpha_2 + \beta_2^2 \cos^2 \alpha_2 \right) + 2 \sin \alpha_2 \cos \alpha_2 \left( 1 - \beta_2^2 \right) k_x K_{y_2} + K_{y_2}^2 \left( \cos^2 \alpha_2 + \beta_2^2 \sin^2 \alpha_2 \right), \]

\[ \quad \text{(6.33)} \]

This yields

\[ \vartheta_t(x_2, y_2) = -\frac{\beta_2}{2\pi s_2} \sum_{n_2 = -\infty}^{+\infty} \exp \left\{ -iK_{y_2}(y_2 + X_s \sin \alpha_2) \right\} \int_{-\infty}^{+\infty} \frac{\exp \left\{ -ik_x(x_2 - X_s \cos \alpha_2) \right\}}{D} \, dk_x. \]

\[ \quad \text{(6.34)} \]
The integrand in equation (6.34) is analytic in the $k_x$-complex plane and has poles at

$$k_x = K_y \frac{\pm i \beta_2 - \sin \alpha_2 \cos \alpha_2 (1 - \beta_2^2)}{\beta_2^2 \cos^2 \alpha_2 + \sin^2 \alpha_2} = \kappa_r \pm i \kappa_c, \quad \kappa_c > 0. \quad (6.35)$$

We can thus perform the integration over a closed contour $C$ in the half plane containing one of the poles and calculate its values using the theorem of Residues. Note that we set $x_2 < 0$ in the upstream direction. For upstream interactions the contour is closed in the upper half complex plane containing the pole $\kappa^+ = \kappa_r + i \kappa_c$.

The integration contour $C$ can be decomposed into a line segment along the real axis ($C_R$) between $-R_C$ and $R_C$ and an outer semi-circle ($C_C$) closing the contour, centered at the origin and of radius $R_C$, see figure 6.28. As $R_C \to \infty$, the integral over $C_C$ vanishes and the Residue theorem gives

$$\oint_{C_R} \frac{\exp \{ -i k_x (x_2 - X_s \cos \alpha_2) \}}{D} \, dk_x = 2\pi i \text{Res}_{\kappa^+} \left( \frac{\exp \{ -i k_x (x_2 - X_s \cos \alpha_2) \}}{D} \right). \quad (6.36)$$

![Figure 6.9: Contour integration.](image)

The potential field due to blade thickness produced by a unit strength elementary source is thus

$$\vartheta^t(x_2, y_2) = -\frac{1}{2s_2} \sum_{n_2=-\infty}^{+\infty} \frac{1}{K_{y_2}} \exp \{ K_{y_2} (\nu_1 + i \nu_2) (x_2 - X_s \cos \alpha_2) - iK_{y_2} (y_2 + X_s \sin \alpha_2) \}. \quad (6.37)$$
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where

\[
\nu_1 = \frac{\beta_2}{\beta_2^2 \cos^2 \alpha_2 + \sin^2 \alpha_2}, \quad (6.38)
\]

\[
\nu_2 = \frac{\sin \alpha_2 \cos \alpha_2 (1 - \beta_2^2)}{\beta_2^2 \cos^2 \alpha_2 + \sin^2 \alpha_2}. \quad (6.39)
\]

In order to calculate the velocity potential due to blade thickness for a distributed source, we must determine the source strength \(f^t\), i.e. the thickness distribution function \(G\) must be defined, and evaluate

\[
\phi^t(x_2, y_2) = -\frac{U_{r_2} t_{max}}{2 s_2 \beta_2} \sum_{n_2=-\infty}^{+\infty} \frac{1}{K_{y_2}} \exp \left\{ K_{y_2} (\nu_1 + i \nu_2) x_2 - iK_{y_2} y_2 \right\}

\times \int_{c_2^2}^{X_m^2} \exp \left\{ -K_{y_2} (\nu_1 + i \nu_2) X_s \cos \alpha_2 - iK_{y_2} X_s \sin \alpha_2 \right\} \frac{\partial G}{\partial X_2} \bigg|_{X_2=X_s} dX_s.
\]

(6.40)

The velocity potential is obtained by inserting equation (6.16) into equation (6.40), i.e.

\[
\phi^t = -\frac{U_{r_2} t_{max} \pi}{4 s_2 \beta_2} \sum_{n=-\infty}^{+\infty} \frac{1}{K_{y_2}} \exp \left\{ K_{y_2} (\nu_1 + i \nu_2) x_2 - iK_{y_2} y_2 \right\} [\mathcal{I}_1 + \mathcal{I}_2], \quad (6.41)
\]

where

\[
\mathcal{I}_1 = \frac{1}{X_m} \int_{c_2^2}^{X_m^2} \cos \left( \frac{\pi}{2X_m} \left( X_s + \frac{c_2}{2} \right) \right) \exp \left\{ -K_{y_2} [ (\nu_1 + i \nu_2) \cos \alpha_2 + i \sin \alpha_2 ] X_s \right\} dX_s,
\]

\[
\mathcal{I}_2 = \frac{-1}{c_2 - X_m} \int_{\frac{c_2^2}{2} + X_m}^{\frac{c_2^2}{2}} \sin \left( \frac{\pi}{2} \frac{X_s + \frac{c_2}{2} - X_m}{c_2 - X_m} \right) \exp \left\{ -K_{y_2} [ (\nu_1 + i \nu_2) \cos \alpha_2 + i \sin \alpha_2 ] X_s \right\} dX_s.
\]

(6.42)
The integrals $I_1$ and $I_2$ can be easily calculated giving

$$I_1 = \frac{2 \exp \left\{ E_2 \left( \frac{c_2}{2} - X_m \right) \right\} \left[ \pi + 2E_2X_m \exp \{ E_2X_m \} \right]}{\pi^2 + 4E_2^2X_m^2}, \quad (6.43)$$

$$I_2 = -\frac{2 \left[ \pi \exp \left\{ E_2 \left( \frac{c_2}{2} + X_m \right) \right\} - 2 \exp \left\{ -\frac{c_2}{2}E_2 \right\} \left( c_2 - X_m \right)E_2 \right]}{\pi^2 + 4E_2^2(c_2 - X_m)^2}, \quad (6.44)$$

where

$$E_2 = K_{y_2} (\nu_1 + i \nu_2) \cos \alpha_2 + i \sin \alpha_2. \quad (6.45)$$

According to equation (D.8) in Appendix D, in the cascade representation the velocity potentials given by model 1 and model 2 are proportional by a factor

$$F = \frac{(I_1 + I_2) (\beta_2 \cos^2 \alpha_2 + \sin^2 \alpha_2)}{\beta_2 K_{y_2} (\beta_2 a + b)(\beta_2 \cos \alpha_2 + i \sin \alpha_2)}, \quad (6.46)$$

with $a = t_{max}/2$ and $b = c_2/2$.

Note that the velocity potential around an ellipse can be determined inserting equations (6.19) into equation (6.40) to give

$$\phi'_{ellipse} = -\frac{U_{r_2}t_{max}}{2s_2\beta_2} \sum_{n=-\infty}^{+\infty} \frac{1}{K_{y_2}} \exp \left\{ K_y (\nu_1 + i \nu_2)x_2 - iK_{y_2}y_2 \right\} I_e, \quad (6.47)$$

where

$$I_e = \pi I_1 \left( \frac{c_2}{2} K_{y_2} (\nu_1 + i \nu_2) \cos \alpha_2 + i \sin \alpha_2 \right), \quad (6.48)$$

and $I_1$ is the modified Bessel function of the first kind with order 1.

### 6.2.1.2 Upwash on the upstream airfoil

Consider an isolated airfoil of the upstream row in the coordinate system defined in figure 6.2. Recall that the suffixes 1 and 2 denote values associated with the
upstream and downstream rotor respectively and that \( x_2 < 0 \) in the upstream direction. The upwash normal to the airfoil is given by

\[
  u_1 = -\sin \alpha_1 \frac{\partial \phi^l}{\partial x_2} + \cos \alpha_1 \frac{\partial \phi^l}{\partial y_2},
\]

where \( \alpha_1 \) is the stagger angle of the upstream airfoil. Applying equation (6.49) to equation (6.47), the upwash on the upstream airfoil is given by

\[
  u_1 = U_r \frac{t_{\max}}{s_2 \beta_2} \left[ (\nu_1 + i\nu_2) \sin \alpha_1 + i \cos \alpha_1 \right] \sum_{n=-\infty}^{+\infty} \exp \left\{ K_y (\nu_1 + i\nu_2)x_2 - iK_y y_2 \right\} [I_1 + I_2].
\]

The \((x_1, y_1)\) and \((x_2, y_2)\) coordinate systems defined in figure 6.2 are related by

\[
  x_2 = x_1 - \left( g - \frac{c_1}{2} \cos \alpha_1 \right),
  \quad
  y_2 = y_1 + \frac{c_1}{2} \sin \alpha_1 - r(\Omega_1 + \Omega_2)t,
\]

where \( g \) is the gap between the midchords of the upstream and downstream rotors and \( c_1 \) is the local chordlength of the upstream airfoil. The \((x_1, y_1)\) and \((X_1, Y_1)\) coordinate systems are related by

\[
  x_1 = X_1 \cos \alpha_1 - Y_1 \sin \alpha_1,
  \quad
  y_1 = X_1 \sin \alpha_1 + Y_1 \cos \alpha_1.
\]

Inserting equations (6.51) and (6.52) into equation (6.50), one harmonic of the upwash normal to the upstream airfoil can be written in the form

\[
  u_1 = W^l_1 \exp \left\{ \lambda_{x_2} X_1 + \lambda_{y_2} Y_1 + i\omega_2 t \right\},
\]

where
\[
\begin{aligned}
W_{t1} &= -\lambda_{y2} \frac{U_{r2} t_{\text{max}}}{8n_2 \beta_2} \exp \{ A_1 \} [I_1 + I_2], \\
A_1 &= -K_{y2} \left( \eta_2 \left( g - \frac{c_1}{2} \cos \alpha_1 \right) + i \frac{c_1}{2} \sin \alpha_1 \right), \\
\eta_2 &= \nu_1 + i \nu_2, \\
\lambda_{x2} &= K_{y2} (\eta_2 \cos \alpha_1 - i \sin \alpha_1), \\
\lambda_{y2} &= -K_{y2} (\eta_2 \sin \alpha_1 + i \cos \alpha_1), \\
\omega_2 &= K_{y2} r (\Omega_1 + \Omega_2).
\end{aligned}
\] (6.54)

Note that the harmonic number $n_2$ is assumed to be positive here. Figure 6.10 shows the ten first harmonics of the upwash at $r \approx 60\% R_{t1}$ at the upstream rotor’s trailing edge calculated using both model 1 and model 2. Since model 2 utilizes the geometry of an ellipse, model 1 is assessed for both the geometry defined by the thickness distribution from equation (6.16) (labelled geometry A) and the geometry of an ellipse (labelled geometry B). When using the latter, there is a good agreement between the two models at low harmonic counts. However at higher frequencies the consideration of a distributed profile creates significant differences in the harmonic content in comparison to the one from Parry’s source representation. Note that there is a good qualitative agreement between the results calculated using geometries A and B as inputs to model 1.

![Figure 6.10: Magnitude of the first ten harmonics of the upwash velocity normalized by the free stream velocity at the upstream blade’s trailing edge at $r \approx 60\% R_{t1}$. Geometry A: sum of sine functions; Geometry B: ellipse.](image-url)
The effect of varying the airfoil’s geometry is assessed further in figures 6.11 and 6.12 in which the normalized amplitude of, respectively, the first and fifth upwash harmonic is calculated at all spanwise locations along the upstream airfoil’s trailing edge. In figure 6.11 both models provide amplitudes of similar order of magnitude, although model 1 dispenses a difference in the peak amplitude by approximately 12% due to the change of the airfoil’s geometry. Figure 6.12 shows, for the same geometry, a significant difference between the amplitudes calculated using model 1 and model 2 along most of the blade span. This suggests that at high frequencies the airfoil can no longer be assimilated as a point source and that the variation of the airfoil’s thickness along its chordline needs to be accounted for. The figure also exhibits a greater dependance on the geometry at high frequency. One should bear in mind, though, that the geometry A does not have a blunt leading edge and thus does not constitute a realistic representation of the airfoil. Consequently the thickness effects at the leading edge are likely to differ from the reality on the prediction of the potential field.

![Figure 6.11: Amplitude of the first upwash harmonic (normalized against the free stream velocity $U_\infty$) at the upstream blade’s trailing edge. Geometry A: sum of sine functions; Geometry B: ellipse.](image)

Consider now a third geometry for which the airfoil has a blunt leading edge and a sharp trailing edge. The geometry, labelled geometry C, is defined as a combination of geometries A and B as follows.
Figure 6.12: Amplitude of the fifth upwash harmonic (normalized against the free stream velocity $U_\infty$) at the upstream blade’s trailing edge. Geometry A: sum of sine functions; Geometry B: ellipse.

\[
G(X_2) = \begin{cases} 
\sqrt{1 - \left(\frac{X_2 + c_2/2 - X_m}{X_m}\right)^2} & \text{if } -\frac{c_2}{2} < X_2 < -\frac{c_2}{2} + X_m, \\
\sin \left(\frac{\pi X_2 + \frac{c_2}{2} - X_m}{2 c_2 - X_m}\right) & \text{if } -\frac{c_2}{2} + X_m < X_2 < \frac{c_2}{2}.
\end{cases}
\]

The velocity potential of the downstream airfoil becomes

\[
\phi' = -\frac{U_\infty t_{\text{max}}}{2\alpha_2}\sum_{n=-\infty}^{+\infty} \frac{1}{K_{y_2}} \exp \{K_{y_2}(\nu_1 + i \nu_2)x_2 - iK_{y_2}y_2\} [I_{1,C} + I_{2,C}],
\]

where

\[
I_{1,C} = \pi I_1 (X_m E_2) \exp \{-E_2(X_m - c_2/2)\}, \quad I_{2,C} = \frac{\pi}{2} I_2.
\]
The amplitude of the harmonics of the upwash incident on the upstream airfoil follows as

\[ W_{1}^{t} = -\lambda_{y_{2}} \frac{U_{r2} t_{\max}}{4\pi n_{2} \beta_{2}} \exp \{ A_{1} \} [ I_{1,C} + I_{2,C} ] . \]  

(6.59)

The amplitude of the first upwash harmonic at the upstream blade’s trailing edge calculated with equation 6.59 is compared for geometries A, B and C in figure 6.13. It is observed that the geometry C produces stronger perturbations than both geometries A and B with an increase in the peak amplitude by about 22% in comparison to that given by geometry A, as opposed to 12% using geometry B. In fact, in this representation the point of maximum thickness is closer to the upstream blade’s trailing edge than the one in geometry B which is located at the midchord, which may increase the thickness effects at the leading edge. The comparison between the output from geometry A and C also shows a great influence of the shape of the downstream airfoil’s leading edge on the resulting velocity potential. Similar comparisons conducted for the fifth upwash harmonic are shown in figure 6.14. In this case the input of geometry C provokes a considerable increase in the harmonic content at the upper half part of the blade. Hence the geometry and the source location on the airfoil’s chordline play a critical role in the determination of the potential field due to blade thickness.

![Figure 6.13: Amplitude of the first upwash harmonic (normalized against the free stream velocity $U_{\infty}$) calculated by model 1 at the upstream blade’s trailing edge at $r \approx 60\% R_{1}$. Geometry A: sum of sine functions; Geometry B: ellipse; Geometry C: combination of a sine function and an ellipse.](image)
Figure 6.14: Amplitude of the fifth upwash harmonic (normalized against the free stream velocity $U_\infty$) calculated by model 1 at the upstream blade’s trailing edge at $r \approx 60\% R_t$. Geometry A: sum of sine functions; Geometry B: ellipse; Geometry C: combination of a sine function and an ellipse.

### 6.2.2 Potential field due to blade circulation

#### 6.2.2.1 Velocity potential of the downstream airfoil

On the determination of the potential flow due to the blade thickness, the airfoil was considered symmetric and at zero angle of attack. When the body is asymmetric, i.e. with angle of attack or/and camber, lift is generated. In order to satisfy the Kutta condition, stating that the flow leaves tangentially at the trailing edge, we must add circulation. In the present section we consider a thin airfoil with small camber and angle of attack. To formulate the lifting problem, it is convenient to work with the stream function $\psi$ rather than the velocity potential $\phi$. The two functions are related by

\[
\frac{\partial \phi}{\partial \chi_2} = \frac{\partial \psi}{\partial \zeta_2},
\]

\[
\frac{\partial \phi}{\partial \zeta_2} = -\frac{\partial \psi}{\partial \chi_2}.
\] (6.60) (6.61)
Isolated airfoil. The potential field due to blade camber and angle of attack is defined as the field produced by a distribution of elementary sources of strength \( f^l \) along the airfoil chordline, giving, from equation (6.3),

\[
\psi^l(X_2, Y_2) = \frac{c_2}{\pi} \int_{-c_2}^{c_2} f^l(X_s) \vartheta^l((X_2, Y_2)|(X_s, 0)) \, dX_s. \tag{6.62}
\]

where \( \vartheta^l((X_2, Y_2)|(X_s, 0)) \) represents the stream function for an elementary source of unit strength. As described in section 6.2.1, the stream function for an elementary source located at \( \chi_2 = X_s/\beta_2 \) and \( \zeta_2 = 0 \) satisfies the governing equation (6.9).

\[
\frac{\partial^2 \vartheta^l}{\partial \chi_2^2} + \frac{\partial^2 \vartheta^l}{\partial \zeta_2^2} = \delta \left( \chi_2 - \frac{X_s}{\beta_2} \right) \delta (\zeta_2). \tag{6.63}
\]

The Boundary condition is given by Ashley and Landahl (1965) as

\[
\frac{\partial \psi^l}{\partial X_2} (-\frac{c_2}{2} < X_2 < \frac{c_2}{2}, Y_2 = 0) = \frac{U_{r_2}}{\beta_2} \left[ \theta \frac{\partial h}{\partial X_2} - \gamma \right]. \tag{6.64}
\]

An elementary singular solution to equation (6.63) is

\[
\vartheta^l((X_2, Y_2)|(X_s, 0)) = \frac{1}{2\pi} \log \left( \frac{1}{\beta_2} \sqrt{(X_2 - X_s)^2 + \beta_2^2 Y_2^2} \right). \tag{6.65}
\]

Thus we can write

\[
\psi^l(X_2, Y_2) = \frac{1}{2\pi} \int_{-c_2}^{c_2} f^l(X_s) \log \left( \frac{1}{\beta_2} \sqrt{(X_2 - X_s)^2 + \beta_2^2 Y_2^2} \right) \, dX_s. \tag{6.66}
\]

Inserting equation (6.66) into (6.64) yields

\[
- \int_{-c_2}^{c_2} \frac{f^l(X_s)}{X_s - X_2} \, dX_s = 2\pi \frac{U_{r_2}}{\beta_2} \left[ \theta \frac{\partial h}{\partial X_2} - \gamma \right]. \tag{6.67}
\]
The solution to equation (6.67) is

\[ f^I(X_2) = \frac{2U_{r_2}}{\pi \beta_2} \sqrt{\frac{c_2}{c_2^2} - X_2} \int_{-c_2^2}^{c_2^2} \left[ \theta \frac{\partial h}{\partial X_2} - \gamma \right] \sqrt{\frac{c_2}{c_2^2} + \frac{t}{c_2^2} - t - X_2} \, dt. \]  

(6.68)

Note that for an uncambered airfoil (i.e. a symmetric airfoil), the integral from equation (6.68) evaluates as \(-\gamma \pi\). Thus \(f^I\) is given by

\[ f^I(X_2) = -2\gamma \frac{U_{r_2}}{\beta_2} \sqrt{\frac{c_2}{c_2^2} - X_2}. \]  

(6.69)

![Figure 6.15: Vortex source distribution.](image)

The source strength \(f^I\) corresponds of the vorticity of a vortex.

**Cascade of airfoils.** Using a similar method as in section 6.2.1, the stream function for a cascade of distributions of elementary sources is given by

\[ \vartheta^I(x_2, y_2) = -\frac{1}{2s_2} \sum_{n_2=-\infty}^{+\infty} \frac{1}{K_{y_2}} \exp \{ -iK_{y_2}(y_2 + X_s \sin \alpha_2) \} \times \exp \{ K_{y_2}(\nu_1 + n_2)(x_2 - X_s \cos \alpha_2) \}, \]  

(6.70)

where \(\nu_1\) and \(\nu_2\) are defined by equations (6.38) and (6.39). Hence, the stream function can be determined substituting equations (6.70) and (6.69) into equation (6.62) and evaluating the integral.
\[ \psi^l(X_2, Y_2) = \frac{\gamma U_{r2}}{\beta_2 s_2} \sum_{n_2=-\infty}^{+\infty} \frac{1}{K_{y2}} \int_{c_2}^{c_2} \sqrt{\frac{c_2}{c_2} - X_s} \exp \left\{ -iK_{y2}(y_2 + X_s \sin \alpha_2) \right\} \times \exp \left\{ K_{y2}(\nu_1 + i\nu_2)(x_2 - X_s \cos \alpha_2) \right\} dX_s \]  

(6.71)

Making the change of variable \( X_s = \frac{c_2}{2} \cos \Theta \) gives

\[ \psi^l = \frac{\gamma U_{r2} c_2}{\beta_2 s_2} \frac{2}{2} \sum_{n_2=-\infty}^{+\infty} \frac{\exp \left\{ K_{y2}(\nu_1 + i\nu_2)x_2 - iK_{y2}y_2 \right\}}{K_{y2}} \times \int_0^\pi (1 - \cos \Theta) \exp \left\{ -K_{y2} \frac{c_2}{2} ((\nu_1 + i\nu_2) \cos \alpha_2 + i \sin \alpha_2) \cos \Theta \right\} d\Theta. \]

(6.72)

The above equation evaluates as

\[ \psi^l = \frac{\pi \gamma U_{r2} c_2}{\beta_2 s_2} \frac{2}{2} \sum_{n_2=-\infty}^{+\infty} \frac{\exp \left\{ K_{y2}(\nu_1 + i\nu_2)x_2 - iK_{y2}y_2 \right\}}{K_{y2}} \left[ J_0 \left( i \frac{c_2}{2} E_2 \right) - iJ_1 \left( i \frac{c_2}{2} E_2 \right) \right], \]

(6.73)

where \( E_2 \) is defined in equation (6.45). Equation (6.73) gives the stream function related to the lifting flow about an infinite cascade of distributions of vortices.

### 6.2.2.2 Upwash on the upstream airfoil

Consider an isolated airfoil from the downstream blade row. Again, the subscripts 1 and 2 denote the values associated with the upstream and downstream airfoil respectively. Assume \( n_2 > 0 \). The upwash normal to the upstream airfoil is

\[ u_1 = -\sin \alpha_1 \frac{\partial \psi}{\partial y_2} - \cos \alpha_1 \frac{\partial \psi}{\partial x_2}, \]

(6.74)

where \( \alpha_1 \) is the stagger angle of the upstream airfoil. Using a similar method as in section 6.2.1.2, the upwash \( u_1 \) can be written of the form
\[ u_1 = W' \exp \{ \lambda_{x_2} X_1 + \lambda_{y_2} Y_1 + i \omega_2 t \}, \quad (6.75) \]

where

\[
\begin{aligned}
W'_{1} &= - \lambda_{x_2} \frac{\gamma U_{r} c_2}{4n_2 \bar{\beta}_2} \exp \{ A_1 \} \left[ J_0 (\lambda_J) - i J_1 (\lambda_J) \right], \\
\lambda_J &= \frac{c_2}{2} K_{y_2} (i \eta_2 \cos \alpha_2 - \sin \alpha_2),
\end{aligned}
\]

(6.76)

and \( \lambda_{x_2} \), \( \lambda_{y_2} \), \( A_1 \), \( \eta_2 \) and \( \omega_2 \) are defined in equation (6.54). The gust amplitude (6.80) is proportional to the that given by model 2, which is expressed in equation (D.14), by a factor equal to

\[
\frac{\eta_2 \cos \alpha_1 - i \sin \alpha_1}{\cos \alpha_1 - i \sin \alpha_1} \left[ J_0 \left( i \frac{c_2}{2} E_2 \right) - i J_1 \left( i \frac{c_2}{2} E_2 \right) \right].
\]  

(6.77)

The ten first upwash harmonics at the upstream rotor’s trailing edge are plotted in figure 6.10 at \( r \approx 60\% R_{t_1} \). The two models decay at a similar rate however model 1 provides higher amplitudes than model 2. Indeed in the former model the source strength is characterized by a discontinuity at the leading edge which causes the disturbances to be significant and dominant at this location. In the case of upstream interactions this can be interpreted as a reduction of the distance from the source to the upstream airfoil’s trailing edge by half the chordlength of the downstream airfoil, resulting in an increase of the strength of the downstream blade’s velocity potential seen at the upstream airfoil’s trailing edge.

Remark that in both figures 6.10 and 6.16 the first harmonic has a very large amplitude in comparison to the other harmonics. Because the high harmonics have very low amplitude, they may not have significant effects on the total noise produced by the advanced open rotor and the potential field source may mostly be effective for the first harmonic.

In figure 6.17, the amplitude of the first upwash harmonic is plotted over all blade radii. The levels calculated considering model 1 are much higher that those calculated using model 2 along most of the blade span. Note that the contribution of the potential field due to blade thickness is very small in comparison to the component of the potential field due to the blade camber and angle of attack. Therefore the resulting perturbation will be essentially dominated by the loading.
Because it greatly influences the resulting flow field, it is thus important to consider the chordwise distribution of loading acting on the downstream airfoil in the predictions.

Figure 6.16: Upwash velocity magnitude normalized against the free stream velocity $U_\infty$ at $r \approx 60\% R_{t1}$.

Figure 6.17: Amplitude of the first upwash harmonic normalized against the free stream velocity $U_\infty$ at the front rotor trailing edge.
The total amplitude of the gust incident on the upstream airfoil is the sum of the gust amplitudes of the thickness and the loading components, which are defined in equations (6.54) and (6.80). The evolution of the “total” upwash velocity in the inter-rotor region is assessed in figure 6.18 in which the amplitude of the first upwash harmonic provided by the two analytical models is compared with the CFD data at various axial locations at $r \approx 60\%R_t$. Both the analytical models and the CFD results show evidence of the exponential decay of the velocity potential with axial distance in most of the inter-rotor region. Nevertheless the CFD exhibits a notable drop in the gust amplitude close to the upstream airfoil’s trailing edge which could be due to interactions with the complex flow field surrounding the blade. The numerical simulation also displays a discontinuity which is in fact located at the position of the sliding plane. Particularly, it is clear in figure 6.19 that the upwash velocities measured one cell upstream and downstream of the sliding plane both differ in phase and amplitude. Those disparities could be caused by several sources such as numerical dissipation across the sliding plane due to the interpolated mesh, difficulties numerically capturing the features of the flow between the contra-rotative blade rows at the sliding plane, difficulties extracting the bound potential field of the downstream rotor at either side of the sliding plane, or the different extraction procedures upstream and downstream of the sliding plane (such as time averaging in different frames or reference).

Figure 6.18: Amplitude of the first upwash harmonic normalized against the free stream velocity $U_\infty$ in the inter-rotor region. The analytical models are compared with the CFD data.
Figure 6.20 shows the axial component of the velocity potential of the downstream airfoil determined from the CFD data on a cylindrical surface with axis collinear with the engine’s axis. Clearly, the velocity field upstream of the sliding plane (represented by the dotted line) contains an additional component which originates from the viscous wake. Indeed, to fully isolate the vortical component from the potential component is a critical process. Though, the separation between the front rotor’s wake and the bound potential field of the downstream rotor seems to be more successful downstream of the sliding plane, in the frame of reference of the downstream blade row. The upwash velocity measured on the same surface is shown in figure 6.21. The field is visibly discontinuous at the sliding plane as if the velocity potential was shifted upstream when passing across it. The reasons of such discontinuities should be subject to further investigation of the CFD data in future analysis.

![Graph showing upwash velocity captured by the CFD one cell upstream and downstream of the sliding plane for one rear rotor’s blade passage.](image-url)
Figure 6.20: Axial component of the downstream airfoil’s velocity potential normalized against the free-stream velocity captured by the CFD at $r \approx 60\% R_t$ in the inter-rotor region.

Figure 6.21: Upwash velocity normalized against the free-stream velocity captured by the CFD at $r \approx 60\% R_t$ in the inter-rotor region.
6.2.3 Response of the upstream airfoil

This section refers to the high frequency response of the upstream airfoil to the potential field of the adjacent blade. The latter being decomposed into thickness and lift contributions, the resultant upwash on the upstream airfoil is the linear sum of these different components. The expression used in this section was derived by Parry for a semi-infinite flat plate, extending to upstream infinity, using the Wiener-Hopf technique. Since the trailing edge problem is considered, the Kutta condition was imposed at the trailing edge of the upstream airfoil. This representation nonetheless neglects the inverse square root singularity at the leading edge where the pressure tends to infinity. Parry (1988) showed that in the case of incompressible flow, neglecting the leading edge effects led to errors in the determination of the unsteady pressure, but that in the compressible flow case the leading edge correction had relatively small impact on the total airfoil response. However, it has been decided that any singularity be addressed, therefore the leading edge correction will be applied to the semi-infinite response. The latter is given by (Parry, 1988, equation 7.4.48)

\[
\Delta p(X_1) = -\frac{i 2 \rho U_{r1} W_1 (\sigma - \mu)}{\beta_1^{0.5}(i \sigma^* + i \mu)^{0.5}(i \bar{\sigma} - i \mu)^{0.5}} \left\{ \exp \left\{ i (X_1 - 1) w[i ((1 - X_1)(i \mu + i \sigma^*))^{0.5}] \right\} \right.
- \exp \{-i \mu (X_1 - 1)\} \exp \{i \omega_2 t\},
\]

(6.78)

where the function \(w\) is defined by (Abramowitz and Stegun, 1964, equation 7.1.3) as

\[
w(z) = \exp \left\{ -z^2 \right\} \text{erfc}(-iz),
\]

(6.79)

and

\[
\begin{align*}
W_1 &= W^t_1 + W^l_1, \\
\sigma &= \frac{k c_1}{2 M_{r1}}, \\
\bar{\sigma} &= \frac{M_{r1}}{1 + M_{r1}} \sigma, \\
\sigma^* &= \frac{\sigma M_{r1}}{1 - M_{r1}}, \\
k &= \frac{\omega_2}{c_0}, \\
\mu &= \frac{i \lambda_{x2} c_1}{2}.
\end{align*}
\]

(6.80)
The acoustically weighted lift per unit span is obtained by integrating the pressure over the blade chord, including the non-compactness factor \( \exp \left\{ -i k_X \frac{2 X_1}{c_1} \right\} \) and is given by Parry by

\[
\frac{dL_1}{dr} = \rho U_r c_1 (\sigma - \mu) \exp \left\{ i \omega t - i k_X \right\} \left[ - \exp \left\{ i 2 (\mu + k_X) \right\} \right. \\
\left. + w \left[ i (2 (i \mu + i \sigma^*))^{0.5} \right] \right. \\
\left. + \exp \left\{ - \frac{i \pi (i \mu + i \sigma^*)^{0.5}}{4 (\sigma^* - k_X)^{0.5}} \right\} \right. \\
\left. \times \left( 1 - \exp \left\{ -i 2 (\sigma^* - k_X) \right\} w \left[ (i - 1) (\sigma^* - k_X)^{0.5} \right] \right) \right].
\]

(6.81)

To account for the leading edge effects, Parry introduced a correction term based on a technique developed by Landahl (1989) and Adamczyk (1971). The pressure jump associated with the leading edge contribution was given by

\[
\Delta p_{LE} (X_1) = \left( \frac{2}{\pi} \right)^{0.5} \frac{i \rho W_1 U_r (\sigma - \mu) \exp \left\{ i \omega t + i \bar{\sigma} (1 - X_1) - 4 i \bar{\sigma} \right\}}{(i \mu + i \sigma^*) (i \bar{\sigma} - i \mu)^{0.5} (1 - M_{r_1}^2)^{0.5}} \\
\times \left[ - \exp \left\{ -i \frac{\pi}{4} \right\} (2 \bar{\sigma} (1 + X_1))^{0.5} \right] - \left( \frac{2}{\pi \sigma'' (1 + X_1)} \right)^{0.5} \exp \left\{ -i \frac{\pi}{4} \right\}.
\]

(6.82)

where

\[
\begin{cases}
\bar{\sigma} = \frac{\sigma M_{r_1}}{1 - M_{r_1}^2} \\
\sigma'' = \frac{1 + M_{r_1}}{M_{r_1} (1 - M_{r_1})} \sigma.
\end{cases}
\]

(6.83)

The lift per unit span associated with the leading edge term is given by (Parry, 1988, equation 7.5.12),
\[
\frac{dL_{LE}}{dr} = -\rho W U r_1 c_1 (\sigma - \mu) \exp \{i \omega t + ik_X - i2\sigma^*\} \\
\times \left[ 1 - w \left[ -2 \exp \left\{ -i \frac{2}{3} \tilde{\sigma}^{0.5} \right\} \exp \left\{ -i2(\tilde{\sigma} + k_X) \right\} \right] + \left( \frac{1}{\sigma^{0.5}} - \tilde{\sigma}^{0.5} - k_X \right) \right] \\
\times 2^{0.5} \left( 1 - w \left[ - \exp \left\{ -i \frac{3}{2} \right\} \left( \frac{2(\tilde{\sigma} + k_X)^{0.5}}{(\tilde{\sigma} + k_X)^{0.5}} \right) \exp \left\{ -i2(\tilde{\sigma} + k_X) \right\} \right] \right) 
\]

(6.84)

The response of the upstream airfoil to the incident gusts is the sum of equations (6.78) and (6.82), and similarly, the total lift per unit span is the sum of equations (6.81) and (6.84). The first harmonic of the unsteady lift per unit span \((k_X = 0)\) acting on the blade is plotted in figure 6.22. The predictions from model 1 which used both geometry A and geometry B show an increase in the lift force acting on the upstream blade compared to that predicted by model 2. As expected, an important contribution to the lift originates from the potential field caused by the loading acting on the downstream blade. This contribution is dominant over that of the potential field due to blade thickness which has little effect on the resulting lift at low harmonic count but can become significant at high frequency, see figure 6.23.
A comparison of the analytical methods with the CFD simulation was conducted. The comparisons show a reasonable agreement for the first harmonic in terms of upwash velocity and lift per unit span, as shown in figures 6.18 and 6.24 respectively. However the agreement becomes very poor at higher harmonics. In figure 6.25 a comparison between the magnitude of the first three harmonics of the viscous wake and the potential field contributions extracted from the CFD data in the inter-rotor region is shown. Upstream of the sliding plane, only the first upwash harmonic related to the potential field was captured properly in the CFD simulation. Significant errors are observed for the second and third harmonics in this region, underlining difficulties separating the potential field contribution from the numerical solution in the given frame of reference. The results are also strongly affected by the discontinuity at the sliding plane. However, the potential field is perfectly captured close to the downstream blade row. Thus the use of the analytical model would be a useful asset to calculate the related perturbation at the upstream blade’s trailing edge avoiding those numerical errors. Note that the viscous wake contribution is also subject to discontinuities at the sliding plane and inaccuracies downstream of the sliding plane. The origin of those inaccuracies should be investigated further to determine if they are a numerical artefact or the result of some non-linear interaction between both fields. It is interesting to notice that in the analysis of Jaron et al. (2014) the amplitude of the Fourier harmonics
of the simulated velocity field also showed a decreasing then increasing behaviour at some axial locations.

Figure 6.24: Comparison between the first harmonic of the lift per unit span acting on the upstream blade provided by the analytical models and the CFD.

Figure 6.25: Amplitude of the first three upwash harmonics determined from the CFD data.
6.3 Downstream interactions

In the present section we derive the expressions to model the interaction of the potential field of the upstream rotor with the downstream rotor. Again, the suffixes 1 and 2 denote values associated with the upstream and downstream rotor respectively. The analysis is similar to that in section 6.2, replacing $\alpha_2$ by $-\alpha_1$, as the upstream blade is staggered in the opposite direction, and considering $x_1 > 0$ in the downstream direction. The coordinate system used to model the downstream interactions is shown in figure 6.26. In this configuration, the $\{x_1, y_1\}$ and $\{X_1, Y_1\}$ coordinate systems are related by

$$
\begin{align*}
X_1 &= \cos \alpha_1 x_1 + \sin \alpha_1 y_1, \\
Y_1 &= -\sin \alpha_1 x_1 + \cos \alpha_1 y_1.
\end{align*}
$$

and

$$
\begin{align*}
x_1 &= \cos \alpha_1 X_1 - \sin \alpha_1 Y_1, \\
y_1 &= \sin \alpha_1 X_1 + \cos \alpha_1 Y_1.
\end{align*}
$$

Figure 6.26: Coordinate system for the interaction of the upstream potential field with the downstream row.
6.3.1 Potential field due to blade thickness

6.3.1.1 Velocity potential of the upstream airfoil

Isolated airfoil. We consider an isolated airfoil of the upstream row. The non-dimensional thickness function \( G(X_1) \) is expressed as

\[
\begin{align*}
G(X_1) &= \sin \left( \frac{\pi}{2X_m} \left( X_1 + \frac{c_1}{2} \right) \right) \quad \text{if} \quad -\frac{c_1}{2} < X_1 < -\frac{c_1}{2} + X_m, \\
G(X_1) &= \sin \left( \frac{\pi}{2} + \frac{\pi X_1 + c_1/2 - X_m}{c_1 - X_m} \right) \quad \text{if} \quad -\frac{c_1}{2} + X_m < X_1 < \frac{c_1}{2}.
\end{align*}
\]

The thickness effects are thus modelled by a distribution of sources of strength \( f^t \) which is given by

\[
\begin{align*}
f^t(X_1) &= \frac{U_{r_1} t_{max} \pi}{2\beta_1 X_m} \cos \left( \frac{\pi}{2X_m} \left( X_1 + \frac{c_1}{2} \right) \right) \quad \text{if} \quad -\frac{c_1}{2} < X_1 < -\frac{c_1}{2} + X_m, \\
f^t(X_1) &= -\frac{U_{r_1} t_{max} \pi}{2\beta_1 (c_1 - X_m)} \sin \left( \frac{\pi X_1 + c_1/2 - X_m}{c_1 - X_m} \right) \quad \text{if} \quad -\frac{c_1}{2} X_m < X_1 < \frac{c_1}{2},
\end{align*}
\]

where \( X_m \) is now the local chordwise coordinate of the point of maximum thickness \( t_{max} \) of the upstream airfoil, and \( \beta_1 = \sqrt{1 - M_{r_1}^2} \). The potential of a point source located at \( \chi_1 = X_s \) and \( \zeta_1 = 0 \) satisfies the following governing equation and boundary condition

\[
\beta_1^2 \frac{\partial^2 \phi^t}{\partial X_1^2} + \frac{\partial^2 \phi^t}{\partial Y_1^2} = \beta_1 \delta(X_1 - X_s) \delta(Y_1),
\]

\[
\lim_{Y_1 \to 0^\pm} \frac{\partial \phi^t}{\partial Y_1} \left( -\frac{c_1}{2} < X_1 < \frac{c_1}{2}, Y_1 \right) = \pm U_{r_1} \frac{t_{max}}{2} \frac{\partial G}{\partial X_1}.
\]

An elementary singular solution to equation (6.90) is

\[
\phi^t((X_1, Y_1)| (X_s, 0)) = \frac{1}{2\pi} \log \left( \frac{1}{\beta_1 \sqrt{(X_1 - X_s)^2 + \beta_1^2 Y_1^2}} \right).
\]
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Hence the potential due to blade thickness of a single airfoil is given by, see equation (6.18),

\[
\phi^t = \frac{U_{r_1} t_{max}}{4\beta_1} \left[ \frac{1}{X_m} \int_{-\frac{c_1}{2}}^{\frac{c_1}{2}+X_m} \cos \left( \frac{\pi}{2X_m} \left( \frac{c_1}{2} + X_s \right) \right) \log \left( \frac{1}{\beta_1} \sqrt{(X_1 - X_s)^2 + \beta_1^2 Y_1^2} \right) dX_s \right. \\
\left. - \frac{1}{c_1 - X_m} \int_{-\frac{c_1}{2}+X_m}^{\frac{c_1}{2}} \sin \left( \frac{\pi}{2} \frac{X_1 + c_1/2 - X_m}{c_1 - X_m} \right) \log \left( \frac{1}{\beta_1} \sqrt{(X_1 - X_s)^2 + \beta_1^2 Y_1^2} \right) dX_s \right] .
\] (6.92)

**Cascade of airfoils.** Consider now an infinite cascade of airfoils equally spaced by a distance \( s_1 = \frac{2\pi r}{B_1} \) as shown in figure 6.27. From section 6.2.1 we show that the potential produced by an infinite cascade of elementary sources satisfy the equation

\[
\nabla_c^2 \phi^t = \beta_1 \sum_{n_1 = -\infty}^{+\infty} \delta(x_1 - X_s \cos \alpha_1) \delta(y_1 - X_s \sin \alpha_1 - n_1 s_1),
\] (6.93)

with

\[
\nabla_c^2 = \left( \sin^2 \alpha_1 + \beta_1^2 \cos^2 \alpha_1 \right) \frac{\partial^2}{\partial x_1^2} - 2 \sin \alpha_1 \cos \alpha_1 \left( 1 - \beta_1^2 \right) \frac{\partial^2}{\partial x_1 \partial y_1} + \left( \cos^2 \alpha_1 + \beta_1^2 \sin^2 \alpha_1 \right) \frac{\partial^2}{\partial y_1^2}.
\] (6.94)

A solution to equation (6.93) is given by

\[
\phi^t(x_1, y_1) = -\frac{\beta_1}{2\pi s_1} \sum_{n_1 = -\infty}^{+\infty} \exp \{-iK y_1 (y_1 - X_s \sin \alpha_1)\}
\times \int_{-\infty}^{+\infty} \frac{\exp \{-iK x (x_1 - X_s \cos \alpha_1)\}}{D} dk_x,
\] (6.95)
where \( K_{y_1} = \frac{2\pi n_1}{s_1} \) and

\[
D = k_x^2 \left( \sin^2 \alpha_1 + \beta_1^2 \cos^2 \alpha_1 \right) - 2 \sin \alpha_1 \cos \alpha_1 \left( 1 - \beta_1^2 \right) k_x K_{y_1} + K_{y_1}^2 \left( \cos^2 \alpha_1 + \beta_1^2 \sin^2 \alpha_1 \right). \tag{6.96}
\]

The integrand from equation (6.95) has poles at

\[
k_x = \pm i \beta_1 + \sin \alpha_1 \cos \alpha_1 \left( 1 - \beta_1^2 \right) = \kappa_r \pm i \kappa_c, \ \kappa_c > 0. \tag{6.97}
\]

Recall that \( x_1 > 0 \) in the downstream direction. Thus, in order for the solution to decay away from the airfoil, we perform the integration over a close contour \( C \) encircling the pole \( \kappa^- = \kappa_r - i\kappa_c \), see figure 6.9. Similarly to the previous case, the integrand vanishes on \( C_C \) as \( R_C \to \infty \) and the theorem of Residues is expressed as follows

Figure 6.27: Thickness source cascade distribution.
\[
\oint_{C_R} \mathrm{exp} \left\{ -ik \left( x_1 - X_s \cos \alpha_1 \right) \right\} \frac{dk_x}{D} = -2\pi i \text{Res}_{\kappa^-} \left( \frac{\exp \left\{ -ik \left( x_1 - X_s \cos \alpha_1 \right) \right\}}{D} \right),
\]

(6.98)

Figure 6.28: Contour integration.

Note that the negative sign at the left hand side of equation (6.98) is due to the fact that the outer circle \( C_C \) is going clockwise around \( \kappa^- \).

The potential field due to blade thickness produced by a unit strength elementary source is thus

\[
\vartheta^t(x_1, y_1) = -\frac{1}{2s_1} \sum_{n_1 = -\infty}^{+\infty} \frac{1}{K_{y_1}} \exp \left\{ -K_{y_1} (\mu_1 + i\mu_2)(x_1 - X_s \cos \alpha_1) - iK_{y_1} (y_1 - X_s \sin \alpha_1) \right\},
\]

(6.99)

where

\[
\mu_1 = \frac{\beta_1}{\beta_1^2 \cos^2 \alpha_1 + \sin^2 \alpha_1}, \quad \mu_2 = \frac{\sin \alpha_1 \cos \alpha_1 (1 - \beta_1^2)}{\beta_1^2 \cos^2 \alpha_1 + \sin^2 \alpha_1}. \quad (6.100)
\]

The analytical expression of the potential follows immediately as

\[
\phi^t = -\frac{U_{r_1} t_{r_{\text{max}}}^4 \pi}{4\beta_1 s_1} \sum_{n_1 = -\infty}^{+\infty} \frac{1}{K_{y_1}} \exp \left\{ -K_{y_1} \eta_1 x_1 - iK_{y_1} y_1 \right\} \left[ \mathcal{I}_1 + \mathcal{I}_2 \right],
\]

(6.101)

where \( \eta_1 = (\mu_1 + i\mu_2) \), and \( \mathcal{I}_1 \) and \( \mathcal{I}_2 \) are expressed as
\[ I_1 = \frac{1}{X_m} \int_{\frac{-c_1}{2} + X_m}^{\frac{c_1}{2}} \cos \left( \frac{\pi}{2X_m} \left( X_s + \frac{c_1}{2} \right) \right) \exp \{ K_{y_1} [\eta_1 \cos \alpha_1 + i \sin \alpha_1] X_s \} \, dX_s. \quad (6.102) \]

\[ I_2 = -\frac{1}{c_1 - X_m} \int_{\frac{-c_1}{2} + X_m}^{\frac{c_1}{2}} \sin \left( \frac{\pi}{2} \frac{X_s + c_1/2 - X_m}{c_1 - X_m} \right) \exp \{ K_{y_1} [\eta_1 \cos \alpha_1 + i \sin \alpha_1] X_s \} \, dX_s. \]

The above integrals are equal to

\[ I_1 = 2 \exp \left\{ -E_1 \left( \frac{c_1}{2} - X_m \right) \right\} \left[ \pi - 2E_1X_m \exp \left\{ -E_1X_m \right\} \right], \quad (6.103) \]

\[ I_2 = -2 \left[ \pi \exp \left\{ E_1 \left( -\frac{c_1}{2} + X_m \right) \right\} + 2 \exp \left\{ \frac{c_1}{2}E_1 \right\} (c_1 - X_m)E_1 \right] \frac{\pi^2}{\pi^2 + 4E_1^2(c_1 - X_m)^2}, \quad (6.104) \]

where

\[ E_1 = K_{y_1} [\eta_1 \cos \alpha_1 + i \sin \alpha_1]. \quad (6.105) \]

### 6.3.1.2 Upwash on the downstream airfoil

Now consider an isolated airfoil of the downstream row in the coordinate system shown in figure 6.26. The upwash normal to such airfoil is given by

\[ u_2 = \sin \alpha_2 \frac{\partial \phi^t}{\partial x_1} - \cos \alpha_2 \frac{\partial \phi^t}{\partial y_1}, \quad (6.106) \]

where \( \alpha_2 \) is the local stagger angle of the downstream airfoil. Applying the above equation to equation (6.101) yields

\[ u_2 = -\frac{U_{r_t} \pi}{4s_2 \beta_2} \left[ -\eta_1 \sin \alpha_2 + i \cos \alpha_2 \right] \exp \left\{ -K_{y_1} \eta_1 x_1 - iK_{y_1} y_1 \right\} \left[ I_1 + I_2 \right]. \quad (6.107) \]

The \( \{x_1, y_1\} \) and \( \{x_2, y_2\} \) coordinate systems defined in figure 6.26 are related by
\[
x_1 = x_2 + g - \frac{c_2}{2} \cos \alpha_2, \\
y_1 = y_2 + \frac{c_2}{2} \sin \alpha_2 + r(\Omega_1 + \Omega_2)t.
\] (6.108)

Inserting equation (6.108) into equation (6.107) and expressing the upwash in terms of \(\{X_2, Y_2\}\) coordinates using equation (6.21), one harmonic of the upwash normal to the reference airfoil of the downstream row can be written of the form

\[
u_2 = W'_2 \exp \{\lambda_x X_2 + \lambda_y Y_2 - i\omega_1 t\},
\] (6.109)

where

\[
\begin{cases}
W'_2 &= -\lambda^* \eta_1 \frac{U_{r1} t_{max}}{8n_1 \beta_1} \exp \{A_2\} [I_1 + I_2] , \\
A_2 &= -K \eta_1 \left( g - \frac{c_2}{2} \cos \alpha_2 + i \frac{c_2}{2} \sin \alpha_2 \right) , \\
\lambda_x &= -K \eta_1 \left( \eta_1 \cos \alpha_2 - i \sin \alpha_2 \right) , \\
\lambda_y &= -K \eta_1 \left( \eta_1 \sin \alpha_2 + i \cos \alpha_2 \right) , \\
\omega_1 &= K r(\Omega_1 + \Omega_2). 
\end{cases}
\] (6.110)

The term \(\lambda^* \eta_1\) corresponds to the conjugate of \(\lambda_\eta\). The magnitude of the first harmonic of the upwash velocity incident on the downstream airfoil’s leading edge calculated using models 1 and 2 is shown in figure 6.29. In the latter, geometry A refers to the profile represented by the thickness function defined in equation (6.87) and geometry B corresponds to that of an ellipse. On the determination of the potential field of an ellipse, the consideration of the thickness distribution along the chordline provides greater amplitudes than the point source representation. Recall that the point source representation is a far-field approximation thus the distance between the two blade rows should be large enough for the models to converge. It is interesting to observe that in the present case the change of geometry has a notable influence on the amplitudes predicted by model 1, causing a difference of approximately 22 % in the peak amplitude of the upwash harmonic. This increase is essentially due to the fact that the thickness distribution of the elliptic profile is defined over a wider area (with, in particular, a blunt trailing edge) and has its point of maximum thickness closer to the downstream airfoil than the one of geometry A. The amplitude of the fifth harmonic of the upwash incident on the
downstream airfoil is plotted in figure 6.30. Again, the source representation and the type of profile considered strongly influences the solution.

![Figure 6.29](image1.png)

Figure 6.29: Amplitude of the first upwash harmonic normalized against the free stream velocity at the downstream blade’s leading edge (thickness contribution).

![Figure 6.30](image2.png)

Figure 6.30: Amplitude of the fifth upwash harmonic normalized against the free stream velocity at the downstream blade’s leading edge (thickness contribution).
6.3.2 Potential field due to blade circulation

6.3.2.1 Velocity potentials of the upstream airfoil and of the cascade

The velocity potential of the upstream airfoil is obtained using the same method as in section 6.2.2, using the coordinates associated with the upstream airfoil with $x_1 > 0$ in the upstream direction and replacing $\alpha_2$ by $-\alpha_1$. The source strength is given by

$$f^l(X_1) = -2\gamma \frac{U_{r_1}}{\beta_1} \sqrt{x_1 - \frac{c_1 - X_1}{X_1}}.$$ \hspace{1cm} (6.111)

The stream function for a cascade of distribution of elementary sources is obtained by applying the theorem of Residues as in equation (6.98), performing the $k_x$-integral over the close contour $\mathcal{C}$ defined in figure 6.28

$$\vartheta^l(x_1, y_1) = -\frac{1}{2s_1} \sum_{n_1=-\infty}^{+\infty} \frac{1}{K_{y_1}} \exp \left\{ -K_{y_1} \eta_1 (x_1 - X_s \cos \alpha_1) - iK_{y_1} (y_1 - X_s \sin \alpha_1) \right\},$$ \hspace{1cm} (6.112)

and the stream function associated with the distribution of sources of strength $f^l$ is given by

$$\psi^l = \frac{\pi \gamma U_{r_1} c_1}{\beta_1 s_1} \sum_{n_1=-\infty}^{+\infty} \frac{1}{K_{y_1}} \exp \left\{ -K_{y_1} \eta_1 x_1 - iK_{y_1} y_1 \right\}$$
$$\times \left[ J_0 \left( K_{y_1} \frac{c_1}{2} [-i\eta_1 \cos \alpha_1 + \sin \alpha_1] \right) - iJ_1 \left( K_{y_1} \frac{c_1}{2} [-i\eta_1 \cos \alpha_1 + \sin \alpha_1] \right) \right].$$ \hspace{1cm} (6.113)

6.3.2.2 Upwash on the downstream airfoil

Consider an isolated airfoil from the upstream blade row in the coordinate system defined by figure 6.26. Using the same method as in section 6.3.1.2, the upwash normal to the downstream blade is given by
\[ u_2 = W_{l2} \exp \{ \lambda_{x_1} X_2 + \lambda_{y_1} Y_2 + i \omega_1 t \}, \quad (6.114) \]

where

\[
\begin{align*}
W_{l2} &= -\lambda_{x_1} \frac{\gamma U_{r_2} c_1}{4 n_1 \beta_1} \exp \{ A_2 \} [J_0(\lambda_{J_1}) - iJ_1(\lambda_{J_1})], \\
\lambda_{J_1} &= K y_1 c_1 \left( -i \eta_1 \cos \alpha_1 + \sin \alpha_1 \right), \\
A_2, \lambda_{y_1}, \text{ and } \lambda_{x_1} \text{ are defined in equation (6.110).}
\end{align*}
\]

Figure 6.31 shows the magnitude of the first upwash harmonic calculated along the downstream blade’s leading edge. As observed in section 6.2.2, there are significant differences in amplitude between the results provided by model 1 and those provided by model 2. In the case of downstream interactions the latter produces stronger perturbations at the vicinity of the downstream row than the former. This trend is expected owing to the fact that the point source is defined at the upstream airfoil’s midchord whereas in the distributed profile the dominant source is located at the leading edge. Because the velocity potential decays exponentially with axial distance the difference in the source representation has a great influence on the predicted levels.

### 6.3.3 Response of the downstream airfoil

Since we now consider the response of the downstream airfoils to the unsteady perturbations produced by the upstream row, the airfoil response is expected to be dominated by the leading edge singularity. Parry derived another expression to treat only the leading edge problem under the assumption that the trailing edge region is negligible, leading to the following solution.

\[
\Delta p(X_2) = \frac{-2 \rho W_{l2} U_{r_2} \exp \{ i \omega_1 t \}}{(1 - M_{r_2}^2)^{0.5}(i \mu + i \sigma^*)^{0.5}} \left[ \left( \exp \{ -i \sigma (X_2 + 1) \} \right) w \left[ i(X_2 + 1)^{0.5}(-i \mu + i \sigma)^{0.5} \right] \\
- \left( \exp \{ -i \mu (X_2 + 1) \} \right) \right] \left( \frac{i(\sigma - \mu)}{(-i \mu + i \sigma)^{0.5}} \frac{\exp \{ -i \sigma (X_2 + 1) \}}{\pi^{0.5}(X_2 + 1)^{0.5}} \right), \quad (6.116)
\]
Chapter 6 Potential field interactions

<table>
<thead>
<tr>
<th>Model 1</th>
<th>Model 2</th>
</tr>
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Figure 6.31: Amplitude of the first upwash harmonic (loading contribution) normalized against the free stream velocity at the downstream blade’s leading edge.

where

\[
\begin{align*}
W_2 & = W_2^t + W_2^l, \\
\sigma & = \frac{k c_2}{M_{r_2}}, \\
\tilde{\sigma} & = \frac{M_{r_2}}{1 + M_{r_2}} \sigma, \\
\sigma^* & = \frac{\sigma M_{r_2}}{1 - M_{r_2}}, \\
k & = \frac{\omega_1}{c_0}, \\
\mu & = \frac{i \lambda x^2 c_2}{2}.
\end{align*}
\] (6.117)

Again, the lift per unit span is obtained by integrating the above equation over the chord accounting for the non-compactness factor. This yields
\[
\frac{dL_2}{dr} = \frac{\rho W_2 U_r c_2 \exp \{ ik_X + i \omega_1 t \}}{(1 - M_{r_2}^2)^{0.5} (i \mu + i \sigma^*)^{0.5}(\mu + k_X)} \\
\times \left[ \exp \left\{ -i \frac{\pi}{4} \right\} \frac{(\sigma + k_X)}{0.5 (\sigma + k_X)^{0.5}} \left( 1 - w \left[ (1 - i)(\tilde{\sigma} + k_X)^{0.5} \exp \{ -i 2(\tilde{\sigma} + k_X) \} \right] \right) \right. \\
+ \left. \frac{\sigma - \mu}{(i \tilde{\sigma} - i \mu)^{0.5}} \left( w [ i 2^{0.5}(i \tilde{\sigma} - i \mu)^{0.5}] \exp \{ -2i(\tilde{\sigma} + k_X) \} - \exp \{-i 2(\mu + k_X)\} \right) \right]. 
\]

(6.118)

The lift per unit span calculated at the downstream blade’s radius range is shown in figure 6.32. It is observed in this case that, because the potential field due to the blade loading is weaker, the airfoil’s geometry has greater influence on the resulting lift in comparison to the upstream interaction case.

![Figure 6.32: First harmonic of the lift per unit span acting on the downstream blade.](image-url)
6.4 Potential field interaction noise prediction

The radiated noise caused by the interaction between the potential field of one rotor and the adjacent rotor was calculated using the radiation formula from equation (4.36). The polar directivity of different interaction tones was investigated and the contributions of the forward and rearward potential fields were compared to that of the viscous wakes. The analysis also included a brief comparison between the noise levels calculated by model 1 and model 2. Finally, a cross-validation of the analytical models with wind tunnel measurements and CFD-based noise predictions was conducted. In this section the forward potential refers to the interaction between the potential field of the downstream rotor with the upstream row, and the rearward potential refers to the interaction between the potential field of the upstream rotor with the downstream row. Note that, unless otherwise specified, the results presented in the following used the geometry B.

The relative contributions of the wake and of the rotors’ potential field to the sound pressure level of the first interaction tone \{1,1\} are shown in figure 6.33 for a given take-off case. The wake interaction noise evidently constitutes the principal source of tonal noise. Nevertheless the forward potential appears to be dominant at some observer locations. The addition of the contribution of the forward potential provides a better agreement with the wind tunnel measurements at those particular locations. In this case the rearward potential has no effect on the emitted noise.

As mentioned in section 6.2.2.2 the potential field due to blade thickness is likely to have little effect on the first interaction tone produced by the potential field interactions. In the given take-off case the change of geometry in model 1 only engender differences in sound pressure levels up to around 1.5\,dB for the upstream interactions and up to approximately 3\,dB for downstream interactions at few observer locations, see figure 6.34.

A comparison between the noise levels predicted by model 1 and model 2 is shown in figure 6.35 for the tone \{1,1\}. As a logical continuation of previous observations, model 1 produces higher noise levels than model 2 in the case of upstream interactions, and lower levels in the case of downstream interactions. Differences from 2\,dB to about 8\,dB are observed between the results from the two methods. The use of a distributed source model could thus have non negligible effect on the predicted noise levels at some observer locations if the potential field interaction becomes a predominant source of tonal noise.
Figure 6.33: Relative contribution of the potential field and viscous wake effects for the polar directivity of the tone \{1, 1\} compared with wind tunnel measurements and CFD-based calculations at a given take-off case.

Figure 6.34: Effect of the airfoil’s geometry on the polar directivity of the tone \{1, 1\} caused by potential field interactions.

The predictions performed using the CFD blade response in the analytical noise
radiation formulae were also compared with the analytical model in figure 6.35 for the case of upstream interactions. There is a good agreement between the CFD-based calculation and both analytical models at most of the emission polar angles. In figure 6.36 the sum of the contributions from the viscous wake and the downstream rotor’s potential field provided by the CFD-based calculations and by model 1 are compared with wind tunnel measurements. Both methods show a reasonable agreement with the measurements in terms of absolute levels within a range of 10dB at most observer locations. Yet some more important discrepancies between the predicted and measured levels (≈ 15dB) remain present, in particular between 110° and 115°. Note that the analytical model does not take into account other flow disturbances that could influence the results such as the swirl or induced velocity in the inter-rotor region, the tip of hub vortices or the interactions between the blades of the same row or between the flow velocity fields. The results could also be affected by measurements uncertainties or numerical dissipation in the CFD simulation. Nonetheless the noise levels provided by each method lie in the same ballpark thus prove to be promising for future studies.

Figure 6.35: Polar directivity of the tone \{1,1\} due to the potential field interactions calculated using analytical models and the CFD blade response.
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The polar directivity of the tone \( \{1, 1\} \) is shown in figure 6.37 and is decomposed into the different contributions from the interaction sources of interest. As expected the potential field interactions have no effect on the emitted noise levels due to the small order of magnitude of the perturbations. Although it is interesting to note that in this case the effect of the rearward potential becomes dominant other than that of the forward potential downstream of the engine.

Figure 6.36: Polar directivity of the tone \( \{1, 1\} \) due to the potential field and wake interactions calculated using model 1 and the CFD blade response compared with wind tunnel measurements.
6.5 Concluding remarks

An analytical method for predicting the far-field tonal noise produced by the interaction of the bound potential field of one rotor with the adjacent rotor was developed. The results showed that, for the potential field due to blade loading, the consideration of a distribution of sources along the airfoil’s chordline greatly affects the predicted levels in comparison to a point source model. Varying the airfoil’s geometry visibly influenced the predicted potential field due to blade thickness, particularly at high frequencies. However this contribution proved to have negligible effect on the noise radiated by the advanced open rotor. The forward potential appeared to be a dominant source of tonal noise at some observer locations, and its inclusion in the analytical model provided an amelioration in the agreement with both the wind tunnel measurements and the CFD. A cross validation of the analytical method with CFD-based calculations and wind tunnel measurements showed promising results for the improvement of prediction methods to calculate the tonal noise produced by an advanced open rotor. However, the analysis of the numerical results was limited to the first interaction tone due to some difficulties extracting the higher harmonics of the potential flow contribution from the CFD simulation.
Chapter 7

Conclusions

7.1 Research outcomes

The study presented in this thesis is the product of an active collaboration with the project sponsor Rolls-Royce plc. The aim of the project was to develop fast prediction methods to calculate the far-field tonal noise produced by an advanced open rotor. Of particular interest was the development of analytical models to calculate the tonal noise caused by various sources (such as the flow disturbances due to the angle of attack, the front rotor’s viscous wakes and the rotors’ bound potential fields) and the cross-validation of those models against wind tunnel measurements and numerical simulations. For that purpose the author carried out various types of analysis which led to the research outcomes listed below.

An extensive analysis of experimental data was conducted in order to understand, quantify and optimize the noise produced by an advanced open rotor, and to evaluate the effectiveness of the analytical models. A first part of the data analysis was recently published by Kingan, Ekoule and Parry (2014). Comparisons with existing prediction methods for an isolated advanced open rotor showed a satisfactory agreement. A second part of the analysis was presented and used as the basis for comparison of the prediction methods developed in Chapters 5 and 6. A third part of the analysis was provided to the project sponsor via a proprietary deliverable. The investigation of the wind tunnel measurements provided good insights about the acoustic performance of the advanced open rotor at various operating conditions, and is a valuable asset for the project sponsor to validate the new design parameters of the engine.

The post-processing method developed by the author, and used in this analysis,
provides polar directivities of rotor-alone and interaction tones produced by an advanced open rotor, as well as narrow band and one-third octave band noise spectra, for analysis of the broadband and tonal noise at various observer locations. The method can also be used to generate noise maps around the engine. It constitutes a useful tool for the project sponsor as it is adaptable to the analysis of the measurements from Rolls-Royce’s future test campaigns.

An analytical method for predicting the noise produced by a propeller at angle of attack has been developed. The method uses a simplified airfoil geometry in order to enable rapid analysis of the effect of the angle of attack on the tonal noise emitted by an advanced open rotor. The technique showed promising results in its ability to capture the changes in sound pressure levels due to the flow perturbations caused by the angle of attack. It has been implemented into one of Rolls-Royce’s prediction tools for the assessment of open rotor noise.

A hybrid analytical/computational method has been developed in order to predict the tonal noise caused by the impingement of the front rotor viscous wakes onto the rear rotor’s blades. The method utilized Computational Fluid Dynamics (CFD) data provided by the Rolls-Royce University Technology Centre in Gas Turbine Aerodynamics, and post-processed by the author. This data was used as an input to a simple two-dimensional analytical model for which the wake characteristics were measured on a CFD plane between the two blade rows, then convected analytically to the leading edge of the downstream blades. The analytic propagation of the wakes from the numerical measurement plane constitutes a novel method which enables one to counteract possible losses in the wake content due to numerical dissipation. This provides a comparative reference to assess the evolution of the wakes in the inter-rotor region, and to quantify the differences between the CFD-based calculations and the analytical model. Comparisons at different propagation distances showed good agreement between the analytical and numerical wake profiles at a wide range of radii. Additional comparisons of the unsteady loading on the downstream blade, calculated using both the CFD simulation and the hybrid method, showed similarities in terms of the general behaviour of the flow field around the blades. However, the results exhibited some discrepancies at localized regions in the wake profile and the blade response which could be due to numerical errors or physical phenomena such as the bound potential fields of both blade rows, the tip vortex of the front rotor, vortices at the hub, or stream tube contraction. Nevertheless the noise predictions showed satisfactory agreement between the wind tunnel measurements, the analytical model, and the CFD-based method in a wide range of observer locations and frequencies. The hybrid method
can be a valuable tool because it enables one to predict wake interaction noise using only a limited amount of numerical data, at one axial plane downstream of the front rotor, without the need of running a full CFD simulation.

In addition, an analytical method for predicting the noise produced by the interactions between the bound potential field of one rotor with the adjacent rotor has also been developed. The method includes derivations of the potential field due to both the airfoil’s thickness and its camber and angle of attack (loading effects). The results showed that the first interaction tones were essentially dominated by the loading component. On the prediction of the latter, modelling the airfoil as a distribution of sources and sinks along its chordline proved to have a significant influence on the predicted levels compared to the point source representation. The thickness effects were mostly significant at high frequencies, however at these frequencies the potential field interactions have little effects on the total noise produced by the advanced open rotor. The forward potential appeared to be a dominant source of tonal noise at some observer locations. Comparisons of its contribution with the CFD-based noise calculation showed a very good agreement for the first interaction tone. The addition of the effect of the rotor’s bound potential fields to the wake interaction noise predictions provided an improvement in the agreement with both the wind tunnel measurements and the CFD-based predictions, and thus gives promising results for the prediction of advanced open rotor tone noise. However some difficulties capturing the higher harmonics of the potential field components from the CFD simulation led to a poor agreement with the analytical results at higher frequency. For that matter the analytical method can become a useful asset to overcome the limitations of the numerical simulations. The method can also be beneficial to the engine manufacturers if the gap between the two blades rows were to be reduced.
7.2 Further research

The research carried out throughout this PhD is amenable to further research, some areas of future work are suggested below.

In the investigation of the effects of the angle of attack on the tonal noise produced by a propeller only the loading component was accounted for, as it constitutes the dominant source. However the blade thickness can be significant at some observer locations and can become important at high frequencies. A direct extension to this model would be the inclusion of the thickness effects in the prediction method. Another possible future area of research is the investigation of the angle of attack on the interaction tones produced by the advanced open rotor.

In Chapters 5 and 6, the tip vortex in the wake and the potential field models is not included because, for contra-rotating open rotors with cropped (or clipped) rear rotors, the effect of the tip vortex on interaction noise is minimal. Nonetheless, further work by the author will add the tip vortex to the current viscous wake model using the analytical derivation of Kingan and Self (2009) or Roger et al. (2012). Moreover the discrepancies observed between the numerical simulations and the analytical models will be investigated further by comparisons with additional CFD simulations of higher-order accuracy in time and space. An interesting extension to this work would be a comparison of the present noise prediction methods with that obtained via CFD/CAA methods. Further research also includes extension of the current hybrid approach to apply it to the rotor’s potential fields, the front rotors tip vortex, and the associated unsteady loading and radiated noise field.

In this thesis the sweep was only accounted for in the noise radiation formulation and in the wake’s propagation distance, but not in the blade calculation itself. Further work will include the addition of the blade’s sweep in the calculation. A preliminary study was carried out making use of the Wiener-Hopf technique. This method should be investigated further.

Finally, a relevant area of future work is the consideration of the physical properties of the flow such as the swirl, which can cause the wakes to skew tangentially (Brookfield et al. (1997)), and the stream tube contraction which, as well as the angle of attack effects, can cause the front rotor’s tip vortex to impinge on the downstream blade’s tip (Vion (2013)).
Appendix A

Evaluation of $I_x$ and $I_\phi$

This section details the derivation of equations (3.38) and (3.39). Starting with the latter, which is initially given as equation (3.37) by

\[ I_{y_1} = \frac{1}{2\pi} \int_0^{2\pi} \exp \{ i(mB - k)\phi_y \} \exp \{ -i k_m S_s r_y \cos(\pm \phi' - \phi_y) \} d\phi_y, \quad (A.1) \]

we can write

\[ I_{y_1} = \frac{1}{2\pi} \int_0^{2\pi} \exp \{ i(mB - k)\phi_y \} \exp \{ i k_m S_s r_y \cos(\pm \phi' + \phi_y + \pi) \} \times \exp \left\{ i(mB - k) \left( \pm \phi' - \frac{\pi}{2} \right) \right\} \exp \left\{ i(mB - k) \left( \pm \phi' + \frac{\pi}{2} \right) \right\} \times \exp \left\{ i(mB - k) \frac{\pi}{2} \right\} \exp \left\{ -i(mB - k) \frac{\pi}{2} \right\} d\phi_y. \quad (A.2) \]

This yields the following expression

\[ I_{y_1} = \exp \left\{ i(mB - k) \left( \pm \phi' - \frac{\pi}{2} \right) \right\} \frac{1}{2\pi i mB - k} \times \int_0^{2\pi} \exp \{ i k_m S_s r_y \cos(\pm \phi' + \phi_y + \pi) \} \exp \{ i(mB - k)(\pm \phi' + \phi_y + \pi) \} d\phi_y. \quad (A.3) \]
According to the definition of the Bessel first integral

\[ J_n(z) = \frac{1}{2\pi i} \int_0^{2\pi} \exp\{iz \cos \theta\} \exp\{in\phi\} d\phi, \quad (A.4) \]

equation can be directly written

\[ I_{y_1} = \exp\left\{i(mB - k)(\pm \phi' - \frac{\pi}{2})\right\} J_{mB-k}(k_m S_s r_y). \quad (A.5) \]

The initial expression of \( I_\phi \) is given by equation as:

\[ I_{\phi_y} = \frac{1}{2\pi} \int_0^{2\pi} \sin(\pm \phi' - \phi'_y) \exp\{i(mB - k)\phi_y\} \exp\{-ik_m S_s r_y \cos(\pm \phi' - \phi_y)\} d\phi_y. \quad (A.6) \]

Integrating by part and using the same method as for the calculation of \( I_x \) yields

\[ I_{\phi_y} = \frac{-1}{k_m S_s r_y} \left[ \frac{1}{2\pi} \exp\{-ik_m S_s r_y\} \{\exp\{2\pi i(mB - k)\} - 1\} \right] \]
\[ - \frac{1}{2\pi i^{mB-k}} \left\{ i(mB - k) \exp\{i(mB - k)(\pm \phi' - \frac{\pi}{2})\} \frac{1}{2\pi i^{mB-k}} \right\} \]
\[ \times \int_0^{2\pi} \exp\{ik_m S_s r_y \cos(\pm \phi' + \phi_y + \pi)\} \exp\{i(mB - k)(\pm \phi' + \phi_y + \pi)\} d\phi_y \]. \quad (A.7) \]
Appendix B

Coordinate systems

In Hansons notations, the relationship between the source coordinates in the system aligned with the flight \((y_1, y_2, y_3)\) and the source coordinates in the propeller system \((y'_1, y'_2, y'_3)\) are defined by:

\[
y_1 = c_{11}y'_1 + c_{12}y'_2 + c_{13}y'_3, \quad \text{(B.1)}
\]
\[
y_2 = c_{21}y'_1 + c_{22}y'_2 + c_{23}y'_3, \quad \text{(B.2)}
\]
\[
y_3 = c_{31}y'_1 + c_{32}y'_2 + c_{33}y'_3, \quad \text{(B.3)}
\]

where the coefficients \(c_{ij}\) are defined from the yaw, pitch and roll angles as:

\[
c_{11} = \cos \alpha_y \cos \alpha_p, \quad \text{(B.4)}
\]
\[
c_{12} = \cos \alpha_y \sin \alpha_p \sin \alpha_r - \sin \alpha_y \cos \alpha_r,
\]
\[
c_{13} = \sin \alpha_y \sin \alpha_r + \cos \alpha_y \sin \alpha_p \cos \alpha_r,
\]
\[
c_{21} = \sin \alpha_y \cos \alpha_p, \quad \text{(B.5)}
\]
\[
c_{22} = \cos \alpha_y \cos \alpha_r + \sin \alpha_y \sin \alpha_p \sin \alpha_r,
\]
\[
c_{23} = \sin \alpha_y \sin \alpha_p \cos \alpha_r - \cos \alpha_y \sin \alpha_r,
\]


\[ c_{31} = -\sin \alpha_p, \]
\[ c_{32} = \cos \alpha_p \sin \alpha_r, \]
\[ c_{33} = \cos \alpha_p \cos \alpha_r. \]  

(B.6)

This can be applied to the relationship between the retarded observer coordinates \((x_{1r}, x_2, x_3)\) in the system aligned with the flight and the observer coordinates in the propeller system \((x_1', x_2', x_3')\) as follows:

\[ x_{1r} = c_{11} x_1 + c_{12} x_2' + c_{13} x_3', \]
\[ x_2 = c_{21} x_1' + c_{22} x_2' + c_{23} x_3', \]
\[ y_3 = c_{31} x_1' + c_{32} x_2' + c_{33} x_3'. \]  

(B.7)

with

\[ x_{1r} = x_1 + Mr. \]  

(B.8)

Note that in figures 2.7(a) and 2.7(b), \(x_r = x_{r1}\).

In retarded coordinates system, relationships between the spherical angles in the propeller and in the flight system are the following:

\[ \cos \theta' = c_{11} \cos \theta + c_{21} \sin \theta \cos \phi + c_{31} \sin \theta \sin \phi, \]
\[ \sin \theta' \cos \phi' = c_{12} \cos \theta + c_{22} \sin \theta \cos \phi + c_{32} \sin \theta \sin \phi, \]
\[ \sin \theta' \sin \phi' = c_{13} \cos \theta + c_{23} \sin \theta \cos \phi + c_{33} \sin \sin \phi. \]  

(B.9)

\[ \cos \theta = c_{11} \cos \theta' + c_{12} \sin \theta' \cos \phi' + c_{13} \sin \theta' \sin \phi', \]
\[ \sin \theta \cos \phi = c_{21} \cos \theta' + c_{22} \sin \theta' \cos \phi' + c_{23} \sin \theta' \sin \phi', \]
\[ \sin \theta \sin \phi = c_{31} \cos \theta' + c_{32} \sin \theta' \cos \phi' + c_{33} \sin \theta' \sin \phi'. \]  

(B.10)
In Chapter 3 we assume that

\[
\begin{align*}
\alpha_y &= 0, \\
\alpha_p &= -\gamma, \\
\alpha_r &= 0.
\end{align*}
\]  \hspace{1cm} (B.11)

where \(\gamma\) is the angle of attack.
Appendix C

Profile drag and $\beta$

This section details the calculation of the profile drag associated with the formation of the wake, which will enable the determination of the wake parameter $\beta$.

Consider an isolated airfoil defined in the coordinate system shown in figure C.1. The flow velocity downstream of the airfoil $\bar{u}$ is defined as

$$\bar{u} = U_\infty - \bar{u}' ,$$

where $U_\infty$ and $\bar{u}'$ are respectively the free stream velocity and the perturbation velocity corresponding to the velocity deficit due to the propagation of the wake downstream of the airfoil’s trailing edge. The drag coefficient is calculated using the momentum theorem. Indeed, Newton’s second law of motion states that the rate of change of linear momentum of a solid body is equal the net force acting on the body, and takes place in the direction of the force. In this section, the blades are assumed to be far enough from each other for each single airfoil to be treated separately.

To apply the momentum theorem, we start from the Navier-stokes equations

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} = \frac{\partial \sigma_{ij}}{\partial x_j}$$

Using the conservation of mass,

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_j)}{x_j} = 0,$$
equation (C.2) can be written

\[ u_i \frac{\partial \rho}{\partial t} + u_i \frac{\partial (\rho u_j)}{\partial x_j} + \rho \left[ \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right] = \frac{\partial \sigma_{ij}}{\partial x_j} \]  \hspace{1cm} (C.4)

i.e.

\[ \left[ \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right] = \frac{\partial \sigma_{ij}}{\partial x_j} \]  \hspace{1cm} (C.5)

The conversion to the moving frame yields

\[ x_i = x'_i + U_it' \]  \hspace{1cm} (C.6)
\[ t = t' \]  \hspace{1cm} (C.7)
\[ u_j = u'_j + U_j. \]  \hspace{1cm} (C.8)

Thus the partial derivatives can be written

\[ \frac{\partial}{\partial x_i} = \frac{\partial}{\partial x'_i} \]  \hspace{1cm} (C.9)
\[ \frac{\partial}{\partial t} = \frac{\partial}{\partial t'} - U_j \frac{\partial}{\partial x'_j}. \]  \hspace{1cm} (C.10)

thus equation (C.5) becomes

\[ \rho \left[ \frac{\partial u_i}{\partial t'} + (u_j - U_j) \frac{\partial u_i}{\partial x'_j} \right] = \frac{\partial \sigma_{ij}}{\partial x'_j}. \]  \hspace{1cm} (C.11)

Applying the conservation of mass with the convection time derivative we have

\[ \frac{\partial \rho}{\partial t'} - U_j \frac{\partial \rho}{\partial x_j} + \frac{\partial (\rho(u'_j + U_j))}{\partial x_j} = 0 \]  \hspace{1cm} (C.12)

i.e.

\[ \frac{\partial \rho}{\partial t'} + \frac{\partial (\rho u'_j)}{\partial x'_j} = 0 \]  \hspace{1cm} (C.13)

(C.13) + (C.11) yields
Appendix C Profile drag and $\beta$

\[
\frac{\partial (\rho u'_i)}{\partial t'} + \frac{\partial (\rho u'_i u'_j)}{\partial x'_j} = \frac{\partial \sigma_{ij}}{\partial x'_j} \tag{C.14}
\]

The flow is steady in the moving coordinate system so the first right term of equation (C.14) is equal to zero. Consider a control volume $V$ as shown in figure 2.1. The upper and lower surfaces of the control volume are respectively the flat plate surface and a streamline outside the wake region. Let the inlet plane be at the leading edge from the flat plate, from the point $Y = 0$ up to the point $Y = h_{in}$. The outlet plane is located at an arbitrary point downstream of the airfoil, from the airfoil surface $Y = 0$ up to a point $Y = h_{in} + \delta^*$. The quantity $\delta^*$ denotes the displacement thickness, distance by which the streamlines are displaced outward due to the presence of the wake.

Integrating on the control volume $V$ - that is moving with velocity $U_i$ and not varying in size - we deduce from equation (C.14)

\[
\int_V \frac{\partial}{\partial x'_j} (\rho u'_i u'_j - \sigma_{ij}) dV = 0 \tag{C.15}
\]

Applying the divergence theorem yields

\[
\int_S (\rho u'_i u'_j - \sigma_{ij}) n_j dS = 0 \tag{C.16}
\]

The surface $S$ is the sum of the outer surface of $V$ and the airfoil surface $S = S_a + S_o$. The velocity $u'_i$ is equal to zero on the airfoil surface. Thus, the net forces acting on the blade are

\[
F_i = L_i + D_i = \int_{S_o} \rho u'_i u'_j n_j dS - \int_{S_o} \sigma_{ij} n_j dS. \tag{C.17}
\]

$S_o$ can be decomposed in a number of surfaces as (see figure 2.1)

\[
S_o = S_{o,I} + S_{o,o} + S_{o,u}. \tag{C.18}
\]

As the upper surface follows a streamline, the flow is tangential to the local velocity and there is no flow across the surface. Thus, equation (C.17) becomes
Assume that the static pressure downstream of the airfoil is equal to the pressure in the free stream, and that the flow is incompressible. The drag per unit length acting on the blade in the control volume $V$ is given by

$$dD_V = \int_{h_{in}}^{h_{in} + \delta^*} \rho U_\infty^2 dY - \int_{0}^{h_{in} + \delta^*} \rho (\bar{u}_i^2 - U_\infty^2) dY - \int_{0}^{h_{in} + \delta^*} \rho U_\infty^2 dY. \quad (C.20)$$

The above equation can be re-written

$$\frac{dD_V}{dr} = \int_{0}^{h_{in}} \rho U_\infty^2 dY - \left[ \int_{0}^{h_{in} + \delta^*} \rho (\bar{u}_i^2 - U_\infty^2) dY - \int_{0}^{h_{in} + \delta^*} \rho U_\infty^2 dY \right]. \quad (C.21)$$

Setting $h_{in} + \delta^* = \delta$ where $u_i' = U_\infty$ on the outlet surface, outside of the wake region, and by definition of the thickness displacement we have

$$\delta^* = \frac{1}{U_\infty} \int_{0}^{\delta} (U_\infty - \bar{u})dY. \quad (C.22)$$

Thus the drag per unit length on the control volume $V$ is given by

$$\frac{dD_V}{dr} = \int_{0}^{\delta} \rho \bar{u}(U_\infty - \bar{u})dY \quad (C.23)$$
Appendix C Profile drag and $\beta$

The drag per unit length due to the wake is then

$$\frac{dD}{dr} = \int_{-\delta}^{\delta} \rho \bar{u} (U_\infty - \bar{u}) dY \quad (C.24)$$

Assume that there is no drag outside the wake region, the integration can be performed at infinity.

$$\frac{dD}{dr} = \rho \int_{-\infty}^{+\infty} \bar{u} (U_\infty - \bar{u}) dY. \quad (C.25)$$

According to equation (C.1) equation (C.25) becomes

$$\frac{dD}{dr} = \rho U_\infty^2 \int_{-\infty}^{+\infty} \left(1 - \frac{\bar{u}'}{U_\infty}\right) \frac{\bar{u}'}{U_\infty} dY. \quad (C.26)$$

The drag per unit length is defined as

$$\frac{dD}{dr} = \frac{1}{2} \rho U_\infty^2 c C_D. \quad (C.27)$$

Thus the drag coefficient is given by

$$C_D = \frac{dD}{dr} \frac{1}{2 \rho U_\infty^2 c}. \quad (C.28)$$

In order to compare the analytical models with the CFD data, given at fixed axial location, the integration must be performed over the tangential direction $x_2 = r \phi$. In this coordinate, equation (C.26) becomes

$$\frac{dD}{dr} = \rho U_\infty^2 \int_{-\infty}^{+\infty} \left(1 - \frac{\bar{u}'}{U_\infty}\right) \frac{\bar{u}'}{U_\infty} dx_2 \cos \alpha. \quad (C.29)$$

Note that the wake parameter $\beta$ can be deduced from the above equation by: (1) calculating the drag per unit span numerically by performing the integration in (C.29) over one blade passage; (2) replacing the wake velocity deficit $\bar{u}'$ by its analytical expression (5.3) in equation (C.29) and solving the latter, yielding

$$\beta = \pi \left( u_c \left( -2 + \frac{u_c \sqrt{2}}{U_r} \right) \frac{\rho_0 U_r}{dD/dr 2 \cos \alpha_1} \right)^2. \quad (C.30)$$
Appendix D

Parry’s method for the potential field interactions prediction

D.1 Potential due to blade thickness for an elliptic airfoil

Since Parry’s model (Parry (1988)) is a far field-approximation of Milne-Thomson’s exact solution (Milne-Thomson (1968)) of the potential field of an ellipse, we first verify the extent of its validity. We define the complex coordinate $z = X + iY$ and the complex potential $w$ such that

$$w = \phi^t + i\psi^t, \quad (D.1)$$

where $\phi^t$ and $\psi^t$ are respectively the potential and stream function. The ellipse is defined using a conformal mapping from a circle of radius $r$ in a $Z-$plane to an ellipse of major axis $2b$ and minor axis $2a$ in the $z-$plane. The complex potential around such ellipse in a flow of velocity $U_r$ is given by Milne-Thomson (1968) as

$$w(z) = -U_rZ - U_r \frac{(a + b)^2}{4Z}, \quad (D.2)$$

where

$$Z = \frac{z}{2} \pm \frac{1}{2} \sqrt{z^2 - (b^2 - a^2)} \quad (D.3)$$
Appendix D Parry’s method for the potential field interactions prediction

Note that in the above expression, the flow direction is opposite to Milne-Thomson’s definition. In the far field, Parry approximates the $Z$ coordinate as

$$Z \sim z - \frac{l^2}{z} + o\left(\frac{1}{z^3}\right), \quad (D.4)$$

where

$$l^2 = \frac{b^2 - a^2}{4}. \quad (D.5)$$

This yields to the following expression of the complex potential

$$w(z) = -U_r z - U_r \frac{a(a + b)}{2z}. \quad (D.6)$$

The velocity potential $\phi^t$ is thus given by (see equation A6.1.9 in Parry (1988))

$$\phi^t = -U_r X - U_r \frac{a(a + b)}{2} \frac{\partial}{\partial X} \ln(\sqrt{X^2 + Y^2}). \quad (D.7)$$

In this far-field approximation the ellipse can be perceived as a point source. Figure D.1 shows the potential field around a given ellipse calculated using Milne-Thomson’s formulation, setting $a = t_{\text{max}}/2$ and $b = c/2$. The potential field is similar to that of a dipole with poles located at the trailing edge and the leading edge of the elliptic airfoil. The potential field around an identical ellipse calculated using Parry’s expression is plotted in figure D.2. Clearly, the far field approximation concentrates the source strength at the origin where the amplitudes are much greater than those observed in the previous figure. Further away from the airfoil, the two models provide similar amplitudes. A comparison between the two models at constant $Y$ coordinates is shown in figure D.3. This shows that the models agree from a distance as we move further away from the origin. Parry’s model can thus be used to predict the potential field interaction noise of an AOR, provided that the gap between the upstream and downstream blades is large enough to approach the exact solution. Note that for all figures the free-stream potential $U_r X$ is not accounted for.

For a cascade of airfoils in a compressible flow, Parry expressed the potential field due to blade thickness as

$$\phi^t = \frac{\pi U_r a(\beta a + b)}{2s(\sin^2 \alpha + \beta^2 \cos^2 \alpha)} \left[\beta \text{sgn}(x) \cos \alpha - i \text{sgn}(n) \sin \alpha\right] E_x E_y, \quad (D.8)$$
Appendix D Parry’s method for the potential field interactions prediction

Figure D.1: Potential field around an ellipse calculated using Milne-Thomson’s model.

Figure D.2: Potential field around an ellipse calculated using Parry’s model.

where

\[
E_x = \exp \left\{ -\frac{2\pi \beta |n||x|}{s(\sin^2 \alpha + \beta^2 \cos^2 \alpha)} + \frac{i2\pi (1 - \beta^2)nx \sin \alpha \cos \alpha}{s(\sin^2 \alpha + \beta^2 \cos^2 \alpha)} \right\}, \quad (D.9)
\]

\[
E_y = \exp \left\{ -i2\pi \frac{ny}{s} \right\}, \quad (D.10)
\]
and $\beta$, $n$, $\alpha$ and $s$ are respectively the compressibility factor, the harmonic number, the blade stagger angle at the given radius and the space between two airfoils of the same row.

The upwash on the upstream airfoil has a gust amplitude

$$W_{1}^{*} = - \frac{n_{2}B_{2}^{2}U_{r_{1}}a_{1}(\beta_{2}a_{1} + b_{1})}{4r^{2}(1 - M_{x}^{2})} \left( \sin \alpha_{2} - i\beta_{2} \cos \alpha_{2} \right) \left( \cos \alpha_{1} - i\eta_{2} \sin \alpha_{1} \right) \times \exp \left\{ - \frac{n_{2}B_{2}}{r} \left[ \eta_{2} \left( g - \frac{c_{1}}{2} \cos \alpha_{1} \right) + i\frac{c_{1}}{2} \sin \alpha_{1} \right] \right\}. \quad (D.11)$$

The gust amplitude of the upwash on the downstream airfoil is expressed as

$$W_{2}^{*} = \frac{n_{1}B_{1}^{2}U_{r_{1}}a_{2}(\beta_{1}a_{2} + b_{2})}{4r^{2}(1 - M_{x}^{2})} \left( \sin \alpha_{1} + i\beta_{1} \cos \alpha_{1} \right) \left( \cos \alpha_{2} + i\eta_{1} \sin \alpha_{2} \right) \times \exp \left\{ - \frac{n_{1}B_{1}}{r} \left[ \eta_{1} \left( g - \frac{c_{2}}{2} \cos \alpha_{2} \right) - i\frac{c_{2}}{2} \sin \alpha_{2} \right] \right\}. \quad (D.12)$$
Appendix D Parry’s method for the potential field interactions prediction

D.2 Potential due to blade circulation

Parry modeled each airfoil by a point vortex of circulation $\Gamma$ and expressed the velocity potential as

$$\phi^l = \frac{i\Gamma}{4\pi \beta} \sum_{n=-\infty}^{+\infty} \frac{1}{n} E_x E_y,$$  \hspace{1cm} (D.13)

where $E_x$ and $E_y$ are defined in equations (D.10) and (D.10). In the coordinate system defined in section 6.2, the upwash on the upstream airfoil has a gust amplitude

$$W^l_1 = -\frac{\Gamma B_2}{4\pi r} (\cos \alpha_1 - i\eta_2 \sin \alpha_1) \exp \left\{ -K_{y_2} \left[ \eta_2 \left( g - \frac{c_1}{2} \cos \alpha_1 \right) + i \frac{c_1}{2} \sin \alpha_1 \right] \right\}. \hspace{1cm} (D.14)$$

In the coordinate system defined in section 6.3, Parry defines the upwash on the downstream airfoil with the following gust amplitude

$$W^l_2 = \frac{\Gamma B_1}{4\pi r} (\cos \alpha_2 + i\eta_1 \sin \alpha_2) \exp \left\{ -K_{y_1} \left[ \eta_1 \left( g - \frac{c_2}{2} \cos \alpha_2 \right) - i \frac{c_2}{2} \sin \alpha_2 \right] \right\}. \hspace{1cm} (D.15)$$

D.3 Simplification of the distributed source model for an ellipse

In section 6.2.1, the velocity potential due to blade thickness around an isolated airfoil (which is modelled as a distribution of point sources) is given by equations (6.91) and (6.15) and is stated below.

$$\phi^l = \frac{U_t l_{max}}{2\pi \beta} \int_{\mathbb{R}} \frac{\partial G}{\partial X} \bigg|_{X=X_s} \log \left( \frac{1}{\beta} \sqrt{(X - X_s)^2 + \beta^2 Y^2} \right) dX_s \hspace{1cm} (D.16)$$

Integrating by parts leads to
Appendix D Parry’s method for the potential field interactions prediction

\[
\phi^t = \frac{U_r a}{\pi \beta} \left( \left[ \mathcal{G} \log \left( \frac{1}{\beta} \sqrt{(X - X_s)^2 + \beta^2 Y^2} \right) \right]^{\frac{3}{2}} - \int_{\frac{c}{2}}^{\frac{c}{2}} \mathcal{G} \frac{X - X_s}{(X - X_s)^2 + \beta^2 Y^2} \, dX_s \right),
\]

(D.17)

where \( a = \frac{t_{\text{max}}}{2} \). The term in parenthesis at the right hand side is equal to zero. The non-dimensional thickness distribution \( \mathcal{G} \) for an ellipse is given by equation (6.19) as follows.

\[
\mathcal{G} = \sqrt{1 - \left( \frac{X}{b} \right)^2},
\]

where \( b = c/2 \). In the far-field assumption, we consider that the source is far enough from the observer such that its position along the blade chordlength doesn’t affect the results. Thus we can set \( X_s = 0 \).

\[
\phi^t = -\frac{U_r a}{\pi \beta} \frac{X}{\sqrt{X^2 + \beta^2 Y^2}} \int_{\frac{c}{2}}^{\frac{c}{2}} \sqrt{1 - \left( \frac{X_s}{b} \right)^2} \, dX_s.
\]

(D.18)

Making the change of variable \( \frac{X_s}{b} = \cos \Theta \) gives

\[
\phi^t = \frac{U_r a b}{\pi \beta} \frac{X}{\sqrt{X^2 + \beta^2 Y^2}} \int_0^\pi \sin^2 \Theta \, d\Theta.
\]

(D.19)

The above integral evaluates as \( \pi/2 \) thus

\[
\phi^t = \frac{U_r a b}{2 \beta} \frac{X}{\sqrt{X^2 + \beta^2 Y^2}} = \frac{U_r a b}{2 \beta} \frac{\partial}{\partial X} \ln(\sqrt{X^2 + \beta^2 Y^2}).
\]

(D.20)

In the limit of a thin airfoil we have \( a << b \), thus in the far-field the above equation is equivalent to equation (D.7) (if we are only considering the component of the velocity potential due to the blade thickness and not the free-stream potential and set \( \beta = 1 \)).
References


REFERENCES


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