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Topics on Modern String Phenomenology

by

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Abstract

FACULTY OF PHYSICAL SCIENCES AND ENGINEERING

PHYSICS

Doctor of Philosophy Topics on Modern String Phenomenology

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In this thesis we present phenomenological consequences of the modern non-perturbative regimes of String Theory, M- and F-Theory. The origins of $SO(10)$ from M-Theory on G_2 manifolds are discussed, accompanied by a detailed discussion on a rank-breaking mechanism and consequences for neutrino masses. The Minimal Supersymmetric Standard Model is derived from F-Theory compactifications exhibiting a spectral cover equation with Klein monodromy and a geometric parity that endows the spectrum with an effective matter parity. A dedicated and systematic study on R-Parity violating couplings in F-Theory is also presented, where we find these couplings to be generic, and we compute their magnitudes.

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Declaration of Authorship

I, Miguel Crispim Romão, declare that this thesis, entitled 'Topics on Modern String Phenomenology', and the work presented in it are my own and has been generated by me as the result of my own original research.

I confirm that

- This work was done wholly or mainly while in candidature for a research degree at this University
- Where I have consulted the published work of others, this is always clearly attributed
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work
- I have acknowledged all main sources of help
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself
- Work contained in this thesis has previously been published in [1–4]

Signed:
Date:

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To Avó Guida
Who would have been the proudest of this achievement

Chapter 1

Introduction and Motivation

After over four decades eluding detection, a remarkably Standard Model-like Higgs was finally discovered at the Large Hadron Collider with a mass of around $m_H \simeq 125$ GeV, along with no new strong evidence for Beyond the Standard Model Physics [5–7].

The discovery of the Higgs boson represents the final piece of a large picture portrayed by the Standard Model, which remarkable successes cannot blind us from some of its flaws. For instance, the Standard Model is failing to match observations from which we came to understand that the neutrinos are massive, in contradiction with the Standard Model predictions. In addition, we would expect a model describing the fundamental degrees of freedom of our universe to be less arbitrary about its parameters and to be set in a framework where General Relativity and Quantum Mechanics cohabit, which is not the case in the Standard Model.

We are then compelled to look for extensions of the Standard Model that would provide a more satisfactory explanation of its details. In this quest, a crucial question that arises is why is the energy scale of Standard Model, usually referred as the Electroweak Scale $M_{EW} \simeq 10^2$ GeV, so light in comparison with the only known fundamental scale in nature, the (reduced) Planck Scale $m_{Pl} \simeq 10^{18}$ GeV? In fact, the Higgs doublet scalar mass term, which sets the overall scale of the Standard Model, receives quantum corrections that are quadratic in the cut-off scale, leading to the question why is the Electroweak Scale stable?

This question is famously addressed by Supersymmetry [8–10], which provides a theoretical framework where scalar masses are protected against quantum corrections. While naturalness arguments suggest light Supersymmetric partners to be light, we have failed to detect them after the

Run I of the Large Hadron Collider pushed some of the mass limits to the TeV range [11–13]. However, it is important to notice that naturalness arguments and current bounds arise within the Minimal Supersymmetric Standard Model or simple extensions thereof, disregarding any Ultra-Violet physics that would offer a natural description for heavier Supersymmetric partners or provide different bounds. The search for more general and complete models is then very well motivated by current experimental constraints on particle physics.

In order to pursue the motivation to go beyond minimal and simpler models, it would be good to have a guiding principle in this search for more complete models, as the possibilities are virtually uncountable. It is in this task that String Theory [14–18] excels, as a powerful framework that unifies Quantum Mechanics and General Relativity that provides the playing grounds for rich particle physics, a task we call String Phenomenology [19–21].

The quest for a realistic description of our universe from String Theory is not a novel task. Indeed, String Phenomenology has received considerable research effort as String Theory naturally incorporates not only the basic ingredients and building blocks we witness in our universe, but also the more customary extensions to the Standard Model such as Supersymmetry and Grand Unification Theories [22, 23]. Nevertheless, it is still a growing and thriving area of research, and its scope is widening as modern advances in String Theory in non-perturbative regimes, such as M- [24] and F-Theory [25], have opened a new frontier for model building and phenomenology [26–36].

In M-Theory, an eleventh dimension emerges in the strong coupling regime of Heterotic $E_8 \times E_8$ and Type IIA String Theory. Hence, connection with the real world requires a need to compactify one extra dimension compared to the ten-dimensions we have to compactify in perturbative regimes. This is accomplished by considering the extra seven dimensions to be compactified on a G_2 manifold.

In F-Theory, we take Type IIB String Theory where the dilaton, which sets the String coupling constant, is not fixed and instead pairs up with the RR 0-form to constitute a complex field behaving as the complex modulus of a torus. In this description, the dilaton is not fixed and so we cannot describe this regime in perturbative Type IIB String Theory. Instead, we embrace the geometric description of the torus and consider a theory which seemingly is a compactification of a 12-dimensional Supergravity on an elliptically fibred Calabi-Yau four-fold.

In both cases, the non-perturbative nature is captured by the geometrification of the new dynamics. Consequently, we are led to new compactification paradigms that are not present in perturbative regimes. Therefore, we expect new avenues for particle physics arising from these modern approaches to String Theory.

In this thesis we present work conducted in these novel directions of String Phenomenology, and is structured as follows. For the remainder of this Chapter we will review the Standard Model of Particle Physics in Section 1.1, after which we introduce two important Beyond the Standard Model extensions: Grand Unification Theories in Section 1.2, and Supersymmetry in Section 1.3. The review provided will follow very closely the discussion presented in [20]. With this we can finally contextualise in Section 1.4 the dawn of M- and F-Theory from perturbative String Theory. In Chapters 2 and 3 we will discuss how $SO(10)$ Supersymmetric Models arise from M-Theory and how they accommodate neutrino masses and symmetry breaking. In Chapters 4 and 5 we will discuss some low-energy implications of F-Theory model building, namely how the Minimal Supersymmetric Standard Model can arise from the details of the compactification data and how R-Parity violation can provide an important window into F-Theory compactifications. Finally, Chapter 6 provides the conclusions.

1.1 The Standard Model of Particle Physics

In this section we review the structure of the the Standard Model of Particle Physics (SM) and discuss some of its shortcomings and challenges. For complete reviews we refer to [37–40].

The SM can be briefly described as a Quantum Field Theory with a non-Abelian gauge symmetry (the so-called Yang-Mills theory), chiral matter spectrum, and a mechanism for spontaneous symmetry breaking. We will go through each of these ingredients, which together describe one of the most successful scientific ideas of all times.

The gauge interactions of the SM are described by the non-Abelian group which is composed of three factors

$$G_{\mathrm{SM}} = SU(3)_c \times SU(2)_L \times U(1)_Y \ . \tag{1.1}$$

The first factor accounts for Quantum Chromodynamics (QCD) and is responsible for the strong nuclear interactions that bind nuclei together. The second and third factors account for the Glashow-Salam-Weinberg [41–43] Electroweak theory, and is responsible for nuclear decays. The electromagnetic interaction is the remnant of these two groups after the Higgs mechanism, as we will see below. Each interaction is mediated by gauge vector bosons, that are valued in the Lie algebra of the corresponding group.

The fermionic content accounts for the matter fields, which form three distinct families, or generations, that are indistinguishable from one another except for their masses below the symmetric phase. In each family we identify the Quarks and Leptons, where the former transforms non-trivially under QCD while the latter does not. Each fermion state is a Weyl two-component spinor, and we list all of them and respective quantum numbers in Table 1.1. We use the customary charge conjugation convention[1] of the $SU(2)_L$ singlets in order to take all matter to be Left-handed Weyl spinors, but we might abuse terminology by referring to $SU(2)_L$ singlets as Right-handed fields.

Fields	Quantum Numbers
$Q_L^i = \begin{pmatrix} u^i & d^i \end{pmatrix}_L$	$\left(\mathbf{3}, \mathbf{2}, \frac{1}{6}\right)$
$L_L^i = \begin{pmatrix} \nu^i & e^i \end{pmatrix}_L$	$\left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}\right)$
$(u^c)_L^i$	$\left(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3}\right)$
$(e^c)_L^i$	$\left(\mathbf{1}, \mathbf{1}, 1\right)$
$(d^c)_L^i$	$\left(\overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3}\right)$

Table 1.1: Standard Model Fermions. The quantum numbers are in terms of G_{SM} factors. The index $i = 1, 2, 3$ accounts for the three families

The chirality of the spectrum, which is the statement that matter fields do not form real representations of G_{SM}, is a paramount feature of the SM as

[1]Where a Left-handed spinor can be described in terms of a Right-hand spinor by $(\psi^c)_L = -i\gamma_2\psi_R^*$.

tree-level Dirac masses are forbidden due to gauge invariance. From a model building perspective, the existence of a chiral set of fermions is an important ingredient to look for in an Ultra-Violet (UV) completion candidate.

The third advertised ingredient of the SM is the Brout-Englet-Higgs-Kibble Mechanism [44–46], which we will refer to from this point on as the Higgs Mechanism. This is a mechanism of spontaneous symmetry breaking, where $SU(2)_L \times U(1)_Y$ is broken down to the electromagnetism subgroup, $U(1)_{em}$. This is achieved by a complex scalar, H, transforming as a doublet of $SU(2)_L$ with hypercharge $YH = -1/2H$. The potential for this scalar doublet takes the form

$$V = -\mu^2 |H|^2 + \lambda |H|^4 \ , \tag{1.2}$$

and for $\mu^2 > 0$ it exhibits a tachyonic behaviour, meaning that the initially assumed configuration with vacuum expectation value (vev) $\langle H \rangle = 0$ is not stable and we need to re-expand the theory around a stable vacuum. This happens at the minimum of the potential, where the vev is

$$v^2 = \langle |H| \rangle = \frac{\mu^2}{2\lambda} \ . \tag{1.3}$$

We can make use of the G_{SM} gauge symmetry to rotate H into its H^0 component and take the corresponding vev to be real, in what is called the unitary gauge. This vev is immediately responsible to generate masses to three of the four gauge bosons of $SU(2)_L \times U(1)_Y$, whose observation fixes the value of v

$$v = \langle H^0 \rangle \simeq 170 \text{ GeV} \ . \tag{1.4}$$

This value is usually used as definition of the Electroweak Scale (EWS) at which the SM gauge symmetry is broken.

The remaining massless gauge boson accounts for a remnant, surviving symmetry group generated by the unbroken linear combination

$$Q_{em} = T_3 + Y \ , \tag{1.5}$$

where T_3 is the diagonal generator of $SU(2)_L$ and Y the generator of $U(1)_Y$. The surviving symmetry group accounts for the electromagnetic interactions, under which the fields transform with charges given by Q_{em}. The surviving gauge group below the EWS is

$$SU(3)_c \times U(1)_{em} \ . \tag{1.6}$$

Three of the four real components of H are *eaten* by the now-massive gauge fields, called $W^{\pm}$ and Z^0, as longitudinal polarisations, while a surviving real scalar, h, describes the excitations around the new vacuum

$$H^0 = v + \frac{1}{\sqrt{2}} h \ , \tag{1.7}$$

which we call *Higgs boson.* Its detection eluded us for 40 years, but has finally been detected at the Large Hadron Collider (LHC) [5–7] with a mass of

$$m_h \simeq 125 \text{ GeV} \ . \tag{1.8}$$

Tree-level Dirac fermion masses are also obtained through the Higgs mechanism. The most general allowed renormalisable couplings between the Higgs doublet and the fermionic states by gauge invariance are

$$\mathcal{L}_{Yuk} = Y_u^{ij} Q^i (u^c)^j \overline{H} + Y_d^{ij} Q^i (d^c)^j H + Y_e^{ij} L^i (e^c)^j H + \text{ h.c.} \ , \tag{1.9}$$

where $\overline{H} = i\sigma_2 H^*$. We dropped the subscript L for readability, and the notation leaves implicit Lorentz and gauge contractions. Since $\langle H \rangle \neq 0$, tree-level masses are generated. Under the smaller subgroup $U(1)_{em}$ the fermions no longer form a chiral set, pairing up into Dirac fermion masses. This means that the matter content at low-energies form Dirac spinors where the Right-handed component is formed of the $SU(2)_L$ singlets, and the Left-handed component is formed of $SU(2)_L$ doublets. The exception to this is the neutrino states, ν_i, which do not have a singlet partner and therefore remain massless within the SM.

Upon closer inspection of the SM Lagrangian, we notice it also allows for Baryon and Lepton number symmetries. These are considered accidental as they do not hold against quantum corrections or non-renormalisable interactions. On the other hand, $G_{\rm SM}$ is a consistent gauge symmetry with vanishing triangular anomalies. This is often presented as an argument for the Y assignments in Table 1.1, which are unique for the presented spectrum to be anomaly-free for each individual generation.

The mass matrices $m_a^{ij} = \langle H \rangle Y_a^{ij}$, for $i, j = 1, 2, 3$ and $a = u, d, e$, are not diagonal. By diagonalising them, we find the mass eigenstates for charged Leptons and the two types of Quarks that can be seen in Table 1.2, where we find an explicit hierarchy between the three families that is left unexplained in the SM.

up	charm	top
$m_u = 2 \times 10^{-3}$	$m_c = 5 \times 10^{-1}$	$m_t = 1.73 \times 10^2$
down	strange	bottom
$m_d = 4 \times 10^{-3}$	$m_c = 10^{-1}$	$m_b = 3$
electron	muon	tau
$m_e = 5.1 \times 10^{-4}$	$m_\mu = 1.05 \times 10^{-1}$	$m_\tau = 1.7$

Table 1.2: Standard Model Fermionic Masses in GeV [40]

Rotating the mass matrices into their diagonal form will transfer the mixing to the charge currents, i.e. couplings between u- and d-type Quarks to $W^\pm$ massive bosons. If we take V_u, V_d to be the matrices that diagonalise the (hermitian square of the) mass matrices of u- and d-type Quarks, respectively, the mixing is given the Cabibbo-Kobayashi-Maskawa (CKM) [47,48] matrix, which is fixed by experimental data [40]

$$|V_u V_d^\dagger| = |V_{CKM}| = \begin{pmatrix} 0.974 & 0.22 & 0.003 \\ 0.22 & 0.973 & 0.04 \\ 0.008 & 0.04 & 0.999 \end{pmatrix} . \tag{1.10}$$

The matrix above is not the identity and therefore illustrates the mixing between different flavours of Quarks. Namely, the lighter two families have considerable mixing in comparison to the mixing between these families and the heavier one. Furthermore, the matrix is not real and has one complex phase that cannot be rotated away. This phase is responsible for CP-violation in the Quark sector. In the Lepton sector, the absence of neutrino masses in the SM produces no analogue mixing. This picture, however, is changed with the experimental observation of neutrino masses.

This brief definition of the SM fits decades of experimental data. There are, however, some challenges and shortcomings that indicate that it cannot be the ultimate description of Nature's fundamental degrees of freedom. Indeed, nowadays we take the SM as an effective field theory, which has a cut-off at most at the Planck scale, where Quantum Gravity effects are expected to become relevant, and a new picture marrying Quantum Mechanics with General Relativity comes into place. Furthermore, the SM has many parameters fixed only by experiment. These are the fermion masses and mixing angles, the QCD θ CP-Violating parameter (see below), gauge couplings, and Higgs potential parameters. We would expect to be able to draw a more compelling picture of Nature where such parameters can be determined from basic principles.

1.1.1 Challenges and Shortcomings of the Standard Model

The picture drawn above is very robust and has been successfully tested against experimental data over many decades. Nonetheless, the SM has some loose ends and challenges that indicate that it cannot be the ultimate description of the fundamental degrees of freedom of our universe. Apart from the *a priori* free parameters listed above, there are other less than satisfactory aspects of the SM worth discussing. Here we describe a few in order to motivate further studies Beyond the Standard Model (BSM) physics.

Neutrino Oscillations and Masses

The brief introduction to the SM above does not include neutrino masses by construction. This is in direct contradiction with experimental observation as neutrino oscillations provide evidence for the massive nature of neutrinos. For a full and modern review see [49].

At the time of this work, two neutrino squared-mass differences are known. For the so-called Normal Ordering, where $m_1 < m_2 < m_3$ these are

$$\Delta m^2_{21} = 7.5 \times 10^{-5}\ \mathrm{eV}^2 \qquad \Delta m^2_{31} = 2.4 \times 10^{-3}\ \mathrm{eV}^2 \tag{1.11}$$

and we notice that this leaves room for one of the neutrino masses to be identically equal to zero.

The existence of masses also means that there is the possibility for mixing. The analogous of the CKM matrix for the neutrino mixing is known as

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [50, 51] mixing matrix [52, 53]

$$|V_{PMNS}| = \begin{pmatrix} 0.82 \pm 0.01 & 0.54 \pm 0.02 & -0.15 \pm 0.03 \\ -0.35 \pm 0.06 & 0.70 \pm 0.06 & 0.62 \pm 0.06 \\ 0.44 \pm 0.06 & -0.45 \pm 0.06 & 0.77 \pm 0.06 \end{pmatrix} , \qquad (1.12)$$

which exhibits a far more pronounced mixing between neutrinos than on Quarks.

The existence of non-vanishing neutrino masses can not be explained within the SM picture presented in the previous section. In order to account for these masses and mixings, new BSM physics is needed.

A simple extension to the SM to account for neutrino masses would be to consider the existence of extra singlet states, the right-handed conjugated neutrino N. If such states exist, the neutrinos would be endowed with a Yukawa coupling like those in eq. (1.9). In this picture, we would have purely Dirac neutrinos and would preserve Lepton number, but we would have to accept the fact that their Yukawa couplings would be so much more suppressed than the rest of the Yukawa couplings of the SM in order to account for the smallness of their masses.

Alternatively, we could consider that neutrino masses break Lepton number by allowing them a Majorana mass term

$$m_\nu^{ij} \nu^i \nu^j \ . \qquad (1.13)$$

Given the smallness of the mass eigenstates of the neutrino masses, m_ν^{ij}, this assumption would fit 't Hooft's prescription for a naturally small parameter [54], since their vanishing limit would reinstate the Lepton number symmetry.

In order to generate the Majorana mass we consider the so-called non-renormalisable Weinberg operator [55]

$$\frac{h^{ij}}{\Lambda} \nu^i \nu^j H H + \text{ h.c.} \ , \qquad (1.14)$$

where h^{ij} is a coupling matrix arising from integrating out some heavier mode at a scale Λ. Below EWS this coupling generates Majorana masses for the Left-handed neutrinos, which under the assumption $\Lambda \gg \langle H \rangle$ would be naturally smaller than the other SM masses. We notice that this operator breaks Lepton number by $\Delta L = 2$.

Depending on if the mode being integrated out is a fermion singlet, a scalar triplet, or a fermion triplet, this neutrino mass generation mechanism takes the name of See-saw mechanism of type I, II, or III, respectively.

The Hierarchy Problem

Given that the only known fundamental mass scale in nature is given by the Planck scale, $M_{Pl} \simeq 10^{18}$ GeV, we could ask why are the SM masses so light? There are two sides to this question, which have puzzled particle physicists for a long time. The first is why the EWS is at $\mathcal{O}(10^2)$ GeV, a value set by the Higgs doublet scalar mass term μ. The second side of the question is why this value is stable, which is a more technical question that still needs to be addressed.

To understand this we notice that the EWS breaking order-parameter is the Higgs vev, v^2, which is parametrically dependent on the Higgs potential mass parameter, μ^2. In the limit that this is vanishing, there are no tree-level mass parameters in the SM. Explaining the tree-level value of μ^2 would then explain the observed SM masses. This is the first side of the puzzle, which questions the tree-level value of μ^2.

The second side of the puzzle arises when we understand that scalar masses are very sensitive to quantum corrections. In fact, μ^2 receives corrections which are quadratic on the cut-off scale[2], Λ,

$$\mu^2 \propto \Lambda^2 \ . \tag{1.15}$$

Of course we could argue that the bare tree-level μ^2 parameter could be such that it would cancel the magnitude of the cut-off contribution, leading to a small physical value. Unfortunately, if we take the cut-off to be the Planck scale, for example, this would require a large amount of fine-tuning. This argument alone suggests to us that new physics could be just around the TeV scale.

[2]It is important to mention that this argument is independent of the choice of regularization procedure. Here we take the usual discussion using a Wilsonian cut-off, which provides a more intuitive discussion.

The Strong CP Problem

The full SM Lagrangian, derived by imposing gauge and Lorentz-Poincaré invariance, contains the CP-Violating term

$$\mathcal{L} \supset \frac{\theta}{32\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu} \ , \tag{1.16}$$

where θ is the so-called QCD θ-angle, $F_{\mu\nu}$ the QCD field strength tensor and $\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ its dual. This term is a total derivative and therefore does not appear in perturbation theory, i.e. in Feynmann diagrams. However, its presence leads to too large contributions to the electric dipole moment of the neutron if $\theta \simeq 1$, as a naïve estimate would suggest. Experimental constraints on the neutron electric dipole moment tell us that $\theta < 10^{-10}$, and therefore θ requires a certain level of suppression. This is called the Strong CP problem, where Strong refers to the fact it emerges in the QCD gauge part of the SM Lagrangian.

A solution to this problem is to promote θ to a dynamical field, a, called axion. The axion would be stabilised $\langle a \rangle = \theta f_a$, where f_a is the axion decay constant, in such a way that the smallness of θ can be explained dynamically. In order to implement this mechanism we need to extend the global symmetries of the SM with what is known as a Peccei-Quinn [56] symmetry. This symmetry is not present in the canonical SM but appears naturally in extensions of the SM.

Cosmological Shortcomings

If the SM was to be the ultimate picture of the fundamental degrees of freedom of our universe, it ought to contribute to our understanding of cosmology. Namely, the current Cosmological Standard Model, Λ-CDM, requires certain ingredients that are not provided by the SM of particle physics. We briefly discuss some of these issues.

We start with one of the biggest puzzles in modern theoretical physics: the cosmological constant. In the SM, the energy of the ground state $\langle V \rangle = V_0 \neq 0$ has no meaning, as we only study excitations of the ground state. On the other hand, in cosmology, this plays a role as a contribution to the energy-momentum tensor, namely

$$\langle T_{\mu\nu} \rangle = -V_0 g_{\mu\nu} \tag{1.17}$$

and so we identify the cosmological constant with the vacuum of the SM $\Lambda_{c.c.} = V_0$. In the full Quantum theory, the Einstein field equations in the vacuum are sensitive to this contribution

$$G_{\mu\nu} = -8\pi G_N V_0 g_{\mu\nu}, \tag{1.18}$$

and the value of the vacuum-energy – the cosmological constant – can be inferred from observation. The observed and computed value are off by many orders of magnitude. If we take $M_{\text{cut-off}}$ as the cut-off scale of the SM, we then have to compare

$$\begin{aligned} \Lambda^{obs}_{c.c.} &\simeq (10^{-3}\ \text{eV})^4 \\ \Lambda^{SM}_{c.c.} &\simeq (M_{\text{cut-off}})^2\ . \end{aligned} \tag{1.19}$$

It is then clear that even if we take the cut-off scale to be just above the EWS, we find a discrepancy of many orders of magnitude.

We could argue, just like with the Higgs potential mass parameter instability discussed above, that there is a tree-level cosmological constant that cancels out the quantum correction. This would turn the issue into a fine-tuning problem, although more severe than the Higgs fine-tuning discussion.

Another aspect where the SM is in direct confrontation with cosmological observations is that it does not provide a candidate for Dark Matter (DM). According to observation, around 26% of the current energy density of our universe is composed of matter that does not interact with the SM matter, which only accounts for around 5% of the total energy density [57]. Therefore, cosmology points to the need for further states not included in the SM.

Our current best understanding of the onset of the Hot Big Bang requires a period of rapid expansion, called Inflation [58–61]. While inflation can take many forms, there is a convenient description of its dynamics in a field theory context, where a dynamical degree of freedom called the Inflaton provides the required ingredients for an inflationary epoch in the early universe. The Inflaton, which is normally taken to be a scalar field, is not present within the SM.

Finally, in order to provide a successful picture of Baryogenesis, three conditions proposed by Sakharov [62] need to be met. These are Baryon number asymmetry, C and CP violating interactions, and interactions out-side of thermal equilibrium. While the SM does indeed provide Baryon number, C, and CP violation, it falls short of its contributions to these asymmetries.

1.2 Grand Unification Theories

As we saw in our description, the SM does not provide an explanation for its gauge group and the matter representations transforming under these symmetries. For example, the fact that the SM provides an anomaly-free gauge theory and that the charge of the electron is the exact opposite of that of the proton, guide us to enquire whether there is a common ancestry between the Leptons and Quarks. In addition, at high energies the SM gauge couplings seem to converge. To see this we first consider the Renormalisation Group Equations (RGEs) for the gauge couplings, g_i,

$$\frac{dg_i}{d\mu} = \frac{1}{16\pi^2}\beta_{g_i} = \frac{1}{16\pi^2} g_i^3 b_i \ , \tag{1.20}$$

where μ is the renormalisation scale, the first equality defines the so-called β-functions, the numbers b_i depend on the representations acting under the gauge factor i, and we define the fine-structure constants $\alpha_i = g_i^2/4\pi$. These equations have the solutions

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M) + \frac{b_i}{4\pi}\log\frac{M^2}{\mu^2} \ , \tag{1.21}$$

where M represents a boundary scale, and in the SM the b_i numbers are

$$(b_1, b_2, b_3) = \left(\frac{41}{10}, \frac{-19}{6}, -7\right) \ . \tag{1.22}$$

Using low-energy data to fix the values of the gauge couplings at the EWS, meaning setting $M \simeq M_Z$, we can run the couplings to higher energies. The running can be seen in Figure 1.1, where we can see that the couplings do not meet at the same point, but suggest for the possibility of unification around 10^{16} GeV.

The idea that the gauge couplings are indeed unified at a higher scale, at which the SM gauge group is a subgroup of a larger group is called Grand Unification Theory (GUT). The first GUT being proposed based on a simple Lie group was the Georgi-Glashow model based on $SU(5)$ [22],

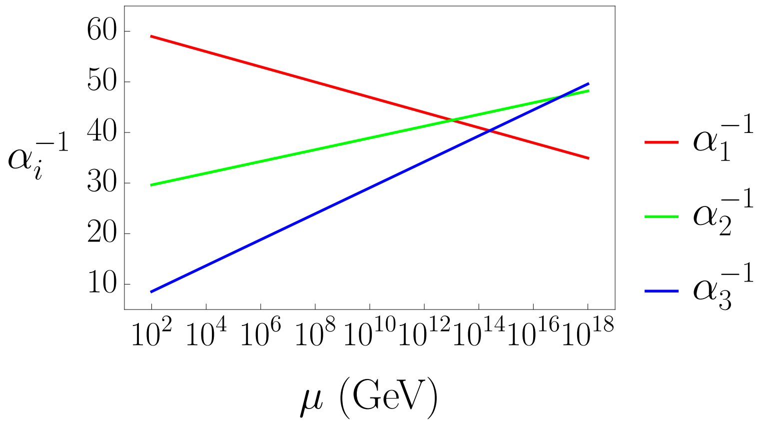


Figure 1.1: Near unification of the gauge couplings in the Standard Model

which we will review below, that followed the footsteps of Pati-Salam [63] on extending the SM gauge symmetry. Afterwards, we will also introduce the GUT based on $SO(10)$ [23].

1.2.1 $SU(5)$ GUTs

First we notice that the SM gauge group has rank four. The only simple Lie group with rank four that has non-real representations is $SU(5)$, with dimension 24. The gauge bosons are in the adjoint, **24**, which accounts for twice as many than the SM. Since the SM gauge group is a maximal subgroup of $SU(5)$, we can express the tensor representation indices of $SU(5)$ as split into $SU(3)$ and $SU(2)$ indices of the SM. In this decomposition, it is easy to see that the extra vector gauge bosons are triplets of $SU(3)$ and doublets of $SU(2)$. These bosons are usually termed Leptoquarks, as to indicate the fact that they can mediate interactions that transform Leptons into Quarks, and vice-versa. This interaction leads to Proton decay as it breaks Lepton and Baryon number. In order for Proton decay rates to be within experimental bounds, the mediating Leptoquarks need to be as heavy as $M_{GUT} \simeq 10^{16}$ GeV, which, as we have seen above, is the scale at which expect unification to happen.

The fermionic states of a single family all fit into two different representations of $SU(5)$, a $\overline{\mathbf{5}}$ and a $\mathbf{10}$, inside which the SM states read

$$\overline{\mathbf{5}} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ l^- \\ -\nu_l \end{pmatrix} \qquad \mathbf{10} = \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & l^+ \\ -d_1 & -d_2 & -d_3 & -l^+ & 0 \end{pmatrix} , \tag{1.23}$$

while the SM Higgs doublet transforms in a $\overline{\mathbf{5}}$

$$\overline{\mathbf{5}}_H = H \oplus T \tag{1.24}$$

and as such comes with a coloured triplet partner, which can mediate Proton decay. Following the same considerations as for the Leptoquarks, this coloured triplet has to be heavy in order to prevent too fast Proton decay. Avoiding a light T, while keeping H light enough to produce the EWS is known as the Doublet-Triplet splitting problem [64–66] and plagues every GUT where Leptons and Quarks share the same representations.

Since the GUT group is broken below the GUT scale, there is at least another Higgs field we have to consider that breaks the GUT group down to the SM gauge group. This is achieved by scalar field transforming as a **24**, which breaking patterns preserve the rank.

The model shown here has many interesting features. We already mentioned Proton decay, which is a generic prediction of any GUT model where the Leptons and Quarks share representations. While still unobserved, the detection of Proton decay would provide staggering support for GUTs. In addition, the fact that Quarks and Leptons share the same representations provides a common ancestry between them. In fact, tracelessness of the $SU(5)$ generators implies $Q_{em} d^c = -1/3 Q_{em} l^-$.

The extended symmetry provided by the GUT group also changes the structure of the Yukawa interactions. The Yukawa interactions will have the schematic form[3]

$$\mathcal{L} \supset Y_{\mathbf{10}}^{ij} \mathbf{10}^i \mathbf{10}^j \overline{\mathbf{5}}_H^* + Y_{\overline{\mathbf{5}}}^{ij} \overline{\mathbf{5}}^i \mathbf{10}^j \overline{\mathbf{5}}_H + \text{h.c.} , \tag{1.25}$$

which immediately implies that the down-type Quarks and charged Leptons

[3]Where by schematic we mean abuse of terminology and we omit the symmetry contractions.

have the same Yukawa couplings

$$Y_d^{ij} = Y_l^{ij} \ . \tag{1.26}$$

Furthermore, although the $SU(5)$ invariant contraction in the first term is antisymmetric, the fact that the **10** are antisymmetric means that Y_u is a symmetric matrix.

Since the Yukawa β-functions are different for Quarks and Leptons, the above results do not mean a $m_d = m_l$ prediction. Nonetheless, running the Yukawa RGEs in the SM shows the the low-energy data is in contradiction with the minimal $SU(5)$ prediction for Leptons and down-type Quarks Yukawa unification. Therefore, despite the many positive features, the minimal $SU(5)$ suffers from a series of challenges.

1.2.2 $SO(10)$ GUTs

Going one rank higher we find $SO(10)$ and $SU(6)$. Since $SU(6)$ mostly reproduces the dynamics of $SU(5)$ models, we will focus on $SO(10)$, which has the appealing feature of fitting an entire family into a single **16** representation. In terms of $SU(5)$ representations, a single family lives inside a **16** as

$$\mathbf{16} = \mathbf{10} \oplus \overline{\mathbf{5}} \oplus \mathbf{1} \ , \tag{1.27}$$

where **1** is an $SU(5)$ singlet that is not present in either the SM or minimal $SU(5)$ models. In terms of SM states we have

$$\mathbf{16} = \left(\nu_l, u_1, u_2, u_3, l^-, d_1, d_2, d_3, d_3^c, d_2^c, d_1^c, l^+, u_3^c, u_2^c, u_1^c, N\right) \ , \tag{1.28}$$

and we identify N as the Right-handed (conjugated) neutrino. This means that $SO(10)$ is also a theory of neutrino masses, which makes this GUT very attractive.

The Higgs content gets further expanded as there are no $\overline{\mathbf{5}}$ representations of $SO(10)$. The SM Higgs will live in a **10**

$$\mathbf{10} = \mathbf{5} \oplus \overline{\mathbf{5}} \ , \tag{1.29}$$

which means it will be partnered with another doublet and two triplets

$$\mathbf{10} = H_u \oplus H_d \oplus T \oplus \overline{T} \ . \tag{1.30}$$

This means that $SO(10)$ is also a Double Higgs model. In a Supersymmetric formulation, which we will present in the next section, this is a good feature as we need to consider two distinct Higgs fields of opposite Hypercharge that come for free in $SO(10)$. In addition, we would have to consider a GUT-scale Higgs mechanism to break the GUT group down to the SM. For $SO(10)$ this is achieved by two GUT-Higgs fields, an adjoint **45** and a **16** scalars. The first preserves the rank, and so it does not break $SO(10)$ down to the SM gauge group, while the second scalar can break the rank. These two scalars provide a plenitude of different breaking path patterns from $SO(10)$ down to the SM, which provides rich model-building phenomenology. A special feature of these braking patterns is that the rank-preserving breaking patterns will have an explicit gauged $B-L$ symmetry, which is a subgroup of $SO(10)$. This feature provides some insight into why the SM has global accidental B and L symmetries.

Since $SO(10)$ is a larger symmetry group than $SU(5)$, we expect the Yukawa couplings to be further constrained. Indeed, the Yukawa couplings are now schematically

$$\mathcal{L} \supset Y^{ij}\mathbf{16}^i\mathbf{16}^j\mathbf{10} \ , \tag{1.31}$$

which imply

$$Y_u^{ij} = Y_d^{ij} = Y_l^{ij} = Y_\nu^{ij} \ . \tag{1.32}$$

This prediction is in complete disagreement with low-energy data, as $Y_u = Y_d$ would lead to the CKM matrix being too close to the identity matrix.

Just like with $SU(5)$, we find that the minimal $SO(10)$ models have many challenges, although they do provide an attractive playground for model building. In fact, every minimal GUT will be, in one way or another, in contradiction with some experimental bound. This means that realistic GUT models will need to be some variation of the models presented above. We will see that String Theory provides a more general framework for GUT model building, where extra ingredients and dynamics provide natural solutions for minimal GUT models.

1.3 Supersymmetry

It is time to review Supersymmetry [8–10] (SUSY). SUSY is an extension of the Lorentz-Poincaré symmetry group of space-time relating fermions and bosons. This happens as SUSY is described by a set of new fermionic generators in order to circumvent the Coleman-Mandula theorem [67]. If we take these new generators to be schematically Q, SUSY then allows for the transformation between fermionic and bosonic states

$$Q|boson\rangle = |fermion\rangle \qquad Q|fermion\rangle = |boson\rangle \tag{1.33}$$

and so we have a new set of irreducible representations that include bosonic and fermionic degrees of freedom, called supermultiplets.

In more detail, in four dimensions, minimal SUSY accounts for one copy, $\mathcal{N} = 1$, of two spinor generators, Q and $\overline{Q}$, such that they extend the Lorentz-Poincaré algebra

$$\{Q_\alpha, \overline{Q}_{\dot{\alpha}}\} = 2\sigma^\mu_{\alpha\dot{\alpha}} P^\mu \ , \tag{1.34}$$

where α, $\dot{\alpha}$ are indices for the Left- and Right-handed Weyl two-spinor representations, respectively, and $\sigma^\mu = (1, \sigma^i)$, P^μ the linear momentum generator, and

$$\{Q_\alpha, Q_\beta\} = \{\overline{Q}_{\dot{\alpha}}, \overline{Q}_{\dot{\beta}}\} = [Q_\alpha, P^\mu] = [\overline{Q}_{\dot{\alpha}}, P_\mu] = 0 \ . \tag{1.35}$$

While SUSY is a crucial theoretical tool and an integral part of Superstrings, we will motivate it from a particle physics model building perspective. The fact that fermions and bosons share the same supermultiplet, together with the fact that the Q-generators commute with the momentum operator P_μ, mean that the fermionic and bosonic degrees of freedom in a certain supermultiplet share the same mass. Furthermore, we know that a fermion, ψ, has quantum corrections to its mass, m_ψ, that are logarithmic with the cut-off scale, Λ, schematically

$$\Delta m_\psi \propto m_\psi \log \Lambda / m_\psi \ , \tag{1.36}$$

and so the fermionic masses are naturally protected against quantum corrections. Therefore, if the SM Higgs doublet has a Supersymmetric partner, Higgsino, with which form a supermultiplet, then the scalar potential mass term, μ^2 would be protected against quantum corrections as well, providing a solution to the hierarchy problem.

In the following sub-sections we will go through all the main results on how to construct Supersymmetric theories. We will also provide the main results from Supergravity (SUGRA), before we finish the section with a quick description of the Minimal Supersymmetric Standard Model (MSSM).

1.3.1 $\mathcal{N} = 1$ Global SUSY Lagrangians

As mentioned above, in SUSY we have supermultiplets that contain fermionic and bosonic degrees of freedom. For example, a SM fermion, ψ, would be in a multiplet composed of

$$(\Phi, \psi, F) \tag{1.37}$$

where Φ is the scalar partner, and F an off-shell auxiliary field with mass dimension two.

A very convenient way of representing SUSY is to use what is called the superspace formalism. In this formalism we consider space-time, with coordinates x^μ, to be as if extended by two grassmanian coordinates, θ and $\overline{\theta}$, such that the superspace coordinates read

$$(x^\mu, \theta_\alpha, \overline{\theta}_{\dot{\alpha}}) \tag{1.38}$$

and the grassmanian coordinates are anti-commuting

$$\{\theta_\alpha, \theta_\beta\} = 0 \ , \tag{1.39}$$

which also means that each component of θ and $\overline{\theta}$ is nilpotent

The main reason the superspace formalism is so appealing to formulate SUSY theories relies on integration properties of the grassmanian coordinates. We have

$$\int d\theta_\alpha = 0 \ , \tag{1.40}$$

where the integral formally covers the full range of θ_α coordinate. In addition, integration and derivation are interchangeable for grassmanian coordinates

$$\int d\theta_\alpha \theta_\alpha = \frac{\partial \theta_\alpha}{\partial \theta_\alpha} = 1 \ . \tag{1.41}$$

With the characteristics above, it is possible to construct superspace covariant derivatives

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}}\overline{\theta}^{\dot{\alpha}}\partial_\mu \qquad \overline{D}_{\dot{\alpha}} = -\frac{\partial}{\partial \overline{\theta}^{\dot{\alpha}}} - i\theta^\alpha \sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu \ , \tag{1.42}$$

which commute with SUSY transformations. The relevance of these SUSY covariant derivatives is that they allow us to construct irreducible representations of the SUSY algebra in the superspace formalism. For example, we could start with the most general superfield, $S(x, \theta, \overline{\theta})$, by expanding it in the grassmanian coordinates. This expansion would be finite due to the nilpotency of the grassmanian coordinates, but would still have too many components when compared to irreducible representations of the SUSY algebra. By applying constraints on a general superfield, S, we reduce the number of components, and by which we obtain an irreducible representation.

There are two types of supermultiplets that are of interest and can be constructed as described above: the chiral superfield and the vector superfield. The former will include the SM fermions and Higgs, while the latter will include the SM vector bosons.

The chiral superfield, Φ, is a function on superspace that respects

$$\overline{D}_{\dot{\alpha}} \Phi = 0 \ . \tag{1.43}$$

In the Wess-Zumino gauge it can be written as

$$\Phi(x, \theta, \overline{\theta}) = \Phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \ , \tag{1.44}$$

where $y^\mu = x^\mu + i\theta\sigma^\mu\overline{\theta}$ and we notice that we use the same symbol for the superfield and its scalar component.

The vector superfield is defined as a real function in superspace

$$V = V^\dagger \tag{1.45}$$

and in the Wess-Zumino gauge it will have the θ expansion

$$V(x, \theta, \overline{\theta}) = -\theta\sigma^\mu\overline{\theta}A_\mu(x) + i\theta\theta\overline{\theta}\overline{\lambda}(x) - i\overline{\theta}\overline{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\overline{\theta}\overline{\theta}D(x) \ , \tag{1.46}$$

where each component is valued in the Lie algebra of the symmetry group mediated by A_μ. The fields λ and $\overline{\lambda}$ form an on-shell Majorana spinor, while D is an off-shell auxiliary field, analogous to the F field for the chiral superfield. The corresponding gauge strength-field tensor, $F_{\mu\nu}$, is obtained from the chiral superfields defined as

$$\mathcal{W}_\alpha = -\frac{1}{4}\overline{D}\overline{D}e^{-V}D_\alpha e^V \ , \tag{1.47}$$

where it is possible to show $\overline{D}_{\dot{\alpha}}\mathcal{W}_\alpha = 0$.

Defining the algebraic expressions to derive the so-called F- and D-terms

$$[X]_F = \int d^2\theta X \qquad\qquad [X]_D = \int d^2\theta d^2\overline{\theta} X \tag{1.48}$$

a generic global $\mathcal{N} = 1$ SUSY Lagrangian takes the form

$$\mathcal{L} = \frac{1}{16\pi i}\left([g_a^2\tau_a\mathcal{W}_a^\alpha\mathcal{W}_{a\alpha}]_F + \text{h.c.}\right) + [K(\Phi^{*i}, \tilde{\Phi}_i)]_D + ([W(\Phi_i)]_F + \text{h.c.}) \ , \tag{1.49}$$

where τ_a is a holomorphic coupling, g_a the coupling of the gauge symmetry labelled with a, K is the Kähler potential, which is a real function of mass dimension two, with

$$\tilde{\Phi}_i = (e^{2g_aT^aV^a})_j^{\ i}\Phi_i \ , \tag{1.50}$$

and W is the superpotential, a holomorphic function of superfields with mass dimension three.

Expanding the superfields in the grassmanian variables and performing the integrations, we find that the auxiliary fields D and F are not dynamical and appear only linearly and bilinearly. Integrating them out, by solving for their equations of motion, we find that the scalars have the potential

$$V(\Phi_i) = V_F + V_D = |F_i|^2 + \frac{1}{2}|D^a|^2 = \left|\frac{\partial W}{\partial\Phi_i}\right|^2 + \frac{g_a^2}{2}|\Phi^{*j}T^{ai}_{\ \ j}\Phi_i|^2 \ , \tag{1.51}$$

where T^a and g_a are generators and the couplings of the gauge symmetries.

1.3.2 SUSY Breaking

The above construction is SUSY invariant, but since we do not observe Supersymmetric partners with the same masses as the SM fields, we know that SUSY has to be broken at low energies. Just like any symmetry, SUSY is broken if the vacuum state is not annihilated by the generator

$$Q_\alpha|0\rangle \neq 0 \ . \tag{1.52}$$

Furthermore, from the SUSY algebra we know that we can write the Hamiltonian as

$$H = P^0 = \frac{1}{4}\left(\overline{Q}_1Q_1 + Q_1\overline{Q}_1 + \overline{Q}_2Q_2 + Q_2\overline{Q}_2\right) \tag{1.53}$$

and therefore

$$\cancel{SUSY} \Rightarrow \langle H \rangle \neq 0 \ , \tag{1.54}$$

meaning that the vacuum expectation value of the Hamiltonian reads

$$\langle H \rangle = \langle V_F + V_D \rangle \neq 0 \ , \tag{1.55}$$

which translates to non-vanishing F- and D-terms

$$\begin{aligned} \langle F_i \rangle &\neq 0 \\ \langle D^a \rangle &\neq 0 \ . \end{aligned} \tag{1.56}$$

It is interesting to see that, since the scalar potential is positive semi-definite, broken SUSY also means a non-vanishing contribution to the cosmological constant. Therefore, while unbroken SUSY could explain the smallness of the cosmological constant, the observational necessity for SUSY to be broken makes things worse.

1.3.3 $\mathcal{N} = 1$ Supergravity

The most robust symmetries in nature are gauged. SUSY can be gauged, meaning its transformations can be made local, into what is known as Supergravity (SUGRA). Here we present the very generic results and formulae without going into detail on how to derive them. For our purpose it suffices to note that in SUGRA the graviton is in a supermultiplet with a spin 3/2 fermionic partner called gravitino. The gravitino plays the role of the gauge field in the sense that it is defined up to a shift arising from derivative terms upon a SUGRA transformation of the Lagrangian.

Fortunately, SUGRA can be written in the superspace formalism described above. All the interactions between chiral fields are described by a real function called Kähler function

$$G(\Phi_i, \Phi^{*i}) = \frac{1}{m_{Pl}^2} K(\Phi_i, \Phi^{*i}) + \log \left| \frac{W(\Phi_i)}{m_{Pl}^3} \right|^2 \ , \tag{1.57}$$

where m_{Pl} is the reduced Planck mass, and K and W are the already defined Kähler potential and superpotential. The full SUGRA theory is invariant under the shift

$$K \to K + f + \bar{f} \ , \tag{1.58}$$

where f $(\bar{f})$ is a holomorphic (antiholomorphic) function of chiral fields.

The F-term scalar potential in SUGRA is

$$V_F = F_i K^i{}_j F^{*j} - 3e^{\frac{K}{m_{Pl}^2}} \frac{|W|^2}{m_{Pl}^2} , \tag{1.59}$$

where we identify the Kähler metric

$$K^i{}_j = \frac{\partial^2 K}{\partial \Phi_i \partial \Phi^{*j}} \tag{1.60}$$

and the SUGRA version of the F-term

$$F_i = -e^{\frac{K}{2m_{Pl}^2}} (K^{-1})_i^j \left(W_j^* + \frac{1}{m_{Pl}^2} W^* K_j \right) , \tag{1.61}$$

where $(K^{-1})_i^j$ is the inverse of the Kähler metric.

The F-term scalar potential explicitly exhibits the global SUSY limit when we decouple gravity, $m_{Pl} \to \infty$. We can then conclude that the second term in the F-term potential is a special feature of SUGRA, and the F-term potential is no longer positive semi-definite as in the global case.

From the full SUGRA Lagrangian, not shown here, the gravitino mass can be derived to take the form

$$m_{3/2} = e^{\frac{K}{2m_{Pl}^2}} \frac{|W|}{m_{Pl}^2} , \tag{1.62}$$

which means that imposing $V = 0$ at the minimum of the theory, the gravitino mass becomes an order parameter for SUSY breaking

$$V = 0 \Rightarrow m_{Pl}^2 = \frac{\langle F_i K^i{}_j F^{*j} \rangle}{3m_{Pl}^2} . \tag{1.63}$$

The D-term contribution to the scalar potential is also modified with the presence of Gravity, but retains a more familiar form

$$V_D = \frac{1}{2} [\mathrm{Re} f^- 1]^{ab} (K_i T^{ai}{}_j \Phi^j)^2 (K_k T^{bk}{}_j \Phi^l)^* , \tag{1.64}$$

where f is gauge kinetic function. This is an upgraded version of the holomorphic coupling τ_a, in the sense that it is holomorphic on chiral fields and can, in principle, mix $U(1)$ factors. Again, taking $m_{Pl} \to \infty$ returns the D-term scalar potential we encountered before in the global case.

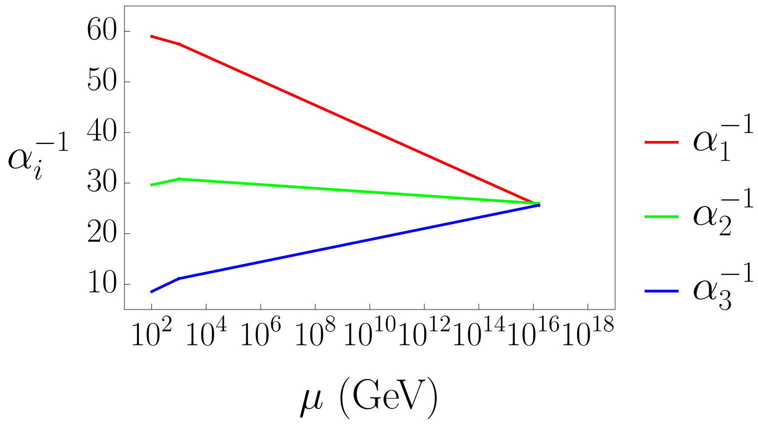


Figure 1.2: Unification of the gauge couplings in the Minimal Symmetric Standard Model

1.3.4 The Minimal Supersymmetric Standard Model

To finish our lightning review on SUSY we briefly introduce the Minimal Supersymmetric Standard Model (MSSM). The matter content from the SM will be extended by the Supersymmetric partners. The SM fermions will have a complex scalar partner, which we refer to by the same name as the SM state with the prefix *s*-, e.g. a fermion is partnered with a sfermion. The gauge bosons will have a Majorana fermion partner, which we refer to by the same name as the vector boson with the added suffix *-ino*, e.g. the B^μ Hypercharge boson is partnered by the Bino. The Higgs scalar will also have a chiral fermionic partner, denoted Higgsino.

The content above has a couple of issues. The first is since in the SM the complex conjugate of the Higgs is used to generate the up-type Yukawas, holomorphicity of the superpotential will not allow these. Furthermore, the addition of a new fermion with the Higgs gauge quantum numbers will make the spectrum exhibit both a chiral and a Witten [68] anomaly. The solution for these issues is to allow for a new Higgs doublet with opposite Hypercharge than the SM Higgs. The two Higgses are then distinguished by their Yukawa couplings: H_u for the up-type Yukawa couplings, and H_d for the down-type Yukawa couplings. The superpotential, W, for the MSSM reads

$$W = \mu H_u H_d + Y_u^{ij} H_u Q^i u^{cj} + Y_d^{ij} H_d Q^i d^{cj} + Y_e^{ij} H_d L^i e^{cj} \ . \tag{1.65}$$

With the full spectrum enumerated above, the MSSM provides a compelling unification scenario, as can be seen in Figure 1.2.

There are, however, some issues with the simple picture drawn above.

First, if we consider only the SM symmetry group, then there should be no reason to have ignored the following B and L violating terms

$$W_{\not{R}} = \frac{1}{2}\lambda L L e^c + \lambda' L Q d^c + \kappa H_u L + \frac{1}{2}\lambda'' u^c u^c d^c \ , \tag{1.66}$$

where we dropped the family indices for readability. The first three terms break L number, as they evidence a difficulty in distinguishing H_d from a Lepton doublet L, while the last term breaks B number. These terms break what we call R-parity, or equivalently matter-parity that is defined as

$$P_M = (-1)^{3(B-L)} \tag{1.67}$$

and therefore matter superfields transform with -1, while the Higgses transform with $+1$. The MSSM requires this symmetry to be present *ad hoc*, while a more complete theory should provide an explanation. For example, it is clear from the definition of P_M that a theory incorporating $B-L$ symmetry, like an $SO(10)$ GUT, will be effectively endowed with matter parity.

Another challenge of the MSSM is that, while providing an explanation of why the Higgs μ term is protected against quantum corrections, it does not offer an explanation for its tree-level value. This is called the μ-problem, and we expect a more complete theory to provide a suitable prediction for this parameter.

The phenomenology of the MSSM, and small variations thereof, is an active area of research. We are mainly interested in producing the ingredients of the MSSM from String Theory while offering solutions for its shortcomings, in order to provide semi-realistic models of particle physics.

1.4 String Theory Phenomenology

Having presented two of the most famous extensions of the SM, we are now in place to present a broader framework where they naturally arise: String Theory.

String Theory [14–18], the theory of quantum mechanical relativistic extended objects, is arguably the best candidate for a theory of quantum gravity. Indeed, one of its main features is that it *post*dicts gravity by providing a consistent quantum description and its classical limit, as the graviton arises as an oscillation mode of the closed String.

While an impressive success, the merits of String Theory are not restricted to its description of quantum gravity. In fact, String Theory provides all the ingredients and building blocks required to describe our universe, e.g. non-Abelian gauge theories and fermions. In addition, String Theory provides a broad framework for BSM physics as it naturally incorporates ideas such as GUTs, SUSY, extra-dimensions, etc and as such it is a logical step to study BSM models arising from String Theory, in what we call String Phenomenology [19–21].

String Theory is an extensive subject and many excellent reviews and lectures can be found in the references already provided. In this section we will only enumerate the highlights and results of String Theory in order to contextualise the modern developments on M- and F-Theory.

In 1984, following the work by Green and Schwarz [69], it was understood that there are only five consistent Superstring theories, which we detail in Table 1.3, marking the dawn of the First Superstring Revolution. By perturbative we mean that the String coupling constant, g_s, is small and that fundamental degrees of freedom are freely propagating strings. All of them live in ten-dimensional space-time, as required by consistency, and exhibit space-time SUSY. In order to make contact with our universe, where we experience a four-dimensional space-time, we need to account for the extra six dimensions. This introduces the notion of compactification, where we consider the extra six dimensions to be compact and having characteristic dimensions small enough to have escaped observation until now. The general picture is then that at large distances we cannot perceive the small extra dimensions

$$\mathbb{R}^{1,9} \to \mathbb{R}^{1,3} \times X \ , \tag{1.68}$$

where $\mathbb{R}^{1,d-1}$ are Minkowski d-dimensional space-times, and X is a compact six-dimensional manifold. The geometric and topological details of X define the four-dimensional effective field theory, which is obtained through Kaluza-Klein reduction. For example, looking at Table 1.3 we see that the different perturbative regimes have either 16 or 32 supercharges, while $\mathcal{N} = 1$ SUSY in four-dimensions has four supercharges. In order to reduce the amount of low-energy SUSY, X needs to be a so-called Calabi-Yau (CY) manifold, which is an $SU(3)$ Holonomy Kähler manifold. If X is a CY then only a quarter of the ten-dimensional supercharges survive, which leads to $\mathcal{N} = 1$ for CY compactifications of the Type-I and Heterotic regimes. This

Theory	Supercharges	String Content	Bosonic Spectrum	
Heterotic $E_8 \times E_8$	16 ($\mathcal{N}=1$)	Closed		ϕ,B_{MN}, G_{MN} A_M^{ij}
Heterotic $SO(32)$	16 ($\mathcal{N}=1$)	Closed		ϕ,B_{MN}, G_{MN} A_M^{ij}
I	16 ($\mathcal{N}=1$)	Closed and Open	NS-NS	ϕ, G_{MN}, A_M^{ij}
			R-R	B_{MN}
IIA	32 ($\mathcal{N}=2$)	Closed	NS-NS	ϕ,B_{MN}, G_{MN}
			R-R	C_M, C_{MNP}
IIB	32 ($\mathcal{N}=2$)	Closed	NS-NS	ϕ, B_{MN}, G_{MN}
			R-R	C, C_{MN}, C_{MNPQ}

Table 1.3: All the known perturbative and consistent String-Theory regimes. NS stands for Neveau-Schwarz, R stands for Ramond. The indices run through the ten dimensions. The common spectrum is the Dilaton ϕ, a 2-form B_{MN}, and the graviton G_{MN}. All fields C are n-forms, with n given by the amount of indices.

fact has made the Heterotic regimes phenomenologically attractive, which has led to extensive and on-going research.

While the perturbative regimes presented in Table 1.3 provide a rich framework for both formal and phenomenological work, there are other objects that do not possess a perturbative description – the so-called D-branes. These are extended objects where String endpoints can end, hence D for Dirichlet in accordance to the Dirichlet boundary condition imposed on String endpoints.

Although naturally motivated by open String endpoints, D-branes are also present in regimes that only have closed strings. The presence of n-forms indicates that are conserved electric and magnetic charges sourcing the n-forms on higher-dimensional objects, which we identify as D-branes. Following the Dirac quantisation relation we find that an n-form is electrically and magnetically coupled to $p = n - 1$ and $p = 7 - n$ dimensional D-branes, respectively. This means that the Type-II regimes have extended Branes despite only allowing for free closed strings.

For many years D-branes were disregarded, as non-free String endpoints would not preserve Lorentz invariance. Nowadays, however, we take them as fundamental non-perturbative objects that are at the core of the most recent String Theory developments.

1.4.1 Non-perturbative limits: F- and M-Theory

Above we have introduced the highlights of perturbative String Theory regimes, and now it is time to present the non-perturbative regimes. We are specially interested to study those arising from non-perturbative limits of Type-IIA and Type-IIB, known as M- and F-Theory, respectively. For a modern pedagogical study on how these vacua are related see [70].

M-Theory

In 1995, Witten [24], in the sequence of previous studies on 11-dimensional SUGRA [71, 72] noticed that an 11-dimensional SUGRA

$$S_{11d} = \frac{1}{2\kappa_{11}^2} \int d^{11}x\sqrt{-G}(R - \frac{1}{2}|G_4|^2) - \frac{1}{6} \int C_3 \wedge G_4 \wedge G_4 \ , \qquad (1.69)$$

where $G_4 = dC_3$, C_3 is a 3-form, G the 11-dimensional metric with Ricci scalar R, and κ some dimensionful constant related to the 11-dimensional Planck mass, can be dimensionally reduced to Type-IIA and Heterotic $E_8 \times E_8$ if compactified on S^1 or S^1/Z_2, respectively. The size of the new eleventh dimension will be proportional to the String coupling constant, and as such it is not visible in the perturbative limit.

The theories are then speculated to be dual, meaning that we can use each to infer results on another theory in certain regimes that would otherwise be impossible to probe. The emergence of dualities relating different types of String Theory regimes is so powerful that it was branded the Second Superstring Revolution.

This 11-dimensional SUGRA theory is called M-Theory. It has 32 Supercharges and cannot be microscopically described as a String Theory, although it allows for $M2$- and $M5$-Branes that are lifts of the Type-IIA String and D4-Brane, respectively.

In order to make a connection between M-Theory and the real world we have two options: to either go through the perturbative limit and work with a perturbative String regime, or to look for a compactification scenario starting from the full 11-dimensional theory. Since the former choice leads to the perturbative regimes, an exciting new branch of String Phenomenology awaits us on the latter case.

Just like CY manifolds are paramount to obtain $\mathcal{N} = 1$ four-dimensional effective actions from Heterotic strings, for M-Theory we need to find what family of seven-dimensional compact manifolds can provide similar semi-realistic compactifications. It happens that in order to have $\mathcal{N} = 1$ four-dimensional theories, M-Theory has to be compactified on a manifold with G_2 holonomy [73] and, while we still do not have explicit examples with all the required features [74, 75], there has been considerable progress on deriving phenomenologically viable solutions. We will review and apply some general results of M-Theory vacua on G_2-manifolds in Chapters 2 and 3.

F-Theory

In 1996, Vafa [25] suggested that a similar non-perturbative regime could be found in Type-IIB String Theory. This opens up the possibility for a class of non-perturbative compactifications which we call F-Theory. These are characterised by exploring cases where the dilaton, ϕ, is not constant and therefore the String Coupling constant is not fixed. In more detail, the dilaton and the RR 0-form, C, can be combined into a complex field

$$\tau = C_0 + ie^{-\phi} \ , \tag{1.70}$$

which transforms non-linearly under $SL(2, \mathbb{Z})$ just like the modular parameter of a torus

$$\tau \to \frac{a\tau + b}{c\tau - d} \tag{1.71}$$

with $ad - bc = 1$. Under these transformations, the two 2-forms transform as a doublet, while everything else is invariant.

The fact that τ seemingly behaves as a torus has led to the description of F-Theory as a 12-dimensional SUGRA. Although there is no sensible 12-dimensional SUGRA, this geometric description has been proven to be very useful in making a connection with the real world. In particular, we can take this 12-dimensional theory to be compactified on an eight-dimensional CY space with an elliptical fibration, which will only preserve $\mathcal{N} = 1$ SUSY in four dimensions, thus providing a phenomenologically rich new scenario. For reviews on F-Theory model building we refer to [76–79] and to the introductions in Chapters 4 and 5.

The Web of Dualities

Since the first proposals for M- and F-Theory, there has been a huge development on connecting all different regimes of String Theory with dualities. This has led to the all-encompassing picture: what once was thought as many independent possibilities, different String Theories are in fact just different manifestations of the same underlying theory as can be seen in Figure 1.3.

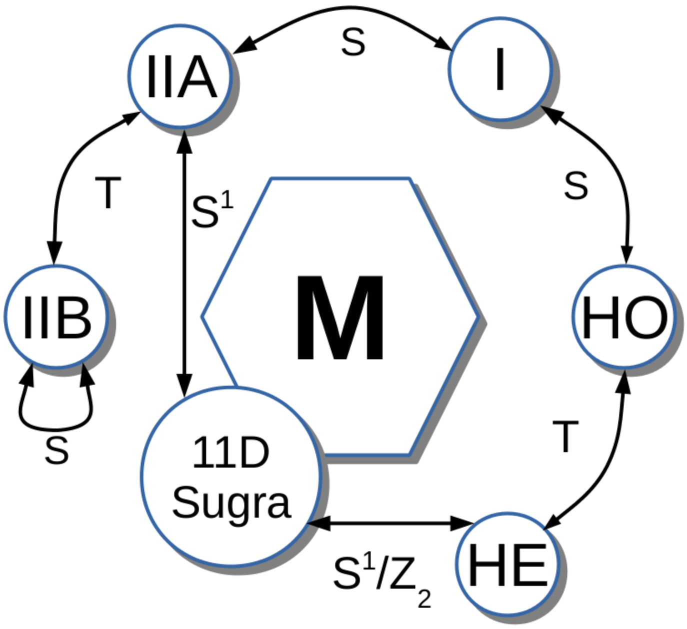


Figure 1.3: The web of dualities in String Theory. T and S refer to T and S dualities, respectively. S^1 refers to a compactification on a circle, while S^1/Z_2 refers to a compactification on the circle with antipodal points identified.

Chapter 2

$SO(10)$ SUSY GUT from M Theory on G2

The hierarchy or naturalness problem which is the question, *what physics stabilises the electroweak scale of the Standard Model (SM) to be so low ($\mathcal{O}$(100 GeV))?*, has become even sharper after run I of the CERN Large Hadron Collider (LHC). A remarkably Standard Model-like Higgs boson was discovered with a mass around 125 GeV, with no evidence for new physics beyond the Standard Model [5–7]. Supersymmetry in principle solves this problem, but the limits from run I of the LHC can be of order a few TeV for the superparticle masses [11–13], whilst naturalness arguments suggest that such particles would have been seen by now.

Since many of the results and limits from LHC searches have been in the context of the Minimal Supersymmetric Standard Model (MSSM) or very special, simple, subsets of the MSSM parameter space, one could ask: what limits would we obtain in more general supersymmetric models? However, since there are literally infinite choices to be made in constructing such models, we might first ask, what reasonable guides do we have to go beyond the MSSM?

Results over the past decade or so have shown that the simple combination of supersymmetry breaking moduli stabilisation and string/M theory can in fact be a very useful guide to constructing models [19,26–29]. Namely, the progress in understanding supersymmetry breaking and moduli stabilisation in string/M theory has been shown to lead to effective models with distinctive features and very few parameters.

One is thus led to consider supersymmetric grand unified theories (GUTs) based on simple groups, such as $SU(5)$ which explain the fermion quantum numbers and unify the three Standard Model forces, in the string/M theory context. In doing so, however, we have to face the basic problem of GUTs – the Higgs doublet-triplet splitting problem: the Standard Model Higgs doublet is unified into a GUT multiplet containing colour triplets which can mediate proton decay too quickly. In many models, including those originating in string/M theory, this problem is often solved by making the colour-triplets very massive [64–66], something often achieved with a discrete symmetry whose effective action on the triplets is different from that on the doublets.

The main purpose of this chapter is to extend the scope of the M theory approach from the previously considered $SU(5)$/MSSM case arising from M theory on G_2 manifolds [27, 80] to $SO(10)$, where an entire fermion family Q, u^c, d^c, L, e^c, N, including a charge conjugated right-handed neutrino N, is unified within a single **16** representation denoted **16**. In particular we focus on the Higgs doublet-triplet splitting problem, whose solution turns out to be necessarily quite different in the $SO(10)$ case, leading to distinct phenomenological constraints and predictions. In the remainder of this chapter, we first review some basic ideas and results from M theory, followed by their application in the $SU(5)$/MSSM context, before embarking on a discussion of the new $SO(10)$ case.

M theory on a manifold of G_2 holonomy [1] leads elegantly to four dimensional models with supersymmetry. In such models, both Yang-Mills fields and chiral fermions arise from very particular kinds of singularities in the extra dimensions [83, 84]. Yang-Mills fields are localised along three-dimensional subspaces of the seven extra dimensions along which there is an orbifold singularity. Chiral fermions, which couple to these Yang-Mills fields, arise from additional localised points at which there is a conical singularity. Therefore, different GUT multiplets are localised at different points in the extra dimensions. The GUT gauge group can be broken to $SU(3) \times SU(2) \times U(1)$ (possibly with additional $U(1)$ factors) by Wilson lines on the three-dimensional subspace supporting the gauge fields. Compact manifolds of G_2 holonomy – being Ricci flat and having a finite fundamental group – can not have continuous symmetries, but could have discrete symmetries. If present, such symmetries play a very important role in the physics. In particular, for the $SU(5)$ case, Witten showed that such symmetries can solve the doublet-triplet splitting problem [80].

[1] Constructions of compact G_2 holonomy manifolds are described in [81, 82].

The reminder of this chapter is organised as follows. In the next section we will review all relevant data and low-energy constraints for phenomenology from M Theory with G_2 vacua for. In Section 2.2 we present the G_2-MSSM, an $SU(5)$ realisation. Next, in Section 2.3 we present our proposed model in [1] for an $SO(10)$ model in this framework. Finally, in Section 2.4 we conclude and discuss.

2.1 Review of M Theory with G_2 vacua

M Theory compactified on a G_2-manifold leads to a 4 dimensional theory with $\mathcal{N} = 1$ SUSY, where gauge fields and chiral fermions are supported by different types of singularities in the compactified space [83, 84]. Yang-Mills fields are supported in three dimensional subspaces of the compactified space, along which there is an orbifold singularity, while chiral fermions will be further localised on conical singularities localised on these three dimensional spaces and interact with the gauge fields.

One of the key features of M Theory compactified on G_2-manifolds without fluxes is that it provides a hierarchical framework. To understand the reason behind this notice that in M Theory, the moduli fields, s_i, are paired with the axions, a_i, in order to form a complex scalar component of a superfield Φ_i

$$\Phi_i = s_i + ia_i + \text{fermionic terms} \ . \tag{2.1}$$

In the absence of fluxes, the axions enjoy an exact shift-symmetry, which is remnant of the higher dimensional gauge symmetry,

$$a_i \to a_i + c_i \tag{2.2}$$

where c_i is an arbitrary constant. This Peccei-Quinn symmetry, in conjugation with holomorphicity of the superpotential, prevents a perturbative superpotential for the moduli. As such, terms which are polynomial in the moduli and matter fields are forbidden at tree-level in superpotential, appearing only in the Kähler potential.

Since non-perturbative effects break the above shift symmetry, the superpotential involving moduli and matter is not vanishing. Interactions will be generated by membrane instantons, whose actions are given by exponentials of the moduli. As the moduli stabilise and acquire vevs, these exponentials will turn out to be small, and the vev of the hidden sector superpotential naturally leading to an hierarchical generation of masses, as we will see below.

It was shown before [26, 27, 85] that the dynamics of a strongly coupled hidden sector can stabilise the moduli and spontaneously break supersymmetry. To do so, we consider that there are two different hidden sectors localised on two different 3-dimensional subspaces of the compactified space. Each of these spaces support a different (asymptotically free) non-abelian gauge group. One of the hidden sectors is assumed to supports a light (massless) vector-like pair matter on appropriate conical singularities, which is effectively described by a meson field ϕ.

At low energies the hidden sector dynamics is strongly coupled, generating the non-perturbative superpotential[2]

$$W_{hid} = m_p^3 \left(C_1 P \phi^{-2/P} e^{i b_1 f_1} + C_2 Q e^{i b_2 f_2} \right) \tag{2.3}$$

where m_p is the 4-dimensional Planck mass, C_1, C_2 are normalisation constants defined by the specific geometry of the G_2 manifold, f_1, f_2 are the gauge kinetic functions of the two hidden sectors, and $b_1 = \frac{2\pi}{P}$, $b_2 = \frac{2\pi}{Q}$ such that effectively the theory above is describing by $SU(Q) \times SU(P+1)$ gauge theory where ϕ is composed of chiral superfields charged under $SU(P+1)$.

Under the assumption that both hidden sector gauge fields lie on the same homology class, in that case the kinetic functions are the same, $f_1 = f_2 = f_{hid}$, reading

$$f_{hid} = \sum_i^N N_i (i s_i + a_i) \tag{2.4}$$

where s_i are the N geometric moduli of the G_2-manifold, a_i the axions originated from the KK reduction of the 11-dimensional 3-form, and N_i are integers specified by the homology class of the hidden sector 3-cycles. It is clear from the above expression that as the moduli acquire a high-scale vev, the superpotential will be vanishingly small in comparison to the Planck scale.

The theory is only fully described when provided with a Kähler potential. While its form is hard to derive from first principles, the fact that chiral supermultiplets are localised in 3-dimensional subspaces, leads us to make

[2]In this section the meson field is not canonically normalised and is massless, hence the factor m_p^3 in the expression.

an educated guess that the Kähler potential takes the form

$$\hat{K}/m_p^2 = -3 \ln \left(4\pi^{1/3} V_7\right) + \overline{\phi}\phi \tag{2.5}$$

where V_7 is the volume of the compactified space and can be parametrised by $V_7 = \prod_{i=1}^{N} s_i^{n_i}$, where n_i are positive integers constrained by $\sum_i^N n_i = 7/3$.

It was shown [26, 85] that in the above scenario all the moduli s_i are stabilised and acquire a vev $\langle s_i \rangle \simeq \mathcal{O}(0.1) m_p$, and the vacuum corresponds to de Sitter 4-dimensional space-time with vanishingly small cosmological constant. Furthermore, the original $\mathcal{N} = 1$ SUSY is spontaneously broken by the moduli vevs. We also note that in these vacua, the F-term of the meson field, F_ϕ, can be found to be greater than the moduli F-terms, $F_{s_i} \simeq \mathcal{O}(0.01) m_{3/2} m_p$.

The values for the parameters N, P, Q, N_i, n_i, C_1, C_2 completely specify the compactification and the 4-dimensional theory, but can only be computed from first principles if we are given an explicit example of a compact G_2-manifold. Since explicit examples with singularities are still unknown, the discussion presented here provides a definition of a framework for compactifications of M theory on G_2-manifolds without fluxes.

Visible matter is localised on a different 3-dimensional subspace, where the GUT group is supported. Previous work with the GUT group $SU(5)$ has been developed [86], and the scope of this work is to extend the framework to $SO(10)$. The full 4-dimensional $\mathcal{N} = 1$ supergravity theory is then described at the GUT scale by

$$K/m_p^2 = \hat{K}/m_p^2 + \tilde{K}_{\overline{\alpha}\beta}(s_i)\overline{\Phi}^{\overline{\alpha}}\Phi^\beta + \left(Z(s_i)_{\alpha\beta}\Phi^\alpha\Phi^\beta + \text{h.c.}\right) + \mathcal{O}(\Phi^3) \tag{2.6}$$

$$W = W_{hid} + Y'_{\alpha\beta\gamma}\Phi^\alpha\Phi^\beta\Phi^\gamma \tag{2.7}$$

where Φ are visible chiral superfields, $Y'_{\alpha\beta\gamma}$ the un-normalised Yukawa couplings, and the Kähler potential involving visible matter is expanded to leading order in visible superfields.

The un-normalised Yukawas, $Y'_{\alpha\beta\gamma}$ are given by non-perturbative effects from membrane instantons action on the 3-dimensional subspace where the superfields Φ^α, Φ^β, Φ^γ are supported. More explicitly, the trilinear couplings take the form

$$Y'_{\alpha\beta\gamma} \simeq C_{\alpha\beta\gamma} e^{i2\pi \sum_i l_i^{\alpha\beta\gamma}(is_i + a_i)} \tag{2.8}$$

where $C_{\alpha\beta\gamma}$ are complex numbers with $\mathcal{O}(1)$ magnitude, and $l_i^{\alpha\beta\gamma}$ are integers characterizing (the homology class of) the 3-cycle encapsulating the three singularities supporting the chiral supermiltiplets Φ^α, Φ^β, Φ^γ. This means that the values for the Yukawas are parametrised by the exponential of the volume of these 3-cycles in the GUT 3-dimensional subspace of the internal space. In principal one needs an explicit example of a compact G_2 manifold in order to compute $l_i^{\alpha\beta\gamma}$, which makes these numbers in practice not computable. Instead, one can use low-energy data to parametrize these data of the compact manifold.

So far we presented the vacua framework for M theory on a G_2-manifold with $\mathcal{N} = 1$ SUSY. Below we will show how moduli vev spontaneously break SUSY and generate soft breaking terms.

2.1.1 Wilson line and Witten's Proposal

A remarkable result from M Theory on G_2 manifolds noticed by Witten [80] is that the resulting theory has a natural discrete symmetry that does not commute with the GUT group. This can then be used to prevent fast coloured triplet mediated proton-decay channels, and relies on the presence of a geometric discrete symmetry whose action will be enhanced by Wilson line phases as we explain below.

A Wilson line, $\mathcal{W}$, is an element of the GUT group, G, that furnishes a representation of the fundamental group of the compactified manifold, K, which we take to be $\pi_1(K) = \mathbf{Z_N}$, and it is defined by

$$\mathcal{W} = \mathcal{P} \exp i \oint_K A_k dx^k \ , \tag{2.9}$$

where $k = 5, ..., 11$ runs through the compactified dimensions, $\mathcal{P}$ means path ordering, and the integral is performed along topological inequivalent closed paths – the so called 1-cycles.

The above quantity is completely defined by the topology of the compactified manifold, and it cannot be gauged away. It can however be absorbed into a chiral supermultiplet which is localised along the 1-cycle where $\mathcal{W}$ is defined. This breaks the gauge group as the chiral supermultiplet will no longer transform homogeneously under the whole original GUT group. In fact the surviving gauge group, H, is going to be the centraliser of $\mathcal{W}$ in G, meaning

$$H = \{g \in G : [g, \mathcal{W}] = 0\} \ . \tag{2.10}$$

As the Wilson line furnishes a representation of the fundamental group, for the abelian case, $\pi(K) = \mathbf{Z_N}$, we have a set of mutually commuting matrices. Since the surviving group, H, is composed of the elements of G that commute with $\mathcal{W}$, the matrices $\mathcal{W}$ will also be elements of H. As all $\mathcal{W}$ commute with each other, we then conclude that the Wilson lines are elements of the centre of the surviving group. If the surviving group has n U(1) factors a Wilson line can then be conveniently represented as

$$\mathcal{W} = \exp\left(\frac{i2\pi}{N}\sum_{i}^{n} a_i Q_i\right) = \sum_{m=0}^{\infty}\frac{1}{m!}\left(\frac{i2\pi}{N}\sum_{i=1}^{n} a_i Q_i\right)^m , \tag{2.11}$$

where Q_i are the generators of the U(1) factors, a_i are coefficients of the linear combination that are only constrained by $\mathcal{W}^N = 1$ (since $\pi_1(K) = \mathbf{Z_N}$) which completely specify the embedding $\mathcal{W}$. Given that Q_i are diagonal generators, the matrix $\mathcal{W}$ will also be diagonal. The diagonal entries will be of the form η^α with η being the n^{th} root of unity, and α some integer related to the coefficients a_i and the eigenvalues of the generators Q_i.

Witten [80] suggested that if K admits a discrete symmetry of the geometry isomorphic to the fundamental group, i.e. a $\mathbf{Z_N}$ symmetry in our case, the discrete symmetry action and the Wilson line charges would mix leading to a discrete symmetry that does not commute with the GUT group. This means we have non-GUT preserving selection rules, which will constraint our Lagrangian below the GUT scale. This was successfully applied to $SU(5)$ [86] in providing a doublet-triplet splitting mechanism as well as avoiding proton-decay, in the so-called G_2-MSSM.

2.1.2 Spontaneously SUSY breaking and soft terms

As the moduli stabilise, their vevs spontaneously break $\mathcal{N} = 1$ Supergravity. The order parameter for the breaking is given by the gravitino mass

$$m_{3/2} = m_{Pl}^{-2} e^{K/2m_{Pl}^2}|W|, \tag{2.12}$$

which will depend on the specific hidden sector gauge groups and the volume of the compactified space.

In our framework it was found to naturally be [26]

$$m_{3/2} \simeq \mathcal{O}(10 - 100 \text{ TeV}). \tag{2.13}$$

The gravitino mass plays a crucial role as it sets the scale for the soft breaking parameters. For the soft scalar masses squared, under the assumption that the main non flavour diagonal contribution comes from the moduli F terms, F_i, which being smaller than the F_ϕ gives

$$m^2_{\overline{\alpha}\beta} \simeq m^2_{3/2}\delta_{\overline{\alpha}\beta} \tag{2.14}$$

a dominantly diagonal and universal soft-scalar masses squared matrix. This means that the scalar partners are naturally heavy and should not play a major role in loop corrections or have any signals in current accelerator experiments.

The trilinear soft-terms can be derived in a similar way. Under the same assumptions, and that the un-normalised Yukawa couplings and the visible sector Kähler metric are not expected to depend on the meson field, the un-normalised trilinear soft-terms read

$$A'_{\alpha\beta\gamma} \simeq \frac{W^*_{hid}}{|W_{hid}|} e^{\hat{K}} F^\phi \hat{K}_\phi Y'_{\alpha\beta\gamma} \tag{2.15}$$

and the normalised terms can be found to be

$$A_{\alpha\beta\gamma} \simeq \mathcal{O}(1) m_{3/2} Y_{\alpha\beta\gamma} \tag{2.16}$$

and so both the soft-masses squared and the soft trilinear couplings are set by the gravitino mass.

Finally we have the gaugino masses. The total gaugino mass at the GUT scale has three contributions: tree-level mass from SUGRA breaking, anomaly mediation contribution, and threshold effects from integrating out the KK modes of the gauge fields and other heavy states. A detailed study was carried out [27], and showed that at the unification scale one should expect

$$m^a_{1/2} \simeq \mathcal{O}(100 \text{ GeV}), \tag{2.17}$$

the suppression relative to the other soft-breaking parameters is understood as the tree-level contribution does not include the hidden-sector meson field, whose F term is greater than the moduli F terms that represent the biggest contribution to the gaugino tree-level mass.

Furthermore, since the tree-level contribution and the anomaly mediation correction have the same order of magnitude, the gaugino masses are not universal. The suppression in comparison to the other soft-terms is traced back to the fact that the moduli F-terms are suppressed relative to $m_{3/2}^2$.

2.1.3 Effective μ-terms and trilinear couplings

In M Theory compactified on a G_2 manifold without fluxes there is a natural way of generating effective μ terms. An effective μ-term of order TeV scale can be generated by moduli vev from interactions in the Kähler potential, the mechanism is closely resemblant to Giudice-Masiero mechanism [87].

To see how the above considerations lead to a natural $\mathcal{O}(1 \text{ TeV})$ μ term consider the Kähler potential interaction

$$K \supset \frac{s}{m_{Pl}} X\overline{X} + \text{h.c.} \;, \tag{2.18}$$

here we take the coefficient to be or order one, and s symbolically represents a modulus field, and X a chiral supermultiplet in some gauge irrep, with $\overline{X}$ another chiral supermultiplet in the charge conjugated irrep. As moduli have charges under the discrete symmetry, and in principle there are many of them, the above coupling is generally allowed even if $X\overline{X}$ is forbidden by the same discrete symmetry.

As a consequence of the moduli stabilisation and associated vevs, an effective μ parameter for the X field will be generated. This effective parameter appearing in the superpotential is derived from the usual supergravity mass formulae [88] [9] when taking the flat – *global SUSY* – limit of supergravity.

In the end one finds that the effective superpotential μ term is given by

$$\mu_X = \langle m_{3/2} K_{X\overline{X}} - F^{\overline{\kappa}} K_{X\overline{X}\kappa} \rangle \;, \tag{2.19}$$

which leads to

$$\mu_X = \frac{\langle s \rangle}{m_{Pl}} m_{3/2} + \frac{\langle F_s \rangle}{m_{Pl}} \;, \tag{2.20}$$

and since $F_s \ll m_{3/2}\langle s \rangle$ the moduli vev dominates. One finds

$$\mu_X \sim 0.1 m_{3/2} \;, \tag{2.21}$$

and since $m_{3/2} \sim \mathcal{O}(10 \text{ TeV})$, we have $\mu_X \simeq \mathcal{O}(1 \text{ TeV})$. Notice that this analysis is valid for all vector-like pairs X, $\overline{X}$. This means that if one adds extra vector-like states to the model, beyond the MSSM spectrum, one has to worry about possible mixings and LHC-reachable extra fermionic matter. This will be studied below when we construct the SO(10) model.

Similar to the above procedure to generate μ parameters, our framework can generate effective trilinear terms in the superpotential. They will mediate proton decay but also provide an LSP decay channel, since they can be R-parity violating. These interactions play an important role for the low-energy model, and have to be studied in detail.

Trilinear interactions are generated in the same way as the effective μ terms dsicussed above. Consider the Kähler potential contribution

$$K \supset \frac{s}{m_{Pl}^2} XYZ + \text{h.c.} \ , \tag{2.22}$$

where s does not have to be the same moduli as above, and X, Y, and Z are chiral supermultiplets. As one can see, these interactions are more suppressed than the effective μ-terms studied before, they will, however, still appear as effective trilinear couplings in the superpotential as

$$W_{eff} \supset \frac{(\langle s \rangle m_{3/2} + F_s)}{m_{Pl}^2} m_{3/2} XYZ \ , \tag{2.23}$$

Again, since $F_s \ll \langle s \rangle m_{3/2}$, the F-term contribution is sub-leading and we can estimate the order of magnitude of effective coupling. This turns out to be small

$$\frac{\langle s \rangle}{m_{Pl}^2} m_{3/2} \to 0.1 \frac{m_{3/2}}{m_{Pl}} \sim 10^{-15} \ , \tag{2.24}$$

but it will have a deep impact on the LSP lifetime, as it will be discussed below.

We also note that in principle these cannot be big enough to generate realistic Yukawa couplings. One is then led to expect the Yukawa couplings to be generated by tree-level interactions as discussed above.

2.2 The G_2-MSSM

In Witten's M theory approach to $SU(5)$, the combination of the discrete symmetry, the Wilson lines and the fact that GUT multiplets are localised at points, allows one to prevent the MSSM Higgs doublets, H_u and H_d, from having a mass (the μ-term) whilst the colour triplets D and $\overline{D}$ could have large masses. For simplicity we assume that the symmetry is $\mathbf{Z_N}$. We use the following notation: $\overline{\mathbf{5}}^w$ is the multiplet containing H_d and $\overline{D}$ and is localised along the Wilson line (which is a circle in the extra dimensions); $\mathbf{5}^h$ is the multiplet containing H_u; $\overline{\mathbf{5}}^m$ and $\mathbf{10}^m$ are the matter multiplets. Then the transformation rules for these multiplets under $\mathbf{Z_N}$ are:

$$\begin{aligned} \overline{\mathbf{5}}^w &\to \eta^\omega \left(\eta^\delta H_d^w \oplus \eta^\gamma \overline{D}^w\right), \\ \mathbf{5}^h &\to \eta^\chi \mathbf{5}^h, \\ \overline{\mathbf{5}}^m &\to \eta^\tau \overline{\mathbf{5}}^m, \\ \mathbf{10}^m &\to \eta^\sigma \mathbf{10}^m, \end{aligned} \tag{2.25}$$

where $\eta \equiv e^{2\pi i/N}$, $2\delta + 3\gamma = 0 \bmod N$. By requiring that Yukawa couplings, Majorana neutrino masses, and colour-triplet masses must be present, we obtain constraints on the charges as can be seen in Table 2.1 where we chose $\omega = 0$.

Coupling	Constraint
$H_u^h \mathbf{10}^m \mathbf{10}^m$	$2\sigma + \chi = 0 \bmod N$
$H_d^w \mathbf{10}^m \overline{\mathbf{5}}^m$	$\sigma + \tau + \delta = 0 \bmod N$
$H_u^w H_u^w \overline{\mathbf{5}}^m \overline{\mathbf{5}}^m$	$2\chi + 2\tau = 0 \bmod N$
$\overline{D}^w D^h$	$\chi + \gamma = 0 \bmod N$

Table 2.1: Couplings and charges for $SU(5)$ operators.

One can solve these by writing all angles in terms of, say, σ

$$\begin{aligned} \chi &= -\gamma = -2\sigma \quad \bmod N, \\ \delta &= -3\sigma + N/2 \quad \bmod N, \\ \tau &= 2\sigma + N/2 \quad \bmod N, \end{aligned} \tag{2.26}$$

which *automatically forbids the μ-term and dimension four and five proton decay operators.*

The discrete symmetry forces $\mu = 0$, however phenomenologically, $\mu \geq \mathcal{O}(100)$ GeV from direct limits on the masses of charged Higgsinos from colliders. The symmetry must therefore be broken. Since the discrete symmetry is a geometric symmetry of the extra dimensions, the moduli fields are naturally charged under it. Moduli stabilisation for G_2-manifolds was considered in [73], and it was shown that (asymptotically free) gauge interactions in the hidden sector can generate a moduli potential capable of spontaneously breaking supersymmetry at a hierarchically small scale which *stabilises all the moduli.* A key point behind the success of this mechanism and which plays a crucial role in the following, is that, in M theory compactifications on G_2-manifolds *without* fluxes, all of the moduli fields s_i reside in chiral superfields which contain axions. The shift symmetries enjoyed by these axions, combined with holomorphy, prevent terms in the superpotential which are polynomial in the moduli [84].

Generically the vacua of the potential will spontaneously break the $\mathbf{Z_N}$ symmetry. This then generates an effective μ term from, e.g. Kähler potential operators of the form

$$K \supset \frac{s}{m_{pl}} H_u H_d \quad + \quad h.c., \tag{2.27}$$

à la Giudice-Masiero [87], where s generically denotes a modulus field of the appropriate charge and m_{pl} is the Planck scale. Note that such terms are forbidden in the superpotential due to holomorphy and the axion shift symmetries. From [19, 26, 27, 73] we know that the moduli vevs are approximately $\langle s \rangle \sim 0.1 m_{pl}$, $\langle F_s \rangle \sim m_{1/2} m_{pl}$ and from the standard supergravity Lagrangian [88] we get an effective μ-term:

$$\mu = \langle m_{3/2} K_{H_u H_d} - F^k K_{H_u H_d k} \rangle, \tag{2.28}$$

which leads to

$$\mu \sim \frac{\langle s \rangle}{m_{pl}} m_{3/2} + \frac{\langle F_s \rangle}{m_{pl}}. \tag{2.29}$$

Since gaugino masses are suppressed [19, 26, 27, 73], the F-term vev is subleading and we get

$$\mu \sim 0.1 m_{3/2} \sim \mathcal{O}(TeV) \ . \tag{2.30}$$

2.3 $SO(10)$ from M Theory on G_2 Manifolds

Following this recap, we now turn to our M theory approach to $SO(10)$, where a novel solution to the doublet-triplet splitting problem seems to be required: since the Wilson line is in the adjoint representation, it can break $SO(10)$ to $SU(3) \times SU(2) \times U(1)_Y \times U(1)$ and the Wilson line itself is a combination of $U(1)_Y$ and the additional $U(1)$. If we consider a fundamental of $SO(10)$ localised along a Wilson line, then its transformation properties under the $\mathbf{Z_N}$ symmetry are in analogy to Eq. (2.25),

$$\mathbf{10}^w \to \eta^\omega \left(\eta^{-\alpha} H_d^w \oplus \eta^\beta \overline{D}^w \oplus \eta^\alpha H_u^w \oplus \eta^{-\beta} D^w\right). \tag{2.31}$$

In minimal $SO(10)$ the μ-term arises from a term in the superpotential of the form

$$W \supset \mu \mathbf{10}^w \mathbf{10}^w = \mu \left(H_u^w H_d^w + D^w \overline{D}^w\right), \tag{2.32}$$

with the triplet mass $m_D = \mu$. Clearly if $2\omega = 0 \mod N$ both terms are allowed, otherwise both terms are forbidden. *Therefore generically, they will both be forbidden.*

We briefly digress to consider the effect of adding additional **10** multiplets. In this case, one can forbid some couplings between the different members of the various **10** multiplets, but one can see that there will typically be more than one pair of light Higgs doublets which tend to destroy gauge coupling unification. Consider one additional **10**, denoted $\mathbf{10}^h$ without Wilson line phases: $\mathbf{10}^h \to \eta^\xi \mathbf{10}^h$. We have eight possible gauge invariant couplings with a $\mathbf{10}^w$ and $\mathbf{10}^h$ that can be written in matrix form as

$$W \supset \mathbf{H}_d^T \cdot \mu_H \cdot \mathbf{H}_u + \overline{\mathbf{D}}^T \cdot M_D \cdot \mathbf{D}, \tag{2.33}$$

where μ_H and M_D are two 2×2 superpotential mass parameters matrices, $\mathbf{H}_{u,d}^T = \left(H_{u,d}^w, H_{u,d}^h\right)$, $\overline{\mathbf{D}}^T = \left(\overline{D}^w, \overline{D}^h\right)$, and $\mathbf{D}^T = \left(D^w, D^h\right)$. The entries of the matrices are non-vanishing depending on which of the following discrete charge combinations are zero (mod N)

$$\begin{aligned} D^w\overline{D}^w,\ H_u^w H_d^w &: \ 2\omega, \\ D^h\overline{D}^h,\ H_u^h H_d^h &: \ 2\xi, \\ H_u^w H_d^h &: \ \alpha + \omega + \xi, \\ H_u^h H_d^w &: \ -\alpha + \omega + \xi, \\ D^w \overline{D}^h &: \ -\beta + \omega + \xi, \\ D^h \overline{D}^w &: \ \beta + \omega + \xi. \end{aligned} \tag{2.34}$$

The naive doublet-triplet splitting solution would be for μ_H to have only one zero eigenvalue, with M_D having all non-zero eigenvalues. One finds that there is no choice of constraints in Eq. (2.34) that accomplishes this.

Henceforth we shall only consider a single light $\mathbf{10}^w$, without any extra $\mathbf{10}$ multiplets at low energies.

Assuming a single light $\mathbf{10}^w$, it is possible to use the discrete symmetry to forbid certain couplings, namely to *decouple* D^w *and* $\overline{D}^w$ *from matter*. Such couplings arise from the operator $\mathbf{10}^w\mathbf{16}^m\mathbf{16}^m$, with $\mathbf{16}^m$ denoting the three $SO(10)$ multiplets, each containing a SM family plus right handed neutrino N. If $\mathbf{16}^m$ transforms as $\eta^\kappa\mathbf{16}^m$, the couplings and charge constraints are in Table 2.2, where we allow for up-type quark Yukawa couplings together with couplings to the right-handed neutrinos,

$$y_u^{ij} H_u^w \mathbf{16}_i^m \mathbf{16}_j^m \equiv y_u^{ij} H_u^w (Q_i u_j^c + L_i N_j + i \leftrightarrow j), \tag{2.35}$$

and similarly for down-type quarks and charged leptons.

Table 2.2: Couplings and charges for $SO(10)$ operators.

Coupling	Constraint
$H_u^w\mathbf{16}^m\mathbf{16}^m$	$2\kappa + \alpha + \omega = 0 \bmod N$
$H_d^w\mathbf{16}^m\mathbf{16}^m$	$2\kappa - \alpha + \omega = 0 \bmod N$
$D^w\mathbf{16}^m\mathbf{16}^m$	$2\kappa - \beta + \omega \neq 0 \bmod N$
$\overline{D}^w\mathbf{16}^m\mathbf{16}^m$	$2\kappa + \beta + \omega \neq 0 \bmod N$

We emphasise that there exist solutions to these constraints, e.g.

$$(N, \kappa, \alpha, \omega, \beta) = (6, 1, 3, 1, 2). \tag{2.36}$$

The suppression of colour triplet couplings to matter was previously considered by Dvali in [89] and also [90–92] from a bottom-up perspective.

Next we consider the breaking of the discrete symmetry via the moduli vevs as discussed above, leading to proton decay. For proton decay, the relevant operators can be generated in the Kähler potential, schematically, writing $D = D^w$,

$$\begin{aligned} K \;\supset\; & \tfrac{s}{m_{pl}^2} DQQ + \tfrac{s}{m_{pl}^2} De^c u^c + \tfrac{s}{m_{pl}^2} DNd^c + \\ & + \tfrac{s}{m_{pl}^2}\overline{D}d^c u^c + \tfrac{s}{m_{pl}^2}\overline{D}QL. \end{aligned} \tag{2.37}$$

Just like the μ-term, the effective potential may be calculated from supergravity to be

$$\begin{aligned} W_{eff} \ \supset & \ \lambda DQQ + \lambda De^c u^c + \lambda DNd^c + \\ & + \lambda \overline{D} d^c u^c + \lambda \overline{D} QL, \end{aligned} \tag{2.38}$$

where

$$\lambda \approx \frac{1}{m_{pl}^2} \left(\langle s \rangle m_{3/2} + \langle F_s \rangle \right) \sim 10^{-15}. \tag{2.39}$$

Notice that unlike the case of SU(5), there is no SO(10) invariant bilinear term $\kappa L H_u$ whose presence would lead to fast proton decay. We estimate the scalar triplet induced proton decay rate to be

$$\Gamma_p \approx \frac{|\lambda^2|^2}{16\pi^2} \frac{m_p^5}{m_D^4}. \tag{2.40}$$

Generically, the mass of the colour triplets is of the same order as μ, i.e., $m_D \sim 10^3$ GeV, so the proton lifetime is

$$\tau_p = \Gamma_p^{-1} \sim 10^{42} \text{ yrs}, \tag{2.41}$$

which exceeds the current experimental limit.

Now consider the D triplet decay rate:

$$\Gamma_D \sim \lambda^2 m_D \sim (0.1 \text{ sec})^{-1}. \tag{2.42}$$

The associated lifetime of 0.1 sec is (just) short enough to be consistent with BBN constraint. They will also give interesting collider signatures due to their long-lived nature.

Gauge coupling unification is in general spoiled by light colour triplets, unless they are also accompanied by additional light doublet states. In the present framework, the only way we know of to circumvent this issue is the presence of light additional states which complete the triplets into complete GUT multiplets. Happily, this can also be achieved by use of the discrete symmetry. First we introduce a vector-like pair of **16**'s, labelled as $\mathbf{16}_X + \overline{\mathbf{16}}_X$. Next a GUT-scale mass is given to their colour triplet components $d^c_X, \overline{d^c}_X$ whilst keeping the remaining particles light. Suitable charges under the discrete symmetry can forbid the appropriate mass terms and the large mass can arise from membrane instantons if the $\mathbf{16}_X$ and $\overline{\mathbf{16}}_X$ are close by on the G_2 manifold [64].

We take $\mathbf{16}_X$ to be localised along a Wilson line, and find that it transforms under the discrete symmetry as

$$\begin{aligned}\mathbf{16}_X \to &\eta^x \left(\eta^{-3\gamma} L \oplus \eta^{3\gamma+\delta} e^c \oplus \eta^{3\gamma-\delta} N \oplus \eta^{-\gamma-\delta} u^c \oplus \right. \\ &\left. \oplus\, \eta^{-\gamma+\delta} d^c \oplus \eta^{\gamma} Q\right). \end{aligned} \tag{2.43}$$

Assuming $\overline{\mathbf{16}}_X$ transforms without Wilson line phases, $\overline{\mathbf{16}}_X \to \eta^{\overline{x}}\, \overline{\mathbf{16}}_X$, the condition for the mass term is

$$\overline{d^c}_X d^c_X : x - \gamma + \delta + \overline{x} = 0 \mod N, \tag{2.44}$$

whilst forbidding all the other self couplings that would arise from $\mathbf{16}_X \overline{\mathbf{16}}_X$.

The light D^w and $\overline{D}^w$ from the original $\mathbf{10}^w$ then "complete" the $\mathbf{16}_X + \overline{\mathbf{16}}_X$ pair, since they have the same SM quantum numbers as the missing $d^c_X, \overline{d^c}_X$. The light states in the $\mathbf{16}_X$ and $\overline{\mathbf{16}}_{\mathbf{X}}$ also obtain masses via the Kähler potential of order a TeV via the Giudice-Masiero mechanism. Gauge unification is clearly restored, albeit with a larger gauge coupling at the GUT scale due to the extra low energy matter content (relative to the MSSM).

Effective μ-terms induced by moduli vevs of the form $\mu \mathbf{16}^m \overline{\mathbf{16}}_X$ are then generated and one might be concerned about too much mixing with quarks and leptons. However, one finds that all the light components of the extra matter decouple from ordinary matter, with mixings supressed by terms of order (2.39). For example, consider the up-type quark sector. The superpotential contribution to the mass matrix is, schematically, $W \supset \overline{U}^T \cdot M_U \cdot U$, where $U^T = (u_i, \overline{u^c}_X, u_X)$, $\overline{U}^T = (u^c_i, \overline{u}_X, u^c_X)$, with $i = 1, 2, 3$, where

$$M_U \sim \begin{pmatrix} y_u^{ij}\langle H_u \rangle & \mu_X^i & \lambda_X^i \langle H_u\rangle \\ \mu_X^j & \lambda_{XX}\langle H_u\rangle & \mu_{XX} \\ \lambda_X^j \langle H_u \rangle & \mu_{XX} & \lambda_{XX}\langle H_u\rangle \end{pmatrix}. \tag{2.45}$$

Here $\mu^i_X, \mu^j_X, \mu_{XX}$ are moduli induced μ-type parameters of $\mathcal{O}(\text{TeV})$ while $\lambda^i_X, \lambda^j_X, \lambda_{XX}$ are moduli induced trilinear interactions which are vanishingly small, $\mathcal{O}(10^{-14})$, causing the determinant of M_U to be approximately independent of μ^i_X, μ^j_X, i.e. of the mixing masses. This means the eigenvalues are, to leading order in $\lambda^i_X, \lambda^j_X, \lambda_{XX}$, independent of the mixings so the up-type quarks decouple from the new particles.

Introducing the $\mathbf{16}_X$ and $\overline{\mathbf{16}}_X$ will play a crucial role in breaking the extra $U(1)$ subgroup of $SO(10)$ and generating right-handed neutrino masses. Assuming a D-flat direction, they acquire vevs in their right-handed neutrino components, $\langle N_X \rangle = \langle \overline{N}_X \rangle = v_X$, breaking the rank. Here we do not specify the details of this symmetry breaking mechanism. However the scale v_X is constrained, as discussed below.

Presence of the $\mathbf{16}_X$ and $\overline{\mathbf{16}}_X$ with vevs in their right-handed neutrino components gives us the possibility of having a see-saw mechanism for light physical neutrino masses. Such a mechanism is welcome since representations larger than the $\mathbf{45}$ are absent in M theory [83]. In the present framework, a Majorana mass term for the right handed neutrino in $\mathbf{16}^m$ is generated by letting the discrete symmetry to allow the Planck suppressed operator $\frac{1}{m_{pl}}\overline{\mathbf{16}}_X\overline{\mathbf{16}}_X\mathbf{16}^m\mathbf{16}^m$. This requires charges to satisfy $2\overline{x} + 2\kappa = 0$ mod N, and leads to the Majorana mass $M \sim \frac{{v_X}^2}{m_{pl}}$.

Due to the nature of $SO(10)$, the neutrinos will have the same Yukawa coupling as the up-type quarks y_u^{ij}, as in Eq. (3.13), leading to their Dirac masses being the same as the up-quark masses. For the case of the top quark mass we would need $M \sim 10^{14}$ GeV in order to give a realistic neutrino mass. Such a high value can only be achieved by the above see-saw mechanism if $v_X \gtrsim 10^{16}$ GeV.

The magnitude of v_X is also constrained by R-parity violating (RPV) dynamically generated operators, due to moduli and $N_X, \overline{N}_X$ vevs, arising from the Kähler interactions

$$K_{RPV} \supset \frac{s}{m_{pl}^3}\mathbf{16}_X\mathbf{16}^m\mathbf{16}^m\mathbf{16}^m + \frac{s}{m_{pl}^2}\mathbf{10}^w\mathbf{16}_X\mathbf{16}^m. \tag{2.46}$$

Since s and N_X acquire vevs, these operators generate the effective superpotential terms (otherwise forbidden by the discrete symmetry),

$$\begin{aligned} W_{RPV}^{eff} \supset\ & \lambda\frac{v_X}{m_{pl}}LLe^c + \lambda\frac{v_X}{m_{pl}}LQd^c + \lambda\frac{v_X}{m_{pl}}u^c d^c d^c + \\ & +\lambda v_X L H_u, \end{aligned} \tag{2.47}$$

with $\lambda \sim \mathcal{O}(10^{-14})$. One can absorb the last term into $\mu H_d H_u$ by a small rotation $\mathcal{O}(v_X/m_{pl})$ in (H_d, L) space,

$$W_{RPV}^{eff} \supset y_e\frac{v_X}{m_{pl}}LLe^c + y_d\frac{v_X}{m_{pl}}LQd^c + \lambda\frac{v_X}{m_{pl}}u^c d^c d^c, \tag{2.48}$$

where the first two terms originate from the Yukawa couplings $y_e H_d L e^c$, etc., and we have dropped the $\mathcal{O}(\lambda)$ contributions to these terms since now the Yukawa rotated contributions are much larger.

The last term in Eq. (2.47) gives an important RPV limit that constrains the value of v_X [93, 94], as it induces an extra contribution to the neutrino masses after the rotation. Neutrino masses limit the bilinear contribution to the mixing to be $\lambda v_X \lesssim \mathcal{O}(1\text{ GeV})$, which leads to the upper bound $v_X \lesssim 10^{14}$ GeV in contradiction with the see-saw requirement $v_X \sim 10^{16}$ GeV assumed in the above estimates.

In order to suppress RPV induced neutrino masses which are too large, while maintaining $v_X \sim 10^{16}$ GeV, we shall suppose that there is some suppression in the last coupling in Eq. (2.47) of order $\mathcal{O}(10^{-2})$. Such a suppression could occur if the discrete symmetry does not allow the term, i.e., there is no modulus with appropriate charge coupling with $\mathbf{10}^w\mathbf{16}_X\mathbf{16}^m$ in the Kähler potential. However higher dimension operators, for example $\frac{s^2}{m_{pl}^3}\mathbf{10}^w\mathbf{16}_X\mathbf{16}^m$, may be allowed providing the necessary extra suppression in order to relax the bound. Even though such accidental symmetries might be non-generic, there is no reason for them not to be present in examples.

The RPV terms in Eq. (2.48) induce the lightest supersymetric particle (LSP) decay. We can estimate its lifetime as [86]:

$$\tau_{LSP} \simeq \frac{10^{-13}\,\text{sec}}{(v_X/m_{pl})^2}\left(\frac{m_0}{10\text{ TeV}}\right)^4\left(\frac{100\text{ GeV}}{m_{LSP}}\right)^5. \tag{2.49}$$

Since, as discussed above, $v_X/m_{pl} \sim 10^{-2}$, one finds $\tau_{LSP} \sim 10^{-9}\,\text{sec}$. This value is compatible with current bounds $\tau_{LSP} \lesssim 1$ sec [95], from BBN. Clearly the LSP is not a good DM candidate. However, M theory usually provides axion dark matter candidates [96, 97]

2.4 Conclusions

We have discussed the origin of an $SO(10)$ SUSY GUT from M theory on a G_2 manifold. We were naturally led to a novel solution of the doublet-triplet splitting problem involving an extra $\mathbf{16}_X+\overline{\mathbf{16}}_X$ vector-like pair where discrete symmetries of the extra dimensions were used to prevent proton decay by suppressing the Yukawa couplings of colour triplets. Such models maintain gauge coupling unification but with a larger GUT coupling than predicted by the MSSM. We argue that these extra multiplets, also required to break the additional $U(1)$ gauge symmetry, inevitably lead to R-parity violating operators . Even though the moduli potential generically breaks the discrete symmetry, we have seen that one naturally satisfies the constraints from the proton lifetime and decays affecting BBN. We also have found a consistent scenario for neutrino masses arising from the high scale see-saw mechanism, with sufficiently suppressed RPV contributions. We emphasise the main prediction of this approach, namely light states with the quantum numbers of a $\mathbf{16}_X + \overline{\mathbf{16}}_X$ vector-like pair which might be accessible in future LHC searches.

Chapter 3

Symmetry Breaking and Neutrino Masses in M Theory $SO(10)$

The discovery of neutrino mass and lepton mixing provides key evidence for new physics beyond the Standard Model (SM) [98–102]. The seesaw mechanism [23, 103–106] is an attractive possibilty to account for the origin of neutrino mass and lepton mixing in terms of right-handed neutrinos with large Majorana masses. $SO(10)$ Grand Unified Theories (GUTs) [107] predict such right-handed neutrinos which appear along with SM matter fields in a single **16** multiplet. When the $SO(10)$ gauge group is broken to that of the SM, neutrino mass is an inevitable consequence. In order to satisfy the constraint of gauge coupling unification, we shall here assume low energy supersymmetry (SUSY) [8]. However to also account for gravity, one needs to go beyond gauge theories, and here we shall focus on an M theory version of string theory [24, 108].

Recently we showed how $SO(10)$ SUSY GUTs could emerge from M Theory compactified on a G_2-manifold [1]. In this framework, discrete symmetry and Wilson lines [80] were used to prevent proton decay while maintaining gauge unification. In contrast to the $SU(5)$ version [27, 86], the Wilson line symmetry breaking mechanism in $SO(10)$ requires additional matter at the TeV scale, with the quantum numbers of an extra $\mathbf{16}_X$ plus $\overline{\mathbf{16}}_X$ [1]. In addition, there were a number of unresolved issues in this approach, notably the mechanism for breaking the extra gauged $U(1)_X$ which accompanies the SM gauge group after the Wilson line symmetry breaking mechanism in $SO(10)$. This gauge group is the usual one in the maximal

$SO(10)$ subgroup $SU(5) \times U(1)_X$ [1], where $SU(5)$ embeds the SM gauge group. The key point is that, since Abelian Wilson line symmetry breaking preserves the rank of the gauge group, the $U(1)_X$ gauge group needs to be broken by some other mechanism in the low energy effective field theory. Since right-handed Majorana neutrino masses can only arise once the $U(1)_X$ is broken, the origin of neutrino mass is therefore linked to this symmetry breaking.

In this chapter we address the problem of $U(1)_X$ breaking and neutrino masses arising from the $SO(10)$ M theory, following the construction in [1] and presented above in Chapter 2, although our approach to solving these problems may be more general than the specific example studied. To break the $U(1)_X$ gauge symmetry, we employ a (generalised) Kolda-Martin mechanism [109], where higher order operators can break the symmetry, inducing vacuum expectation values (VEVs) in the scalar right-handed neutrino components of both the matter $\mathbf{16}$ and the extra $\mathbf{16}_X$, as well as their conjugate partners. The subsequent induced R-parity violation [110] provides additional sources of neutrino mass, in addition to that arising from the see-saw mechanism [23, 103–106]. The resulting 11×11 neutrino mass matrix is analysed for one neutrino family (nominally the third family) and it is shown how a phenomenologically acceptable neutrino mass can emerge. We defer any discussion of flavour mixing to a possible future study of flavour from M theory. Here we only show that symmetry breaking and viable neutrino masses can arise within the framework of M theory $SO(10)$, which is a highly non-trivial result, given the constrained nature of M theory constructions.

It is worth remarking that there are other alternative ways that have been proposed to study neutrino masses in string theory, which are complementary to the approach followed here. For example, it is possible to obtain large Majorana mass terms from instanton effects [85, 111–114], large volume compactification [115], or orbifold compactfications of the heterotic string [114]. However the origin of Majorana mass terms in $SO(10)$ has been non-trivial to realise from the string theory point of view. In GUTs all matter fields are unified in $\mathbf{16}$ multiplets whereas Higgs fields and triplet scalars are unified in $\mathbf{10}$. Since string theory does not predict light particles in representations larger than the adjoint, the traditional renormalisable terms involving $\mathbf{126}, \overline{\mathbf{126}}, \mathbf{210}$, e.g., $W \sim \mathbf{126}\,\mathbf{16}\,\mathbf{16}$, are not possible. The

[1] The $U(1)_X$ is also commonly called $U(1)_\chi$ in the literature.

dominant higher order operators are quartic ones such as $W = \overline{\mathbf{16}}\,\overline{\mathbf{16}}\,\mathbf{16}\,\mathbf{16}$. Assuming that the supersymmetric partner of the right handed neutrino singlet gets a VEV, the Majorana mass is given by $M \sim \frac{\langle \widetilde{N} \rangle^2}{M_{PL}}$. However, the required values of neutrino mass imply $M > 10^{14}$ GeV, which gives $\langle \widetilde{N} \rangle \sim \sqrt{M m_{Pl}} \sim 10^{16}$ GeV. The implementation of the seesaw mechanism [23, 103–106] in other corners of string compactification has also been discussed [116–119].

The layout of the remainder of the chapter is as follows. In the next section we briefly review the model already discussed in Chapter 2. In section 3.2, the mechanism proposed in [2] for $U(1)_X$ breaking will be given. The neutrino mass matrix will be analysed in section 3.3, and the numerical results presented in section 3.3.2. Finally we conclude in section 3.4.

3.1 SO(10) SUSY GUTS from M Theory on G_2-manifolds

In this section, we focus on the $SO(10)$ SUSY GUT from M Theory on G_2 manifolds which we proposed in [1] and discussed in 2. The breaking patterns of an abelian Wilson line are the same as the ones of an adjoint Higgs. The simplest case of a surviving group that is the most resembling to the SM is

$$SO(10) \to SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X \ , \tag{3.1}$$

under which the branching rules of the GUT irreps read

$$\mathbf{10} : H_u = (\mathbf{1}, \mathbf{2})_{\left(\frac{1}{2}, 2\right)} \oplus H_d = (\mathbf{1}, \mathbf{2})_{\left(-\frac{1}{2}, -2\right)} \oplus D = (\mathbf{3}, \mathbf{1})_{\left(-\frac{1}{3}, 2\right)} \oplus \overline{D} = (\overline{\mathbf{3}}, \mathbf{1})_{\left(\frac{1}{3}, -2\right)} \ , \tag{3.2}$$

$$\begin{aligned} \mathbf{16} : L = (\mathbf{1}, \mathbf{2})_{\left(-\frac{1}{2}, 3\right)} \oplus e^c = (\mathbf{1}, \mathbf{1})_{(1, -1)} \oplus N = (\mathbf{1}, \mathbf{1})_{(0, -5)} \oplus u^c = (\overline{\mathbf{3}}, \mathbf{1})_{\left(-\frac{2}{3}, -1\right)} \oplus \\ \oplus d^c = (\overline{\mathbf{3}}, \mathbf{1})_{\left(\frac{1}{3}, 3\right)} \oplus Q = (\mathbf{3}, \mathbf{2})_{\left(\frac{1}{6}, -1\right)} \ , \end{aligned} \tag{3.3}$$

and the subscripts are the charges under $U(1)_Y \times U(1)_X$, which are normalised as $Q_Y = \sqrt{\frac{5}{3}} Q_1, \ Q_X = \sqrt{40} \tilde{Q}_X$, where $Q_1, \ \tilde{Q}_X$ are $SO(10)$ generators.

The Wilson line can be conveniently represented as

$$\mathcal{W} = \exp\left[\frac{i2\pi}{N}\left(aQ_Y + bQ_X\right)\right] = \sum_{m=0}^{\infty}\frac{1}{m!}\left(\frac{i2\pi}{N}\right)^m \left(aQ_Y + bQ_X\right)^m \ , \quad (3.4)$$

where the coefficients a, b are constrained by the requirement that $\mathcal{W}^N = 1$ and specify the parametrisation of the Wilson line. Under the linear transformation

$$\frac{1}{2}a + 2b \to \alpha \ , \quad (3.5)$$
$$\frac{1}{3}a - 2b \to \beta \ , \quad (3.6)$$

its action on the fundamental irrep then reads

$$\mathcal{W}10 = \eta^{\alpha} H_u \oplus \eta^{-\alpha} H_d \oplus \eta^{-\beta} D \oplus \eta^{\beta}\overline{D} \ , \quad (3.7)$$

where η is the Nth root of unity.

Likewise the Wilson line matrix acts on the 16 irrep as

$$\mathcal{W}16 = \eta^{-\frac{3}{2}\beta}L \oplus \eta^{\alpha+\frac{3}{2}\beta}e^c \oplus \eta^{-\alpha+\frac{3}{2}\beta}N \oplus \eta^{-\alpha-\frac{1}{2}\beta}u^c \oplus \eta^{\alpha-\frac{1}{2}\beta}d^c \oplus \eta^{\frac{1}{2}\beta}Q \ , \quad (3.8)$$

which could be simplified a bit further by replacing $\beta \to 2\beta$ without loss of generality, in order for the parameters to read as integers.

The effective discrete charges – of different states on a chiral supermultiplet that absorbs Wilson line phases – will be the overall charge of the discrete symmetry (common to all states belonging to the same GUT irrep) in addition to the Wilson line phases (different for each state inside the GUT irrep).

Having all the ingredients required to employ Witten's discrete symmetry proposal, we would like to have a consistent implementation of a well-motivated doublet-triplet splitting mechanism as it was done for $SU(5)$. Unfortunately the customary approach to the problem does not seem to work with $SO(10)$, as shown in [1]. To understand this first notice that Witten's splitting mechanism can only work in order to split couplings between distinct GUT irreps. This is understood as $\mathcal{W}$ has the form of a gauge transformation of the surviving group and so it will never be able to split self bilinear couplings of a GUT irrep. For example, if one takes a **10** with Wilson line phases to contain the MSSM Higgses, we can see from eq. (3.7) that both mass terms for the Higgses and coloured triplets are trivially allowed. We could consider that in order to split the Higgses, H_u and H_d, from the coloured triplets – D, $\overline{D}$ – we would need to add another **10**, but it was shown that this cannot be achieved and so we are ultimately left with light coloured triplets.

In order to allow for light D, $\overline{D}$ we need to guarantee that they are sufficiently decoupled from matter to prevent proton-decay. To accomplish this, we can use the discrete symmetry to forbid certain couplings, namely to *decouple D and $\overline{D}$ from matter*. Such couplings arise from the $SO(10)$ invariant operator **10 16 16**, with **16** denoting the three $SO(10)$ multiplets, each containing a SM family plus right handed neutrino N. If **16** transforms as $\eta^{\kappa}\mathbf{16}$, the couplings and charge constraints are

$$H_u\mathbf{1616}\ : 2\kappa+\alpha+\omega=0 \bmod N \tag{3.9}$$

$$H_d\mathbf{1616}\ : 2\kappa-\alpha+\omega=0 \bmod N \tag{3.10}$$

$$D\mathbf{1616}\ : 2\kappa-\beta+\omega\neq 0 \bmod N \tag{3.11}$$

$$\overline{D}\mathbf{1616}\ : 2\kappa+\beta+\omega\neq 0 \bmod N, \tag{3.12}$$

where we allow for up-type quark Yukawa couplings together with couplings to the right-handed neutrinos,

$$y_u^{ij}H_u^w\mathbf{16}_i\mathbf{16}_j \equiv y_u^{ij}H_u^w(Q_iu_j^c+L_iN_j+i\leftrightarrow j), \tag{3.13}$$

and similarly for down-type quarks and charged leptons.

The couplings forbidden at a renormalisable tree-level by the discrete symmetry are generically regenerated from Kähler interactions through the Giudice-Masiero mechanism [87]. While this provides the Higgsinos a TeV scale μ-term mass, it also originates effective trilinear couplings with an $\mathcal{O}(10^{-15})$ coefficient. As these are generic, we need to systematically study their physical implications at low energies, such as proton-decay, R-parity violation, and flavour mixing. s pointed out in section 2.1.3.

For proton decay, effective superpotential will be generate by the following Kähler potential

$$K\supset \frac{s}{m_{Pl}^2}\overline{D}d^cu^c+\frac{s}{m_{Pl}^2}De^cu^c+\frac{s}{m_{Pl}^2}DQQ+\frac{s}{m_{Pl}^2}\overline{D}QL+\frac{s}{m_{Pl}^2}DNd^c+\text{h.c.}\ , \tag{3.14}$$

where we assume $\mathcal{O}(1)$ coefficients. As the moduli acquire non-vanishing

VEVs, these become

$$\begin{aligned} W_{eff} \quad &\supset \lambda DQQ + \lambda De^c u^c + \lambda DNd^c + \\ &\qquad + \lambda\overline{D}d^c u^c + \lambda\overline{D}QL, \end{aligned} \tag{3.15}$$

where we considering all couplings to be similar and taking one family for illustrative purposes. Notice that contrary to $SU(5)$ case, there is no extra contribution from rotation of L and H_u as the bilinear term κLH_u is not allowed by gauge invariance.

We estimate the scalar triplet mediated proton decay rate to be

$$\Gamma_p \simeq \frac{|\lambda^2|^2}{16\pi^2}\frac{m_p^5}{m_D^4} \simeq \left(10^{42}\ \text{yrs}\right)^{-1}, \tag{3.16}$$

where we took the mass of the colour triplets to be $m_D \simeq 10^3$ GeV.

Another limit for triplet scalar comes from the cosmological constraints on its decay. As we have seen from proton-decay operators, triplet scalars can decay into quarks. If they start to decay during the Big Bang Nucleosynthesis (BBN) then nucleons could be disassociated, spoiling the predictions for light element abundances. We can estimate another limit on the triplet scalar mass by calculating its lifetime as it decay through the processes $D \to e^c u^c, QQ, QL, d^c u^c$, and we get

$$\Gamma \simeq \lambda^2 m_D \simeq (0.1\ \text{sec})^{-1}, \tag{3.17}$$

which is approximately consistent with BBN constraint. They will also give interesting collider signatures due to their long-lived nature.

3.1.1 The vector-like family splitting

Because the presence of a light vector-like pair coloured triplets spoils unification, we need a workaround that will preserve unification while keeping the presented doublet-triplet problem solution. We achieve this by considering the presence of extra matter that would form a complete GUT irrep with the coloured triplets, and hence restore unification. Unification constraints requires heavy states with equivalent SM gauge numbers, say d^c_X and $\overline{d^c}_X$, that have to be subtracted from the spectrum. This can be achieved by adding a vector-like family pair, $\mathbf{16}_X\overline{\mathbf{16}}_X$, and splitting its mass terms using Wilson line phases.

Furthermore, as the Wilson line breaking pattern is rank-preserving, we still need to break the extra abelian gauge factor $U(1)_X$. This can be achieved if a scalar component of the right-handed conjugated neutrino pair of an extra vector-like family $\mathbf{16}_X$, $\overline{\mathbf{16}}_X$ acquires VEVs. On top of this, this VEV can generate a Majorana mass for the matter right-handed conjugated neutrinos, providing a crucial ingredient for a type I see-saw mechanism.

In order to preserve gauge coupling unification, we notice that the down-type quarks – d^c_X, $\overline{d^c}_X$ – have the same SM quantum numbers as the coloured triplet pair – D, $\overline{D}$ – coming from the $\mathbf{10}$. We take $\mathbf{16}_X$ to be localised along a Wilson line, and find that it transforms under the discrete symmetry as

$$\mathbf{16}_X \to \eta^x \left(\eta^{-3\gamma} L \oplus \eta^{3\gamma+\delta} e^c \oplus \eta^{3\gamma-\delta} N \oplus \eta^{-\gamma-\delta} u^c \oplus \ \eta^{-\gamma+\delta} d^c \oplus \eta^{\gamma} Q\right). \tag{3.18}$$

On the other hand, we let $\overline{\mathbf{16}}_X$ transform without Wilson line phases, $\overline{\mathbf{16}}_X \to \eta^{\overline{x}}\,\overline{\mathbf{16}}_X$, and the condition for the mass term that will split the vector-like family is

$$\overline{d^c}_X d^c_X : x - \gamma + \delta + \overline{x} = 0 \mod N, \tag{3.19}$$

whilst forbidding all the other self couplings that would arise from $\mathbf{16}_X\overline{\mathbf{16}}_X$. The d^c_X, $\overline{d^c}_X$ quarks will then be naturally endowed a GUT scale mass through membrane instantons, provided that the singularities supporting $\mathbf{16}_X$, $\overline{\mathbf{16}}_X$ are close enough to each other in the compactified space. The remaining states of $\mathbf{16}_X$, $\overline{\mathbf{16}}_X$ will have a μ term of order TeV through the Giudice-Masiero mechanism. The coloured triplets – D, $\overline{D}$ – and the light components of $\mathbf{16}_X$, $\overline{\mathbf{16}}_X$ will effectively account for a full vector-like family. The light spectrum is then the one of MSSM in addition to this vector-like family, which in turn preserves unification, with a larger unification coupling at the GUT scale.

3.1.2 R-parity violation

Despite the existence of an effective matter parity symmetry inside $SO(10)$, the presence of a vector-like family will lead to R-parity violating (RPV) interactions though the VEV of the N_X, $\overline{N}_X$ components in the presence of moduli generated interactions. Furthermore, as we will see in detail in Section 3.2, the scalar component of the matter conjugate right-handed neutrino, N, will also acquire a VEV. These VEVs break $SO(10)$ and will inevitably generate RPV. These interactions will mediate proton-decay, enable the lightest supersymmetric particle (LSP) to decay, and generate extra contributions to neutrino masses. In our framework RPV is generic, not only arising from allowed superpotential terms but as well from Kähler interactions involving moduli fields.

The interactions that break R-parity can either be trilinear or bilinear (B-RPV), and have different origins in our framework. The first contribution we can find comes from the tree-level renormalisable superpotential allowed by the discrete symmetry. Since we will encounter $\langle N \rangle \neq 0$, this means that even in a minimal setup, there will be an R-RPV contribution from matter Dirac mass coupling

$$W \supset y_\nu N H_u L \ , \tag{3.20}$$

reading

$$W \supset y_\nu \langle N \rangle H_u L \ . \tag{3.21}$$

Next we turn our attention to the Kähler potential, where interactions otherwise forbidden by the discrete symmetry might arise if there is a modulus with required charge. In such case, there is another contribution arising from the non-vanishing VEVs of N_X, $N \ \overline{N}_X$ in conjugation with moduli VEVs. To see this, notice that in the Kähler potential there are generically interactions of the form

$$K \supset \frac{1}{m_{Pl}} N H_u L + \frac{s}{m_{Pl}^2} \overline{N}_X H_u L + \frac{s}{m_{Pl}^2} \overline{N}_X^\dagger H_u L + \text{ h.c.} \ , \tag{3.22}$$

where while the first term exists in zeroth order in moduli (otherwise there would be no neutrino Dirac mass in the superpotential), the last two are otherwise forbidden by the discrete symmetry, and s denotes a generic modulus for each coupling. These terms will generate contributions to B-RPV as N_X, $N \ \overline{N}_X$, s acquire VEVs.

There are two types of contribution arising from the terms above. The first is generates through the Giudice-Masiero mechanism. As the moduli acquire VEVs, new holomorphic couplings will appear in the superpotential

$$W_{eff,1} = \frac{m_{3/2}}{m_{Pl}} \langle N \rangle H_u L + 0.1 \frac{m_{3/2}}{m_{Pl}} \langle N_X \rangle H_u L + 0.1 \frac{m_{3/2}}{m_{Pl}} \langle \overline{N}_X^\dagger \rangle H_u L \ , \tag{3.23}$$

where $m_{3/2} \simeq \mathcal{O}(10^4)$ GeV, and since $s/m_{Pl} \simeq 0.1$ in M Theory. Notice that in principle we would also have a term in the Kähler potential involving N, but this can be found to be subleading in comparison to the term arising from the Dirac mass Eq. (3.21).

The second contribution arises if the F-terms of the fields N_X, N, $\overline{N}_X$ are non-vanishing. In this case, we expect the appearance of the contributions

$$W_{eff,2} = \frac{\langle F_N \rangle}{m_{Pl}} H_u L + 0.1 \frac{\langle F_{N_X} \rangle}{m_{Pl}} H_u L + 0.1 \frac{\langle F_{\overline{N}_X^\dagger} \rangle}{m_{Pl}} H_u L \ , \tag{3.24}$$

and its magnitude will depend on how much F-breaking provoked by our symmetry breaking mechanism. Here we are considering that the case where $\overline{N}_X^\dagger H_u L$ cannot exist in the Kähler potential in zeroth order in a modulus field.

Putting all together, the B-RPV interactions account to the B-RPV paramter

$$W \supset \kappa H_u L \tag{3.25}$$

with

$$\begin{aligned} \kappa = & \left(y_\nu + \frac{m_{3/2}}{m_{Pl}} \right) \langle N \rangle + 0.1 \frac{m_{3/2}}{m_{Pl}} \langle N_X \rangle + 0.1 \frac{m_{3/2}}{m_{Pl}} \langle \overline{N}_X^\dagger \rangle \\ & + \frac{\langle F_N \rangle}{m_{Pl}} + 0.1 \frac{\langle F_{N_X} \rangle}{m_{Pl}} + 0.1 \frac{\langle F_{\overline{N}_X^\dagger} \rangle}{m_{Pl}} \ , \end{aligned} \tag{3.26}$$

and the relative strength of each contribution is model detail dependent, namely on neutrino Yukawa textures, symmetry breaking details, and F-flatness deviation.

In a similar manner, trilinear RPV couplings will be generated when N, N_X, $\overline{N}_X$, s acquire VEVs. In order to systematically study this, we notice that the trilinear RPV couplings come from the term

$$\mathbf{16\ 16\ 16\ 16},\ \mathbf{16}_X\mathbf{16\ 16\ 16},\ \overline{\mathbf{16}}_X^\dagger\mathbf{16\ 16\ 16} \tag{3.27}$$

as the scalar component of N_X, N acquires non-vanishing VEVs. Notice that the last term lives in the Kähler potential. These are made forbidden at tree-level using the discrete symmetry of the compactified space. However, just like the μ terms and the B-RPV terms shown above, these terms will in general be present in the Kähler potential and will effectively be generated as the moduli acquire VEVs. This happens again through the Giudice-

Masiero mechanism and we will find

$$\mathcal{O}\left(\frac{m_{3/2}}{m_{Pl}^2}(\langle N\rangle + \langle N_X\rangle + \langle \overline{N}_X^\dagger\rangle)\right)\{LLe^c,\ LQd^c,\ u^cd^cd^c\}, \tag{3.28}$$

where $m_{3/2}/m_{Pl} \simeq \mathcal{O}(10^{-14})$. The apparent suppression of trilinear RPV is understood as these terms can only be generated by non-renormalizable terms in an $SO(10)$ context.

Similarly to the B-RPV case, there will be further contributions if the F-terms of N_X, N, $\overline{N}_X$ are non-vanishing. Namely we find

$$\mathcal{O}\left(\frac{\langle F_N\rangle + \langle F_{N_X}\rangle + \langle F_{\overline{N}_X^\dagger}\rangle}{m_{Pl}^2}\right)\{LLe^c,\ LQd^c,\ u^cd^cd^c\}, \tag{3.29}$$

and again we expect these to be sub-leading even if the F-terms are not vanishing.

We see then that the values of all RPV coupling are strictly related to the details of the breaking mechanism employed to break the extra $U(1)_X$. This will be studied in great detail in Section 3.2. Furthermore, the bilinear B-RPV term generates a contribution to the physical neutrino masses [94,110]. The complete picture of neutrino masses, including B-RPV operators, will be discussed in Section 3.3.

We can study now some direct effects of RPV in the dynamics of our class of models. Under the assumption that $\kappa \ll \mu$, performing a small rotation, of $\mathcal{O}(\kappa/\mu)$, in (H_d, L) space, the last term can be absorbed $\mu H_d H_u$. As a consequence, the first two terms will be enhanced by the Yukawa couplings $y_e H_d L e^c$, etc., leading to

$$W \supset y_e\frac{\kappa}{\mu}LLe^c + y_d\frac{\kappa}{\mu}LQd^c + \lambda\frac{v}{m_{Pl}}u^cd^cd^c, \tag{3.30}$$

and we have dropped the $\mathcal{O}(1/m_{Pl})$ contributions to the first two terms since now the Yukawa rotated contributions are much larger. Also, we kept the last term with the parametrization v describing all contributions. These will be very small, for example in the case the VEVs are high-scale, $\langle N_X\rangle \simeq 10^{16}$ GeV, the trilinear RPV coupling strength is of $\mathcal{O}(10^{-16})$. A direct consequence of this result is that proton decay will be slow, even when the $\Delta L = 1$ terms are enhanced.

While the proton is relatively stable, the enhanced terms will provide a decay channel for the LSP, which is now unstable. In the limit that we can take the final states to be massless, and considering that the LSP is a neutralino mainly composed of neutral gauginos, the LSP lifetime through the decay $\tilde{\chi}^0 \to d^c Q L$ can be estimated from a tree-level diagram involving a virtual $\tilde{d}^c$ with mass m_0,[2]

$$\tau_{LSP} \simeq \left(3.9 \times 10^{-15}\right) \left(\frac{\mu}{g_w y_d \kappa}\right)^2 \left(\frac{m_0}{10 \text{ TeV}}\right)^4 \left(\frac{100 \text{ GeV}}{m_{LSP}}\right)^5 \text{sec}, \qquad (3.31)$$

where g_w is a weak gauge coupling. The LSP lifetime is bounded to be either $\tau_{LSP} \lesssim 1$ sec or $\tau_{LSP} \gtrsim 10^{25}$ sec [94, 95], from Big Bang Nucleosynthesis (BBN) and indirect Dark Matter (DM) experiments, respectively. If we take $m_{LSP} \simeq 100$ GeV, $m_0 \simeq 10$ TeV, $y_d = y_b \simeq 10^{-2}$, $g_w \simeq 0.1$, we find that the VEV v_X is constrained to be either

$$\kappa \gtrsim 6 \times 10^{-2} \text{ GeV} \qquad (3.32)$$

$$\text{or } \kappa \lesssim 2 \times 10^{-14} \text{ GeV}, \qquad (3.33)$$

for a short- and long-lived LSP, respectively. In the above estimate we used the fact that the decay involving the bottom Yukawa is the largest contribution to the decay width.

We can use the above result to infer some parametric dependence on the scale of the $U(1)_X$ breaking. If we have the leading contribution to the B-RPV coupling to be $\kappa \simeq \langle N_X \rangle \lambda \Rightarrow \langle N_X \rangle \gtrsim 10^{12}$ GeV. In this case, the LSP is too short lived to be a good DM candidate, but decays quickly enough to not spoil BBN predictions. On the other-hand, a low-scale VEV is bound to be $\langle N_X \rangle \lesssim 1$ GeV in order to allow for a long-lived LSP. This would imply the Abelian gauge boson associated with extra $U(1)_X$ to be light, $m_{Z'} < \mathcal{O}(1)$ GeV. This last scenario is completely excluded from experimental searches.

The lack of a good DM candidate in the visible sector indicates us that DM is realised elsewhere. For instance, it has been recently suggested that in the context of String/M Theory, the generic occurrence of hidden sectors could account for the required DM mechanics [120].

[2]See, for example, the diagrams in [8].

3.1.3 The see-saw mechanism

The relevance of the bounds on the rank-breaking VEV is only fully understood when studying the details of symmetry breaking mechanism and neutrino masses. For example, if we start with an $SO(10)$ invariant theory the Yukawas are unified for each family leading to at least one very heavy Dirac neutrino mass, m_ν^D. However, if the right-handed conjugated neutrino has a heavy Majorana mass, then the physical left-handed neutrino mass will be small through a type I see-saw mechanism. In order to accomplish this, one has to allow the following terms in the superpotential

$$W \supset y_\nu H_u L N + M N N, \tag{3.34}$$

where y_ν are the neutrino Yukawas, L the matter lepton doublets, N the right-handed conjugated neutrino, and M its Majorana mass, which we take $M \gg m_\nu^D = y_\nu \langle H_u \rangle$. With the above ingredients, a mostly left-handed light neutrino will have a physical mass

$$m^\nu_{phy} \simeq -\frac{(m_\nu^D)^2}{M}. \tag{3.35}$$

One of the most appealing features of $SO(10)$ models is that each family is in a **16** which includes a natural candidate for the right-handed conjugated neutrino, the N. In order to employ a type I see-saw mechanism, we need to generate a Majorana mass term for the matter right-handed conjugated neutrino through the operator $W \supset \overline{\mathbf{16}}_X \overline{\mathbf{16}}_X \mathbf{16}\,\mathbf{16}$ [3] leading to the operator

$$\frac{1}{m_{Pl}} \overline{N}_X \overline{N}_X N N \ , \tag{3.36}$$

from which the Majorana mass for the (CP conjugated) right-handed neutrino field N is emerges as

$$M \simeq \frac{\langle \overline{N}_X \rangle^2}{m_{Pl}} \ . \tag{3.37}$$

We can now relate the bounds on the value of the D-flat VEVs $\langle \overline{N}_X \rangle = \langle N_X \rangle$ from both RPV and the requirement of a realistic see-saw mechanism. Since

[3] Given that in M Theory one does not account for irreps larger than the adjoint, this is the lowest order term that can generate a right-handed neutrino Majorana mass.

the physical neutrino mass in type I see-saw mechanism is given by

$$m_\nu^{phy} \simeq \frac{(m_\nu^D)^2}{M} \ , \tag{3.38}$$

assuming $m_\nu^D \simeq \mathcal{O}(100 \text{ GeV})$, and knowing that the upper bound on the neutrino masses $m_\nu^{phy} \lesssim 0.1$ eV, one finds

$$M \gtrsim 10^{14} \text{ GeV} \Rightarrow \langle N_X \rangle \gtrsim 10^{16} \text{ GeV}. \tag{3.39}$$

The above argument suggests that we need to break the $U(1)_X$ close to the GUT scale. Since the Wilson line breaking mechanism is rank-preserving, we need to look for an alternative solution. Although the neutral fermion mass matrix will be considerably more intricate, obscuring the relations and hierarchies amongst different contributions to the neutrino masses, the above estimate motivates the need for a high-scale $U(1)_X$ breaking mechanism.

3.1.4 Effective light families

For a simple SUSY $SO(10)$ model where each family is unified into a single irrep with universal soft masses, it is well known that electroweak symmetry is difficult to break [121–126]. Since the two Higgs soft masses are unified at GUT scale and have similar beta function due to Yukawa unification, either both masses are positive at electroweak scale and symmetry is not broken or both masses are negative and the potential becomes unbounded from below. Another aspect of Yukawa unification problem lies in the fact that low energy spectrum of quarks and leptons requires some degree of tuning in parameter space when their RG runnings are considered. A customary solution to these problems it the use of higher dimensional representations [127], which are not present in our framework.

The EWSB and Yukawa textures issues are naturally solved if each family is not contained in one single complete **16**, but is instead formed of states from different Ultra Violet (UV) complete **16**s. In order to implement this in our framework, first we assume the existence of multiple **16** with independent and different Wilson Line phases, alongside the existence of multiple $\overline{\mathbf{16}}$. Second, we employ Witten's proposal to turn on some vector-like masses such that three effective light **16** survive. Since in M Theory the strength of the Yukawa couplings is given by membrane instantons, and are therefore related to distances between the singularities supporting the respective superfields, by constructing effective families from different UV **16**s one can obtain different Yukawa couplings within each family.

Such solution can be achieved if one considers M complete $\overline{\mathbf{16}}_j$ and $M+3$ complete $\mathbf{16}_i$ UV irreps. Allowing for masses between different states of these UV irreps to appear, one has schematically the mass terms in the superpotential

$$\mathbf{16}_i \mu_{ij} \overline{\mathbf{16}}_j, \tag{3.40}$$

but since $i = 1, ..., M$ while $j = 1, ..., M+3$ the mass matrix μ_{ji} can only have at most rank M and hence there will be three linear combinations composing three **16** that will remain massless. If these masses are truly $SO(10)$ invariant, i.e.

$$\mathbf{16}_i \mu_{ij} \overline{\mathbf{16}}_j = \mu_{ij} \left(Q_i \overline{Q}_j + L_i \overline{L}_j + \ldots \right), \tag{3.41}$$

each effective light family will be $SO(10)$ invariant. Consequently each family will retain unified Yukawa textures, and so this does not solve our problem of splitting the Yukawa couplings within each family.

However, Witten's proposal endows our framework with a GUT breaking discrete symmetry which can be employed to ensure that the superpotential mass matrices between the UV states

$$\mu^Q_{ij} Q_i \overline{Q}_j + \mu^L_{ij} L_i \overline{L}_j + \ldots, \tag{3.42}$$

are not the same, leading to different diagonalisations of Q, L, etc which in turn break the Yukawa $SO(10)$ invariance. In order to accomplish that, take for example that the $\mathbf{16}_i$ absorb distinct and independent Wilson line phases, while $\overline{\mathbf{16}}_j$ do not, i.e. the UV irreps will transform under the discrete symmetry as

$$\mathbf{16}_i \rightarrow \eta^{m_i} \left(\eta^{-3\gamma_i} L_i \oplus \eta^{3\gamma_i + \delta_i} e^c_i \oplus \eta^{3\gamma_i - \delta_i} N_i \oplus \eta^{-\gamma_i - \delta_i} u^c_i \oplus \eta^{-\gamma_i + \delta_i} d^c_i \oplus \eta^{\gamma_i} Q_i \right) \tag{3.43}$$

$$\overline{\mathbf{16}}_j \rightarrow \eta^{\overline{m}_i} \overline{\mathbf{16}}_j, \tag{3.44}$$

and look for solutions for the discrete charges where different states have different mass matrices. Since explicit examples can only be given by solving extensive modular linear systems, which are computationally prohibitive, a fully working example with three light-families is not provided.

3.2 $U(1)_X$ Breaking scenarios and mechanisms

In this section we are interested in implementing a symmetry breaking mechanism for the extra $U(1)_X$ in which the breaking VEV is stabilised at high values, more or less close to the GUT scale. In order to do so, we will look into the D-flat direction of the potential that breaks the extra $U(1)_X$. It was shown [109, 128] that in the D-flat direction, non-renormalisable operators can provide such scenario. In its simplest inception, the Kolda-Martin (KM) mechanism [109] relies on a vector-like pair which lowest order term allowed in the superpotential is non-renormalizable

$$W = \frac{c}{m_{Pl}}(\Phi\bar{\Phi})^2 \tag{3.45}$$

and alongside the soft-term Lagrangian

$$-\mathcal{L}_{soft} = m_{\Phi}^2|\Phi|^2 + m_{\bar{\Phi}}^2|\bar{\Phi}|^2, \tag{3.46}$$

it is immediate to find that along the D-flat direction the potential has a non-trivial minimum which fixes the VEVs at a high scale

$$\Phi^2 = \sqrt{-\frac{(m_{\Phi}^2 + m_{\bar{\Phi}}^2)m_{Pl}^2}{12c}}, \tag{3.47}$$

where if we take $m \simeq 10^4$ the VEVs are estimated at $\Phi \simeq 10^{11}$ GeV.

There are some caveats to this mechanism as presented above. First, there is significant F-breaking as $\langle F \rangle \simeq \mathcal{O}(10^{15})$ GeV. While this is not a problem if the vector-like family does not share gauge interactions with ordinary matter, in our case non-vanishing F-terms will originate undesirable interactions, c.f. Section 3.1.2. We shall therefore focus on F-flat solutions.

Second, the mechanism is not complete in the absence of the full soft-terms Lagrangian, which has to include

$$-\mathcal{L}_{soft} \supset C\frac{1}{m_{Pl}}\Phi^2\bar{\Phi}^2 + \text{ h.c.}. \tag{3.48}$$

As we estimate $C \simeq \mathcal{O}(m_{3/2})$ at the GUT scale from the SUGRA [88], at the VEV scale this term is competing with the non-renormalisable terms in the potential arising from the superpotential, and therefore cannot be ignored.

Finally the model presented differs from ours as μ-terms are generically generated by moduli VEVs even if they are disallowed by the discrete symmetry of the compactified space.

In order to proceed, we turn to a more complete version of the mechanism. To do so, we include the μ-term

$$W = \mu\Phi\bar{\Phi} + \frac{c}{m_{Pl}}(\Phi\bar{\Phi})^2 \tag{3.49}$$

and the more complete soft Lagrangian,

$$-\mathcal{L}_{soft} = m_{\Phi}^2|\Phi|^2 + m_{\bar{\Phi}}^2|\bar{\Phi}|^2 - (B\mu\Phi\bar{\Phi} + \text{ h.c.}) + \frac{C}{m_{Pl}}\Phi^2\bar{\Phi}^2 + \text{ h.c.}. \tag{3.50}$$

Due to the presence of the μ-term, the F-term

$$F_{\Phi} = \mu\bar{\Phi} + \frac{2c}{m_{Pl}}\Phi\bar{\Phi}^2 \tag{3.51}$$

can be set to zero for two different field configurations

$$F_{\Phi} = 0 \Rightarrow \begin{cases} \bar{\Phi} = 0 \\ \Phi\bar{\Phi} = -\frac{\mu m_{Pl}}{2c} \end{cases} \tag{3.52}$$

and the non-trivial VEV can be estimated. Taking $\mu \simeq \mathcal{O}(10^3)$ GeV, this leads to $|\Phi| = 10^{10.5}$ GeV. This looks very similar to the original Kolda-Martin case, with the exception being that the F-term can vanish, and the parametric dependence on the VEV is now on μ instead of a soft-mass. In general there might be a non-SUSY preserving vacuum elsewhere in field space, but we will work under the assumption that the SUSY vacua discovered with this approach are at least stable enough to host phenomenologically viable models.

We wish to assess if we can minimise the potential in this SUSY-preserving field configuration. For that, we need to check if the above field configuration will also extremise the soft-term Lagrangian. To see this we take

$$-\partial_{\Phi}\mathcal{L}_{soft} = m_{\Phi}^2\Phi^* - B\mu\bar{\Phi} + \frac{2C}{m_{Pl}}\Phi\bar{\Phi}^2 = 0 \tag{3.53}$$

and, in the limit the VEVs are real, we find a trivial and a non-trivial solutions

$$-\partial_{\Phi}\mathcal{L}_{soft}=0\Rightarrow\begin{cases}\Phi=0\\ \Phi^2=-\frac{(m_\Phi^2-B\mu)m_{Pl}}{2C}\end{cases}\tag{3.54}$$

and the second one seems very similar to the non-trivial configuration derived through the F-term. In fact, both conditions can be met. To see this, we re-parametrise the soft-terms by factoring out their dimensionful dependence on $m_{3/2}$

$$B\mu=m_{3/2}\mu b\tag{3.55}$$

$$C=m_{3/2}\tilde{c}\tag{3.56}$$

$$m_\Phi=m_{3/2}a,\tag{3.57}$$

where a, b, $\tilde{c}$ are dimensionless, and from SUGRA formulae they are $\mathcal{O}(1)$ at the GUT scale. Of course they will evolve with the scale through RGE evolution, so they need not to be always of the same order. The condition that both the F-flatness and soft-term stabilisation are jointly achieved boils down to be a relation between parameters

$$\frac{\tilde{c}}{c}=\frac{2am_\Phi-\mu b}{\mu},\tag{3.58}$$

which is generically valid.

In order for the above non-trivial VEV be a minimum, we need the trivial VEV solution to account for a maximum. This is to say that the mass matrix for the system $(\Phi,\bar{\Phi}^*)$ evaluated at the origin has a negative eigen-value. In our case this accounts for allowing its determinant to be negative

$$(|\mu|^2+m_\Phi^2)(|\mu|^2+m_{\bar{\Phi}}^2)-B\mu^2<0.\tag{3.59}$$

We notice as well that the above discussion can be immediately extended for the case that the lowest order non-renormalisable term allowed by the discrete symmetry

$$W\supset\frac{c}{m_{Pl}^{2n-3}}(\Phi\bar{\Phi})^n\Rightarrow\Phi\simeq(\mu m_{Pl}^{2n-3})^{\frac{1}{2n-2}}\tag{3.60}$$

happens for $n\geq 2$, and not only for $n=2$. Even so, the presented implementation of the Kolda-Martin mechanism only accounts for a vector-like pair of superfields, while in our case the system breaking the extra $U(1)_X$ is composed of N, N_X, $\overline{N}_X$ states.

Therefore, we want to find similar solutions starting with the superpotential

$$W = \mu^N_{Xm} N\overline{N}_X + \mu^N_X N_X\overline{N}_X + \frac{c_{2,2}}{m_{Pl}}(N\overline{N}_X)^2 + \frac{c_{n,k}}{m_{Pl}^{2n-3}}(N_X\overline{N}_X)^{n-k}(N\overline{N}_X)^k \tag{3.61}$$

where $n \geq 2$ and $k < n$. The third term generates a Majorana mass for the matter right-handed conjugated neutrino, N. The full soft-term Lagrangian for this theory is

$$\begin{aligned} -\mathcal{L}_{soft} =& m_N^2|N|^2 + m_{N_X}^2|N_X|^2 + m_{\overline{N}_X}^2|\overline{N}_X|^2 - (B\mu^N_{Xm}N\overline{N}_X + \text{ h.c.}) \\ & - (B\mu^N_X N_X\overline{N}_X + \text{ h.c.}) + \left(\frac{C_{2,2}}{m_{Pl}}(N\overline{N}_X)^2 + \text{ h.c.}\right) \\ & + \left(\frac{C_{n,k}}{m_{Pl}^{2n-3}}(N_X\overline{N}_X)^{n-k}(N\overline{N}_X)^k + \text{ h.c.}\right) \end{aligned} \tag{3.62}$$

where again $C_{i,j}$ coefficients are $\mathcal{O}(m_{3/2})$ at the GUT scale.

The F-terms now read

$$F_N = \mu^N_{Xm}\overline{N}_X + \frac{2c_{2,2}}{m_{Pl}}N\overline{N}_X^2 + \frac{kc_{n,k}}{m_{Pl}^{2n-3}}N_X^{n-k}N^{k-1}\overline{N}_X^n \tag{3.63}$$

$$F_{N_X} = \mu^N_X\overline{N}_X + \frac{(n-k)c_{n,k}}{m_{Pl}^{2n-3}}N_X^{n-k-1}N^k\overline{N}_X^n \tag{3.64}$$

$$F_{\overline{N}_X} = \mu^N_{Xm}N + \mu^N_X N_X + \frac{2c_{2,2}}{m_{Pl}}N^2\overline{N}_X + \frac{nc_{n,k}}{m_{Pl}^{2n-3}}N_X^{n-k}N^k\overline{N}_X^{n-1} \tag{3.65}$$

which have a significantly more challenging look than the simplified version presented above. Nonetheless, the same conclusions hold. The above F-terms become more tractable for the $k = 0$ and $k = n-1$ cases. In these cases it is possible to get algebraic expressions for the VEVs estimates. For the $k = 0$, the F-flatness conditions alone give us

$$N\overline{N}_X = -\frac{\mu^N_{Xm}m_{Pl}}{2c_{2,2}} \tag{3.66}$$

$$N_X\overline{N}_X = \left(-\frac{\mu^N_X m_{Pl}^{2n-3}}{nc_{n,o}}\right)^{\frac{1}{n-1}} \tag{3.67}$$

while for $k = n-1$, analogous expressions can be obtained

$$|N\overline{N}_X| \simeq (\mu^N_{Xm}m_{Pl}^{2n-3})^{\frac{1}{n-1}} \tag{3.68}$$

$$|N_X\overline{N}_X| \simeq ((\mu^N_X)^{3-n}m_{Pl}^{3n-5})^{\frac{1}{n-1}} \tag{3.69}$$

where the approximations mean we dropped $\mathcal{O}(1)$ parameters and took all μ-terms to be of the same order, which is expected.

In both cases, the ratio between the N_X and N VEV is follows the same dependency on n

$$\left|\frac{N_X}{N}\right| \simeq \left(\frac{m_{Pl}}{\mu}\right)^{\frac{n-2}{n-1}} \simeq \begin{cases} 1 & n=2 \\ 10^{7.5} & n=3 \\ 10^{10} & n=4 \end{cases} \tag{3.70}$$

where we μ is an $\mathcal{O}(\mu_X^N, \mu_{Xm}^N)$ parameter. This result shows that there is a hierarchy between N_X and N VEVs, which is very desirable as N VEVs can generate large B-RPV couplings, c.f. Section 3.1.2.

Just like before, we use the D-flat direction

$$\left|\frac{\overline{N}_X}{N_X}\right|^2 = \left|\frac{N}{N_X}\right|^2 + 1, \tag{3.71}$$

which sets the magnitude of the three VEVs. The results for $k = 0$ and $k = n - 1$ can be immediately estimated algebraically, in contrast to the other cases. The full result of SUSY preserving configurations can be seen in Table 3.2. It is important to note that for $n = 4$, the only viable scenario is for $k = 0$, while for $n = 3$ the $k = 2$ is not viable as there are super-GUT VEVs. In the end we are only interested in the sensible cases, where the VEVs are below the GUT scale and therefore the mechanism is self-consistent.

The SUSY configurations above are expected stabilise the soft-terms Lagrangian just before. The stabilisation conditions are

$$m_N^2 N^* - B\mu_{Xm}^N \overline{N}_X + \frac{2C_{2,2}}{m_{Pl}} N\overline{N}_X^2 + \frac{kC_{n,k}}{m_{Pl}^{2n-3}} N_X^{n-k}\overline{N}_X^n N^{k-1} = 0 \tag{3.72}$$

$$m_{N_X}^2 N_X^* - B\mu_X^N \overline{N}_X + \frac{(n-k)C_{n,k}}{m_{Pl}^{2n-3}} N_X^{n-k-1} N^k \overline{N}_X^n = 0 \tag{3.73}$$

$$m_{\overline{N}_X}^2 \overline{N}_X^* - B\mu_{Xm}^N N - B\mu_X^N N_X + \frac{2C_{2,2}}{m_{Pl}} N^2 \overline{N}_X + \frac{nC_{n,k}}{m_{Pl}^{2n-3}} N_X^{n-k} N^k \overline{N}_X^{n-1} = 0 \tag{3.74}$$

and re-parametrising the dimensionful soft-terms just as before, the above

n	k	N (GeV)	N_X (GeV)	$\overline{N}_X$ (GeV)
2	0	$10^{10.5}$	$10^{10.5}$	$10^{10.5}$
	1	$10^{10.5}$	$10^{10.5}$	$10^{10.5}$
3	0	$10^{6.5}$	$10^{14.25}$	$10^{14.25}$
	1	$10^{10.2}$	$10^{15.5}$	$10^{15.5}$
	2	$10^{10.5}$	10^{18}	10^{18}
4	0	$10^{5.5}$	$10^{15.5}$	$10^{15.5}$
	1	$10^{10.1}$	$10^{16.5}$	$10^{16.5}$
	2	$10^{10.3}$	10^{18}	10^{18}
	3	$10^{10.5}$	$10^{20.5}$	$10^{20.5}$

Table 3.1: Estimate of the magnitude of the VEVs in SUSY vacua for different implementations of the modified Kolda-Martin mechanism. In all cases the scalar component of the (CP conjugated) right-handed neutrino field N develops a VEV, breaking R-parity, in addition to the N_X and $\overline{N}_X$ VEVs.

conditions will resemble the F-flatness conditions in form and so they'll be jointly respected taken the parameters of the theory respect relations between them.

As before, the condition that the above extrema are minima is that the potential has a runaway direction around the origin. This is the same to say that, when close to the origin the potential takes the form

$$V \simeq \mathbf{N}^* \cdot M_N \cdot \mathbf{N} \tag{3.75}$$

with $\mathbf{N} = (N, N_X, \overline{N}^*_X)$, such that M_N at least one negative eigenvalue to account for a run-away behaviour at the trivial extremum. i of the potential in the D-flat direction is achieved by noticing that – for each field direction – at least a quadratic term from the non-renormalisable interactions becomes the leading contribution, while keeping a run-away behaviour at the origin.

3.3 Neutrino-neutralino mass matrix

The different breaking scenarios discussed in the previous section rely on different superpotential terms, which are either present or suppressed depending the discrete symmetry of the compactified G_2 space. Furthermore, the generic presence of a matter field VEV, $\langle N \rangle$, will generate B-RPV terms, as seen in Section 3.1.2. In turn, these provide a new source of neutrino masses which has to be taken into account.

To be more precise we enumerate all the interactions that contribute to neutrino masses. First, we let the matter neutrino to have a Yukawa coupling at tree-level, of the form

$$W_{tree} \supset y_\nu N L H_u \ . \tag{3.76}$$

Next we have to consider the non-renormalizable terms that employ the KM mechanism for each scenario. Alongside this, we also keep a term that can generate a Majorana mass for the matter right-handed conjugated neutrino, N. On top of these, we include a set of non-renormalizable terms involving the Higgses or L-type fields, in first order of $1/m_{Pl}$. The non-renormalizable terms that will affect the neutral fermion mass matrix are then

$$\begin{aligned} W_{non.ren.} \supset & \frac{c_{2,2}}{m_{Pl}} (NN) (\overline{N}_X \overline{N}_X) + \frac{c_{n,k}}{m_{Pl}^{2n-3}} \left(N_X \overline{N}_X\right)^{n-k} \left(N \overline{N}_X\right)^k \\ & + \frac{1}{m_{Pl}} \big(b_1 H_d H_u L \overline{L}_X + b_2 L L \overline{L}_X \overline{L}_X + b_3 H_d H_u L_X \overline{L}_X \\ & + b_4 L L_X \overline{L}_X \overline{L}_X + b_5 L_X L_X \overline{L}_X \overline{L}_X + b_6 H_d H_u N \overline{N}_X \\ & + b_7 L \overline{L}_X N \overline{N}_X + b_8 L_X \overline{L}_X N \overline{N}_X + b_9 H_d H_u N_X \overline{N}_X \\ & + b_{10} L \overline{L}_X N_X \overline{N}_X + b_{11} L_X \overline{L}_X N_X \overline{N}_X \big) \ . \end{aligned} \tag{3.77}$$

The terms that are disallowed by discrete symmetry are generically regenerated as the moduli acquire VEVs. As such, the following Kähler potential terms will have an important contribution for neutrino masses

$$\begin{aligned} K \supset & \frac{s}{m_{Pl}} \overline{L}_X L_X + \frac{s}{m_{Pl}} \overline{L}_X L + \frac{s}{m_{Pl}} \overline{N}_X N_X + \frac{s}{m_{Pl}} \overline{N}_X N + \frac{s}{m_{Pl}} \overline{H}_u H_d \\ & + \frac{s}{m_{Pl}^2} N_X L_X H_u + \frac{s}{m_{Pl}^2} N L H_u + \frac{s}{m_{Pl}^2} N_X L H_u + \frac{s}{m_{Pl}^2} N L_X H_u \\ & + \frac{s}{m_{Pl}^2} \overline{N}_X \overline{L}_X H_d, \end{aligned} \tag{3.78}$$

where s denotes a generic modulus fields that counterbalances the discrete charge. This modulus field needs not to be the same for each coupling. As the moduli acquire VEVs as they are stabilised, the above terms will generate the effective superpotential

$$\begin{aligned}W_{eff} \supset &\mu^L_{XX}\overline{L}_X L_X + \mu^L_{Xm}\overline{L}_X L + \mu^N_{XX}\overline{N}_X N_X + \mu^N_{Xm}\overline{N}_X N + \mu H_u H_d \\ &+ \lambda_{\overline{XX}} H_d \overline{L}_X \overline{N}_X + \lambda_\nu H_u L N + \lambda_{mX} H_u L N_X \\ &+ \lambda_{Xm} H_u L_X N + \lambda_{XX} H_u L_X N_X \end{aligned} \tag{3.79}$$

where the parameters can be estimated to lie inside the orders of magnitude

$$\mu \simeq m_{3/2}\frac{s}{m_{Pl}} \simeq \mathcal{O}(10^3)\text{ GeV} \tag{3.80}$$

$$\lambda \simeq m_{3/2}\frac{s}{m_{Pl}^2} \simeq \mathcal{O}(10^{-15}). \tag{3.81}$$

Therefore, the total superpotential, which includes all the interactions that contribute to the neutral fermion mass matrix is give by

$$W_{total} \supset W_{tree} + W_{non.ren.} + W_{eff}. \tag{3.82}$$

In our framework we have VEVs of the N-type fields that can be significantly large, depending on which implementation of the KM mechanism we assume. As such, B-RPV couplings, mixing Higgses superfields with L-type superfields, appear in the superpotential as

$$\kappa_m H_u L + \kappa_X H_u L_X + \kappa_{\overline{X}} H_d \overline{L}_X \tag{3.83}$$

where the κ-parameters read

$$\kappa_m \simeq (y_\nu + \lambda_\nu)\langle N\rangle + \lambda_{mX}\langle N_X\rangle \tag{3.84}$$

$$\kappa_X \simeq \lambda_{Xm}\langle N\rangle + \lambda_{XX}\langle N_X\rangle \tag{3.85}$$

$$\kappa_{\overline{X}} \simeq \lambda_{\overline{XX}}\langle \overline{N}_X\rangle \tag{3.86}$$

where we are dropping the F-terms contribution as the solutions for our KM mechanism presented in Section 3.2 are aligned in the D and F directions. We also note that we are assuming no tree-level Yukawa couplings involving extra vector-like N_X, $\overline{N}_X$ for the KM scenarios.

Furthermore, the presence of B-RPV induces a sub-EWS VEV on the scalar components of the ν-type fields. In our case, below the EWS, we expect all ν-type scalars to acquire a non-vanishing VEV, generating a mixing between N-type fermions and Higgsinos through

$$\epsilon_m H_u^0 N + \epsilon_X H_u^0 N_X + \epsilon_{\overline{X}} H_d^0 N_{\overline{X}} \tag{3.87}$$

where the coefficients read

$$\epsilon_m \simeq (y_\nu + \lambda_\nu)\langle\nu\rangle + \lambda_{mX}\langle\nu_X\rangle \tag{3.88}$$
$$\epsilon_X \simeq \lambda_{Xm}\langle\nu\rangle + \lambda_{XX}\langle\nu_X\rangle \tag{3.89}$$
$$\epsilon_{\overline{X}} \simeq \lambda_{\overline{XX}}\langle\nu_{\overline{X}}\rangle \tag{3.90}$$

and, as expected, they have the same generic form as the κ-parameters since both set of parameters arise from trilinear, Yukawa, couplings in the superpotential.

Finally, as in the MSSM, the presence of VEVs will mix some fermions with gauginos through kinetic terms, namely the Higgsinos with $\tilde{B}_1$, $\tilde{W}^0$ due to the Higgses VEVs. In our case we also have N-type and ν-type scalar VEVs, which will mix gauginos with matter fermions through kinetic terms. We have, for the $SU(2)$ states,

$$g'\widetilde{B}\langle\widetilde{\nu}_i\rangle\nu_i, \quad g\widetilde{W}^0\langle\widetilde{\nu}_i\rangle\nu_i, \quad g''\widetilde{B}_X\langle\widetilde{\nu}_i\rangle\nu_i \tag{3.91}$$

while for the N-states, which are singlets under the SM gauge group, the mixing with the gaugino of the extra $U(1)_X$ gauge group

$$g''\widetilde{B}_X\langle\widetilde{N}_i\rangle N_i \tag{3.92}$$

where, in both expressions, we used the shorthand $g' = \sqrt{\frac{5}{3}}g_1$ and $g'' = \frac{1}{2\sqrt{10}}g_X$.

With all the above considerations, we can now construct the 11×11 mass matrix for neutral fermions of our model. We define this matrix in the basis

$$\psi = (\widetilde{B}, \widetilde{W}^0, \widetilde{B}_X, \widetilde{H}_d^0, \widetilde{H}_u^0, \nu, \nu_X, \nu_{\overline{X}}, N, N_X, N_{\overline{X}}), \tag{3.93}$$

and it has the schematic form

$$\mathbf{M}_{\chi-\nu} = \begin{pmatrix} \mathbf{M}_{\chi^0}^{5\times5} & \mathbf{M}_{\chi\nu}^{5\times6} \\ (\mathbf{M}_{\chi\nu}^{5\times6})^T & \mathbf{M}_{\nu}^{6\times6} \end{pmatrix}. \tag{3.94}$$

The usually called neutralino part of the matrix includes only mass terms involving gauginos and Higgsinos, and its form is very similar to the MSSM, except we have an extended gauge group with one more $U(1)_X$ factor. It reads

$$\mathbf{M}_{\chi^0}^{5\times 5} = \begin{pmatrix} M_1 & 0 & 0 & -\frac{1}{\sqrt{2}}g'v_d & \frac{1}{\sqrt{2}}g'v_u \\ 0 & M_2 & 0 & \frac{1}{\sqrt{2}}gv_d & -\frac{1}{\sqrt{2}}gv_u \\ 0 & 0 & M_X & -2\sqrt{2}g''v_d & 2\sqrt{2}g''v_u \\ -\frac{1}{\sqrt{2}}g'v_d & \frac{1}{\sqrt{2}}gv_d & -2\sqrt{2}g''v_d & 0 & -\mu \\ \frac{1}{\sqrt{2}}g'v_u & -\frac{1}{\sqrt{2}}gv_u & 2\sqrt{2}g''v_u & -\mu & 0 \end{pmatrix} \tag{3.95}$$

The next block is the one involving terms mixing the usual neutralino states with matter states. As such, they include B-RPV masses that mix matter with higgses. The matrix reads

$$\mathbf{M}_{\chi\nu}^{5\times 6} = \begin{pmatrix} -\frac{1}{\sqrt{2}}g'\nu & -\frac{1}{\sqrt{2}}g'\nu_X & \frac{1}{\sqrt{2}}g'\overline{\nu}_X & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}}g\nu & \frac{1}{\sqrt{2}}g\nu_X & -\frac{1}{\sqrt{2}}g\overline{\nu}_X & 0 & 0 & 0 \\ 3\sqrt{2}g''\nu & 3\sqrt{2}g''\nu_X & -3\sqrt{2}g''\overline{\nu}_X & -5\sqrt{2}g''N & -5\sqrt{2}g''N_X & 5\sqrt{2}g''\overline{N}_X \\ 0 & 0 & \kappa_{\overline{X}} & 0 & 0 & \epsilon_{\overline{X}} \\ \kappa_m & \kappa_X & 0 & \epsilon_m & \epsilon_X & 0 \end{pmatrix} \tag{3.96}$$

where, in order to de-clutter notation, we are taking the fields names as to represent the VEVs. We notice that the B-RPV couplings κ and ϵ are superpotential terms, while the top three rows is generated by kinetic terms only.

The lower-right 6×6 block is purely from the superpotential, and includes only the masses involving ν-type and/or N-type fermions. To obtain the mass, one performs the usual SUSY rule for fermionic masses

$$\left(\mathbf{M}_{\nu}^{6\times 6}\right)_{ij} = -\frac{1}{2}\frac{\partial^2}{\partial\psi_i\partial\psi_j}W_{total} \tag{3.97}$$

where $i,\, j = \{\nu, \nu_X, \overline{\nu}_X, N, N_X, \overline{N}_X\}$.

This 6×6 matrix has three main blocks: the $\nu\nu$ block, νN block, and NN block. Schematically they are arranged, in our basis, as

$$\mathbf{M}_{\nu}^{6\times 6} = -\frac{1}{2}\left(\begin{array}{c|c} M_{\nu\nu} & M_{\nu N} \\ \hline M_{\nu N}^T & M_{NN} \end{array}\right) \tag{3.98}$$

The actual form of the matrix is obtained using the full superpotential in Eq. (3.82). Doing so, one gets the following sub-blocks. First we have the $\nu\nu$ block that has mixing between $\overline{\nu}_X$ and ν, ν_X. In the sub-basis $(\nu, \nu_X, \overline{\nu}_X)$ this reads

$$M_{\nu\nu} = \begin{pmatrix} 0 & 0 & \frac{b_7 \overline{N}_X N}{m_{Pl}} + \frac{b_{10} \overline{N}_X N_X}{m_{Pl}} + \mu^L_{Xm} \\ & 0 & \frac{b_8 \overline{N}_X N}{m_{Pl}} + \frac{b_{11} \overline{N}_X N_X}{m_{Pl}} + \mu^L_X \\ & & 0 \end{pmatrix} \tag{3.99}$$

where we dropped the terms ν^2/m_{Pl}, $v^2_{u/d}/m_{Pl}$ as they are irrelevant and to de-clutter, and since this block is symmetric we omit the lower left triangular part. But notice that the terms with coefficients b_7, b_8, b_{10}, b_{11} can play an important role as they can generate heavy Dirac masses, depending on the KM mechanism.

Next we have the νN block, where one can find the neutrino Dirac masses generated by the Higgses VEV at the EWS. Taking the rows to be along the basis $(\nu, \nu_X, \overline{\nu}_X)$, while the columns along $(N, N_X, \overline{N}_X)$, this block reads

$$M_{\nu N} = \begin{pmatrix} v_u y_\nu + \frac{b_7 \overline{N}_X \overline{\nu}_X}{m_{Pl}} & \frac{b_{10} \overline{N}_X \overline{\nu}_X}{m_{Pl}} & \frac{b_7 N \overline{\nu}_X}{m_{Pl}} + \frac{b_{10} N_X \overline{\nu}_X}{m_{Pl}} \\ \frac{b_8 \overline{N}_X \overline{\nu}_X}{m_{Pl}} & \frac{b_{11} \overline{N}_X \overline{\nu}_X}{m_{Pl}} & \frac{b_8 N \overline{\nu}_X}{m_{Pl}} + \frac{b_{11} N_X \overline{\nu}_X}{m_{Pl}} \\ \frac{b_7 \overline{N}_X \nu}{m_{Pl}} + \frac{b_8 \overline{N}_X \nu_X}{m_{Pl}} & \frac{b_{10} \overline{N}_X \nu}{m_{Pl}} + \frac{b_{11} \overline{N}_X \nu_X}{m_{Pl}} & \frac{b_7 N \nu}{m_{Pl}} + \frac{b_{10} N_X \nu}{m_{Pl}} + \frac{b_8 N \nu_X}{m_{Pl}} + \frac{b_{11} N_X \nu_X}{m_{Pl}} \end{pmatrix} \tag{3.100}$$

where we dropped the sub-leading terms $v_{u/d}\lambda \simeq \mathcal{O}(10^{-12})$ GeV.

Finally we have the NN block, that involves Dirac and Majorana masses generated through the first two terms in Equation (3.77). Ignoring the terms generated by Higgses and sneutrino VEVs, in the sub-basis $(N, N_X, \overline{N}_X)$ this block reads

$$M_{NN} = \begin{pmatrix} \frac{c_{n,k}(k-1)k}{m^{2n-3}_{Pl}} \overline{N}^n_X N^{n-k}_X N^{k-2} + \frac{2c_{2,2}}{m_{Pl}} \overline{N}^2_X & \frac{c_{n,k} k(n-k)}{m^{2n-3}_{Pl}} \overline{N}^n_X N^{k-1} N^{-k+n-1}_X & \frac{c_{n,k} kn}{m^{2n-3}_{Pl}} \overline{N}^{n-1}_X N^{n-k}_X N^{k-1} + \mu^N_{Xm} + \frac{4c_{2,2}}{m_{Pl}} \overline{N}_X N \\ & \frac{c_{n,k}(-k+n-1)(n-k)}{m^{2n-3}_{Pl}} \overline{N}^n_X N^k N^{n-k-2}_X & \frac{c_{n,k} n(n-k)}{m^{2n-3}_{Pl}} \overline{N}^{n-1}_X N^{-k+n-1}_X N^k + \mu^N_{XX} \\ & & \frac{c_{n,k}(n-1)n}{m^{2n-3}_{Pl}} \overline{N}^{n-2}_X N^{n-k}_X N^k + \frac{2c_{2,2}}{m_{Pl}} N^2 \end{pmatrix} \tag{3.101}$$

where the orders of magnitude of each entry will largely depend on which KM scenario is being considered. The matrix is symmetric so only the upper diagonal entries are displayed.

3.3.1 The mass matrix hierarchies

Following the description of the mass matrix above, we will now try to infer the hierarchies between the entries of the matrix. First we notice that, regardless of the case (i.e. the allowed Kolda-Martin operators), the biggest entry in the mass matrix is always in the Gaugino-N mixing block. [4] This result is understandable as we expect the breaking of the extra $U(1)_X$ to transform a chiral superfield and a massless vector superfield into a single massive vector superfield. The degrees of freedom add up correctly, and would mean that below the $U(1)_X$ breaking scale we can take $\tilde{B}_X$ and the linear combination of N-states that break the $U(1)_X$ to be integrated out jointly. The linear combination that breaks the extra $U(1)_X$ depends on the exact values of the VEVs, but we can highlight some characteristics and how the mass-matrix will look like after this is integrated out.

In order to single out the correct liner combination that breaks the extra $U(1)_X$, one can perform a rotation in the last three states – N, N_X, $\overline{N}_X$ – in order to retain only one mixing mass between these states and the $\tilde{B}_X$. In order to do so, in the limit the mass matrix is real, the rotation is

$$U = \begin{pmatrix} 1 & 0 & & & & \\ 0 & 1 & \cdots & & & \\ & \vdots & \ddots & & & \\ & & & \cos(\theta) & -\sin(\theta)\cos(\phi) & \sin(\theta)\sin(\phi) \\ & & & \sin(\theta) & \cos(\theta)\cos(\phi) & -\cos(\theta)\sin(\phi) \\ & & & 0 & \sin(\phi) & \cos(\phi) \end{pmatrix}$$

where the angles are determined by the strength of the mixing mass parameters. For instance, in the $n = 2, k = 0$ Kolda-Martin mechanism presented before, the VEVs of the scalar components of N, N_X, $\overline{N}_X$ are all of same order. In such case, taking $\theta \simeq 3\pi/4$ and $\phi \simeq \arctan\sqrt{2}$ will leave only one state mixing with $\tilde{B}_X$. For the other Kolda-Martin implementations, the N_X, $\overline{N}_X$ VEVs are much larger than N VEV and so we can take $\theta \simeq 0$ with $\phi \simeq 3\pi/4$ to accomplish the same.

The rotation above affects only the last three columns and rows. Since the matrix is unitary (orthogonal in the case the masses are real), the entries of last three columns of a given row will be mixed with at-most order 1 coefficients, and whilst there might be cancellations there will be no order of magnitude enhancements. Once the rotation is performed one can then

[4] The caveat to this statement is if we allow for an order 1 Neutrino Yukawa, in that case the κ entry originated from $y_\nu \langle N \rangle L H_u$, will have the same order of magnitude. But since the B-RPV coupling above does not involve $\tilde{B}_X$, N, N_X, or $\overline{N}_X$, the magnitude of this coupling does not change the following discussion. We will return to B-RPV couplings further below.

integrate out $\tilde{B}_X$ jointly with its Dirac partner. This in turn will affect all the remainder of the matrix. For example, the entry i, j will receive a contribution from integrating out a Dirac mass at position a, b of order

$$-\frac{M_{i3}M_{bj}}{M_{3b}}$$

with some order one coefficients from the rotation. In this case we are setting one of the indices to 3 as this is the position of $\tilde{B}_X$ in our basis. The remaining index, b, refers to the position of the linear combination that breaks the extra $U(1)_X$. If, for example, the breaking linear combination that breaks the extra $U(1)_X$ is mostly composed of N_X, $\overline{N}_X$ states, the main contribution to the ν Majorana mass is given by

$$\frac{b_{10}}{m_{Pl}}\langle\nu\rangle\langle\overline{\nu}_X\rangle \ll 10^{-10}\text{ GeV}$$

even if we let the respective coupling on, i.e. $b_{10} \simeq \mathcal{O}(1)$. Therefore, after the above rotation and integrating out , the mass matrix remains schematically the same, but with the absence of $\tilde{B}_X$ and a linear combination composed of N, N_X, $\overline{N}_X$.

After integrating out the Dirac fermion originated by the breaking, one can see that the Majorana and Dirac masses – generated at the $U(1)_X$ breaking scale – involving only the surviving terms of the N, N_X, $\overline{N}_X$ system are the leading entries of the mass matrix. These are present in the bottom-right-most 2×2 block. These states will then be responsible for a type of see-saw mechanism involving the lighter $SU(2)$ doublet states ν, ν_X, $\overline{\nu}_X$, with EW scale Dirac mass terms. In order to make sense of this see-saw mechanism, the ν-states need to be protected from too much mixing with the remaining gauginos and higgsinos, such that the lightest mass eigenstate is dominantly composed of ν. Actually the mixing between the ν-type states with gauginos is negligible since it is generated by ν-type VEVs and are therefore sub-EWS. But the mixing with Higgsinos is parametrically dependent on N-type VEVs through B-RPV terms, the so called κ mass parameters.

The κ parameters defined in Equations (3.84), (3.85) and (3.85) can have other potentially undesirable consequences as they can spoil Higgs physics. Take for example the matter B-RPV interaction, with κ_m significantly larger than any other mass involving H_u. If were to happens, then L and H_u

superfields would pair up to produce a heavy vector-like pair. Then H_u would be much heavier than the EWS physics and would spoil Higgs physics, where H_u and H_d are identified as a vector-like pair. In order to preserve viable Higgs physics, we need all κ-parameters to be much smaller than the remaining masses appearing in the Higgs potential.

Finally, there is risk that ν, ν_X, $\overline{\nu}_X$ states will mix with each other too much. To see this consider the 3×3 sub-block of the matrix as shown in Eq. (3.99). If all b_i couplings are suppressed, this matrix will maximally mix ν and ν_X through the μ-terms. But it is important to note that while most of the b_i interactions will be generated by Higgses and ν-type VEVs (making them naturally sub-leading even if they are allowed by discrete symmetry) there are two terms that can have important contributions

$$\frac{b_{10}}{m_{Pl}} N_X \overline{N}_X \nu \overline{\nu}_X, \ \frac{b_{11}}{m_{Pl}} N_X \overline{N}_X \nu_X \overline{\nu}_X, \tag{3.102}$$

which for the KM cases can generate Dirac masses much greater than μ-terms if the respective b_i coefficients are unsuppressed. This can then provide a natural mechanism to split ν from ν_X, $\overline{\nu}_X$, if the coupling b_{10} is forbidden while b_{11} is allowed. In this case, we define

$$\mu_{11} = \frac{b_{11}}{m_{Pl}} N_X \overline{N}_X \tag{3.103}$$

and the leading entries for Eq. (3.99) will take the form

$$\begin{pmatrix} 0 & 0 & \mu^L_{Xm} \\ 0 & 0 & \mu_{11} \\ \mu^L_{Xm} & \mu_{11} & 0 \end{pmatrix}, \tag{3.104}$$

which will then lead to ν_X, $\overline{\nu}_X$ to pair up and decouple from ν.

3.3.2 Numerical Results

As the full mass matrix presents an intricate structure of relations and hierarchies between different states, it is ultimately impossible to obtain a simple and revealing analytic expression that describes how one should obtain good neutrino physics. Instead, we perform a numerical scan over space, ensuring that the above constraints are satisfied. In so doing, we divided the analysis into different realisations of the Kolda-Martin mechanism, parametrised by different values of (n,k), corresponding to the scenarios in Table 3.2.

In all the cases, we considered a point of the parameter space to be good if the mass of the lightest eigenstate of the mass matrix, identified as a physical neutrino, has a mass in the range

$$[50, 100] \text{ meV}, \tag{3.105}$$

and in addition that the corresponding eigenstate is mostly composed of the left-handed doublet component ν (i.e. the state arising from $(\nu \ e)^T$). In order to do so, we compute the decomposition of the eigenstate in the original basis

$$|\nu_{light}\rangle = \alpha|\nu\rangle + \dots \tag{3.106}$$

and impose α to be the largest of the coefficients. As discussed in the previous section, the prevalence of ν as the largest component of ν_{light} will depend greatly on the parameters of the mass matrix that mix different states, i.e. Dirac masses. For definiteness, we shall also require that the second lightest mass eigenstate (essentially the lightest non-neutrino-like neutralino) to be at least 100 GeV.

For each example, we only allow the particular desired Kolda-Martin operator while preventing all tree-level Yukawas involving states of the extra vector-like family. Furthermore, unless otherwise stated we assume that all quadratic terms in Eq. (3.77) involving large VEVs are turned off. As expected within the M Theory framework, the disallowed tree-level couplings are regenerated through moduli VEVs, and so the respective coupling strength was set to be of $\mathcal{O}(10^{-15})$. Along the same line, the μ-terms generated by moduli VEVs were set to $\mathcal{O}(1)$ TeV.

Below we will show our findings for the only promising cases, which are $(n,k) = (2,0),\ (2,1),\ (3,0)$. The other (n,k) assignments either returned to little points or no viable correlation to enhance α. This happens as for the $(3,1)$, $(4,0)$ cases, since $N_X,\ \overline{N}_X \simeq 10^{15.5}$ GeV, the B-RPV coupling is generically greater than 1 GeV. As we will see below, the only viable regions of the parameter space coincide with a naturally suppressed B-RPV parameter.

3.3.3 ν component of the lightest state

From the discussion above, we expect the value of α to be correlated with some parameters of the theory. Namely, we expect α to be enhanced if b_{11} is not suppressed and if the B-RPV coupling κ_m is much smaller than any other mass involving Higgsinos. Since any disallowed tree-level coupling can be regenerated through moduli VEVs with a $\lambda \simeq 10^{-15}$ suppression, we started our numerical study by looking at the behaviour of α as we let b_{11} vary in the range

$$b_{11} \in [10^{-15}, 1], \tag{3.107}$$

which, in conjugation with a non-vanishing N_X, $\overline{N}_X$ VEVs will lead to non-vanishing μ_{11} as defined in Eq. (3.103).

In order to assess the strength of the B-RPV term, κ_m, allowed in the regions of the parameter space that return good neutrinos, we also registered the value of κ_m at each point which returned the mass inside the bounds stated.

$(2,0)$ and $(2,1)$ cases

For these two Kolda-Martin implementation cases, the three $(N, N_X, \overline{N}_X)$ VEVs are all of order $\mathcal{O}(10^{10.5})$ GeV. As such, we allowed these VEVs to take values around

$$N,\ N_X,\ \overline{N}_X \in [10^{9.5}, 10^{11.5}] \text{ GeV} \tag{3.108}$$

to cover the range of expected values. Since with these values the mass matrix is very similar for both $(2,0)$ and $(2,1)$ cases, we present them together.

As a consequence of the values of the VEVs above, the μ_{11} Dirac mass between ν_X, $\overline{\nu}_X$, defined in Eq. (3.103), will take values spanning

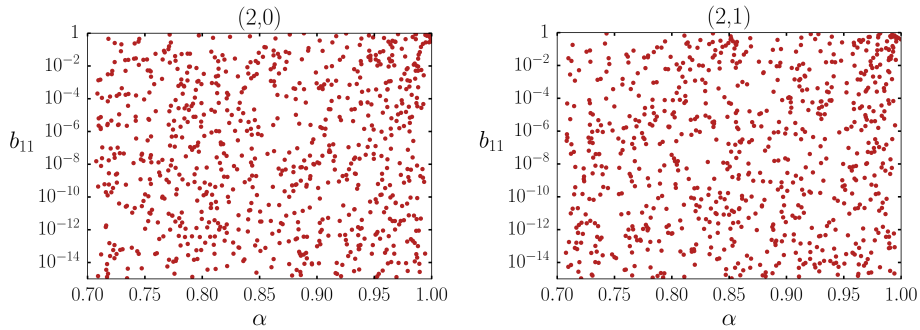
$$\mu_{11} = b_{11}\frac{N_X\overline{N}_X}{m_{Pl}} = b_{11}[10, 10^5] \text{ GeV} \tag{3.109}$$

which means that, only for non-suppressed b_{11} we expect

$$\mu_{11} > \mu^L_{Xm} \tag{3.110}$$

as required to split ν from ν_X, as discussed in Section 3.3.1.

The above considerations indicate us that the mechanism to split ν from ν_X will only work for large values of b_{11}. This can be seen in Figures 3.1(a) and 3.1(b), where a slight agglomeration of points around $(\alpha, b_{11}) \simeq (1, 1)$ can be identified.



(a) $(2, 0)$ case

(b) $(2, 0)$ case

Figure 3.1: Scatter plots showing the amplitude α of the left-handed doublet state ν in the lightest mass eigenstate ν_{light} as b_{11} varies for the $(2, 0)$ and $(2, 1)$ cases. The points are fairly evenly distributed with a slight clustering near the desired value of $\alpha \approx 1$ for $b_{11} \approx 1$.

On the other hand, we find that the κ_m parameter is mostly bounded to be smaller than 1 GeV, as is shown in Figures 3.2(a) and 3.2(b). Although such small values of κ_m are welcome, the fact that there is no clear preference for $\kappa_m \gtrsim 10^{-2}$ GeV suggests this class of models is challenged by BBN constraints, c.f. Eq. (3.32).

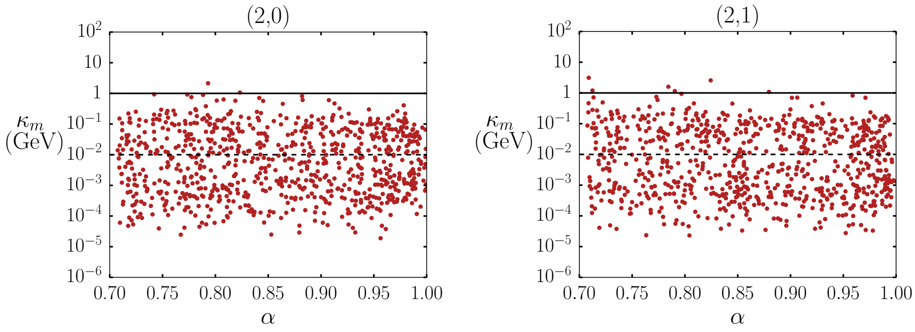
$(3, 0)$ **case**

For the $(3, 0)$ Kolda-Martin realisation, we found much promising results. Since the N_X, $\overline{N}_X$ VEVs are expected to be around $\mathcal{O}(10^{14.25})$ GeV, if we allow them to be in the range

$$N_X,\ \overline{N}_X \in [10^{13.25}, 10^{15.25}] \text{ GeV} \tag{3.111}$$

we find

$$\mu_{11} \in b_{11}[10^{8.25}, 10^{12.25}] \text{ GeV} \tag{3.112}$$



(a) $(2, 0)$ case (b) $(2, 1)$ case

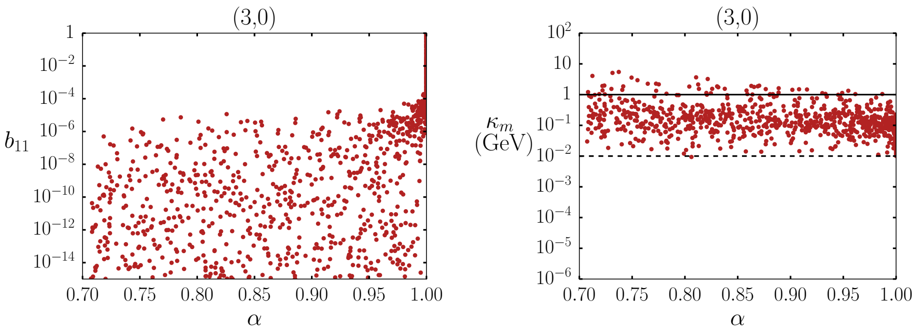
Figure 3.2: Scatter plots showing the amplitude α of the left-handed doublet state ν in the lightest mass eigenstate ν_{light} as κ_m varies for the $(2, 0)$ and $(2, 1)$ cases. The points are fairly evenly distributed with a slight clustering near the desired value of $\alpha \approx 1$. The horizontal dashed line represents the bound on the LSP lifetime, c.f. Eq. (3.32).

which implies that it is natural to achieve

$$\mu_{11} \gg \mu^L_{Xm} \tag{3.113}$$

and consequently ν will decouple easily from the other ν-type states.

The above expectations are confirmed by the numerical results, and the lightest state will be mostly composed of ν even for values of b_{11} below $\mathcal{O}(1)$. This behaviour can be seen in Figure 3.3(a).



(a) Scatter of (α, b_{11}) plane (b) Scatter of (α, κ_m) plane

Figure 3.3: Scatter plots showing the amplitude α of the left-handed doublet state ν in the lightest mass eigenstate ν_{light} as κ_m varies for the $(3, 0)$ case. The points are fairly evenly distributed except for a significant clustering near the desired value of $\alpha \approx 1$ for larger values of b_{11}. The horizontal dashed line represents the bound on the LSP lifetime, c.f. Eq. (3.32). The right panel shows that nearly all the points satisfy $\kappa_m \gtrsim 10^{-2}$ GeV.

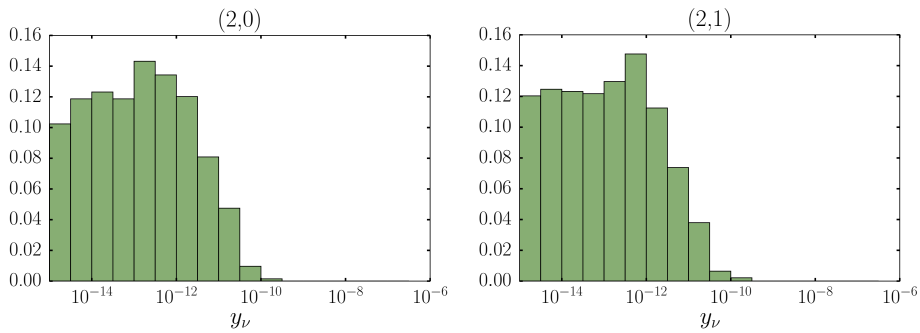
Interestingly, in the (α, κ_m) plane, shown in Figure 3.3(b) we can see again that the mass matrix prefers $\kappa_m < 1$ GeV in order to reproduce a mostly-ν lightest state. This is a nice result which ensures that whenever we have good physical neutrinos, we also find sufficiently suppressed B-RPV. Furthermore, all the good points also suggest $\kappa_m \gtrsim 10^{-2}$ GeV, satisfying the requirement for successful BBN physics, c.f. Eq. (3.32).

3.3.4 Matter Neutrino Yukawas and B-RPV couplings

From the above analysis we learned that for the $(2,0)$, $(2,1)$ and $(3,0)$ cases we expect a non-suppressed b_{11} to enhance the component of ν in the lightest state. As such, we will now consider this coupling to be of order 1 and re-run the analysis for these cases, with the goal being to assess what typical values κ_m and y_ν should take for a successful implementation of the proposed Kolda-Martin mechanism.

$(2,0)$ and $(2,1)$ cases

In Figures 3.4(a) and 3.4(b) we see that the preferred points are those with $y_\nu \lesssim 10^{-10}$. This suggests that for theses cases, the see-saw mechanism does not take a great role in explaining the light neutrino masses.



(a) $(2,0)$ case

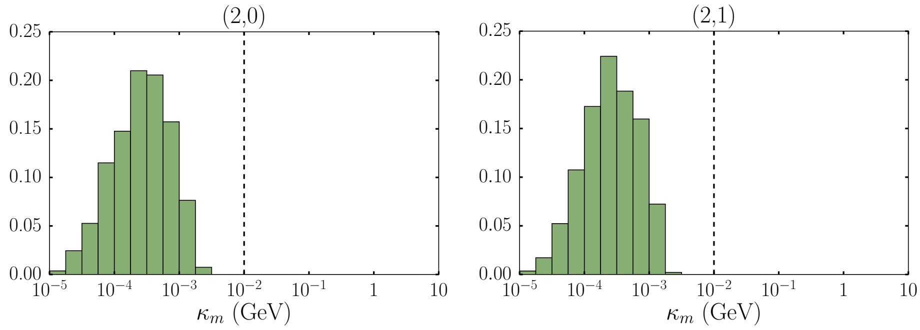
(b) $(2,1)$ case

Figure 3.4: Histograms for the values of y_ν for the $(2,0)$ and $(2,1)$ cases with unsuppressed b_{11}

In Figures 3.5(a) and 3.5(b) we see that for these cases, the B-RPV parameter κ_m is naturally very small. This result is easy to understand, considering the main contribution to κ_m to be

$$\kappa_m \simeq y_\nu v_m,$$

and given the range of values that we are allowing the VEVs to take, κ_m is expected to be small. Unfortunately, all points returning good neutrino physics also return $\kappa_m > 10^{-2}$ GeV, which means that these classes of models spoil BBN, c.f. (3.32). Although not shown here one can also find that κ_X, $\kappa_{\overline{X}}$ parameters, which mix L_X, $\overline{L}_X$ with H_u, H_d respectively, are also constrained to be smaller than 1 GeV.



(a) Histogram for values of κ_m (b) Histogram for values of κ_m

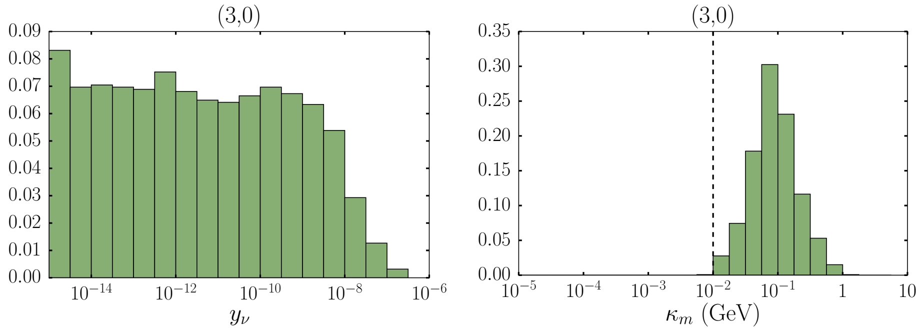
Figure 3.5: Histograms for the values of κ_m for the $(2,0)$ and $(2,1)$ cases with unsuppressed b_{11}. The vertical dashed line represents the bound on the LSP lifetime, c.f. Eq. (3.32).

$(3,0)$ case

For this realisation of the Kolda-Martin mechanism, the results are slightly different but in line with our expectations. In Figure 3.6(a) we can see that the matter Yukawa coupling is allowed to take values larger than in the previous case. This indicates that the see-saw mechanism is having an effect on reducing the contribution of the matter neutrino Dirac mass to the lightest eigenstate.

In Figure 3.6(b) we see that κ_m is bound to be smaller than 1 GeV. The fact that κ_m takes larger values for $(3,0)$ case than for the $n=2$ cases is easily understandable. The main contributions to κ_m are

$$\kappa_m \simeq y_\nu N + \lambda N_X \tag{3.114}$$



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(a) Histogram for values of y_ν

(b) Histogram for values of κ_m. The vertical dashed line represents the bound on the LSP lifetime, c.f. Eq. (3.32).

Figure 3.6: Histograms for the values of y_ν and κ_m for the $(3,0)$ case with unsuppressed b_{11}

where the VEVs are expected as in Table 3.2. These contributions are in general greater than those in $n = 2$ cases, but they are still bounded to be smaller than 1 GeV. This is fortunate, as $\kappa_m \gtrsim 10^{-2}$ GeV and hence this class of models retain the successful predictions of BBN, c.f. (3.32). As before, although not shown here also finds that κ_X, $\kappa_{\overline{X}}$ parameter are also constrained to be smaller than 1 GeV.

3.4 Conclusions and Discussion

We have studied the origin of neutrino mass from $SO(10)$ SUSY GUTs arising from M Theory compactified on a G_2-manifold. We have seen that this problem is linked to the problem of $U(1)_X$ gauge symmetry breaking, which appears in the $SU(5) \times U(1)_X$ subgroup of $SO(10)$, and remains unbroken by the Abelian Wilson line breaking mechanism. In order to break the $U(1)_X$ gauge symmetry, we considered a (generalised) Kolda-Martin mechanism. Our results show that it is possible to break the $U(1)_X$ gauge symmetry without further SUSY breaking while achieving high-scale VEVs that play a crucial role in achieving the desired value of neutrino mass.

The subsequent induced R-parity violation provides an additional source of neutrino mass, in addition to that arising from the seesaw mechanism from non-renormalisable terms. The resulting 11×11 neutrino mass matrix was analysed for one neutrino family and it was shown how a phenomenologically acceptable neutrino mass can emerge. This happens easily for the

$(n, k) = (3, 0)$ case of the Kolda-Martin mechanism we developed. For this class of models, not only is the neutrino masses phenomenologically viable, but also the physical light neutrino eigenstate is almost entirely composed of the left-handed (weakly charged) state ν in the same doublet as the electron (ν, e), as desired. Furthermore, our analysis showed that the B-RPV parameters, which play an important role in neutrino masses and low-energy dynamics, are in the required range, being smaller than 1 GeV. Finally, we notice that contrary to the $n = 2$ cases, the $n = 3$ type of Kolda-Martin mechanism immediately preserves the successful predictions of BBN by allowing the LSP to decay quickly in early universe.

In conclusion, we have shown that $SO(10)$ SUSY GUTs from M Theory on G_2 manifolds provides a phenomenologically viable framework, in which the rank can be broken in the effective theory below the compactification scale, leading to acceptable values of neutrino mass, arising from a combination of the seesaw mechanism and induced R-parity breaking contributions. In principle the mechanism presented here could be extended to three neutrino families and eventually could be incorporated into a complete theory of flavour, based on M Theory $SO(10)$, however such questions are beyond the scope of the present work.

Chapter 4

MSSM from F Theory with Klein Monodromy

Over the last decades string theory GUTs have aroused considerable interest. Recent progress has been focused in F-theory [25, 31] effective models [32]- [36] which incorporate several constraints attributed to the topological properties of the compactified space. Indeed, in this context the gauge symmetries are associated to the singularities of the elliptically fibred compactification manifold. As such, GUT symmetries are obtained as a subgroup of E_8 and the matter content emerges from the decomposition of the E_8-adjoint representation (for reviews see [76]).

As is well known, GUT symmetries, have several interesting features such as the unification of gauge couplings and the accommodation of fermions in simple representations. Yet, they fail to explain the fermion mass hierarchy and more generally, to impose sufficient constraints on the superpotential terms. Hence, depending on the specific model, several rare processes -including proton decay- are not adequately suppressed. We may infer that, a realistic description of the observed low energy physics world, requires the existence of additional symmetry structure of the effective model, beyond the simple GUT group.

Experimental observations on limits regarding exotic processes (such as baryon and lepton number as well as flavour violating cases) and in particular neutrino physics seem to be nicely explained when the Standard Model or certain GUTs are extended to include abelian and discrete symmetries. On purely phenomenological grounds, $U(1)$ as well as non-abelian discrete symmetries such as $A_n, S_n, SLP_2(n)$ and so on, have already been successfully implemented. However, in this context there is no principle to single out the family symmetry group from the enormous number of possible finite groups. Moreover, the choice of the scalar spectrum and the Higgs vev alignments introduce another source of arbitrariness in the models.

In contrast to the above picture, F-theory constructions offer an interesting framework for restricting both the gauge (GUT and discrete) symmetries as well as the available Higgs sector. In the elliptic fibration we end up with an 8-dimensional theory with a gauge group of ADE type. In this work we will focus in the simplest unified symmetry which is $SU(5)$ GUT. In the present geometric picture, the $SU(5)$ GUT is supported by 7-branes wrapping an appropriate (del Pezzo) surface S on the internal manifold, while the number of chiral states is given in terms of a topological index formula. Moreover, there is no use of adjoint Higgs representations since the breaking down to the Standard Model symmetry can occur by turning on a non-trivial $U(1)_Y$ flux along the hypercharge generator [33]. At the same time this mechanism determines exactly the Standard Model matter content. Further, if the flux parameters are judiciously chosen they may provide a solution to the well known doublet triplet splitting problem of the Higgs sector. In short, in F-theory one can in principle develop all those necessary tools to determine the GUT group and predict the matter content of the effective theory.

In the present work we will revisit a class of $SU(5)$ SUSY GUT models which arise in the context of the spectral cover. The reason is that the recent developments in F-theory provide now a clearer insight and a better perspective of these constructions. For example, developments on computations of the Yukawa couplings [129]- [130] have shown that a reasonable mass hierarchy and mixing may arise even if more than one of the fermion families reside on the same matter curve. This implies that effective models left over with only a few matter curves after certain monodromy identifications could be viable and it would be worth reconsidering them. More specifically, we will consider the case of the Klein Group monodromy $V_4 = Z_2 \times Z_2$ [131–134]. Interestingly, with this particular spectral cover, there are two main ways to implement its monodromy action, depending on whether V_4 is a transitive or non-transitive subgroup of S_4, where an action of a group on a non-empty set is called transitive if there is exactly one orbit. A significant part of the present work will be devoted to the viability of the corresponding two kinds of effective models. Another ingredient related to the predictability of the model, is the implementation of R-parity conservation, or equivalently a Z_2 Matter Parity, which can be realised with the introduction of new geometric symmetries [135] respected from the spectral cover. In view of these interesting features, we also investigate in more detail the superpotential, computing higher non-renormalisable corrections, analysing the D and F-flatness conditions and so on.

The chapter is based on our work presented in [3] and is organised as follows. In the next section we give a short description of the derivation of $SU(5)$ GUT in the context of F-theory. In Section 4.2 we describe the action of monodromies and their role in model building. We further focus on the Klein Group monodromy and the corresponding spectral cover factorisations which is our main concern in the present work. In section 4.3 we review a few well known mathematical results and theorems which will be used in model building of the subsequent sections. In section 4.4 we discuss effective field theory models with Klein Group monodromy and implement the idea of matter parity of geometric origin. Section 4.5 deals with the particle spectrum, the Yukawa sector and other properties and predictions of the effective standard model obtained from the above analysis. Finally we present our conclusions in Section 4.6.

4.1 The origin of SU(5) in F-theory

In this section we explain the main setup of these class of models. Focusing in the case under consideration, i.e. the GUT $SU(5)$, the effective four dimensional model can be reached from the maximal E_8 gauge symmetry through the decomposition

$$E_8 \supset SU(5)_{GUT} \times SU(5)_{\perp}$$

In the elliptic fibration, we know that an $SU(5)$ singularity is described by the Tate equation

$$y^2 = x^3 + b_0 z^5 + b_2 x z^3 + b_3 y z^2 + b_4 x^2 z + b_5 x y \tag{4.1}$$

where the homologies of the coefficients in the above equation are given by:

$$[b_k] = \eta - k c_1$$
$$\eta = 6c_1 - t$$

where c_1 and t are the Chern classes of the Tangent and Normal bundles respectively.

The first $SU(5)$ is defining the GUT group of the effective theory, the second $SU(5)_\perp$ incorporates additional symmetries of the effective theory while it can be described in the context of the spectral cover. Indeed, in this picture, one can depict the non-abelian Higgs bundle in terms of the adjoint scalar field configuration [35] and work with the Higgs eigenvalues and eigenvectors. For $SU(n)$ these emerge as roots of a characteristic polynomial of n-th degree. Thus the $SU(5)$ spectral surface C_5 is represented by the fifth order polynomial

$$C_5 = b_0 s^5 + b_1 s^4 + b_2 s^3 + b_3 s^2 + b_4 s + b_5 \; = \; b_0 \prod_{i=1}^{5} (s - t_i) \tag{4.2}$$

Since the roots are associated to the $SU(5)$ Cartan subalgebra their sum is zero, $\sum_i t_i = 0$, thus we have put $b_1 = 0$.

The $5 + \bar{5}$ and $10 + \overline{10}$ representations are found at certain ehnancements of the $SU(5)$ singularity. In particular, for this purpose the relevant quantities are [35]

$$\mathcal{P}_{10} \;=\; b_5 = \prod_i t_i \tag{4.3}$$

$$\mathcal{P}_5 \;=\; b_3^2 b_4 - b_2 b_3 b_5 + b_0 b_5^2 \propto \prod_{i \neq j} (t_i + t_j) \tag{4.4}$$

At the $\mathcal{P}_{10} = 0$ locus the enhanced singularity is $SO(10)$ and the intersection defines the matter curve accommodating the 10's. Fiveplets are found at a matter curve defined at an $SU(6)$ enhancement associated to the locus $\mathcal{P}_5 = 0$.

In practive, we are interested in phenomenologically viable cases where the spectral cover splits in several pieces. Consider for example the splitting expressed through the breaking chain

$$E_8 \to SU(5) \times SU(5) \to SU(5) \times U(1)^4$$

where we assumed breaking of $SU(5)_\perp$ along the Cartan, $\sum_i t_i = 0$. The presence of four $U(1)$'s in the effective theory leaves no room for a viable superpotential, since many of the required terms, including the top Yukawa coupling, are not allowed. Nevertheless, monodromies imply various kinds of symmetries among the roots t_i of the spectral cover polynomial which can be used to relax these tight constraints. The particular relations among these roots depend on the details of the compactification and the geometrical properties of the internal manifold. All possible ways fall into some Galois group which in the case of $SU(5)_\perp$ is a subgroup of the corresponding Weyl group, i.e., the group S_5 of all possible permutations of the five Cartan weights t_i. It is obvious that there are several options and each of them leads to models with completely different properties and predictions of the effective field theory. Before starting our investigations on the effective models derived in the context of the aforementioned monodromy, we will analyse these issues in the next section.

4.2 The Importance of Monodromy

For the $SU(5)_{GUT}$ model, we have seen that any possible remnant symmetries (embedable in the E_8 singularity) must be contained in $SU(5)_\perp$. We have already explained that in the spectral cover approach we quotient the theory by the action of a finite group [131] which is expected to descend from a geometrical symmetry of the compactification. Starting form an C_5 spectral cover, the local field theory is determined by the $SU(5)$ GUT group and the Cartan subalgebra of $SU(5)_\perp$ modulo the Weyl group $W(SU(5)_\perp)$. This is the group S_5, the permutation symmetry of five elements which in the present case correspond to the Cartan weights $t_{1,\ldots 5}$.

Depending on the geometry of the manifold, C_5 may slit to several factors

$$C_5 = \prod_j C_j$$

For the present work, we will assume two cases where the compactification geometry implies the splitting of the spectral cover to $C_5 \to C_4 \times C_1$ and $C_5 \to C_2 \times C_2' \times C_1$. Assuming the splitting $C_5 \to C_4 \times C_1$, the permutation takes place between the four roots, say $t_{1,2,3,4}$, and the corresponding Weyl group is S_4. Notwithstanding, under specific geometries to be discussed in the subsequent sections, the monodromy may be described by the Klein group $V_4 \in S_4$. The latter might be either transitive or non transitive. This second case implies the spectral cover factorisation $C_4 \to C_2 \times C_2'$. As a result, there are two non-trivial identifications acting on the pairs (t_1, t_2) and (t_3, t_4) respectively while both are described by the Weyl group

	S_4 cycles	Transitive A_4	Transitive V_4
4-cycles	(1234), (1243), (1324), (1342), (1423), (1432)	No	No
3-cycles	(123), (124), (132), (134), (142), (143), (234), (243)	Yes	No
2+2-cycles	(12)(34), (13)(24), (14)(23)	Yes	Yes
2-cycles	(12), (13), (14), (23), (24), (34)	No	No
1-cycles	e	Yes	Yes

Table 4.1: A summary of the permutation cycles of S_4, categorised by cycle size and whether or not those cycles are contained within the transitive subgroups A_4 and V_4. This also shows that V_4 is necessarily a transitive subgroup of A_4, since it contains all the $2+2$-cycles of A_4 and the identity only.

$W(SU(2)_\perp) \sim S_2$. Since $S_2 \sim Z_2$, we conclude that in this case the monodromy action is the non-transitive Klein group $Z_2 \times Z_2$. Next, we will analyse the basic features of these two spectral cover factorisations.

4.2.1 S_4 Subgroups and Monodromy Actions

The group of all permutations of four elements, S_4, has a total of 24 elements.[1] These include 2,3,4 and 2+2-cycles, all of which are listed in Table 4.1. These cycles form a total of 30 subgroups of S_4, shown in Figure 4.1. Of these there are those subgroups that are transitive subgroups of S_4: the whole group, A_4, D_4, Z_4 and the Klein group.

We focus now in compactification geometries consistent with the Klein group monodromy $V_4 = Z_2 \times Z_2$. We observe that there are three non-transitive V_4 subgroups within S_4 and only one transitive subgroup. This transitive Klein group is the subgroup of the A_4 subgroup. Considering Table 4.1, one can see that A_4 is the group of all even permutations of four elements and the transitive V_4 is that group excluding 3-cycles. The significance of this is that in the case of Galois theory, to be discussed in Section 4.3, the transitive subgroups A_4 and V_4 are necessarily irreducible quartic polynomials, while the non-transitive V_4 subgroups of S_4 should be reducible.

In terms of group elements, the Klein group that is transitive in S_4 has the elements:

$$\{(1), (12)(34), (13)(24), (14)(23)\} \tag{4.5}$$

which are the 2+2-cycles shown in Table 4.1 along with the identity. On

[1]The order of an S_N group is given by $N!$

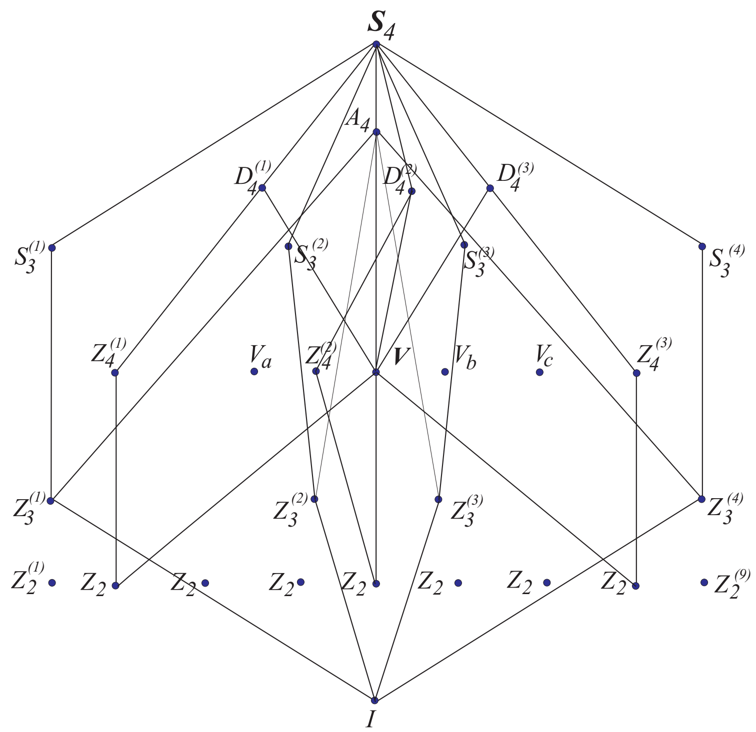


Figure 4.1: Pictorial summary of the subgroups of S_4, the group of all permutations of four elements - representative of the symmetries of a cube.

the other hand, the non-transitive Klein groups within S_4 are isomorphic to the subgroup containing the elements:

$$V_4 = \{(1), (12), (34), (12)(34)\} \tag{4.6}$$

The distinction here is that the group elements are not all within one cycle, since we have two 2-cycles and one 2+2-cycle. These types of subgroup must lead to a factorisation of the quartic polynomial, as we shall discuss in Section 4.3. Referring to Figure 4.1, these Klein groups are the nodes disconnected from the web, while the central V_4 is the transitive group.

4.2.2 Spectral cover factorisation

In this section we will discuss the two possible factorisations of the spectral surface compatible with a Klein Group monodromy, in accordance with the previous analysis. In particular, we shall be examining the implications of a monodromy action that is a subgroup of S_4 - the most general monodromy action relating four weights. In particular we shall be interested in the chain of subgroups $S_4 \rightarrow A_4 \rightarrow V_4$, which we shall treat as a problem in Galois theory.

$\mathcal{C}_4$ spectral cover

This set of monodromy actions require the spectral cover of Equation (4.2) to split into a linear part and a quartic part:

$$\mathcal{C}_5 \to \mathcal{C}_4 \times C_1 \tag{4.7}$$
$$\mathcal{C}_5 \to (a_5 s^4 + a_4 s^3 + a_3 s^2 + a_2 s + a_1)(a_6 + a_7 s) \tag{4.8}$$

The $b_1 = 0$ condition must be enforced for $SU(5)$ tracelessness. This can be solved by consistency in Equation (4.8),

$$b_1 = a_5 a_6 + a_4 a_7 = 0\,. \tag{4.9}$$

Let us introduce a new section a_0, enabling one to write a general solution of the form:

$$\begin{aligned} a_4 &= \pm a_0 a_6 \\ a_5 &= \mp a_0 a_7 \end{aligned}$$

Upon making this substitution, the defining equations for the matter curves are:

$$\mathcal{C}_5 \ := a_1 a_6 \tag{4.10}$$
$$\mathcal{C}_{10} \ := (a_2^2 a_7 + a_2 a_3 a_6 \mp a_0 a_1 a_6^2)(a_3 a_6^2 + (a_2 a_6 + a_1 a_7) a_7) \tag{4.11}$$

which is the most general, pertaining to an S_4 monodromy action on the roots. By consistency between Equation (4.8) and Equation (4.2), we can calculate that the homologies of the coefficients are:

$$\begin{aligned} [a_i] &= \eta - (i-6)c_1 - \chi \\ i &= 1,2,3,4,5 \\ [a_6] &= \chi \\ [a_7] &= c_1 + \chi \\ [a_0] &= \eta - 2(c_1 + \chi) \end{aligned}$$

The $C_2 \times C_2' \times C_1$ case

If the V_4 actions are not derived as transitive subgroups of S_4, then the Klein group is isomorphic to:

$$A_4 \not\supset V_4 : \ \{(1), (12), (12)(34), (34)\} \tag{4.12}$$

This is not contained in A_4, but is admissible from the spectral cover in the form of a monodromy $C_5 \to C_2 \times C_2' \times C_1$.

Then, the $\mathbf{10} \in SU(5)$ GUT ($\in SU(5)_\perp$) spectral cover reads

$$C_5 : (a_1 + a_2 s + a_3 s^2)(a_4 + a_5 s + a_6 s^2)(a_7 + a_8 s) \tag{4.13}$$

We may now match the coefficients of this polynomial in each order in s to the ones of the spectral cover with the b_k coefficients:

$$\begin{aligned}
b_0 &= a_{368} \\
b_1 &= a_{367} + a_{358} + a_{268} \\
b_2 &= a_{357} + a_{267} + a_{348} + a_{258} + a_{168} \\
b_3 &= a_{347} + a_{257} + a_{167} + a_{248} + a_{158} \\
b_4 &= a_{247} + a_{157} + a_{148} \\
b_5 &= a_{147}
\end{aligned}$$

following the notation $a_{ijk} = a_i a_j a_k$ in [132]. In order to find the homology classes of the new coefficients a_i, we match the coefficients of the above polynomial in each order in s to the ones of Equation (4.2) such that we get relations of the form $b_k = b_k(a_i)$.

Comparing to the homologies of the unsplit spectral cover, a solution for the above can be found for the homologies of a_i. Notice, though, that we have 6 well defined homology classes for b_j with only 8 a_i coefficients, therefore the homologies of a_i are defined up to two homology classes:

$$\begin{aligned}
&= \chi_1 + (n-3)c_1 \\
[a_{n=4,5,6}] &= \chi_2 + (n-6)c_1 \\
[a_{n=7,8}] &= \eta + (n-8)c_1 - \chi_1 - \chi_2
\end{aligned}$$

We have to enforce the $SU(5)$ tracelessness condition, $b_1 = 0$. An Ansatz for the solution was put forward in [132],

$$
\begin{aligned}
a_2 &= -c(a_6a_7 + a_5a_8) \\
a_3 &= ca_6a_8
\end{aligned}
$$

which introduces a new section, c, whose homology class is completely defined by

$$[c] = -\eta + 2\chi_1 \tag{4.14}$$

With this anstaz for the solution of the splitting of spectral cover, P_{10} reads

$$P_{10} = a_1a_4a_7 \tag{4.15}$$

while the P_5 splits into

$$
\begin{aligned}
P_5 =& a_5(a_6a_7 + a_5a_8)(a_6a_7^2 + a_8(a_5a_7 + a_4a_8))(a_1 - a_5a_7c) \\
&(a_1^2 - a_1(a_5a_7 + 2a_4a_8)c + a_4(a_6a_7^2 + a_8(a_5a_7 + a_4a_8))c^2),
\end{aligned} \tag{4.16}
$$

An extended analysis of this interesting case will be presented in the subsequent sections.

4.3 A little bit of Galois theory

So far, we have outlined the properties of the most general spectral cover with a monodromy action acting on four of the roots of the perpendicular $SU(5)$ group. This monodromy action is the Weyl group S_4, however a subgroup is equally admissible as the action. Transitive subgroups are subject to the theorems of Galois theory, which will allow us to determine what properties the coefficients of the quartic factor of Equation (4.8) must have in order to have roots with a particular symmetry [136]- [137]. In this chapter we shall focus on the Klein group, $V_4 \cong Z_2 \times Z_2$. As already mentioned, the transitive V_4 subgroup of S_4 is contained within the A_4 subgroup of S_4, and so shall share some of the same requirements on the coefficients.

While Galois theory is a field with an extensive literature to appreciate, in the current work we need only reference a handful of key theorems. We shall omit proofs for these theorems as they are readily available in the literature and are not required for the purpose at hand.

Theorem 1. *Let K be a field with characteristic different than 2, and let $f(X)$ be a separable, polynomial in $K(X)$ of degree n.*

- *If $f(X)$ is irreducible in $K(X)$ then its Galois group over K has order divisible by n.*
- *The polynomial $f(X)$ is* irreducible *in $K(X)$* if and only if *its Galois group over K is a transitive subgroup of S_n.*

This first theorem offers the key point that any polynomial of degree n, that has non-degenerate roots, but cannot be factorised into polynomials of lower order with coefficients remaining in the same field must necessarily have a Galois group relating the roots that is S_n or a transitive subgroup thereof.

Theorem 2. *Let K be a field with characteristic different than 2, and let $f(X)$ be a separable, polynomial in $K(X)$ of degree n. Then the Galois group of $f(X)$ over K is a subgroup of A_n* if and only if *the discriminant of f is a square in K.*

As already stated, we are interested specifically in transitive V_4 subgroups. Theorem 2 gives us the requirement for a Galois group that is A_4 or its transitive subgroup V_4 - both of which are transitive in S_4. Note that no condition imposed on the coefficients of the spectral cover should split the polynomial ($C_4 \to C_2 \times C_2$), due to Theorem 1. We also know by Theorem 2 that both V_4 and A_4 occur when the discriminant of the polynomial is a square, so we necessarily require another mechanism to distinguish the two.

4.3.1 The Cubic Resolvent

The so-called Cubic Resolvent, is an expression for a cubic polynomial in terms of the roots of the original quartic polynomial we are attempting to classify. The roots of the cubic resolvent are defined to be,

$$x_1 = (t_1 t_2 + t_3 t_4),\ x_2 = (t_1 t_3 + t_2 t_4),\ x_3 = (t_1 t_4 + t_2 t_3) \tag{4.17}$$

and one can see that under any permutation of S_4 these roots transform between one another. However, in the event that the polynomial has roots

Group	Discriminant	Cubic Resolvent
S_4	$\Delta \neq \delta^2$	Irreducible
A_4	$\Delta = \delta^2$	Irreducible
D_4/Z_4	$\Delta \neq \delta^2$	Reducible
V_4	$\Delta = \delta^2$	Reducible

Table 4.2: A summary of the conditions on the partially symmetric polynomials of the roots and their corresponding Galois group.

with a Galois group relation that is a subgroup of S_4, the roots need not all lie within the same orbit. The resolvent itself is defined trivially as:

$$(x-(t_1t_2+t_3t_4))(x-(t_1t_3+t_1t_4))(x-(t_1t_4+t_3t_2)) = g_3x^3+g_2x^2+g_1x+g_0 \tag{4.18}$$

The coefficients of this equation can be determined by relating of the roots to the original $\mathcal{C}_4$ coefficients. This resulting polynomial is:

$$g(x) = a_5^3x^3 - a_3a_5^2x^2 + (a_2a_4 - 4a_1a_5)\,a_5x - a_2^2a_5 + 4a_1a_3a_5 - a_1a_4^2 \tag{4.19}$$

Note that this may be further simplified by making the identification $y = a_5x$.

$$g(y) = y^3 - a_3y^2 + (a_2a_4 - 4a_1a_5)\,y - a_2^2a_5 + 4a_1a_3a_5 - a_1a_4^2 \tag{4.20}$$

If the cubic resolvent is factorisable in the field K, then the Galois group does not contain any three cycles. For example, if the Galois group is V_4, then the roots will transform only under the 2+2-cycles:

$$V_4 \subset A_4 = \{(1),(12)(34),(13)(24),(14)(23)\}\,. \tag{4.21}$$

Each of these actions leaves the first of the roots in Equation (4.17) invariant, thus implying that the cubic resolvent is reducible in this case. If the Galois group were A_4, the 3-cycles present in the group would interchange all three roots, so the cubic resolvent is necessarily irreducible. This leads us to a third theorem, which classifies all the Galois groups of an irreducible quartic polynomial (see also Table 4.2).

Theorem 3. *The Galois group of a quartic polynomial $f(x) \in K$, can be described in terms of whether or not the discriminant of f is a square in K and whether or not the cubic resolvent of f is reducible in K.*

4.4 Klein monodromy and the origin of matter parity

In this section we will analyse a class of four-dimensional effective models obtained under the assumption that the compactification geometry induces a $Z_2 \times Z_2$ monodromy. As we have seen in the previous section, there are two distinct ways to realise this scenario, which depends on whether the corresponding Klein group is transitive or non-transitive. In the present work we will choose to explore the rather promising case where the monodromy Klein group is non-transitive. In other words, this essentially means that the spectral cover admits a $C_2 \times C_2' \times C_1$ factorisation. The case of a transitive Klein group is more involved and it is not easy to obtain a viable effective model, hence we will consider this issue in a future work.

Hence, turning our attention to the non-transitive case, the basic structure of the model obtained in this case corresponds to one of those initially presented in [131] and subsequently elaborated by other authors [132]- [134]. This model possesses several phenomenologically interesting features and we consider it is worth elaborating it further.

4.4.1 Analysis of the $Z_2 \times Z_2$ model

To set the stage, we first present a short review of the basic characteristics of the model following mainly the notation of [132]. The $Z_2 \times Z_2$ monodromy case implies a $2+2+1$ splitting of the spectral fifth-degree polynomial which has already been given in (4.13). Under the action (4.12), for each element, either x_2 and x_3 roots defined in (4.17) are exchanged or the roots are unchanged.

The effective model is characterised by three distinct 10 matter curves, and five 5 matter curves. The matter curves, along with their charges under the perpendicular surviving $U(1)$ and their homology classes are presented in table 4.3.

Knowing the homology classes associated to each curve allows us to determine the spectrum of the theory through the units of abelian fluxes that pierce the matter curves. Namely, by turning on a flux in the $U(1)_X$ directions, we can endow our spectrum with chirality and break the perpendicular group. In order to retain an anomaly free spectrum we need to

Curve	$U(1)$ Charge	Defining Equation	Homology Class
10_1	t_1	a_1	$-2c_1+\chi_1$
10_3	t_3	a_4	$-2c_1+\chi_2$
10_5	t_5	a_7	$\eta-c_1-\chi_1-\chi_2$
5_1	$-2t_1$	$a_6a_7+a_5a_8$	$\eta-c_1-\chi_1$
5_{13}	$-t_1-t_3$	$a_1^2-a_1(a_5a_7+2a_4a_8)c+a_4(a_6a_7^2+a_8(a_5a_7+a_4a_8))c^2$	$-4c_1+2\chi_1$
5_{15}	$-t_1-t_5$	$a_1-a_5a_7c$	$-2c_1+\chi_1$
5_{35}	$-t_3-t_5$	$a_6a_7^2+a_8(a_5a_7+a_4a_8)$	$2\eta-2c_1-2\chi_1-\chi_2$
5_3	$-2t_3$	a_5	$-c_1+\chi_2$

Table 4.3: Matter curves and their charges and homology classes

allow for

$$\sum M_5+\sum M_{10}=0, \tag{4.22}$$

where M_5 (M_{10}) denote $U(1)_X$ flux units piercing a certain 5 (10) matter curve.

A non-trivial flux can also be turned on along the Hypercharge. This will allow us to split GUT irreps, which will provide a solution for the doublet-triplet splitting problem. In order for the Hypercharge to remain ubroken, the flux configuration should not allow for a Green-Schwarz mass, which is accomplished by

$$F_Y\cdot c_1=0,\ \ F_Y\cdot\eta=0. \tag{4.23}$$

For the new, unspecified, homology classes, χ_1 and χ_2 we let the flux units piercing them to be

$$F_Y\cdot\chi_1=N_1,\ \ F_Y\cdot\chi_2=N_2, \tag{4.24}$$

where N_1 and N_2 are flux units, and are free parameters of the theory.

For a fiveplet, 5 one can use the above construction as a *doublet-triplet splitting solution* as

$$\begin{aligned} n(3,1)_{-1/3}-n(\overline{3},1)_{1/3}&=M_5,\\ n(1,2)_{1/2}-n(1,2)_{-1/2}&=M_5+N, \end{aligned} \tag{4.25}$$

where the states are presented in the SM basis. For a 10 we have

$$\begin{aligned} n(3,2)_{1/6}-n(\overline{3},2)_{-1/6}&=M_{10},\\ n(\overline{3},1)_{-2/3}-n(3,1)_{2/3}&=M_{10}-N,\\ n(1,1)_1-n(1,1)_{-1}&=M_{10}+N. \end{aligned} \tag{4.26}$$

In the end, given a value for each M_5, M_{10}, N_1, N_2 the spectrum of the theory is fully defined as can be seen in Table 4.4

Curve	Weight	Homology	N_Y	N_X	Spectrum
10_1	t_1	$-2c_1+\chi_1$	N_1	M_{10_1}	$M_{10_1}Q+(M_{10_1}-N_1)u^c+(M_{10_1}+N_1)e^c$
10_3	t_3	$-2c_1+\chi_2$	N_2	M_{10_3}	$M_{10_3}Q+(M_{10_3}-N_2)u^c+(M_{10_3}+N_2)e^c$
10_5	t_5	$\eta-c_1-\chi_1-\chi_2$	$-N_1-N_2$	M_{10_5}	$M_{10_5}Q+(M_{10_5}+N)u^c+(M_{10_5}-N)e^c$
5_1	$-2t_1$	$\eta-c_1-\chi_1$	$-N_1$	M_{5_1}	$M_{5_1}\overline{d^c}+(M_{5_1}-N_1)\overline{L}$
5_{13}	$-t_1-t_3$	$-4c_1+2\chi_1$	$2N_1$	$M_{5_{13}}$	$M_{5_{13}}\overline{d^c}+(M_{5_{13}}+2N_1)\overline{L}$
5_{15}	$-t_1-t_5$	$-2c_1+\chi_1$	N_1	$M_{5_{15}}$	$M_{5_{15}}\overline{d^c}+(M_{5_{15}}+N_1)\overline{L}$
5_{35}	$-t_3-t_5$	$2\eta-2c_1-2\chi_1-\chi_2$	$-2N_1-N_2$	$M_{5_{35}}$	$M_{5_{35}}\overline{d^c}+(M_{5_{35}}-2N_1-N_2)\overline{L}$
5_3	$-2t_3$	$-c_1+\chi_2$	N_2	M_{5_3}	$M_{5_3}\overline{d^c}+(M_{5_3}+N_2)\overline{L}$

Table 4.4: Matter curve spectrum. Note that $N = N_1 + N_2$ has been used as short hand.

4.4.2 Matter Parity

It was first proposed before [135], in local F Theory constructions there are geometric discrete symmetries of the spectral cover that manifest on the final field theory. To see this note that the spectral cover equation is invariant, up to a phase, under the transformation $\sigma : s \mapsto \sigma(s)$ of the fibration coordinates, such that

$$\begin{aligned} s &\to s e^{i\phi} \\ b_k &\to b_k e^{i\chi} e^{i(k-6)\phi}. \end{aligned} \tag{4.27}$$

As detailed in [134] this can be associated to a symmetry of the matter fields residing on the various curves. We can use the equations relating $b_k \propto a_l a_m a_n$, with $l+m+n=17$, to find the transformation rules of the a_k such that the spectral cover equation respects the symmetry (4.27). This implies that the coefficients a_n should transform as

$$a_n \to e^{i\psi_n} e^{i(11/3-n)\phi} a_n. \tag{4.28}$$

We now note that the above transformations can be achieved by a Z_N symmetry if $\phi = 3\frac{2\pi}{N}$. In that case one can find, by looking at the equations

(4.14) for $b_k \propto a_l a_m a_n$ that we have

$$\begin{aligned}
\psi_1 &= \psi_2 = \psi_3 \\
\psi_4 &= \psi_5 = \psi_6 \\
\psi_7 &= \psi_8
\end{aligned} \tag{4.29}$$

meaning that there are three distinct cycles, and

$$\chi = \psi_1 + \psi_4 + \psi_7. \tag{4.30}$$

Furthermore, the section c introduced to split the matter conditions (4.14) has to transform as

$$c \rightarrow e^{i\phi_c} c, \tag{4.31}$$

with

$$\phi_c = \psi_3 - \psi_6 - \psi_7 + \left(-\frac{11}{3} + 11\right)\phi \quad , \quad \phi_c = \psi_2 - \psi_5 - \psi_8 + \left(-\frac{11}{3} + 11\right)\phi \tag{4.32}$$

We can now deduce what would be the matter parity assignments for Z_2 with $\phi = 3(2\pi/2)$. Let $p(x)$ be the parity of a section (or products of sections), x. We notice that there are relations between the parities of different coefficients, for example one can easily find

$$\frac{p(a_1)}{p(a_2)} = -1 \tag{4.33}$$

amongst others, which allow us to find that all parity assignments depend only on three independent parities

$$\begin{aligned}
p(a_1) &= i \\
p(a_4) &= j \\
p(a_7) &= k \\
p(c) &= ijk,
\end{aligned} \tag{4.34}$$

where we notice that $i^2 = j^2 = k^2 = +$. The parities for each matter curve – both in form of a function of i, j, k and all possible assignments – can are presented in the table 4.5.

As such, models from $Z_2 \times Z_2$ are completely specified by the information present in table 4.6.

Curve	Charge	Parity	All possible assignments							
10_1	t_1	i	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$
10_3	t_3	j	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$
10_5	t_5	k	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$
5_1	$-2t_1$	jk	$+$	$+$	$-$	$-$	$-$	$-$	$+$	$+$
5_{13}	$-t_1 - t_3$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
5_{15}	$-t_1 - t_5$	i	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$
5_{35}	$-t_3 - t_5$	j	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$
5_3	$-2t_3$	$-j$	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$+$

Table 4.5: All possible matter parity assignments

Curve	Charge	Matter Parity	Spectrum
10_1	t_1	i	$M_{10_1}Q + (M_{10_1} - N_1)u^c + (M_{10_1} + N_1)e^c$
10_3	t_3	j	$M_{10_3}Q + (M_{10_3} - N_2)u^c + (M_{10_3} + N_2)e^c$
10_5	t_5	k	$M_{10_5}Q + (M_{10_5} + N_1 + N_2)u^c + (M_{10_5} - N_1 - N_2)e^c$
5_1	$-2t_1$	jk	$M_{5_1}\overline{d^c} + (M_{5_1} - N_1)\overline{L}$
5_{13}	$-t_1 - t_3$	$+$	$M_{5_{13}}\overline{d^c} + (M_{5_{13}} + 2N_1)\overline{L}$
5_{15}	$-t_1 - t_5$	i	$M_{5_{15}}\overline{d^c} + (M_{5_{15}} + N_1)\overline{L}$
5_{35}	$-t_3 - t_5$	j	$M_{5_{35}}\overline{d^c} + (M_{5_{35}} - 2N_1 - N_2)\overline{L}$
5_3	$-2t_3$	$-j$	$M_{5_3}\overline{d^c} + (M_{5_3} + N_2)\overline{L}$

Table 4.6: All the relevant information for model building with $Z_2 \times Z_2$ monodromy

4.4.3 The Singlets

For the singlets on the GUT surface we start by looking at the splitting equation for singlet states, P_0. For $SU(5)$ these are found to be

$$\begin{aligned}
P_0 = {} & 3125b_5^4b_0^4 + 256b_4^5b_0^3 - 3750b_2b_3b_5^3b_0^3 + 2000b_2b_4^2b_5^2b_0^3 + 2250b_3^2b_4b_5^2b_0^3 \\
& - 1600b_3b_4^3b_5b_0^3 - 128b_2^2b_4^4b_0^2 + 144b_2b_3^2b_4^3b_0^2 - 27b_3^4b_4^2b_0^2 + 825b_2^2b_3^2b_5^2b_0^2 \\
& - 900b_2^3b_4b_5^2b_0^2 + 108b_3^5b_5b_0^2 + 560b_2^2b_3b_4^2b_5b_0^2 - 630b_2b_3^3b_4b_5b_0^2 \\
& + 16b_2^4b_4^3b_0 - 4b_2^3b_3^2b_4^2b_0 + 108b_2^5b_5^2b_0 + 16b_2^3b_3^3b_5b_0 - 72b_2^4b_3b_4b_5b_0
\end{aligned}$$

Applying the solution for the $Z_2 \times Z_2$ monodromy from Eq.(4.14,4.14) the above splits into 13 factors as follows

$$\begin{aligned}
P_0 = {} & a_6^2a_8^2c\left(a_5^2 - 4a_4a_6\right)\left(a_8(a_4a_8 - a_5a_7) + a_6a_7^2\right)^2\left(c(a_5a_8 + a_6a_7)^2 - 4a_1a_6a_8\right) \\
& (a_1a_8 + a_7c(a_5a_8 + 2a_6a_7))^2\left(a_1^2a_6 + a_1c\left(-2a_4a_6a_8 + 2a_5^2a_8 + a_5a_6a_7\right)\right. \\
& \left.+a_4c^2\left(a_6a_8(a_4a_8 + 3a_5a_7) + 2a_5^2a_8^2 + a_6^2a_7^2\right)\right)^2
\end{aligned}$$

Their homologies and geometric parities can be founded by applying the results from the previous section, and are presented in Table 4.7

Equation	Power	Charge	Homology Class	Matter Parity
a_6	2	$\pm(t_1 - t_3)$	χ_2	j
a_8	2	$\pm(t_1 - t_5)$	$\eta - \chi_1 - \chi_2$	$-k$
c	1	0	$-\eta + 2\chi_1$	ijk
$a_5^2 - \ldots$	1	0	$-2c_1 + \chi_2$	$+$
$a_8(a_4 a_8 - \ldots$	2	$\pm(t_3 - t_5)$	$2\eta - 2c_1 - 2\chi_1 - \chi_2$	j
$c(a_5 a_8 + \ldots$	1	0	$\eta - 2c_1$	ijk
$(a_1 a_8 + \ldots$	2	$\pm(t_1 - t_5)$	$\eta - 2c_1 - \chi_2$	$-ik$
$(a_1^2 a_6 + \ldots$	2	$\pm(t_1 - t_3)$	$-4c_1 + 2\chi_1 + \chi_2$	j

Table 4.7: Defining euations, multiplicity, homologies, matter parity, and perpendicular charges of singlet factors

4.4.4 Application of Geometric Matter Parity

We study now the implementation of the explicit $Z_2 \times Z_2$ monodromy model presented in [132] alongside the matter parity proposed above. The model under consideration is defined by the flux data

$$\begin{aligned} N_1 &= M_{5_{15}} = M_{5_{35}} = 0 \\ N_2 &= M_{10_3} = M_{5_1} = 1 = -M_{10_5} = -M_{5_3} \\ M_{10_1} &= 3 = -M_{5_{13}} \end{aligned} \tag{4.35}$$

which leads to the spectrum presented in Table 4.8 alongside all possible geometric parities.

Curve	Charge	Spectrum	All possible assignments							
10_1	t_1	$3Q + 3u^c + 3e^c$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$
10_3	t_3	$Q + 2e^c$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$
10_5	t_5	$-Q - 2e^c$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$
5_1	$-2t_1$	$D_u + H_u$	$+$	$+$	$-$	$-$	$-$	$-$	$+$	$+$
5_{13}	$-t_1 - t_3$	$-3\overline{d^c} - 3\overline{L}$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
5_{15}	$-t_1 - t_5$	0	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$
5_{35}	$-t_3 - t_5$	$-\overline{H}_d$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$
5_3	$-2t_3$	$-\overline{D}_d$	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$+$

Table 4.8: Spectrum and allowed geometric parities for the $Z_2 \times Z_2$ monodromy model

Name	Charge	All possible assignments							
θ_1	$\pm(t_1 - t_3)$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$
θ_2	$\pm(t_1 - t_5)$	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$+$
θ_3	0	$+$	$-$	$-$	$+$	$-$	$+$	$+$	$-$
θ_4	0	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
θ_5	$\pm(t_3 - t_5)$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$
θ_6	0	$+$	$-$	$-$	$+$	$-$	$+$	$+$	$-$
θ_7	$\pm(t_1 - t_5)$	$-$	$+$	$-$	$+$	$+$	$-$	$+$	$-$
θ_8	$\pm(t_1 - t_3)$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$

Table 4.9: Singlet curves and their perpendicular charges and geometric parity

Inspecting Table 4.8 one can arrive at some conclusions. For example, looking at the spectrum from each curve it's immediate that all matter is contained in 10_1 and 5_{13}, while the Higgses come from 5_1 and 5_{35}, and the rest of the states are exotics that come in vector-like pairs. Immediately we see that there will be R-Parity violating terms since 5_{13} has positive parity.

In order to fully describe the model one also has to take into account the singlets, whose perpendicular charges and all possible geometric parities can be seen in Table 4.9, where we included the same field with its charge conjugated partner in the same row - i.e. θ_i has the same parity as $\overline{\theta}_i$.

Of the possible combinations $\{i, j, k\}$ for the geometric parity assignments, the only choices that allow for a tree-level top quark mass are:

$$\begin{aligned} \{i, j, k\} &= \{+, +, +\} \\ \{i, j, k\} &= \{-, +, +\} \\ \{i, j, k\} &= \{+, -, -\} \\ \{i, j, k\} &= \{-, -, -\} \end{aligned} \tag{4.36}$$

The option that most closely resembles the R-parity imposed in the model [132] corresponds to the choice $i = -$, $j = k = +$. However, if R-parity has a geometric origin the parity assignments of matter curves cannot be arbitrarily chosen. Using the Mathematica package presented in [138], it is straight forward to produce the spectrum of operators up to an arbitrary mass dimension. One can readily observe that its implementation allows a number of operators that could cause Bilinear R-Parity Violation (BRPV) at unacceptably high rates. For example, the lowest order operators are:

$$H_uL\theta_1,\ H_uL\theta_8,\ H_uL\theta_1\theta_4,\ H_uL\theta_4\theta_8,\ H_uL\overline{\theta}_5\theta_7 \tag{4.37}$$

with higher order operators also present, amplifying the scale of the problem. In order to avoid problems, we must forbid vacuum expectations for a number of singlets, especially θ_1 and θ_8. This does not immediately appear to be a model killing issue, however we must look to the exotic masses. Considering the Higgs triplets $D_{u/d}$, the only mass terms are:

$$\begin{aligned} &D_uD_d\theta_1\theta_1\theta_3,\ D_uD_d\theta_1\theta_1\theta_6,\ D_uD_d\theta_1\theta_2\overline{\theta}_5,\ D_uD_d\theta_1\theta_3\theta_8,\\ &D_uD_d\theta_1\theta_6\theta_8,\ D_uD_d\theta_2\overline{\theta}_5\theta_8,\ D_uD_d\theta_3\theta_8\theta_8,\ D_uD_d\theta_6\theta_8\theta_8 \end{aligned} \tag{4.38}$$

As can be seen each of these terms contains θ_1 or θ_8. Since these are required to have no vacuum expectation value, it follows that the Higgs triplets cannot become massive. Since this is a highly disfavoured feature, we must rule out this model.

It transpires that in a similar way, all the models with this flux assignment must be ruled out when we apply this geometric parity. This is due to the tension between BRPV terms and exotic masses, which seem to always be at odds in models with this novel parity. This motivates one to search for models without any exotics, as these models will not have any constraining features coming from exotic masses, and we shall analyse one such model in the subsequence.

4.5 Deriving the MSSM with the seesaw mechanism

The parameter space of models is very large, given the number of reasonable combinations of fluxes, multiplicities and choices of geometric parities. There are a number of ways to narrow the parameter space of any search, for example requiring that there be no exotics present in the spectrum, or contriving there to be only one tree-level Yukawa (to enable a heavy top quark), or perhaps allowing only models with standard matter parity be considered. This last option is quite difficult to search for, but can be constructed.

Let us make a choice for the flux parameters that enables this standard

Curve	Charge	Matter Parity	Spectrum
10_1	t_1	$-$	$Q_3 + Q_2 + u_3^c + 3e^c$
10_3	t_3	$+$	$-$
10_5	t_5	$-$	$Q_1 + u_2^c + u_1^c$
5_1	$-2t_1$	$-$	$-\overline{L}_1$
5_{13}	$-t_1 - t_3$	$+$	$2H_u$
5_{15}	$-t_1 - t_5$	$-$	$-\overline{d}_2^c - \overline{d}_1^c - \overline{L}_2$
5_{35}	$-t_3 - t_5$	$+$	$-2\overline{H}_d$
5_3	$-2t_3$	$-$	$-\overline{d}_3^c - \overline{L}_3$
$1_{15} = \theta_7$	$t_1 - t_5$	$-$	N_R^a
$1_{51} = \overline{\theta}_7$	$t_5 - t_1$	$-$	N_R^b

Table 4.10: Matter content for a model with the standard matter parity arising from a geometric parity assignment.

matter parity:

$$\begin{aligned}
&\{N_1 = 1, N_2 = 0\} \\
&M_{10_1} = -M_{5_{13}} = 2 \\
&M_{10_5} = -M_{5_3} = 1 \\
&M_{10_3} = M_{5_1} = M_{5_{13}} = M_{5_{35}} = 0 \\
&i = -j = k = -
\end{aligned} \tag{4.39}$$

The matter spectrum of this model is summarised in Table 4.10. With this choice, Table 4.9 will select the column with only the singlets θ_7 and $\overline{\theta}_7$ having a negative matter parity. Provided this singlet does not acquire a vacuum expectation it will then be impossible for Bilinear R-parity violating terms due to the nature of the parity assignments. This will also conveniently give us candidates for right-handed neutrinos, θ_7 and $\overline{\theta}_7$.

4.5.1 Yukawas

Having written down a spectrum that has the phenomenologically preferred R-parity, we must now examine the allowed couplings of the model. The model only allows Yukawa couplings to arise at non-renormalisable levels, however the resulting couplings give rise to rank three mass matrices. This is because the perpendicular group charges must be canceled out in any Yukawa couplings. For example, the Yukawa arising from $10_1 \cdot 10_1 \cdot 5_{13}$ has a charge $t_1 - t_3$, which may be canceled by the $\theta_{1/8}$ singlets. Consider the

Yukawas of the Top sector,

$$\begin{aligned}10_1 \cdot 10_1 \cdot 5_{13} \cdot (\overline{\theta}_1 + \overline{\theta}_8) &\to (Q_3 + Q_2) u_3 H_u (\overline{\theta}_1 + \overline{\theta}_8) \\ 10_1 \cdot 10_5 \cdot 5_{13} \cdot \theta_5 &\to ((Q_3 + Q_2)(u_1 + u_2) + Q_1 u_3) H_u \theta_5 \\ 10_5 \cdot 10_5 \cdot 5_{13} \cdot \theta_2 \cdot \theta_5 &\to Q_1 (u_1 + u_2) H_u \theta_2 \theta_5\end{aligned} \tag{4.40}$$

where the numbers indicate generations (1, 2 and 3). The resulting mass matrix should be rank three, however the terms will not all be created equally and the rank theorem [139] should lead to suppression of operators arising from the same matter curve combination:

$$M_{u,c,t} \sim v_u \begin{pmatrix} \epsilon\theta_2\theta_5 & \theta_2\theta_5 & \theta_5 \\ \epsilon^2\theta_5 & \epsilon\theta_5 & \epsilon(\overline{\theta}_1 + \overline{\theta}_8) \\ \epsilon\theta_5 & \theta_5 & \overline{\theta}_1 + \overline{\theta}_8 \end{pmatrix} \tag{4.41}$$

where each element of the matrix has some arbitrary coupling constant. We use here ϵ to denote suppression due to the effects of the Rank Theorem [139] for Yukawas arising from the same GUT operators. The lightest generation will have the lightest mass due to an extra GUT scale suppression arising from the second singlet involved in the Yukawa. There are a large number of corrections at higher orders in singlet VEVs, which we have not included here for brevity. These corrections will also be less significant compared to the lowest order contributions.

In a similar way, the Down-type Yukawa couplings arise as non-renormalisable operators, coming from four different combinations. The operators for this sector often exploit the tracelessness of $SU(5)$, so that the sum of the GUT charges must vanish. The leading order Yukawa operators,

$$\begin{aligned}10_1 \cdot \overline{5}_3 \cdot \overline{5}_{35} \cdot (\theta_1 + \theta_8) &\to (Q_3 + Q_2) d_3 H_d (\theta_1 + \theta_8) \\ 10_1 \cdot \overline{5}_{15} \cdot \overline{5}_{35} \cdot \theta_5 &\to (Q_3 + Q_2)(d_1 + d_2) H_d \theta_5 \\ 10_5 \cdot \overline{5}_3 \cdot \overline{5}_{35} \cdot (\theta_1 + \theta_8)\theta_2 &\to Q_1 d_3 H_u (\theta_1 + \theta_8)\theta_2 \\ 10_5 \cdot \overline{5}_{15} \cdot \overline{5}_{35} \cdot \theta_2 \cdot \theta_5 &\to Q_1 (d_1 + d_2) H_u \theta_2 \theta_5\end{aligned} \tag{4.42}$$

The resulting mass matrix will, like in the Top sector, be a rank three matrix, with a similar form:

$$M_{d,s,b} \sim v_d \begin{pmatrix} \epsilon\theta_2\theta_5 & \theta_2\theta_5 & (\theta_1+\theta_8)\theta_2 \\ \epsilon^2\theta_5 & \epsilon\theta_5 & \epsilon(\theta_1+\theta_8) \\ \epsilon\theta_5 & \theta_5 & \theta_1+\theta_8 \end{pmatrix} \tag{4.43}$$

The structure of the Top and Bottom sectors appears to be quite similar in this model, which should provide a suitable hierarchy to both sectors.

The Charged Leptons will have a different structure to the Bottom-type quarks in this model, due primarily to the fact the e_i^c matter is localised on one GUT tenplet. The Lepton doublets however all reside on different $\overline{5}$ representations, which will fill out the matrix in a non-trivial way, with the operators:

$$\begin{gathered} 10_1 \cdot \overline{5}_3 \cdot \overline{5}_{35} \cdot (\overline{\theta}_1 + \overline{\theta}_8) \rightarrow L_3(e_1^c + e_2^c + e_3^c)H_d(\overline{\theta}_1 + \overline{\theta}_8) \\ 10_1 \cdot \overline{5}_{15} \cdot \overline{5}_{35} \cdot \theta_5 \rightarrow L_2(e_1^c + e_2^c + e_3^c)H_d\theta_5 \\ 10_1 \cdot \overline{5}_1 \cdot \overline{5}_{35} \cdot (\theta_1 + \theta_8) \rightarrow L_1(e_1^c + e_2^c + e_3^c)H_d(\theta_1 + \theta_8) \end{gathered} \tag{4.44}$$

The mass matrix for the Charged Lepton sector will be subject to suppressions arising due to the effects discussed above.

4.5.2 Neutrino Masses

The spectrum contains two singlets that do not have vacuum expectation values, which protects the model from certain classes of dangerous operators. These singlets, $\theta_7/\overline{\theta}_7$, also serve as candidates for right-handed neutrinos. Let us make the assignment $\theta_7 = N_R^a$ and $\overline{\theta}_7 = N_R^b$. This gives Dirac masses from two sources, the first of which involve all lepton doublets and N_R^a:

$$\begin{gathered} \overline{5}_3 \cdot 5_{13} \cdot \theta_7 \cdot \overline{\theta}_5 \rightarrow L_3 N_R^a H_u \overline{\theta}_5 \\ \overline{5}_{15} \cdot 5_{13} \cdot \theta_7 \cdot (\overline{\theta}_1 + \overline{\theta}_8) \rightarrow L_2 N_R^a H_u (\overline{\theta}_1 + \overline{\theta}_8) \\ \overline{5}_1 \cdot 5_{13} \cdot \theta_7 \cdot (\overline{\theta}_1 + \overline{\theta}_8) \cdot \theta_2 \rightarrow L_1 N_R^a H_u (\overline{\theta}_1 + \overline{\theta}_8)\theta_2 \end{gathered} \tag{4.45}$$

This generates a hierarchy for neutrinos, however the effect will be mitigated by the operators arising from the N_R^b singlet:

$$
\begin{aligned}
\overline{5}_3 \cdot 5_{13} \cdot \overline{\theta}_7 \cdot (\overline{\theta}_1 + \overline{\theta}_8) \cdot \theta_2 &\to L_3 N_R^b H_u (\overline{\theta}_1 + \overline{\theta}_8)\theta_2 \\
\overline{5}_{15} \cdot 5_{13} \cdot \overline{\theta}_7 \cdot \theta_2 \cdot \theta_5 &\to L_2 N_R^b H_u \theta_2 \theta_5 \\
\overline{5}_1 \cdot 5_{13} \cdot \overline{\theta}_7 \cdot \theta_5 &\to L_1 N_R^b H_u \theta_5
\end{aligned} \tag{4.46}
$$

If all these Dirac mass operators are present in the low energy spectrum, then the neutrino sector should have masses that mix greatly. This is compatible with our understanding of neutrinos from experiments, which requires large mixing angles compared to the other sectors.

A light mass scale for the neutrinos can be generated using the see-saw mechanism [23, 103–106], which requires large right-handed Majorana masses to generate light physical left-handed Majorana neutrino mass at low values. The singlets involved in this scenario has perpendicular charges that must be canceled out, as with the quark and charged lepton operators. Fortunately, this can be achieved, in part due to the presence of $\theta_2/\overline{\theta}_2$, which have the same charge combinations as $N_R^{a,b}$. The leading contribution to the mass term will come from the off diagonal $\theta_7\overline{\theta}_7$ term, however there are diagonal contributions:

$$
\frac{\langle \theta_2 \rangle^2}{\Lambda}\overline{\theta}_7^2 \;+\; \frac{\langle \overline{\theta}_2 \rangle^2}{\Lambda}\theta_7^2 \;+\; M\theta_7\overline{\theta}_7 \tag{4.47}
$$

Two right-handed neutrinos are sufficient to generate the appropriate physical light masses for the neutrinos required by experimental constraints [140, 141].

4.5.3 Other Features

An interesting property of this model is the requirement of extra Higgs fields. Due to the flux factors, under doublet-triplet splitting it is necessary to have two copies of the up and down-type Higgs. This insures that the model is free of Higgs colour triplets, D_u/D_d in the massless spectrum, while also allowing the designation of + parity to Higgs matter curves. As a consequence of this, the μ-term for the Higgs mass would seem to give four Higgs operators of the same mass: $M_{ij}H_u^i H_d^j$, with $i, j = 1, 2$. However, since for both the up and down-types there are two copies on the matter curve, we can call upon the rank theorem [139]. Consider the operator for the μ-term:

$$5_{13} \cdot \overline{5}_{35} \cdot \theta_2 \to M_{ij} H_u^i H_d^j \to M \begin{pmatrix} \epsilon_h^2 & \epsilon_h \\ \epsilon_h & 1 \end{pmatrix} \begin{pmatrix} H_u^1 \\ H_u^2 \end{pmatrix} \begin{pmatrix} H_d^1 & H_d^2 \end{pmatrix} \quad (4.48)$$

This operator will give a mass that is naturally large for one generation of the Higgs, while the second mass should be suppressed due to non-perturbative effects. This is parameterised by ϵ_h, which is required to be sufficiently small as to allow a Higgs to be present at the electroweak scale, while the leading order Higgsl must be heavy enough to remain at a reasonably high scale and not prevent unification. Thus we should have a light Higgs boson as well as a heavier copy that is as of yet undetected.

The spectrum is free of the Higgs colour triplets D_u/D_d, however we must still consider operators of the types $QQQL$ and $d^c u^c u^c e^c$, since the colour triplets may appear in the spectrum at the string scale. Of these types of operator, most are forbidden at leading order due to the charges of the perpendicular group. However, one operator is allowed and we must consider this process:

$$10_1 10_1 10_5 \overline{5}_3 \to (Q_3+Q_2)(Q_3+Q_2)Q_1 L_3 + (u_2^c+u_1^c)u_3^c d_3^c(e_1^c+e_2^c+e_3^c) \quad (4.49)$$

None of the operators arising are solely first generation matter, however due to mixing they may contribute to any proton decay rate. The model in question only has one of each type of Higgs matter curve, which means any colour triplet partners must respect the perpendicular charges of those curves. The result of this requirement is that the vertex between the initial quarks and the D_u colour triplet must also include a singlet to balance the charge, with the same requirement for the final vertex. The resulting operator should be suppressed by some high scale where the colour triplets are appearing in the spectrum - Λ_s. The most dangerous contribution of this operator can be assume to be the $Q_2 Q_1 Q_2 L_3$ component, which will mix most strongly with the lightest generation. It can be estimated that, given the quark mixing and the mixing structure of the charged Leptons in particular, the suppression scale should be in the region $\sim 10^{4-6}\Lambda_s$. This estimate seems to place the suppression of proton decay at too small a value, though not wildly inconsistent.

However, if we consider Figure 4.5.3, we can see that while the external legs of this process give an overall adherence to the charges of the per-

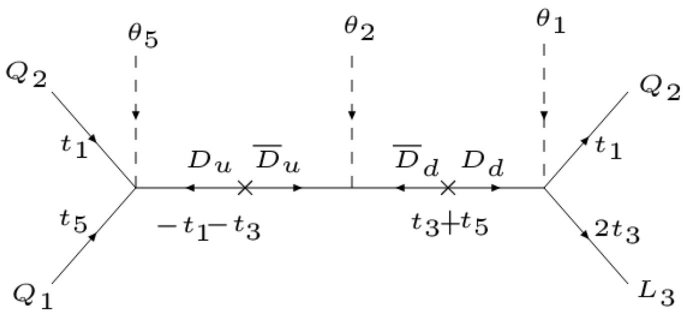


Figure 4.2: Proton Decay graph

pendicular group charges, the vertices require singlet contributions. For example, the first vertex is $Q_2Q_1D_u\theta_5$, which is nonrenormalisable and we cannot write down a series of renormalisable operators to mediate this effective operator. This is because the combination of perpendicular group and GUT charges constrain heavily the operators we can write down, which means proton decay can be seen to be suppressed here by the dynamics as well as the symmetries required by the F-theory formalism. The full determination of the coupling strengths of any process of this type in F-theory should be found through computing the overlap integral of the wavefunctions involved [142], and this will be discussed in upcoming work on R-parity violating processes.

4.6 Conclusions

We have revisited a class of $SU(5)$ SUSY GUT models which arise in the context of the spectral cover with Klein Group monodromy $V_4 = Z_2 \times Z_2$. By investigating the symmetry structures of the spectral cover equation and the defining equations of the mater curves it is possible to understand the F-theory geometric origin of matter parity, which has hitherto been just assumed in an *ad hoc* way. In particular, we have shown how the simplest Z_2 matter parities can be realised via the new geometric symmetries respected by the spectral cover. By exploiting the various ways that these symmetries can be assigned, there are a large number of possible variants.

We have identified a rather minimal example of this kind, where the low energy effective theory below the GUT scale is just the MSSM with no exotics and standard matter parity. Furthermore, by deriving general properties of the singlet sector, consistent with string vacua, including the

D and F-flatness conditions, we were able to identify two singlets, which provide suitable candidates for a two right-handed neutrinos. We were thus able to derive the MSSM extended by a two right-handed neutrino seesaw mechanism. We also computed all baryon and lepton number violating operators emerging from higher non-renormalisable operators and found all dangerous operators to be forbidden.

Chapter 5

R-parity Violation from F-Theory

The quest for a unified theory of elementary particles has led to numerous extensions of the successful Standard Model (SM) of electroweak and strong interactions. During the last decades, string theory has been proven to be a powerful approach to describing gravity, which also enforces restrictions on the particle physics theory. Grand Unified Theories (GUTs) [22] may be embedded in string scenarios, while supersymmetry (SUSY) is also incorporated in a consistent way, leading to a natural solution of the hierarchy problem. Although string theory does not provide a unique prediction for the precise GUT symmetry and matter content, it enables a classification of possible solutions in a well defined and organised way. Moreover, it provides computational tools for various parameters such as the Yukawa couplings and potentials which would otherwise be left unspecified in more arbitrary extensions of the Standard Model.

Among other restrictions imposed by string theory principles, of particular importance are those on the massless spectrum. In many string constructions only small representations such as the fundamental and spinorial of the GUT group are available while the adjoint or higher ones are absent in the massless spectrum. In some cases this puts model building in a precarious position since the spontaneous breaking of most successful GUTs requires Higgs fields in the adjoint representation. But it was precisely this difficulty which gave rise to the invention of new symmetry breaking mechanisms and other alternative ways to obtain the Standard Model. In the case of $SU(5)$ for example [22], one manages to circumvent this obstacle by replacing it with the flipped -$SU(5) \times U(1)$- version of the model [143], [144], while in the case of Pati-Salam symmetry $SU(4) \times SU(2) \times SU(2)$ [63] the adjoint Higgs field, which transforms under the gauge group as $(15, 1, 1)$, is replaced by the vector-like Higgs pair of fields which transform as $(4, 1, 2) + (\bar{4}, 1, 2)$ [145], [146]. Analogously, a way out of this difficulty in F-theory models [32–34, 147], where the singularity is realised on a del Pezzo surface, is the use of fluxes to break the GUT symmetry. Indeed, in the last decade or so, a considerable amount of work has been devoted to the possibility of successfully embedding GUTs such as $SU(5)$ as well as exceptional $E_{6,7,8}$ in an F-theory framework, leading to new features [76–79].

Recently, some of us have analysed various phenomenological aspects of F-theory effective models using the spectral cover description [3, 137, 148]. While, in F-constructions, R-parity conservation (RPC) can emerge either as a remnant symmetry of extra $U(1)$ factors, or it can be imposed by appealing to some geometric property of the internal manifold and the flux [135], there is no compelling reason to assume this. Moreover, experimental bounds permit R-parity violating (RPV) interactions at small but non-negligible rates, providing a generic signature of F-theory models. In the field theory context, RPV proved to be the *Achilles heel* of many SUSY GUTs. The most dangerous such couplings induce the tree-level operators $QLd^c, d^cd^cu^c, e^cLL$ and in the absence of a suitable symmetry or displacement mechanism, all of them appearing simultaneously can lead to Baryon and Lepton (B and L) violating processes at unacceptable rates [149]. On the other hand, in F-theory constructions, parts of GUT multiplets are typically projected out by fluxes, giving rise only to a part of the above operators. In other cases, due to symmetry arguments, the Yukawa couplings relevant to RPV operators are identically zero. As a result, several B/L violating processes, either are completely prevented or occur at lower rates in F-theory models, providing a controllable signal of RPV. This observation motivates a general study of RPV in F-theory, which is the subject of this chapter.

In the present chapter, then, we consider RPV in local F-theory, trying to be as general as possible, with the goal of making a bridge between F-theory and experiment. An important goal of the chapter is to compute the strength of the RPV Yukawas couplings, which mainly depend on the topological properties of the internal space and are more or less independent of many details of a particular model, enabling us to work in a generic local F-theory setting. We focus on F-theory SU(5) constructions, where a displacement mechanism, based on non-trivial fluxes, renders several GUT multiplets incomplete. This mechanism has already been suggested to eliminate the colour triplets from the Higgs five-plets, so that dangerous dimension-5 proton decay operators are not present. However, it turns out that, in several cases, not only the Higgs but also other matter multiplets are incomplete, while the superpotential structure is such that it implies RPV terms. In this context, it is quite common that not all of the RPV operators appear simultaneously, allowing observable RPV effects without disastrous proton decay.

Our goal in this chapter is twofold. Firstly, to present a detailed analysis of all possible combinations of RPV operators arising from a generic semi-local F-theory spectral cover framework, assuming an $SU(5)$ GUT. This includes a detailed analysis of the classification of all possible allowed combinations of RPV operators, originating from the $SU(5)$ term $10 \cdot \bar{5} \cdot \bar{5}$, including the effect of $U(1)$ fluxes, with *global* restrictions, which are crucial in controlling the various possible multiplet splittings. Secondly, using F-theory techniques developed in the last few years, we perform explicit computations of the bottom/tau and RPV Yukawa couplings, assuming only *local* restrictions on fluxes, and comparing our results with the present experimental limits on the coupling for each specific RPV operator. The ingredients for this study have already appeared scattered through the literature, which we shall refer to as we go along.

We emphasise that the first goal is related to the nature of the available *global* Abelian fluxes of the particular model and their restrictions on the various matter curves, hence, on its specific geometric properties. The second goal requires the computation of the strengths of the corresponding Yukawa couplings. This in turn requires knowledge of the wavefunctions' profiles of the particles participating in the corresponding trilinear Yukawa couplings and, as we will see, these involve the *local* flux data. Once such couplings exist in the effective Lagrangian, we wish to explore the regions of the available parameter space where these couplings are sufficiently suppressed and are compatible with the present experimental data.

Our aim in this dedicated study is to develop and extend the scope of the existing results in the literature, in order to provide a complete and comprehensive study, which make direct contact with experimental limits on RPV, enabling F-theory models to be classified and confronted with experiment more easily and directly than previously. We emphasise that this is the first study of its kind in the literature which focusses exclusively on RPV in F-theory.

The remainder of this chapter presents the work in [4] and is divided into two parts: in the first part, we consider semi-local F-theory constructions where global restrictions are imposed on the fluxes, which imply that they take integer values. In Section 5.1 we show that RPV is a generic expectation of semi-local F-theory constructions. In Section 5.1.1 we classify F-theory $SU(5)$ models in the spectral cover approach according to the type of monodromy which dictates the different curves on which the matter and Higgs fields can lie, with particular attention of the possibility for RPV operators in each case at the level of $10 \cdot \bar{5} \cdot \bar{5}$ operators, involving complete $SU(5)$ multiplets, focussing on which multiplets contain the Higgs fields H_u and H_d. In Section 5.1.2 we introduce the notion of flux, quantised according to global restrictions, which, when switched on, leads to incomplete $SU(5)$ multiplets in the low energy (massless) spectrum, focussing on missing components of the multiplets projected out by the flux, and tabulating the type of physical process (RPV or proton decay) can result from particular operators involving different types of incomplete multiplets. Appendix A details all possible sources of R-parity violating couplings for all models classified with respect to the monodromies in semi-local F-theory constructions.

In the second part of the chapter, we relax the global restrictions of the semi-local constructions, and allow the fluxes to take general values, subject only to local restrictions. In Section 5.2 we describe the calculation of a Yukawa coupling originating from an operator $10 \cdot \bar{5} \cdot \bar{5}$ at an $SO(12)$ local point of enhancement in the presence of general local fluxes, with only local (not global) flux restrictions. In Section 5.3 we apply these methods to calculate the numerical values of Yukawa couplings for bottom, tau and RPV operators, exploring the parameter space of local fluxes. In Section 5.4 we finally consider RPV coupling regions and calculate ratios of Yukawa couplings from which the physical RPV couplings at the GUT scale can be determined and compared to limits on these couplings from experiment. Section 5.5 concludes the chapter. Appendix B details the local F-theory constructions and local chirality constraints on flux data and RPV operators.

5.1 R-parity violation in semi-local F-theory constructions

5.1.1 Multi-curve models in the spectral cover approach

In the present F-theory framework of $SU(5)$ GUT, third generation fermion masses are expected to arise from the tree-level superpotential terms $10_f \cdot \bar{5}_f \cdot \bar{5}_{\bar{H}}$, $10_f \cdot 10_f \cdot 5_H$ and $5_H \cdot \bar{5}_f \cdot 1_f$, where the index f stands for fermion, H for Higgs and we have introduced the notation

$$10_f = (Q, u^c, e^c),\ \bar{5}_f = (d^c, L),\ 1_f = \nu^c,\ 5_H = (D, H_u),\ \bar{5} = (\bar{D}, H_d) \quad (5.1)$$

The lighter generations receive masses from higher order terms, involving the same invariants, although suppressed by powers of $\langle\theta_i\rangle/M$, with θ_i representing available singlet fields with non-zero vacuum expectation values (vevs), while M is the GUT scale. The 4-d RPV couplings are obtained similarly with the replacements $\bar{5}_{\bar{H}} \to \bar{5}_f$ (provided that the symmetries of the theory permit the existence of such terms). At the level of the minimal supersymmetric standard model (MSSM) superpotential the RPV couplings read [110]:

$$W \supset 10_f \cdot \bar{5}_f \cdot \bar{5}_f \to \mu_i H_u L_i + \frac{1}{2}\lambda_{ijk} L_i L_j e_k^c + \lambda'_{ijk} L_i Q_j d_k^c + \frac{1}{2}\lambda''_{ijk} u_i^c d_j^c d_k^c \quad (5.2)$$

in the conventional notation for matter multiplets $Q_i, u_i^c, d_i^c, L_i, e_i^c$ where $i = 1, 2, 3$ is a flavour index. Notice that in the presence of vector-like pairs, $5_f + \bar{5}_f$, additional RPV couplings appear from the following decompositions

$$W \supset 10_f \cdot 10_f \cdot 5_f \to \kappa Q u^c \bar{L} + \kappa' u^c \bar{d}^c e^c + \frac{1}{2}\kappa'' Q Q \bar{d}^c \quad (5.3)$$

where we have introduced the notation $5_f = (\bar{d}^c, \bar{L})$ and dropped the flavour indices here for simplicity. However, as we will analyse in detail, Abelian fluxes and additional continuous or discrete symmetries which are always present in F-theory models, eliminate several of these terms. We will perform the analysis in the context of the spectral surfaces whose covering group is $SU(5)_\perp$ (dubbed usually as perpendicular) and is identified as the commutant to the GUT $SU(5)$ in the chain

$$E_8 \supset SU(5) \times SU(5)_\perp \to SU(5) \times U(1)_\perp^4$$

where E_8 is assumed to be the highest singularity in the elliptically fibred compact space. Then, a crucial rôle on the RPV remaining terms in the effective superpotential is played by the specific assignment of fermion and Higgs fields on the various matter curves and the remaining perpendicular $U(1)_\perp$'s after the monodromy action.

A classification of the set of models with simple monodromies that retain some perpendicular $U(1)_\perp$ charges associated with the weights t_i has been put forward in [131, 132, 136], where we follow the notation of Dudas and Palti [132] . In the following, we categorize these models in order to assess whether tree-level, renormalizable, perturbative RPV is generic if matter is allocated in different curves. More specifically, we present four classes, characterised by the splitting of the spectral cover equation. These are:

- $2+1+1+1$-splitting, which retains three independent perpendicular $U(1)_\perp$. These models represent a Z_2 monodromy ($t_1 \leftrightarrow t_2$), and as expected we are left with seven **5** curves, and four **10** curves.

- 2+2+1-splitting, which retains two independent perpendicular $U(1)_\perp$. These models represent a $Z_2 \times Z_2$ monodromy ($t_1 \leftrightarrow t_2$, $t_3 \leftrightarrow t_4$), and as expected we are left with five **5** curves, and three **10** curves.

- 3+1+1-splitting, which retains two independent perpendicular $U(1)_\perp$. These models represent a Z_3 monodromy ($t_1 \leftrightarrow t_2 \leftrightarrow t_3$), and as expected we are left with five **5** curves, and three **10** curves.

- $3 + 2$-splitting, which retains a single perpendicular $U(1)_\perp$. These models represent a $Z_3 \times Z_2$ monodromy ($t_1 \leftrightarrow t_2 \leftrightarrow t_3$, $t_4 \leftrightarrow t_5$), and as expected we are left with three **5** curves, and two **10** curves.

In Appendix A we develop the above classes of models, identifying which curve contains the Higgs fields and which contains the matter fields, in order to show that RPV is a generic phenomenon in semi-local F-theory constructions. Of course, if all the RPV operators are present, then proton decay will be an inevitable consequence. In the next subsection we show that this is generally avoided in semi-local F-theory constructions when fluxes are switched on, which has the effect of removing some of the RPV operators, while leaving some observable RPV in the low energy spectrum.

5.1.2 Hypercharge flux with global restrictions and R-parity violating operators

In F-theory GUTs, when the adjoint representation is not found in the massless spectrum, the alternative mechanism of flux breaking is introduced to reduce the GUT symmetry down to the SM gauge group. In the case of $SU(5)$ this can happen by turning on a non-trivial flux along the hypercharge generator in the internal directions. At the same time, the various components of the GUT multiplets living on matter curves, interact differently with the hypercharge flux. As a result, in addition to the $SU(5)$ symmetry breaking, on certain matter curves we expect the splitting of the 10 and $5, \bar{5}$ representations into different numbers of SM multiplets.

In a minimal scenario one might anticipate that the hyperflux is non-trivially restricted only on the Higgs matter curves in such a way that the zero modes of the colour triplet components are eliminated. This would be an alternative to the doublet-triplet scenario since only the two Higgs doublets remain in the light spectrum. The occurrence of this minimal scenario presupposes that all the other matter curves are left intact by the flux. However, in this section we show that this is usually not the case. Indeed, the common characteristic of a large class of models derived from the various factorisations of the spectral cover are that there are incomplete $SU(5)$ multiplets from different matter curves which comprise the three known generations and eventually possible extraneous fields. Interestingly, such scenarios leave open the possibility of effective models with only a fraction of RPV operators and the opportunity of studying exciting new physics implications leading to suppressed exotic decays which might be anticipated in the LHC experiments.

To analyse these cases, we assume that m_{10}, m_5 integers are units of $U(1)$ fluxes, with n_Y representing the corresponding hyperflux piercing the matter curves. The integer nature of these fluxes originates from the assumed *global* restrictions [131, 132, 136]. Then, the tenplets and fiveplets split according to:

$$10_{t_i} = \begin{cases} \text{Representation} & \text{flux units} \\ n_{(3,2)_{1/6}} - n_{(\bar{3},2)_{-1/6}} & = \; m_{10} \\ n_{(\bar{3},1)_{-2/3}} - n_{(3,1)_{2/3}} & = \; m_{10} - n_Y \\ n_{(1,1)_{+1}} - n_{(1,1)_{-1}} & = \; m_{10} + n_Y \end{cases} \tag{5.4}$$

$$5_{t_i} = \begin{cases} \text{Representation} & \text{flux units} \\ n_{(3,1)_{-1/3}} - n_{(\bar{3},1)_{+1/3}} & = \; m_5 \\ n_{(1,2)_{+1/2}} - n_{(1,2)_{-1/2}} & = \; m_5 + n_Y \end{cases} \tag{5.5}$$

The integers $m_{10,5}, n_Y$ may take any positive or negative value, leading to different numbers of SM representations, however, for our purposes it is enough to assume the cases [1] $m, n_Y = \pm 1, 0$. Then, substituting these numbers in Eqs. (5.4,5.5) we obtain the cases of Table 5.1. Depending on

10	Flux units	10 content	$\bar{5}$	Flux units	$\bar{5}$ content
10_1	$m_{10} = 1, n_Y = 0$	$\{Q, u^c, e^c\}$	$\bar{5}_1$	$m_5 = 1, n_Y = 0$	$\{d^c, L\}$
10_2	$m_{10} = 1, n_Y = 1$	$\{Q, -, 2e^c\}$	$\bar{5}_2$	$m_5 = 1, n_Y = 1$	$\{d^c, 2L\}$
10_3	$m_{10} = 1, n_Y = -1$	$\{Q, 2u^c, -\}$	$\bar{5}_3$	$m_5 = 1, n_Y = -1$	$\{d^c, -\}$
10_4	$m_{10} = 0, n_Y = 1$	$\{-, \bar{u}^c, e^c\}$	$\bar{5}_4$	$m_5 = 0, n_Y = 1$	$\{-, L\}$
10_5	$m_{10} = 0, n_Y = -1$	$\{-, u^c, \bar{e}^c\}$	$\bar{5}_5$	$m_5 = 0, n_Y = -1$	$\{-, \bar{L}\}$

Table 5.1: Table of MSSM matter content originating from $10, \overline{10}, 5, \bar{5}$ of $SU(5)$ for various fluxes

the specific choice of m, n_Y integer parameters, we end up with incomplete $SU(5)$ representations. For convenience we collect all distinct cases of incomplete $SU(5)$ multiplets in Table 5.1.

We now examine all parity violating operators formed by trilinear terms involving incomplete representations. Table 5.2 summarises the possible cases emerging form the various combinations $10_a\bar{5}_b\bar{5}_c$ of the incomplete representations shown in Table 5.1.

In the last column of Table 5.2 we also show the dominant RPV processes, which lead to baryon and/or lepton number violation. We notice however, that there exist other rare processes beyond those indicated in the tables which can be found in reviews (see for example [110].) We have already stressed, that in addition to the standard model particles, some vector-like pairs may appear too. For example, when fluxes are turned on, we have seen in several cases that the MSSM spectrum is accompanied in vector like states such as:

$$u^c + \bar{u}^c, L + \bar{L}, d + \overline{d}^c, Q + \overline{Q} \dots$$

Of course they are expected to get a heavy mass but if some vector-like pairs remain in the light spectrum they may have significant implications

[1] Of course there are several combinations of (m, n_Y) values which do not exceed the total number of three generations. Here, in order to illustrate the point, we consider only the cases with $m, n_Y = \pm 1, 0$.

$SU(5)$-invariant	matter content	operators	Dominant $\not{R}$-process
$10_1 \cdot \bar{5}_1 \cdot \bar{5}_1$	$(Q, u^c, e^c)(d^c, L)^2$	All	proton decay
$10_1 \cdot \bar{5}_2 \cdot \bar{5}_2$	$(Q, u^c, e^c)(d^c, 2L)^2$	All	proton decay
$10_1 \cdot \bar{5}_3 \cdot \bar{5}_3$	$(Q, u^c, e^c)(d^c, -)^2$	$u^c d^c d^c$	$n - \bar{n}$-osc.
$10_1 \cdot \bar{5}_4 \cdot \bar{5}_4$	$(Q, u^c, e^c)(-, L)^2$	LLe^c	$L_{e,\mu,\tau}$-violation
$10_1 \cdot \bar{5}_5 \cdot \bar{5}_5$	$(Q, u^c, e^c)(-, \bar{L})^2$	None	None
$10_2 \cdot \bar{5}_1 \cdot \bar{5}_1$	$(Q, -, e^c)(d^c, L)^2$	QLd^c, LLe^c	$L_{e,\mu,\tau}$-violation
$10_2 \cdot \bar{5}_2 \cdot \bar{5}_2$	$(Q, -, e^c)(d^c, 2L)^2$	QLd^c, LLe^c	$L_{e,\mu,\tau}$-violation
$10_2 \cdot \bar{5}_3 \cdot \bar{5}_3$	$(Q, -, e^c)(d^c, -)^2$	None	None
$10_2 \cdot \bar{5}_4 \cdot \bar{5}_4$	$(Q, -, e^c)(-, L)^2$	LLe^c	$L_{e,\mu,\tau}$-violation
$10_2 \cdot \bar{5}_5 \cdot \bar{5}_5$	$(Q, -, e^c)(-, \bar{L})^2$	None	None
$10_3 \cdot \bar{5}_1 \cdot \bar{5}_1$	$(Q, 2u^c, -)(d^c, L)^2$	$QLd^c, d^c d^c u^c$	proton decay
$10_3 \cdot \bar{5}_2 \cdot \bar{5}_2$	$(Q, 2u^c, -)(d^c, 2L)^2$	$QLd^c, d^c d^c u^c$	proton decay
$10_3 \cdot \bar{5}_3 \cdot \bar{5}_3$	$(Q, 2u^c, -)(d^c, -)^2$	$d^c d^c u^c$	$n - \bar{n}$-osc.
$10_3 \cdot \bar{5}_4 \cdot \bar{5}_4$	$(Q, 2u^c, -)(-, L)^2$	None	None
$10_3 \cdot \bar{5}_5 \cdot \bar{5}_5$	$(Q, 2u^c, -)(-, \bar{L})^2$	None	None

Table 5.2: Fluxes, incomplete representations and $\not{R}$-processes emerging from the trilinear coupling $10_a \bar{5}_b \bar{5}_c$ for all possible combinations of the incomplete multiplets given in Table 5.1.

in rare processes, such as contributions to diphoton events which are one of the primary searches in the ongoing LHC experiments.

5.2 Yukawa couplings in local F-theory constructions: formalism

In this section (and subsequent sections) we relax the global constraints on fluxes, and consider the calculation of Yukawa couplings, imposing only local flux restrictions. The motivation for doing this is to calculate the Yukawa couplings associated with the RPV operators in a rather model independent way, and then compare our results to the experimental limits. Flavour hierarchies and Yukawa structures in F-theory have been studied in a large number of papers [129]- [150]. In this section we shall discuss Yukawa couplings in F-theory, following the approach of [151–153].

In the previous section we assessed how chirality is realised on different curves due to flux effects. These considerations take into account the global flux data and are therefore called semi-local models. The flux units considered in the examples above are integer valued as they follow from the Dirac flux quantisation

$$\frac{1}{2\pi}\int_{\Sigma\subset S} F = n \tag{5.6}$$

where n is an integer, Σ a matter curve (two-cycle in the divisor S), and F the gauge field-strength tensor, i.e. the flux. In conjugation with the index theorems, the flux units piercing different matter curves Σ will tell us how many chiral states are globally present in a model.

While the semi-local approach defines the full spectrum of a model, the computation of localised quantities, such as the Yukawa couplings, requires appropriate description of the local geometry. As we will see below, a crucial quantity in the local geometry is the notion of *local* flux density, understood as follows.

First we notice that the unification gauge coupling is related to the compactification scale through the volume of the compact space

$$\alpha_G^{-1} = m_*^4 \int_S 2\omega\wedge\omega = m_*^4 \int \mathrm{dVol}_S = \mathrm{Vol}(S) m_*^4 \tag{5.7}$$

where α_G is the unification gauge coupling, m_* is the F-Theory characteristic mass, S the GUT divisor with Kähler form

$$\omega = \frac{i}{2}(dz_1\wedge d\bar{z}_1 + dz_2\wedge d\bar{z}_2) \tag{5.8}$$

that defines the volume form

$$\mathrm{dVol}_S = 2\omega\wedge\omega = dz_1\wedge dz_2\wedge d\bar{z}_1\wedge d\bar{z}_2. \tag{5.9}$$

As the volume of Σ is bounded by the volume of S, we assume that

$$\mathrm{Vol}(\Sigma) \simeq \sqrt{\mathrm{vol}(S)}, \tag{5.10}$$

and if we now consider that the background of F is constant, we can estimate the values that F takes in S by

$$F \simeq 2\pi\sqrt{\alpha_G} m_*^2 n. \tag{5.11}$$

This means that, in units of m_*, the background F is an $\mathcal{O}(1)$ real number. Since in the computation of Yukawa couplings it's the local values of F – and not the global quantisation constraints – that matter, we will from now on abuse terminology and refer to flux densities, F, as fluxes. Furthermore, as we will see later, the local values of F also define what chiral states are supported locally. This will be crucial to study the full plenitude of RPV couplings in different parts of the parameter space.

Before dealing with the particular rare reaction, it is useful to recall a few basic facts about the Yukawa couplings.

5.2.1 The local $SO(12)$ model

In F-theory matter is localised along Riemann surfaces (matter curves), which are formed at the intersections of D7-branes with the GUT surface S. Yukawa couplings are then realised when three of these curves intersect at a single point on S, while, at the same time, the gauge symmetry is enhanced. The computation relies on the knowledge of the profile of the wavefunctions of the states participating in the intersection. When a specific geometry is chosen for the internal space (and in particular for the GUT surface) these profiles are found by solving the corresponding equations of motion [139]- [153]. Their values are obtained by computing the integral of the overlapping wavefunctions at the triple intersections.

In $SU(5)$ two basic Yukawa terms are relevant when computing the Yukawa matrices and interactions. These are $\mathbf{y_u 10 \cdot 10 \cdot 5}$ and $\mathbf{y_d 10 \cdot \bar{5} \cdot \bar{5}}$. The first one generates the top Yukawa coupling while the symmetry at this intersection enhances to the exceptional group E_6. The relevant couplings that we are interested in, are related to the second coupling. This one is realised at a point where there is an $SO(12)$ gauge symmetry enhancement[2]. To make this clear, next we highlighted some of the basic analysis of [153].

The 4-dimensional theory can be obtained by integrating out the effective 8-dimensional one over the divisor S

$$W = m_*^4 \int_S \mathrm{Tr}(F \wedge \Phi) \tag{5.12}$$

where $F = dA - iA \wedge A$ is the field-strength of the gauge vector boson A and Φ is a $(2,0)$-form on S.

From the above superpotential, the F-term equations can be computed by varying A and Φ. In conjugation with the D-term

$$D = \int_S \omega \wedge F + \frac{1}{2}[\Phi, \bar{\Phi}], \tag{5.13}$$

[2]For a general E_8 point of enhancement that containing both type of couplings see [154,155]. Similar, an E_7 analysis is given in [130].

where ω is the Kähler form of S, a 4-dimensional supersymmetric solution for the equations of motion of F and Φ can be computed.

Both A and Φ, locally are valued in the Lie algebra of the symmetry group at the Yukawa point. In the case in hand, the fibre develops an $SO(12)$ singularity at which point couplings of the form $\mathbf{10} \cdot \bar{\mathbf{5}} \cdot \bar{\mathbf{5}}$ arise. Away from the enhancement point, the background Φ breaks $SO(12)$ down to the GUT group $SU(5)$. The rôle of $\langle A \rangle$ is to provide a 4d chiral spectrum and to break further the GUT gauge group.

More systematically, the Lie-Algebra of $SO(12)$ is composed of its Cartan generators H_i with $i = 1, ..., 6$, and 60 step generators E_ρ. Together, they respect the Lie algebra

$$[H_i, E_\rho] = \rho_i E_\rho \tag{5.14}$$

where ρ_i is the i^{th} component of the root ρ. The E_ρ generators can be completely identified by their roots

$$(\underline{\pm 1, \pm 1, 0, 0, 0, 0, 0}) \tag{5.15}$$

where underline means all 60 permutations of the entries of the vector, including different sign combinations. To understand the meaning of this notation it is sufficient to consider a simpler example:

$$(0, \underline{1, 0, 0,} 0, 0, 0) \equiv \{(0, 1, 0, 0, 0, 0), (0, 0, 1, 0, 0, 0), (0, 0, 0, 1, 0, 0)\} \tag{5.16}$$

The background of Φ will break $SO(12)$ away from the $SO(12)$ singular point. In order to see this consider it takes the form

$$\Phi = \Phi_{z_1 z_2} dz_1 \wedge dz_2 \tag{5.17}$$

where it's now explicit that it parametrises the transverse directions to S. The background we are considering is

$$\langle \Phi_{z_1 z_2} \rangle = m^2 \left(z_1 Q_{z_1} + z_2 Q_{z_2} \right) \tag{5.18}$$

where m is related to the slope of the intersection of 7-branes, and

$$Q_{z_1} = -H_1 \tag{5.19}$$

$$Q_{z_2} = \frac{1}{2}\sum_i H_i. \tag{5.20}$$

The unbroken symmetry group will be the commutant of $\langle \Phi_{z_1 z_2} \rangle$ in $SO(12)$. The commutator between the background and the rest of the generators is

$$[\langle \Phi_{z_1 z_2} \rangle, E_\rho] = m^2 q_\Phi(\rho) E_\rho \tag{5.21}$$

where $q_\Phi(\rho)$ are holomorphic functions of the complex coordinates z_1, z_2. The surviving symmetry group is composed of the generators that commute with $\langle \Phi \rangle$ on every point of S. With our choice of background, the surviving step generators are identified to be

$$E_\rho : (0, \underline{1, -1, 0, 0, 0}), \tag{5.22}$$

which, together with H_i, trivially commute with $\langle \Phi \rangle$, generating $SU(5) \times U(1) \times U(1)$.

When $q_\Phi(\rho) = 0$ in certain loci we have symmetry enhancement, which accounts for the presence of matter curves. This happens as at these loci, extra step generators survive and furnish a representation of $SU(5) \times U(1) \times U(1)$. For the case presented we identify three curves joining at the $SO(12)$ point, these are

$$\Sigma_a = \{z_1 = 0\} \tag{5.23}$$

$$\Sigma_b = \{z_2 = 0\} \tag{5.24}$$

$$\Sigma_c = \{z_1 = z_2\}, \tag{5.25}$$

and defining a charge under a certain generator as

$$[Q_i, E_\rho] = q_i(\rho) E_\rho \tag{5.26}$$

all the data describing these matter curves are presented in Table 5.3. Since the bottom and tau Yukawas come from such an $SO(12)$ point, in order to have such a coupling the point must have the a^+, b^+, and c^+.

In order to both induce chirality on the matter curves and break the two $U(1)$ factors, we have to turn on fluxes on S valued along the two Cartan

Curve	Roots	q_Φ	$SU(5)$ irrep	q_{z_1}	q_{z_2}
$\Sigma_{a^\pm}$	$(\pm 1, \underline{\mp 1, 0, 0, 0, 0})$	$\mp z_1$	$\bar{\mathbf{5}}/\mathbf{5}$	∓ 1	0
$\Sigma_{b^\pm}$	$(0, \underline{\pm 1, \pm 1, 0, 0, 0})$	$\mp z_2$	$\mathbf{10}/\bar{\mathbf{10}}$	0	± 1
$\Sigma_{c^\pm}$	$(\mp 1, \underline{\mp 1, 0, 0, 0, 0})$	$\pm(z_1 - z_2)$	$\bar{\mathbf{5}}/\mathbf{5}$	± 1	∓ 1

Table 5.3: Matter curves and respective data for an $SO(12)$ point of enhancement model with a background Higgs given by Equation 5.18. The underline represent all allowed permutations of the entries with the signs fixed

generators that generate the extra factors.

We first consider the flux

$$\langle F_1 \rangle = i(M_{z_1} dz_1 \wedge d\bar{z}_1 + M_{z_2} dz_2 \wedge d\bar{z}_2) Q_F, \tag{5.27}$$

with

$$Q_F = -Q_{z_1} - Q_{z_2} = \frac{1}{2}(H_1 - \sum_{j=2}^{6} H_j). \tag{5.28}$$

It's easy to see that the $SU(5)$ roots are neutral under Q_F, and therefore this flux does not break the GUT group. On the other hand, the roots on a, b sectors are not neutral. This implies that this flux will be able to differentiate $\bar{\mathbf{5}}$ from $\mathbf{5}$ and $\mathbf{10}$ from $\bar{\mathbf{10}}$

$$\int_{\Sigma_a,\ \Sigma_b} F_1 \neq 0 \Rightarrow \text{ Induced Chirality.} \tag{5.29}$$

This flux does not induce chirality in $c^\pm$ curves as $q_F = 0$ for all roots in $c^\pm$. To induce chirality in $c^\pm$ one needs another contribution to the flux

$$\langle F_2 \rangle = i(dz_1 \wedge d\bar{z}_2 + dz_2 \wedge d\bar{z}_1)(N_a Q_{z_1} + N_b Q_{z_2}) \tag{5.30}$$

that does not commute with the roots on the $c^\pm$ sectors for $N_a \neq N_b$.

Breaking the GUT down to the SM gauge group requires flux along the Hypercharge. In order to avoid generating a Green-Schwarz mass for the Hypercharge gauge boson, this flux has to respect global constraints. Locally we may define it as

$$\langle F_Y \rangle = i[(dz_1 \wedge d\bar{z}_2 + dz_2 \wedge d\bar{z}_1)N_Y + (dz_2 \wedge d\bar{z}_2 - dz_1 \wedge d\bar{z}_1)\tilde{N}_Y]Q_Y \tag{5.31}$$

and the Hypercharge is embedded in our model through the linear combination

$$Q_Y = \frac{1}{3}(H_2 + H_3 + H_4) - \frac{1}{2}(H_5 + H_6). \tag{5.32}$$

Since this contribution to the flux does not commute with all elements of $SU(5)$, only with its SM subgroup, distinct SM states will feel this flux differently. This known fact is used extensively in semi-local models as a mechanism to solve the doublet-triplet splitting problem. As we will see bellow, it can also be used to locally prevent the appearance of certain chiral states and therefore forbid some RPV in subregions of the parameter space.

The total flux will then be the sum of the three above contributions. It can be expressed as

$$\begin{aligned}\langle F\rangle =& i(dz_2 \wedge d\bar{z}_2 - dz_1 \wedge d\bar{z}_1)Q_P \\ &+ i(dz_1 \wedge d\bar{z}_2 + dz_2 \wedge d\bar{z}_1)Q_S \\ &+ i(dz_2 \wedge d\bar{z}_2 + dz_1 \wedge d\bar{z}_1)M_{z_1 z_2}Q_F\end{aligned} \tag{5.33}$$

with the definitions

$$Q_P = MQ_F + \tilde{N}_Y Q_Y \tag{5.34}$$

$$Q_S = N_a Q_{z_1} + N_b Q_{z_2} + N_Y Q_Y \tag{5.35}$$

and

$$M = \frac{1}{2}(M_{z_1} - M_{z_2}) \tag{5.36}$$

$$M_{z_1 z_2} = \frac{1}{2}(M_{z_2} + M_{z_1}). \tag{5.37}$$

As the Hypercharge flux will affect SM states differently, breaking the GUT group, we will be able to distinguish them inside each curve. The full split of the states present in the different sectors, and all relevant data, is presented in Table 5.4.

5.2.2 Wavefunctions and the Yukawa computation

In general, the Yukawa strength is obtained by computing the integral of the overlapping wavefunctions. More precisely, according to the discussion on the previous section one has to solve for the zero mode wavefunctions for

Sector	Root	SM	q_F	q_{z_1}	q_{z_2}	q_S	q_P
a_1	$(1,\underline{-1,0,0},0,0)$	$(\bar{\mathbf{3}},\mathbf{1})_{-\frac{1}{3}}$	1	-1	0	$-N_a-\frac{1}{3}N_Y$	$M-\frac{1}{3}\tilde{N}_Y$
a_2	$(1,0,0,0,\underline{-1,0})$	$(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	1	-1	0	$-N_a+\frac{1}{2}N_Y$	$M+\frac{1}{2}\tilde{N}_Y$
b_1	$(0,\underline{1,1,0},0,0)$	$(\bar{\mathbf{3}},\mathbf{1})_{\frac{2}{3}}$	-1	0	1	$N_b+\frac{2}{3}N_Y$	$-M+\frac{2}{3}\tilde{N}_Y$
b_2	$(0,\underline{1,0,0},\underline{1,0})$	$(\mathbf{3},\mathbf{2})_{-\frac{1}{6}}$	-1	0	1	$N_b-\frac{1}{6}N_Y$	$-M-\frac{1}{6}\tilde{N}_Y$
b_3	$(0,0,0,0,1,1)$	$(\mathbf{1},\mathbf{1})_{-1}$	-1	0	1	N_b-N_Y	$-M-\tilde{N}_Y$
c_1	$(-1,\underline{-1,0,0},0,0)$	$(\bar{\mathbf{3}},\mathbf{1})_{-\frac{1}{3}}$	0	1	-1	$N_a-N_b-\frac{1}{3}N_Y$	$-\frac{1}{3}\tilde{N}_Y$
c_2	$(-1,0,0,0,\underline{-1,0})$	$(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	0	1	-1	$N_a-N_b+\frac{1}{2}N_Y$	$\frac{1}{2}\tilde{N}_Y$

Table 5.4: Complete data of sectors present in the three curves crossing in an $SO(12)$ enhancement point considering the effects of non-vanishing fluxes. The underline represent all allowed permutations of the entries with the signs fixed

the sectors a, b and c presented in Table (5.4). The physics of the D7-Branes wrapping on S can be described in terms of a twisted 8-dimensional $\mathcal{N}=1$ gauge theory on $R^{1,3}\times S$, where S is a Kähler submanifold of elliptically fibered Calabi-Yau 4-fold X. One starts with the action of the effective theory, which was given in [33]. The next step is to obtain the equations of motion for the 7-brane fermionic zero modes. This procedure has been performed in several of papers including [152–154] and we will not repeat it here in detail. In order for this chapter to be self-contained we highlight the basic computational steps.

The equations for a 4-dimensional massless fermionic field are of the Dirac form:

$$\mathcal{D}_A\Psi = 0 \tag{5.38}$$

where

$$\mathcal{D}_A = \begin{pmatrix} 0 & D_1 & D_2 & D_3 \\ -D_1 & 0 & -D_{\bar{3}} & D_{\bar{2}} \\ -D_2 & -D_{\bar{3}} & 0 & -D_{\bar{1}} \\ -D_3 & -D_{\bar{2}} & D_{\bar{1}} & 0 \end{pmatrix} \quad , \quad \Psi = \Psi E_\rho = \begin{pmatrix} -\sqrt{2}\eta \\ \psi_{\bar{1}} \\ \psi_{\bar{2}} \\ \chi_{12} \end{pmatrix}. \tag{5.39}$$

The indices here are a shorthand notation instead of the coordinates

z_1, z_2, z_3. The components of Ψ are representing 7-brane degrees of freedom. Also the covariant derivatives are defined as $D_i = \partial_i - i[\langle A_i\rangle, \ldots]$ for $i = 1, 2, \bar{1}, \bar{2}$ and as $D_{\bar{3}} = -i[\langle \Phi_{12}\rangle, \ldots]$ for the coordinate z_3. It is clear from equations (5.38,5.39) that we have to solve the equations for each sector. According to the detailed solutions in [153] the wavefunctions for each sector have the general form

$$\Psi \sim f(az_1 + bz_2)e^{M_{ij}z_iz_j} \tag{5.40}$$

where $f(az_1 + bz_2)$ is a holomorphic function and M_{ij} incorporates flux effects. In an appropriate basis this holomorphic function can be written as a power of its variables $f_i \sim (az_1 + bz_2)^{3-i}$ and in the case where the generations reside in the same matter curve, the index-i can play the rôle of a family index. Moreover the Yukawa couplings as a triple wavefunction integrals have to respect geometric $U(1)$ selection rules. The coupling must be invariant under geometric transformations of the form: $z_{1,2} \to e^{i\alpha}z_{1,2}$. In this case the only non-zero tree level coupling arises for $i = 3$ and by considering that, the index in the holomorphic function f_i indicates the fermion generation we obtain a non-zero top-Yukawa coupling. Hierarchical couplings for the other copies on the same matter curve can be generated in the presence of non commutative fluxes [139] or by incorporating non-perturbative effects [152]- [130].

The RPV couplings under consideration emerge from a tree level interaction. Hence, its strength is given by computing the integral where now the rôle of the Higgs $\bar{5}_H$ is replaced by $\bar{5}_M$. We consider here the scenario where the generations are accommodated in different matter curves. In this case the two couplings, the bottom/tau Yukawa and the tree level RPV, are localised at different $SO(12)$ points on $\mathcal{S}_{GUT}$, (see Figure 5.1). In this approach, at first approximation we can take the holomorphic functions f as constants absorbed in the normalization factors.

As a first approach, our goal is to calculate the bottom Yukawa coupling as well as the coupling without hypercharge flux and compare the two values. So, at this point we write down the wavefunctions and the relevant parameters in a more detailed form as given in [153] but without the holomorphic functions. The wavefunctions in the holomorphic gauge have the

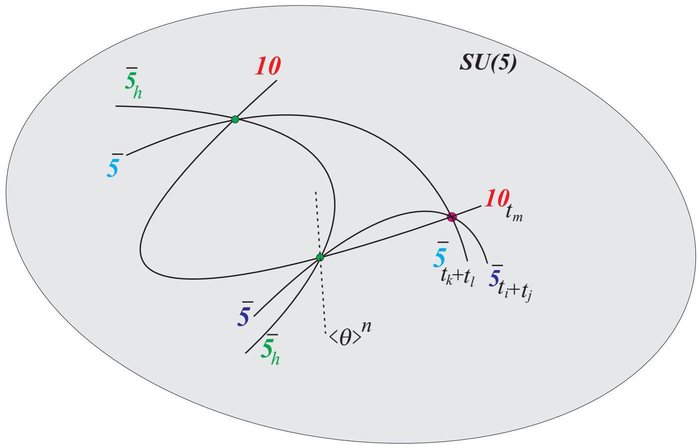


Figure 5.1: Intersecting matter curves on the GUT divisor. While the Yukawa coupling arises on intersections of curves hosting two matter and one Higgs representations, the RPV coupling arises when all three curves host matter representations.

following form

$$\vec{\psi}^{(b)hol}_{10_M} = \vec{v}^{(b)}\chi^{(b)hol}_{10_M} = \vec{v}^{(b)}\kappa^{(b)}_{10_M}\, e^{\lambda_b z_2(\bar{z}_2-\zeta_b\bar{z}_1)} \tag{5.41}$$

$$\vec{\psi}^{(a)hol}_{5_M} = \vec{v}^{(a)}\chi^{(a)hol}_{5_M} = \vec{v}^{(a)}\kappa^{(a)}_{5_M}\, e^{\lambda_a z_1(\bar{z}_1-\zeta_a\bar{z}_2)} \tag{5.42}$$

$$\vec{\psi}^{(c)hol}_{5_H} = \vec{v}^{(c)}\chi^{(c)hol}_{5_H} = \vec{v}^{(c)}\kappa^{(c)}_{5_H}\, e^{(z_1-z_2)(\zeta_c\bar{z}_1-(\lambda_c-\zeta_c)\bar{z}_2)} \tag{5.43}$$

$$\vec{\psi}^{(c)hol}_{5_M} = \vec{v}^{(c)}\chi^{(c)hol}_{5_H} = \vec{v}^{(c)}\kappa^{(c)}_{5_M}\, e^{(z_1-z_2)(\zeta_c\bar{z}_1-(\lambda_c-\zeta_c)\bar{z}_2)}. \tag{5.44}$$

where

$$\zeta_a = -\frac{q_S(a)}{\lambda_a - q_P(a)} \tag{5.45}$$

$$\zeta_b = -\frac{q_S(b)}{\lambda_b + q_P(b)} \tag{5.46}$$

$$\zeta_c = \frac{\lambda_c(\lambda_c - q_P(c) - q_S(c)}{2(\lambda_c - q_S(c))} \tag{5.47}$$

and λ_ρ is the smallest eigenvalue of the matrix

$$m_\rho = \begin{pmatrix} -q_P & q_S & im^2 q_{z_1} \\ q_S & q_P & im^2 q_{z_2} \\ -im^2 q_{z_1} & -im^2 q_{z_2} & 0 \end{pmatrix}. \tag{5.48}$$

To compute the above quantities we make use of the values of q_i from Table 5.4. It is important to note that the values of the flux densities in this table depend on the $SO(12)$ enhancement point. This means that one can in principle have different numerical values for the strength of the interactions at different points.

The column vectors are given by

$$\vec{v}^{(b)} = \begin{pmatrix} -\frac{i\lambda_b}{m^2}\zeta_b \\ \frac{i\lambda_b}{m^2} \\ 1 \end{pmatrix}, \vec{v}^{(a)} = \begin{pmatrix} -\frac{i\lambda_a}{m^2} \\ \frac{i\lambda_a}{m^2}\zeta_a \\ 1 \end{pmatrix}, \vec{v}^{(c)} = \begin{pmatrix} -\frac{i\zeta_c}{m^2} \\ \frac{i(\zeta_c-\lambda_c)}{m^2} \\ 1 \end{pmatrix}. \tag{5.49}$$

Finally, the κ coefficients in equations (5.41-5.42) are normalization factors. These factors are fixed by imposing canonical kinetic terms for the matter fields. More precisely, for a canonically normalized field χ_i supported in a certain sector (e), the normalization condition for the wavefunctions in the real gauge is

$$1 = 2m_*^4 ||\vec{v}^{(e)}||^2 \int (\chi^{(e)real})_i^* \chi_i^{(e)real} \mathrm{dVol}_S \tag{5.50}$$

where $\chi_i^{(e)real}$ are now in the real gauge, and in our convention $\mathrm{Tr}E_\alpha^\dagger E_\beta = 2\delta_{\alpha\beta}$. The wavefunctions in real and holomorphic gauge are related by

$$\psi^{real} = e^{i\Omega}\psi^{hol} \tag{5.51}$$

where

$$\Omega = \frac{i}{2}\left[\left(M_{z_1}|z_1|^2 + M_{z_2}|z_2|^2\right)Q_F - \tilde{N}_Y\left(|z_1|^2 - |z_2|^2\right)Q_Y + \left(z_1\bar{z}_2 + z_2\bar{z}_1\right)Q_S\right], \tag{5.52}$$

which only transforms the scalar coefficient of the wavefunctions, χ, leaving the $\vec{v}$ part invariant.

With the above considerations, one can find the normalization factors to be

$$|\kappa_{5_M}^{(a)}|^2 = -4\pi g_s \sigma^2 \cdot \frac{q_P(a)(2\lambda_a + q_P(a)(1+\zeta_a^2))}{\lambda_a(1+\zeta_a^2) + m^4}, \tag{5.53}$$

$$|\kappa_{10_M}^{(b)}|^2 = -4\pi g_s \sigma^2 \cdot \frac{q_P(b)(-2\lambda_b + q_P(b)(1+\zeta_b^2))}{\lambda_b(1+\zeta_b^2) + m^4}, \tag{5.54}$$

$$|\kappa_{5_H}^{(c)}|^2 = -4\pi g_s \sigma^2 \cdot \frac{2(q_P(c)+\zeta_c)(q_P(c)+2\zeta_c-2\lambda_c) + (q_S(c)+\lambda_c)^2}{\zeta_c^2 + (\lambda_c-\zeta_c)^2 + m^4}, \tag{5.55}$$

$$|\kappa_{5_M}^{(c)}|^2 = -4\pi g_s \sigma^2 \cdot \frac{2(q_P(c)+\zeta_c)(q_P(c)+2\zeta_c-2\lambda_c) + (q_S(c)+\lambda_c)^2}{\zeta_c^2 + (\lambda_c-\zeta_c)^2 + m^4}. \tag{5.56}$$

where we used the relation $\left(\frac{m}{m_*}\right)^2 = (2\pi)^{3/2} g_s^{1/2}\sigma$, making use of the dimensionless quantity $\sigma = (m/m_{st})^2$, where m_{st} the string scale. The expressions (5.53-5.56) above can be shown numerically to be always positive.

The superpotential trilinear couplings can be taken to be in the holomorphic gauge. For the bottom Yukawa, we consider that ψ_{10_M} and ψ_{5_M} contain the heaviest down-type quark generations. In this case the bottom and tau couplings can be computed:

$$\begin{aligned} y_{b,\tau} =& m_*^4\ t_{abc} \int_S \det(\vec{\psi}^{(b)hol}_{10_M}, \vec{\psi}^{(a)hol}_{5_M}, \vec{\psi}^{(c)hol}_{5_H}) \mathrm{dVol}_S \\ =& m_*^4\ t_{abc}\ \det(\vec{v}^{(b)}, \vec{v}^{(a)}, \vec{v}^{(c)}) \int_S \chi^{(b)hol}_{10_M} \chi^{(a)hol}_{5_M} \chi^{(c)hol}_{5_H} \mathrm{dVol}_S. \end{aligned} \tag{5.57}$$

The bottom and tau Yukawa couplings differ since they have different SM quantum numbers and arise from different sectors, leading to different q_S and q_P as shown in Table 5.4.
A similar formula can be written down for the RPV coupling

$$\begin{aligned} y_{RPV} =& m_*^4\ t_{abc} \int_S \det(\vec{\psi}^{(b)hol}_{10_M}, \vec{\psi}^{(a)hol}_{5_M}, \vec{\psi}^{(c)hol}_{5'_M}) \mathrm{dVol}_S \\ =& m_*^4\ t_{abc}\ \det(\vec{v}^{(b)}, \vec{v}^{(a)}, \vec{v}^{(c)}) \int_S \chi^{(b)hol}_{10_M} \chi^{(a)hol}_{5_M} \chi^{(c)hol}_{5_M} \mathrm{dVol}_S. \end{aligned} \tag{5.58}$$

Here this RPV Yukawa coupling can in principle refer to any generations of squarks and sleptons, and may have arbitrary generation indices (suppressed here for simplicity).
The factor t_{abc} represents the structure constants of the $SO(12)$ group. The integral in the last term can be computed by applying standard Gaussian techniques. Computing the determinant and the integral, the combined result of the two is a flux independent factor and the final result reads:

$$y_{b,\tau} = \pi^2 \left(\frac{m_*}{m}\right)^4 t_{abc} \kappa^{(b)}_{10_M} \kappa^{(a)}_{5_M} \kappa^{(c)}_{5_H}. \tag{5.59}$$

This is a standard result for the heaviest generations. As we observe the flux dependence is hidden on the normalization factors.

We turn now our attention in the case of a tree-level RPV coupling of the form $10_M \cdot \bar{5}_M \cdot \bar{5}_M$. This coupling can be computed in a different $SO(12)$ enhancement point p. As a first approach we consider that the hypercharge flux parameters are zero in the vicinity of p. From a different point of view, $\bar{5}_M$ replaces the Higgs matter curve in the previous computation. The new wavefunction ($\psi^{(c)}_{5_M}$) can be found by setting all the Hypercharge flux parameters on $\psi^{(c)}_{5_H}$, equal to zero. The RPV coupling will be given by an equation similar to that of the bottom coupling :

$$y_{RPV} = \pi^2 \left(\frac{m_*}{m}\right)^4 t_{abc}\kappa^{(b)}_{10_M}\kappa^{(a)}_{5_M}\kappa^{(c)}_{5_M}. \tag{5.60}$$

and we notice that family indices are understood and this coupling is the same for every type of RPV interaction, depending on which SM states are being supported at the $SO(12)$ enhancement point. Notice that the κ's in equations (5.59, 5.60) are the modulus of the normalization factors defined in equations (5.53-5.56).

In the next section, using equations (5.59) and (5.60), we perform a numerical analysis for the couplings presented above with emphasis on the case of the RPV coupling. We notice that in our conventions for the normalization of the $SO(12)$ generators, the gauge invariant coupling supporting the above interactions has $t_{abc} = 2\cdot$.

5.3 Yukawa couplings in local F-theory constructions: numerics

Using the mathematical machinery developed in the previous section, we can study the behaviour of $SO(12)$ points in F-theory - including both the bottom-tau point of enhancement and RPV operators. The former has been well studied in [153] for example. The coupling is primarily determined by five parameters - N_a, N_b, M, N_Y and $\tilde{N}_Y$. The parameters N_a and N_b give net chirality to the c-sector, while N_Y and $\tilde{N}_Y$ are components of hypercharge flux, parameterising the doublet triplet splitting. M is related to the chirality of the a and b-sectors. There is also the $N_b = N_a - \frac{1}{3}N_Y$ constraint, which ensures the elimination of Higgs colour triplets at the Yukawa point. This can be seen by examining the text of the previous section, based on the work found in [153].

For a convenient and comprehensive presentation of the results we make the following redefinitions. In Eq. (5.59) and (5.60), one can factor out $4\pi g_s\sigma^2$ from inside Eq. (5.53),(5.54), and (5.55). In addition by noticing that $\left(\frac{m}{m_*}\right)^2 = (2\pi)^{3/2} g_s^{1/2} \sigma$, we obtain

$$y_{b,\tau} = 2 g_s^{1/2} \sigma \; y'_{b,\tau} \tag{5.61}$$

$$y_{RPV} = 2 g_s^{1/2} \sigma \; y'_{RPV} \tag{5.62}$$

where $y'_{b,\tau}$ and y'_{RPV} are functions of the flux parameters. Furthermore, we set the scale $m = 1$ and as such the remainder mass dimensions are given in units of m. The presented values for the strength of the couplings are then in units of $2g_s^{1/2}\sigma$.

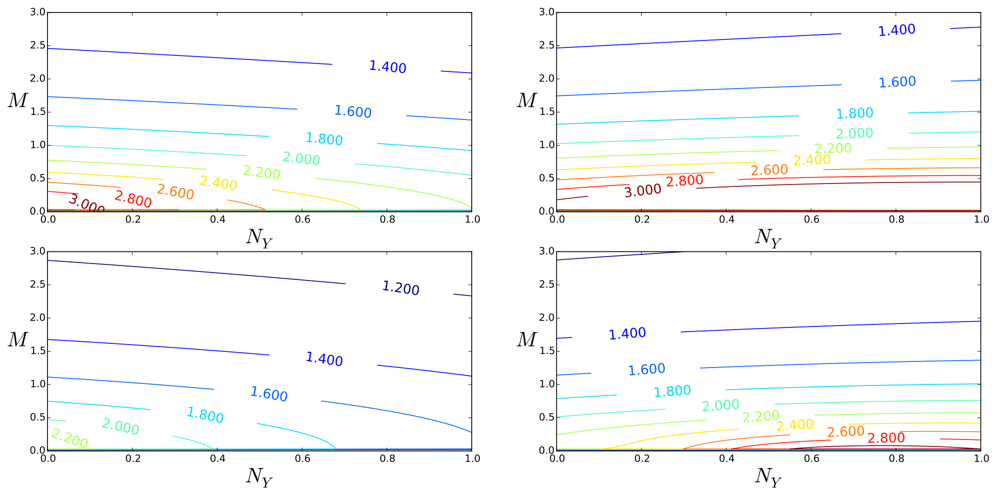


Figure 5.2: Ratio between bottom Yukawa and tau Yukawa couplings, shown as contours in the plane of local fluxes. The requirement for chiral matter and absence of coloured Higgs triplets fixes $N_b = N_a - \frac{1}{3}N_Y$

Figure 5.2 shows the ratio of the bottom and tau Yukawa couplings at a point of $SO(12)$ in a region of the parameter space with reasonable values. These results are consistent with those in [153]. Note that the phenomenological desired ratio of the couplings at the GUT scale is $Y_\tau / Y_b = 1.37 \pm 0.1 \pm 0.2$ [156], which can be achieved within the parameter ranges shown in Figure 5.2. Having shown that this technique reproduces the known results for the bottom to tau ratio, we now go on to study the behaviour of an RPV coupling point in $SO(12)$ models.

5.3.1 Behaviour of $SO(12)$ points

The simplest scenario for an $SO(12)$ enhancement generating RPV couplings, would be the case where all three of the types of operator, QLD, UDD, and LLE arise with equal strengths, which would occur in a scenario

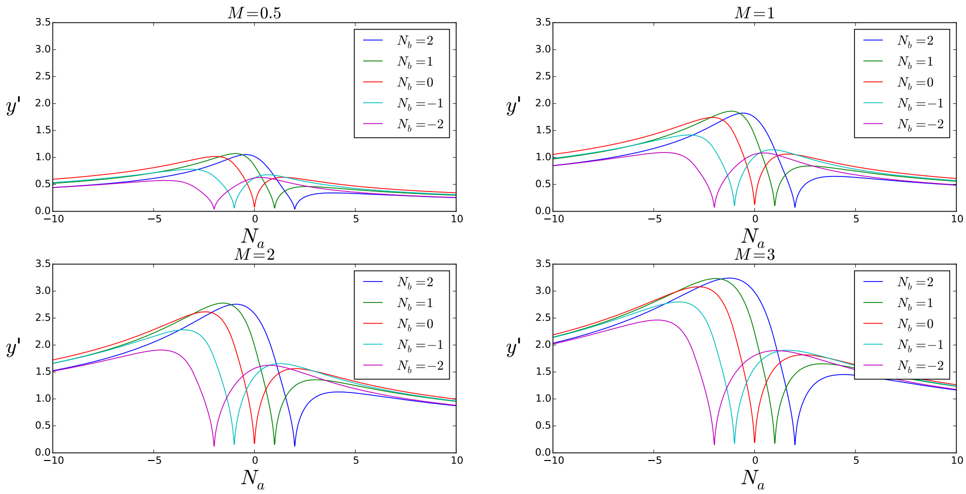


Figure 5.3: Dependency of the RPV coupling (in units of $2g_s^{1/2}\sigma$) on N_a in the absence of hypercharge fluxes, for different values of M and N_b.

with vanishing hypercharge flux, leading to an entirely "unsplit" scenario. This assumption sets N_Y and $\tilde{N_Y}$ to vanish, and we may also ignore the condition $N_b = N_a - \frac{1}{3}N_Y$. The remaining parameters determining are then N_a, N_b and M. Figure 5.3 shows the coupling strength in the N_a plane for differing N_b and M values. The general behaviour is that the coupling strength is directly related to M, while the coupling vanishes at the point where $N_a = N_b$. This latter point is due to the flip in net chirality for the c-sector at this point in the parameter space - $N_a > N_b$ gives the c^+ part of the spectrum.

Figure 5.4 and Figure 5.5 also demonstrate this set of behaviours, but for contours of the coupling strength. Figure 5.4, showing all combinations of the three non-zero parameters, shows that in the $N_a - N_b$ plane there is a line of vanishing coupling strength about the $N_a = N_b$, chirality switch point for the c-sector. The figure also reinforces the idea that small values of M correspond to small values of the coupling strength, as close to the point of $M = 0$ the coupling again reduces to zero. Figure 5.5 again shows this behaviour, with the smallest values of M giving the smallest values of the coupling. From this we can infer that an RPV $SO(12)$ point is most likely to be compatible with experimental constraints if M takes a small value.

Figure 5.6(a) (and Figure 5.6(b)) shows the RPV coupling strength in the absence of flux for the N_a (N_b) plane, along with the "bottom" coupling strength for corresponding values. The key difference is that the Hy-

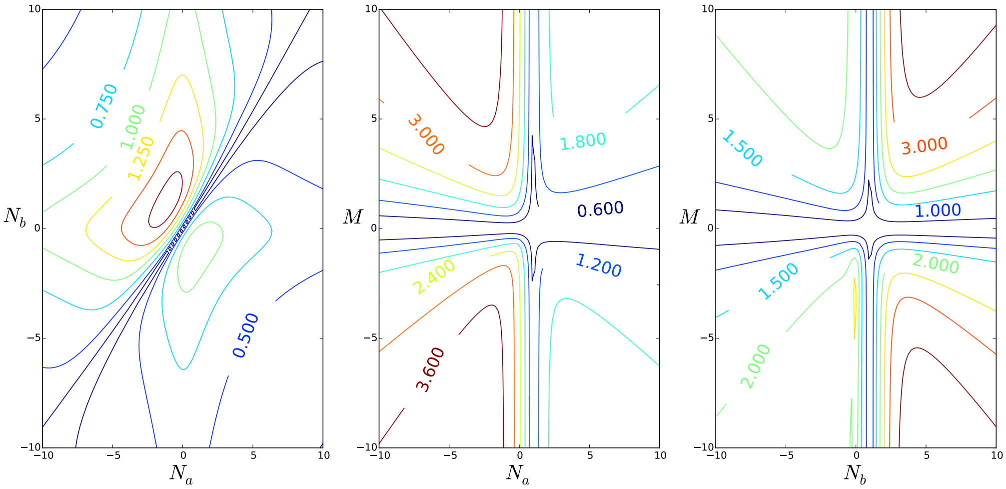


Figure 5.4: Dependency of the RPV coupling (in units of $2g_s^{1/2}\sigma$) on different flux parameters, in absence of Hypercharge fluxes. Any parameter whose dependency is not shown is set to zero.

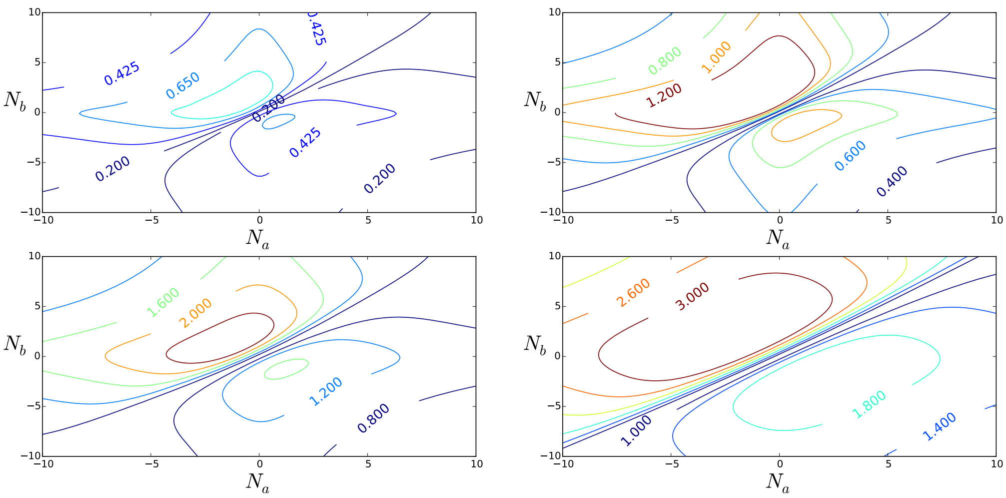
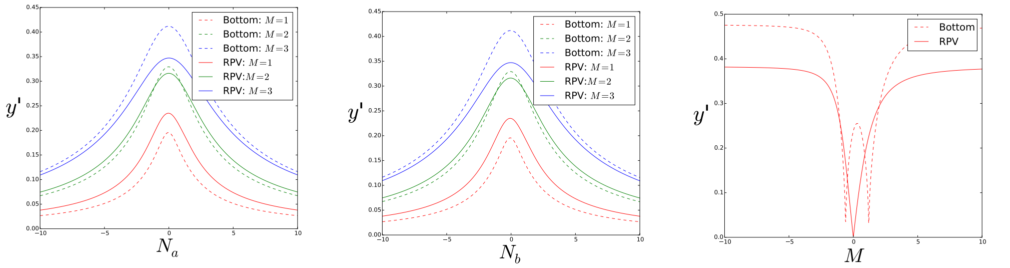


Figure 5.5: Dependency of the RPV coupling (in units of $2g_s^{1/2}\sigma$) on the (N_a, N_b)-plane, in absence of hypercharge fluxes and for different values of M. Top: left $M = 0.5$, right $M = 1.0$. Bottom: left $M = 2.0$, right $M = 3.0$.

percharge flux is switched on at the bottom $SO(12)$ point, with values of $N_Y = 0.1$ and $\tilde{N}_Y = 3.6$. The figures show that for the bottom coupling, the fluxes always push the coupling higher, similarly to increasing the M values.

Figure 5.6(c) plots out the two couplings in the M-plane, showing that the bottom Yukawa goes to zero for two values of M, while the RPV point has only one. Considering the form of Equation (5.59), we can see that the factors κ_{5_M} and κ_{10_M} are proportional to the parameter q_p. Referring



(a) Varying N_a with fixed M (b) Varying N_b with fixed M (c) Varying M with fixed N_a=1, $N_b = 29/30$, $N_Y = 0.1$, $\tilde{N_Y} = 3.6$

Figure 5.6: Dependency of the RPV and bottom Yukawa couplings (in units of $2g_s^{1/2}\sigma$) on different parameters at different regions of the parameter space

to Table 5.4, one can see which values these take for each sector - namely, $q_p(a_1) = M - \frac{1}{3}\tilde{N_Y}$ and $q_p(b_2) = -M - \frac{1}{6}\tilde{N_Y}$. Solving these two equations shows trivially that zeros should occur when $M = \frac{1}{3}\tilde{N_Y}$ and $-\frac{1}{6}\tilde{N_Y}$, which is the exact behaviour exhibited in Figure 5.6(c).

5.4 R-parity violating Yukawa couplings: allowed regions and comparison to data

In this section we focus on calculating the RPV Yukawa coupling constant at the GUT scale, which may be directly compared to the experimental limits, using the methods and results of the previous two sections. As a point of notation, we have denoted the RPV Yukawa coupling at the GUT scale to be generically y_{RPV}, independently of flavour or operator type indices. This coupling may be directly compared to the phenomenological RPV Yukawa couplings at the GUT scale λ_{ijk}, λ'_{ijk} and λ''_{ijk} as defined below.

Recall that, in the weak/flavour basis, the superpotential generically includes RPV couplings, in particular those from Eq. 5.2:

$$W \supset \frac{1}{2}\lambda_{ijk}L_iL_je^c_k + \lambda'_{ijk}L_iQ_jd^c_k + \frac{1}{2}\lambda''_{ijk}u^c_id^c_jd^c_k \tag{5.63}$$

In the local F-theory framework, each of the above Yukawa couplings (generically denoted as y_{RPV}) is computable through Eq. (5.60). What distinguishes different RPV couplings, say λ from λ', are the values of the flux densities, namely the hypercharge flux. This is because the normalization of matter curves depends on the hypercharge flux density. As such, different SM states will have different hypercharges and consequently different respective normalization coefficient.

Even though a given $SO(12)$ enhancement point can in principle support different types of trilinear RPV interactions, the actual effective interactions arising at such point depend on the local chiral spectrum present at each curve. For example, in order to have an LLe^c interaction, both Σ_a and Σ_c curves need to have chiral L states, and the Σ_b curve an e^c state at the enhancement point. In Figure 5.7 we show contours on the (N_a, N_b) plane for the different types of trilinear RPV couplings.

The local spectrum is assessed by local chiral index theorems [154]. In Appendix B we outline the results for the constraints on flux densities such that different RPV points are allowed at a given $SO(12)$ enhancement point. These results are graphically presented in Figure 5.8 and may be compared to the operators presented in Table 5.2 in the semi-local approach. Thus, the green coloured region is associated with the $10_3\bar{5}_1\bar{5}_1$ operator of this Table, the blue colour with $10_1\bar{5}_3\bar{5}_3$, the pink with $10_2\bar{5}_4\bar{5}_4$ and so on. Thus different regions of the parameter space can support different types of RPV interactions at a given enhancement point. We can then infer that in F-theory the allowed RPV interactions can, in principle, be only a subset of all possible RPV interactions.

In the limiting cases where only one coupling is turned on, one can derive bounds on its magnitude at the GUT scale from low-energy processes [157]. In order to do so, one finds the bounds at the weak scale in the mass basis, performs a rotation to the weak basis and then evaluates the couplings at the GUT scale with the RGE. Since the effects of the rotation to the weak basis in the RPV couplings requires a full knowledge of the Yukawa matrices, we assume that the mixing only happens in the down-quark sector as we are not making any considerations regarding the up-quark sector in this work. Table 5.5 shows the upper bounds for the trilinear RPV couplings at the GUT scale.

The bounds presented in Table 5.5 have to be understood as being derived under certain assumptions on mixing and points of the parameter space [110, 158]. For example, the bound on λ_{12k} can be shown to have an explicit dependence on

$$\frac{\tilde{m}_{e_{k,R}}}{100\text{ GeV}} \tag{5.64}$$

where $\tilde{m}_{e_{k,R}}$ refers to a 'right-handed' selectron soft-mass. The values presented in Table 5.5, as found in [157], were obtained by setting the soft-masses to 100 GeV, which are ruled out by more recent LHC results [159–164] . By assuming heavier scalars, for example around 1 TeV, we would then get the bounds in Table 5.5 to be relaxed by one order of magnitude.

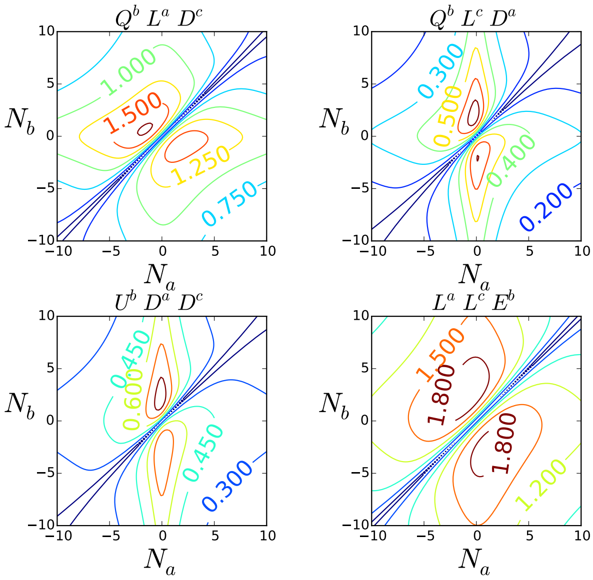


Figure 5.7: Strength of different RPV couplings (in units of $2g_s^{1/2}\sigma$) in the (N_a, N_b)-plane in the presence of Hypercharge fluxes $N_Y = 0.1$, $\tilde{N}_Y = 3.6$, and with $M = 1$. The scripts a, b, c refer to which sector each state lives.

The results show that the λ type of coupling, corresponding to the LLe^c interactions, is bounded to be < 0.05 regardless of the indices taken. The red regions of Figures 5.11(a) and 5.9 show the magnitude of the coupling where it is allowed. A similar analysis can be carried out for the remaining couplings. The λ' coupling, which measures the strength of the LQd^c type of interactions, can be seen in the yellow regions of Figure 5.10. Finally, the derived values for λ'' coupling, related to the $u^cd^cd^c$ type of interactions, are shown in the blue regions of Figures 5.10 and 5.11(b). However these couplings shown are all expressed in units of $2g_s^{1/2}\sigma$, and so cannot yet be directly compared to the experimental limits.

In order to make contact with experiment we must eliminate the $2g_s^{1/2}\sigma$ coefficient. We do this by taking ratios of the couplings computed in this framework where the $2g_s^{1/2}\sigma$ coefficient cancels in the ratio. The ratio between any RPV coupling and the bottom Yukawa at the GUT scale is given

ijk	λ_{ijk}	λ'_{ijk}	λ''_{ijk}
111	-	1.5×10^{-4}	-
112	-	6.7×10^{-4}	4.1×10^{-10}
113	-	0.0059	1.1×10^{-8}
121	0.032	0.0015	4.1×10^{-10}
122	0.032	0.0015	-
123	0.032	0.012	1.3×10^{-7}
131	0.041	0.0027	1.1×10^{-8}
132	0.041	0.0027	1.3×10^{-7}
133	0.0039	4.4×10^{-4}	-
211	0.032	0.0015	-
212	0.032	0.0015	(1.23)
213	0.032	0.016	(1.23)
221	-	0.0015	(1.23)
222	-	0.0015	-
223	-	0.049	(1.23)
231	0.046	0.0027	(1.23)
232	0.046	0.0028	(1.23)
233	0.046	0.048	-
311	0.041	0.0015	-
312	0.041	0.0015	0.099
313	0.0039	0.0031	0.015
321	0.046	0.0015	0.099
322	0.046	0.0015	-
323	0.046	0.049	0.015
331	-	0.0027	0.015
332	-	0.0028	0.015
333	-	0.091	-

Table 5.5: Upper bounds of RPV couplings (ijk refer to flavour/weak basis) at the GUT scale under the assumptions: 1) Only mixing in the down-sector, none in the Leptons; 2) Scalar masses $\tilde{m} = 100$ GeV; 3) $\tan\beta(M_Z) = 5$; and 4) Values in parenthesis refer to non-perturbative bounds, when these are stronger than the perturbative ones. This Table is reproduced from [157].

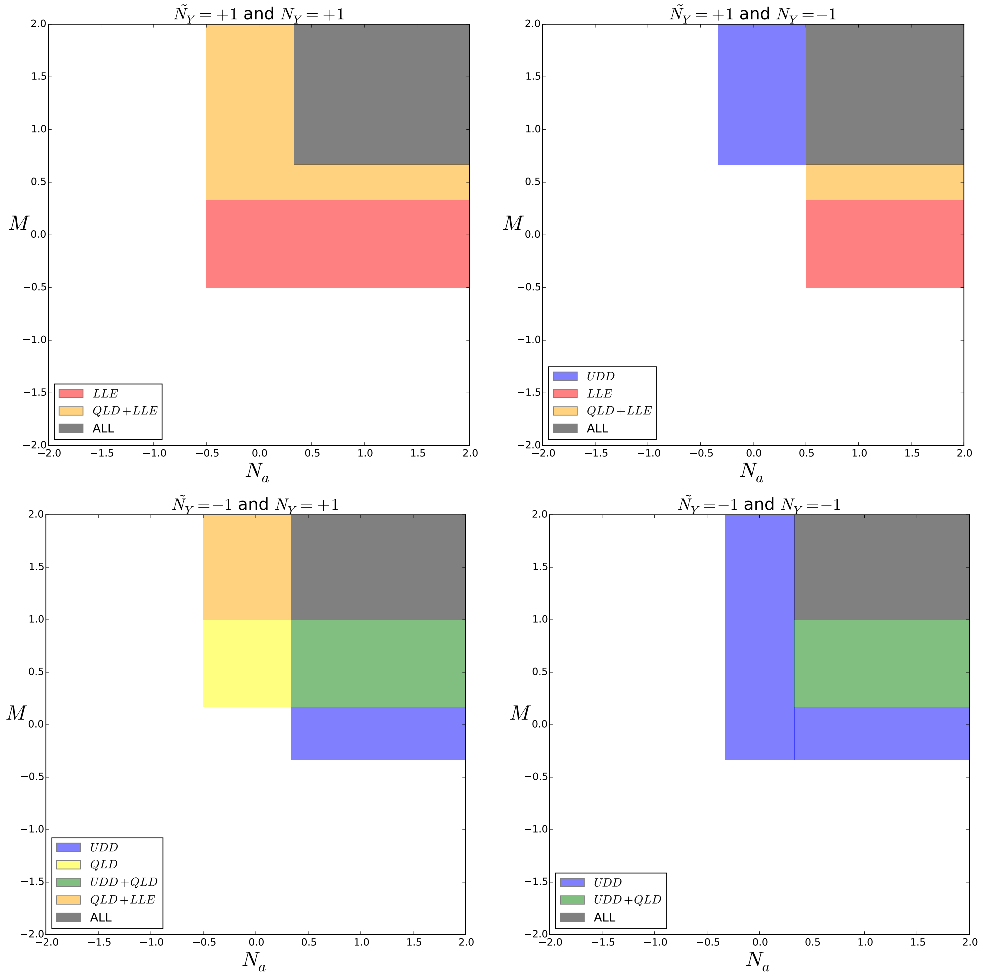


Figure 5.8: Allowed regions in the parameter space for different RPV couplings. These figures should be seen in conjunction with the operators presented in Table 5.2.

by

$$r = \frac{y_{RPV}}{y_b} = \frac{y'_{RPV}}{y'_b}, \tag{5.65}$$

as defined in Equation (5.61) and Equation (5.62). This ratio can be used to assess the absolute strength of the RPV at the GUT scale as follows.

First we assume that the RPV interaction is localised in an $SO(12)$ point far away from the bottom Yukawa point. This allows us to use different and independent flux densities at each point. We can then compute y'_b at a point in the parameter space where the ratio y_b/y_τ takes reasonable values, following [153]. Finally we take the ratio, r. In certain regions of the parameter space, r is naturally smaller than 1. This suppression of the RPV coupling in respect to the bottom Yukawa is shown in Figures 5.12(a), 5.12(b), 5.12(c), and 5.12(d), for different regions of the parameter space

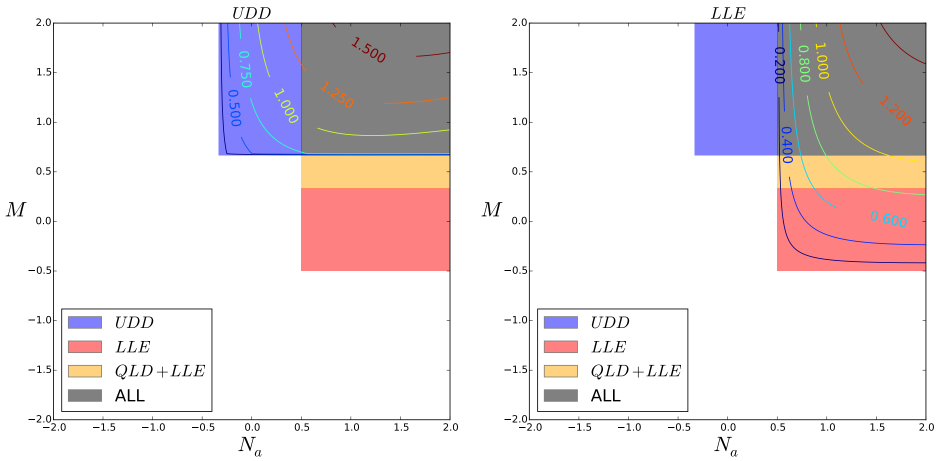


Figure 5.9: Allowed regions in the parameter space for different RPV couplings with $\tilde{N}_Y = -N_Y = 1$. We have also include the corresponding contours for the $u^c d^c d^c$ operator (left) and LLe^c (right).

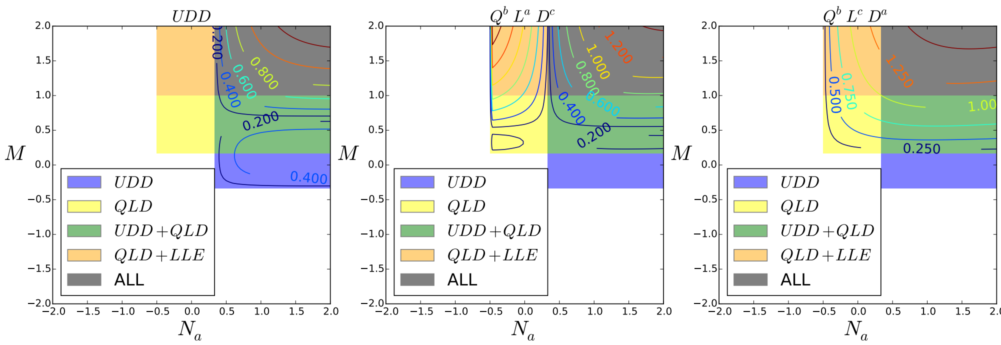
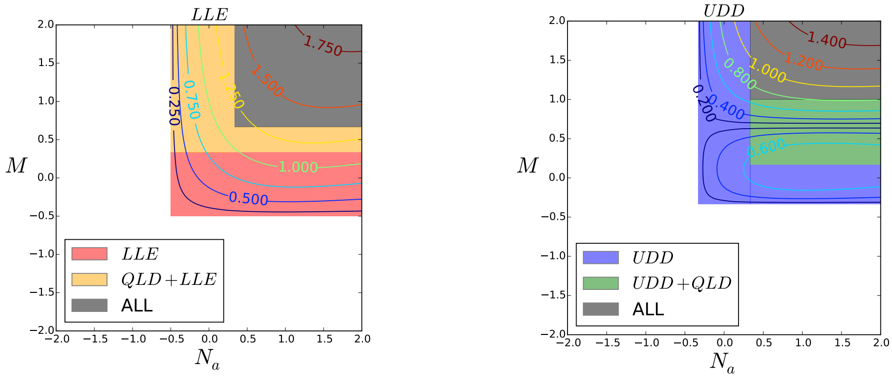


Figure 5.10: Allowed regions in the parameter space for different RPV couplings with $N_Y = -\tilde{N}_Y = 1$. We have also include the corresponding contours for the $u^c d^c d^c$ operator (left) and QLd^c (middle and right). The scripts a, b and c refer to which sector each state lives.

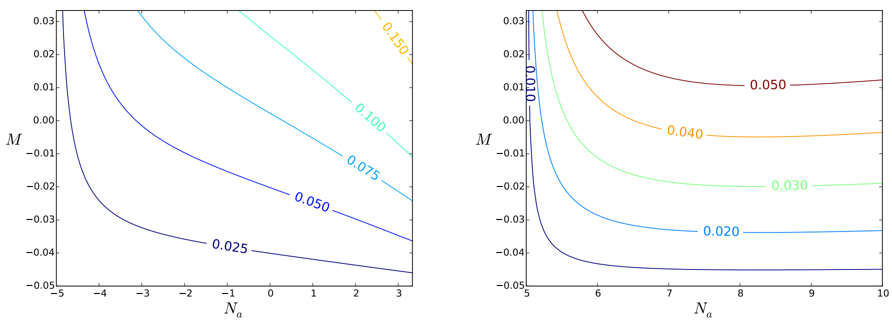
that allows for distinct types of RPV interactions.

Since r is the ratio of both primed and unprimed couplings, respectively unphysical and physical, at the GUT scale, we can extend the above analysis to find the values of the physical RPV couplings at the GUT scale. To do so, we use low-energy, experimental, data to set the value of the bottom Yukawa at the weak scale for a certain value of $\tan\beta$. Next, we follow the study in [156] to assess the value of the bottom Yukawa at the GUT scale through RGE runnings.

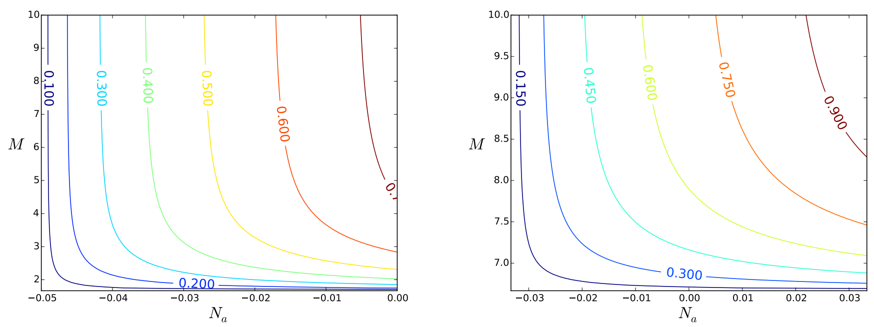


(a) LLe^c regions with $\tilde{N}_Y = N_Y = 1$ (b) $u^c d^c d^c$ regions with $\tilde{N}_Y = N_Y = -1$

Figure 5.11: Allowed regions in the parameter space for different RPV couplings.



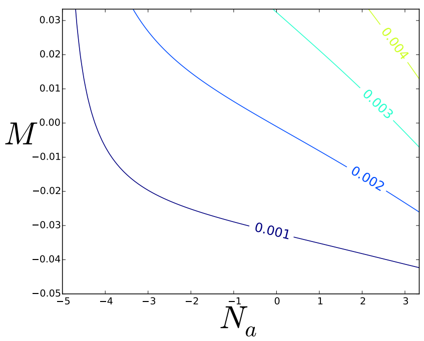
(a) LLe^c region with $N_Y = 10$, $\tilde{N}_Y = 0.1$ (b) LLe^c region with $N_Y = -10$, $\tilde{N}_Y = 0.1$



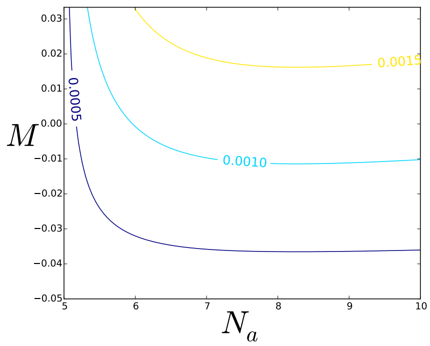
(c) QLd^c region with $N_Y = 0.1$, $\tilde{N}_Y = -10$ (d) $u^c d^c d^c$ region with $N_Y = -0.1$, $\tilde{N}_Y = -10$

Figure 5.12: y_{RPV}/y_b ratio. The bottom Yukawa was computed in a parameter space point that returns a reasonable y_b/y_τ ratio [153]

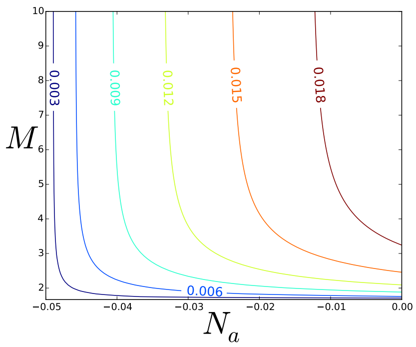
In order to make a connection with the bounds in Table 5.5, we pick $\tan\beta = 5$ and we find $y_b(M_{GUT}) \simeq 0.03$. The results for the value of the



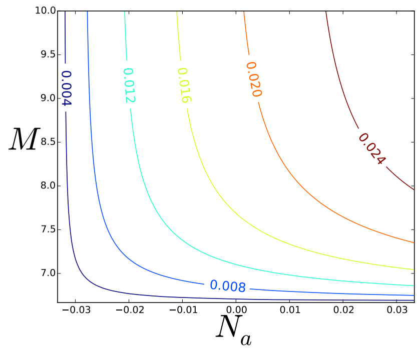
(a) λLLe^c region with $N_Y = 10$, $\tilde{N}_Y = 0.1$



(b) λLLe^c region with $N_Y = -10$, $\tilde{N}_Y = 0.1$



(c) $\lambda' QLd^c$ region with $N_Y = 0.1$, $\tilde{N}_Y = -10$



(d) $\lambda'' u^c d^c d^c$ region with $N_Y = -0.1$, $\tilde{N}_Y = -10$

Figure 5.13: y_{RPV} at GUT scale for $\tan\beta = 5$. The values here can be compared directly to the bounds presented in Table 5.5.

RPV couplings in different regions in the parameter space at the GUT scale are presented in Figures 5.13(a), 5.13(b), 5.13(c), and 5.13(d). These results show that, for any set of flavour indices, the strength of the coupling λ related to an LLe^c interaction is within the bounds. This means that this purely leptonic RPV operator, which violates lepton number but not baryon number, may be present with a sufficiently suppressed Yukawa coupling, according to our calculations. Therefore in the future lepton number violating processes could be observed.

By contrast, only for a subset of possible flavour index assignments for baryon number violating (but lepton number conserving) $u^c d^c d^c$ couplings are within the bounds in Table 5.5. The constraint on the first family up quark coupling λ''_{1jk} for the $u^c_1 d^c_j d^c_k$ interaction is so stringent, that this operator must only be permitted for the cases $u^c_2 d^c_j d^c_k$ and $u^c_3 d^c_j d^c_k$ (corresponding to the two heavy up-type quarks c^c, t^c), assuming no up-type quark mixing.

However, if up-type quark mixing is allowed, then such operators could lead to an effective $u_1^c d_j^c d_k^c$ operator suppressed by small mixing angles, in which case it could induce $n - \bar{n}$ oscillations [137].

Finally the LQd^c operator with Yukawa coupling λ' apparently must be avoided, since according to our calculations, the value of λ' that we predict exceeds the experimental limit by about an order of magnitude for all flavour indices, apart from λ'_{333} coupling corresponding to the $L_3Q_3d_3^c$ operator. This implies that we should probably eliminate such operators which violate lepton number, using the flux mechanism that we have described. However in some parts of parameter space, for certain flavour indices, such operators may be allowed leading to lepton number violating processes such as $K^+ \to \pi^- e^+ e^+$ and $D^+ \to K^- e^+ e^+$.

5.5 Conclusions

In this chapter we have provided the first dedicated study of R-parity violation (RPV) in F-theory semi-local and local constructions based on the $SU(5)$ grand unified theory (GUT) contained in the maximal subgroup $SU(5)_{GUT} \times SU(5)_\perp$ of an E_8 singularity associated with the elliptic fibration. Within this framework, we have tried to be as general as possible, with the primary aim of making a bridge between F-theory and experiment.

We have focussed on semi-local and local F-theory $SU(5)$ constructions, where a non-trivial hypercharge flux breaks the GUT symmetry down to the Standard Model and in addition renders several GUT multiplets incomplete. Acting on the Higgs curves this novel mechanism can be regarded as the surrogate for the doublet-triplet splitting of conventional GUTs. However, from a general perspective, at the same time the hyperflux may work as a displacement mechanism, removing certain components of GUT multiplets while accommodating fermion generations on other matter curves.

In the first part of the chapter we considered semi-local constructions, focussing on F-theory $SU(5)_{GUT}$ models which are classified according to the discrete symmetries – acting as identifications on the $SU(5)_\perp$ representations – and appearing as a subgroup of the maximal $SU(5)_\perp$ Weyl group S_5. Furthermore, we considered phenomenologically appealing scenarios with the three fermion generations distributed on different matter curves and showed that RPV couplings are a generic feature of such models. Upon introducing the flux breaking mechanism, we classified all possible cases of incomplete GUT multiplets and examined the implications of their associated RPV couplings. Then we focused on the induced MSSM plus RPV Yukawa sector which involves only part of the MSSM allowed RPV operators as a consequence of the missing components of the multiplets projected out by the flux. Next, we tabulated all distinct cases and the type of physical process (RPV or proton decay) that can arise from particular operators involving different types of incomplete multiplets.

In the second part of the chapter we computed the strength of the RPV Yukawa couplings, which mainly depend on the topological properties of the internal space and are more or less independent of many details of a particular model, enabling us to work in a generic local F-theory setting. Due to their physical relevance, we paid special attention to those couplings originating from the $SU(5)$ operator $10 \cdot \bar{5} \cdot \bar{5}$ in the presence of general fluxes, which is realised at an $SO(12)$ point of enhancement. Then, we applied the already developed F-theory techniques to calculate the numerical values of Yukawa couplings for bottom, tau and RPV operators. Taking into account flux restrictions, which limit the types of RPV operators that may appear simultaneously, we then calculated ratios of Yukawa couplings, from which the physical RPV couplings at the GUT scale can be determined. We have explored the possible ranges of the Yukawa coupling strengths of the $10 \cdot \bar{5} \cdot \bar{5}$-type operators in a five-dimensional parameter space, corresponding to the number of the distinct flux parameters/densities associated with this superpotential term. Varying these densities over a reasonable range of values, we have observed the tendencies of the various Yukawa strengths with respect to the flux parameters and, to eliminate uncertainties from overall normalization constants, we have computed the ratios of the RPV couplings to the bottom Yukawa one. This way, using the experimentally determined mass of the bottom quark, we compared our results to limits on these couplings from experiment.

The results of this chapter show firstly that, in semi-local F-theory constructions based on $SU(5)$ GUTs, RPV is a generic feature, but may occur without proton decay, due to flux effects. Secondly, our calculations based on local F-theory constructions show that the value of the RPV Yukawa couplings at the GUT scale may be naturally suppressed over large regions of parameter space. Furthermore, we found that the existence of LLe^c type of RPV interactions from F-Theory are expected to be within the current bounds. This implies that such lepton number violating operators could be present in the effective theory, but simply below current experimental limits, and so lepton number violation could be observed in the future. Similarly, the baryon number violating operators $c^c d^c_j d^c_k$ and $t^c d^c_j d^c_k$ could also be present, leading to $n - \bar{n}$ oscillations. Finally some QLd^c operators could be present leading to lepton number violating processes such as $K^+ \to \pi^- e^+ e^+$ and $D^+ \to K^- e^+ e^+$. In conclusion, our results suggest that RPV SUSY consistent with proton decay and current limits may be discovered in the future, shedding light on the nature of F-theory constructions.

Chapter 6

Conclusions

In this thesis we presented recent progress on modern String Phenomenology model building. The work presented highlights some of the potential for new branches of String Theory to provide low-energy phenomena while addressing many of the issues of the Standard Model as discussed in Chapter 1.

In Chapter 2 we introduced an $SO(10)$ model realised from M-Theory compactified on a G_2 manifold. We found that the suggested solution for the Doublet-Triplet by Witten on G_2 compactifications could not be employed, in contrast to the $SU(5)$ case called G_2-MSSM. Instead, we considered the Higgs coloured triplet partners to be light, as long as the symmetry proposed by Witten prevents them from coupling to matter and therefore avoiding too fast Proton decay. These two coloured triplets do not form a full GUT irrep, thus unification is spoiled unless in the presence of extra states. Since the coloured triplets have the same SM quantum numbers as a vector like pair of $SU(2)_L$ singlets down-type Quarks, we proposed a solution where we allowed for an effective light vector-like family, which we referred to as $\mathbf{16}_X + \overline{\mathbf{16}}_X$, which down-type $SU(2)_L$ singlets are allowed to be heavy and as such the remainder of the states complete a GUT irrep alongside with the Higgses coloured triplets.

We proposed that these extra states would also be responsible for breaking the extra $U(1)$ and would be crucial to generate neutrino masses. The vevs of the conjugated Right-handed neutrino states within this new extra vector-like family are also expected to generate B-RPV, leading to unstable LSP.

Just like in the $SU(5)$ case, in the $SO(10)$ model vector-like pairs are naturally and generically endowed with a μ-type mass term. In M-Theory on G_2 manifolds these masses are of $\mathcal{O}(1)$ TeV and therefore the first and main prediction of the $SO(10)$ model is a vector-like family that can be discovered at the LHC.

In Chapter 2 we did not study the details of the extra $U(1)$ symmetry breaking and neutrino mass generation. These were presented in Chapter 3, where we extended and adapted the so-called Kolda-Martin mechanism such that it could be employed to the $SO(10)$ model. The generalised mechanism allowed us to break the extra gauge factor without sourcing new SUSY breaking, while allowing for high-scale vevs that are crucial for neutrino mass generation. However, these high-scale vevs provide a source of B-RPV, which contributes to the low-scale neutral fermion mass matrix, from which the neutrino masses are obtained. Due to the complexity of this mass matrix, we performed a numerical analysis for a single family. We found that the phenomenological viable neutrino masses arise for the $(n, k) = (3, 0)$ implementation of the Kolda-Martin mechanism, which once again features the importance of Witten's discrete symmetry proposal. For this scenario, the light neutrino state is decoupled from the remaining neutral fermion states and B-RPV couplings induce an instability on the LSP, forcing it to decay within cosmological bounds but stripping away its potential as a dark matter candidate. An important note is that the mechanism presented can not only be extended to incorporate three families, but it can also be used in different scenarios other than M-Theory compactifications on G_2 manifolds.

Proceeding to Chapter 4, we began presenting our work on low-energy consequences of F-Theory. In this chapter we employed some results from Galois theory to classify the models in the spectral cover approach when its monodromy group was the Klein group, $V = Z_2 \times Z_2$. This study clarified certain details on monodromy actions from Galois groups of the spectral cover polynomial, where we found that distinct matter curves arise for different possible Klein groups. With this result, we presented an $SU(5)$ model with the same matter content as the MSSM. Furthermore, we made use of an assumed geometric parity to justify that this model has the standard matter parity assignments, which is a crucial ingredient as the perpendicular charges selection rules do not suffice in preventing RPV. Furthermore, we showed that the singlet states, which although not living in the GUT divisor still possess perpendicular charges, could be employed as Right-handed conjugated neutrinos while not producing dangerous couplings.

With Chapter 4 we touched on the idea that RPV couplings are rather generic in F-Theory compactifications. Namely, when semi-local models are considered, it becomes a challenging task to find an MSSM realisation that does not possess some type of Baryon or Lepton number violation. This motivated us to systematically study RPV in F-Theory.

In Chapter 5 we presented the first dedicated study of RPV in F-theory compactifications. We started by extensively describing the generic nature of RPV in $SU(5)$ semi-local models. In these models, GUT irreps often appear incomplete on matter curves. While this is often used as a solution for the Doublet-Triplet problem, it can also be responsible for RPV couplings when the spectrum does not facilitate the identification between the Higgs and Leptons.

Following this conclusion, we then proceeded to estimate the strength and details of the RPV couplings by using local techniques that were insofar used only to compute Yukawa couplings. These techniques require the computation of the wave function profiles around an $SO(12)$ enhancement singularity in the internal space, and the results are therefore largely model-agnostic.

With this study we started by finding that RPV couplings can arise isolated, depending on the local values of the flux parameters. This opens the possibility for single Baryon and Lepton number violating processes, or only smaller subsets of all possible RPV couplings. Next we found that the values of these couplings could be suppressed at the GUT scale, although LLe^c interactions seem to be well within current bounds regardless of the family indices assigned. For the Baryon number violating processes, depending on the family structure, we might still have some RPV processes that would lead to exotic interactions that could be observed in current and near future experiments, like neutron anti-neutron oscillations. Therefore, RPV from F-Theory could provide an experimental window into F-Theory compactifications.

Concluding, in this thesis we have illustrated how modern non-perturbative regimes of String Theory provide a rich and exciting avenue for model builders and string phenomenology, with strong low-energy consequences and predictions. While much work has already been conducted into these new regimes, there is still much to be done, and we hope that the work presented in this thesis has shed some light on how our universe might be described by extended fundamental degrees of freedom, such as Strings and Branes.

Appendix A

Semi-local F-theory constructions: R-Parity violating couplings for the various monodromies

In this Appendix we examine the semi-local F-theory models in detail in order to demonstrate that RPV couplings are generic or at least common. To this end we note that:

1. We want models with matter being distributed on different curves. This setup we call multi-curve models, in contrast to the models presented section 4 of [132] and usually considered in other papers that compute Yukawa couplings.

2. The models defined in this framework "choose" the H_u assignment for us, since a tree-level, renormalizable, perturbative top-Yukawa requires the existence of the coupling

$$\mathbf{10}_a\mathbf{10}_a\mathbf{5}_b \tag{A.1}$$

   such that the perpendicular charges cancel out. As such, all the models listed above will have a definite assignment for the curve supporting H_u, and we do not assign the remaining MSSM states to curves, i.e. all the remaining $\mathbf{5}$ curves will be called $\overline{\mathbf{5}}_a$, making clear that they are either supporting some $\overline{\mathbf{5}}_M$ or H_d. Furthermore, we will refer to the $\mathbf{10}$ curve containing the top quark as $\mathbf{10}_M$.

3. The indication for existence of tree-level, renormalizable, perturbative RPV is given by the fact we can find two couplings of the form

$$\mathbf{10}_a\overline{\mathbf{5}}_b\overline{\mathbf{5}}_c \tag{A.2}$$

$$\mathbf{10}_d\overline{\mathbf{5}}_e\overline{\mathbf{5}}_f \tag{A.3}$$

for $(b,c) \neq (e,f)$, and a,d unconstrained. This happens as H_d cannot be both supported in one of the $\overline{\mathbf{5}}_b$, $\overline{\mathbf{5}}_c$ and at the same in one of the $\overline{\mathbf{5}}_e$, $\overline{\mathbf{5}}_f$.

4. We do not make any comment on flux data. The above criteria can be evaded by switching off the fluxes such that the RPV coupling (once the assignment of H_d to a curve is realised) disappears.

With this in mind we study the possible RPV realisations in multi-curve models.

A.1 $2+1+1+1$

In this case the spectral cover polynomial splits into four factors, three linear terms and a quadratic one. Also, due to the quadratic factor we impose a Z_2 monodromy. The bestiary of matter curves and their perpendicular charges (t_i) is given in the Table 6.

Curve :	5_{H_u}	5_1	5_2	5_3	5_4	5_5	5_6	10_M	10_2	10_3	10_4
Charge :	$-2t_1$	$-t_1-t_3$	$-t_1-t_4$	$-t_1-t_5$	$-t_3-t_4$	t_3-t_5	$-t_4-t_5$	t_1	t_3	t_4	t_5

Table A.1: Matter curves and the corresponding $U(1)$ charges for the case of a $2+1+1+1$ spectral cover split. Note that because of the Z_2 monodromy we have $t_1 \longleftrightarrow t_2$.

In this model RPV is expected to be generic as we have the following terms

$$\mathbf{10}_4\overline{\mathbf{5}}_1\overline{\mathbf{5}}_2,\ \mathbf{10}_3\overline{\mathbf{5}}_1\overline{\mathbf{5}}_3,\ \mathbf{10}_M\overline{\mathbf{5}}_1\overline{\mathbf{5}}_6,\ \mathbf{10}_2\overline{\mathbf{5}}_2\overline{\mathbf{5}}_3,\ \mathbf{10}_M\overline{\mathbf{5}}_2\overline{\mathbf{5}}_5,\ \mathbf{10}_M\overline{\mathbf{5}}_3\overline{\mathbf{5}}_4 \tag{A.4}$$

A.2 $2+2+1$

Here the spectral cover polynomial splits into three factors, it is the product of two quadratic terms and a linear one. We can impose a $Z_2 \times$

Z_2 monodromy which leads to the following identifications between the weights,$(t_1 \leftrightarrow t_2)$ and $(t_3 \leftrightarrow t_4)$. In this case there are two possible assignments for H_u (and $\mathbf{10}_M$), as we can see in Table 7.

case 1								
Curve	5_{H_u}	5_1	5_2	5_3	5_4	10_M	10_2	10_3
Charge	$-2t_1$	$-t_1-t_3$	$-t_1-t_5$	$-t_3-t_5$	$-2t_3$	t_1	$-t_3$	t_5
case 2								
Curve	5_{H_u}	5_1	5_2	5_3	5_4	10_M	10_2	10_3
Charge	$-2t_3$	$-t_1-t_3$	$-t_1-t_5$	$-t_3-t_5$	$-2t_1$	t_3	$-t_1$	t_5

Table A.2: The scenario of a $2+2+1$ spectral cover split with the corresponding matter curves and $U(1)$ charges. Note that we have two possible cases.

A.2.1 $2+2+1$ case 1

The bestiary of matter curves and their perp charges is given in the upper half table of Table 7.

In this model RPV is expected to be generic as we have the following terms

$$\mathbf{10}_2\overline{\mathbf{5}}_1\overline{\mathbf{5}}_2,\ \mathbf{10}_M\overline{\mathbf{5}}_1\overline{\mathbf{5}}_3,\ \mathbf{10}_M\overline{\mathbf{5}}_2\overline{\mathbf{5}}_4,\ \mathbf{10}_3\overline{\mathbf{5}}_1\overline{\mathbf{5}}_1 \tag{A.5}$$

Notice that if $\overline{\mathbf{5}}_1$ contains only one state, then the last coupling is absent due to anti-symmetry of SU(5) contraction.

A.2.2 $2+2+1$ case 2

The bestiary of matter curves and their perp charges is given in the lower half table of Table 7.

In this model RPV is expected to be generic as we have the following terms

$$\mathbf{10}_M\overline{\mathbf{5}}_1\overline{\mathbf{5}}_2,\ \mathbf{10}_2\overline{\mathbf{5}}_1\overline{\mathbf{5}}_3,\ \mathbf{10}_M\overline{\mathbf{5}}_3\overline{\mathbf{5}}_4,\ \mathbf{10}_3\overline{\mathbf{5}}_1\overline{\mathbf{5}}_1 \tag{A.6}$$

Notice that if $\overline{\mathbf{5}}_1$ contains only one state, then the last coupling is absent due to anti-symmetry of SU(5) contraction.

A.3 $3+1+1$

In this scenario the splitting of the spectral cover leads to a cubic and two linear factors. We can impose a Z_3 monodromy for the roots of the cubic polynomial. The bestiary of matter curves and their perpendicular charges is given in Table 8:

Curve	5_{H_u}	5_1	5_2	5_3	10_M	10_2	10_3
Charge	$-2t_1$	$-t_1-t_4$	$-t_1-t_5$	$-t_4-t_5$	t_1	t_4	t_5

Table A.3: Matter curves and the corresponding $U(1)$ charges for the case of a $3+1+1$ spectral cover split. Note that we have impose a Z_3 monodromy.

In this model R-parity violation is not immediately generic as we only have

$$\mathbf{10}_2\overline{\mathbf{5}}_1\overline{\mathbf{5}}_2,\ \mathbf{10}_M\overline{\mathbf{5}}_1\overline{\mathbf{5}}_3 \tag{A.7}$$

and as such assigning H_d to $\overline{\mathbf{5}}_1$ avoids tree-level, renormalizable, perturbative RPV.

A.4 $3+2$

These type of models are in general very constrained because of the large monodromies which leads to a low number of matter curves.

In this case there are two possible assignments for H_u (and $\mathbf{10}_M$), as described in Table 9.

	case 1				
Curve	5_{H_u}	5_2	5_3	10_M	10_2
Charge	$-2t_1$	$-t_1-t_3$	$-2t_3$	t_1	t_3
	case 2				
Curve	5_{H_u}	5_2	5_3	10_M	10_2
Charge	$-2t_3$	$-t_1-t_3$	$-2t_1$	t_3	t_1

Table A.4: The two possible cases in the scenario of a $3+2$ spectral cover split, the matter curves and the corresponding $U(1)$ charges.

A.4.1 $3+2$ case 1

The matter curves content is given in the upper half of Table 9 (case 1).

Possible RPV couplings are

$$\mathbf{10}_M\overline{\mathbf{5}}_2\overline{\mathbf{5}}_3\,,\ \mathbf{10}_2\overline{\mathbf{5}}_2\overline{\mathbf{5}}_2 \tag{A.8}$$

Notice that if $\overline{\mathbf{5}}_2$ contains only one state, then the last coupling is absent due to anti-symmetry of SU(5) contraction.

A.4.2 $3+2$ case 2

This second scenario is referred as case 2 in the lower half of Table 9.

Only one coupling

$$\mathbf{10}_M\overline{\mathbf{5}}_2\overline{\mathbf{5}}_2 \tag{A.9}$$

which is either RPV or is absent. Notice that if $\overline{\mathbf{5}}_2$ contains only one state, then the last coupling is absent due to anti-symmetry of SU(5) contraction.

Appendix B

Local F-theory constructions: local chirality constraints on flux data and R-Parity violating operators

The chiral spectrum of a matter curve is locally sensitive to the flux data. This is happens as there is a notion of local chirality due to local index theorems [154, 165]. The presence of a chiral state in a sector with root ρ is given if the matrix

$$m_\rho = \begin{pmatrix} -q_P & q_S & im^2 q_{z_1} \\ q_S & q_P & im^2 q_{z_2} \\ -im^2 q_{z_1} & -im^2 q_{z_2} & 0 \end{pmatrix}$$

with q_i presented in Table 5.4, has positive determinant

$$\det m_\rho > 0. \tag{B.1}$$

As such, if we want a certain RPV coupling to be present, then the above condition has to be satisfied for the three states involved in the respective interaction at the $SO(12)$ enhancement point. For example, in order for the emergence of an QLd^c type of RPV interaction, locally the spectrum has to support a Q, a L, and a d^c states. The requirement that at a single point Equation (B.1) hold for each of these states imposes constraints on the values of the flux density parameters.

Therefore, while RPV effects in general include all three operators -

QLd^c, $u^cd^cd^c$, LLe^c - there are regions of the parameter space that allow for the elimination of some or all of the couplings. These are in principle divided into four regions, depending on the sign of the parameters $\tilde{N}_Y$ and N_Y. In the appendix we present the resulting regions of the parameter space and which operators are allowed in each.

B.1 $\tilde{N}_Y \leq 0$

For $\tilde{N}_Y \leq 0$, the conditions on the flux density parameters for which each RPV interaction is turned on are

$$
\begin{aligned}
QLd^c: \ & M > \frac{-\tilde{N}_Y}{6} \\
& N_a - N_b > \frac{-N_Y}{2} \\
u^cd^cd^c: \ & M > \frac{\tilde{N}_Y}{3} \\
& N_a - N_b > -\frac{N_Y}{3} \\
LLe^c: \ & M > -\tilde{N}_Y \\
& N_a - N_b > \frac{-N_Y}{2}
\end{aligned}
$$

Depending on the sign of N_Y, the above conditions define different regions of the flux density parameter space. These are presented in Tables B.1 and B.2.

$-$	$M < \frac{\tilde{N}_Y}{3}$	$\frac{\tilde{N}_Y}{3} < M < \frac{-\tilde{N}_Y}{6}$	$\frac{-\tilde{N}_Y}{6} < M < -\tilde{N}_Y$	$-\tilde{N}_Y < M$
$(N_a - N_b) < \frac{-N_Y}{2}$	None	None	None	None
$\frac{-N_Y}{2} < (N_a - N_b) < \frac{N_Y}{3}$	None	None	QLd^c	QLd^c, LLe^c
$\frac{N_Y}{3} < (N_a - N_b)$	None	$u^cd^cd^c$	QLd^c, $u^cd^cd^c$	All

Table B.1: Regions of the parameter space and the respective RPV operators supported for $\tilde{N}_Y \leq 0$, $N_Y > 0$

$-$	$M < \frac{\tilde{N}_Y}{3}$	$\frac{\tilde{N}_Y}{3} < M < \frac{-\tilde{N}_Y}{6}$	$\frac{-\tilde{N}_Y}{6} < M < -\tilde{N}_Y$	$-\tilde{N}_Y < M$
$(N_a - N_b) < \frac{N_Y}{3}$	None	None	None	None
$\frac{N_Y}{3} < (N_a - N_b) < \frac{-N_Y}{2}$	None	$u^cd^cd^c$	$u^cd^cd^c$	$u^cd^cd^c$
$\frac{-N_Y}{2} < (N_a - N_b)$	None	$u^cd^cd^c$	QLd^c, $u^cd^cd^c$	All

Table B.2: Regions of the parameter space and the respective RPV operators supported for $\tilde{N}_Y \leq 0$, $N_Y < 0$

B.2 $\tilde{N}_Y > 0$

For $\tilde{N}_Y > 0$, the conditions on the flux density parameters for which each RPV interaction is turned on are

$$
\begin{aligned}
QLd^c: \; M &> \frac{\tilde{N}_Y}{3} \\
N_a - N_b &> \frac{-N_Y}{2} \\
u^c d^c d^c: \; M &> \frac{2\tilde{N}_Y}{3} \\
N_a - N_b &> -\frac{N_Y}{3} \\
LLe^c: \; M &> \frac{-\tilde{N}_Y}{2} \\
N_a - N_b &> \frac{-N_Y}{2}
\end{aligned}
$$

Depending on the sign of N_Y, the above conditions define different regions of the flux density parameter space. These are presented in Tables B.3 and B.4.

–	$M < -\frac{-\tilde{N}_Y}{2}$	$-\frac{-\tilde{N}_Y}{2} < M < \frac{\tilde{N}_Y}{3}$	$\frac{\tilde{N}_Y}{3} < M < \frac{2\tilde{N}_Y}{3}$	$\frac{2\tilde{N}_Y}{3} < M$
$(N_a - N_b) < \frac{-N_Y}{2}$	None	None	None	None
$\frac{-N_Y}{2} < (N_a - N_b) < \frac{N_Y}{3}$	None	LLe^c	QLd^c, LLe^c	QLd^c, LLe^c
$\frac{N_Y}{3} < (N_a - N_b)$	None	LLe^c	QLd^c, LLe^c	All

Table B.3: Regions of the parameter space and the respective RPV operators supported for $\tilde{N}_Y > 0$, $N_Y > 0$

–	$M < -\frac{-\tilde{N}_Y}{2}$	$-\frac{-\tilde{N}_Y}{2} < M < \frac{\tilde{N}_Y}{3}$	$\frac{\tilde{N}_Y}{3} < M < \frac{2\tilde{N}_Y}{3}$	$\frac{2\tilde{N}_Y}{3} < M$
$(N_a - N_b) < \frac{N_Y}{3}$	None	None	None	None
$\frac{N_Y}{3} < (N_a - N_b) < \frac{-N_Y}{2}$	None	None	None	$u^c d^c d^c$
$\frac{-N_Y}{2} < (N_a - N_b)$	None	LLe^c	QLd^c, LLe^c	All

Table B.4: Regions of the parameter space and the respective RPV operators supported for $\tilde{N}_Y > 0$, $N_Y < 0$

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