An Investigation into Tensile Membrane Action as a Means of Emergency Load Redistribution

by

Peter P. Smith

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Project Supervisor: Prof. S.S. Moy
Abstract

This thesis presents a review of structural robustness in conventional (non-hardened, non-governmental and non-military) buildings subjected to malicious actions and has also investigated the capability of tensile membrane (catenary) action as a means of emergency load redistribution in reinforced concrete (RC) framed buildings, designed in accordance with BS 8110 and Eurocode 2.

Case studies and forensic data relating to the response of conventional blast damaged buildings have been investigated. Based on the conclusions drawn from this investigation a simple ranking system has been presented that identifies the level of confidence offered by various forms of direct and indirect robust design.

Catenary action has been shown to be a fundamental mechanism of emergency load redistribution for which research has been relatively limited, particularly in relation to RC frame constructions. An experimental investigation has been carried out; this involved the testing to failure of a series of half scale RC strip specimens, detailed to conventional detailing practices, to investigate the large-displacement behaviour and ultimate collapse resistance of laterally restrained RC floor components following the loss of intermediate column support. The results showed characteristic compressive and tensile membrane responses. However, tensile membrane action was found to provide the largest reserve of strength and to offer the highest potential as a means of emergency load redistribution. By testing specimens to outright collapse the investigation identified factors influencing catenary performance and demonstrated that the total energy absorption by work done in catenary action was influenced by the tensile properties and detailing of the specimens.

The current ultimate limit criteria and theoretical approach to ultimate load prediction in catenary response have been critically reviewed using results obtained in testing. The findings have been implemented in an analytical study of collapse resistance by ultimate catenary response. A series of exemplar floor systems, of different size and detailing arrangements have been examined. The conclusions presented indicate that safe load redistribution by catenary action is dependent on the area of the unsupported structural bays and the area of the bottom reinforcement specified at or across structural connections, for which design parameters such as moment redistribution, detailing rules and tie force requirements are governing factors.
Acknowledgements

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The support and guidance of Prof. Stuart Moy has been invaluable to the completion of this thesis. I would like to express my sincere gratitude for his time and encouragement.

A large component of the thesis entailed testing in the University’s heavy structures lab. As such Ben, Dave, Mike and Cliff deserve my thanks and acknowledgement for their technical support. Ben requires a special mention as it was his assistance and collaboration that made the concept of large-scale testing a reality and his thesis (Punton, 2015) constitutes a valuable extension to this research.

Finally, thank you to my family, friends and colleagues for all your support. You know who you are.

Declaration

The work presented herein was done wholly or mainly while in candidature for a research degree at the University of Southampton. No part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution. Where I have consulted the published work of others, this is always clearly attributed and where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work. I have acknowledged all main sources of help. Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Parts of this work have been published as:


## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of Figures</td>
<td>vi</td>
</tr>
<tr>
<td>List of Tables</td>
<td>xii</td>
</tr>
<tr>
<td>Nomenclature</td>
<td>xiii</td>
</tr>
<tr>
<td>1 Introduction</td>
<td>1-1</td>
</tr>
<tr>
<td>1.1 Progressive &amp; Disproportionate Collapse</td>
<td>1-2</td>
</tr>
<tr>
<td>1.2 Structural Robustness</td>
<td>1-3</td>
</tr>
<tr>
<td>1.3 The Research Aims &amp; Objectives</td>
<td>1-6</td>
</tr>
<tr>
<td>1.4 Thesis Layout</td>
<td>1-7</td>
</tr>
<tr>
<td>2 Literature Review – Structural Robustness</td>
<td>2-1</td>
</tr>
<tr>
<td>2.1 High Explosive Blast</td>
<td>2-1</td>
</tr>
<tr>
<td>2.1.1 Basic characteristics of high explosive blast</td>
<td>2-1</td>
</tr>
<tr>
<td>2.1.2 Fundamental factors that influence load intensity</td>
<td>2-2</td>
</tr>
<tr>
<td>2.1.3 Scale distance</td>
<td>2-4</td>
</tr>
<tr>
<td>2.1.4 TNT equivalency</td>
<td>2-5</td>
</tr>
<tr>
<td>2.2 In-Field Performance of Buildings Subject to Blast</td>
<td>2-5</td>
</tr>
<tr>
<td>2.2.1 Progressive collapse resistance</td>
<td>2-6</td>
</tr>
<tr>
<td>2.2.2 Performance of structural members under blast</td>
<td>2-15</td>
</tr>
<tr>
<td>2.3 Review of Current Protective Measures &amp; Robustness Schemes</td>
<td>2-21</td>
</tr>
<tr>
<td>2.3.1 Safe stand-off distance</td>
<td>2-22</td>
</tr>
<tr>
<td>2.3.2 Specific local resistance</td>
<td>2-23</td>
</tr>
<tr>
<td>2.3.3 Direct alternative load paths</td>
<td>2-25</td>
</tr>
<tr>
<td>2.3.4 Accidental Limit State (ALS) load cases</td>
<td>2-31</td>
</tr>
<tr>
<td>2.3.5 Notional column removal &amp; key element design</td>
<td>2-33</td>
</tr>
<tr>
<td>2.3.6 Structural segmentation</td>
<td>2-35</td>
</tr>
<tr>
<td>2.3.7 Indirect alternative load paths – effective tying</td>
<td>2-36</td>
</tr>
<tr>
<td>2.3.8 Relative performance</td>
<td>2-38</td>
</tr>
<tr>
<td>2.4 Conclusions</td>
<td>2-41</td>
</tr>
<tr>
<td>3 Literature Review – Tensile Membrane (Catenary) Action in RC Systems</td>
<td>3-1</td>
</tr>
<tr>
<td>3.1 Research into Large Displacement Behaviour in Laterally Restrained Reinforced Concrete Assemblies – Origin &amp; Background</td>
<td>3-1</td>
</tr>
<tr>
<td>3.2 Characteristic Membrane Action Response</td>
<td>3-2</td>
</tr>
<tr>
<td>3.2.1 Compressive membrane action (CMA) response</td>
<td>3-4</td>
</tr>
<tr>
<td>3.2.2 Snap-through</td>
<td>3-6</td>
</tr>
<tr>
<td>3.2.3 Tensile membrane action (TMA) or catenary response</td>
<td>3-7</td>
</tr>
<tr>
<td>3.2.4 ‘Primary’ TMA response, ‘Secondary’ TMA response &amp; typical failure modes</td>
<td>3-8</td>
</tr>
<tr>
<td>3.2.5 Principal displacements</td>
<td>3-11</td>
</tr>
<tr>
<td>3.3 Analysis of Catenary Action in RC Assemblies</td>
<td>3-12</td>
</tr>
<tr>
<td>3.3.1 Uniformly loaded 2-way systems</td>
<td>3-12</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
</tr>
<tr>
<td>---------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>3.3.2</td>
<td>Uniformly loaded 1-way systems</td>
</tr>
<tr>
<td>3.4</td>
<td>Ultimate Load Criteria &amp; Failure Limits</td>
</tr>
<tr>
<td>3.5</td>
<td>Emergency Load Redistribution by Catenary Action</td>
</tr>
<tr>
<td>3.5.1</td>
<td>Large-scale experimental investigations of emergency response in RC systems</td>
</tr>
<tr>
<td>3.5.2</td>
<td>Area of reinforcement &amp; maximum tension membrane force</td>
</tr>
<tr>
<td>3.5.3</td>
<td>Reinforcement detailing &amp; arrangement</td>
</tr>
<tr>
<td>3.5.4</td>
<td>Rotation capacity, extensibility &amp; predicting failure</td>
</tr>
<tr>
<td>3.5.5</td>
<td>Lateral restraint stiffness</td>
</tr>
<tr>
<td>3.6</td>
<td>Summary</td>
</tr>
<tr>
<td>4</td>
<td>Experimental Study of Catenary Action in RC Framed Buildings</td>
</tr>
<tr>
<td>4.1</td>
<td>Experimental Programme</td>
</tr>
<tr>
<td>4.2</td>
<td>Scale Effects</td>
</tr>
<tr>
<td>4.3</td>
<td>Test Specimen &amp; Material Specification</td>
</tr>
<tr>
<td>4.3.1</td>
<td>Edge Beam &amp; Slab-strip specimens</td>
</tr>
<tr>
<td>4.3.2</td>
<td>Flat-slab strip specimens</td>
</tr>
<tr>
<td>4.3.3</td>
<td>Reinforcement specification</td>
</tr>
<tr>
<td>4.3.4</td>
<td>Concrete specification</td>
</tr>
<tr>
<td>4.4</td>
<td>Test Rig</td>
</tr>
<tr>
<td>4.4.1</td>
<td>End reactions – ( R_H ) &amp; ( R_V )</td>
</tr>
<tr>
<td>4.4.2</td>
<td>Support reactions – ( R_1, R_2 ) &amp; ( P )</td>
</tr>
<tr>
<td>4.4.3</td>
<td>Applied load &amp; midspan displacement</td>
</tr>
<tr>
<td>4.4.4</td>
<td>Specimen &amp; column displacements</td>
</tr>
<tr>
<td>4.4.5</td>
<td>Rig-specimen interface</td>
</tr>
<tr>
<td>4.5</td>
<td>Experimental Procedure</td>
</tr>
<tr>
<td>5</td>
<td>Test Results &amp; Analysis</td>
</tr>
<tr>
<td>5.1</td>
<td>Results Summary – General Large-Displacement Performance &amp; Definitions</td>
</tr>
<tr>
<td>5.2</td>
<td>Ultimate Load Capacity, Enhancement &amp; Performance in Catenary Action</td>
</tr>
<tr>
<td>5.3</td>
<td>Specimen Response &amp; Catenary Mechanisms</td>
</tr>
<tr>
<td>5.3.1</td>
<td>Edge Beam specimens</td>
</tr>
<tr>
<td>5.3.2</td>
<td>Slab-strip specimens</td>
</tr>
<tr>
<td>5.3.3</td>
<td>Flat-slab strip specimens</td>
</tr>
<tr>
<td>5.4</td>
<td>Rotation Limits &amp; Specimen Extensibility</td>
</tr>
<tr>
<td>5.4.1</td>
<td>CMA-TMA transition, ( \theta_0 )</td>
</tr>
<tr>
<td>5.4.2</td>
<td>Ultimate plastic chord &amp; joint rotation, ( \theta'u ) &amp; ( \phi'u )</td>
</tr>
<tr>
<td>5.4.3</td>
<td>Secondary ultimate chord rotation, ( \theta'u )</td>
</tr>
<tr>
<td>5.4.4</td>
<td>Elongation at ultimate secondary displacement</td>
</tr>
<tr>
<td>5.5</td>
<td>Load Resistance &amp; Membrane Forces in Catenary Action</td>
</tr>
<tr>
<td>5.5.1</td>
<td>Horizontal restraint &amp; membrane force, ( RH ) &amp; ( N )</td>
</tr>
<tr>
<td>5.5.2</td>
<td>System equilibrium at ultimate load carrying capacity, ( PTMA )</td>
</tr>
<tr>
<td>5.6</td>
<td>Discussion &amp; Conclusions</td>
</tr>
<tr>
<td>6</td>
<td>Emergency Load Redistribution by Catenary Action</td>
</tr>
</tbody>
</table>
List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Image Source</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1</td>
<td>Remains of the Alfred P. Murrah Federal Building</td>
<td><a href="http://dart2.arc.nasa.gov">http://dart2.arc.nasa.gov</a></td>
<td>1-1</td>
</tr>
<tr>
<td>1-2</td>
<td>Schematic illustration of basic forms of emergency load redistribution found in framed structures following single column loss.</td>
<td></td>
<td>1-3</td>
</tr>
<tr>
<td>2-1</td>
<td>Idealised free-field pressure-time history of high explosive blast wave.</td>
<td></td>
<td>2-2</td>
</tr>
<tr>
<td>2-2</td>
<td>Pressure-time histories recorded for a given detonation of high explosives at 1, 5 and 10m intervals from blast source (Mainstone, 1974).</td>
<td></td>
<td>2-3</td>
</tr>
<tr>
<td>2-3</td>
<td>The four forms of high explosive aerial ordnance recognised by the Allies during WWII, Christopherson (1945).</td>
<td></td>
<td>2-6</td>
</tr>
<tr>
<td>2-4</td>
<td>Load redistribution due to masonry infill panels (Baker et al. 1948).</td>
<td></td>
<td>2-7</td>
</tr>
<tr>
<td>2-5</td>
<td>Connection failure modes as reported by Baker et al. (1948).</td>
<td></td>
<td>2-10</td>
</tr>
<tr>
<td>2-6</td>
<td>External wall system forced from its abutting floor under blast loading (Baker et al. 1948).</td>
<td></td>
<td>2-11</td>
</tr>
<tr>
<td>2-7</td>
<td>Typical failure modes of RC-frame building joints subject to internal blast loading (Rhodes, 1974).</td>
<td></td>
<td>2-13</td>
</tr>
<tr>
<td>2-8</td>
<td>Collapse damage observed in long-span roof truss structure following loss of internal column (Baker et al. 1948).</td>
<td></td>
<td>2-14</td>
</tr>
<tr>
<td>2-9</td>
<td>Images of RC column shear failure following lateral blast loading (Baker et al. 1948).</td>
<td></td>
<td>2-16</td>
</tr>
<tr>
<td>2-10</td>
<td>Heave in a floor slab after being subject to a confined explosion at its soffit (Thomas, 1948).</td>
<td></td>
<td>2-17</td>
</tr>
<tr>
<td>2-11</td>
<td>Tension failure at RC column base (Baker et al. 1948; Thomas, 1948).</td>
<td></td>
<td>2-18</td>
</tr>
<tr>
<td>2-12</td>
<td>Load-deflection curves from pairs of identical RC beams (Christopherson, 1945).</td>
<td></td>
<td>2-19</td>
</tr>
<tr>
<td>2-14</td>
<td>Images taken of the incident column in the Club El Nogal Building, Bogota, Columbia, (a) under construction and (b) following the incident (Garcia et al. 2006).</td>
<td></td>
<td>2-24</td>
</tr>
<tr>
<td>2-15</td>
<td>Collapse sustained by Murrah Federal Building, Oklahoma (1996), following attack by VBIED.</td>
<td></td>
<td>2-26</td>
</tr>
<tr>
<td>2-16</td>
<td>Schematic illustration of damage from September 11 attacks and subsequent load redistribution (diagram adapted from FEMA 403).</td>
<td></td>
<td>2-27</td>
</tr>
<tr>
<td>2-17</td>
<td>1940’s steel frame having sustained local damage under near blast effects (Baker et al. 1948).</td>
<td></td>
<td>2-29</td>
</tr>
<tr>
<td>2-18</td>
<td>Ronan Point collapse, 1968 (image obtained from: <a href="http://www.ace.ac.uk/">http://www.ace.ac.uk/</a>).</td>
<td></td>
<td>2-34</td>
</tr>
</tbody>
</table>
Figure 2-19 – Partial collapse of Terminal E2 superstructure, Charles de Gaulle Airport, Paris (2004). ................................................................. 2-36
Figure 2-20 – Schematic of tie arrangement as illustrated in UFC 4-023-03 (DoD, 2005)........ 2-37
Figure 3-1 – Schematic illustration of membrane force development in a centrally loaded RC element, subject to large displacement and rigid lateral restraint......................................................... 3-3
Figure 3-2 - Idealised axial force and load-displacement curves showing membrane actions in reinforced concrete member fixed against all translation (consolidated from Park and Gamble, 2000; Black, 1975; Rankin and Long, 1998; Wang, 2010).......................................................... 3-3
Figure 3-3 – Ultimate moment-axial force diagram for typical symmetrically reinforced section (Park and Gamble, 2000). ................................................................. 3-5
Figure 3-4 – Image taken of the underside of a slab specimen subject to tensile membrane action (Park, 1964b). ................................................................. 3-7
Figure 3-5 – Primary TMA response (a), secondary TMA response (b) and collapse (c) as observed in doubly reinforced RC Catenary mechanisms ......................................................... 3-9
Figure 3-6 – Extract from Mitchell and Cook (1984) showing initial punching shear failure and subsequent distress at column headers, as observed in flat-plate RC systems......................................................... 3-10
Figure 3-7 – Plan of plastic membrane and free body diagram of small segment, as shown by Park (1964). .............................................................................. 3-13
Figure 3-8 – Comparison of experimental and theoretical load-displacement history (Park, 1964b). 3-14
Figure 3-9 – Circular deflection profile assumed by Hawkins and Mitchell (1979). ................. 3-15
Figure 3-10 – Deflection profiles and free-body diagrams implemented by Regan (1975). .......... 3-17
Figure 3-11 – Load-displacement history of a one-way spanning slab specimen, showing secondary TMA response – Woodson and Garner (1985). ......................................................... 3-20
Figure 3-12 – Schematic illustration of emergency load redistribution in RC frame sub-assembly following column loss............................................................... 3-20
Figure 3-13 – Typical detailing of double span slab-slab specimens tested by Regan (1975). .... 3-22
Figure 3-14 – Elevation of the test arrangement used by Regan (1975), showing the left-hand restraint detail and removable central support. ................................................................. 3-23
Figure 3-15 – Results obtained by Regan (1975) in testing.................................................. 3-24
Figure 3-16 – Typical beam-column sub-assembly detailing, as tested by Yu and Tan (2013a and 2013b). .............................................................................. 3-25
Figure 3-17 – Test set-up implemented by Yu and Tan (2013a)............................................... 3-25
Figure 3-18 – Load-displacement histories recorded by Yu and Tan (2013b) for specimens S4 and S6. .............................................................................. 3-26
Figure 3-19 – Load-displacement histories recorded by Yu and Tan (2013b) for specimens S3, S4, S5, S7 and S8. .............................................................................. 3-26
Figure 3-20 – Extract from Gouverneur (2013a) showing test arrangement and failure of test specimen ‘Slab 2’ .......................................................................................................................................................................................... 3-27
Figure 3-21 – Load-displacement histories recorded by Gouverneur et al. (2013a) in double-span slab strip specimens featuring continuous (Slab 1) and curtailed (Slab 2) reinforcement arrangements. ................................................................. 3-28
Figure 3-22 – Longitudinal reinforcement strain recorded at ¼ span from supports (extract from Lew et al. 2011) ........................................................................................................................................................................ 3-29
Figure 3-23 – Load-displacement and membrane force histories recorded by Gouverneur et al. (2013a) in double-span slab strip specimens featuring continuous (Slab 1) and curtailed (Slab 2) reinforcement arrangements. ........................................................................................................................ 3-30
Figure 3-24 – Load and horizontal reaction force histories recorded by Yu and Tan (2013b)........ 3-31
Figure 3-25 – Load and horizontal reaction force histories recorded by Yu and Tan (2013b)........ 3-32
Figure 3-26 – Extract from Regan (1975) showing reinforcement arrangement and modes of response observed in double-span test specimen........................................................................................................... 3-33
Figure 3-27 – Beam-column assemblage detailing of tests carried out by Kang and Tan (2015). .. 3-35
Figure 3-28 – Direct tension tests documented by Regan (1975), featuring discontinuous critical reinforcement .......................................................................................................................................................... 3-36
Figure 3-29 – Ultimate displacement data, recorded at maximum primary load resistance, obtained from open-source TMA and Catenary Action experimental investigations. ...................................................... 3-39
Figure 3-30 – Photograph of large-scale double span test specimen, as tested by Lew et al. (2011) and Sadek et al. (2011). ........................................................................................................................................................................ 3-40
Figure 3-31 – Component-based joint model proposed by Yu and Tan (2014). ....................... 3-47
Figure 3-32 – Macro-FEA developed by Yu and Tan (2014). .................................................. 3-47
Figure 3-33 – Emergency load redistribution by catenary action in penultimate structural bay following perimeter column loss (Dat and Hai, 2011). .......................................................................................................................... 3-48
Figure 3-34 – Illustration of ‘compression thrust ring’ development in slab diaphragms (Dat and Hai, 2011). ........................................................................................................................................................................ 3-49
Figure 3-35 – Emergency load redistribution by TMA, observed in ½ scale flat-slab test specimen (Yi et al. 2014). ........................................................................................................................................................................ 3-49
Figure 3-36 – Photograph of test specimen investigated by Yi et al. (2008)............................. 3-51
Figure 4-1 – Emergency load carrying mechanisms in RC frame having sustained peripheral column loss. ........................................................................................................................................................................ 4-2
Figure 4-2 – Floor plan of beam-slab test building................................................................. 4-6
Figure 4-3 – Longitudinal sections showing edge beam and slab-strip 1:2 scale test specimen reinforcement arrangement and curtailment (dimensions in mm). ................................................................. 4-8
Figure 4-4 – Longitudinal section showing column and middle strip test specimen reinforcement arrangement.................................................................................................................................................. 4-10
Figure 5-12 – Load-rotation record of edge-beam specimens, E01, E02 and E03. .......................... 5-14
Figure 5-13 – Photographs of E02 after collapse (at $\Delta = 585\,\text{mm}$), showing (a) collapse mechanism and (b) close-up of B2 rebar fracture and scabbing of soffit cover. ................................................................. 5-15
Figure 5-14 – Hinge development observed at the support (a) and midspan (b) of S02 at $\Delta = 270\,\text{mm}$. .................................................................................................................................. 5-16
Figure 5-15 – Schematic elevation of deflection profile and crack distribution observed in specimens S02 and S03 at $\Delta = 670\,\text{mm}$ (end of test) – elevation not to scale. ................................................................. 5-16
Figure 5-16 – Images of S02 at $\Delta = 670\,\text{mm}$ (end of test), showing (a) tension cracking and change in slope at support hinge and (b) final catenary profile. ................................................................. 5-17
Figure 5-17 – Schematic elevation of deflection profile and crack distribution observed in S01 at $\Delta = 372\,\text{mm}$ (prior to failure) – elevation not to scale. ................................................................. 5-19
Figure 5-18 – Load-rotation record of slab-strip specimens S01, S02 and S03. ................................ 5-20
Figure 5-19 – Typical deflection profile of flat-slab specimens at failure (elevation – not to scale). . 5-21
Figure 5-20 – Load-rotation record of column-strip flat-slab specimens C01, C02, C03 and C04. 5-22
Figure 5-21 – Load-rotation record of middle-strip flat-slab specimens M01 and M02. ............ 5-23
Figure 5-22 – Test record midspan displacements at transition of membrane force ($R_H = 0\,\text{kN}$) against specimen thickness ($h$). Note: data points with no fill denote results obtained following primary failure of reinforcement (E01, E03, C02, C04, M01 and M02). ................................................................. 5-24
Figure 5-23 – Relationship between midspan displacement at transition of membrane force ($R_H = 0\,\text{kN}$) and specimen span-thickness ratio ($2L/h$). Note: data points with no fill denote results obtained following primary failure of reinforcement (E01, E03, C02, C04, M01 and M02). ................................................................. 5-24
Figure 5-24 – Ultimate plastic chord rotation recorded for each test specimen at first fracture. .... 5-26
Figure 5-25 – Sketch showing derivation of chord and joint rotation angles in (a) three-pin and (b) four-pin mechanisms........................................................................................................... 5-27
Figure 5-26 – Ultimate joint rotation against reinforcement ratio of the local extreme tension rebar. 5-27
Figure 5-27 – Experimental record of failure chord rotation in TMA response. ....................... 5-28
Figure 5-28 – Relationship of recorded ultimate secondary chord rotation against (a) area of critical reinforcement and (b) ratio of critical reinforcement.............................................. 5-30
Figure 5-29 – Specimen percentage elongation, recorded at incipient collapse. ..................... 5-31
Figure 5-30 – Force-rotation plot for test specimen S03 showing applied load and restraint force. 5-32
Figure 5-31 – Catenary mechanism assumed for assessment – shown as three-hinged mechanism BDF. ........................................................................................................................................... 5-33
Figure 5-32 – Ratios of recorded membrane force upon measured yield and tensile strength of the critical reinforcement. ................................................................................................................ 5-36
Figure 5-33 – Experimental moment-axial force interaction found at midspan and support of specimen E02 shown against theoretical ultimate M-N interaction envelope. ...................................................... 5-40
Figure 5-34 – Experimental vs. predicted values of ultimate secondary catenary load capacity. ...5-44
Figure 5-35 – Experimental vs. predicted values of ultimate secondary catenary load capacity. ...5-47
Figure 6-1 – Comparison of experimental and theoretical load-rotation relationship for Edge Beam test specimens. ........................................................................................................................................6-2
Figure 6-2 – Comparison of experimental and theoretical load-rotation relationship for Slab Strip test specimens. ........................................................................................................................................6-3
Figure 6-3 – Free-body diagrams and notations supporting analysis of a double bay bilinear catenary mechanism. ........................................................................................................................................6-5
Figure 6-4 – Tributary area assumed for emergency load redistribution by catenary action..............6-7
Figure 6-5 – Reference floor plan. ........................................................................................................................................6-8
Figure 6-6 – Reinforcement layout and bar reference. .............................................................................6-10
Figure 6-7 – FOS results obtained for floor systems where $A_{s-crit} = A_{s3}$. ........................................6-11
Figure 6-8 – FOS results obtained for floor systems where $A_{s-crit} = A_{s4}$. ............................................6-13
Figure 6-9 – FOS results obtained for floor systems where $A_{s-crit} = A_{s5}$. .............................................6-14
Figure 6-10 – FOS results obtained for floor systems where $A_{s-crit} = A_{s10}$. ........................................6-14
Figure 6-11 – FOS results obtained for floor systems where $A_{s-crit} = A_{s15}$. ........................................6-14
List of Tables

Table 2-1 – Proposed robustness strategy ranking .......................................................... 2-39
Table 4-1 – Summary of 1:2 scale specimen detailing and geometry .................................. 4-5
Table 4-2 – Derivation of edge beam and slab-strip specimen dimensions and detailing at 1:2 scale. 4-7
Table 4-3 – Derivation of column and middle-strip slab specimen dimensions and detailing. ...... 4-10
Table 5-1 – Experimental results summary table ..................................................................... 5-4
Table 5-2 – Summary of primary and secondary TMA load resistance data .......................... 5-7
Table 5-3 – Horizontal restraint and tension membrane force data, recorded at ultimate secondary TMA load .................................................................................................................. 5-35
Table 5-4 – Horizontal restraint and tension membrane force data, recorded at ultimate primary TMA load .......................................................................................................................... 5-38
Table 5-5 – Summary table of experimental and theoretical values of ultimate load resistance in secondary TMA response ........................................................................................................... 5-42
Table 5-6 – Summary table of experimental and theoretical values of ultimate load resistance in primary TMA response ............................................................................................................. 5-45
# Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_c$</td>
<td>Area of concrete</td>
</tr>
<tr>
<td>$A_{gt}$</td>
<td>Percentage elongation at maximum force</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Area of reinforcement</td>
</tr>
<tr>
<td>$A_s'$</td>
<td>Area of compression reinforcement</td>
</tr>
<tr>
<td>$A_{s-crit}$</td>
<td>Area of critical reinforcement engaged in secondary TMA response</td>
</tr>
<tr>
<td>$A_{s-TMA}$</td>
<td>Area of reinforcement effective in tensile membrane force</td>
</tr>
<tr>
<td>$A_{sn}$</td>
<td>Area of reinforcement allocation ‘n’</td>
</tr>
<tr>
<td>$A_{s-0.5}$</td>
<td>50% of total area of longitudinal reinforcement</td>
</tr>
<tr>
<td>$A_{s-1.0}$</td>
<td>100% of total area of longitudinal reinforcement</td>
</tr>
<tr>
<td>$A_5$</td>
<td>Percentage elongation at fracture</td>
</tr>
<tr>
<td>$b$</td>
<td>Breadth of cross section</td>
</tr>
<tr>
<td>$d$</td>
<td>Effective depth to tension reinforcement centroid</td>
</tr>
<tr>
<td>$d'$</td>
<td>Effective depth to compression reinforcement centroid</td>
</tr>
<tr>
<td>$FOS$</td>
<td>Factor of safety</td>
</tr>
<tr>
<td>$f_{bu}$</td>
<td>Ultimate anchorage bond strength (Regan, 1975)</td>
</tr>
<tr>
<td>$f_{ck}$</td>
<td>Concrete compressive (cylinder) strength</td>
</tr>
<tr>
<td>$f_{cu}$</td>
<td>Concrete compressive (cube) strength</td>
</tr>
<tr>
<td>$f_{cu}'$</td>
<td>Characteristic (factored) concrete compressive (cube) strength</td>
</tr>
<tr>
<td>$f_{cm}$</td>
<td>Mean concrete compressive (cube) strength</td>
</tr>
<tr>
<td>$f_d$</td>
<td>Design reinforcement stress</td>
</tr>
<tr>
<td>$f_y; f_{yk}$</td>
<td>Yield reinforcement stress</td>
</tr>
<tr>
<td>$f_u$</td>
<td>Ultimate reinforcement tension stress</td>
</tr>
</tbody>
</table>
\( G_k; G_{k,j} \) Characteristic permanent action (dead load)

\( g_k \) Characteristic distributed permanent action (dead load)

\( k \) Aspect ratio of floor bay \((L_y/L_x)\)

\( h \) Total depth/height of concrete cross section

\( L; l \) Span

\( L_{ALS}; L_{EM} \) Accidental limit state or emergency span (following intermediate support removal)

\( L_x; l_x \) Span in \( x \) (longest) orientation

\( L_y; l_y \) Span in \( y \) (shortest) orientation

\( l_n \) Clear span (Mitchell and Cook, 1984)

\( l_2 \) Slab width (Mitchell and Cook, 1984)

\( l_{y/x,i} \) Discrete length ‘i’

\( M_{RES} \) Moment resistance (sagging)

\( M'_{RES} \) Moment resistance (hogging)

\( N \) Membrane force

\( N' \) Ultimate primary tensile membrane force

\( N'' \) Ultimate secondary tensile membrane force

\( N_d \) Design tensile membrane force

\( P \) Mid-span load or reaction

\( P_{CMA} \) Ultimate load in CMA response

\( P_{FA} \) Ultimate theoretical flexural resistance

\( P_{TMA} \) Ultimate load in TMA response

\( P'_{TMA} \) Ultimate load in primary TMA response

\( P''_{TMA} \) Ultimate load in secondary TMA response
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P'_{TMA-theory}$</td>
<td>Ultimate theoretical load in primary TMA response</td>
</tr>
<tr>
<td>$P''_{TMA-theory}$</td>
<td>Ultimate theoretical load in secondary TMA response</td>
</tr>
<tr>
<td>$P_0$</td>
<td>Load at CMA-TMA transition</td>
</tr>
<tr>
<td>$Q_{ALS}$</td>
<td>Accidental limit state load</td>
</tr>
<tr>
<td>$Q_k; Q_{k,1}$</td>
<td>Characteristic leading variable action (live load)</td>
</tr>
<tr>
<td>$Q_{k,i}$</td>
<td>Additional variable action</td>
</tr>
<tr>
<td>$Q_{Sn}$</td>
<td>Characteristic snow load</td>
</tr>
<tr>
<td>$q$</td>
<td>Uniformly distributed load</td>
</tr>
<tr>
<td>$q_{ALS}$</td>
<td>Accidental limit state distributed load</td>
</tr>
<tr>
<td>$q'_{TMA}$</td>
<td>Ultimate distributed load resistance in primary TMA response</td>
</tr>
<tr>
<td>$q''_{TMA}$</td>
<td>Ultimate distributed load resistance in secondary TMA response</td>
</tr>
<tr>
<td>$R_e$</td>
<td>Recorded yield strength of reinforcement</td>
</tr>
<tr>
<td>$R_H$</td>
<td>In-test Lateral restraint reaction</td>
</tr>
<tr>
<td>$R_m$</td>
<td>Recorded ultimate tensile strength of reinforcement</td>
</tr>
<tr>
<td>$R_{V1}; R_{V2}$</td>
<td>In-test rotational restraint reaction</td>
</tr>
<tr>
<td>$R_1; R_2$</td>
<td>In-test interior support reaction</td>
</tr>
<tr>
<td>$R_{0.5(e+m)}$</td>
<td>Average of yield and ultimate tensile strength of reinforcement</td>
</tr>
<tr>
<td>$T_x$</td>
<td>Membrane force in $x$ plane (Park, 1964b; Michell and Cook, 1984)</td>
</tr>
<tr>
<td>$T_y$</td>
<td>Membrane force in $y$ plane (Park, 1964b; Michell and Cook, 1984)</td>
</tr>
<tr>
<td>$W$</td>
<td>Work done</td>
</tr>
<tr>
<td>$W_{CMA}$</td>
<td>Work done in CMA response</td>
</tr>
<tr>
<td>$W_k$</td>
<td>Characteristic wind load</td>
</tr>
<tr>
<td>$W_{TMA}$</td>
<td>Work done in TMA response</td>
</tr>
<tr>
<td>$w$</td>
<td>Distributed unit load (Park, 1964b; Michell and Cook, 1984)</td>
</tr>
</tbody>
</table>
\( \beta \)  
Ratio of elastic moment redistribution

\( \Delta \)  
Mid-span displacement

\( \Delta_{CMA} \)  
Mid-span displacement at peak CMA load

\( \Delta_{cr} \)  
Mid-span displacement at cracking and plastic hinge development

\( \Delta_f \)  
Ultimate catenary displacement at rebar fracture (Merola, 2009)

\( \Delta_i \)  
Mid-span displacement at discrete location ‘i’

\( \Delta_{TMA} \)  
Mid-span displacement at peak TMA load

\( \Delta_{TMA}' \)  
Ultimate mid-span displacement in primary TMA load

\( \Delta_{TMA}'' \)  
Ultimate mid-span displacement in primary TMA load

\( \Delta_{ult}; \Delta_u \)  
Mid-span displacement at incipient collapse

\( \Delta_0 \)  
Mid-span displacement at CMA-TMA transition

\( \delta L \)  
Longitudinal extension

\( \delta L_u \)  
Ultimate longitudinal extension

\( DLF; \Omega \)  
Dynamic load factor

\( \varepsilon \)  
Strain in reinforcement

\( \varepsilon_x \)  
Strain in \( x \) orientation longitudinal reinforcement

\( \varepsilon_y \)  
Strain in \( y \) orientation longitudinal reinforcement

\( \varepsilon_f \)  
Rupture strain of the reinforcement

\( \varepsilon_u \)  
Ultimate strain of the reinforcement

\( \phi \)  
Joint rotation

\( \phi_u' \)  
Ultimate plastic joint rotation

\( \psi_{1,1}; \psi_{2,1}; \psi_{2,i} \)  
Case dependent variable action factors (Eurocode 0)

\( \rho \)  
Reinforcement ratio (tension reinforcement allocation)
\( \rho' \) Reinforcement ratio (compression reinforcement allocation)

\( \rho_{\text{crit}} \) Reinforcement ratio (critical reinforcement allocation)

\( \theta \) Chord (end) rotation \( (\tan^{-1} \Delta/L) \)

\( \theta_{\text{CMA}} \) Chord rotation at peak CMA load

\( \theta_{\text{TMA}} \) Chord rotation at peak TMA load

\( \theta'; \theta'_{\text{TMA}} \) Ultimate chord rotation in primary TMA response

\( \theta''; \theta''_{\text{TMA}} \) Ultimate chord rotation in secondary TMA response

\( \theta'_y \) Chord rotation at yield of tension membrane in primary TMA response

\( \theta''_y \) Chord rotation at yield of tension membrane in secondary TMA response

\( \theta'_u \) Chord rotation at first fracture of extreme tension reinforcement

\( \theta''_u \) Chord rotation at fracture of critical reinforcement (total collapse)

\( \theta_0 \) Chord rotation at CMA-TMA transition

\( \gamma_f \) Load factor (British Standards)
1 Introduction

The consequences associated with catastrophic building collapse have been well documented; a high potential of mass casualties and the loss or destruction of assets located within the building (Mallonee et al. 1996). As indicated by Elliott et al. (1992 and 1994) these attributes have made structural collapse an effective tactic in present day terrorism and to those set on achieving political or socio-economic impact by violent means. Perhaps the most notorious instances of terrorist actions against buildings have been those involving the use of improvised explosive devices (IEDs), impact and or fire to cause collapse; the Beirut US Marine Corps Barracks (Beirut, 1983), Alfred P. Murrah Federal Building (Oklahoma, 1995; see Figure 1-1) and Manhattan Twin Towers (New York, 2001). Such cases are rare and, whilst these are amongst the most severe, they demonstrate the susceptibility of some buildings to collapse following the loss of key structural components as the result of malicious actions.

![Figure 1-1 – Remains of the Alfred P. Murrah Federal Building – image source: http://dart2.arc.nasa.gov.](http://dart2.arc.nasa.gov)

In this context a malicious action can be described as a deliberate attempt to cause serious damage to a structure. Governmental and military facilities, which traditionally represent high value targets to terrorists and insurgents, are designed in anticipation of malicious actions. Such facilities are augmented with pre-emptive measures designed to keep aggressors at a safe distance from buildings of importance. In certain cases, the buildings themselves are hardened to meet the plausible threats; key structural elements are designed to withstand predicted malicious loads or the building is arranged in such a way as to sustain damage without the onset of collapse. However, few conventional structures (non-hardened, non-governmental and non-military) feature this level of security or physical protection as malicious actions are generally not anticipated. These buildings therefore represent comparatively ‘soft’ targets, whereby the likelihood of an attack reaching and damaging the building is significantly greater.
Between 2011 and 2013, English-language media demonstrated a dramatic rise in the use of IEDs with reports of related civilian casualties increasing by 70%, impacting 66 countries and territories (AOAV, 2014). Analysis of global terrorist attacks highlight that 58% of these incidents involve vehicle borne improvised explosive devices (VBIEDs; Department of Homeland Security, 2015) and increasingly target civilian (non-governmental or non-military) infrastructures. The targeting of buildings that are not designed in anticipation of malicious actions highlights a pressing need to understand how such structures respond to an appropriate level of damage. Once known, such knowledge has a spectrum of applications, from informing best practice guidelines when designing new buildings for abnormal loads to informed risk reduction by identification of vulnerable infrastructures.

The resulting initiative has identified a need for analytical and experimental research into validating or improving structural robustness within present day infrastructure and construction practice (Ellingwood et al., 2007; Stevens, 2008; DCLG, 2011). This thesis presents a contribution to the field of research by investigating emergency load redistribution in reinforced concrete (RC) framed buildings following hypothetical column loss.

### 1.1 Progressive & Disproportionate Collapse

Local structural failure can, in the case of damaged columns and load bearing walls, lead to part of the building being left without support causing instability in the structure and possible collapse. There are two recognised forms of collapse:

**Progressive collapse** is a cascading failure that propagates from its source to adjacent elements; spreading either in a horizontal direction, from bay to bay, vertically, from floor to floor, or as a combination of the two. In the context of malicious actions, this form of failure may be attributed to primary damage that results in the successive overload of structural components under redistribution of load and, in the extreme, can lead to complete collapse.

**Disproportionate collapse** describes a collapse whose extents are disproportionate to the initiating event. Whether a collapse may be termed as disproportionate is ultimately dependent on the judgement of the observer and their perception. UK regulations (ODPM, 2013) define a collapse as disproportionate when loss of a single load bearing element leads to a collapse exceeding the lesser of 100m² or 15% of the immediate floor area.

The Beirut US Marine Corps barracks (Beirut, 1983) and Alfred P. Murrah Federal Building (Oklahoma, 1995) are examples of catastrophic progressive collapse. However, given the extreme extent of the initiating damage, whether these examples can be deemed disproportionate is less clear.
1.2 Structural Robustness

For a structure to resist the onset of collapse it must be capable of safely redistributing the load previously associated with its damaged components – referred to herein as the emergency load. Most malicious actions occur at ground floor level and bridging requires emergency loads to be redistributed through the structure above the zone of damage. Figure 1-2 provides schematic illustrations of the principal forms of emergency response observed in structural systems immediately following the loss of local support (Ellingwood 2008 and Cormie 2009). The images shown are of framed structures that have sustained the loss of a single column and demonstrate the potential mechanisms by which the emergency load may be redistributed to the remaining structure. Each mechanism is a form of alternative load path and may be identified as follows:

Figure 1-2 – Schematic illustration of basic forms of emergency load redistribution found in framed structures following single column loss.

Arching Action (see Figure 1-2a) is a form of response found to occur in horizontal spanning elements during the early stages of displacement (less than the depth of the member), when the system features high lateral restraint. Displacement of the element facilitates an in-plane compression force known as compressive membrane action (CMA) that effectively forms a compression arch between abutments. This form of redistribution in reinforced concrete structures requires a low
span-depth ratio and high degree of restraint. Arching action has also be observed in prestressed concrete slabs which have higher span-depth ratios.

*Vierendeel or Frame Action* (see Figure 1-2b) is unique to continuous framed systems with moment resistant connection. In this case the emergency load is redistributed to the surrounding structure by flexural redundancy in the frame. This requires a high degree of continuity between structural components and reserve flexural strength.

*Catenary Action* (see Figure 1-2c) is a form of load carrying mechanism akin to a cable bridge. The emergency load is sustained by in-plane catenary or tensile forces but substantial displacements may be required for sufficient catenary force to be developed. This form of response demands ductile structural members and connections and a suitable level of lateral restraint from the surrounding structure.

*Direct Alternative Load Path* is achieved by the emergency load being redirected via an arrangement of adjacent structural or non-structural elements that form an emergency truss type system. Figure 1-2d shows this mechanism can achieved by the presence of a simple truss system in the upper portion of the frame. However, shear panels, masonry infill panels and adequately tied prefabricated modular units are each capable of sustaining this type of response provided they have appropriate strength and level of connectivity.

The ability of a building to safely redistribute load and resist the onset of disproportionate or progressive collapse is dependent upon two factors: 1) that the structure is capable of redistributing load by at least one of the emergency mechanisms detailed above and 2) that the extent of initial damage does not impose an emergency load that exceeds the capacity of the alternative load path or paths. Some forms of construction, such as monolithic framed structures with infill panelling, support a number of the emergency mechanisms introduced above but others, such as gravity load bearing buildings, feature little interconnection between structural components and are more susceptible to collapse and primary damage (Loizeaux and Osborne, 2006).

The susceptibility of certain building types to progressive collapse was first officially recognised following the Ronan Point incident in Newham, London in 1968 – an accidental explosion resulted in the loss of an external load bearing panel, on the eighteenth floor, and subsequent disproportionate collapse within the twenty-two storey residential building (see Figure 1-3). The investigation that followed revealed the existing codes of practice took no account of the vulnerability of structures to progressive collapse following the loss of key structural components under abnormal loads and that certain forms of construction were inherently vulnerable to this form of failure (Griffiths *et al.*, 1968).
International engineering practice has since seen the introduction of a series of requirements intended to ensure structural robustness – ‘the ability of a structure to withstand events like fire, explosions, impact or the consequences of human error, without being damaged to an extent disproportionate to the original cause’ BSI (2006). These robustness requirements were intended to reduce the vulnerability of buildings to disproportionate collapse by a process of direct or indirect robust design whereby a building is either designed to prevent initiating damage or provided with the means to form an effective emergency mechanism.

Direct robust design requires the deliberate consideration of progressive and disproportionate collapse. In the UK, this predominantly entails a process of notional element removal whereby the design engineer is required to demonstrate the response of a building to the removal of individual load bearing elements, such as walls and columns. The designer is required to demonstrate that, with use of direct alternative load paths, the failure attributed to the removal of an individual structural component does not exceed disproportionate collapse limits (ODPM, 2013). In cases where this is not achievable, the vulnerable elements can be designed as key elements capable of withstanding a prescribed static out-of-plane load of 34kPa, applied to each face and those components to which it provides support.

Indirect robust design requires no deliberate consideration of structural response following the removal of load bearing components. Instead, this process entails the application of prescribed effective tying rules that regulate the strength and arrangement of structural connections. By designing
connections to meet the tie rules it is assumed that a building’s robustness is sufficient and that disproportionate and progressive collapse will be prevented by catenary action.

Robustness measures introduced to UK structural design practice in the 1970s have since been incorporated within building regulations and guidance documents internationally. However, the adequacy of the robustness measures in preventing disproportionate collapse following accidental actions has been the subject of scrutiny in recent years – mainly since the collapse of the World Trade Centre (2001). The effective tying rules, key element design and provision of direct alternative load paths have remained largely unchanged for over four decades, since their introduction with the Fifth Amendment of the UK Building Regulations (Department of the Environment, 1971) and CP110 (BSI, 1972). Within modern infrastructure, where advances in fabrication methods, design theory and spatial demands have led to increased spans and less redundant structural systems, these measures may now be outdated and inadequate. Recent studies, such as Ellingwood et al. (2007), Stevens (2008) and DCLG (2011), have demonstrated a lack of experimental and analytical support for the various robustness measures and indicate that research is required in order to verify their adequacy.

1.3 The Research Aims & Objectives

In view of current confidence in the robustness of convention buildings and an accompanying increase in malicious actions, the aim of the research presented in this thesis was to investigate the performance of conventionally designed buildings that sustain damage characteristic of VBIED attack. The study was intended to contribute to the current field of research by determining whether current practice is adequate or whether further recommendations are required. However, it became apparent that the aim had to be reduced in scope to concentrate on a particular aspect of resistance to disproportional collapse, namely tensile membrane (sometimes called catenary) action.

The study objectives explored within this research are:

1. To carry out an investigation into the applicability of tensile membrane (catenary) action as a means of preventing disproportionate collapse in the hypothetical situation where an external column in a structural frame is instantaneously removed.

2. To carry out an experimental programme to investigate the parameters that affect tensile membrane action at large span-depth ratios which result from the removal of an intermediate column.

3. To use the experimental results to produce a means of predicting the load bearing capacity resulting from tensile membrane action.
1.4 Thesis Layout

The objectives of this study were explored by undertaking several research investigations, some of which are review based in nature (Chapters 2 and 3), experimental (Chapters 4 and 5) or analytical (Chapter 6).

Chapter 2 consists of two main sections. The first is a review of forensic research literature concerned with the effects of high explosive blast on conventional buildings. The investigation identifies characteristic blast damage, typical modes of structural response and collapse and general trends in the resilience of different structural materials and construction types. This is followed by a review of the current methods implemented in protective and robust design of conventional buildings. This is mainly concerned with code and institutional guidance based recommendations for the design, analysis and implementation of mitigation measures. Case studies and current research findings are used to critically review the feasibility and effectiveness of these procedures and an attempt is made to rank each in terms of the relative level of robustness and protection provided. The bulk content of this chapter was published in following journal and conference paper Smith et al. (2010).

By investigating the current confidence in the each of the measures currently used in progressive collapse and protective design (Chapter 2), emergency load redistribution by tensile membrane (catenary) action in reinforced concrete (RC) structures was identified as a predominant means of direct and in-direct robust design but a field with limited supporting research. Chapter 3 provides a literature review of the experimental and analytical research available on the subject. The investigation identifies the current understanding of catenary performance in RC structures, in the event of instantaneous column removal, and demonstrates that current practice is largely based on experimental investigations of RC slabs subject to overload or misuse. By comparison with the limited research available, the investigation establishes inaccuracies in the current approaches to the analyses and design of RC catenary systems for support loss conditions and demonstrates that the empirical limit criteria are not representative of ultimate catenary response in modern RC constructions. Furthermore, the investigation identifies that the ultimate resistance of double-span catenaries, in emergency load redistribution, is frequently sustained immediately prior to incipient collapse but that this is a level of response that has only been documented by two independent studies, one of which is out-dated.

Chapter 4 describes the design and fabrication of a test programme devised for the experimental investigation of conventional RC beam and slab sub-assemblies subject to large in-elastic displacements under double-span and laterally restrained conditions. Twelve ½-scale RC test specimens were fabricated and tested as part of a collaborative effort (Punton, 2015). This chapter describes key aspects of the experimental rig, test procedure and test specimens that were specifically designed to address gaps identified in the current understanding of catenary response under support-loss conditions. Namely; the span-depth ratios of the test specimens were consistent with conventional full-scale buildings and a gap identified in the current data stock; the test specimens featured different
reinforcement detailing to establish the effect of complex rebar arrangements on catenary performance; the experimental rig was instrumented to allow direct assessment of in-plane and out-of-plane forces and specimen behaviour, from initial central support removal to displacement at collapse; a system was devised to allow displacement of all specimens through 16° end-rotation, such that out-right collapse was achieved or the ultimate failure mechanism could be identified. This test arrangement supported the collection of experimental data in primary and secondary catenary response, for large-scale test specimens featuring different area of reinforcement, detailing and span-depth ration.

Chapter 5 presents the results obtained in testing. The results are analysed to draw conclusions regarding ultimate load resistance criteria in primary and secondary catenary response. Trends between area of reinforcement, reinforcement detailing, span-depth ratio and catenary performance are identified and analysed. The experimental results are implemented to review the validity of current analytical approaches used in the assessment and design of catenary mechanisms in RC beam and slab systems. The chapter presents one of the only current studies of secondary catenary response and identifies a need for the revision of current analytical and design practice, specifically; the review of ultimate displacement and membrane force criteria.

Chapter 6 consolidates the findings and conclusions made in testing to present an analytical study of secondary catenary response. An analytical approach is proposed for the assessment of ultimate load resistance in catenary action and a sensitivity study is undertaken to determine factors influencing the robustness of exemplar RC framed systems in resisting the loss of intermediate columns by catenary action.

Conclusions, observations and areas for further research identified throughout the thesis are consolidated in Chapter 7.
2 Literature Review – Structural Robustness

This research was originally concerned with the performance of conventional framed buildings subject to the effects of high explosive (HE) blast, improvised explosive devices (IEDs) in particular. The following chapter aims to identify typical primary damage attributed to the direct effects of the HE blast and the secondary behaviour of buildings in redistributing emergency load after local damage has been sustained. This is achieved by the review of blast damage data, case studies and current research and design guidance. The fundamental properties of blast loading are introduced, highlighting properties that are known to influence the degree of structural damage and considered critical for technical discussion.

2.1 High Explosive Blast

High explosive (HE) blast has routinely been employed as a means of causing damage or outright collapse by destabilising or overwhelming the ultimate strength of structural systems. This is evident from accounts provided by authors such as Christopherson (1945), Baker et al. (1948), Thomas (1948), Rhodes (1974), Elliott (1990) and more recently Davis (2008). Accordingly a large volume of resources are available related to the prediction of blast loads and primary structural damage (Biggs, 1964; Baker et al. 1983; DoD, 2014; etc.). As the precise prediction of blast damage is beyond the scope of this thesis, the following section provides a brief overview to the subject of high explosive blast, identifying the main factors that influence blast load and severity of damage to allow interpretation of terminology and conclusions drawn throughout the thesis.

2.1.1 Basic characteristics of high explosive blast

Explosive blast is formed by the violent expansion of gasses upon initiation of the source material, resulting in highly impulsive pressure wave and rarefaction effects. To give an idea of the volatility of this process, the gasses associated with initiation of a charge of trinitrotoluene (TNT) form at some 1.54GPa and a temperature of about 3,000 degrees absolute, the reaction travelling at approximately 6.7km/s (Christopherson, 1945). The surrounding air is driven outward by this initial expansion of explosive gasses to form a pressure pulse that matures from what is initially a sinusoidal wave to the distinctive shock front and pressure wave shown in Figure 2-1.

Figure 2-1 shows an idealisation of the pressure-time history of a blast wave that propagates spherically from its source. Velocity of propagation varies but is always greater than the speed of sound (approximately 340m/s at sea level). The discontinuous increase in pressure, from ambient to the peak-overpressure, is characteristic of the arrival of the positive phase; a short period of extreme positive pressure that decays rapidly. The subsequent period of sub-atmospheric pressure is known as the negative phase, a partial vacuum, not exceeding one atmosphere, which gradually dissipates before stabilising at ambient atmospheric pressure. In addition to this extreme variation of pressure, objects...
within the blast radius will experience a drag force associated with rarefaction winds that lag from the passage of the blast wave. This anomaly is known as dynamic pressure which exerts a force at first away from the source before then reversing toward the point of detonation.

![Idealised free-field pressure-time history of high explosive blast wave.](image)

**Figure 2-1** – Idealised free-field pressure-time history of high explosive blast wave.

For a given explosive material, at a given altitude, the velocity, magnitude and wavelength of the associated blast wave are directly proportional to the load experienced by an object. For this reason blast loading is expressed in terms of the peak-overpressure and the integral of the pressure-time history, the impulse. The response of individual structural components to this form of impulsive load may be determined by a process of linear or nonlinear dynamic analysis (Smith and Hetherington, 1994; Biggs, 1964).

### 2.1.2 Fundamental factors that influence load intensity

The following factors are known to influence the peak overpressure, impulse and thus load intensity of a hemispherical blast.

*Charge size* – The peak overpressure, phase duration and thus net impulse increases with the calorific yield of the charge and thus, for a given explosive, the total weight of charge detonated.

*Stand-off* – As shown in Figure 2-2, the peak pressure and damage potential associated with a blast decay with distance from the source of the explosion. Closer inspection demonstrates an increase in phase duration with increasing standoff but Christopherson (1945) documents an almost exponential decay of net impulse with scaled distance (scaled distance is defined below). This rapid attenuation indicates the significant benefit found in providing stand-off between the device and the target building.
Angle of incidence – The magnitude of load experienced at a surface is known to vary with angle of incidence, owing to reflection of the blast. Completely oblique surfaces, with a ninety degree angle of incidence, are subject to a side-on pressure and impulse defined by free-field blast characteristics. Smith and Hetherington (1994) and the document UFC 3-340-02 (DoD, 2014) show that normal surfaces, of zero angle of incidence, are subject to a reflected load of between 2 and 12.5 times the peak side-on or free-field pressure. Thus, the orientation of a building or structural component with respect to the blast origin is critical to the magnitude of blast load and damage sustained.

Diffraction – Research conducted by Christopherson (1945) demonstrated that the pressure-time history of a blast wave is significantly altered when forced to diffract about an up-wind obstructing surface; in the immediate shadow of the obstruction the peak overpressure and duration of the positive phase is reduced and the discontinuous increase of pressure, associated with the shock front, becomes more gradual. This observation is supported by Jarrett (1968) who claims a twenty percent reduction in impulse and duration at all distances, when the blast is obstructed by a suitably designed vertical traverse. Therefore whilst objects in the shadow of an obstruction will still experience a time variable overpressure of substantial magnitude, loading and damage will be less severe.

Confinement – In ‘free-field’ an explosion is unconstrained; the blast wave and explosive gasses are able to expand freely. The idealised pressure-time record shown in Figure 2-1 is that of an unconfined explosion. When confined, the opposite is true. Provided with complete confinement, the walls are subject to a significant gas pressure and the blast wave reverberates, reflecting from opposing surfaces. Whilst the severity of load enhancement is dependent on the degree of...
ventilation, volume of the enclosed space and proximity of the reflecting surfaces, Baker et al. (1948) claimed that the impulse associated with each phase would be as much as 10-20 times that experienced in the open. Whilst this form of amplification may seem limited to the case of internal explosions, it is typical for blast load associated with free-field blast to be significantly increased in re-entrant corners, concave profiles and confined alleys which hinder the propagation of the blast and cause it to coalesce (Mays and Smith, 1995).

**Permeability & Clearing** – Authors such as Tyas et al. (2011) have documented a reduced reflected overpressure and impulse at the face of components that offer a small surface area to a blast wave, such as isolated columns. This is due to the ability of the blast to equalize about an element that presents relatively minor surface area with respect to the wavelength of the blast load and an action known as ‘clearing’ whereby rarefaction at the edges of the reflecting surface acts to reduce net impulse (Baker et al. 1948). This would suggest that isolated columns and key structural components suffer less damage than those supporting rigid tributary wall systems loaded by blast, thus promoting highly permeable structural systems. However, FEMA (2003b) issues caution against arrangements that may be susceptible to detrimental heave and uplift.

Considering these factors, it is evident that the intensity of a blast load and thus the extent of primary damage incurred by a building is dependent upon the charge weight and position of the device and the geometry and scale of the structure. However, there is a general consensus that the most important factor with regards to primary damage is the scale-distance found between the device and structure (Bangash and Bangash, 2006; Christopherson, 1945; Dusenberry, 2010; Krauthammer, 2008).

### 2.1.3 Scale distance

The distance from the source of an explosion is routinely expressed in terms of the scaled distance (Z), see Equation 2-1. This expression is derived from the Hopkinson-Cranz or ‘cube-root’ scaling law which states that two charges of similar geometry and explosive material will produce blast waves of similar characteristic properties at a common scale distance, Z. More specifically, the impulse and phase duration experienced at distance Z will be directly proportional to the ratio of the two charge sizes but the peak-overpressure, of both explosions, will be the same (Baker et al. 1983).

\[ Z = \frac{R}{W^{1/3}} \]

Where; \( W \) is the charge weight (kg) and \( R \) is the respective distance from the explosion epicentre (m).

The cube-root law is used extensively to interpret the recorded blast characteristics of small experimental explosions so as to determine the properties of blast waves produced by larger, in-field devices. This practice has limitations in terms of interpolating between differing explosive sources and charge weights that differ greatly in size or shape. Furthermore, amplification factors must be introduced when scaling from spherical air blasts to hemispherical explosions, like VBIEDs (a multiplication factor of 1.8 was suggested by Mays and Smith 1995 for the hemispherical case).
However, by expressing stand-off as a scaled distance, cautious comparisons can be made between the structural damage caused to different buildings by devices of varying size and proximity.

### 2.1.4 TNT equivalency

The magnitude, duration and strength of a blast are dependent upon the mass specific energy of the explosive material – its potential energy per unit mass and density (Smith and Hetherington, 1994; Baker et al. 1983). TNT equivalency is a method whereby alternative forms of explosive or explosive mixture are scaled with respect to the mass specific energy of the universal constant Trinitrotoluene (TNT); 4,520kJ/kg at a density of 1.6Mg/m³. Application of a ‘Charge Factor’ (C.F.), permits any form of high explosive to be expressed in terms of the mass required to yield an equivalent specific energy of TNT, for example; 50kg of Nitro Glycerine, of mass specific energy 6,700kJ/kg, may be described as having an equivalent charge weight of 74.1kg TNT (C.F. of 1.48). Expressing different devices in terms of their equivalent TNT charge weight assists in the comparison of explosions and attributed structural loading/damage from different source materials.

Alternative methods of calculating the TNT equivalency of a device are also used – based upon the peak-overpressure or impulse of a given weight of TNT at a given stand-off – which can be inconsistent with charge factors determined from the mass specific energy.

### 2.2 In-Field Performance of Buildings Subject to Blast

During WWII a considerable amount of research was undertaken to understand weapons effects on conventional structures. The work was conducted in a combined effort between US and British engineers. The effort involved extensive field trials and, given the bombing of the British mainland and Allied advance, numerous forensic investigations; some 60,000 basic and 5,000 more detailed surveys of bomb damaged buildings.

Research carried out in the USA was relatively limited and has been summarised in the report *Effects of Impact and Explosion*, NDRC (1946). However, results from the WWII structural engineering research have been reported in a series of reports with the prefix ‘R.C.’, with the main findings consolidated into *RC450 – Structural Defence* (Christopherson, 1945). Other key publications by Baker et al. (1948) and Thomas (1948), together with the Francis Walley Collection held at the Institution of Civil Engineers archive (London, UK), provided valuable first-hand evidence of primary blast damage and structural robustness in conventional buildings. Much of the contents of these publications have been superseded by advances in computing, plastic theory and the understanding of structural dynamics, however, their basic ‘rules of thumb’ are still valid today and can be traced to current practice in blast and protective engineering.

The research was centred upon building damage attributed to the four main types of aerial ordnance classified by the Allies, namely; armour-piercing, semi-armour-piercing, medium capacity (general...
purpose) and high capacity 'blockbuster' bombs (see Figure 2-3). Armour-piercing and semi-armour-piercing types had low charge-to-casing weight ratios to facilitate penetration and delayed fusing as well as pronounced fragmentation effects. Such fragmentation was generally found to cause only superficial structural damage (Christopherson, 1945). In contrast, the medium and high capacity bombs had much higher charge-to-casing weight ratios. These devises were typically contact or barometrically fused and posed a significant threat to buildings by free-field blast effects. Importantly, with charge weights ranging from 450kg to 1,815kg (1000lb to 4000lb) such devices can be compared with current blast threats, such as the VBIED, recognised by NATO (1999), ATF (1999), FEMA (2003) and NYCPD (2009).

![Figure 2-3 – The four forms of high explosive aerial ordnance recognised by the Allies during WWII, Christopherson (1945).](image)

This section provides a review of the observations made during the 1940’s to identify lessons learnt with regard to the performance of non-military and non-hardened buildings subject to the effects of high explosive blast. The investigation was intended to provide some insight into the threat of terrorist devices; the extent of primary blast damage and characteristic secondary building response. However, it must be acknowledged that the nature of the aerial ordnance identified and its mode of delivery is dissimilar in many ways from threats such as the VBIED and assessments of blast damage and relative structural robustness should be conducted on a case-by-case basis.

### 2.2.1 Progressive collapse resistance

The surveys conducted during the 1940's investigations categorised structural failures into three forms; direct damage, primary collapse and progressive collapse. The most common accounts were of direct damage caused by either blast or impact of a weapon. Cases of primary collapse, detailing the local failure of structural systems immediately dependent upon damaged elements for support, were less
common. Progressive collapse, whereby collapse would spread from the point of damage, was less common still; largely confined to load bearing masonry buildings with the majority of well-constructed framed structures demonstrating a good resilience against the onset of progressive collapse. Interpretation of the evidence collected from bomb damaged buildings during WWII suggests that much of this robustness can be attributed to the use of elastic design theory, close column spacing and the presence of masonry infill panelling – all features that are rare in modern construction.

![Image of load redistribution due to masonry infill panels](image-url)  

*Figure 2-4 – Load redistribution due to masonry infill panels (Baker et al. 1948).*

Most framed structures of the 1940's era featured the extensive use of 100mm or 240mm (4” or 9\(\frac{1}{2}\)”) masonry infill panelling. With the exception of those incorporated in shear walls and perhaps central cores, the brick/blockwork panels were implemented as redundant non-structural partitioning and external cladding elements – not contributing to the strength of the structure in its service state. In the event of a close proximity explosion the panels proved effective in two ways; helping to segregate the building and isolate the effects of the blast whilst providing multiple alternative load paths, by the formation of diagonal compression struts that facilitate load redistribution in the event of severe direct damage and column loss. The images of Figure 2-4 demonstrate this form of emergency response. Figure 2-4a shows the entire front face of a steel framed building unsupported due to the loss of a girder at the first floor, whilst Figure 2-4b and Figure 2-4c provide aspects of a ten-storey framed structure that sustained loss of its corner columns due to a near miss. In both cases damage can be
seen to be relatively localised and extensive primary and progressive collapse were prevented by the alternative load paths mobilised in the masonry panelling.

The additional robustness afforded a structure by infill panelling was indisputable and has been more recently demonstrated by the experimental investigations of Li *et al.* (2016) and Sasani (2008). However, Baker *et al.* (1948), Christopherson (1945) and their contemporaries identified a significant drawback to masonry in-filled frame (MIF) construction. Whether framed by reinforced concrete or steel, it was found that by flanking the columns of a structure a blast would be unable to pass freely about (or ‘clear’) stanchions and columns. Baker (1948b) observed that by obstructing *clearing*, even with masonry panels not bonded to the column face, the impulse load could be fifty to one hundred times that experienced by a similar but isolated column. Despite this amplification and its significant effect on the likely extent of direct damage attributed to any given explosion, Christopherson and Baker remained advocates of infill panelling. Christopherson stated that use of 220mm (9”) masonry panels would provide substantial gains in racking resistance without compromising the permeability of the frame. The idea of insetting the columns away from the line of any external walling was considered but later dismissed as an inferior construction method; the masonry being more vulnerable to blast effects because of a lack of segmentation and structural stability significantly reduced by additional eccentricities.

### 2.2.1.1 Steel framed buildings

Examples of progressive collapse were relatively limited for multi-storey steel framed buildings. Due to the discontinuous and modular form of this type of construction (slabs being fixed to the beams using a wide variety of techniques and walls tending to be independent), direct damage attributed to medium sized bombs (approximately 500lb) was found to be highly localised. Whilst subsequent primary collapse was found to be common, any spread of failure thereafter was generally confined to that caused by either the excessive debris loading of floor systems or column eccentricities incurred by ground shock – accounts of progressive collapse following column loss were very few. Collapse due to debris and overload seems to have been the principle concern as design recommendations made by Christopherson and Baker *et al.*, intended to reduce the risk of progressive collapse, entailed the strengthening of floor systems in anticipation of floors falling from above. Overload precautions are not typically observed in modern robust design as malicious and accidental loading is generally considered to occur at ground level. Further, engineering practice is more concerned with preventing the collapse of overbearing floors, for which the behaviour of the frame under column loss is more relevant.

Given the extensive use of MIF systems, it is difficult to interpret the robustness of steel framed buildings in their bare form, without the alternative load paths made available by masonry infill. However, scrutiny of direct damage and primary collapse evidence points to areas of weakness that might indicate the robustness and response of an unclad steel-frame. The principle weakness identified
by Baker (1948), Baker et al. (1948), Thomas (1948) and Christopherson (1945) was found to be the inter-system connections between walls and floors and individual steel components.

Forensic evidence showed a distinct weakness of the flange-cleat and web-cleat beam to column connections commonly used in these buildings, proving vulnerable to direct damage and fundamental to primary collapse. Fixed with bolts and rivets and designed as simple 'shear only' connections, these assemblies were found to be particularly susceptible to shear failure under high loading rates – although tensile failures were found to occur if the shear strength of the bolts greatly outweighed the tensile strength – and exhibited a marked inferiority in ductility and continuity compared to the members that they connected. Figure 2-5 provides schematic representations of the recorded typical failure modes of each of the semi-rigid connections common to the era. Column 'a' of the table shows the complete shear failure of bolt/rivet groups at the column face for each type of connection; a failure mode that was very rare and typical of assemblies overwhelmed by blast loads transmitted by floor beams. Failures featured in column 'c' are of a similar nature but were due to the lateral blast loading of wall systems and were found to be quite common (see Figure 2-6). However with regard to the emergency mechanisms adopted by a steel-framed building, in which a degree of rotation is likely in the connections, the modes of failure shown in column 'b' are most appropriate. Christopherson (1945) accredited the performance of a connection to its ductility and rotation capacity – its ability to sustain end rotation whilst under load. And whilst column 'b' demonstrates the susceptibility of flange-cleat assemblies to bolt/rivet shear and prying of the cleats, Christopherson stated that the ductility of the assembly was significantly greater than that of the double angle web cleats which would to fail by tearing of the web under excessive bending. Marginal benefit was noted in the performance of double angle web-cleats with multiple bolt groups at the web but, where welded connection was not possible, Christopherson (1945) advocated the installation of additional flange-cleats. Of the assemblies considered by Christopherson and his compatriots, the double angle web-cleat was found to be the worst, which is of particular interest because of its similarity to the modern fin-plate connection.

Baker (1948) noted that of the numerous steel-framed buildings that he surveyed almost all collapses were a result of the inadequate connection between perimeter columns and beams, a conclusion consolidated by Christopherson (1945); "in those few cases in which serious collapses have occurred, the cause could always be traced to weakness in the connections". The use of moment resistant connections was heavily promoted by both Baker et al. (1948) and Christopherson (1945), who stated that beams should be made continuous for the length and breadth of the building and columns continuous for its full height. Though such connections were rarely used during the period, the added continuity and ductility afforded by such joints were deemed to provide better performance and resilience than the simple semi-rigid connections. It was also recommended that, despite their performance being documented as satisfactory, the riveted column splices of the day should be designed for the full moment capacity of the stanchions they joined to achieve the desired continuous
frame. No direct preference was specified between the form of fixed beam-column connection used, whether haunched, site welded or fixed with more specialist bolts.

<table>
<thead>
<tr>
<th>Load Scenario</th>
<th>a). Very Close Proximity Blast to Floor System (pos. &amp; neg.)</th>
<th>b). Near Proximity Blast to Floor System (pos. &amp; neg.)</th>
<th>c). Lateral Blast Loading Sustained by Wall System</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Typical Modes of Failure</strong></td>
<td>Complete shear failure of column-cleat bolt group:</td>
<td>Combined shear failure of bolts at column and beam:</td>
<td>Shear failure of landing bolts (beam-cleat):</td>
</tr>
<tr>
<td>Flange-Cleat Connection</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Web-Cleat Connection</td>
<td>Complete shear failure of column-cleat bolt group:</td>
<td>Shear failure of landing bolts and bending in leeward cleat:</td>
<td></td>
</tr>
<tr>
<td>Web-Cleat Connection (extended rivet group)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Web-Cleat Connection (extended rivet group)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Likelihood</strong></td>
<td>Extremely rare</td>
<td>Commonly documented</td>
<td>Commonly documented</td>
</tr>
</tbody>
</table>

Figure 2-5 – Connection failure modes as reported by Baker et al. (1948).

Although advocated by Christopherson (1945), site welding was very uncommon at the time. Similarly, high strength friction grip bolts and partial depth end-plates, later recommended by Rhodes
(1974), had not been developed by the 1940’s, so quite complex riveted or bolted arrangements would have been needed. Interestingly, no mention was made as to whether the frame itself should be designed as moment resisting. However, Baker et al. (1948) implied that lateral bracing should remain and connections should be designed to the capacity of the members supported, advantage being drawn from greater economy in the section sizes used.

![Figure 2-6](image)

Figure 2-6 – External wall system forced from its abutting floor under blast loading (Baker et al. 1948).

The importance of continuity to the resilience of framed structures includes the connections between floor and wall systems. This is particularly so in steel-framed structures, whose construction is distinctly segmented. Buildings featuring weak joints between floor slabs and their surrounding beams were found to be extremely susceptible to direct damage and primary collapse. Of particular fragility were buildings whose perimeter walls ran parallel to the direction of the slab span. In such cases the interface between the floor slabs and the wall beams were found to be particularly weak, leaving only the connection between the column and the floor beam perpendicular to the building edge effective in restraining the column against outward lateral loading (as shown in Figure 2-6). Christopherson (1945) and Baker (1948) documented a number of instances in which internal and external explosions caused the simultaneous failure of wall-column connections (see Figure 2-6) for the height of the structure, leading to floor collapse in several storeys. Thus emphasis was placed on the anchoring of floor slabs to their supporting beams, regardless of their bearing. Baker advised that, for anchorage to be sufficient, the reinforcement of the supported slabs should be continuous through intermediate primary beams and heavily tied about wall beams in a manner to provide substantial tying force. In addition, Baker concluded that the most satisfactory fixity of floor and wall systems was achieved with
filler-joist and in-situ floor systems, whilst the performance of timber and precast concrete systems were found to be particularly unsatisfactory.

2.2.1.2 RC-framed buildings

Investigations into reinforced concrete (RC) framed structures was less insightful than with the steel as the case studies were limited to ten individual accounts of direct hits by medium sized ordnance. From those cases surveyed, it was found that the area of direct damage was remarkably small and primary and progressive collapse proved extremely rare. Christopherson (1945) attributes this to a high level of continuity that would permit the frame to bridge damaged areas with little difficulty by frame or vierendeel action. Of the few examples of collapse, the integrity of the structure was found to be undermined by poor workmanship, the use of substandard concrete, poor detailing not adhering to the code requirements, a low factor of safety with respect to the design to service load ratio or a combination of these factors.

The weaknesses identified by Baker et al. (1948) to contribute most to collapse of RC-frames are of little relevance to modern practice as construction standards and design practice have changed significantly over the years. To highlight possible areas of weakness it is necessary to refer to the work of Rhodes (1974) who presents the findings of structural surveys undertaken for various buildings subjected to high explosive attack during the early stages of the terrorist campaign in Northern Ireland. Rhodes concluded that the standard detailing of RC-frame joints was insufficient to prevent some cases of primary collapse. Direct damage to supporting columns were often found to result in joint failure, as shown in Figure 2-7. Typically the arrangement of rebar rendering the concrete vulnerable to crushing due to stress concentrations induced by the emergency loads. Such observations are perhaps of greater relevance to the modern assessment of collapse risk as the buildings surveyed during this period were likely to have been designed using early versions of limit state theory rather than the permissible stress methods used until the replacement of CP114 by CP110 1972-1986, BSI (1972). Furthermore frames designed using the elastic methods used pre-1948 were claimed (Marshall, 1952) to resist twice their design load before ultimate failure.

A note must be made concerning the relative performance of the various forms of RC construction. In the WWII period RC frames were either flat slab (a single depth of construction over the whole floor area) or fully framed beam-slab assemblies and each performed differently under the effects of blast loading. Of the two, flat-slab structures were found to be the most resilient to both direct damage and primary collapse as the floors possessed a greater relative mass and level of continuity than the fully framed variant (Christopherson, 1945).
2.2.1.3 Steel frame shed-structures

The bombing of industrial and manufacturing infrastructure was a significant feature of the WWII aerial campaigns. Thus surveys of bomb damaged industrial and factory buildings were numerous and identification of vulnerable structural configurations seems to have been a priority. In general, these buildings were of single-storey steel framed construction, featuring roof truss systems to facilitate large column spacing and open floor areas. Christopherson (1945) and Baker et al. (1948) remark that severe blast damage to primary structural components was rare, owing to the high permeability of the layout and use of lightweight roof cladding that was found to fail prior to the transfer of significant

Figure 2-7 – Typical failure modes of RC-frame building joints subject to internal blast loading (Rhodes, 1974).
blast load to the structural system. Further, cases of severe damage were typically limited to near-miss and direct hit events, whereby columns were either severed or uprooted from their foundations. However, this form of construction was found to be susceptible to progressive collapse in the event of column loss and provides evidence of catenary action as an emergency mechanism that can mitigate but also exacerbate the spread of collapse.

Figure 2-8 is a photograph taken of a progressive collapse suffered by a 22x4 column grid valley-beam roof truss system following the loss of two internal columns. The columns lost their load bearing capability when uprooted by the crater of a medium capacity device (shown centre of image). The subsequent response of the building was for a catenary to develop in the purlins. This was found to be a typical mode of emergency response in truss roofs, in a valley-beam arrangement. In the case of short-span truss systems this type of emergency response was favourable. Baker et al. suggests that for valley-beam roof systems with spans less than 18m (60ft) the trusses of adjacent bays would provide adequate restraint to facilitate emergency load redistribution by catenary action in the purlins. Further, the strength of splicing and connection provided between purlins was typically such that the catenary would fail prior to overloading the surrounding structure. However, in long-span systems, defined by spans greater than 18m (60ft), load redistribution by catenary action in the purlins was found to destabilise and fail adjacent roof trusses, as the compression chord of each truss was typically reliant on the purlins for consistent buckling restraint. The robustness of long-span systems was therefore dependent on the strength and lateral restraint provided at gable ends and by rafter bracing. In cases where insufficient restraint was available, collapse was found to spread from the point of initial damage and, in catastrophic cases, result in the remaining structure being dragged down by the weight of the catenary.

![Figure 2-8 - Collapse damage observed in long-span roof truss structure following loss of internal column (Baker et al. 1948).](image)

The WWII research provides relatively few accounts of catenary response being sustained along multiple trusses or valley beams. Baker et al. suggest that this is due to inadequate connection between these larger components. However, of those cases documented the resultant lateral thrust was almost always found to result in the external walls and gable ends being pulled into the building. Similarly, Baker et al. (1948) and Christopherson (1948) identify further variants of truss roof systems such as
the ‘Umbrella’, North-Light Lattice Girder and Appex Lattice Girder variants. Several variants were found to possess sufficient robustness to sustain singular column loss without subsequent progressive collapse – by catenary response or alternative means of load redistribution – although multiple column loss was typically found to result in a spreading collapse. This is the only form of building that Baker et al. stress the need for implicit design consideration of column loss scenarios. Their subsequent recommendations, aside from promoting the use of minimal span lengths, are similar to that of direct robust design for which the design should assess the emergency load path and stability of the remaining structure for a combination of support loss scenarios.

2.2.1.4 Precast concrete construction

Whilst there is little about the robustness of precast concrete construction, authors such as Rhodes (1974) and Krauthammer (2008) have expressed concerns regarding methods of connection. Whether the building was framed or panelled, the obvious comparison would be with that of an in-situ RC structure, which has established proven high performance due to its mass, redundancy and level of continuity. Where the precast units were joined in a continuous manner their performance was likely to be of a similar high standard but the use dowel bars and other forms of standard connection were deemed by Rhodes (1974) to produce structures that were inherently susceptible to primary damage and progressive collapse. Krauthammer (2008) acknowledged the distinct advantage of panelled construction over framed variants, owing to the pronounced lateral stiffness and availability of alternative load paths, but stated that, even when enhanced by modern robustness measures, the resilience of such forms of construction remained inferior to in-situ RC and simple steel counterparts.

2.2.2 Performance of structural members under blast

During the 1940’s a substantial amount of the research effort was dedicated to understanding the behaviour of individual structural elements subject to the effects of blast and impact. The observations made from the numerous forensic surveys of bomb damage were combined with experimental tests to characterise behaviour and numerically model response under the transient dynamic load induced by an explosion. Whilst the accuracy of the modelling techniques developed was undermined by the underdeveloped representation of plastic material behaviour, the physical data has proved valuable in assessing the relative robustness of the main structural materials and the variations of design, construction and detailing practice employed for each.

2.2.2.1 RC members

At close proximities, an explosion is capable of devastating an RC member by overwhelming the tensile and crushing strength of the concrete. Such effects are known as spalling, scabbing and brisance and in severe cases can lead to the complete local disintegration of the element. RC elements and members, as with the steel-frame connections mentioned above, were found to be particularly susceptible to shear failure under the large transient dynamic loads attributed to more remote blast. This was attributed to speed of response since flexural (and, to some extent tensile) failures require an
ultimate load to be applied for a sufficient period for the member/material to respond, whereas shear failure happens quickly and is more suited to the very short duration pressures exerted by a blast. Such response was found most pronounced in the columns of wall systems subject to the lateral blast loading of a confined explosion and aided by the ability of the blast to cause uplift in the building, reducing preload in the column. Contrary to the failure modes shown in Figure 2-7, columns were frequently found to have failed by shear at floor and ceiling level. Such failure modes are shown in Figure 2-9 and clearly show the shear cracks to be complete, permitting the member to be displaced laterally from its original position. However it should be noted that general (or even local) collapse has not occurred, despite the failure of the column.

![Figure 2-9 – Images of RC column shear failure following lateral blast loading (Baker et al. 1948).](image)

The beams of RC-framed structures were also found to be potentially vulnerable shear failure but, presumably due to their higher design shear resistance, such members were rarely found to manifest such failures. RC beams and slabs instead proved more vulnerable to flexural effects, especially if damage occurred when the blast loaded the elements in a direction opposite to that for which they had been designed (as demonstrated by the heave evident in the Figure 2-10). The vulnerability of RC elements to reverse loading was regarded by Christopherson (1945), Rhodes (1974) and Krauthammer (2008) as the most pronounced shortcoming found in RC elements subject to blast and placed the material at a disadvantage in comparison to steel construction.

Slabs supported on all four edges and loaded by a concentrated force at the centre of the slab were found to exhibit classic yield line behaviour, with diagonal cracking followed by circumferential shear cracking (punching) when loaded statically. However when loaded by blast there was no diagonal cracking, only punching shear, highlighting the importance of experimental testing and/or dynamic analysis in understanding blast. This was true, however, when the shock front impinged unevenly, i.e.
at close range. With regard to two-way spanning slabs under uniformly distributed loading, as would occur at larger scaled distances, the modes of failure and type of distortion observed were similar whether the slab was loaded dynamically or statically.

![Figure 2-10](image1.png)

*Figure 2-10 – Heave in a floor slab after being subject to a confined explosion at its soffit (Thomas, 1948).*

Though few accounts of such failure have been documented probably due to secondary collapses masking such failures by far the most common form of response recorded was that of tension failure in columns following uplift (Baker *et al.* 1948; Christopherson, 1945; Thomas, 1948). Figure 2-11 shows a column stem that has sustained tension failure after the upper storeys of the building were lifted some 375mm (15”). It can be seen that the main reinforcement has been pulled clear from its laps with the column starters by the uplift. Whilst in this instance general collapse has not occurred, Christopherson (1945) claimed that in most cases the column would fail in buckling on return of the load. The reinforcement used during the 1940s was ‘plain bar’, rather than the deformed rebar used in current practice, and this contributed to the lack of bond at the lap. However, both Christopherson (1945) and Baker *et al.* (1948) remained advocates of non-deformed reinforcement – claiming that it aided robustness and ductility in RC members by helping to prevent tension stiffening and allowing greater spread of yield along the length of the steel. They recommended an increase in lap length from 45 diameters (as indicated by Reynolds, 1948) to 80-120 diameters (Baker, 1948) or by using hooked bars to overcome the lack of tension resistance in the bond. This is reflected by the recommendations made by the UK MoD for a lap length of 72 diameters, to allow for the decomposition of concrete under extreme in-elastic response (MoD, 2013).

Baker’s recommendations to retain the use of plain reinforcement bars were endorsed by Christopherson (1945) who supported the claim that the mild steel bar in service at the time possessed clear benefit against the high yield reinforcement used today. The reduced bond between steel and concrete would result in limited slip and greater spread of plasticity and strain along the rebar that,
particularly for mild steel, led to a reduced susceptibility to localised fracture and premature failure. This is reflected by Christopherson’s further recommendation for the use of multiple small diameter bars, in favour of fewer larger bars, thus further reducing bond strength and promoting greater continuity of ductility along a reinforced concrete member. This has been incorporated into modern recommendations for blast resistant design (MoD, 2013).

**Figure 2-11 – Tension failure at RC column base (Baker et al. 1948; Thomas, 1948).**

*RC450* (Christopherson, 1945) reported tests on several design alternatives with a mind to increase the blast resistance of reinforced concrete. These alternatives can be summarised as follows:

*High-yield reinforcement* – The typical high yield rebar at that time had a yield stress of approximately 430MPa, which is akin to the 500MPa grade bar used today. Such high yield steel was found to reduce blast resistance because it had lower ductility in comparison with the mild steel rebar used at the time. Christopherson stated that use of high-tensile steel ‘would be out of the question for reinforced concrete beams’.

*Percentage of tension reinforcement* – Christopherson noted that whilst the moment capacity increased with percentage of reinforcement, the ductility reduced. For this reason he concluded that increasing the tension reinforcement content beyond 0.5% did little to increase the potential for energy absorption (defined by the area under the load-deflection curve). Addition of steel beyond the 0.5% limit was considered possibly counterproductive. The results of some of the tests on which he based this conclusion are shown in Figure 2-12.

*Use of high strength concrete* – Structural grade concrete at that time typically had a strength of 17.5 to 22.5MPa. Whilst higher grades were recognised as providing greater energy absorption
before cracking. Christopherson (1945) concluded that once developed, there was no discernible improvement in performance and thus the use of high grade concrete was deemed ineffective. A series of impact tests on RC beams in the USA supported this conclusion, showing that compressive strength of concrete had only a secondary influence on strength, NDRC (1946).

*Form of reinforcement* – Various (unspecified) non-standard forms of reinforcement were trialled and these were demonstrated to be no more effective than the plane and typical deformed reinforcement commonly used during the era.

In general, those factors of most significant influence to the performance of RC are related to strength and ductility. Of particular interest is Christopherson’s claim that reinforcement areas greater than 0.5% provide no significant improvement in performance. It can be assumed that this points to the tendency for over-reinforced sections to fail in a brittle fashion by concrete crushing, with no

![Figure 2-12 – Load-deflection curves from pairs of identical RC beams (Christopherson, 1945).](image-url)
significant gain in out-of-plane strength. In which case the research carried out in the U.S. prior to the issue of *TM5-1300* (DoD, 1990) and its sister manuals is of importance as it was found that higher reinforcement percentages would provide gains in out-of-plane resistance when accompanied by intense lacing bars or blast links – to provide improved confinement of the concrete in order to achieve a greater ultimate crushing strength (Raja Iyengar *et al.* 1971) and support larger joint rotations (FEMA, 2000). This practice is also advocated by the UK MoD (MoD, 2013). The additional consideration of catenary or tension membrane actions resulting from effective lateral restraint has also been recognised to enhance performance, with the potential of second-order load resistance and further increased end-rotation capacities (see Chapters 3, 5 and 6).

### 2.2.2.2 Steel members

Under statically applied load the steel used in the 1940’s had similar elastic properties to today’s S275 structural steels. However dynamic test programmes reported in *RC450*, which showed that the material strength of steel was increased under dynamically applied loads, were plagued with premature failures. Therefore Christopherson (1945) warned against the general application of a dynamic increase factor to steel under blast loads. The problems related to unreliability of steel under dynamic loads have been confirmed by the work of Munoz-Garcia *et al.* (2005) which showed that bolts demonstrate a significant reduction in their tensile load capacity and ductility when dynamically loaded in tension. However this contradicts modern practice as set out in the various versions of *TM5-1300* (DoD, 1990).

The response of steel members to blast was found to be highly dependent on the position and orientation of both the member and its adjacent ancillary elements with respect to the origin of the explosion. Beams subject to close proximity blast were commonly found to be severed or torn from their connections (see Figure 2-5), whilst columns tended to be cut or exhibited severe buckling. Christopherson recommended that any opportunity to brace the compression flange of beams should be exploited so that the potential ultimate bending capacity could be fully realised. It is interesting to note the effect of stress reversals (due to negative phase blast forces and to elastic rebound), which suggest that both flanges should be restrained.

### 2.2.2.3 Mass concrete & brick

As a general rule, it was found that load bearing masonry panels could sustain substantial loads up to deflections of approximately half the wall thickness before arching action of the brick was exceeded and collapse occurred. The 225mm (9”) thick panel wall was the most common during the 1940’s, with typical bay dimensions of approximately 2.6m (8’ 6”) square. Such elements were found, by Christopherson (1945), to be demolished under mean impulse of 500-840kPa.ms (73-122psig.ms); an impulse easily attained by a medium proximity unconfined blast from high explosive ordnance. Given the dependency of unreinforced masonry on characteristic tensile strength the 343mm (13.5”) thick masonry units were found to exhibit only a marginal increase in strength.
The damage to masonry houses subjected to blast from 475kg (1,050lb) high capacity (V2) flying bombs was extensively surveyed. A total of 849 blast damaged buildings were surveyed within a blast radius of 52m (170ft). Of these 34% incurred damage ranging from at least 25% of external walls demolished to complete demolition (less than 25% of external walls remaining) at a mean damage radius of 47m (155ft). These observations can be seen consolidated in the general stand-off distances prescribed by Jarrett (1968), for which the author provides general empirical terms to specify the scale stand-off to unreinforced masonry buildings for various states of damage, from complete demolition to ‘inhabitable after repair’.

In general, the use of mass concrete or brick was deemed to be unsuited to blast load applications due to its brittle nature. This is reflected in modern practice of blast resistant design whereby structural use of mass concrete and masonry is not advised (MoD, 2013).

2.3 Review of Current Protective Measures & Robustness Schemes

The design philosophy advocated by Mays and Smith (2001) and Elliot et al. (1992) for the protection of buildings and their occupants from the threat of explosive devices is shown below. Inspection of the design strategy demonstrates the lengths advocated in providing effective protection against this form of malicious attack.

- **Deflect** Reduce hostile attraction to the building (or key components) by either disguising its potential as a target or demonstrating a low likelihood of success by providing visual deterrent.
- **Disguise** Distract from the valued aspects of a building/facility such that attacks are concentrated on small, low-value assets, away from areas of importance where a successful attack may cause mass casualties or significant impact.
- **Disperse** Distribute valuable assets across a large area such that the risk of a major loss is significantly reduced.
- **Stop** Deny proximity to the target by stopping the threat from reaching effective range.
- **Blunt** Minimise the direct effects of the attack and protect personnel and valued assets within by hardening and strengthening aspects of the target.

Documents such as *FEMA-452* (FEMA, 2005), *FEMA-426* (FEMA, 2003a and 2003b) and *UFC* (DoD, 2003; 2008; 2014) can be used to support the implementation of this philosophy of protective engineering. Both documents provide guidance to facilitate a risk-consequence assessment of a structure or facility and prescribe minimum protective measures in relation to the graded value of the target. Guidance for the protective planning, design and engineering of U.S. facilities can be undertaken with guidance from the Interagency Security Committee (ISC) and restricted source documents (not reviewed here).
Strategies intended to *deflect, disguise, disperse* and *stop* are beyond the scope of this thesis as they typically fall under the remit of planning and usage. However, current codes of practice and other documented guidance are available to facilitate *blunting* the effects of malicious or accidental loads by reducing primary damage and increasing collapse resistance through structural design. This section provides a critical review of the protective measures and robustness schemes available to the practicing engineer in conventional design. A series of case studies have been used in an effort to identify the strengths and flaws associated with each. All case studies are of buildings subject to severe structural damage. Some of the buildings featured survived extreme primary loading whilst others suffered progressive collapse disproportionate to the initial structural damage.

### 2.3.1 Safe stand-off distance

Safe stand-off distance is the only *stop* scheme considered herein and is applicable to vehicle impact and blast threats. Obstructions are strategically placed around the structure in order to prevent or minimise the effects of abnormal or malicious loads. In the case of HE blast and VBIEDs this is implemented to exploit the rapid decay of pressure with distance (with the cube of the distance; see Section 2.1.3). A perimeter is introduced to maintain a predetermined safe stand-off distance between the structure and the identified threat. Typically this involves the specification of a design charge-weight for which the stand-off is determined from the strength of the building and damage acceptance criteria (Smith and Tyas, 2008; Krauthammer, 2008; Dusenberry, 2010).

Methods used in the calculation of safe stand-off distance rely upon the prediction of structural response and vary considerably in terms of computational complexity. The most advanced approach entails finite element methods and multi-degree-of-freedom systems employed to model the building and structural components under idealised pressure-time histories (Bangash and Bangash, 2006; Krauthammer, 2008). The use of single-degree-of-freedom models is an alternative and less intensive approach in which, in the simplest terms, the ultimate resistance of the system is assessed against the external dynamic load (Biggs, 1964; Mays and Smith, 1995). Recent research at University of Southampton (Paramasivam, 2008) has developed a method, and associated software, to determine the safe stand-off distance for a given column size and design charge weight of TNT. Whilst this procedure is explicit to the survivability of RC columns, the safe stand-off distances of various structures can be calculated based upon ISO damage charts (DoD, 2014) to various levels of conservatism. Whichever of these strategies is employed, the provision of stand-off distance is hugely beneficial and significantly reduces the risk of collapse.

The 1996 bombing of the Khobar Towers in Saudi Arabia (see Figure 2-13) and the Islamabad Marriot Hotel bombing in 2008 are practical demonstrations of the level of protection afforded by the provision of stand-off. The device detonated outside the Khobar Towers resulted in a crater 26m wide and 11m deep and was estimated by the The Defense Special Weapons Agency (DSWA) to have been at least 9,000kg (TNT equivalent). The Khobar Towers were defended by a simple arrangement of precast concrete ‘Jersey Barriers’ which provided a 32m stand-off. Despite the severe nature of the attack the
survival of the building was in large part due to the provision of an impassable perimeter, without which the internal structure of the building would have been destroyed at ground level. A similar level of protection was achieved for the Islamabad Marriott Hotel (2008) which was subjected to an attack that produced a 20m diameter and 6m deep crater. Again, the stand-off provided proved effective with only minimal structural damage due to the immediate effects of the blast.

![Image](image_url)

Figure 2-13 – Khobar Towers, Dharan, Saudi Arabia (1996).

The problem with the safe stand-off approach is that suicide bombers have shown a consistent ability to penetrate security check posts, as was demonstrated at the Pearl Continental Hotel in Peshawar, Pakistan on 9th June 2009. Suicide bombers stormed the security gate and were able to deliver a device close enough to cause the progressive collapse of multiple bays of the building. The most infamous failure to secure a perimeter occurred in 1983 when the US Marine Corps and the French military forces were attacked simultaneously in West Beirut. The modern RC-framed building housing the US forces suffered a catastrophic progressive collapse following detonation of a VBIED within the footprint of the building.

### 2.3.2 Specific local resistance

Specific local resistance is an event specific protection method (for *blunting*) that requires the design of structural components to withstand the loading from an impact or blast of specified size and proximity. The result is the design of a hardened structure that can be a relatively expensive option.

The author has not been able to find any public domain case studies of buildings designed for blast that were in fact subjected to attack. Instead reference is drawn to buildings designed for exceptional circumstances that might be considered hardened in comparison to conventional structures due design considerations for enhanced ductility and/or strength. The HSBC Headquarters in Istanbul has been
taken as an example of a hardened structure as the building was designed and detailed for high-seismicity. Detailed to withstand the dynamic effects of an earthquake the structure featured a high level of strength and ductility and performed extremely well when the building was subject to the blast effects of a close proximity VBIED in 2003. Inspection of photographs taken in the aftermath show that the face of the building sustained significant but largely superficial damage, with none of the columns lost. Another example of a building able to withstand severe blast loading from a close proximity explosion was the Club El Nogal Building in Bogota, Columbia. In 2003 an estimated 160kg (TNT equivalent) VBIED was detonated within 3m of two structural columns in the fourth floor car park with the specific intention of causing the collapse of the building (Garcia et al. 2006). However, structural damage to the thirteen storey structure was limited to demolition of the floor slabs and joists for a number of storeys immediately above and below the explosion. The near columns were found to have sustained only superficial damage – limited to spalling of the concrete cover – and the building did not collapse. Garcia et al. (2006) provides a retrospective assessment of the building and concludes that performance was attributed to the high shear strength, stiffness and mass of the columns, as dictated by seismic design requirements for extreme lateral resistance. Figure 2-14b shows an image of a column located adjacent to the blast and the damage sustained by the building in the near field. The reinforcement cage shown in Figure 2-14a demonstrates the intensity of detailing used in the subject column and close inspection of the heavy use of stirrups draws similarity with shear and confinement detailing practice found in blast resistant design, such as JSP 482 (MoD, 2013).

These incidents illustrate that it is possible to design structural elements to withstand very intense loading and limit or prevent primary damage. Further, the additional strength provided to columns to resist seismic actions appears to have been sufficient to resist high-explosive blast loads – as supported by more recent research by Corley (2008) and Parisi (2015).
Specific load resistant design for the effects of high explosive blast can be carried out using the guidance in TM5-1300 (US Department of the Army, 1990), the more recent UFC 3-340-02 (DoD, 2002; 2014), Smith and Hetherington (1997) and Biggs (2009). In general the use of stand-off measures is preferred to specific local resistance (FEMA, 2005) but in urban areas and situations where stand-off cannot be guaranteed, this represents a viable alternative for new builds and existing structures where strengthening upgrades are feasible. However, the drawbacks of specific local resistance are twofold. Generally blast resistant design for close proximity blast will lead to increased construction cost (although Mays and Smith, 1995, suggest that this increase may be as little as 2-3% of the overall project cost). Furthermore, the safety of a blast resistant design is highly reliant on the accurate identification and siting of a governing explosive threat and estimate of design blast loads, adequate for the full design life of the building (Conrath et al. 1999).

2.3.3 Direct alternative load paths

The use of ‘direct alternative load paths’ is set out in the British Standards (BSI, 2000; 2005b), the Eurocodes (BSI, 2010), the American Society of Civil Engineers (ASCE, 2006), US General Services Administration (GSA, 2003; 2013) and US Department of Defense (DoD, 2005; 2009; 2014).

Rather than design structural components to resist accidental or malicious loads, this approach assumes localised damage to load-bearing members. If a viable alternative load path exists to redistribute the resulting emergency load, the sensitivity of the building to local damage is significantly reduced thus presenting an economical alternative to ‘specific load resistance’ whilst reducing the risk associated with under-prediction of likely threats. The level of protection afforded by this approach is potentially quite high but design guidance recommends several alternative design methods, a number of which are reviewed below.

2.3.3.1 The double span method

This approach requires a full-moment resistant frame designed to accommodate the loss of any individual key element from the frame grid. Each bay is designed for double the service span allowing the building to bridge a damaged area by redistribution of the emergency load by flexure to the adjacent structure, following the loss of a key member. Many rigidly jointed structures can achieve this requirement with little difficulty since the ultimate limit state load used for the design of the service span is significantly greater than the accidental limit state load used in consideration of the double span. Sasani (2008), Sasani and Sagiroglu (2008) and Sasani et al. (2011) provide experimental evidence supporting this conclusion, demonstrating the capacity of RC framed structures to withstand multiple column removal owing largely to vierendeel action. This is of particular interest given that the subject buildings were not designed using the double span method.

The importance of incorporating full moment connections throughout a structural frame has been recognised since the Second World War. Christopherson (1945), Baker (1948) and Rhodes (1978) recognised the benefits of using connections capable of adopting the full moment capacity of the
members they connect. When compared with equivalent structures of simple construction, significant gains in local and overall robustness are achieved. These findings support the use of double-span design in collapse mitigation and resilient design.

![Figure 2-15 – Collapse sustained by Murrah Federal Building, Oklahoma (1996), following attack by VBIED.](image)

Guidance for the implementation of this approach is provided in detail in the US document GSA (2003; 2013). This code provides detailed requirements for design, including dynamic load factors to account for sudden column loss that are not found in other design codes. However the collapse of the Alfred P. Murrah Federal Building, Oklahoma demonstrates the primary concern regarding the method; by designing for single column loss the structure may be more vulnerable in the event of multiple column failures than if it were of simple construction. In 1996 the Murrah building was subject to the blast of an estimated 1,800kg device (TNT equivalent). The vehicle bourn explosion created a crater 9.15m in diameter and 2.45m deep and was initiated at a distance of only 4.8m from the nearest stanchion. It resulted in the failure of three columns at the front of the building and a fourth interior column. The subsequent progressive failure resulted in the loss of almost 50% of the total floor area of the building making the event a significant disproportionate collapse (Figure 2-15). This example supports the argument that design for the loss of a single key element may be insufficient under the threat of VBIEDs and, furthermore, that design under the double-span method may in the instance of multiple column loss, induce progressive failure. The economics of such a scheme can also be a problem as material and construction costs will be substantially greater than in alternative structures of simple construction.

### 2.3.3.2 Truss systems

A favoured method in the robust design of Class 3 buildings, this scheme is the introduction of truss systems at intervals throughout the height of a building. The truss systems are designed such that an
unsupported section of building can be effectively suspended and the emergency load transferred safely to the ground. This is usually achieved by means of out-rigger trusses designed to cantilever from a protected core structure. Compliance with the majority of codes available requires that the alternative load path need only consider the notional removal of a single column. Therefore, as with the double span method, there is a significant risk of underestimating the potential emergency load. Should the structural core be subject to an emergency load greater than the design accidental load, a major collapse would be possible.

![Figure 2-16](image)

Figure 2-16 – Schematic illustration of damage from September 11 attacks and subsequent load redistribution (diagram adapted from *FEMA 403*).

The World Trade Centre towers are good examples of truss system performance. Outrigger trusses were located at the top of both buildings, between the 106th and 110th floors (FEMA, 2002). After aircraft impact, both buildings survived subsequent blast and fire damage for more than 50 minutes before they collapsed. Figure 2-16 is a schematic illustration of the damage experienced during the 11th September 2001 attacks and demonstrates how the load was probably redistributed. The splice connections of the perimeter columns were capable of full axial continuity and load reversal, allowing the transfer of the columns’ tensile capacity over the height of the building. The ‘hat-truss’, although principally incorporated to increase structural stiffness under wind loading and to support a transmission mast on the roof of each structure, provided the buildings with significant redundancy and the ability to suspend the unsupported region. The towers each had a central core which acted with the remaining perimeter stanchions to enable the truss systems to cantilever and redistributed load to the ground.
Whilst the collapse of the two towers in 2001 is regarded as one of the worst building disasters in history, the extended period before collapse allowed a significant number of the occupants to escape and demonstrated a high level of immediate structural robustness against the primary damage and column loss associated with initial impact. This can be attributed directly to the alternate load paths provided by the truss systems within the structure. However, their eventual collapse highlights the importance of considering the vulnerability of such alternative load paths to secondary effects, such as fire.

2.3.3.3 Shear panels – masonry & reinforced concrete infill

The use of infill panelling was strongly advocated during the 1940s. It was found that buildings with infill panels were very stiff and offered numerous alternative load paths which, in practice, proved remarkably effective in resisting the onset of collapse and greatly increased the capacity of the structure to bridge a damaged area. Christopherson (1945), Baker (1948) and Walley (2001) commented on the added robustness afforded by the inclusion of shear panels in structures subjected to aerial bombardment. They remarked that the presence of masonry panels tended to compartmentalise a building, helping to localise direct blast damage, and reduce sensitivity to loss of support. They noted that the likelihood of direct damage to columns, and possible primary collapse, was generally increased by the inclusion of such panels but the advantages of their presence significantly outweighed the detrimental effects.

Figure 2-17 shows direct blast damage and primary collapse following the direct hit of a medium capacity bomb during the Second World War. The building was a seven storey high steel frame with masonry infill panelling throughout which sustained the loss of four peripheral columns. Primary collapse was limited to the unseating of the transfer plate-girder at the first floor. It can be seen that the remaining storeys were left unsupported and that the masonry infill panels assisted in sustaining the building by corbelling from the undamaged structure. Examples such as this are also documented by the observations made by Rhodes (1974) in surveying blast damaged buildings during the Anglo-Irish conflicts and Faella and Nigro (2005), who examine the robustness of MIF buildings following mud-slide damage.

There is, as yet, no established design guidance for the use of masonry infill panels in providing emergency load paths. Possible disadvantages may be the weight of the material, the potential debris, architectural inflexibility and vulnerability to out-of-plane buckling, but the potential redundancy provided by such panels presents a cheap and rudimentary method for increasing structural robustness that would be ideal in retrofitting if not new design.
Appendix A provides a detailed review of in-plane compression strut theory as currently used in seismic analysis for the quantification of lateral stiffness afforded by masonry infill panels in inter-storey drift, under seismic actions. The subject is of significant importance given the vulnerability of MIF structures to soft-storey-collapse. Thus extensive research has been conducted for the development of macro-model theory, to facilitate dynamic modelling of global building response. The recent work of Li et al. (2016) offers perhaps the first investigation into the adequacy of using these seismic analytical terms for the prediction of robustness in column loss scenarios. The experimental and numerical investigation emphasises the added robustness afforded by MIF arrangements and suggests that seismic modelling methods can adequately support robust design. Nevertheless, with no additional information or guidance, the infill panels constructed during the 1940’s were typically of solid 230mm thick brickwork, and whilst bay-widths were smaller than in modern design, this suggests a minimum.

The use of shear panels is not confined to masonry infill. Alternative materials such as reinforced concrete and stud-panels of carefully considered in-plane strength should be considered together with proprietary bracing systems and prefabricated modular units which are able to provide alternative load paths. Whilst more expensive, these alternatives are viable in direct robust design as their behaviour is more quantifiable and their bracing capacities are easily calculated, Lawson et al. (2007). Further, these systems have a clear advantage in out-of-plane ductility making them favourable over unreinforced masonry alternatives. The work of Loizeaux and Osborn (2006) gave more support to the use of shear panels, showing that they afford a high degree of resilience against collapse.

Figure 2-17 – 1940’s steel frame having sustained local damage under near blast effects (Baker et al. 1948).
2.3.3.4 Arching Action

Arching action, otherwise known as compressive membrane action (CMA), is a phenomenon known to enhance the strength of laterally restrained RC elements. The beneficial effects of this internal compression arch have been seen in beams, in-situ slabs, composite metal decking and bridge elements. Though dependant on lateral restraint stiffness, span-depth ratio, area of reinforcement and concrete compressive strength, tests have shown (Ockleston, 1955; Park and Gamble, 2000; Peel-Cross, 2001) that the ultimate strength of an element may be enhanced by some 50-300% above the predictions of yield line theory.

The fundamental limitations regarding this form of emergency response are the need for sufficient in-plane restraint to sustain CMA and the lack of practical design methods for systems of common span-depth (Punton, 2015). In-plane restraint can be provided by an appropriate degree of stiffness in the adjacent structure, which is easily achievable mid-structure but is less so along the perimeter of buildings.

2.3.3.5 Catenary Action

The word Catenary comes from the Latin for ‘chain’ and is a mathematical expression that describes the shape formed by a chain or cable of uniform linear mass suspended between two points. The system is assumed perfectly flexible (i.e. with negligible bending strength) and forms under the influence of gravity. In this context, catenary action constitutes a secondary form of structural response that is generated at large deflections (typically greater than the depth of the section), beyond those considered in elastic or serviceable design. Therefore, as the ideological definition would imply, when applied as an alternative load path in construction significant flexural ductility is required; the specific design of connections to sustain extreme rotations and combined tensile forces. However, this is not a requirement generally enforced by existing guidance.

Recent studies, such as those by Byfield and Paramasivam (2007b, 2010), Tyas (2010), Kuhlmann et al. (2007), Demonceau and Jaspart (2008), Astaneh-Asl et al. (2002), Tan and Astaneh (2003) indicated that it is unlikely that catenary action can be achieved in conventional steel construction by the consideration of tensile requirements in isolation. The studies show that in steel buildings significant rotation is required at the connections for the mobilisation of the tying force stipulated in BS 5950 or Eurocode 1 and that standard simple connections are not ductile enough to sustain the displacement required to satisfy load redistribution. The ductility of RC beam-column connections has been reviewed by Merola (2009). His study showed that the required rotation can be achieved but that the top layer of reinforcement would fracture leaving a single layer of reinforcement to resist the tensile force required to sustain the catenary. Further, the author suggests that for catenary action to be effective, the more ductile European Grade C reinforcement is required.
The work conducted by Astaneh-Asl et al. (2002) and Tan and Astaneh (2003) offers a more optimistic outlook for buildings designed with special consideration of catenary action. Their research at Berkley demonstrates, by means of full scale testing, the effective use of elastic response catenary cable systems, to compensate for the deficiency of conventional steel construction by adopting and effectively redistributing emergency load. Their recommendations suggesting that such a system should be designed to accommodate emergency load at deflections prior to connection failure – documented to be as low as 0.026 radians for a seven bolt shear tab arrangement – is likely to result in an impractical lateral demand on the supporting structure. Nevertheless, this demonstrates that custom catenary systems may be feasible provided that large in-elastic deformations are acceptable.

By review of the available research, a distinct lack of confidence is evident in the ability of conventional steel and RC floor systems to sustain the joint rotations required for safe redistribution of emergency load. The studies presented for braced steel buildings seems conclusive. However, authors concerned with catenary action in RC construction (Yi et al. 2008; Dat and Hai, 2011; Gouverneur et al. 2013a; Merola, 2009; DCLG, 2011) point to a significant lack of supporting experimental data that undermines conclusion.

2.3.4 Accidental Limit State (ALS) load cases

The design codes that allow robustness design by the use of direct alternative load paths and notional column removal stipulate an accidental limit state (ALS) load case under which the stability of the structure must be checked. A summary of ALS load cases is provided below for which there is seen to be significant variance.

Eurocodes

BS EN 1990:2002+A1:2005 (BSI, 2010), the Eurocodes, recommend structural assessment under the following gravity load:

\[ \sum_{j \geq 1} G_{k,j} + P + A_d + (\psi_{1,1} \text{ or } \psi_{2,1}) Q_{k,1} + \sum_{i > 1} \psi_{2,i} Q_{k,i} \]

Equation 2-2

Given that the situation under consideration is that following an accidental event, \( A_d = 0 \). Furthermore, unless the system under consideration is pre-stressed, the representative value of prestressing action, \( P = 0 \). In the analysis of notional column removal situations, other accompanying variable actions (\( Q_{k,i} \); such as wind load) are ignored. Therefore the ALS load case becomes:

\[ G_{k,j} + (\psi_{1,1} \text{ or } \psi_{2,1}) Q_{k,1} \]

Equation 2-3

Where \( G_{k,j} \) and \( Q_{k,1} \) are defined as the permanent action (dead loads) and the leading variable action (live loads) respectively. The accompanying UK National Annex (BSI, 2009c) gives various values for factors \( \psi_{1,1} \) and \( \psi_{2,1} \) which depend upon the longevity of the load carried by the building or floor area under consideration. For domestic, residential and office spaces values of 0.5 and 0.3 are
specified. Whereas, in the case of storage areas in which the superimposed load may be assumed permanent, a higher characteristic load is required. Therefore values of 0.9 and 0.8 are specified.

**British Standards & UK Building Regulations**

The ALS load case incorporated in *BS 5950* and *BS 8110* (BSI, 2008b and 2005b) requires:

\[
\gamma_f G_k + \gamma_f (Q_k + Q_{k,i})/3
\]

Equation 2-4

The load factor, \( \gamma_f = 1.05 \) or 0.9 gives the designer discretion in assessing system stability with reduced restoring moments. As with the *Eurocode* procedure; \( Q_{k,i} \), specified by the *British Standards* as wind load, can be neglected at the designers discretion.

**US General Services Administration (GSA, 2003)**

The following ALS load case is recommended by GSA (2003):

\[
DLF(G_k + 0.25Q_k)
\]

Equation 2-5

DLF denotes a dynamic load factor that may be taken as \( 1.0 \leq DLF \leq 2.0 \), where 1.0 is to be adopted when undertaking a nonlinear dynamic analysis and 2.0 when conducting linear or nonlinear static analysis for instantaneous loss of support.

The above ALS load case has since seen revision in GSA (2013), which recommends use of the following:

\[
\Omega(1.2G_k + (0.5Q_k \text{ or } 0.2Q_{SN}))
\]

Equation 2-6

Live loads \( (Q_k) \) are limited to 244kPa and \( Q_{SN} \) defines characteristic snow load. The load increase factor \( \Omega \) is prescribed in accordance with structural material, form of analysis implemented (linear static, nonlinear dynamic, etc.) and whether performance is force or displacement dependent. However, values of \( \Omega \) generally appear to approach 2.0 for most material disciplines and static procedures, demonstrating an increased ALS gravity load compared with 2003 guidance.

**American Society of Civil Engineers**

The ALS load case specified in *ASCE 7* (ASCE, 2006) for notional element removal procedures:

\[
(0.9\text{or}1.2)G_k + (0.5Q_k\text{or}0.2Q_{SN}) + 0.2W_k
\]

Equation 2-7

This approach implements alternative factors for the dead load and the inclusion of wind load \( (W_k) \), to allow the designer to examine advantageous restoring moments and thereby identify a worst-case emergency load condition.

**US Department of Defense**

The ALS load case advocated by *UFC 4-023-03* (DoD, 2009) for linear and nonlinear static procedures is detailed below.
\[ \Omega(0.9 \text{ or } 1.2 G_k + (0.5 Q_k \text{ or } 0.2 Q_{SN})) \]

Equation 2-8

It can be seen that the ALS load case is based upon ASCE 7-05 but with the additional consideration of a load increase factor (\( \Omega \)) for load and displacement controlled dynamic actions. Prior to the 2009 revision of the document \( \Omega \) had to be implemented as indicated in the GSA (2003) guidance (see above), however, the current guidance provides factors that are dependent upon both the type of analysis used and the rotational ductility of the material under consideration. It should be noted, rather than stipulate the use of a characteristic wind load (as in ASCE 7-05), UFC 4-023-03 stipulates a notional horizontal load that is to be used with the specified ALS load case to verify the lateral stability of the building in its hypothetical damaged condition.

2.3.5 Notional column removal & key element design

Key Element design entails the notional removal of individual columns and load bearing elements (with particular attention paid to transfer elements) about a structure to establish the extent of subsequent collapse – the system is commonly assessed using linear-static analysis under an accidental gravity load (see Section 2.3.4). If a direct alternative load path cannot be established, those elements found to cause disproportionate collapse must be designed as key elements; which must sustain a 34kPa static load applied simultaneously on each available face and to those components to which it provides support (BSI, 2014). The local failure acceptance criteria beyond which collapse is deemed disproportionate are stipulated as follows:

- **Eurocode 1** (BSI, 2014) - Lesser of 100m² or 15% of the floor area on two adjacent storeys immediately adjacent to the notionally removed element (reflecting that stipulated by ODPM, 2010).

- **BS 5950** (BSI, 2000) - Lesser of 70m² or 15% of the floor or roof area immediately adjacent to the lost element and the storey immediately above or below.

- **GSA** (2013) - 15% of the total floor area above the notionally removed element. Superseding the GSA (2003) requirement of 167m² of the floor area immediately above the notionally removed element.

- **UFC 4-023-03** (DoD, 2009) - The lesser of 70m² or 15% attributed to the perimeter member and the underlying floor should not fail under consequent debris loading.

Note that the above criteria are specific to the loss of an external column or load bearing element. The collapse floor area for the assessment of internal elements tends to be double that shown above.

Key element design was introduced in 1970 following the Ronan Point collapse (1968). The 23-storey residential high-rise, constructed of load-bearing precast units, was subject to a significant progressive collapse initiated by an internal gas explosion on the 18th floor. The blast resulted in the loss of an external wall panel leaving the four apartments above, which were reliant on direct transfer of gravity loads, unsupported. As can be seen from the images in Figure 2-18, the consequent collapse resulted
in the loss of the entire South-East corner of the building; debris from above successively overloading each of the underlying units. The statutory accidental load of 34kPa (5psi) was an estimate of the blast pressure that initiated the event – empirically derived from the dent found in the face of the fuse box in the source flat (Burnett, 1975).

![Figure 2-18 – Ronan Point collapse, 1968 (image obtained from: http://www.ace.ed.ac.uk/)](image)

Key element design is unlike specific load design in that it does not require an accurate prediction of accidental or malicious loads and the dynamic response of the element is not considered. Concerns regarding this approach have centred on the 34kPa static accidental load which, whilst specific to the incident gas explosion of Ronan Point, may not be sufficient under certain circumstances.

The collapse of the Alfred P. Murrah Federal building (1996; see Section 2.3.3.1) is a good example of this possible shortfall. The three columns lost to the blast were subjected to a peak-over pressure of the order of 10,000kPa (Paramasivam, 2008). This load may have been applied for only a few milliseconds but the nearest column in question failed by brisance, and the two further from the blast origin failed in shear. The structure was not designed for seismic action and whilst treating each lost column as a key element would have resulted in an increased shear capacity, it is unlikely that they would have been able to sustain such an extreme load, certainly the near column would still have succumbed to brisance.

This case raises concerns with regards to the ability of key element design in providing a comprehensive form of protection. Whilst a reasonable resilience in the event of a gas explosion and perhaps collision might be expected, a close proximity high explosive blast is unlikely to be accommodated by this type of robust design – specific load resistance would be more appropriate. Furthermore, the inclusion of a key element effectively increases the sensitivity of a building to local
damage and disproportionate collapse. It is only when the accidental load can be guaranteed to be less than a pressure of 34kN/m², or equivalent, that this approach is effective. This point is emphasised by inspection of UFC 4-023-03 (DoD, 2009 and 2013) and GSA (2003 and 2013), which do not support key element design in this form and even require direct alternative load paths following specific local resistance design.

It should be noted that although endorsed by the New York City Building Code (NYCPD, 2009), British Building Regulations (ODPM, 2013) and Eurocodes (BSI, 2014), the notional accidental static load of 34kPa is intended for cases in which all other methods of developing robustness have been exhausted. Eurocode 1 (BSI, 2014) provides guidance for a generalised form of static specific local resistant design for cases in which the threat of an explosion or vehicle impact is specifically anticipated.

2.3.6 Structural segmentation

The collapse resistance offered by any alternative load path is limited. In cases of severe damage in which the associated emergency load is excessive, systems that feature extensive tying and continuity can worsen the extent of the initial collapse by causing it to spread (Starossek, 2009). This phenomenon is known as ‘drag-down’ – a form of progressive or spreading collapse caused by the unsupported region of a building overwhelming and pulling down the adjacent structure to which it is tied causing collapse to propagate horizontally and vertically rather than just vertically. The most effective method of preventing drag-down is to ensure that the extent of local damage does not exceed the capacity of the alternative load path available. However, with the exception of the provision of complete stand-off, accurate damage control is difficult to achieve as it requires a precise prediction of emergency load paths and control of the accidental load. As an alternative, a building can be designed with horizontal and vertical discontinuities which arrest the spread of collapse – akin to a fire break. Originally implemented to prevent the horizontal progression of collapse in bridges, this principle is known as structural segmentation, (Alexander, 2004; Loizeaux and Osborne, 2006; Ellingwood et al., 2007; and Starossek, 2009) and is an effective measure in progressive collapse mitigation.

Starossek (2009) used the Terminal E2 collapse at Charles de Gaulle Airport, Paris (2004) as an example of the use of structural segmentation in preventing progressive collapse (see Figure 2-19). Failure under service load was attributed to fatigue within the inner precast concrete shell structure, (Jonathan, 2005). The resulting collapse was found to propagate along the structure. By site inspection it was found that the horizontal progression of collapse was arrested by structural discontinuities – a solid precast section at one end, boasting superior strength, and a glazed section at the other which possessed insufficient continuity to allow drag-down (see Figure 2-19). It is worth noting that such discontinuities were found to be of significant benefit in the prevention of spreading collapse in steel-shed structures in the bombing raids of the Second World War (see Section 2.2.1.3).
Structural segmentation can be implemented by the design of regions of higher strength or reduced continuity. For example, fracture lines can be introduced at construction joints or independent frames and strong floors can be included through the height and floor plan of a building. Whilst this presents an economic solution to structural robustness, no official guidance for its use in conventional construction has been found.

2.3.7 Indirect alternative load paths – effective tying

The provision of effective tying, as stipulated by the British Building Regulations (ODPM, 2013), Eurocode 1 (BSI, 2014) and UFC 4-023-03 (DoD, 2005; 2009; 2013), originated from the British Standards (BSI, 1972) and was intended to provide a degree of continuity throughout a structure and a minimum level of robustness (DCLG, 2011). Effective ties need only be applied in a horizontal direction for most structures whilst larger buildings may be designed with effective horizontal and vertical ties as an alternative to alternative load path and key element design.

As shown in Figure 2-20, horizontal ties are incorporated around the periphery and at internal structural intersections between the walls, columns, beams and slabs to provide a continuous tensile diaphragm at each storey level. Vertical ties are applied to vertical load bearing components and are detailed to provide a degree of continuity between each other and intersecting horizontal components. Effective ties for both the horizontal and vertical directions are designed in accordance with minimum requirements specified in the structural codes and provide a form of indirect robust design whereby an intrinsic level of continuity is assumed to reduce sensitivity to accidental loading and local damage by facilitating catenary action in the event of key element loss.
2.3.7.1 Minimum Horizontal Ties

The importance of effectively tying structural systems was demonstrated by research conducted during the Second World War (see Section 2.2.1.1). Surveys of bomb damaged buildings showed that insufficiently tied systems were the cause of the majority of primary and secondary collapses as inadequate continuity, especially between floor and wall systems, tended to exaggerate direct blast damage. These findings are reinforced by the collapse of the Droppin Well Bar, Ballykelly, Northern Ireland (1982). The low-rise construction featured precast hollow-core floor units that spanned from an external load-bearing wall to an internal steel frame. A device, estimated at 5kg Semtex, was placed on the lower level beside one of the steel uprights within the building. Whilst the stanchion was relatively undamaged, the explosion resulted in the collapse of a significant proportion of the floor above as the internal blast displaced an external masonry wall, unseating the precast units from their bearing at each end. Preliminary investigations indicated that the floor units were discontinuous through the structure. If the floor units had been tied together (continuous over one or more supports) and effectively anchored to the external masonry, it is likely that the collapse would have been reduced. The precast units would have pitched into the populated space beneath rather than falling bodily.

Burnett (1975), Alexander (2004) and Li et al. (2011) link the current tie force requirements to early tensile membrane research, to show that the tie force requirements may have been derived facilitate emergency load redistribution by catenary action. However, modern research indicates that conventional steel and reinforced concrete is may be incapable of sustaining the joint rotations required to support this mechanism of response (Ellingwood et al. 2007). It should be noted that despite the
considerable horizontal reactions incurred by a catenary system, tying requirements are not accompanied by recommendations for ensuring sufficient lateral restraint or subsequent stability. Furthermore, the requirements are solely based upon tensile capacity, with no provision of rotational ductility. Therefore, provisions of indirect robust design do not overcome the fundamental issues identified in Section 2.3.3.5. Whilst the use of horizontal ties unquestionably provides a good level of basic robustness, research suggests that it cannot be relied upon to provide any form of comprehensive collapse resistance (Li et al. 2011; Tohidi et al. 2014a and 2014b).

2.3.7.2  **Horizontal & Vertical Ties**

The benefits of effective horizontal tying are enhanced by the addition of vertical ties. By providing a degree of continuity between components in both the vertical and horizontal planes, a fundamental level of robustness is provided that permits transfer of abnormal and emergency loads to adjacent structural elements thus increasing the resilience of the structure against accidental damage and primary collapse. However, the ability of such a tying system in preventing disproportionate or progressive collapse is questionable.

As with the ‘effective horizontal tie’ method, this is a form of indirect design that encourages the development of a catenary to prevent collapse in the event of support loss. However, the benefit is no greater than with the use of horizontal ties alone. The use of vertical ties does, in the event of column loss, mobilise the structure above and encourages catenary action throughout the building height. Whilst this is likely to enhance the resilience of the unsupported structure, the lateral stiffness of the adjacent structure is called into question and the risk of drag-down is increased, Loizeaux and Osborn (2006). It seems that the only clear advantage from including this measure would be if it is tied into an ancillary alternative load path that would reliably redistribute the emergency load.

The arguments presented for effective tying providing no practical alternative load path and an increased risk of drag-down is reflected by the recent revision of GSA guidance (GSA, 2013), whereby the document specifically states that effective tying provisions and means of indirect robust design are removed from consideration.

2.3.8  **Relative performance**

This review of case studies has revealed a wide variation in the ability of buildings to survive localised damage from blast or impact without subsequent collapse, depending on the scheme of robustness or protection used. It is therefore possible, based on the case studies presented, to rank the different design strategies, from those that provided a high level of confidence in robustness and protection to those which provide only a nominal level of confidence; see Table 2-1.
In terms of the protection against VBIED’s, the Khobar Towers demonstrated that simple barrier systems can provide an impressive level of protection provided that the threat and respective stand-off is identified. For this reason the provision of a safe stand-off distance is regarded as the best method of protecting buildings and is considered to provide a high level of confidence in protection. However, since suicide bombers have a history of penetrating lightly secured check posts this rating only applies when perimeters are tightly secured and impermeable to likely threats.

It is not always possible to provide adequate safe stand-off distances for buildings in congested urban environments. If VBIED’s are considered a threat then the engineer and client may wish to consider the option of designing a hardened structure and, given the extent of design guidance available for the design of blast resistant buildings, it is indeed possible to design structural and non-structural systems (windows, doors, etc.) to survive severe blasts. This method is considered to provide a high level of confidence provided that the hardened design is based on a thorough and current threat assessment that identifies appropriate malicious actions. The evidence from buildings designed to survive earthquakes which were subsequently subjected to explosions from bomb blast suggests that seismic design achieves a high degree of robustness against blast. Therefore seismic design arguably provides an improved level of confidence in structural robustness over buildings designed with no consideration of seismic actions. However, structural and non-structural systems not designed to meet a specified malicious load can inevitably result in casualties related to debris and movement of the building itself. Therefore additional measures may be needed to minimise the impact of such affects.

The provision of emergency bracing systems in the form of out-rigger trusses has a proven ability of to accommodate multiple column loss. Such systems have been ranked with a high confidence level as it is assumed that the alternative load path is created in an area not affected by primary damage.
The use of shear panelling along column grid lines is ranked ‘moderate’. This is due to the proven ability of internal walls to inhibit the passage of blast waves inside buildings, as well as being able to redistribute loads following damage to multiple columns. Infill panels constructed during the 1940’s were typically of solid brickwork, and whilst bay-widths were smaller than in modern design, more recent cases of infilled structures resisting collapse support this conclusion (Faella and Nigro, 2005). This system could provide a high level of confidence if the integrity of critical panels following primary damage was guaranteed.

In 1996, the Murrah Building was subject to a blast that caused the failure of three columns along the front face of the building. This illustrates the possibility of multiple column loss and therefore the double span method (or notional column removal) is ranked ‘moderate’, due to the inability of the method to cope with multiple column loss. An adverse effect of building continuity into a frame to accommodate notional column removal is the danger of drag down. A further drawback is that fabrication costs can be higher than those of similar structures featuring simple connections.

Catenary action as applied to steel framed structures with industry standard “pinned” connections has been largely discredited, with joints shown to rip apart due to insufficient ductility when subjected to the demands of catenary action. This problem of joint rotation capacity will also occur in partial-strength connections since beams will remain elastic and therefore place all rotation demands onto the connections. Steel framed structures incorporating full moment joints would survive the demands of catenary action although the continuity may introduce dangers from drag-down. The mechanics of catenary action in RC framed structures are not yet fully understood. For these reasons catenary action is considered to provide a low level of confidence in structural robustness. Similarly, case studies of buildings featuring RC floor systems of exceptionally small span-depth ratio indicate that arching action can potentially redistribute emergency loads. However, this mechanism in column loss conditions, and for typical span-depth ratios, is not fully understood at present and the physical and geometric requirements impose substantial limitations. For this reason it is shown to give low confidence.

The key element method load of 34kPa is not representative of the peak pressures or impulses associated with blast loads from HE devices and therefore it is unlikely to provide adequate resistance to VBIED’s. This is of great importance given that key elements are by definition identified to result in disproportionate collapse. Although the current guidance does not exist for conventional design, given the wide use of alternative methods of Specific Local Resistance in modern practiced, it is more appropriate that the notional 34kPa design load be replaced with a case specific design load quantified by a supporting risk assessment. For this reason it is considered to provide low confidence as a scheme of protective design.

The use of horizontal ties unquestionably provides a good level of basic robustness against blast loading. However, for the reasons stated earlier its ability to support catenary action and emergency
load redistribution around damaged columns has been largely discredited. Moreover, tying members together can create risks from drag down. The final combination of robustness measures is to tie members both horizontally and vertically. However, the method is only effective if the columns are anchored to a stiff bearing somewhere higher up in the structure, as illustrated in Figure 2-16, but the Design Codes include no requirement for this anchorage. Effective tying provision is therefore considered to provide low confidence as a protective design measure.

2.4 Conclusions

This chapter provides a review of structural robustness for buildings subject to the effects of HE blast. The use of case studies and blast damage survey data demonstrates the capacity for explosive devices to inflict local damage on buildings of various construction material. The secondary response of buildings in redistributing load thereafter was also investigated. In general, it can be seen that blast and secondary damage is linked to weaknesses in continuity and ductility between structural components and systems. Further, it is evident that there is a trade-off between providing a high level of continuity and the risk of encouraging drag-down or the spread of collapse, in circumstances where the extent of primary local damage is extreme or inadequate alternative load paths are present.

Protective strategies that can be implemented in accordance with current design practice to manage primary structural damage or secondary building response have been considered for the threat of HE attack. The critical review provided indicates a significant disparity between the capability of measures that are event specific (implemented for a defined blast threat) and those that are derived for arbitrary accidental or ‘abnormal’ loads. Typically the latter is intended to account for the loss of single load bearing elements and it is evident that the analytical procedures supporting direct alternative load path design are well established. However, research shows a lack of confidence in a number of the mechanisms commonly employed as alternative load paths. The most significant concerns seem to be focused on catenary action and inadequate joint ductility.

The breadth of the problem of structural response to accidental loading has been highlighted by the literature review presented. It was decided that the research had to concentrate on one particular aspect of structural response, namely tensile membrane (catenary action). Catenary action research in steel framed buildings is well developed, with several studies concluding that the semi-rigid connections of braced steel frames possess inadequate ductility to safely facilitate load redistribution by catenary action. Further, recent investigations now extend to the behaviour of moment resistant steel assemblages subject to large displacements. However, with the exception of a numerical study by Merola (2009), ductility and catenary action in RC framed construction is a subject that has seen only limited investigation. As a result of this change in focus it was decided to present the literature relevant to tensile membrane action in a separate chapter (see Chapter 3).
3 Literature Review – Tensile Membrane (Catenary) Action in RC Systems

As discussed in Chapter 2 it was decided that the research should concentrate on the specific secondary response due to tensile membrane (catenary) action. It was felt that in order to do justice to this topic a further literature review was required. Consequently that review was kept separate from what had gone before and is presented in this chapter.

This chapter presents an investigation into the current understanding of large displacement behaviour and catenary action in laterally restrained reinforced concrete (RC) structural systems. A summary of the historical and current analytical interpretation of RC catenary systems is provided, together with a review of supporting experimental work. The chapter identifies the gaps and limitations found in the research that compromise our understanding of whether conventional reinforced concrete systems are capable of safely redistributing the emergency load associated with single or multiple column loss by catenary action and our ability to accurately model such circumstances.

The subject of Arching and Compressive Membrane Action is discussed in the following sections as it is a characteristic of the behaviour of laterally restrained RC systems. However, this mode of response is outside the scope of the current research. Therefore, all discussion is high-level and limited to the possible impact this phase of response may have on the performance of subsequent catenary action.

3.1 Research into Large Displacement Behaviour in Laterally Restrained Reinforced Concrete Assemblies – Origin & Background

Given the availability of lateral restraint provided by floor diaphragms and adjacent framing (Mitchell and Cook, 1984), the large displacement behaviour of laterally restrained RC flexural systems is recognised to be of significant importance when concerned with outright collapse resistance (Black, 1969; Regan, 1975). In such conditions in-plane ‘membrane forces’ have been found to develop with displacement (Park and Gamble, 2000) – a compressive membrane force that develops as displacement increases to approximately half the section depth and a tensile membrane force that develops with displacements typically greater than the section depth. The two phases of membrane response are commonly referred to as ‘Compressive Membrane Action’ (CMA) and ‘Tensile Membrane Action’ (TMA) and are synonymous with two-way Arching and catenary action (as introduced in Sections 2.3.3.4 and 2.3.3.5). Both modes of response are well documented as providing enhanced out-of-plane resistance over unrestrained counterpart beams or slabs and a greater load resistance than predicted by ultimate bending theory (Park and Gamble, 2000). They therefore constitute a mechanism of secondary collapse resistance following primary failure in bending or shear (Mitchell and Cook, 1984).
Westergaard and Slater (1921) provide what might be the first documented account of ‘membrane actions’. Their report documented RC panels that exhibit an enhanced flexural resistance to the order of three times the working loads of the era, recorded sometime after yield of the reinforcement and the onset of substantial cracking. However, this early account failed to adequately identify the mechanism behind this form of secondary response and did not acknowledge it as a beneficial and reliable reserve strength. It was not until a subsequent study by Ockleston (1955) that membrane action seems to have been fully recognised. Ockleston (1955) reported on the pre-demolition testing of a three-storey RC framed Dental Hospital, in Johannesburg. The study documented a collapse resistance over four times that predicted by conventional yield line theory, owing to CMA response (Ockleston, 1958).

Ockleston’s studies were the first of a series of experimental and analytical investigations conducted throughout the 1960’s and 1970’s (Powell, 1956; Wood, 1961; Park, 1964a and 1964b; Christiansen, 1963; Brotchie and Holley, 1971; Black, 1975; Hopkins, 1969; etc). This early work constitutes the most intensive period of membrane action research to date and was mainly concerned with CMA response and collapse resistance in overload situations. However, the findings of the limited TMA investigations appear to form the basis of our current understanding of limitations in structural robustness by catenary action (Burnett, 1975; Alexander, 2004) and feature as many of the design criteria used in modern inelastic structural design practices, such as blast and progressive collapse resistant design (DoD, 1990, 1998, 2005, 2008, 2010; GSA, 2003; USACE, 2008a and 2008b), for which the effects of membrane actions on ultimate collapse resistance are of significant importance.

As the subject of structural robustness has come under increased scrutiny in recent years, this area of research has seen renewed interest, with a focus on large displacement response in support loss conditions. However, experimental studies are still limited in number and numerous numerical investigations indicate a distinct need for further physical testing (Gouverneur et al. 2013a; Li et al. 2011; Dat and Hai, 2011; Merola, 2009). As a result, the current understanding of catenary action appears to be reliant on early membrane action testing. However, the literature investigation summarised by this chapter demonstrates that the use of analytical rule sets developed in early membrane research, for overload conditions, may be inaccurate and unsafe for robustness applications, for which the current understanding seems to be incomplete.

### 3.2 Characteristic Membrane Action Response

The large displacement response in laterally restrained RC systems, as described above, is shown schematically by the sketch Figure 3-1. The modes of response are shown in series; bending response (Figure 3-1a), CMA response (Figure 3-1b) and TMA response (Figure 3-1c). Figure 3-2 is an idealised representation of the characteristic membrane force and load-deflection behaviour in uniformly loaded laterally restrained flexural RC slabs – as observed by Park and Gamble (2000), Rankin and Long (1998) and Wang (2010).
a). Minor elastic displacement ($\Delta \leq \Delta_{cr}$): element acts in pure flexure as net length of element is remains unchanged prior to cracking and plastic hinge development.

b). In-elastic displacements less than the section depth ($\Delta_{cr} \leq \Delta \leq h$); bending capacity is supplemented by compressive membrane forces that develop due to spatial demand and axial shortening, imposed by edge restraint.

c). In-elastic displacements exceeding the section depth ($\Delta \geq h$); tensile membrane forces develop with displacement as the edge restraint results in a demand for axial lengthening.

Figure 3-1 – Schematic illustration of membrane force development in a centrally loaded RC element, subject to large displacement and rigid lateral restraint.

Figure 3-2 - Idealised axial force and load-displacement curves showing membrane actions in reinforced concrete member fixed against all translation (consolidated from Park and Gamble, 2000; Black, 1975; Rankin and Long, 1998; Wang, 2010).
The load history shows the development of load resistance and membrane force with displacement and demonstrates the two peak resistances associated with CMA and TMA. An idealised load-deflection curve of a counterpart slab, with rotational but no lateral restraint, is shown to demonstrate enhancement.

The following section provides an account of the fundamental physical response of laterally restrained RC floor systems subject to large inelastic displacements and details the characteristic membrane actions. Critical factors understood to affect the development of membrane response and outright collapse resistance are identified. The account provided is based on early membrane research, which provides a fundamental narrative of the mechanism of membrane response for further consideration under support loss conditions (see Section 3.5).

3.2.1 Compressive membrane action (CMA) response

There is good general agreement behind the mechanism that forms CMA response. Development of compressive membrane force is attributed to a net gain in in-plane length along a flexural member that is resisted by the presence of lateral edge restraint (Park, 1964a). Initially this is the product of bending, whereby the strain compatibility of a RC section dictates a net tensile strain at the centroid under equilibrium conditions (Vecchio and Tang, 1990). Where rigid lateral restraint is present, the consequential elongation in the member is resisted by the prevention of lateral translation and expansion at the supports, which results in in-plane axial compression. Keenan (1969) remarked that with the onset of cracking and migration of the neutral axis to the compression face, sectional expansion becomes more pronounced and Beeby and Fathibitaraf (2001) suggested that, due to the geometric nonlinearity of compression regions at the support and mid-span critical sections, a compression arch forms between the supports.

At larger inelastic displacements, displacement is mainly attributed to plastic hinge rotation at the critical sections and yield lines – essentially the system forms a mechanism of two elastic segments that rotate about three hinge locations, two of which are fixed against lateral and vertical translation (see Figure 3-1b). Authors such as Park (1964a) attribute membrane force development at this stage to further spatial demand imposed by the geometry of the system. Park and Gamble (2000) describe the mechanism as ‘jamming’ between lateral restraints, causing the system to develop as an in-plane arch or compressive membrane.

The development of load resistance and compression membrane force is illustrated in Figure 3-2. Point A denotes the onset of cracking at critical sections. The peak load achieved in CMA is denoted by Point B and corresponds approximately with the maximum compressive membrane force (Park, 1964a), at displacements of approximately 0.5\(h\) (Park and Gamble, 2002), where \(h\) is the overall depth of the element. Inspection of slope AB demonstrates the rapid development of compressive membrane force with cracking. Load resistance is shown to develop almost linearly to an enhanced ultimate load carrying capacity and system stiffness over the elastic-plastic region is shown to be greater compared
with the response of unrestrained counterparts (Keenan, 1969). It can be seen that CMA is apparent throughout inelastic response (Park, 1964b).

Powell (1956) and Wood (1961) reported ultimate CMA load capacities of up to 8.2 and 11.22 times the ultimate collapse load predicted using Johansen’s yield line theory, for the respective two-way slab specimens in test conditions. Authors such as Ockleston (1955; 1958), Sasani (2008), Sasani et al. (2011) and Vecchio and Tang (1990) observed collapse loads of approximately 2.0-6.3 times the theoretical ultimate flexural capacity of floor systems (monolithic beam-slab assemblies) in in-situ buildings. Keenan (1969) pointed out that with the development of in-plane compressive forces, CMA also resulted in an enhanced shear resistance.

Enhancement was attributed to the development of a compressive arching mechanism and an enhanced ultimate moment capacity corresponding approximately with maximum membrane force. Park and Gamble (2000) stated that compressive membrane forces are, as a rule, never large enough to prevent yielding of the tension reinforcement. Therefore enhancement of the moment capacity can be rationalised by conventional M-N interaction theory (as previously noted by Keenan, 1969, Beeby, 1999, Park and Gamble, 2000, Beeby and Fathibitaraf, 2001). Inspection of the interaction diagram below (Figure 3-3) demonstrates that in such situations, some degree of enhancement to the ultimate moment will result – the compressive membrane force acting as a stabilising in-plane force that drives the neutral axis towards the centroid of the section.

![Figure 3-3](image)

**Figure 3-3** – Ultimate moment-axial force diagram for typical symmetrically reinforced section (Park and Gamble, 2000).

Keenan (1969) provided an extensive experimental and numerical parametric study investigating factors influencing CMA performance in square slab systems. The author identified four factors found to influence CMA performance:
**Area of reinforcement** – Enhancement increases as tensile steel ratio decreases. Park (1964a) concurs and stated that the highest load resistance is achieved in the situation of lowest percentage area of reinforcement, for all restraint conditions.

**Crushing strain of the concrete** – In-plane thrust is limited by crushing of the concrete therefore higher compressive strain capacity will lead to greater enhancement, which suggests that heavily confined beam sections would do well (Keenan, 1969).

**Span-thickness ratio** – Load enhancement was generally found to be inversely proportional to span-thickness ratio (L/t). However, at L/t ratios greater than 18, load enhancement was found to be limited by inherent geometric instability in the CMA mechanism. When L/t was less than 18, enhancement was found to be limited by the crushing strength of the concrete (Keenan, 1969).

**Edge restraint stiffness** – Enhancement was found to be highly dependent on restraint stiffness – for a given span-thickness ratio, enhancement falls with decreasing restraint stiffness. However, the effect of reduced edge restraint stiffness is most pronounced as span-depth ratio increases. This is consistent with the work of Wood (1961), Park (1964a), Park (1975) and Keenan (1969).

### 3.2.2 Snap-through

Region BC marks the decline of CMA and transition to TMA response (see Figure 3-2). This region is commonly referred to as ‘Snap Through’ (Black, 1975; Mitchell and Cook, 1984). Failure of the CMA phase after Point B can be brittle, due to concrete crushing in flexural-compression or punching shear (Su *et al.* 2008; Hawkins and Michelle, 1979), in the case of flat plate slab systems. However, as displacement progresses beyond Point B, the compressive membrane force and load resistance declines rapidly. The reversal of system stiffness is attributed to the geometric eccentricity of the mechanism, which permits in-plane lengthening with displacement and a subsequent fall in compressive strain (Park, 1964a). In a load controlled situation, this has been found to result in the acceleration of mid-span displacement before the load is ‘caught’ at Point C (Regan, 1975). This marks transition to TMA response and zero membrane force in the system. Beyond Point C any additional displacement results in increased load resistance and tensile strain as the system pulls against its bounding edge restraint in TMA response. Park and Gamble (2000) pointed out that the system is essentially a mechanism after failure of CMA (Point B, Figure 3-2) and displacement is achieved by plastic hinge rotation.

Park and Gamble (2000) claimed that Snap Through is highly dependent on the stiffness of the edge restraint. The load-displacement history shown in Figure 3-2 is representative of a system with perfectly rigid lateral restraint. In this arrangement the transition to TMA response is commonly accepted to occur at a displacement approximately equal to the section depth (Keenan, 1969; Park and Gamble, 2000; Wang, 2011). Authors such as Regan (1975), Rankin and Long (1997) and Wang (2010) claim that the load resistance at this point is approximately equal to the ultimate flexural strength. However, in some instances, concrete crushing has been observed at critical sections during
CMA response (Keenan, 1969), which would suggest a slightly reduced load carrying capacity at Point C (as shown in Figure 3-2b) for perfectly rigid systems. Park and Gamble (2000) noted that in conditions of lower restraint stiffness, the gradient of the load-displacement curve BC tends to be less steep and tensile membrane force development occurs at an earlier stage. This suggests that in conditions of limited edge resistant, whereby outward lateral translation under CMA is not fully prevented, TMA response may develop before load resistance enhancement from CMA is fully dissipated and at lower displacements. This is would be favourable to emergency response, as acceleration of the floor system through Snap Through and the subsequent demand placed on TMA response would be reduced.

3.2.3 Tensile membrane action (TMA) or catenary response

Mitchell and Cook (1984) described TMA response as a mat of reinforcement that acts as a ‘hanging net’ suspended between supports. This mechanism is illustrated by Figure 3-1c. At its onset (Point C, Figure 3-2) the extreme tension reinforcement is assumed to have yielded in bending and cracking, which penetrates the full depth of the section, has been documented at the critical sections (Park, 1964b; Black, 1975; Regan, 1975). In the case of simply supported two-way slab systems, TMA response is described as developing first at the centre of the slab, as the neutral axis migrates to the top fibre of the concrete (Wood, 1961; Desayi and Kulkarni, 1977), however this is likely to be dependent on the rotational restraint present at the boundary.

Figure 3-4 shows an image taken of the underside of a uniformly loaded two-way spanning slab in tensile membrane response and demonstrates the crack patterns and mechanism formed.

![Figure 3-4](image)

Figure 3-4 – Image taken of the underside of a slab specimen subject to tensile membrane action (Park, 1964b).

It can be seen that displacement is accommodated by rotation at yield lines and critical sections, with the regions between remaining largely elastic (Wood, 1961; Park, 1964b; Black, 1975). However, the system eventually tends to a purely tensile response as displacement increases. Park and Gamble
(2000) described this advanced stage of TMA response as being akin to a load carrying cable or chain bridge, whereby the reinforcement forms a ‘plastic tension membrane’ strung between the supports. At extreme displacements, authors have described the development of full-depth traverse tension cracks at frequent intervals along the span as the deformed profile of the system tends to that of a catenary or parabola (Keenan, 1969; Regan, 1975). It was generally noted that both top and bottom reinforcement are mobilised in tension at this stage, although there is some disagreement regarding whether yield is achieved in both (Park, 1964b; Keenan, 1969; Black, 1975), see Section 3.4.

TMA response is characterised by the membrane force and load-displacement curves denoted by CE, as shown in Figure 3-2. Catenary action in this form permits the slab to sustain significant displacement and an enhancement of load carrying capacity over an unrestrained counterpart (Park and Gamble, 2000; Regan, 1975). Inspection of Figure 3-2 demonstrates an almost linear increase of load resistance with displacement throughout catenary response. This corresponds with the development of tensile stress in all reinforcement present (top and bottom layers; Keenan, 1969) until plastic in-plane response is noted (Point D, Figure 3-2). Load resistance continues to increase with displacement until failure (denoted by Point E, Figure 3-2). For heavily reinforced sections, maximum load enhancement was found to significantly exceed that observed in CMA (Wood, 1961; Park, 1964b; Regan, 1975).

From the work of Park, Keenan and Regan (1975) three key factors in the performance of tensile membrane and Catenary response have been identified:

In-plane tensile strength – Load resistance was found to be directly proportional to the tensile strength of the catenary (see Figure 3-2). Regan (1975) and Park (1964b) demonstrated that collapse resistance, at a given displacement, would increase as the area and ratio of reinforcement increases.

Ductility & Extensibility – The ability of a Catenary to sustain large inelastic displacement without loss of in-plane strength is dependent upon its ductility (Beeby, 1997). Since load resistance is found to increase with displacement (see Figure 3-2), the performance of a Catenary is highly dependent on rotational and tensile ductility (Park, 1964b; Regan, 1975).

Edge restraint stiffness – The development of tensile membrane force and load resistance is dependent on tensile stress in the reinforcement. Keenan (1969) demonstrated that TMA is less sensitive to edge restraint stiffness than CMA but remarked that adequate edge restraint is required to ensure ultimate tensile stress can be attained in the reinforcement.

3.2.4 ‘Primary’ TMA response, ‘Secondary’ TMA response & typical failure modes

From the research available, two modes of response can be identified for TMA and catenary action in doubly reinforced RC catenary systems, Figure 3-5. The existing literature does not appear to differentiate between these two modes of response or identify them individually. However, this study
demonstrates that catenary behaviour and performance can differ substantially for each mode. Thus, for clarity, the two modes of response are referred to as primary and secondary TMA response throughout this study.

**Figure 3-5** – *Primary TMA response (a), secondary TMA response (b) and collapse (c) as observed in doubly reinforced RC Catenary mechanisms.*

*Primary* response is defined in this study as large displacement TMA behaviour prior to failure of the extreme tension reinforcement layer (see Figure 3-5a). Two modes of failure are documented.

Experimental studies by Park (1964b), Keenan (1969) and Black (1975) and the subsequent of work of Park and Gamble (2000) are concerned with primary TMA response only. The authors attribute failure and maximum resistance of a tensile membrane to fracture of the extreme tension reinforcement at yield lines and critical sections. Park (1964b) and Keenan (1969) reported this to occur at the mid-span of simply supported two-way slab specimens whereas Black (1975) reported reliable failure by rupture of the extreme tension reinforcement layer along the periphery of the membrane, in slab specimens featuring rotational constraints at the boundary. This suggests that failure is due to combined tension and rotational demand at plastic hinge locations. The mode of failure has been documented more recently by Sadek *et al.* (2011), Lew *et al.* (2011), Yi *et al.* (2008) and Su *et al.* (2009) in the testing of large and full-scale RC beam-column sub-assemblies subject to artificial column loss.

Experimental investigations documented by Mitchell and Cook (1984) demonstrated de-bonding and failure of reinforcement anchorage as an alternative mode of failure. Mitchell and Cook (1984) point to reinforcement anchorage as a principal failure mode. Relevance of the study is limited as the authors were concerned with TMA response in individual bays of flat-plate RC systems following over-load or misuse rather than structural robustness. However, the authors indicate that TMA response was undermined by the top reinforcement being torn from the surface of slabs at joint locations, with subsequent failure of the tension lap (see Figure 3-6). Furthermore, several cases were documented
where the tension laps at support locations were found to fail prior to the development of any significant catenary action.

Figure 3-6 – Extract from Mitchell and Cook (1984) showing initial punching shear failure and subsequent distress at column headers, as observed in flat-plate RC systems.

Kang and Tan (2015) provides further evidence of the potential for anchorage failure. The authors document the testing of five precast concrete beam-column sub-assemblages tested under support-loss conditions. In this case, several of the test specimens were found to fail in primary response due to pull-out/slip along the lap of the extreme tension reinforcement at support and mid-span locations.

The failure modes observed in primary response suggest ultimate displacement is dictated by rotation capacity at critical sections. Furthermore, the tensile membrane force in plastic TMA response (denoted by region DE, Figure 3-2), may be governed by the ultimate tensile strength and ductility of the extreme tension reinforcement or the ultimate bond strength of its anchorage. In either case, the mechanism of failure has been documented as undermining the in-plane tensile strength of the catenary system, resulting in a loss of load resistance and potential collapse.

Regan (1975), Woodson (1990, 1992 and 1994) and Merola (2009) have pointed out that, provided the section is doubly reinforced, the resultant catenary system should be capable of maintaining TMA response and load resistance following initial failure of the extreme tension reinforcement by tension in the remaining ‘critical reinforcement’ layer, at either support or mid-span hinge locations. This mode of secondary response is illustrated schematically by Figure 3-5b and referred to herein as secondary TMA response.

Secondary response has been observed in the overload TMA testing of small-scale slab specimens by Woodson and Garner (1985) and Woodson (1990, 1992 and 1994) and by Regan (1975), Yu and Tan.
and Gouverneur et al. (2013b) in the testing of large-scale double span beam and slab test specimens. Given the mechanism formed (see Figure 3-5b), failure appears to be dictated by the tensile properties of the system – the strength and ductility of reinforcement. The authors point to failure by fracture of the critical reinforcement or anchorage failure and slip or pull-out of the tension lap, where present.

The significance of identifying these modes of response separately is twofold.

By investigating available literature, it is apparent that since the study by Park (1964b) the majority of subsequent experimental and analytical studies of TMA response in RC systems have been limited to primary catenary response – the authors assuming peak load resistance and incipient collapse to occur at or prior to failure of the extreme tension reinforcement. Thus, the current design guidance and analytical interpretation of ultimate progressive collapse resistance in laterally restrained RC systems is typically based upon the limits of primary TMA response and joint rotation capacities (see Sections 3.3 and 3.4). This is therefore an appropriate response threshold that is pertinent to the current research.

The current understanding of ultimate secondary catenary response is relatively limited probably due to the above argument. Furthermore, the understanding of the respective performance and limitations of secondary response in complex RC elements is not well established. However, the experimental studies by Woodson and Garner (1985), Woodson (1990, 1992 and 1994), Regan (1975), Yu and Tan (2013) and Gouverneur et al. (2013b) demonstrate that this represents the final emergency load carrying mechanism before outright collapse is achieved. Moreover, although the respective authors point to the importance of rotational ductility in catenary performance, failure of the extreme tension reinforcement and primary response was not necessarily recognised as a mode of failure that defined maximum catenary resistance or resulted in outright collapse. Rather, maximum TMA resistance was frequently found to occur during secondary response, at displacements well beyond the rotation capacity at critical sections (Section 3.5.1). In fact, the experimental study by Regan (1975), which was specifically related to emergency load redistribution in restrained slab specimens following support loss, was almost entirely concerned with secondary TMA response. Given that this thesis is concerned with the outright collapse resistance of conventional buildings, secondary TMA response is therefore an important emergency load carrying mechanism that should be considered in collapse analyses.

3.2.5 Principal displacements

There is some uncertainty as to the displacement at which each key phase occurs. By geometric reasoning, the peak CMA force would be expected to occur at a displacement of approximately half the section thickness (0.5h). However this is a complex mechanism and, as documented by Keenan (1969) and Park and Gamble (2000), this may vary significantly due to concrete crushing at yield lines and the ongoing in-plane lengthening that results from strain compatibility in bending. Various authors
have documented the peak CMA load to occur at displacements that vary from 0.2h to over 1.0h (Keenan, 1969; Desayi and Kulkarni, 1977) but recent experimental investigations have observed maximum CMA load at deflections of 0.16h (Su et al. 2009). Park and Gamble (2000) and Park (1964b) attributed this variability to the stiffness of edge restraint and demonstrated that systems with lower restraint feature CMA at larger displacements. Further uncertainty is evident in the displacement at which TMA response begins. Generally it is claimed that 1.0h is typical but several tests conducted on emergency span systems have indicated that TMA may not begin to develop until displacements as high as 2h (Regan, 1975).

An investigation of characteristic ultimate displacements in TMA response (primary and secondary) is discussed in Appendix A and Sections 3.4 and 3.5.3.

### 3.3 Analysis of Catenary Action in RC Assemblies

It is widely agreed that there is currently no numerical macro-model that accurately predicts TMA and catenary action behaviour in RC slab assemblies (Park and Gamble, 2000; Desayi and Kulkarni, 1977). The current approach for the calculation of load resistance is reliant on linear analytical expressions derived from simple equilibrium of the catenary mechanism by which load resistance is shown to be a function of geometry, displacement and membrane force. The general indication is that this approach to catenary action modelling results in a conservative prediction of load resistance for a given displacement and is therefore adequate for design and analytical purposes (Park and Gamble, 2000).

The following section details an investigation of the expressions developed for the analysis of large displacement behaviour in laterally restrained conventional RC beams and slabs, as implemented in catenary research and design guidance for overload and robustness applications. Criteria used in the prediction of ultimate load resistance are also investigated and the limitations associated with each are discussed.

#### 3.3.1 Uniformly loaded 2-way systems

Park (1964b) proposed an analytical approach for the prediction of load-displacement TMA behaviour in uniformly loaded two-way spanning rectangular slabs with orthogonal reinforcement. The system is assumed to act as a ‘plastic membrane' with edges restrained against all translation, as illustrated in Figure 3-7. Park showed that it is possible to express equilibrium of a discrete portion of the membrane \((dydx)\) by Equation 3-2.

\[
0 = wdx dy - T_x d y \frac{\partial z}{\partial x} + T_x dy \left( \frac{\partial z}{\partial x} + \frac{\partial^2 z}{\partial x^2} dx \right) - T_y dx \frac{\partial z}{\partial y} + T_y dx \left( \frac{\partial z}{\partial y} + \frac{\partial^2 z}{\partial y^2} dy \right)
\]

Equation 3-1
\[
\frac{\partial^2 z}{\partial x^2} + \frac{T_y}{T_x} \frac{\partial^2 z}{\partial y^2} = - \frac{w}{T_x}
\]

Equation 3-2

Figure 3-7 – Plan of plastic membrane and free body diagram of small segment, as shown by Park (1964).

Park uses terms derived in Prandtl’s 1903 membrane analogy (Timoshenko and Goodier 1951, p276), to satisfy boundary conditions and resolve the differential equation, as given in Equation 3-3.

\[
\frac{wl_x^2}{T_x \Delta} = \frac{\pi^3}{4 \sum_{n=1,3,5...}^{\infty} \frac{1}{n^3} \left( -1 \right)^{\frac{n-1}{2}} \left( 1 - \frac{1}{\cosh \frac{n\pi l_y}{2l_x} \sqrt{\frac{T_x}{T_y}}} \right)}
\]

Equation 3-3

In which the terms uniform load per unit area \((w)\), mid-span deflection \((\Delta)\) and membrane force per unit width \((T)\) are used.

The expression developed by Park assumes that the concrete carries no tension, all reinforcement has reached yield and that no strain hardening occurs within the reinforcement.

By assuming a constant membrane force equal to the yield strength of the reinforcement, Equation 3-3 gives a linear relationship between load and central displacement. By comparison with available test data Park (1964b) and Park and Gamble (2000) demonstrated that this approach provides a conservative estimate of applied load with displacement – underestimating load resistance in the system and therefore providing a ‘safe’ solution for catenary design. Figure 3-8 demonstrates the discrepancy observed between experimental and theoretical results for one of Park’s test specimens. At a prescribed displacement of 0.10\(L_{ALS}\), Park showed Equation 3-3 to predict load resistance at between 40 and 83% that recorded in tests by Park (1946b) and Powell (1956).
Reference to more recent investigations of TMA response demonstrates that the term derived by Park (1964b) is a fundamental and current approach to load-displacement prediction (Merola, 2009). Guidance documents by the U.S. Department of Defense, related to blast resistant design (DoD, 1990; 1998; 2008; 2014) specify Equation 3-3 directly. Similarly, progressive collapse guidance provided by the GSA (GSA, 2003) and DoD (2005) reference Park and Gamble (2000) as an appropriate resource for analytical procedures. In design applications the term is used to compute catenary displacement or load resistance for a given value of the other. DoD (1998) stated that a reasonable approximate solution can be obtained by averaging solutions for \( n = 1, 3, 5 \) and \( n = 1, 3, 5, 7 \).

Keenan (1969) and Black (1975) pointed out that for an isotropically reinforced square slab, the right hand side of Equation 3-3 equates to a dimensionless constant of 13.56. For such an arrangement Keenan and Black suggested that a more accurate interpretation of load-deflection behaviour would be obtained by the use of a dimensionless constant of 20. This approach was derived empirically, based on test data reported by both authors to give an improved correlation with recorded performance. The argument behind the use of this empirical coefficient was that pure tensile membrane action was not observed in testing, rather the initial mechanism formed more of a bilinear profile suggesting residual moment resistance. Keenan and Black both claimed that pure tensile theory is overly conservative unless the catenary is loaded dynamically, whereby response tends to that of a plastic membrane. Black noted that Park’s approach is more representative of dynamic test series data where concrete decomposition, displacement and membrane action were found to be more pronounced.

Hawkins and Mitchell (1979) offered an alternative approach. In this case the authors claimed the assessment of a catenary or parabola profile to be unnecessarily complex and assumed the membrane to take a circular deformed shape, as shown below.
Assuming the concrete to carry no tension, force equilibrium of the above system gives the following expressions:

\[
W = \frac{2T_x \sin \sqrt{6\varepsilon_x}}{l_x} + \frac{2T_y \sin \sqrt{6\varepsilon_y}}{l_y}
\]  

Equation 3-4

\[
\Delta = \frac{3l_x \varepsilon_x}{2 \sin \sqrt{6\varepsilon_x}}
\]  

Equation 3-5

\[
\varepsilon_y = \varepsilon_x \frac{l_x^2}{l_y^2}
\]  

Equation 3-6

Equation 3-4 through Equation 3-6 are complementary expressions that allow prediction of load-deflection response by iteration. Hawkins and Michelle (1975) and Michelle and Cook (1984) showed the approach to be less conservative than that presented by Park (1964) and Park and Gamble (2000), by comparison of theoretical load predictions with those recorded in testing at membrane displacements of 1.0\(L_{ALS}\) and 1.5\(L_{ALS}\). Furthermore, unlike the expressions proposed by Park and Gamble (2000), Keenan (1969) and Black (1975), the authors indicated that this approach has the potential to include geometric and material nonlinearity. However, by inspection of the derivation of Hawkins and Michelle’s approach, it can be seen that reinforcement strain is assumed to be uniform for the length of the membrane, rather than confined to regions local to the critical sections. Thus at the characteristic ultimate displacements advocated by Park and Black (1.0\(L_{ALS}\) and 1.5\(L_{ALS}\)), the resultant strain calculated using Equation 3-4 to Equation 3-6 can be seen to be a fraction of the ultimate tensile strain of modern reinforcement. This strongly suggests instability in the terms.
proposed by Hawkins and Michelle (1975) and Michelle and Cook (1984) and that the terms do not support assessment by material nonlinearity.

By investigation of guidance available in nonlinear analysis of two-way spanning RC slabs in TMA response, Park’s term (Equation 3-3) seems to constitute the industry accepted approach for load-displacement prediction. Furthermore, the term is seen used for the analyses of both primary and secondary TMA responses. Those terms specified by Hawkins and Michelle (1975) and Michelle and Cook (1984) do not appear in subsequent studies of TMA behaviour.

3.3.2 Uniformly loaded 1-way systems

Mitchell and Cook (1984) presented expressions for a one-way spanning catenary:

\[ w = \frac{4T_c \sin \sqrt{6\varepsilon}}{l_n l_2} \]  

Equation 3-7

Where \( T_c \) is the force in the bottom reinforcement of the catenary; \( l_n \) is the clear span, and; \( l_2 \) is the slab width. The deflection corresponding to the load per unit length of the catenary is given by:

\[ \Delta = \frac{3l_n \varepsilon}{2 \sin \sqrt{6\varepsilon}} \]  

Equation 3-8

The above equations are adaptations of the terms derived by Mitchell and Cook (1984) for assessment of square flat slab systems, subject to uniformly distributed loads (see Equation 3-4 through Equation 3-6). Thus the terms are derived from an assumed circular deflection profile (see Figure 3-9). The authors implied that the collapse load and displacement of the system can be determined by using the rupture strain of the reinforcement, rather than use of an empirical displacement limit. However, as noted for the proposed two-way analysis, this results in unrealistically optimistic predictions of ultimate displacement.

Regan (1975) offered a more pragmatic approach. He explained that the catenary behaviour can be predicted with knowledge of the load-extension characteristics of a catenary system and simple equilibrium, giving the following simple equation:

\[ R_H \Delta = \frac{qL^2}{2} \]  

Equation 3-9

Equation 3-9 can be derived by equilibrium of the double span bilinear and parabolic catenary profiles shown in Figure 3-10.
It is an interesting note that Regan (1975) attributed the bilinear three-pin catenary mechanism (Figure 3-10a) to situations where the applied load is less than the flexural capacity of a single span. The author claimed that the more extreme profile of the curved or parabolic catenary did not form unless applied loads are greater. This suggests that all emergency load case situations caused by support loss would result in a bilinear catenary profile.

The base term Equation 3-9 has been used in recent studies of primary and secondary catenary action in one-way spanning concrete assemblies (Merola, 2009; Tohidi et al. 2014a and 2014b). Furthermore, it is consistent with Park’s derivation (Equation 3-3), by consolidating the term for one-way conditions – for example, by assuming the ratio of orthogonal membrane forces to equal infinity Equation 3-3 can be resolved to give Equation 3-9. The term has been recommended by various guidance documents for used in the analysis and design of laterally restrained RC slabs and beams under large inelastic displacement (DoD, 1990; 1998; 2008; 2014; GSA, 2003). From the literature available, the analytical approach and terms developed by Mitchell and Cook (1984) are not used in subsequent studies of catenary response.

### 3.4 Ultimate Load Criteria & Failure Limits

The DoD, USACE and GSA have provided guidance for blast resistant (DoD, 1990, 1998, 2008 and 2014; USACE, 2008a and 2008b) and progressive collapse (DoD, 2005 and 2013; GSA, 2003 and 2013) design and analysis, since the 1960’s. These documents provide empirical design criteria and input parameters to support the direct computation of ultimate load resistance and resistance-
displacement functions for the nonlinear analyses of TMA response. Furthermore, the criteria identified by these documents can be traced to the indirect robustness design (tie force) procedures supported by the Eurocodes, British Standards and UK Building Regulations (Burnett, 1975).

Appendix A provides a detailed investigation of ultimate limit criteria, for implementation with the nonlinear analytical models presented in Section 3.3. The investigation shows that ultimate response limits are typically defined by end-rotation ($\theta$).

$$\theta = \tan^{-1} \frac{\Delta}{L} = 2\tan^{-1} \frac{\Delta}{L_{ALS}}$$  \hspace{1cm} \text{Equation 3-10}

Ultimate limit criteria ($\theta_u$ and $\mu_u$) correspond with the ultimate displacement, associated with maximum load resistance prior to incipient failure, $\Delta = \Delta_u$.

The guidance also stipulates the parameters for membrane force ($N_d$) based on an assumed are of reinforcement ($A_{S-TMA}$) and tension stress ($f_d$).

$$N_d = f_d(A_{S-TMA})$$  \hspace{1cm} \text{Equation 3-11}

The investigation demonstrates that Equation 3-9 is used for the prediction of primary and secondary TMA response. The existing criteria are empirically derived based on predominantly overload testing of small-scale slab specimens [Powell (1956), Park (1964b), Keenan (1969), Black (1975) and Brotchie and Holley (1971), Woodson and Garner (1985), Guice (1986) and Woodson (1990, 1992 and 1994)]. Some confusion was identified with regards the criteria proposed for ultimate membrane force. However, from the information obtained, the conservative assessment of maximum load resistance in primary and secondary TMA response can be summarised.

By substitution of Equation 3-10 into Equation 3-9, TMA load resistance ($q_{TMA}$) becomes:

$$q_{TMA} = \frac{4R_H \tan \theta}{L_{ALS}}$$  \hspace{1cm} \text{Equation 3-12}

By assuming membrane force ($N_d$) is approximately equal to the horizontal restraint force ($R_H$), the existing progressive collapse design guidance (DoD, 2005; GSA, 2003 and Park and Gamble, 2000) suggests that TMA response can be modelled as:

$$q'_{TMA} = \frac{4N_d \tan \theta'}{L_{ALS}} = \frac{4(A_s + A'_s)f_y \tan \theta'}{L_{ALS}}$$  \hspace{1cm} \text{Equation 3-13}
Where ultimate load resistance corresponds with a chord rotation ($\theta'$) of 12° or 8°, for RC elements of $L_{ALS}/h \geq 5$ and $L_{ALS}/h < 5$ respectively.

The above corresponds with primary TMA response. Criteria for secondary TMA response is used in blast resistant design applications only, for which:

$$q''_{TMA} = \frac{4N\tan\theta''}{L_{ALS}} = \frac{2(A_s + A'_s)f_y\tan\theta''}{L_{ALS}}$$

Equation 3-14

In accordance with DoD (1998) and USACE (2008a), ultimate TMA resistance in secondary response and incipient collapse corresponds with a chord rotation ($\theta''$) of 20° or 12°, for elements of $L_{ALS}/h \geq 5$ and $L_{ALS}/h < 5$ respectively.

By considering a RC element of $L_{ALS}/h \geq 5$, with the same type and area of reinforcement ($A_s + A'_s$) and span ($L_{ALS}$), resolving Equation 3-13 and Equation 3-14 gives:

$$q'_{TMA} = 1.168q''_{TMA}$$

Equation 3-15

Thus, the current guidance suggests that ultimate primary TMA load resistance is greater than ultimate secondary TMA load resistance and therefore constitutes the maximum and collapse resistance of a given RC catenary. This may explain why the direct analysis of TMA response is limited to primary response for progressive collapse guidance. However, whilst Park (1964b), Keenan (1969) and Black (1975) suggest that 12° is an achievable end-rotation, the investigations of Woodson and Garner (1985) and Woodson (1994) demonstrate that the theoretical TMA slope given by Equation 3-13 was not attained in testing. Furthermore, maximum load resistance was in fact achieved in secondary TMA response.

This observation is illustrated by Figure 3-11 and suggests that the estimation of collapse resistance in catenary response by Equation 3-13 could be non-conservative and that Equation 3-14 may provide a better approximation of collapse resistance. However, it is unclear whether the discrepancy between the two approaches to the calculation of maximum TMA resistance is attributed to an inaccuracy of the membrane force ($N_d$), ultimate chord rotation ($\theta_u$) or the base term Equation 3-9.
3.5 Emergency Load Redistribution by Catenary Action

This research is concerned with the ultimate collapse resistance of conventional framed buildings following the hypothetical situation of instantaneous column removal. Catenary action has been identified as the ‘last defense mechanism to prevent structural collapse’ (Yu and Tan, 2013b). Thus, the scenario under consideration is that shown below (Figure 3-12); emergency load redistribution by catenary action following the loss of an intermediate column or load bearing element.

Figure 3-11 – Load-displacement history of a one-way spanning slab specimen, showing secondary TMA response – Woodson and Garner (1985).

Figure 3-12 – Schematic illustration of emergency load redistribution in RC frame sub-assembly following column loss.
The previous sections identify the common understanding of structural behaviour in membrane responses (Section 3.2) and shows that existing macro-models and ultimate limit criteria have been established for the prediction of primary and secondary TMA response in RC slab and beam elements (Sections 3.3 and 3.4), as implemented in design and nonlinear analyses. However, it has been shown that there is some dispute between blast resistant and progressive collapse design guidance as to which mode of response constitutes ultimate collapse resistance. Moreover, Regan (1975) presented the first investigation of concrete floor assemblies under support loss conditions. The work constitutes the largest test program of its type and the author suggested that whilst catenary action had been observed under test conditions the ‘overall ability of structures to develop catenary behaviour [in support loss conditions] is much more complex’. Regan points to several instances where slab specimens are found to fail prior to development of any appreciable tensile response. This indicates that the development of TMA response and a reserve strength in support-loss conditions is not as straightforward as the current design guidance would suggest.

From the literature available it can be seen that the current understanding of catenary action is heavily reliant on blast resistance research – Sections 3.2, 3.3 and 3.4 demonstrate that analytical approaches are generally based on experimental results obtained by the testing of small-scale two-way spanning RC slab specimens that sustain TMA response following primary failure due to misuse and/or overload. Furthermore, the bulk of test specimens feature continuous or single span detailing and plain mild-steel or antiquated reinforcement. Whereas, under support loss conditions, a catenary is required to act across multiple structural bays at several times the original design span. These conditions dictate higher span-depth ratios, complex discontinuities in reinforcement arrangement and differing boundary conditions. This suggests that Regan’s concerns are not unfounded as the basis of the current approach to catenary analysis and design in double-span conditions may not address the complexity of large displacement response in robustness applications. This observation is supported by the findings of Ellingwood et al. (2007), Stevens (2008), Yi et al. (2008), Merola (2009) and Dat and Hai (2011) who point to a lack of experimental data and subsequent complications in their analytical investigations of emergency load redistribution by catenary action.

This section investigates factors known to influence the performance of catenary action in RC elements, as introduced in Section 3.2.3. Recent experimental and analytical TMA research were investigated to identify conclusions specific to the response of modern RC systems under support loss conditions and weaknesses in the current approach to catenary analyses.

### 3.5.1 Large-scale experimental investigations of emergency response in RC systems

A number of important experimental studies have been undertaken over the past decade that investigated catenary response in laterally restrained double span RC beam and slab specimens, under support loss conditions. Most have migrated towards testing at large-scale [Gouverneur (2013a and 2013b), Yu and Tan (2013a and 2013b), Lew et al. (2011), Sadek et al. (2011) and Yi et al. (2008), Su et al. (2009), Vecchio and Tang (1990)]. However, a common feature has been to terminate testing
when failure of the extreme tension reinforcement layer occurs. Thus, experimental studies of primary and secondary catenary response are limited to those reported by Gouverneur (2013a and 2013b), Yu and Tan (2013a and 2013b) and Regan (1975). This section therefore focuses on the behaviour observed in these studies.

Regan (1975) documented an experimental investigation of the large-displacement behaviour of laterally restrained RC slab systems. Nineteen double-span floor assemblies were tested. The specimens were each formed of two 375mm x 100mm deep precast concrete slab strips, joined to give a double span \( L_{ALS} \) of 5.5m and span-thickness ratio \( (2L/h) \) of 55, following removal of the central support. Conventional detailing was used for each 2.5m service span \( L \), such that slab-slab joints were located at outer support and mid-span locations and the top reinforcement layer was curtailed to provide intermediate lengths of singly reinforced slab, Figure 3-13.

![Figure 3-13 – Typical detailing of double span slab-slab specimens tested by Regan (1975).](image)

The area of top and bottom longitudinal reinforcement was changed for each test specimen to provide equal and unequal \( A_{S,\text{top}} \) and \( A_{S,\text{bottom}} \) arrangements and investigate the effect of reinforcement area and arrangement on collapse resistance. To investigate the effects of reinforcement deformity and tension properties (strength and ductility) on TMA response, the longitudinal reinforcement and joint ties were varied between smooth pre-stressed, smooth mild, deformed hot-rolled high-yield and twisted high-yield bar.

Figure 3-14 provides an elevation of the quasi-static testing arrangement used for seventeen of the specimens. The test slabs were restrained against all translation at the outer supports and, following removal of the middle support, subject to an approximate uniformly distributed quasi-static load – two jacks applying a load controlled displacement via a roller bearing, in an eight point load arrangement. A critical aspect of the quasi-static testing was that all test specimens were displaced until outright collapse was achieved. This allowed Regan (1975) to investigate the behaviour of the test specimens during all phases of membrane response and draw some conclusions with regards the conditions at outright failure.
The results obtained by Regan (1975) showed a significant difference between the performances of the specimens (Figure 3-15). Specimens with plane-round mild reinforcement were found to sustain significant ultimate deflections. The specimens with deformed hot rolled reinforcement were found to sustain smaller ultimate displacements but demonstrated better efficiency in developing in-plane membrane force. However, a number of specimens were found to collapse without developing any appreciable TMA response. The author therefore demonstrated that ultimate catenary performance was influenced by the area, tension properties and arrangement of reinforcement, as well as the properties of the concrete-reinforcement bond. In that, these factors were found to have a significant influence on the ultimate membrane force, ultimate rotation capacity at hinge locations and the ability of the catenary to sustain elongation without loss of in-plane strength or failure. Sections 3.5.3 and 3.5.3 explore these aspects in more detail.

The most interesting aspect of Regan’s research however is that failure of the extreme tension reinforcement layer was not found to result in collapse of the catenaries. Rather, the majority of test specimens were capable of sustaining catenary response and maximum load resistance even when in a state of extreme distress. This is demonstrated by the load-displacement records provided in Figure 3-15, which show multiple discontinuities attributed to bar fractures, anchorage slip and decomposition of the concrete section. Regan goes on to explain that, due to the number of potential failure mechanisms, the biggest challenge associated with catenary response is the accurate prediction of the failure mode that results in collapse.
Yu and Tan (2013a and 2013b) presented an experimental investigation of eight double-bay RC beam-column sub-assemblage specimens, at ½ scale. The authors used a surrogate five-storey RC framed building as the basis of their study. The building was designed and detailed in full-scale to ACI 318-05, assuming design loads typical of commercial buildings and a 6m column spacing. Two conditions were assumed; standard non-seismic ACI 318-05 detailing and seismic detailing, consistent with a base-shear coefficient of 0.034. The geometry of the specimens (S1-S8) was scaled linearly from the full-scale design, to provide a 150x250mm deep section with a typical clear span ($L_{ALS}$) of 5.75m at ½ scale. The reinforcement detailing of test specimens S1 and S2 were consistent with the ACI 318-05 design, for the seismic and non-seismic full-scale surrogates respectively – the area of each reinforcement allocation (top and bottom) was scaled to provide the same reinforcement ratio as found in the full-scale designs but for the ½ cross-section (Yu and Tan, 2013a). The sub-assemblage specimen is shown in Figure 3-16.

The reinforcement area, curtailment and span (S7 and S8, only) of the additional beam-column specimens S3-S8 were strategically changed to facilitate the investigation of span-depth ratio, detailing and reinforcement ratios on sub-assembly performance (Yu and Tan, 2013b). Thus, the programme provided experimental data for; span-depth ratios ($L_{ALS}/h$) of 23, 18.2 and 13.4 (by comparison of specimens S4, S7 and S8) and reinforcement ratios of 1.87-0.49%, in different arrangements. ASTM
Grade 60 longitudinal reinforcement and a concrete of compressive and tensile strength of 38.2MPa and 3.5MPa, was used throughout.

The test rig used by Yu and Tan (2013a and 2013b) is shown below (Figure 3-17). The test specimens were subject to lateral restraint and a quasi-static load, applied to the middle joint, under displacement controlled conditions of 1mm/sec. The instrumentation allowed the authors to record the load-displacement histories of each test specimen as they were displaced. Characteristic CMA and TMA response was recorded as the mid-joint displacement was increased.

The authors found that TMA typically offered the greatest load resistance. Figure 3-18 shows the load-displacement histories recorded for specimens S4 and S6. The discontinuities in the load-displacement histories, marked with crosses, denote the failure of the extreme tension reinforcement layer at the mid-span joint, either by fracture or pull-out. This was reliably found to occur in close
proximity to the column joint interface. However, despite fracture of the bottom reinforcement to either side of the middle column joint it can be seen that the load resistance of the catenary recovers; both S4 and S6 demonstrate maximum resistance in secondary TMA response.

![Figure 3-18](image)

Figure 3-18 – Load-displacement histories recorded by Yu and Tan (2013b) for specimens S4 and S6.

This trend is generally supported by the remaining test results. However, Figure 3-19 shows additional results obtained for specimens S3, S5, S7 and S8. It should be noted that the test specimens were denoted by name and a suffix defining the reinforcement ratio of the top and bottom allocations at beam-column joint locations and the span-depth ratio, respectively; S1-0.90/0.49/23S, for example. Thus, it can be seen that the specimen with the smallest \( \frac{L_{ALS}}{h} \), specimen S8, was found to exhibit a lower resistance in secondary TMA response than primary response. Given that \( \frac{L_{ALS}}{h} \) is the only attribute defining S8 from counterparts S4 and S7, this suggested that span-thickness ratio may influence ultimate collapse resistance in TMA response. However, it also suggests that the ultimate collapse load cannot reliably form after failure of the primary TMA mechanism.

![Figure 3-19](image)

Figure 3-19 – Load-displacement histories recorded by Yu and Tan (2013b) for specimens S3, S4, S5, S7 and S8.

All specimens (S1-S8) were displaced until the top reinforcement at the exterior support joints failed, either by fracture or pull-out. Therefore the load-displacement records shown above do not show the
potential of the specimens under the mechanism described by Woodson (1994), whereby the catenary is sustained by only the top reinforcement layer at mid-span and bottom reinforcement layer at interior support locations. Furthermore, unlike the tests documented by Regan (1975), the results are not necessarily representative of outright collapse conditions.

Gouverneur et al. (2013a and 2013b) provided one of the most recent experimental investigations of catenary action in RC slab assemblies under support loss conditions and offers the only recent insight of TMA behaviour at collapse. The authors reported the testing of two large-scale one-way slab specimens subject to inbound lateral restraint; slabs 1 and 2. The specimens were subject to removal of a central support and subsequent displacement-controlled load, applied at ¼ and ¾ positions along the double span ($L_{ALS}$). Displacement was increased until the specimens collapsed. The test rig is shown below (Figure 3-20), featuring slab 2 in a failed condition.

Both slab specimens were 160mm deep, with a consistent span-thickness ratios of 50 across the full 8m emergency span. However, Slab 1 was symmetrically reinforced, with 0.52% reinforcement ratio top and bottom, provided by S500 reinforcement strands that were continuous for the full 14.4m specimen length. Whereas, Slab 2 was detailed in accordance with EC2 guidance, with the top reinforcement curtailed at a distance of 0.15$L$ from interior supports. The specimen was doubly but asymmetrically reinforced, with reinforcement ratios at support and mid-span locations of 0.52% and 0.26%, top and bottom respectively. The area of reinforcement ratios at load points were inverted to give 0.26% top and 0.52% bottom reinforcement.

Figure 3-20 – Extract from Gouverneur (2013a) showing test arrangement and failure of test specimen ‘Slab 2’.

Figure 3-21 is an extract from Gouverneur et al. (2013a) that shows a schematic of the four-point test arrangement and the load-deflection history recorded for specimens, ‘Slab 1’ and ‘Slab 2’. Catenary action development is observed at displacements of approximately 1.6$h$. The bilinear load-displacement behaviour at prior displacements is attributed to elastic and plastic bending response. No CMA or arching action is observed as the slabs were not restrained against out-bound translation. The
discontinuity annotated ‘rupture of the top reinforcement bars’ marks the failure of the extreme tension reinforcement at one of the interior support hinge locations and denotes ultimate primary TMA response and the beginning of secondary TMA response.

Inspection of the two specimens shows a high degree of consistency between the response of specimens 1 and 2 during primary TMA response. First failure of the top reinforcement layer at either support was also found to occur at similar chord rotations of 6.3 and 6.6°, for Slab 1 and 2 respectively. However, the behaviour of the two specimens can be seen to diverge in secondary TMA response. The load resistance of Slab 1 recovers rapidly to a new maximum load, at which point the top reinforcement layer at the opposing support was found fail. Sustained by tension in the remaining bottom reinforcement at both supports, the catenary recovers to a maximum ultimate secondary resistance and collapse load at a displacement of $0.08L_{ALS}$ (9.2°). Conversely, maximum load resistance of Slab 2 corresponds with primary response. It can be seen that the specimen sustains a substantial drop in load and whilst load resistance is shown to increase beyond the flexural strength of the system, collapse occurs at a displacement of $0.07L_{ALS}$ (7.4°) before failure of the extreme tension reinforcement at the opposite support could be achieved. The load resistance at this point was approximately 50% maximum load resistance recorded in Slab 1, demonstrating resistance was directly proportional to the area of critical reinforcement (the remaining bottom layer of reinforcement at the interior supports). In both cases collapse was the result of tension failure in the critical reinforcement.

From the test results documented by Regan (1975), Yu and Tan (2013a and 2013b) and Gouverneur (2013a and 2013b) it is evident that the collapse and maximum load resistance of double span RC elements is frequently achieved during secondary TMA response. However, two cases have been identified whereby maximum load resistance was achieved during primary TMA response. This suggests that predictions of emergency load redistribution by TMA response requires consideration of both phases. Thus, factors affecting the response of both are investigated in the following sections.
3.5.2 Area of reinforcement & maximum tension membrane force

The investigation provided in Appendix A and Section 3.4 shows that the quantification of membrane force at ultimate load differs between guidance documents. Moreover, the experimental investigations by Powell (1956), Wood (1961), Park (1964b), Keenan (1969), Black (1975), Regan (1975), Woodson and Garner (1985), Guice (1986) and Woodson (1994) constitute the basis of the current guidance. None of these tests were instrumented to record reinforcement strain data or lateral restraint force \( R_H \) that could be used to validate or confirm the recommendations. Rather, conclusions drawn regarding in-plane membrane force were based on retrospective equilibrium analyses, using the approach developed by Park (1964b), Equation 3-3. This dictates that the interpretation of ultimate membrane force was vulnerable to inaccuracies in the analytical approach and constitutes a significant shortcoming.

Yu and Tan (2013a and 2013b), Gouverneur et al. (2013a), Lew et al. (2011), Sadek et al. (2011), Su et al. (2009) and Yi et al. (2008) each employ strain gauges to investigate stress in the reinforcement and concrete during the testing of large-scale planar RC beam and slab assemblies, acting across double spans. Some of these investigations have reported useful data, demonstrating almost uniform compressive and tensile strain in intermediate regions of the top and bottom reinforcement layers, during arching and catenary action (Figure 3-22; Lew et al. 2011). However, all authors document difficulty in monitoring strain at critical sections and joint locations due to the large in-elastic deformations in these areas. Strain gauges used by Lew et al. were found to fail or provide unreliable data before significant catenary displacements could develop. Alternative references providing reliable strain gauge data acquired at the critical sections are extremely limited and generally unable to support interpretation of the forces sustained by the top and bottom reinforcement at support and mid-span joints.

![Figure 3-22 – Longitudinal reinforcement strain recorded at ¼ span from supports (extract from Lew et al. 2011).](image)

Gouverneur et al. (2013a and 2013b) and Yu and Tan (2013a and 2013b) are the only large-scale experimental investigations that provide lateral restraint force data for primary and secondary TMA response.
Figure 3-23 is an extract from Gouverneur et al. (2013a) that shows the load-deflection and membrane-force history recorded for specimens, ‘Slab 1’ and ‘Slab 2’. Despite Slab 1 having a total reinforcement area 33.3% larger than Slab 2, the load and membrane force associated with ultimate primary TMA response are consistent for the two specimens. This demonstrates that the recommendation employed in the assessment of ultimate primary TMA response, of assuming a membrane force equal to the yield strength of the total area of longitudinal reinforcement [Park and Gamble (2000), GSA (2003) and DoD (2005)], is inaccurate and potentially overestimates ultimate membrane force. Rather, by inspection of the membrane-force history, it can be seen that the membrane force at failure of the extreme tension reinforcement was consistent for both specimens and corresponds approximately with the tensile strength of the top reinforcement layer at support locations only – by scaling the available graphs, approximately 3% greater than ultimate tensile strength in Slab 1 and 2% less than yield strength in Slab 2. This therefore suggests that the tensile strength of the top reinforcement layer at the interior supports was a more suitable criterion for the calculation of ultimate membrane force in primary response.

![Graph](image)

Figure 3-23 – Load-displacement and membrane force histories recorded by Gouverneur et al. (2013a) in double-span slab strip specimens featuring continuous (Slab 1) and curtailed (Slab 2) reinforcement arrangements.

By scaling from Figure 3-23, the ultimate membrane force in secondary response corresponds approximately to 97% the tensile strength of the critical reinforcement layer in Slab 1 and 101% the tensile strength of the critical reinforcement layer in Slab 2. In both cases the critical reinforcement layer can be identified as the bottom reinforcement at support locations, for which Slab 2 featured 50% less reinforcement. Woodson (1994) and DoD (1998) provide the only guidance available for the quantification of ultimate membrane force in secondary TMA response and suggest using the yield or tensile strength of 50% the total reinforcement area. This therefore would have been adequate for Slab 1 but over-predicted membrane force in Slab 2 by up to 50%.

An interesting aspect of the analytical investigation presented by Merola (2009, see Section 3.5.4.2) was the influence of reinforcement placement on emergency load redistribution by TMA response. The author predicted that conventional double-span RC beams were typically unable to sustain emergency load without some reinforcement failure. As a result, Merola recommended that multi-
storey RC frames be designed such that the tie-force requirements be satisfied by the area of bottom reinforcement layer at joint locations alone. This is of significance as BS 8110 and Eurocode 2 allow the minimum tie force requirement to be satisfied by the total area of reinforcement at joint locations. However, Merola found that the increase in area of bottom reinforcement provided a significant improvement in ultimate collapse resistance, particularly when tie requirements of a 10 and 15 storey building were implemented. This study has not been verified by testing but the results obtained by Gouverneur et al. (2013a) corroborate the importance of the bottom reinforcement area to ultimate secondary TMA response.

Yu and Tan (2013b) also provide recorded horizontal restraint force data (see Section 3.5.1). Figure 3-24 presents the load and restraint force histories recorded for specimens S3, S4 and S5. The yield and ultimate tension strength of the top reinforcement is indicated. The authors suggested that ultimate TMA resistance can be conservatively predicted by assuming a membrane force equal to the yield strength of the top reinforcement layer, at support locations. It can be seen that this provides a conservative estimate of the ultimate membrane force in secondary TMA response of these specimens. However, this contradicts the result provided for Slab 2 by Gouverneur et al. (2013a). Moreover, Figure 3-25 shows results recorded for the additional specimens S7 and S8, which featured the same area of reinforcement as S4. It can be seen that the ultimate membrane force recorded in S8 does not follow this trend. Yu and Tan (2013b) attribute this to a premature shear failure at middle beam-column joint interface.

Unlike the behaviour observed by Gouverneur et al. (2013a), Figure 3-24 and Figure 3-25 also demonstrate that the membrane force at first fracture of the mid-span bottom reinforcement layer to be inconsistent. Moreover, it can be seen that this occurred in S3 before tension membrane force could develop. This therefore suggests that primary TMA response may not develop and collapse resistance can be dependent on the secondary catenary mechanism formed, after reinforcement failure.

![Figure 3-24 – Load and horizontal reaction force histories recorded by Yu and Tan (2013b).](image-url)
3.5.3 Reinforcement detailing & arrangement

The catenary testing reported by Regan (1975) demonstrated significantly different performance between test specimens (as shown by Figure 3-15). Regan suggested that the tension and bond strength properties of the different reinforcement types were the most important factors influencing TMA response (Section 3.5.1). However, even specimens with identical reinforcement types were found to exhibit different behaviours, modes of failure and performance, which made the prediction of ultimate displacement and failure impossible. The author suggested that this was related to the reinforcement arrangement and distribution of rebar.

Figure 3-26 is an extract from Regan (1975) that shows the reinforcement arrangement used in all test specimens. Key discontinuities in the longitudinal reinforcement and inflection points are denoted by points A through G, for reference. Regan (1975) observed the development of three different catenary mechanisms that could be related to weighting of top and bottom reinforcement areas, arrangement and the curtailment of the top reinforcement layer. The modes of failure and performance were found to differ with each catenary mechanism. Figure 3-26 documents the chronology of response observed in Test Specimen 1. This constitutes one of the three mechanisms observed in testing and was found to be typical for specimens with reinforcement arrangements of, $A_{S,top} \geq A_{S,bottom}$.

The main feature of this mechanism was that rotational hinges and traverse cracking were found to form at curtailment points (B and F), during CMA response. Thereafter the catenary effectively hung from cantilevers projecting from the supports (AB and FG). With increased displacement the cover at the soffit of the supporting cantilevers began to spall and the bottom reinforcement was torn away.
This tearing of the bottom reinforcement was found to propagate towards the supports (A and G) until the anchorage of the bottom reinforcement failed, resulting in collapse. The cantilevers were found to sustain the catenary without yielding in flexure. The bottom reinforcement of catenary BF was found to yield and maintain a high tension force as tearing of the cover continued. This form of response was referred to by Regan as an ‘incomplete catenary’.

![Figure 3-26 – Extract from Regan (1975) showing reinforcement arrangement and modes of response observed in double-span test specimen.](image)

Specimens featuring mild steel or arrangements where the top and bottom reinforcement layers were of equal area, $A_{s,\text{top}} = A_{s,\text{bottom}}$, were found to develop an alternative catenary mechanism. In this case response was as shown in Figure 3-26 but cantilevers AB and FG were found to undergo increasing deflection as the catenary was loaded. Plastic hinge development was observed at the supports A and G, at which point the system formed a complete catenary (AG), across the full double span. Furthermore, traverse tension cracks were found to develop along the specimen as displacement increased. This happened first along the singly reinforced lengths (BC and EF) and then in the doubly reinforced regions, after which the deflection profile tended from a bilinear 3-pinned mechanism (shown above, Figure 3-26) to that of a gradual curve. These characteristics indicate inelastic tensile response long the full catenary span (AG) and were features not observed in the incomplete catenary mechanism. Collapse was found to occur by tension failure in the bottom reinforcement near interior supports, A or G. No spalling was noted in the soffit cover.

The third mechanism was generally found to occur in specimens featuring a $A_{s,\text{top}} \approx 0.5A_{s,\text{bottom}}$ reinforcement arrangement or the use of mild-steel bar. In this case the catenary was found to immediately form across the full double span, to provide a complete catenary. Traverse tensile cracks
were found to develop frequently along the specimen length, demonstrating inelastic tensile response. In this case, the catenary was found to rapidly adopt a curved profile and sustain load through substantial displacements. Failure was found to occur by bond failure, following disintegration of the concrete section at points of inflection (particularly for smooth mild steel bars), or reinforcement rupture.

It is an important note that rupture of the extreme tension reinforcement, at points A, G or D, was frequently recorded in specimens that sustained complete catenary mechanisms. However, despite an immediate drop in load resistance, the each specimen was found to sustain substantial further displacement and maximum load resistance in secondary catenary response.

Given that the area of reinforcement was varied between specimens, comparison of the performance of each mechanism was made by inspection of the ultimate secondary displacement, at incipient collapse and max load, for specimens detailed with the same type of rebar. Inspection shows that specimens that formed complete catenaries were found to sustain ultimate displacements of $0.10L_{ALS}$ and $0.20L_{ALS}$, whereby failure occurred by reinforcement rupture or de-bonding. Moreover, a number of specimens were found to sustain displacements exceeding the limit of the test rig, without collapse. By comparison, specimens that formed incomplete catenaries were found to collapse at significantly reduced displacements, between $0.05L_{ALS}$ and $0.07L_{ALS}$. Regan (1975) goes as far as to describe this as a semi-brittle form of response. Furthermore, whilst yield was noted by Regan in the longitudinal bottom reinforcement of specimens sustaining incomplete catenary response, tensile membrane force was considered inferior to that in the complete catenary mechanisms. This is evident from the absence of traverse tensile cracking along the specimens and tendency to failure by dowel action rather than rupture of the reinforcement. Thus, any potential gain in tensile membrane force attributed to strain hardening was lost and any ductility gains afforded by in-elastic extension along the specimen was also undermined. Regan concludes that ‘incomplete catenary’ response and subsequent dowel action should be avoided where collapse resistance is to be maximised.

Regan remarks that insufficient data was available to support accurate prediction of ultimate catenary response or failure modes. However, the results do suggest that catenary response is sensitive to reinforcement detailing and that slabs or beams featuring significant discontinuities of rotational stiffness along their length, the system may be prone inferior performance by developing as an ‘incomplete catenary’. Conversely, continuity of reinforcement area for the length of the member and a bias of reinforcement area in the bottom layer may be favourable as it promotes development of the catenary across the full clear span, which is shown to optimise the development of tensile membrane forces and ductility. This suggests that the conventional design practice of using moment redistribution, to reduce of the area hogging reinforcement at interior supports, may aid performance in catenary action. However, it should be noted that the curtailment of top reinforcement to form lengths of singly reinforced concrete was a significant feature of the testing reported by Regan and there are no other large data sets that allow comparison.
Despite the observations made by Regan (1975), the current understanding and approach to TMA analysis suggests that the arrangement of reinforcement and discontinuities in detailing associated with lapping, curtailment and connections do not influence performance, in either primary or secondary response. This is exemplified by the current analytical assumption that a catenary forms as a plastic membrane, typically assumed to be of parabolic profile, thus neglecting all consideration of flexural response prior to and during TMA. The recent studies by Yu and Tan (2014), Merola (2009) and Gouverneur et al. (2013a and 2013b) demonstrate the importance of rotational ductility at critical sections and joint locations. However, these investigations provide no indication that the catenary mechanisms formed in emergency load distribution or ultimate collapse resistance was influenced by reinforcement arrangement. Moreover, Gouverneur et al. (2013a) and Yu and Tan (2014) investigate the influence of reinforcement curtailment on catenary behaviour and suggested that this was of nominal effect to overall performance.

The catenary tests reported by Regan (1975) point to anchorage pull-out is a significant mode of failure – ten of the nineteen test specimens were found to fail by pull-out at hinge and joint locations or by dowel action. The recent testing conducted by Kang and Tan (2015) provides further evidence of the potential for anchorage failure.

Figure 3-27 shows the detailing of the precast concrete beam-column sub-assemblages tested by Kang and Tan (2015). Five specimens featuring bent-up bar and tension lap configurations at the beam-column joints were tested, under laterally restrained conditions, by application of a displacement controlled load to the central column stub. The tests were not advanced beyond primary failure, therefore the failure mode at collapse is unknown. However, the authors found four of the specimens to fail by pull-out in the extreme tension reinforcement at support or mid-span locations. The relative performance of the bent-up bar and tension lap arrangements is not clear as pull-out failure was observed in both. However, this suggested that conventional lapping requirements at connections and
curtailment locations may influence catenary performance by undermining ultimate tension membrane force. Furthermore, these observations support the use of enlarged tension laps and containment reinforcement, as stipulated by DoD (1990; 2002; 2008; 2014, see Appendix A), and raise significant concerns regarding the use of 'compression laps' at inner support locations, which are typically shorter than used in tension lapping.

Regan (1975) points out that tensile anchorage strength should not be the sole consideration for this type of failure. The author documents a series of anchorage tests conducted with deformed reinforcement. These tests demonstrated that conventional design anchorage lengths were adequate in developing the full tensile strength of the rebar under normal tensile conditions. Given the evidence of pull-out failures, obtained in catenary testing, this demonstrated that pure tension testing was not representative of the condition of tension laps and anchorage strength in catenary response. Rather, Regan (1975) points out that significant rotational demand at hinge and lapping locations was found to result in the degradation of the concrete section and reduced anchorage strength. Observations made by Mitchell and Cook (1984) support this finding. The authors go as far as to recommend anchoring critical reinforcement through the supporting columns and into the adjacent spans thereby lapping in regions where rotation demand is minimal – away from critical sections and potential hinge locations.

The use of shear links and stirrups is another aspect of detailing that has seen some focus in catenary research. From the available sources, the use of shear reinforcement has been associated with anchorage strength and catenary performance in secondary response. Figure 3-28 details a series of direct tension tests reported by Regan (1975).

![Figure 3-28](image)

Figure 3-28 – Direct tension tests documented by Regan (1975), featuring discontinuous critical reinforcement.
By applying direct tension to the discontinuous top reinforcement the eccentricity of internal tension forces and resultant moment was found to result in hinge development at the mid-section. The individual sections were found to rotate about the hinge point to attain equilibrium (as shown by Figure 3-28 b and c). Failure of the mechanism was found to occur by ‘unzipping’, as the laps and anchorage of stirrups would fail. The tension stress recorded in the top reinforcement layer was found to increase incrementally between stirrup failures but not reach yield. Regan suggested that the ultimate pull-out force could be adequately calculated by assuming typical dowel action theory. However, this demonstrates that the ultimate membrane force in secondary TMA response or in the event of an incomplete catenary was directly proportional to the intensity of shear or containment reinforcement used.

Woodson and Garner (1985) and Woodson (1994) provide an experimental investigation of the impact of different shear reinforcement configurations on TMA response in small-scale slab specimens. It was found that higher intensities of containment reinforcement would reduce the degradation of the concrete at large displacements and aid continuity of tension membrane force between the top and bottom longitudinal reinforcement allocations. However, the authors report only minor improvement of TMA performance with the presence of stirrups. The use of intense spiral lacing, considered uneconomical in conventional design, was found to improve performance. However, this was suspected to be due to the lacing, which was continuous for the span of the specimens, effectively increasing the area of longitudinal tension reinforcement.

3.5.4 Rotation capacity, extensibility & predicting failure

Regan (1975) used the term ‘extensibility’ to express the capacity of RC elements to sustain the change in length required to achieve large displacements. The author remarked that rotational ductility at joint locations and longitudinal extensibility are the most important factors influencing catenary performance as they govern the ability of a RC floor system to sustain large in-elastic displacements without loss of in-plane strength – a conclusion that is generally supported by all TMA research and the current analytical approaches to TMA prediction (see Section 3.3).

The span-thickness ratio of conventional RC beams and slabs in a double-span condition \( (L_{ALS}/h) \) probably fall within the approximate range of \( 44 \leq L_{ALS}/h \leq 80 \) for slab elements and \( 20 \leq L_{ALS}/h \leq 30 \) for beams (Cobb, 2004). As seen in Appendix A and Section 3.4, for elements of \( L_{ALS}/h > 5 \), design guidance currently suggests that the rotation capacity of conventional RC elements can support primary TMA response to end-rotations of approximately \( 12^\circ \) \((0.10L_{ALS})\) before incipient failure (Park and Gamble, 2000; GSA, 2003; DoD, 2005 and 2014). A characteristic end-rotation limit of \( 20^\circ \) \((0.18L_{ALS})\) is recommended for ultimate secondary TMA response and incipient collapse (DoD, 1998; USACE, 2008a). However, these ultimate displacement limits have been traced to the early TMA testing of small-scale test specimens subject to overload (see Appendix A). Large-scale experimental investigations reported by Regan (1975), Su et al. (2008), Yi et al. (2008), Gouverneur et al. (2013a
and 2013b) and Yu and Tan (2013a and 2013b), each concerned with the testing of double-span beam-column or slab-slab sub-assemblies, demonstrate a significant difference between the ultimate chord rotation achieved by test specimens. Moreover, inspection shows that the test specimens were typically found to fail before reaching the recommended ultimate end-rotation limits.

An investigation of recent experimental and analytical studies was undertaken to establish factors influencing ultimate joint rotation capacity and extensibility in RC elements that sustain emergency load redistribution by TMA response.

### 3.5.4.1 Experimental data

Given its relevance to in-elastic and nonlinear analytical procedures (FEMA, 1997 and 2000), rotational ductility of RC joints has been a subject of much research (Bachmann, 1970; Beeby, 1997; Beeby and Fathibitaraf, 2001; Panagiotakos and Fardis, 2001; etc.). However, investigations rarely account for the combined effects of bending and axial tension force observed in membrane response and joint rotation capacity is defined as the rotation at maximum moment resistance (Merola, 2009). Rotation capacity is defined here by the chord-rotation \( \theta = \tan^{-1} \Delta / L \) at maximum TMA load resistance, prior to rupture of the extreme tension reinforcement – therefore ultimate primary TMA end-rotation. However, Merola (2009) points out that experimental data recorded under these conditions are limited and often provide insufficient information to support analytical interpretation.

Figure 3-29 is a plot showing the joint rotation capacity recorded in RC catenary testing at maximum applied load, prior to or at rupture of the extreme tension reinforcement at critical sections (mid-span or interior support locations). The graph consolidates results reported by early TMA research (Powell, 1956; Park, 1964b; Keenan, 1969; Black, 1975), which form the basis of the current empirical end-rotation limits for primary TMA response, and those documented in recent experimental investigations of RC beam and slab specimens, under support loss conditions (Vecchio and Tang, 1990; Su et al. 2008; Yi et al. 2008; Sadek et al. 2011; Lew et al. 2011; Gouverneur et al. 2013a and 2013b; Yu and Tan, 2013a and 2013b). Results obtained under overload conditions are denoted by triangular data points and give an average ultimate chord rotation of \( 14.6^\circ (0.13L_{ALS}) \). Park (1964b) claimed that yield could reliably be assumed in the extreme tension reinforcement of slab elements at a chord rotation of \( 2.3^\circ (0.02L_{ALS}) \). Using this as a rule of thumb, this suggests a high level of flexural ductility in early testing; the average ultimate displacement being 6.3 times yield displacement, without loss of in-plane strength. Comparatively, results documented in recent TMA research show modern RC double-span specimens to be of inferior ductility; system ductility of between 1.2 and 4.9 times the assumed yield displacement suggested by Park, with an average ultimate chord rotation of \( 7.9^\circ (0.07L_{ALS}) \), a standard deviation of \( 2.0^\circ \). This result suggests that TMA research conducted for overload conditions is not representative of performance in support-loss conditions. Moreover, the current ultimate characteristic limit of \( 11.3^\circ \), recommended by Park and Gamble (2000), and \( 12^\circ \), advocated by progressive collapse guidance GSA (2003) and DoD (2010) and the existing tie-force
requirements, over-estimate the ultimate displacement of primary catenary mechanisms engaged in emergency load redistribution.

![Figure 3-29](image-url) – Ultimate displacement data, recorded at maximum primary load resistance, obtained from open-source TMA and Catenary Action experimental investigations.

The testing documented by Regan (1975), Vecchio and Tang (1990), Su et al. (2009) and Yu and Tan (2013b) raise additional concerns. These authors document reinforcement failure at relatively minor displacements in their respective investigations, some prior to TMA transition. Vecchio and Tang (1990), document the testing of a planar flat slab system in double span condition, with span-thickness ratio ($L_{ALS}/h$) of 30.8. In this case rupture of the bottom reinforcement layer at mid-span was found to occur before catenary response could develop – at a displacement of $0.06L_{ALS}$ (1.8$h$) whilst membrane forces were still compressive. Examples of extreme tension reinforcement failure prior to the development of TMA response are also noted in experimental studies by Regan (1975). However, Regan points out that the two test specimens observed to sustain failure in CMA or snap-through response were detailed with high-yield twisted reinforcement that had brittle tensile characteristic properties.

Su et al. (2009) tested twelve ¼ scale beam-column sub-assemblage specimens and provides one of the largest experimental data sets of membrane response in modern conventional RC assemblies. The span-depth ratio, longitudinal reinforcement ratio and loading rate was varied. However, the investigation showed the plastic rotation capacity of the test population to be highly unpredictable and no trend was found between rotation capacity and the parameters varied in testing. Moreover, primary TMA response was consistently found to offer less resistance than CMA. As such, the authors went as far as to promote CMA as a favourable means of emergency load redistribution. That said, it should be noted that the investigation by Su et al. favours arching action as the beam specimens tested featured
exceptionally small span-thickness ratios in their emergency condition ($2L/h$ of 9.0, 13.5, 14.0 and 19.0) perhaps not representative of conventional constructions.

By inspection of Figure 3-29 it can be seen that the only recent test that demonstrates an ultimate rotation capacity of 11.3° is that reported by Sadek et al. (2011) and Lew et al. (2011). The authors documented the testing of two RC beam-column sub-assemblage specimens in double-span conditions ($L_{ALS} \approx 2L$). A ten storey surrogate building was used as the test case, with a column spacing of 6.1m. The specimens were designed as intermediate and special moment frames (IMFs and SMFs) to seismic categories C and D (ASCE 7-02 and ACI 318-02) and detailed with ASTM A706-Grade 60 reinforcement. Both tests were conducted at full-scale with an emergency span ($L_{ALS}$) of 11.5m, under monotonic vertical displacement of the mid-span joint at a rate of 25mm/min. As shown in Figure 3-30, the limits of the exterior support columns were restrained against lateral translation such that membrane response could be studied. TMA response was shown to provide an enhanced resistance from displacements as low as 0.8$h$ and exceed CMA by 82% in the IMF specimen and 37% in the SMF, pointing to TMA response as the ultimate collapse resistant mechanism. The authors indicate a 125% enhancement at ultimate primary TMA load.

Lew et al. (2011) and Sadek et al. (2011) reported that the tests were terminated at displacements of 0.09$L_{ALS}$ (10.1°) and 0.10$L_{ALS}$ (11.3°) for the IMF and SMF specimens, respectively. This corresponded with the fracture of the bottom reinforcement layers at the mid-span joint and thus ultimate displacement in primary response. However, the test program was not necessarily representative of conventional RC constructions. All reinforcement lapping was conducted mechanically, with use of threaded couplers located at the mid-service-span, to provide continuous reinforcement lengths across the double span. Moreover, the $L_{ALS}/h$ of the specimens were 24 and 18.5, for the IMF and SMF specimens respectively. This therefore demonstrates that the specimens tested by Lew et al. (2011) and Sadek et al. (2011) did not permit interpretation of potential anchorage
failure at lap locations and were of a relatively small span-thickness ratio compared with conventional RC constructions. Moreover, by detailing the specimens for seismic ductility requirements and using low-alloy, high ductility ASTM A706-Grade 60 reinforcement, the rotation capacities of the beam-column joint are likely to be superior to conventional arrangements – an argument also relevant to the test specimen detailed by Yi et al. (2008), see Section 3.5.5. Thus, the results cannot be considered representative of typical emergency response in conventional buildings but demonstrate improved performance over conventional counterparts detailed with no special consideration of seismic actions.

The influence of reinforcement type and ductility on catenary performance was investigated by Regan (1975). It is apparent that Regan’s study was more concerned with extensibility in catenary action, as the author found that this was a more representative characteristic of catenary response at large displacements and defined ultimate collapse load. However, Regan recognised the potential advantage of a high rotation capacity in maintaining structural integrity and maximum membrane force. The author went on to suggest the use of large bar sizes and non-deformed plain round mild steel bar to promote bond-slip and the distribution of plastic strain between crack locations. Regan based this recommendation on the observations made during catenary testing, whereby catenary specimens reinforced with plain round mild steel bar were found to exhibit superior ductility than specimens with deformed high-yield reinforcement (see Figure 3-15). Moreover, these results were supplemented by a series of thirty nine direct tension tests conducted on prismatic RC samples. Sixteen of these tests entailed the application of direct tension to 800mm gauge lengths of longitudinal reinforcement, encased in mass concrete. The reinforcement tested was of varied size and typical of the era; smooth round mild steel bar and 410MPa and 600MPa hot-rolled reinforcement deformed to British (‘type 1’) and Swedish (‘type 2’) specifications, respectively. By testing each sample to failure, Regan found that the ultimate elongation of each specimen was directly dependent upon the bond-strength and tension properties of the reinforcement. The author also cautioned that deformed reinforcements were prone to brittle fracture at locations of high rotational demand, due to the better bond and a tendency for plastic strain to localise at crack locations.

Regan’s findings are of importance given that, on inspection of the data provided by Powell (1956), Park (1964b), Keenan (1969), Black (1975), the test specimens were detailed with longitudinal reinforcement of yield strengths between 210 and 352MPa. This corresponds with mild steel reinforcements; plane round mild steel bar of 250MPa characteristic strength (BS 785: Part 1, 1967 or BS 4449: 1969, 1978 & 1988) or Grade 40 ASTM A615 reinforcing. Whereas, reinforcement used in modern RC construction and recent testing (Su et al. 2008; Yi et al. 2008; Gouverneur et al. 2013a and 2013b; Yu and Tan, 2013a and 2013b) is commonly hot-rolled deformed bar – B 500, in accordance with BS 4449:2005 and Eurocode 2 and Grade 40 or 60 ASTM A615 reinforcing. As an example, Malvar and Crawford (1998) reported the reinforcement properties of ASTM A615 Grade 40 and Grade 60 reinforcing steels. Grade 40 reinforcement was found to have a characteristic ultimate strain of 15.5% and a percentage elongation at fracture of 22.4%. Whereas, the tensile properties of
the Grade 60 was shown to have significant variance between bar sizes, with an ultimate strain of between 7 and 14% and percentage elongation at rupture of between 12 and 20%. This demonstrates a significant difference between the tension properties of the reinforcement used in early TMA research and those modern RC construction. Moreover, based on observations by Regan (1975), the use of plain round mild steel suggests that the tests conducted with this reinforcement would have supported larger ultimate displacements than deformed alternatives. Thus, this provides further evidence that the test results that form the basis of the 12° end-rotation limit used in current progressive collapse design guidance [GSA (2003); DoD (2010); Park and Gamble, 2000] cannot be considered representative of modern RC constructions.

The eight specimen test series presented by Yu and Tan (2013b) used ASTM Grade 60 longitudinal reinforcement and a concrete of 38.2MPa compressive and 3.5MPa tensile strength. This provides an opportunity to examine the rotation capacity of beam-column specimens reinforced with the same reinforcement type and tested under identical conditions (Section 3.5.1). By comparison of results obtained for specimens of different span-depth ratio and reinforcement arrangement, the authors suggested that rotation capacity was influenced both factors. The displacement at which reinforcement failure occurred increased as the depth of the assembly was reduced. By inspection of the relative performance of the exterior and mid-span joints, Yu and Tan reported higher rotation capacities at the mid-span joint. The authors attributed this to the relative weighting of the top and bottom reinforcement layers, suggesting that joint rotation was less dominant in joints featuring a larger area of tension reinforcement than compression reinforcement. These two findings suggest that joint rotation capacity may be dependent on the lever-arm of the top and bottom reinforcement layers, the span and the relative strength of each rebar allocation. However, the authors offer no direct interpretation of these relationships.

It is significant to note that, despite the importance Yu and Tan (2014) placed on span-thickness ratio, Figure 3-29 identifies that there is no experimental data available for specimens of $2L/h > 33$, other than the three data points acquired by Gouverneur et al. (2013a and 2013b). This represents a potential shortcoming of the existing data stock given that span-thickness ratios have been estimated to fall within the approximately range of $44 \leq L_{ALS}/h \leq 80$ for slab elements and $20 \leq L_{ALS}/h \leq 30$ for beams (Cobb, 2004). Moreover, the observations made by Regan (1975) and Yu and Tan (2013b) suggest that representative characteristic ultimate displacement criteria for primary TMA response may require the specification of reinforcement type, reinforcement arrangement and span-depth ratio rather than the current approach that does not differentiate between RC elements for which these properties differ.

Test data for the ultimate displacement of double span RC elements in secondary TMA response, at incipient outright collapse, are much more limited. Regan (1975) provides the largest data set. Eleven specimens were tested using deformed high-yield longitudinal reinforcement, in the main spans and across slab-slab connections. The author reports ultimate chord rotations of between 6.4° and 12.7°,
giving an average ultimate rotation of 9.4°. Gouverneur et al. (2013a) documents collapse at rotations of 9.6 and 7.4°, for slabs 1 and 2 respectively (see Section 3.5.1). The two results obtained by Gouverneur et al. therefore demonstrate good agreement with the displacements reported by Regan at incipient collapse. However, Yu and Tan (2013a and 2013b) demonstrate that eight beam-column sub-assemblages tested achieved an average 13.5° chord rotation, without collapse. These results demonstrate the 20° criteria recommended by DoD (1998) for ultimate secondary TMA response and collapse has not been achieved in recent progressive collapse testing.

Regan (1975) is the only reference to provide a direct investigation of ultimate displacement at incipient collapse (Section 3.5.1). The primary focus of the study was concerned with identifying factors that governed the different performance observed between catenary specimens. Reinforcement type was found to be the most significant factor influencing ultimate displacement. Test specimens featuring plain round mild steel reinforcement at joint locations demonstrated a higher level of performance, with an average chord rotation of 15.9° at maximum load resistance. This demonstrates a displacement capacity 69% higher than the specimens reinforced with high-yield deformed reinforcement. Two specimens reinforced with brittle twisted reinforcement were tested and found to fail at a negligible displacement and 0.6°. This led Regan to suggest that the ductility of the reinforcement and potential for bond-slip were principal factors in catenary performance. However, inspection of the results obtained by Regan (1975) shows a standard deviation of 2.2° and 5.6° for the high-yield and mild steel specimens, demonstrating different ultimate performance between specimens detailed with of the same reinforcement type. Regan proposed that this variance was attributed to the different in-plane tension properties, extensibility and failure modes observed in testing. However, the author showed that defining the extensibility properties of a RC floor system was complex and highly dependent on factors such as; the catenary mechanism in its ultimate condition, changes in the area of reinforcement and thus tension strength and extension properties along the length of the catenary, crack spacing along the element, the area of reinforcement available at critical sections and hinge locations and the mode of failure assumed by the catenary. Regan concluded that ‘much more research is necessary before realistic design recommendations can be drafted. Information is needed regarding bond/slip relationships at large displacements, limits of rotation governed by fracture of reinforcement and the overall behaviour of floor systems’.

3.5.4.2 Failure modelling & analytical investigations

Regan (1975) suggested that ultimate TMA performance could be adequately predicted by consideration of equilibrium and in-plane tensile properties of a catenary. The author demonstrated that terms for extension (δL), over the length of a single span (L), could be obtained for bilinear and parabolic catenary profiles by consideration of their geometry (see Figure 3-10). Equation 3-16 was derived for the bilinear profile:
\[
\frac{\delta L}{L} = \frac{1}{2} \left( \frac{\Delta}{L} \right)^2 \quad \text{or} \quad \frac{\Delta}{L} = \sqrt{2 \frac{\delta L}{L}}
\]

Equation 3-16

Regan suggested that the ultimate load and displacement of a catenary could be defined by its ultimate elongation (\(\delta L_u\)). Thus, by assuming the lateral restraint reaction (\(R_H\)) to be approximately equal to the membrane force (\(N\)), the author suggested the following term for the ultimate response of a bilinear catenary profile, by substitution of Equation 3-9 and Equation 3-16:

\[
q_u = \frac{2N}{L} \sqrt{2 \frac{\delta L_u}{L}}
\]

Equation 3-17

Where:

\[
\Delta = \frac{qL^2}{2N} \quad \text{for} \quad 0 < \Delta < L \sqrt{2 \frac{\delta L_u}{L}}
\]

Equation 3-18

The recent study by Tohidi et al. (2014a and 2014b) shows that this approach can be effective when \(\delta L_u\) and the mode of failure can be reliably identified. In this case the authors were concerned with hollow-core precast concrete floor slabs, acting in double span catenary action following support loss. The slab-slab connection at midspan and supports was facilitated by grouting and reinforcing the hollow-cores between adjoined slabs and, given the composition of the arrangement, the failure mechanism of the resultant catenary was determined to be by pull-out of the grouted cores. The authors used testing and FEA modelling, accounting for the tensile properties and anchorage of the tie bars and the friction resistance and bond along the grout-core interface, to determine characteristic stress-slip parameters for pull-out of the connection arrangement grouted cores. By assuming \(\delta L\) was facilitated at the connections only, the authors were able to implement the stress-slip function associated with failure and thereby define \(\delta L_u\) and the corresponding pull-out membrane force. This was found to provide reasonable approximations of ultimate catenary performance compared with experimental observations.

Merola (2009) proposed a nonlinear static model that would predict the large displacement behaviour and load redistribution in laterally restrained RC beam elements. Merola’s approach was to implement the macro-models presented by Park (1964b) and Park and Gamble (2000) in support of the static push-over analyses to predict CMA and TMA responses. The author developed load-displacement a function for one-way TMA response, consistent with Regan’s interpretation (Equation 3-9), by assuming the equilibrium of a parabolic catenary profile (Figure 3-10b). Notably, the author recognised the importance of predicting the rotation capacity of beam-column joints and fracture of the top and bottom reinforcement layers.
Rather than use empirical end-rotation limits to define failure, steel fracture criteria were integrated to the model to predict the displacement and applied load at reinforcement fracture during CMA and TMA responses. Failure of the extreme tension reinforcement, under combined bending and axial force, was defined by attainment of ultimate strain in the reinforcement across crack widths. Parameters for steel stress, crack spacing and bond stress were implemented in accordance with Marti et al. (1998), a bilinear distribution of reinforcement strain was assumed across crack elements and the plastic hinge length, for which cracking was spread, was taken to be the length of the beam for which the bending moment exceeded the cracking moment. Merola used the experimental test documented by Yi et al. (2008, see Section 3.5.5) to validate the proposed model. Fracture of the mid-span extreme tension reinforcement, which was found to occur in TMA response, was predicted at a displacement 12\% less than that observed in testing, showing the model to be effective but conservative in the estimation of ultimate displacement and load resistance. Merola (2009) implemented a similar approach for the prediction of ultimate displacement in catenary action, following primary reinforcement failure. The proposed model would equate the elongation of a parabolic catenary profile at a given displacement to define the resultant reinforcement strain across crack widths. Failure and ultimate response was then defined by attainment of fracture strain in the reinforcement.

Merola (2009) used the proposed model to investigate a series of prototype RC frames, designed in accordance with Eurocode 2 (BS EN 1992-1-1:2004). The frames were based on 5, 10 and 15 storey arrangements with column grid spacing of 5, 7 and 9m. The study entailed modelling a ground floor and top floor beam element from each prototype frame. Each was designed for the tie-force requirements corresponding to a building height and featured conventional detailing and curtailments. The beam elements of these frames had span-depth ratios (\(L_{ALS}/h\)) of between 14 and 25, following support loss. The response of each beam element was investigated for the three ductility classes; B 500A, B 500B and B 500C, corresponding to the specifications of BS 4449:2005 and BS EN 10080.

Merola (2009) found that beams designed for different grid dimensions behaved similarly. However, by adopting the properties of B 500A reinforcement (\(f_y\) of 500MPa, \(f_t/f_y\) of 1.05 and \(\varepsilon_u\) of 2.5\%), the author showed that the top and bottom reinforcement layers at support locations were predicted to fracture during CMA response. Thus, beam elements reinforced with B 500A steel could not develop primary or secondary catenary response, collapse occurring at relatively minor joint rotations. By assuming B 500B reinforcement (\(f_y\) of 500MPa, \(f_t/f_y\) of 1.08 and \(\varepsilon_u\) of 5.0\%), Merola predicted improved rotation capacities but failure was still found to occur during CMA response. The only documented case where the rotation capacity of a beam element was adequate to sustain primary TMA response was for beams reinforced with B 500C bar (\(f_y\) of 500MPa, \(f_t/f_y\) of between 1.15 and 1.30, and \(\varepsilon_u\) of 7.5\%). However, the results of Merola (2009) indicate that hinge rotation capacity was inadequate to sustain emergency load in primary TMA response and the collapse resistance of beams reinforced with B 500B and B 500C bar was found to be depend upon the area of reinforcement.
available in secondary TMA response, for which the additional strength and ductility of B 500C reinforcement was found to provide significant advantage.

Merola suggests that ultimate secondary TMA response may be adequate to facilitate emergency load redistribution and resist progressive collapse following single column loss, provided B 500B and B 500C reinforcement. However, it is important to note that these findings have not been experimentally verified. Inspection shows that a number of the beam elements reinforced with B 500B and B 500C bar were predicted to sustain deflections of 0.16\(L_{ALS}\) (17.3°) in secondary TMA response without failure of the reinforcement. This was a safety limit imposed by the author rather than an empirical ultimate limit displacement. Whilst this limit is less than the 20° end-rotation limit imposed by DoD (1998), inspection of the limited experimental data available shows that there is no evidence to suggest that catenaries in double-span conditions can sustain this deflection without outright collapse – Gouverneur (2013a and 2013b) show collapse is achieved at less than 10° for both specimens. This suggests that the model proposed by Merola may provide an unrealistic estimation of secondary TMA performance and ultimate response at incipient collapse. However, with such limited experimental data this cannot be confirmed.

The modelling approach taken by Merola (2009) assumed the strain at fracture of the reinforcement \((\varepsilon_f)\) was equal to strain at maximum tension force \((\varepsilon_u)\). The author suggested that this was due to insufficient material test data and acknowledged that this was a conservative approach. It is an interesting aspect of the study that by assuming a fracture strain of \(\varepsilon_f = \varepsilon_u + 0.1\), Merola found that the TMA performance of beams reinforced with B 500B and B 500C reinforcement improved significantly, whilst the B 500A continued to demonstrate brittle failure.

Studies by Yu and Tan (2013a, 2013b and 2014) also acknowledge the importance of accurately predicting failure of the top and bottom reinforcement allocations and demonstrate an effort to address the need for a more technical approach to the assessment of joint rotation capacity. The authors proposed the use of a component based FEA model for the prediction of joint rotation behaviour in RC beam-column sub-assemblages (Figure 3-31). The component model proposed by Yu and Tan (2013 and 2014) is a typical form of mechanical model that discretises the individual mechanisms associated with joint response into a set of isolated components. The components are discrete physical aspects of the beam-column joint that potentially dictate system response, deformation and failure. Each component is defined by nonlinear uniaxial properties, consistent with an equivalent spring resistance and stiffness. Component response is analysed individually for defined boundary conditions and external actions, to determine the global response of the joint. In this case, the model permits interpretation of rotational stiffness, strength and ductility at a global level as well as identification of failure modes at a component level – i.e. failure of the reinforcement, concrete or anchorage. The component joint model was integrated into a macro-FEA model at exterior and mid-joint support locations, as shown below (Figure 3-32).
The macro-FEA model presented was validated against the experimental results documented by Yu and Tan (2013a and 2013b, see Section 3.5.1) by modelling the four test specimens S3, S4, S5 and S6. Comparison with the load-displacement and membrane force histories recorded for the four specimens showed that predictions for specimen response were generally consistent with that seen in testing. Particular attention was paid to the prediction of joint rotation capacity for which reinforcement and anchorage failure was defined by the bar force-slip component. Yu and Tan (2014) suggested that the proposed model provided moderately conservative predictions of reinforcement failure, at a smaller displacements.

Yu and Tan (2014) implemented the established numerical model to investigate the rotation capacity of the beam-column sub-assembly, assuming ‘imperfect’ boundary conditions, reinforcement curtailment and different span-depth ratios, in turn. The format of test specimen S4 (Yu and Tan, 2013b) was retained as the basis of the parametric study. Analysis was described as nonlinear-static and conducted by displacement of the mid-span joint until the assembly failed. The authors found that by varying the cross-section of S4, the prediction of rotation capacity and global response was
significantly was affected. Arrangements of 6m clear span and $2L/h$ of 23, 29 and 38 were considered. By increasing the slenderness of the assembly, CMA response was reduced and TMA performance became more predominant. Furthermore, the model was successful in supporting the conclusions drawings with regards the influence of reinforcement arrangement on rotation capacity (see Section 3.5.4.1).

3.5.5 Lateral restraint stiffness

Regan (1975) pointed out that the importance of adequate lateral restraint is twofold. Firstly, load resistance and membrane force is dependent on the development of stress in the reinforcement of the catenary. Secondly, buckling and drag-down (see Chapter 2) in the surrounding structure needs to be prevented under the inbound lateral thrust associated with the catenary. This suggests that without adequate lateral restraint emergency load redistribution by catenary action may not be viable or could exasperate the spread of collapse. Dat and Hai (2011 and 2014) and Yi et al. (2014) suggested that this is especially pertinent to the case of column loss, where perimeter and penultimate bays possess limited lateral stiffness to act in restraint (see Figure 3-33).

![Figure 3-33 – Emergency load redistribution by catenary action in penultimate structural bay following perimeter column loss (Dat and Hai, 2011).](image)

Dat and Hai (2011) and Yi et al. (2014) suggested that in such situations, the slab diaphragm is critical to the development of adequate lateral restraint.

Figure 3-34 illustrates a ‘compression thrust ring’ that forms around the boundary of a tensile membrane, when developed within a slab diaphragm. This mechanism had been observed by Mitchell and Cook (1984), Desayi and Kulkarni (1977) and Park and Gamble (2000) and is documented as providing effective lateral restraint to two-way tension membranes, even when the RC slab was unrestrained at its edges. However, the development of this mechanism around asymmetrical tensile membranes or in a situation where the compression thrust ring cannot completely surround the membrane, as in the case of perimeter column loss, was uncertain. Moreover, authors such as Park and Gamble (2000) cautioned against the consideration of TMA in slab arrangements without ‘beams or other stiff supports that provide edge restraint’.
Dat and Hai (2011) undertook a series of ¼ scale tests and a supporting numerical study to investigate the response of slab diaphragms following the instantaneous loss of perimeter columns. The authors found compression thrust rings would develop for the situation of penultimate column loss (shown in Figure 3-33) but also in the situation of corner column loss. This mechanism was shown to provide adequate lateral restraint and support TMA response as the prominent mode of emergency load redistribution. Moreover, the authors found the resultant compression thrust rings were stable enough to sustain tension membranes for displacements of up to $0.20L_{ALS}$ ($21.8^\circ$), resulting in a load resistance of over twice the flexural capacity of the emergency span.

This type of behaviour was more recently reported by Yi et al. (2014) who documented the testing of the approximate ½ scale, 90mm thick flat-slab structure shown in Figure 3-35. The test structure was a 2x2 column grid system that was preloaded with approximately twice the design load of the slab and subject to the removal of internal, external and corner columns under quasi-static conditions. Flexural, CMA and TMA response were observed following column removal. The authors reported that

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**Figure 3-34** – Illustration of ‘compression thrust ring’ development in slab diaphragms (Dat and Hai, 2011).

**Figure 3-35** – Emergency load redistribution by TMA, observed in ½ scale flat-slab test specimen (Yi et al. 2014).
maximum load resistance was observed in TMA response, following rupture of a number of reinforcement layers.

Yi et al. (2014) demonstrate by means of the crack patterns recorded in the slab that effective lateral restraint was provided by the development of compression thrust rings. Moreover, the authors stated that the slab was essentially simply supported, as the columns provided nominal lateral restraint. Given that, following perimeter column removal, the system sustained 1.81 times the design load of the slab by TMA response, this result corroborates the findings of Dat and Hai (2011) and demonstrates that the effectiveness of slab diaphragms in providing effective lateral restraint and preventing drag-down.

The numerical analysis implemented by Dat and Hai (2011) was effective in modelling the development of compression thrust rings in slab diaphragms but was unable to identify the failure mode of the tensile membrane. The authors explained that the proposed FEA model was compromised by complex local response, such as concrete crushing at joint locations, fracture of the bottom reinforcement at large displacements and a convergence phenomenon between the hogging moments of the slab and the compression ring that would cause torsional failure in beam elements. Thus, the tensile membrane was modelled by assuming a perfectly plastic membrane and for which displacement was not advanced beyond 0.20LALS (21.8°). Whilst this is adequate for the investigation of lateral restraint demand on the adjacent structure and the integrity of compression thrust ring development, it provides no insight as to the influence of restraint stiffness on catenary performance. An example of this argument might be found in the experimental testing documented by Yi et al. (2008). The authors report the testing of the four-bay three-storey RC frame specimen shown in Figure 3-36. The test specimen was a 1/3 scale planar model of an eight-storey RC framed building, designed to the structural concrete design code of China, GB50010-2002. 25MPa structural concrete and ASTM A706-Grade60 deformed reinforcement (fy of 416MPa and fu of 526MPa) was used throughout. The structural bays consisted of service spans of 100x200mm deep beam elements and 200x200mm columns, arranged to provide a 2.67m column spacing and bay heights of 1.6m, 1.1m and 1.1m. The bottom reinforcement layer of the beams was lapped through each column joint and provided an approximate reinforcement ratio (ρ′) of 1.29%. The top reinforcement layer was of equal area (ρ of 1.29%) and appears to have been lapped within the beam span. A constant 109kN load was applied to the central column, representing the scaled column load associated with the gravity load of the eight storey surrogate. The test was conducted statically by ‘letting’ the resistance force of supportive jacks upon which the middle column was bearing. Thus, load redistribution was induced across the middle two bays, for which the span-thickness ratio (LALS/h) of the beams was 25.7. Lateral restraint of the emergency bays was reliant on the stiffness of the surrounding bays, formed by moment resistant framing.
Yi et al. (2008) state that the purpose of the experimental was to displace the middle column until failure was achieved or the emergency mechanism sustained the 190kN applied load with no assistance. Failure was defined as ‘the complete loss of load-carrying capability of a critical structural component due to material failure […] or uncontrollable progressive increase in the deformation’.

By recording the lateral displacement of the adjacent bays, the authors documented both outbound and inbound translation as the middle-column displaced. However, no CMA enhancement was observed. Rather, the large-displacement response was described as elastic and plastic frame action, followed by TMA response. The middle column was allowed to displace to 456mm (0.09$L_{ALS}$ or 10.1°, chord rotation) before the bottom reinforcement layer was found to fracture at the beam interface with the middle column. Due to the subsequent drop in applied load, the test was terminated. The applied load immediately prior to failure was approximately 108.4kN, allowing the authors to conclude that progressive collapse following column loss would have been avoided by TMA response.

This investigation is valuable in demonstrating that the restraint stiffness provided by the adjacent framing was inadequate to support CMA response but adequate to support emergency load redistribution by TMA response. The ultimate load capacity of the double bay mechanism, assuming plastic hinge formation in all vertical bays, was calculated at 73.2kN. Thus, the enhancement by primary catenary response was approximately 41%. Furthermore, despite the use of a high-alloy reinforcement, this demonstrates an ultimate chord rotation of 10.1°, which is again less than the recommendations made by Park and Gamble (2000), GSA (2003) and DoD (2005). However, an interesting aspect of the testing by Yi et al. (2008) was that reinforcement failure was only recorded in the first floor beam – the catenaries formed in the second and third floor beams did not fail. The authors attribute this to reduced membrane force in the second and third floor catenaries. Whilst the authors provide little explanation for this argument, this could suggest that the reduced restraint
stiffness of the upper stories resulted in some reduction of membrane force and thus resulting in a higher ultimate displacement. However, the sensitivity of catenary action to lateral restraint stiffness has more recently been investigated by Merola (2009) and Yu and Tan (2014) who offer alternative findings.

As part of the author’s frame study, Merola (2009) calculates the axial restraint stiffness afforded a floor beam in prototype buildings of different column grid spacing, number of storeys and number of abutting structural bays. The author predicted that resistant stiffness increased with the number of adjacent bays and the number of storeys; varying from 76N/m at first floor level of a conventional three bay, five storey RC framed structure to 623N/m for the same beam but located in an eight bay fifteen storey building. Moreover, Merola (2009) shows the axial restraint of beams at top floor level to be significantly less than at first floor level; varying from 13% in five storey configurations to 4% in fifteen storey arrangements. This difference in lateral restraint was found to have a significant influence on CMA resistance but the author concluded that there was nominal effect on TMA response. By means of a parametric study, Merola compared the TMA response predicted for a beam subject to infinite axial restraint against that of a nominally restrained arrangement. The author found a 1% difference in the displacement and resistance at ultimate TMA load.

This finding has more recently been confirmed by the numerical parametric study undertaken by Yu and Tan (2014) – see Section 3.5.4.2. The authors investigated the influence of ‘boundary imperfections’ by modelling specimen S4 for different rotational and lateral restraint conditions. However, by comparison with test data and results of the parametric study the authors concluded that these factors had a negligible effect on catenary behaviour or performance.

3.6 Summary

This literature investigation demonstrates that the prediction of ultimate collapse resistance in RC assemblies requires an account of catenary action. A number of analytical methods have been identified that are currently recommended for the prediction of TMA load-displacement response. However, this study shows that criteria currently used to define displacement and membrane force at ultimate load conflict with the test data obtained in large-scale progressive collapse testing. This suggests that further research is required in order to verify the behavioural response of RC beam and slab elements in emergency load redistribution.

By defining primary and secondary TMA response separately, this study has highlighted a significant lack of experimental and analytical investigation at incipient collapse. Moreover, the current understanding of secondary TMA response is limited to the behaviour observed in only two recent test specimens. The remaining experimental investigations can be seen to provide limited insight into
emergency response as specimens tend to be of relatively small span-thickness ratio and frequently feature high-ductility reinforcement not representative of conventional structures.

The findings of this chapter have been implemented to develop an experimental programme (Chapter 4) with the objective of investigating the ultimate collapse resistance and catenary action behaviour of RC floor elements of conventional UK design.
4 Experimental Study of Catenary Action in RC Framed Buildings

Regan (1975), Merola (2009), Yi et al. (2008) and Dat and Hai (2011) have stated that the capabilities and limitations of catenary action as a form of emergency load redistribution in RC constructions is not fully understood. These studies suggested that more experimental data is required before this form of behaviour and its failure modes can be accurately identified and predicted (see Chapter 3). Moreover, the experimental studies documented by Regan (1975), Yu and Tan (2013a and b) and Gouverneur (2013a and b) demonstrate that the ultimate resistance of double-span beam-column and slab-slab assemblies frequently correspond with a secondary mode of TMA response, sustained after fracture of the extreme tension reinforcement at critical sections and joint locations. This mode of response is accepted in blast resistant design and analyses, where large displacement, damage and distress in a structural element can be tolerable provided that outright collapse resistance is avoided. However, secondary TMA response has not been acknowledged by progressive collapse design guidance and, accordingly, has seen limited research attention – Gouverneur et al. (2013a and b) presents the only investigation of TMA response at outright collapse since that reported by Regan (1975).

An experimental programme that was designed and executed to investigate the emergency load redistribution behaviour of laterally restrained RC beam and slab strips at incipient collapse. The programme constitutes the largest investigation of its type since Regan (1975) and is the only recent experimental investigation of British format RC construction under these conditions. This chapter describes the experimental set-up, design of the test rig and test specimens and explains the test methodology. Results obtained in testing are provided in Chapter 5.

4.1 Experimental Programme

This investigation was concerned with the ultimate collapse resistance of conventional RC framed buildings. The situation under consideration is that shown schematically in Figure 4-1; the emergency response of a multi-storey building subject to instantaneous perimeter column loss. Load redistribution is facilitated by the beam and slab elements of the floor system acting across the resulting double or ‘emergency’ span. Chapter 3 indicates that such arrangements have effective lateral restraint and can sustain three key mechanisms that act in series, as displacement increases; Vierendeel or frame action (FA), arching action (or compressive membrane action, CMA) and catenary action (a type of tensile membrane action, TMA). Thus, an experimental programme was devised to investigate the full emergency behaviour of the individual components of conventional RC floor assemblies, from column removal to outright collapse. The objective of the programme was to obtain physical data to support
the study of catenary action response, ultimate collapse resistance and to review existing analytical procedures.

![Diagram of RC frame with column loss](image)

**Figure 4-1 – Emergency load carrying mechanisms in RC frame having sustained peripheral column loss.**

Chapter 3 provides a literature investigation of the current understanding and analytical approach to catenary response. The experimental programme was developed to address aspects of catenary response that were identified as unconfirmed or for which existing studies presented contradictory or inconclusive evidence. Particular focus was paid to those aspects identified as influencing ultimate load resistance in catenary action; in-plane force, ultimate displacement and lateral restraint.

Ultimate tensile membrane force was one aspect of catenary response for which existing research provides differing theories. Thus, a principal consideration for the test rig and program design was to support the investigation and analysis of membrane force with displacement and at ultimate load and collapse. Thus, the test rig was designed to accommodate ½ scale double bay test specimens and instrumented to provide a statically determinate system, with the direct measurement of horizontal restraint force. Each test was initiated with the specimen in its service condition, with three points of vertical restraint. Following removal of the central support the test specimens were subject to a forced displacement by means of a hydraulic actuator applied to the midspan of the specimen. All specimens were constrained by semi-rigid lateral restraint and displaced until collapse or an ultimate failure mode could be identified. The instrumentation of the rig was to support measurement of the applied load and restraint reactions, and analysis of membrane force and TMA response, throughout this process.

The prediction of collapse resistance in catenary action requires knowledge of the displacement at maximum load prior to first failure of the extreme tension reinforcement (the rotation capacity) at
critical sections and maximum load prior to incipient collapse. These displacements define ultimate *primary* and *secondary* TMA response, respectively. Characteristic chord rotations of 12° and 20° are currently suggested as ultimate limits for the two modes of catenary response. However, Chapter 3 showed that there is currently no evidence to support either empirical limit. Moreover, the only experimental study to document displacement at outright collapse was by Gouverneur *et al.* (2013a and b) from which only two data points were obtained. Thus, the stipulation of displacing all specimens to collapse was a principal design consideration and the test rig was designed such that each specimen could be subject to a chord rotation of up to 18°. This was considered adequate for the investigation and identification of displacements at failure of *primary* and *secondary* TMA response or ultimate failure modes. Moreover, to provide an adequate data set, a total of twelve specimens were designed and tested.

Regan (1975) suggested reinforcement arrangement, bar curtailment and detailing were factors in catenary performance (see Chapter 3) but the subject has seen little subsequent research. Therefore, the test programme investigates three groups of test specimen each featuring different reinforcement detailing. The groups are referenced as *edge beam*, *slabstrip* and *flat-slab strip* specimens, as their design and detailing were based on the components of RC framed and flat slab floor systems. To investigate the influence of area of reinforcement and span-depth ratio on catenary performance, these factors were strategically changed within test groups. Three edge beam and three slab-strip specimens were fabricated, each with different area of reinforcement. Six flat-slab strip specimens were fabricated, each with different area of reinforcement and span-depth ratio.

It should be noted that the test programme constitutes an idealised investigation of component based large-displacement response in the double-span condition. In practice, an event resulting in instantaneous perimeter column loss would entail various dynamic effects and potential damage to the floor system and adjacent structure. However, by testing specimens under a quasi-static load and with consistent restraint conditions, these effects were neglected. Also, the component elements of a monolithic floor system would act together in load redistribution, developing two-way response in the slabs and pronounced torsional effects within edge beam elements. By testing strip specimens in isolation, the test programme neglects the effects of such three-dimensional response and assumes that each storey acts independently. It should be noted that this idealised approach is used in current design guidance.

4.2 Scale Effects

By testing specimens at part-scale, subject to a dissimilar loading arrangement and duration, experimental assemblies are subject to dimensional and strain-rate scaling that could affect material behaviour and performance of the system. Research conducted by Krawinkler (1988) suggests that it
is important to consider the extent and manner of scaling so as to ensure the similarity of experimental
and full-scale response.

In testing dimensionally scaled RC specimens complications are found in maintaining the
characteristic properties of both the concrete and reinforcement whilst ensuring that interaction
between the two materials remains consistent with that of the full-scale counterpart. With the careful
use of microconcretes and tailored reinforcement this can be achieved at scales as small as 1:15
(Krawinkler, 1988; Harris and Sabnis, 1999) except for the tensile strength of the concrete. However,
in testing for catenary action and elastic-plastic response, it was considered essential that construction
standard Deformed Type 2 high tensile rebar (BSI 2005b; 2008a) be used rather than introduce an
imitation reinforcement. This form of reinforcing steel is produced to a minimum bar diameter of
6mm. So as to accommodate this size bar and achieve a desired area of reinforcement of between 1.0
and 0.1% all linear dimensions were limited to a minimum scale of 1:2. So as to ensure that anchorage
was not affected by the use of smaller rebar (whose deformations or ‘ribs’ are less pronounced) and to
allow scaled cover to the reinforcement, a microconcrete was engineered with an aggregate graded to
1:2 scale (maximum 10mm) and a 28day cube strength of structural grade concrete. Research
conducted on such forms of microconcrete indicates a good similarity in behaviour with structural
concretes mixed with full-scale aggregates (Krawinkler and Moncarz, 1982).

In fabricating scaled strip specimens the ratio of surface area to volume is greater than that of a full-
scale monolithic assembly. This factor leads to complications with regards to the similarity of complex
internal stresses resulting from creep and thermal shrinkage (Krawinkler, 1988). However at a scale
of 1:2 the test specimens are defined as ‘large-scale’ and manifest similar creep effects to those of the
full-scale element. However, any unnecessary discrepancy was kept to a minimum by placing the
specimen within the test rig shortly before testing. Furthermore, so as reduce the effects of thermal
shrinkage, each specimen was cured in a humid environment to slow the rate of dehydration and
minimise unrepresentative thermal cracking.

The emergency response of full-scale RC sub-assemblies to instantaneous column removal is regarded
as dynamic and nonlinear (Su et al. 2009). Each specimen was loaded by means of a hydraulic actuator
applied at the midspan of the assembly. The system was subject to a controlled displacement at a
constant rate of 1mm/sec so as to permit inspection of the assemblies throughout testing. This form
of loading resulted in a quasi-static rather than the dynamic response. It is well established that
material behaviour varies under these two forms of loading; reinforcement will typically exhibit a
higher yield and tensile strength when loaded at increased strain-rates whilst concretes (including
micro-concretes) exhibit an increased initial stiffness and compressive strength. In both cases material
failure tends to occur at a greater strain when loaded statically (Krawinkler, 1988). It follows that the
results obtained in this test programme are likely under-estimate deformation at collapse where
compared with the dynamic response of a full-scale structure subject to rapid column loss. However,
authors such as Izzudin et al. (2008), Ying and Su (2011), Orton and Kirby (2014) and Yu et al. (2014)
have demonstrated that test data obtained in nonlinear static response can be implemented effectively in dynamic analysis. This topic is considered in more detail in Chapter 6.

4.3 Test Specimen & Material Specification

The section properties and rebar arrangement of the twelve specimens tested are summarised below, in Table 4-1. As-built drawings and sample data for each of the specimens tested can be found in Appendix C.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>(L_{EM})</th>
<th>(L_{EM}/d^*)</th>
<th>h</th>
<th>(A_C)</th>
<th>(\rho_1)</th>
<th>(\rho_2)</th>
<th>(\rho_3)</th>
<th>(\rho_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E01</td>
<td>5,000</td>
<td>35.2</td>
<td>175</td>
<td>39,375</td>
<td>0.713</td>
<td>0.312</td>
<td>0.312</td>
<td>0.492</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(2B12)</td>
<td>(2B8)</td>
<td>(2B8)</td>
<td>(2B10)</td>
</tr>
<tr>
<td>E02</td>
<td>5,000</td>
<td>35.2</td>
<td>175</td>
<td>39,375</td>
<td>1.069</td>
<td>0.469</td>
<td>0.469</td>
<td>0.737</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(3B12)</td>
<td>(3B8)</td>
<td>(3B8)</td>
<td>(3B10)</td>
</tr>
<tr>
<td>E03</td>
<td>5,000</td>
<td>35.2</td>
<td>175</td>
<td>56,000</td>
<td>1.337</td>
<td>0.511</td>
<td>0.511</td>
<td>0.741</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(3B16)</td>
<td>(3B10)</td>
<td>(3B10)</td>
<td>(3B12)</td>
</tr>
</tbody>
</table>

* Span/depth ratio of the emergency span, \(L_{EM} = 2*L\).

** Percentage reinforcement ratio, \(\rho = 100A_s/bd\)

Table 4-1 – Summary of 1:2 scale specimen detailing and geometry.

The test specimens are grouped by reinforcement arrangement (shown schematically in Table 4-1). The three ‘edge beam’ (E-series) specimens were detailed with reinforcement curtailment and a constant span-depth ratio characteristic of edge beam elements. The three ‘slab-strip’ (S-series)
specimens featured detailing and a span-depth ratio consistent with conventional one-way slab elements. Table 4-1 shows that the rebar and reinforcement ratio (ρ) was changed between specimens. The four M-series and two C-series specimens featured continuous top and bottom reinforcement. These six specimens are referenced as flat-slab (C&M-series) strip specimens as their span-depth ratio and area of reinforcement were moderated to be consistent with the middle and column strips of surrogate flat-slab floor designs. Note; the naming convention used of ‘edge beam’, ‘slab-strip’ and ‘flat-slab strip’ specimen groups is purely attributed to the basis of design used in the development of the specimens.

The following section details the design methodology and key considerations in the development of the test specimens.

### 4.3.1 Edge Beam & Slab-strip specimens

Edge beam and slab-strip specimens were based on the component parts of the theoretical 4-storey RC framed structure designed by Higgins and Rogers (1998). A double peripheral bay of the surrogate structure is shown on plan in Figure 4-2. Inspection shows that given removal of a peripheral column, catenary response would occur in the edge-beam and slab elements.

![Figure 4-2](image)

**Figure 4-2 – Floor plan of beam-slab test building.**

The building featured a 9x3 column grid, with bay dimensions 5x6m and 5x8m, and was designed in accordance with BS 8110-1:1997 as a one-way-spanning slab-beam RC frame system. Typical design loads were used, consistent with light-commercial and office use (g_k = 4.7kN/m² and q_k = 4.0kN/m², per BSI, 2002a). Grade C40 concrete and B 500B reinforcement (f_y = 460MPa, as found in standard design prior to the 2007 code amendments of BSI, 2005b) were assumed, with appropriate partial safety factors.

The full-scale theoretical edge beam and slab sections were designed prescriptively for a service span (L) of 5m and load effects were derived from cl.3.4.3 and cl.3.5.2.4, assuming 20% redistribution of the hogging moment. Given the 4-storey height, effective horizontal ties were provided conforming
to the minimum required tensile resistance, 36kN. The resultant dimensions and reinforcement ratios ($\rho$) specified for both elements are provided below, in Table 4-2.

Table 4-2 – Derivation of edge beam and slab-stripe specimen dimensions and detailing at 1:2 scale.

The test specimens were 1:2 scale ‘strip’ models of those designed at full-scale. Three specimens were fabricated for each component. The specification of each is provided in Table 4-2 for comparison with the 1:1 theoretical counterparts. The span ($L$) and section depth ($h$) were dimensioned by scaling linearly from the 1:1 theoretical elements, thus maintaining span-depth ratios approximately equal to the surrogate structure in its service and damaged state.

Table 4-2 shows that the percentage area of reinforcement ($A_S$) was increased between specimens. The edge beam specimens were specified to provide 1.0, 1.5 and 2.0 times the percentage area reinforcement of the 1:1 theoretical counterpart. The slab-strips were detailed for 1.0, 2.0 and 3.0 times the percentage area reinforcement specified in the 1:1 theoretical counterpart. The incremental increase of $A_S$ was arbitrary and intended to facilitate the investigation of catenary performance in specimens of identical span-depth ratio and reinforcement arrangement but differing reinforcement content (moderate to heavily reinforced), within allowable limits (BSI, 2005b; 2014). Target $A_S$ were attained by moderating the reinforcement and section breadth ($b$) between specimens. The effective depth of each bar group was controlled such that $\rho$ was subject to the same increase factor used in sizing the area of reinforcement (i.e. 1.0, 1.5, 2.0 and 3.0 $\rho$).

From the as-built dimensions and reinforcement specifications provided in Table 4-2; the span-depth ratio of each specimen closely matches that of the full-scale elements; the percentage of reinforcement achieved for each rebar group was as close to target as could be practically achieved given geometric limitations and the reinforcement sizes available.
The edge beam and slab-strip test specimens were detailed such that the influence of reinforcement arrangement on catenary performance could be investigated. Moreover, to ensure that experimental results were applicable to typical conventional RC constructions, care was taken to specify and fabricate each specimen with reinforcement detailing consistent with BS 8110 (BSI, 2005b) requirements. Figure 4-3 shows longitudinal sections of each and the reinforcement arrangement implemented. The detailing replicates that specified for the 1:1 theoretical elements but at 1:2 scale – thus corresponding to curtailment and detailing practice laid out by Figure 3.24 and 3.25 of BS 8110. Note; bar-marks used for each reinforcement group correspond with the schedule in Table 4-2. Thus, effective tying requirements were facilitated by the bottom reinforcement layer at interior supports – bars 03 and 02 for the beam and slab elements, respectively.

![Figure 4-3](image)

**Figure 4-3 – Longitudinal sections showing edge beam and slab-strip 1:2 scale test specimen reinforcement arrangement and curtailment (dimensions in mm).**

By inspection of Figure 4-3 and Table 4-2 it can be seen that by detailing the edge beam and slab-strips to BS 8110 the specimens featured significant discontinuities in tension reinforcement along their length. This was a key design consideration for the experimental programme that would support comparison with results obtained by Regan (1975) who found rebar curtailment and discontinuity to improve the in-plane extension capacity and catenary performance of specimens (see Chapter 3).

A principal objective of the experimental investigation was to identify modes of failure in primary and secondary catenary response. Typically failure is attributed to reinforcement rupture (Park and Gamble, 2000). However, Regan (1975), Mitchell and Cook (1984), Su et al. (2009) and Yu and Tan (2013) identified anchorage failure and bar pull-out as an alternative mode of failure (see Chapter 3). Thus, care was taken to ensure that specimens were detailed with reinforcement (see Section 4.3.3) and anchorage consistent with typical RC constructions.
Lap length was a key consideration in the design and specification of specimens with representative anchorage strength. In accordance with code requirements (BSI, 2005b) consistent with the basic anchorage required for a C30 concrete, and accounting for bar spacing and depth of cover, a tension lap of 40 times bar diameter was implemented. The reinforcement was fixed to give the lapping arrangement shown in Figure 4-3a and b.

Due to the experimental load arrangement and double span, the specimens were anticipated to respond in flexure with little likelihood of shear failure. However, shear links were introduced so as to provide realistic bond strength at the lap between the longitudinal steels, aid fabrication and provide containment during extreme deformation. The size and spacing of the shear reinforcement was ascertained from BS 8110 to provide a basic shear capacity. Unlike typical slab construction shear links were introduced to all specimens in areas featuring both B1 and T1 reinforcement (as shown by Figure 4-3a and b). The edge beams were provided with B8 links at 175mm centres and the slab-strips with B6 links at 150 centres. Tests by Yu and Tan (2011) indicate that the spacing of shear links has little influence on catenary response in RC elements. Due to this insensitivity to shear link spacing, the performance of the tests specimens was therefore considered to be representative of the standard full-scale design.

The edge beam and slab-strip specimens were designed and detailed in accordance with BS 8110. This code has been in service since 1985 but was formally superseded in 2010 by Eurocode 2 (EC2, BSI; 2014). Given that the specimens were designed in accordance with a code of more than 25 years of service, the test specimens are representative of a significant proportion of the present day UK building stock. However, to assess the validity of the test programme to current practice, the detailing used was compared with requirements imposed by EC2. It was found that curtailment and general arrangement were approximately the same as tested. The most significant variation was found in lap length, where EC2 requirements would dictate a minimum lap of approximately 46 times bar diameter, for an equivalent C25/30 concrete. Given that this exceeds the lapping provision provided, the test specimens might be described as providing a conservative representation of catenary performance, if failure is governed by anchorage strength. However, the results presented in Chapter 5 demonstrate that this was not the case.

4.3.2 Flat-slab strip specimens

The flat-slab specimens were chosen principally to provide a set of control specimens that would support interpolation of floor system performance at different span-depth ratios. For this purpose a series of test specimens were of varying depth and span, but of constant longitudinal area of reinforcement, were derived. The detailing was purposefully simplistic with continuous top and bottom reinforcement groups anchored directly to end plates of each specimen. The reinforcement arrangement is shown on elevation in Figure 4-4. Detailed construction drawings are provided in Appendix C.
The design of these specimens was conducted by Punton (2015) who, having established a maximum allowable span-depth ratio of 60 (in accordance with BS8110, cl.3.4.6.3), specified a series of specimens at span-depth ratios of 60, 50, 40 and 30 in the double span condition. A suitable percentage of reinforcement was ascertained by designing a series of column and middle strip elements at spans of between 5m and 9m. Average reinforcement ratios ($\rho$) of 0.4% and 0.2% were ascertained for the column and middle strip elements respectively. Given a maximum emergency span of 5m (limited by the size of the test rig), the scale specimens were specified by choosing spans and specimen depths to achieve the target span-depth ratios. The size and number of rebars were then adjusted with the specimen breadth to achieve the target reinforcement ratio.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>$L_{EM}$/d</th>
<th>$L_{EM}$</th>
<th>h</th>
<th>$A_C$</th>
<th>$A_S$</th>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column-strip slab specimen</td>
<td>C01</td>
<td>60.2</td>
<td>5000</td>
<td>110</td>
<td>35200</td>
<td>0.29</td>
<td>0.38 (2B8)</td>
</tr>
<tr>
<td></td>
<td>C02</td>
<td>50.0</td>
<td>4550</td>
<td>118</td>
<td>37760</td>
<td>0.27</td>
<td>0.35 (2B8)</td>
</tr>
<tr>
<td></td>
<td>C03</td>
<td>40.0</td>
<td>4320</td>
<td>140</td>
<td>44800</td>
<td>0.34</td>
<td>0.44 (3B8)</td>
</tr>
<tr>
<td></td>
<td>C04</td>
<td>31.6</td>
<td>4050</td>
<td>160</td>
<td>51200</td>
<td>0.39</td>
<td>0.49 (4B8)</td>
</tr>
<tr>
<td>Middle-strip slab specimen</td>
<td>M01</td>
<td>50.0</td>
<td>3900</td>
<td>105</td>
<td>33600</td>
<td>0.15</td>
<td>0.20 (1B8)</td>
</tr>
<tr>
<td></td>
<td>M02</td>
<td>40.0</td>
<td>4720</td>
<td>145</td>
<td>46400</td>
<td>0.17</td>
<td>0.21 (1B10)</td>
</tr>
</tbody>
</table>

* Percentage reinforcement ratio, $\rho = \frac{100A_s}{bd}$

Table 4-3 – Derivation of column and middle-strip slab specimen dimensions and detailing.

The specification for each column and middle-strip specimen is listed in Table 4-3. The target reinforcement ratios were achieved as closely as practicable given limitations in rebar size and spacing requirements. It should be noted that only two middle-strip specimens were fabricated as their specification was similar to the S01 specimen detailed in the previous section.

Whilst this group of specimens were designed as flat-slab strip specimens, inspection of the span-depth ratios advocated by BS 8110 shows that by testing span-depth ratios of between 30 and 60, the control specimens encapsulated continuous rectangular and flanged section design. Furthermore, Cobb (2004) states that a typical RC beam may be used for spans of between 5m and 15m whilst one-way slabs and flat-slabs can accommodate spans of 5m to 6m and 4m to 8m respectively. Thus, performance of the
control specimens may be taken as indicative of a variety of RC elements at a scale of between 1:2 and 1:3.

It should be noted that the flat slab-strip specimens were detailed with B8 links, located at 200mm centres. These were implemented to aid the fabrication and casting of the specimens.

### 4.3.3 Reinforcement specification

A catenary mechanism sustained by a RC assembly is essentially a tensile net formed by the reinforcement (Park and Gamble, 2000). This suggests that the failure mode of this mechanism is defined by the yield and ultimate strength and elongation properties of the reinforcement. Therefore, care was taken to ensure that the rebar used in the test specimens was representative of that found in conventional construction.

The most common form of high-yield reinforcement used in construction is *BS4449:2005 Grade B500B*, which is cold-formed (BSI, 2007). Cold forming results in different characteristic properties of the reinforcement with different bar diameters. Given the use of scale specimens, it was important to establish whether small diameter rebar could be used in testing and provide similar characteristic properties to the larger bars used in the full-scale structure. A series of tensile tests were conducted on a batch of *deformed type 2 Grade B500B* reinforcement. The tests were carried out in accordance with *BS 15630* and *BS 6892* (BSI 2002; 2009b) on a gauge length of 200mm and under a maximum strain rate of 0.00025s\(^{-1}\). Samples of B6 up to B16 bars, were tested to obtain measurements of yield strength (\(R_e\)), tensile strength (\(R_m\)), percentage elongation at maximum force (\(\Delta g_{t}\)) and percentage elongation at fracture (\(\Delta g_{f}\)).

The results obtained for the B8, B10, B12 and B16 bars are given in Figure 4-5.

![Figure 4-5](image-url)

**Figure 4-5** – Recorded tension properties of reinforcement samples; a). mean yield and tensile strength and b). percentage elongation at maximum force and fracture.
Results obtained for the B6 bar showed lower ductility and strength compared with the larger diameters bars ($R_y = 502\text{MPa}; \ R_e = 556\text{MPa}; \ A_{gt} = 4.0\%$). Hence the B6 rebar was not suitable for the experimental programme and its use as tension reinforcement was ruled out. The B6 results have been omitted from the figures for clarity. A full set of individual results is given in Appendix C.

Inspection of Figure 4-5 demonstrates the variation in mean yield strength, tensile strength, elongation and allows interpretation of the reinforcement ductility. It can be seen that the B8, B10 and B12 reinforcement show similar characteristic properties. The B16 reinforcement is similar with regards to its mean yield and ultimate tensile strength but had elongations at maximum force and fracture lower than the other bar sizes. However, inspection of the BS4449:2005 requirements for Grade B500B reinforcement shows that all specimens performed satisfactorily against minimum criteria specified.

Quality control test data, obtained from various UK manufacturers and the Building Research Establishment (BRE), were consolidated and analysed for additional comparison to the sample test data. Results obtained from a population of some 14,000 test specimens, varying in bar size from B8 to B50, were compared to the behaviour recorded in testing (see Appendix F). The results are shown in Figure 4-6 and Figure 4-7.

Figure 4-6 – Comparison of test and industry reinforcement strength data (error bars denote 2SD).

Figure 4-7 – Comparison of test and industry reinforcement elongation data (error bars denote 2SD).
Comparing the industry rebar data and that obtained in testing, it can be seen that the test mean yield and ultimate tensile strengths fell within the ranges obtained from industry, although the B10 results were close to the bottom limit. Inspection of the elongation results shows greater disparity but this can be explained by problems with the end grips during laboratory testing.

The B8, B10, B12 and B16 bars were selected for use in the test programme. The tensile tests used for investigation of the reinforcement suitability were taken from the stock used in fabrication of the test specimens. The mean properties recorded for each bar size were used in analysis of the test assemblies and catenary response.

Mitchell and Cook (1984) stressed the importance of properly anchoring reinforcement. Their investigation demonstrated that when anchorage was inadequate to mobilise the tensile strength of the reinforcement, the performance of a RC catenary system would be governed by the capacity of the anchorage. Equation 4-1 (BSI, 2005b) shows that ultimate anchorage bond strength \( f_{bu} \) is related to the characteristic compressive strength of concrete \( f'_{cu} \) and the deformation type of the reinforcement (represented by the constant, \( \beta \)).

\[
f_{bu} = \beta \sqrt{f'_{cu}}
\]

Equation 4-1

To ensure that the catenary response recorded during testing was representative of full-scale counterparts, and to ascertain the controlling factor in catenary response (whether anchorage or reinforcement strength), deformed type 2 reinforcement (\( \beta = 0.50 \)) was used and curtailed in accordance with standard detailing practice. However, inspection of Equation 4-1 shows that the characteristic compressive strength of the concrete is the most significant factor influencing the bond strength. The following section details the design and specification of the microconcrete used in testing and provides comment on how the strengths achieved influenced the relevance of the anchorage strength.

### 4.3.4 Concrete specification

A minimum mean compressive strength of 40MPa was enforced for testing. This requirement was imposed to ensure that specimen response in frame, arching and catenary actions was representative of full-scale conventional constructions.

As indicated in the previous section, concrete strength has a significant effect on the anchorage strength of a reinforcement lap (as given in Equation 4-1). It was therefore considered critical that the concrete strength of the test specimens was at least equal to that of a conventional structural concrete. As a practitioner, a grade C30 structural concrete can be considered the weakest currently specified in the design of RC framed superstructures. However, the compressive strength of a structural concrete increases with age and the 28-day characteristic strength used in design and specification represents a
statistical value, corresponding with a confidence interval of 95% (Neville, 1997), rather than its true strength. It follows that the 30MPa 28-day characteristic cube strength associated with a C30 concrete (per BSI, 2005b) is conservative and an underestimate of the mean strength of the mature structural concrete found in serviceable buildings. By taking account of the effect of age and statistical manipulation, together with other factors such as the size of the cubes used for strength testing, the 40MPa imposed as a minimum for the test programme can be shown to be a reasonable approximation of the true mean strength of a grade C30 structural concrete (see Appendix C for supporting commentary).

The concrete mix was designed in accordance with Marsh et al. (1997) to produce a structural grade micro-concrete. The water, cement and sand ratios were tailored for use with a 1:2 scale (10mm) rounded aggregate. To ensure that the target minimum compressive strength was achieved by the forecast test date of each specimen, a 52.5N High Strength Portland cement was used – given its improved curing rate and strength development this cement provided better logistical flexibility compared with the standard 42.5N cement also available. Typically mixes were designed to achieve a 28-day mean compressive cube strength of 45MPa and slump of 30mm to 60mm – the slump criterion was introduced to ensure workability during casting – giving a mix ratio of 0.39:1.00:0.85:2.19 (water:cement:sand:aggregate). However, minor adjustments were made to the mix design for specimens whose projected test date dictated a higher or lower 28 day compressive strength.

The concrete for each specimen was prepared using a pair of concrete mixers. As the concrete was weighed by hand and mixed in batches, precautions were taken to guarantee the quality and consistency of the concrete throughout each specimen. The concrete was mixed to a high standard, using visual inspection and slump tests to verify its consistency before pouring. A poker vibrator was used, in accordance with current good practice guidance, to compact the concrete and ensure a good bond with the reinforcement and consistency of the mix throughout the specimen. In order to check the consistency of the concrete strength, cube samples were taken from each mix, providing between five and eight 100mm cubes per specimen. These were cured and tested in accordance with BS 12390 and BS 5326 (BSI, 2000; 2003), immediately prior to the testing.

The results obtained from testing the cube samples are given in Appendix C. Figure 4-8 presents the mean values of cube strength, together with standard deviation bars, for each of the test specimens.

Inspection of Figure 4-8 shows that all samples were tested with a mean compressive strength in excess of 40MPa. Although hand mixed, samples taken from each mix used in the pour of each specimen demonstrated a high consistency between mixes. With the exception of samples taken for specimens S01 and M02, which were found to have the lowest consistency in concrete strength with (a maximum percentage error of 14.7 and 14.0%, respectively) a maximum percentage error of 10.0% or less was found between the remaining cube samples for respective specimens.
Figure 4-8 – Mean concrete cube strength and standard deviation recorded for experimental specimens.

The variation of mean compressive strengths, between 41MPa and 71MPa, is greater than had been intended at the start of the test programme. This is attributed to over-runs in the curing time of some specimens due to unforeseen logistical demands in the laboratory and delays due to essential repairs and calibration of instrumentation. In retrospect, rather than imposing a minimum concrete strength, it might have been better to follow more usual research practice and use a concrete of consistent strength that would facilitate direct comparison between specimens and other research.

Given the variability of concrete strength and the significance of this factor on reinforcement anchorage, a study was undertaken to establish the approximate concrete grade and age of construction the specimens might correspond with. Commentary and results from this study are provided in Appendix C. The findings indicate that the test programme covers a broad spectrum of RC construction. The weakest specimen (E02) is shown to be consistent with the strength of an approximately six month old C30 grade concrete and the strongest (S02) consistent with a C50 structural concrete approximately 15 years after being cast.

4.4 Test Rig

An experimental rig was designed and fabricated as shown on elevation in Figure 4-9, Figure 4-10 and Figure 4-11. The rig was based on an H-frame composed of a 610x305x149 UKB spanning between two 305x118UKC uprights. Buttressing was introduced to each column and secured with an eight point bolted connection to a strong-floor to minimise lateral displacement under load. The rig was detailed and instrumented such that the 1:2 scale test specimens could be suspended within the rig and tested in the following three phase process:

Test Phase 1 The specimen was located within the rig and restrained vertically and horizontally in such a way that all reactions were equal to those found in the two central spans of a continuous six-bay structure of equivalent self-weight.
Test Phase 2  The central support was removed and the specimen allowed to redistribute load under its own self weight.

Test Phase 3  Load was applied to the centre of the double span under displacement control and the specimen loaded until failure was achieved.

Figure 4-12 illustrates the various phases of the test procedure, showing the specimen subject to a large central displacement whilst ‘hangers’ and ‘end-details’ have been designed to maintain vertical and horizontal restraint throughout. The experimental rig was designed to permit data sampling throughout displacement. Restraint reactions were measured to monitor specimen response in frame action and compressive and tensile membrane action. Load was applied by a hydraulic actuator located at the centre of the double span – this was suspended from the H-frame and lowered into place following the removal of the central hanger. The actuator permitted measurement of applied load and displacement at midspan. The deflection profile of the specimens was measured by an array of potentiometers arranged at intervals along the specimen.

A description of the main features of the rig is provided below. Commentary is provided on the design considerations taken to minimise error and provide appropriate restraint to the test specimens. Additional images of the test assembly can be found in Appendix B.
Figure 4-9 – Elevation of test rig – shown before central support removal.
Figure 4-10 - Details and sections of West End-Detail - load cells and end restraint assembly.

Detail A – West End-Detail Elevation

Section A-A – Section showing axle restraint

Section B-B – Plan of West End-Detail
Figure 4-11 – Test rig details and side elevations of West Hanger – load cell and restraint assembly.
Figure 4-12 – Schematic elevations of test procedure, indicating key test phases, notations of reactions measured and formation of catenary action between hangers.
4.4.1 End reactions – $R_H$ & $R_V$

The end-details form the interface between the H-frame reaction structure and the test specimen. Their design was based upon two key principles: providing effective horizontal and vertical restraint to the end of the specimen and the accurate measurement of the vertical and horizontal reactions, $R_V$ and $R_H$ respectively.

As shown in Figure 4-10 and the photographs in Figure 4-13 and Figure 4-14 the assembly was designed to provide a pinned connection formed by a central pin. The pin was restrained by load cells in vertical and horizontal directions but allowed some degree of end rotation. To monitor the specimen and provide restraint against transverse rotation the load cells were arranged in pairs. In each case the load cells were strain-gauge-based and sensitive to both positive and negative loading – allowing measurement of the reaction forces attributed to both compressive and tensile membrane action. The combined capacity of the two banks of load cells allowed measurement of $\pm 444\text{kN}$ for $R_H$ and $\pm 50\text{kN}$ for $R_V$.

![Figure 4-13 – Images of instrumented West End-detail assembly in use.](image1)

![Figure 4-14 – Images of East End-detail assembly in use.](image2)
The end-detail was designed to ensure that the accuracy of $R_V$ and $R_H$ measurement would not be compromised by bending or shear in the boss of the load cell. Figure 4-10 shows that the pin was connected to each load cell by a rod and pin-plate arrangement – the pin being passed through the pin-plate to exert a force into the load cell via compression or tension in the rod. This arrangement promoted pure axial transmission of the $R_V$ and $R_H$ reaction forces to the load cells. The load cells themselves were calibrated using a certified electromechanical Instron machine. Each load cell was cycled to 90% of the maximum compressive capacity and calibrated to an error of less than 0.5%. Care was taken in detailing and fastening the load cells to the H-frame, making sure that their mounting was level and true.

To reduce any mechanical losses, the pin assembly was machined to a high tolerance. In addition, the connection between the end-detail and the specimen ‘end-plate’ was made using an array of eight threaded rods. The rods were arranged and bolted in such a way as to take up any tolerance between the specimen and the end-detail such that membrane forces were received directly by the $R_H$ load cells.

Steel buttresses were designed and introduced to the outer face of the H-frame to provide the required horizontal restraint and minimise deflection of the columns under horizontal thrust. These were of heavy construction and secured to a strong floor via four 5 tonne holding down bolts (see Figure 4-15).

By inspection of data recorded during testing (see Chapter 5 and Appendix D), maximum restraint reactions $R_H$ were recorded for test E03 in TMA ($R_H = 110.1$ kN) and M02 in CMA ($R_H = -232.8$ kN). The corresponding restraint displacements demonstrate the rig to have provided a lateral restraint stiffness of 101.1N/m and 123.5N/m, under out-bound and in-bound load respectively. This
demonstrates a good level of consistency between CMA and TMA restraint and, by reference to Merola (2009), can be seen to be consistent with upper and lower-bound conditions calculated for an in-situ multi-storey RC framed building.

The end-detail at the eastern column was not instrumented but otherwise replicated the western end-detail described above. This ensured that the specimens were restrained symmetrically about the midspan – featuring the same rotational, horizontal, vertical and torsional restraint conditions at equal distances each side of the midspan. So as to counter the effects of any neglected bending, the specimens were positioned such that the pins at each end corresponded approximately with the points of contraflexure found in a continuous six-bay element. The potential rotation permitted by the end details was not unlike that found in practice as a degree of rotation can be expected across the adjacent bays to an emergency span.

4.4.2 Support reactions – \( R_1, R_2 \) & \( P \)

Figure 4-9 shows the three ‘hanger’ assemblies used to provide vertical restraint at interior support and central locations along the specimens. The hangers each featured 150mm wide bearing plates on which the specimens were landed and effectively suspended from the H-frame cross-member. To support the testing of specimens with different span \( (L) \) and section thickness \( (h) \), each hanger was designed such that the longitudinal and vertical position of restraint could be adjusted and controlled. This was facilitated by the clamp connection at the H-frame and threaded tie rods, shown in Figure 4-11 (the western hanger corresponding with reaction \( R_1 \)).

Reactions \( R_1, R_2 \) and \( P \) were measured by recording the load/tension on the respective hangers. In the case of \( R_1 \), this was accomplished using a pair of load cells fastened at the soffit of the H-frame cross beam. Reactions \( R_2 \) and \( P \) were measured by strain gauging the threaded tie rods – two strain gauges were used per tie rod, applied to flats machined on opposite sides of each rod to ensure a good and reliable bond. In each case the hangers were capable of recording support reactions of up to 50kN.

By instrumenting each leg of the hangers, reactions \( R_1, R_2 \) and \( P \) could be monitored and controlled (by manual adjustment of the bearing plate height and lateral pitch) to achieve longitudinal and lateral symmetry across the specimen during test set-up. This capability was important in order to facilitate Test Phase 1 and the instrumentation provided the data needed for the analysis of specimen response in the later stages of the tests (see Section 4.5).

As with the end-details, precautions were taken to reduce errors in measurement. Each of the load cells and gauged rods were repeat tested and calibrated – the load cells to 90% full-scale load and the gauged rods to 80% of yield strength. The load cells were secured to a level and rigid surface and the lower assembly designed to minimise any bending or shear in the system. During testing, the strain gauged rods were consistently orientated to account for the major axis bending.
Precautions were taken to minimise mechanical losses in the system. High yield and large cross-section threaded rods were specified in order to minimise elongation of the hangers and avoid differential settlement of the bearing plates (across the specimen) when under load. Heavy UK PFC sections were used to provide the 150mm wide bearing surfaces of the hangers and stiffeners used to minimise their deflection under load. All threaded rods were adjusted manually prior to testing in order to take up any tolerance and provide suitable starting reactions at the start of each test. All bolts and Diwidag nuts were tightened and double locked prior to testing in order to secure the position and height of each hanger and prevent movement in the fasteners.

### 4.4.3 Applied load & midspan displacement

The load actuator was located in the same position as the central hanger, at the centre of the double span. During Test Phase 1 the actuator was restrained against the underside of the H-frame cross-member, out of the way. Following removal of the central support in Test Phase 2, the hanger was disassembled and the actuator lowered into position. Care was taken to ensure that the load was applied perpendicular to the span of the specimen.

The load actuator was driven by a hydraulic ring main system and provided a stroke of 150mm. During the loading phase (Test Phase 3) the actuator was used to displace the beam in 20mm intervals at a loading rate of 1mm/sec, providing a quasi-static load. Equipped with a 100kN (full-scale) load cell and linear displacement transducer, the load actuator provided the applied load and displacement record at point P. Calibration of this equipment was conducted by Instron Ltd, prior to testing.

Figure 4-16 – Image of ‘hold-down’ system and actuator packer section in use.
Due to the limited stroke of the actuator a system was devised to allow displacement of specimens through the full clearance between the strong floor and the soffit of the specimen. A ‘hold-down’ system was designed to restrain specimens in their displaced state and allow the stroke of the actuator to be reset (see Figure 4-16). The restraint assembly was formed by a pair of UK RHS sections fastened to the strong floor, parallel to the specimen. Up to four 10tonne cargo straps were used to tie the specimen to the anchored RHS sections and take up the load in the actuator. The stroke was then reset, the resulting clearance safely packed and loading resumed with minimal creep or relaxation, as confirmed by the actuator output before and after resetting the stroke. In this way displacements of up to 750mm were achieved providing the rig with sufficient displacement capacity to allow catenary action to develop.

It should be noted that all specimens were subject to the centrally applied point load described. This arrangement was dissimilar and induced more onerous conditions compared with the uniformly distributed load (UDL) arrangement found in instantaneous support loss and double-span conditions. Several alternative approaches were considered during design that would better represent a UDL arrangement. These included; constant weight sand-bag loading followed by support removal; variable weight water-bag loading; air-bag loading, and; multi-point winch system loading, via the strong floor. However, none of the alternatives investigated allowed the degree of safety or control offered by the point load arrangement used. Moreover, inspection of specimen response during loading would not have been possible. The midspan, displacement controlled, point load system was therefore implemented.

4.4.4 Specimen & column displacements

Five 100mm stroke displacement potentiometers were used to measure displacements along the length of the specimen and act as a check on the central displacement. The potentiometers were attached to an independent frame and positioned at 150-300mm centres between the centre line of the R2 hanger and point P. The spacing of the potentiometers varied in accordance with the span of the specimen.

15mm displacement potentiometers, also attached to independent frames, were used to monitor the horizontal displacement incurred in each of the H-frame columns. These were introduced at the level of the specimens such that the horizontal restraint stiffness of the system could be established.

4.4.5 Rig-specimen interface

The specimens were cast with an end-plate at each end to make a connection with the end-details of the rig (see Figure 4-10, Figure 4-13 and Figure 4-14). During the initial stages of Test Phase 3, when CMA was anticipated, the horizontal force was conveyed to the end-plate via direct contact between the concrete and plate. Sound contact was achieved by incorporating the end plates as permanent shuttering during casting. In the later stages of displacement, once TMA had developed in the specimen, the end-detail instrumentation was reliant on the continuous tensile connection between the specimen reinforcement and the end supports. To achieve this, each end-plate was fabricated with
rebar couplers welded to the concrete facing side. The welds were sized so as to mobilise the failure strength of the rebar and the couplers were located in accordance with the rebar arrangement of each specimen.

4.5 Experimental Procedure

To achieve the aims of the investigation and ensure experimental consistency, each specimen was tested using the following procedure.

*Test Phase 1 – Test set-up*

Test specimens were lifted using a gantry crane and lifting beam – providing a six-point lift to prevent cracking during manhandling – and manoeuvred inside the rig to its approximate testing position between the transfer plates. The specimen was then levelled and positioned before introducing the threaded transfer rods, connecting the end-plates and transfer-plates. The tolerance between transfer-plates and the end-plates was adjusted to allow insertion of the end-detail pins, take up any longitudinal tolerance and ensure the placement of the specimen centrally in the rig. With the transfer pins in place, the vertical components of the end-detail (that provide restraint $R_v$) were adjusted such that a minor clearance was provided between the underside of the baseplate and the surface of the cantilever column stub, to which the baseplate was later fastened. The hangers were then assembled about the specimen and the bearing plates brought into bearing at the soffit of the specimen. The instrumentation and adjustment of each hanger was used to ensure the specimen was supported evenly before lowering and removing the lifting beam. At this stage; the specimen was fully supported by the three hangers; the end-details were assembled, with the transfer pins in place; all longitudinal tolerance had been taken up and the vertical components of the end-detail (the load cells and pin-plate assemblies that provide restraint and measurement of $R_v$) were suspended above the surface of the cantilever column stubs by a very small spacing.

With the specimen located in the test rig, the vertical restraints were adjusted to induce reactions predicted for the central spans of a six-bay continuous system of the same self-weight (target reactions are shown, together with those achieved, in Appendix D). The spacing between the base plates of the vertical components of the end-details and the surface of the cantilever column stubs was taken up and the connection secured. This preloaded the $R_v$ instrumentation with a negative reaction (placing the struts of $R_v$ into tension), securing the connection between the transfer pin and all the pin-plates by removing all vertical tolerance, and resulted in an increased positive reaction in the $R_1$ and $R_2$ hangers. Each of the hangers was then manually adjusted (typically lowering $R_1$, $R_2$ and $P$) to achieve reactions in $R_V1$, $R_1$, $R_2$ and $P$ as close as reasonably practicable to the theoretical target reactions. This iterative procedure, ensured that all specimens were set-up symmetrically with five points of vertical restraint, no height differentials longitudinally or laterally across the supports and with all spacings that could result in mechanical losses eradicated.
Following initial positioning and setup, each test specimen was allowed to settle and undergo short-term creep. Care was taken to ensure that the specimen was subject to zero axial force and the vertical reactions subsequently verified before testing commenced. The final recorded reactions from each test have been provided, together with target values, in Appendix D.

*Test Phase 2 – Support removal*

The displacement potentiometers were placed along the top surface of the specimen prior to removal of the central hanger and set at the top of their stroke (see test images provided in Appendix B). A builder’s line was also introduced in front of the specimen as an additional record of the initial level of the specimen soffit.

With the data logger recording all instrumentation, the central hanger was removed. The central displacement was measured manually with reference to the builder’s line and the specimen observed for cracking and any other significant effects. The reactions achieved following removal of the central support are given for each test in Appendix D. It can be seen from the symmetry of the recorded reactions that the specimens required no further adjustment.

*Test Phase 3 - Loading*

Following removal of the central support, the load actuator was prepared and lowered into place above the midspan of the specimen. Packing was introduced between the specimen surface and the actuator to maximise the potential stroke. Aligned to load the specimen vertically at the midspan and on the specimen centreline, the actuator ram was lowered to just bite on the surface of the specimen before all instrumentation was zeroed.

The specimen was displaced in 20mm increments at a rate of 1mm/sec. The instrumentation and specimen were inspected between increments – a photographic record was made of crack development and keys stages of specimen response. Data was recorded at each millimetre displacement.

At the limit of actuator stroke, a manual record of the central displacement was taken before introducing the ‘holding down’ straps each side of the load point. The holding down straps prevented any upward movement of the specimen while the actuator was reset. Following the introduction of a packer fastened to the underside of the actuator, the actuator load was restored ensuring that the displacement of the specimen was unchanged before recommencing the test. In each test this process was repeated until either failure of the specimen was achieved or the displacement limit of the rig was reached.
5 Test Results & Analysis

The twelve strip specimens were tested by following the procedure reported in the previous chapter. In each case, the experimental data and in-test observations were found to provide evidence of frame (FA), arching (CMA) and catenary action (TMA) during loading. The experimental programme was successful in monitoring specimen behaviour to the point of outright failure or a chord rotation of some 16°, which gave clear evidence of the failure mechanism that was developing. Thus, data was collected for both primary and secondary TMA response.

The objective of this chapter is to present and examine the large-displacement data obtained for each of the twelve test specimens, identify trends in characteristic catenary response and draw conclusions against established TMA theory to verify an analytical approach for the assessment of primary and secondary catenary action in one-way RC systems (implemented in Chapter 6).

It should be noted that the CMA data reported herein is provided purely to differentiate catenary behaviour from CMA response. For a study of the observed CMA response the reader is referred to Punton (2015).

5.1 Results Summary – General Large-Displacement Performance & Definitions

Details of the specimen properties are provided in Chapter 4 and Appendix C, including the section dimensions, reinforcement details and reinforcement ratios, test geometry and recorded material properties for the reinforcement and concrete. Figure 5-1 provides a schematic elevation of an idealised collapse mechanism achieved in catenary response and indicates the notation used for recorded reactions and specimen geometry. The reaction data recorded for each of the specimens tested is provided in Appendix D. The data is plotted against the angle of chord rotation (θ = tan⁻¹ Δ/L), to permit a comparison between specimens with different spans. The sign and name/reference convention shown in Figure 5-1 is used throughout.
As an example, Figure 5-2 shows the applied load \( (P) \) and horizontal restraint force \( (R_H) \) recorded in testing specimen E01. Inspection shows the load-rotation curve is characteristic of a laterally restrained flexural system, exhibiting; a change in gradient attributed to plastic-hinge formation and the development of compressive membrane forces during initial displacement; a peak load corresponding with arching action (CMA) \( (P_{CMA} \text{ found at A}) \); decline in load carrying capacity (AD) following \textit{snap-through}, and; a subsequent increase in flexural stiffness as catenary action (TMA) develops with displacement (DE) to provide a secondary peak load \( (P_{TMA} \text{ found at E}) \). Inspection of the horizontal reaction \( (R_H) \) record shows a compliance with the applied load and evidence of characteristic membrane action behaviour.

![Figure 5-2](image)

Specimen E01 was one of four specimens that did not fail (by outright collapse) within the 16\(^{th}\) rotation limit of the rig. However, Points B and C (Figure 5-2) denote discontinuities in the force-displacement record caused by the fracture of the extreme tension reinforcement at the midspan and interior supports, respectively. Therefore the applied load beyond point D was sustained by \textit{secondary} catenary response only. By developing \textit{secondary} response following fracture of the extreme rebar, the catenary system can be seen to have achieved its ultimate load path as it was sustained by a single remaining layer of reinforcement – referred to in the following sections as the \textit{critical reinforcement} \( (A_{S\text{ crit}}) \).

A chronological account of reinforcement fracture and other in-test physical observations has been provided in Appendix D for each of the test specimens. Key observations are noted on the force-rotation graphs, to support interpretation of the conditions associated with each event. As indicated
by the force-rotation histories and test observations provided in Appendix D, secondary response was achieved in all of the four specimens for which outright failure did not occur.

Table 5-1 provides a summary of the key experimental results obtained for each test specimen. The specimens are listed by increasing area of reinforcement and grouped by their detailing – edge beam, slab and flat-slab column and middle strip specimens. Those tests in which outright failure was achieved are indicated (see column 20). The peak CMA and TMA loads (\(P_{CMA}\) and \(P_{TMA}\)) are provided, together with the corresponding chord rotation (\(\theta_{CMA}\) and \(\theta_{TMA}\)) and horizontal restraint force (\(R_H\)). It should be noted that \(P_{TMA}\) values shown in Table 5-1 are the maximum load resistance recorded in either primary or secondary TMA responses (see Section 5.2). The displacement at CMA-TMA transition, when membrane force was recorded to be zero (\(R_H = 0\)), is denoted \(\Delta_0\). The chord rotation (\(\theta_0\)) and applied load (\(P_0\)) define specimen response at transition (see point D, Figure 5-2).

The theoretical ultimate flexural capacity (\(P_{FA}\)) of each specimen is listed in Table 5-1 and was calculated by assuming a fully rigid-plastic encastred system (Equation 5-1, where \(g_k\) is the measured self-weight of the specimen).

\[
P_{FA} = \frac{4(M'_{RES} + M_{RES})}{L_{EM}} - \frac{g_k L_{EM}}{2}
\]

Equation 5-1

The \(P_{FA}\) values shown in Table 5-1 were calculated using the hogging and sagging moments of resistance (\(M'_{RES}\) and \(M_{RES}\), respectively) of each specimen. Values of \(M'_{RES}\) and \(M_{RES}\) were calculated for the interior support sections by assuming equilibrium of an elastic-cracked RC section with a parabolic concrete stress distribution, rigid-plastic steel behaviour and zero axial force (\(N\)) – detailed commentary provided in Appendix D. In order to provide an accurate prediction of section capacity, the mean experimental values were used for both the concrete strength and the yield stress of each rebar (see Appendix C). Partial safety factors and strain hardening effects were neglected.

It should be noted that the condition of zero axial/membrane force was assumed in the calculation of \(P_{FA}\) to provide a metric for the assessment of specimen performance in TMA response, consistent with existing research.
<table>
<thead>
<tr>
<th>Ref.</th>
<th>Ultimate FA Data</th>
<th>Ultimate CMA Results</th>
<th>Results at $\Delta_0$</th>
<th>Ultimate TMA Results</th>
<th>Work Done</th>
</tr>
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<tr>
<td></td>
<td>$M_{\text{res}}^*$</td>
<td>$M_{\text{res}}$</td>
<td>$P_{\text{FA}}$</td>
<td>$\beta^{**}$</td>
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<td></td>
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<td>[kNm]</td>
<td>[kN]</td>
<td>[%]</td>
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<td>-27.5</td>
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</tr>
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</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>2.4</td>
<td>-2.4</td>
<td>3.7</td>
<td>-3.1</td>
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<td>-6.9</td>
<td>-1.8</td>
<td>-16.9</td>
</tr>
</tbody>
</table>

* Failure of extreme tension reinforcement occurred prior to TMA development, during Snap-Through phase.

** Percentage redistribution ($\beta$) incurred under ultimate frame action load $P_{\text{FA}}$ [kN] and specimen self weight $q$ [kN/m] (‘ve indicates redistribution of elastic sagging moment).
Given the rotational freedom at the supports and non-uniform reinforcement detailing at critical sections, it was of interest to establish what moment redistribution would be incurred at the mid-span or interior supports in attainment of $P_{FA}$. Thus a moment redistribution calculation was conducted for each specimen assuming in-test restraint and double-span conditions. The maximum percentage midspan and interior support moment redistribution ($\beta$) under $P_{FA}$ are provided in Table 5-1. It can be seen that the double-span test arrangement and reinforcement detailing typically resulted in redistribution of elastic moment to the interior supports. E03 required the highest redistribution of elastic moment, sustained at the midspan. At 40%, this is 10% higher than permitted in current design guidance (BSI, 2005b; 2008a).

Inspection of the force-displacement histories recorded (see Appendix D) show that $P_{FA}$ was achieved or exceeded by all specimens, prior to any physical failure.

By comparison of $P_{FA}$ and $P_{CMA}$ (see Table 5-1) it can be seen that all specimens sustained enhanced strength in CMA response, featuring peak load resistances exceeding the theoretical ultimate flexural strength (see column 9). Load enhancement was found to be between 1.1 and 2.4 times the ultimate flexural capacity during CMA response.

Results obtained at CMA-TMA transition give an average $P_0/P_{FA}$ of 0.88. This is consistent with observations made by Park and Gamble (2000) who suggested that typically $P_0 \approx P_{FA}$. However, as identified by Table 5-1, specimens E01, E03, C02, C04, M01 and M02 all sustained fracture of extreme tension reinforcement prior to CMA-TMA transition – the location and conditions of reinforcement rupture are specified in Appendix D. Results obtained for these specimens give an average $P_0$ of 0.56$P_{FA}$. Specimen C04 was the only case in which $P_{FA} \approx P_0$ despite prior failure of reinforcement but the test observations (detailed in Appendix D) show that $P_0$ was recorded after failure of only 25% of the extreme tension reinforcement at midspan. Specimens that sustained $\Delta_0$ without prior reinforcement failure gave an average $P_0$ value of 1.19$P_{FA}$. These results suggest that observations made by Park and Gamble (2000) are under-conservative and that specimen performance at transition and during subsequent TMA response was found to be dependent joint rotation capacity of individual specimens (see Sections 5.2, 5.4.2, 5.5.1 and Chapter 6).

By comparison of $P_{FA}$ and $P_{TMA}$ (see Table 5-1) it can be seen that all specimens sustained enhanced strength in TMA response (see column 16). Load enhancement was found to be between 1.1 and 7.1 times the ultimate flexural capacity during TMA response.

Given the quasi-static load arrangement, work done ($W$) was evaluated as the area under the load-displacement curve. Work done provides a good indication of system performance; accounting for specimen ductility, load resistance and allowing interpretation of factors affecting performance. Table 5-1 provides values for the total work done ($W$) as the sum of the energy absorption recorded during both CMA and TMA response and can be expressed as follows:
\[ W = W_{CMA} + W_{TMA} = \int_0^{\Delta_0} P \, d\Delta + \int_{\Delta_0}^{\Delta_{TMA}} P \, d\Delta \quad \text{Equation 5-2} \]

Where \( \Delta_{TMA} \) is the midspan displacement at peak applied TMA load (see point E, Figure 5-2).

For clarity, work-done in frame action was assumed to be a component of both the CMA and TMA phases and has not been defined separately. Although this approach provides a crude and conservative evaluation of \( W_{TMA} \), inspection shows that catenary action accounts for 25-99% of the total energy absorption associated with double-span response and that, when compared with the flat-slab strip specimens, the edge beam and slab-strip specimens were found to demonstrate greater TMA response.

### 5.2 Ultimate Load Capacity, Enhancement & Performance in Catenary Action

Ultimate load capacity is defined here as the maximum load resistance recorded during displacement of the test specimens. For specimen M01, this was recorded during arching action (CMA) response. For all other specimens, the ultimate load resistance was found to occur during catenary action (TMA) response (see Table 5-1).

Table 5-2 specifies the ultimate load resistance recorded during primary and secondary TMA response. Data specified for primary response \( (P'_{TMA}; \theta'_{TMA}; R_H) \) corresponds to the maximum TMA load recorded prior to the first extreme tension reinforcement failure event (at midspan or support locations). Secondary response data \( (P''_{TMA}; \theta''_{TMA}; R_H) \) corresponds to the maximum TMA load recorded, following first extreme tension reinforcement failure and prior to specimen collapse. As indicated in column 16 (Table 5-2), collapse was not achieved in the testing of four specimens as the displacement limit of the test rig precluded outright failure. It follows that \( P''_{TMA} \) values shown for specimens E01, E03, S02 and S03 were not the ultimate capacities of the specimens and constitute lower-bound results.

For clarity, Table 5-2 shows the area of reinforcement \( (A_s) \) and reinforcement ratio \( (\rho) \) of the rebar layers located at the midspan and interior support locations. The top and bottom reinforcement layers are identified individually. Columns 14-17 identify the condition of the reinforcement at \( P''_{TMA} \). Reinforcement layers that had sustained fracture are indicated. \( A_{S-crit} \) and \( \rho_{crit} \) identify the critical reinforcement that was found to sustain the specimen in secondary catenary response.
### Table 5.2 – Summary of primary and secondary TMA load resistance data.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Ultimate Primary TMA Data</th>
<th>Primary TMA Reinforcement Data</th>
<th>Ultimate Secondary TMA Data</th>
<th>Secondary TMA Reinforcement Condition &amp; Data</th>
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<td>[kN]</td>
<td>[Deg]</td>
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<td><strong>Column-strip specimens – 31.6 ≤ 2L/d ≤ 60.2</strong></td>
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</tr>
<tr>
<td>C02'</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>101</td>
</tr>
<tr>
<td>C03</td>
<td>-30.5</td>
<td>-57.9</td>
<td>6.6</td>
<td>1.7</td>
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<tr>
<td>C04'</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>201</td>
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<tr>
<td><strong>Middle-strip specimens – 40.0 ≤ 2L/d ≤ 50.0</strong></td>
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<tr>
<td>M01'</td>
<td>-</td>
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<td>50</td>
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<tr>
<td>M02'</td>
<td>-</td>
<td>-</td>
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<td>79</td>
</tr>
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</table>

* Failure of extreme tension reinforcement occurred prior to TMA development, during Snap-Through phase.
** $A_{s, crit}$ not consistent with $A_s$ at supports as incomplete catenary mechanism was formed (see Appendix D).
The ultimate load at fracture of the extreme tension reinforcement is, for each of the specimens, specified in Appendix D. This section only reports loads sustained whilst in TMA response. However, the ultimate load at fracture was typically found to be greater than the ultimate flexural capacity of the specimen, whether fracture occurred in snap-through or TMA response. An assessment of the results shows an average enhancement of 97% over ultimate flexural strength. However, M01 and E01 sustained failure of the B2 tension reinforcement at midspan, at 0.65 and 0.89\textit{P}_{\text{FA}}\), after having sustained CMA enhancement.

Table 5-2 identifies the test specimens that did not achieve primary TMA response (specimens E01, E03, C02, C04, M01 and M02) – sustaining fracture of extreme tension reinforcement during snap-through response, whilst membrane forces were still compressive. This result contradicts the recommendations and findings made by Park and Gamble (2000), Keenan (1969) and Black (1975) who indicated that TMA response could be safely achieved in RC slabs prior to the fracture of extreme tension reinforcement. This is therefore a significant finding that suggests the practice of direct catenary analysis for primary response conditions (as specified by, Park and Gamble, 2000; GSA, 2003; DoD, 2005, 2014) may be unsafe.

By comparison of the test data (Table 5-2), it can be seen that ultimate load resistance \(P_{\text{TMA}}\) was predominantly achieved in secondary catenary response. The results obtained provide an average ratio of \(P'_{\text{TMA}}/P_{\text{TMA}}\) = 1.5, with a maximum ratio of 2.1 recorded during the testing of specimen E02. C01 was the only test specimen to exhibit a greater load resistance in primary response, \(P''_{\text{TMA}}/P_{\text{TMA}}\) = 0.8. However, observations made during testing of C01 suggest that the behaviour of this test specimen was irregular, due to an asymmetric load arrangement (see Section 5.3.3).

The load enhancement achieved in catenary action has been taken as the ratio of the ultimate load recorded in TMA response \(P_{\text{TMA}}\) against the ultimate flexural capacity \(P_{\text{FA}}\) of the specimen, where \(P_{\text{TMA}}\) is the greater of \(P'_{\text{TMA}}\) and \(P''_{\text{TMA}}\). From Table 5-1 and Table 5-2 it can be seen that enhancement of the load carrying capacity was observed in each test. \(P_{\text{TMA}}\) was found to exceed the predicted ultimate frame action capacity \(P_{\text{FA}}\) by between 14.3 and 586.2%. The average enhancement was 211.6%. The results demonstrate that maximum enhancement was sustained during secondary TMA response, with the exception of test specimen C01.

Figure 5-3 provides a graphical plot of load enhancement values \(P'_{\text{TMA}}/P_{\text{FA}}\) and \(P''_{\text{TMA}}/P_{\text{FA}}\) for each of the test specimens. Lower-bound results obtained for test \(P''_{\text{TMA}}\) in specimens E01, E03, S02 and S03 are identified from the remaining data points, for clarity.

It is apparent that slab-strip specimens S01, S02 and S03 exhibited the greatest load enhancement over that predicted from pure flexural response, in both primary and secondary TMA response (65.4-463.2% and 243.7-586.2%, respectively). This result indicates that the slab-strip specimens possessed limited flexural resistance but proportionately high tensile membrane performance and the highest
robustness (or reserve strength) recorded in testing. This result also validates observations made in Section 5.3.2 that point to the reinforcement detailing of the slab-strips causing the development of efficient catenary mechanisms, facilitating significant extensibility and rotation capacity.

Figure 5.3 – Load enhancement factors recorded in testing.

The edge beam specimens sustained similar ultimate displacements and out-performed the slab-strip specimens in terms of ultimate TMA resistance. However, maximum enhancement was substantially less – obtaining a maximum enhancement of 170.3% in secondary TMA response. This is due to the disproportionality between the area of reinforcement mobilised in bending and that utilised by the catenary mechanism. These specimens featured a high area of flexural reinforcement and small span-depth ratio. Thus the ultimate flexural resistance of these specimens was of the highest tested. However, observations made in Section 5.3.1 indicate that secondary TMA response was dependent on a relatively small area of critical reinforcement (see Table 5-2).

Inspection of Table 5-1 and Table 5-2 shows that whilst system performance varied substantially between specimens, catenary performance was found to be directly proportional to the area of critical reinforcement. This was found to be the principal factor with regards to TMA response and is illustrated by a plot of peak recorded TMA load \( P_{TMA} \), see Figure 5-4a, for which a regression analysis based on a linear best fit trend line gives a \( R^2 \) value of 91.8%. However, as seen in Figure 5-4a specimens S02, C01 and C02 had an identical area of critical reinforcement but S02 was found to outperform the others. Similarly specimen S03 was found to outperform specimen C03. These anomalies can be accounted for by consideration of the superior rotation capacity found in S02 and S03 (see Figure 5-4b) and demonstrates that, for a given area of tying reinforcement, the load carrying capacity of the catenary was found to be directly proportional to rotational capacity and ultimate displacement.
Figure 5-4 – Ultimate recorded catenary load resistance plotted against (a) critical area of reinforcement and (b) chord rotation.

Figure 5-5 – Variation of energy absorption in catenary response with regards the (a) area of critical reinforcement and (b) span-depth ratio.

Note: lower-bound (E01, E03, C02, C04, M01 and M02) and anomalous (C01) results, whereby the test was terminated at the displacement limit of the test rig are indicated in Figure 5-4 and Figure 5-5 by data points with no fill, for clarity.

Figure 5-5a is a plot of the total work done in TMA response ($W_{TMA}$) against the area of critical reinforcement. It can be seen that whilst all the specimens demonstrate a direct correlation, specimens S02 and S03 were found to exhibit uncharacteristically high energy absorption. This is emphasised following comparison with specimens C01, C02 and C03, which featured identical areas of critical reinforcement and similar span-depth ratios.

Although no direct correlation was found between catenary performance and span-depth ratio, Figure 5-5b shows that it might have some influence. However, by accounting for span-depth ratio in this manner, it can be seen that the performance of the column-strip specimens C01, C02 and C03 was clearly inferior. Given that the variation in performance cannot be accounted for by consideration of area of reinforcement or span-depth ratio alone, this suggests that the detailing of slab-strip specimens
S02 and S03 resulted in a catenary mechanism that could absorb greater strain energy than the column strip specimens which featured only minor discontinuities in reinforcement along their spans (see Section 5.3).

5.3 Specimen Response & Catenary Mechanisms

Observations made in Section 5.2 suggest that the different performance observed between the edge beam, slab-strip and control/flat-slab specimens cannot be explained by consideration of the basic specimen properties alone. The following section provides a commentary on the observed large-displacement response of each specimen tested. In-test observations and recorded data are examined to identify trends in the physical response and mechanisms formed by specimens of similar reinforcement arrangement. The section identifies response characteristics that might account for the different performance of the test specimens in catenary action.

5.3.1 Edge Beam specimens

Figure 5-6 is a schematic elevation of the deflection profiles that occurred in specimens E01, E02 and E03 in the final stages of catenary action. The fundamental mechanism was a three-hinged catenary mechanism (CDE) suspended between two cantilevers (BC/EF). With reference to Regan (1975), this mechanism is described as an incomplete catenary.

During early stages of displacement – typically between 20 and 60mm – minor flexural cracking was found to be distributed along the specimens at centres consistent with the position of the shear links. However, as the displacement was increased – at 140mm (3.4°) and greater – plastic hinges were found to develop at B, C, D, E and F, resulting in much larger crack widths at these locations. In each case, it can be seen that the hinge position was related to a reduction in flexural strength and stiffness owing to curtailment and lapping of the reinforcement. Figure 5-7 provides profile images taken at points B, C and D along specimen E01, at 200m (4.6°) displacement. Inspection shows that cracking was localised at either end of the hogging lap. Furthermore, crack distribution at the mid span can be seen to be limited to the length C2E2, denoting the start of the lap between the tie bar and the service span sagging reinforcement (B2 layer).
Figure 5-7b shows the midspan of specimen E01 following failure of the B2 rebar at D. Typically this was found to occur during the late stages of CMA or early stages of TMA, during transition of the horizontal restraint force. With the exception of E02, this was the first bar to rupture and was followed by failure of the T2 hogging reinforcement at points C_1 and E_1. These observations are as anticipated, as the midspan is subject to twice the rotation demand found at the supports. Furthermore, they suggest that rotation limits are dictated by the ultimate strain of the reinforcement and that the strain can be assumed to occur largely at crack locations.

The failure of the T2 reinforcement at locations C_1 and E_1 can be explained by scrutiny of the flexural-axial stiffness along the beam. The strength and stiffness at C/E was significantly less than that at B/F, owing to curtailment and lapping of the T2 hogging reinforcement. The consequential 55-60% reduction in reinforcement area led to a discontinuity in slope, increased rotation and thus failure at C/E. Had the T2 reinforcement been of constant area and continuous between BF, fracture would probably have occurred at B/F – as observed in the flat slab specimens.

Figure 5-8a and b show the condition of E02 at a midspan displacement of 460mm (10.4°), following failure of the T2 reinforcement (points C_1 and E_1). The photographs show initial development of an incomplete catenary mechanism and the failure mechanism observed in all Edge Beam specimens.

In their ultimate condition E01, E02 and E03 were found to have sustained catenary action by tension in the B2 reinforcement and lap at the interior supports (BC/EF) and T2 reinforcement at the midspan (point D) – denoting secondary catenary response. Inspection of Figure 5-8a and b show initial development of longitudinal cracks at the soffit of the edge beam. The cracking was found to occur at points C_1 and E_1 and develop along the line of the longitudinal B2 reinforcement, resulting in the complete separation of cover across the full width of the specimens. The cracking is attributed to debonding along the B2 reinforcement – as this reinforcement layer was found to sustain all tension membrane force – and dowel action due to the development of the incomplete catenary mechanism and the change-in-slope across points C_1 and E_1 (see deflection profile records found in Appendix D).
Increased midspan displacement was found to result in progressive spalling of concrete cover from the soffit of the specimens, typically propagating towards the interior supports. This process is shown clearly in Figure 5-9 to Figure 5-11. Inspection of the images provided (and recorded deflection profiles in Appendix D) shows that displacement of the cantilevers C1B and E1F was relatively minor.

Figure 5-9 – Catenary profile of E03 at midspan displacements; (a) $\Delta = 500\text{mm}$ and (b) $\Delta = 520\text{mm}$.

Figure 5-10 – Photograph of E03, length BC, showing debonding crack development and scabbing of soffit cover at a midspan displacement of 520mm.
As midspan displacement was increased, spalling of the concrete cover would occur incrementally between links and deterioration of the concrete section was observed (Figure 5-11). However, the shear links or containment reinforcement present at C₁ and E₁ were found to provide effective restraint to the B2 reinforcement, preventing total failure of the concrete cover and failure of the B2 compression lap (found between C₁B and E₁F), therefore supporting catenary mechanism C₁DE₁.

From inspection of the reaction data recorded during testing, spalling and concrete degradation at the beam soffit can be seen to have had only limited influence on performance. Figure 5-12 provides the load-rotation record for each of the edge beam specimens. The points of reinforcement rupture have been identified. The load-rotation plots show a series of additional discontinuities in the load-rotation curve (most notable in the case of E01) that can be described as a sudden but short loss of load resistance from which system resistance recovers rapidly. These are attributed to the incremental and progressive loss of cover and concrete deterioration. This suggests limited slip in the system with each loss of cover. Inspection of the records provided in Section 5.5.1 and Appendix D for horizontal restraint force ($R_H$) show that the edge beams would sustain plastic tensile membrane response in this
condition. The horizontal reaction recorded in specimens E01, E02 and E03 is sustained at 1.0-1.1 times the yield strength of the B2 reinforcement, suggesting that the tensile membrane force was not compromised by anchorage failure.

It is important to note that shear links lapped at the bottom face of the beams would be susceptible to failure as deterioration of the concrete and cover would compromise their anchorage/lap, allowing them to ‘unzip’. Thus, shear reinforcement for containment in catenary response should only be lapped in regions where the cover will not be compromised – i.e. the upper portion of the section and to the sides. This demonstrates that reinforcement detailing is an important consideration in the analysis and design of RC catenary systems.

Figure 5-13 shows the condition of E02 immediately after collapse, which occurred at 585mm displacement (13.2°). Collapse was caused by the rupture of the B2 reinforcement layer at E₁. Rupture occurred at the intersection with the shear link over which the reinforcement was supported. Failure occurred here as the area of reinforcement was 70% of the critical reinforcement area found at the midspan (point D). Furthermore, the acute angle by which the B2 reinforcement at E₁ was bent over the supporting link would have resulted in additional stress in the individual bars probably resulting in a reduced tensile capacity.

Figure 5-13 – Photographs of E02 after collapse (at Δ = 585mm), showing (a) collapse mechanism and (b) close-up of B2 rebar fracture and scabbing of soffit cover.

Collapse was not achieved in tests E01 and E03. These tests were terminated at the deflection limit of the test rig. Both specimens sustained a significant loss of cover, without outright failure. The lap lengths critical to catenary action were inspected after testing and no evidence of slip was found.

It should be noted that test E03 was stopped at a smaller displacement than E01. The test was not advanced to the same displacement as attained in E01 as the specimen had acquired its ultimate failure mechanism and, due to the strain-energy in the system, it was decided that further displacement presented an unnecessary potential hazard.
5.3.2 Slab-stripe specimens

Observations made during testing show a high level of consistency between the response of specimens S02 and S03; little discernible cracking was observed following removal of the central support; flexural cracks developed along the specimen corresponding with the shear link locations within a midspan displacement of 20mm (0.5°), and; pronounced cracking, indicating plastic hinge formation, was recorded at the supports and midspan at 50-70mm displacement (1.2-1.6°) shortly followed by cracks opening at curtailment of the hogging reinforcement (typically at 120-130mm displacement, 2.8-3.0°). Crack patterns and hinge positions observed in specimen S02 are shown in Figure 5-14a and b. It should be noted that the development of a secondary hinge at the end of the curtailed hogging reinforcement was found to reduce rotation demand at the supports but as a result cracking was more pronounced at midspan.

![Figure 5-14 – Hinge development observed at the support (a) and midspan (b) of S02 at Δ = 270mm.](image)

Given the high span-depth ratio ($L_{Ed}/d = 66$), frame and arching action were found to give way to catenary action following relatively minor deformation – tensile membrane forces typically being recorded at a displacement of 130mm (3.0°). This was found to be the predominant mode of response and, whilst displacement was facilitated by rotation at the midspan and supports, severe tension cracks were found to open at frequent intervals within the intermediate spans as displacement increased. The final deflection profile and crack distribution are shown in Figure 5-15 and Figure 5-16.

![Figure 5-15 – Schematic elevation of deflection profile and crack distribution observed in specimens S02 and S03 at Δ = 670mm (end of test) – elevation not to scale.](image)
By inspection of Figure 5-15 it can be seen that the mechanism formed in S02 and S03 was dependent upon the area and arrangement of reinforcement across the double span. Points B, F and D denote the position of principal hinge locations at points of maximum moment. However, scrutiny of the change in slope found at D shows that two hinges were formed at the midspan. The hinge locations were found to correspond with the points of bar curtailment at the extents of the midspan tension lap (see Figure 5-14b). Furthermore, inspection of Figure 5-15 and Figure 5-16 shows that the regions of tension cracking were confined to the intermediate spans C1C2 and E1E2 where, given the curtailment of the T2 reinforcement, the total area of reinforcement was 50% of that elsewhere in the specimen. It follows that slab-strips S02 and S03 formed catenary mechanisms for which displacement was facilitated by chord rotation at four-hinge locations (BDDF) and pronounced tensile extension along regions C1C2 and E1E2. Both the hinge locations and regions of predominant tensile extension were directly related to reinforcement curtailment locations and discontinuities in flexural and in-plane tensile strength and stiffness.

The test results show that ultimate catenary chord rotations recorded for S02 and S03 were the greatest achieved in all the tests. Both tests were terminated at the 16\textdegree rig limit without collapse. This shows a higher rotation capacity and extensibility than E02, which failed at 13.2\textdegree, and the average collapse chord rotation of the flat-slab specimens of 8.9\textdegree (see Section 5.3.3). Fracture of extreme tension reinforcement layers was recorded at the midspan at 497mm (11.2\textdegree) and 415mm (9.4\textdegree) for specimens S02 and S03 respectively. Thus specimens S02 and S03 sustained primary and secondary catenary response. Fracture of the extreme tension reinforcement at the interior support locations (points F and B) was observed in specimen S03 only, at displacements of 565mm (12.7\textdegree) and 669mm (15.0\textdegree). These ultimate plastic rotations were 2.1 times those of the average achieved in the other specimens and all occurred during TMA response.

Figure 5-16 – Images of S02 at $\Delta = 670$mm (end of test), showing (a) tension cracking and change in slope at support hinge and (b) final catenary profile.
The superior ultimate displacement performance observed in slab-strip specimens S02 and S03 can be explained by two key features of the catenary mechanism. Firstly, the development of rotational hinges on either side of the midspan B2 lap provided an advantage over the edge beam and flat-slab specimens, for which midspan rotation was mainly confined to a single hinge. The presence of a second rotational hinge suggests that rotational demand at the midspan hinges would have been up to half that experienced at the single midspan hinge observed in the edge and flat-slab specimens. Furthermore, at a given chord rotation, tensile strain in the reinforcement at hinge locations would have been less than in the edge and flat-slab specimens as the slab-strips featured the highest span-depth ratio used in testing, at $2L/d = 66$. Secondly, the tensile response observed along regions $C_1C_2$ and $E_1E_2$ would have improved the extensibility of the system. Given the reduced area of reinforcement along this region, tension membrane force was adequate to cause frequent tension cracking, suggesting inelastic tensile response in the reinforcement. Assuming that plastic strain in the reinforcement is localised to crack locations, the presence of multiple tension cracks is advantageous as this facilitates inelastic extension across multiple points along the catenary. In comparison, crack development in the edge beam and flat-slab specimens was generally confined to hinge locations. Therefore in-plane extension would have been confined to fewer and high stress locations, resulting in ultimate strain being reached at lower midspan displacements.

The slab-strip specimens featured the same reinforcement detailing and span-depth ratio. Specimen S03 was cast with 50% more reinforcement than S02. However, inspection of the ultimate plastic rotation capacity recorded at the interior supports and midspan do not provide conclusive evidence of any relation with reinforcement area. Furthermore, as collapse was not achieved in either specimen, comparison cannot be made here either.

With an identical span-depth ratio and rebar arrangement, it was anticipated that S01 would develop a similar catenary mechanism to those observed in S02 and S03. However, specimen S01 collapsed in secondary catenary response at a midspan displacement of only 372mm (8.5°), with none of the catenary mechanism characteristics observed in the more heavily reinforced counterparts, S02 and S03. This therefore indicates that the area of reinforcement was critical to the development of the catenary mechanisms observed in the slab specimens.

Figure 5-17 is a sketch of the mechanism observed in specimen S01 immediately prior to collapse. Response showed good consistency with that observed in S02 and S03 at lower displacements – during frame and arching action minor crack development was observed at each side of the tension lap, consistent with S02 and S03 mechanisms. However, as displacement was increased a single plastic-hinge was found to develop at the midspan (point D), central to the B2 lap. This was followed by fracture of the B2 reinforcement layer at 178mm displacement (4.1°) in this location, approximately 0.4 times the chord rotation recorded for midspan B2 reinforcement fracture in the more heavily reinforced specimens. Following this event, as displacement and tensile membrane force increased, significant crack development and change in slope was observed across points $C_2$ and $E_2$. This
response can be described as ‘tensile alignment’, where the specimen deformed to provide a direct line of action between the centroid of tensile strength at the midspan (T2 reinforcement layers) and the supports (bias to the B2 reinforcement layers). Therefore, development of these new hinge locations can be attributed to the in-plane eccentricity of the centroid of tensile strength at points B, D and F, and the resulting second order effects along the specimen, together with the reinforcement curtailment at points C₂ and E₂, which provided nominal hogging moment resistance along regions C₁C₂ and E₁E₂.

Figure 5-17 – Schematic elevation of deflection profile and crack distribution observed in S01 at Λ = 372mm (prior to failure) – elevation not to scale.

Figure 5-17 shows that S01 sustained fracture of the T2 reinforcement at point B, followed by tensile failure of the B2 reinforcement at point C₂, resulting in collapse of the specimen. Collapse of the specimen occurred without the development of traverse tension cracks as seen in specimens S02 and S03.

Scrutiny of the specimen specification showed that the response and mechanism formed by S01 could only be attributed to its smaller area of reinforcement. All slab specimens had the same cross sectional area but specimen S1 was detailed with 50% the area of reinforcement used in S02. As a result, it appears that the rotational strength and stiffness across the midspan lap of the B2 reinforcement was inadequate to prevent development of a single plastic hinge at midspan. Furthermore, the area of reinforcement provided was insufficient to cause traverse tensile cracks along regions C₁C₂ and E₁E₂ – the total ultimate tensile strength of the B2 reinforcement layer (34kN) was approximately 35% the theoretical tensile strength of the un-cracked concrete section (98kN), calculated in accordance with EC2 (BSI, 2008a) using the mean crushing strength of the specimen cube samples. This compared with a ratio of 0.62 and 1.1 for specimens S02 and S03.

The extent to which S01 was found to underperform can be seen in Figure 5-18 and can be related to the failure of S01 to develop a four-hinge mechanism and transverse tension cracking, thus confining in-plane extension to hinge locations and resulting in a lower ultimate displacement than observed in the other specimens. Given that the development of the single hinge at D was the root cause of the primary ductility and thus performance, the importance of accurately predicting hinge development so as to prevent significant over or underestimation of catenary performance becomes clear.
One shortcoming of the testing conducted in this investigation is that the specimens did not feature down-stands (an increased section thickness) at joint locations (points B, D and F) associated with intermediate supporting beams, columns and framing. These thickenings would be present in the span of a double structural bay of a building that sustains support loss and would result in a distinct increase in rotational stiffness at midspan and support locations. This suggests that in-test plastic hinge development at points B and F may be optimistic as the test arrangement allowed higher rotational freedom than would be found in practice. Furthermore, the presence of a down-stand or column-stub at the midspan joint would have made development of a single hinge at point D unlikely. Rather, hinge development would occur at either side of the joint down-stand or column stub. This suggests that the development of a four-hinged catenary mechanism is the most likely form of response in framed double bays and that specimens S02 and S03 are more representative of in-situ catenary performance. However, in the case of an asymmetrical double bay, where the joint is not located at the midspan, a three-hinge catenary mechanism would be more appropriate.

It should be noted that post failure investigations found no evidence of slip at the tension laps. Furthermore, none of the slap-strip specimens exhibited any sign of distress in the region C3D and DE2 following fracture of the B2 reinforcement at D. This suggests that the length of curtailment advocated by BS 8110 and EC2 was sufficient to support the safe transfer of TMA forces between the top and bottom rebar layers. It follows that, despite the different responses, the load carrying capacity of each catenary mechanism was a function of the system geometry and the tensile capacity of the critical reinforcement at D and in the region C1C2/E1E2.

5.3.3 Flat-slab strip specimens

The flat-slab specimens (C01, C02, C03, C04, M01 and M02) featured no discontinuities in the longitudinal reinforcement – all lapping of the rebar occurred outside the double span and the T2 and B2 layers of reinforcement were of equal area. The response of each specimen was consequently found to be less complex than that observed in the edge beam and slab-strip test specimens.
Figure 5-19 – Typical deflection profile of flat-slab specimens at failure (elevation – not to scale).

Figure 5-19 shows the typical deflection profile and mode of response at failure. Although hinge formation was observed at the supports and midspan, the slope across these locations demonstrated a greater spread of plastic rotation than found in the edge-beam and slab specimens. Flexural cracks were found to coincide with the locations of the shear links and inspection of the size of these flexural cracks showed the crack widths reduced with distance from the points of maximum moment (points B, D and F). This three-hinged system was found to develop throughout frame and arching action and resulted in the fracture of the B2 reinforcement at D. With the exception of C03, fracture of the extreme tension reinforcement at midspan was found to occur prior to TMA response, typically during snap-through. The recorded chord rotations were between 3.1° and 6.6°, demonstrating a significant variation in recorded plastic rotation capacity.

In secondary catenary response, following the fracture of the B2 reinforcement layer at point D, increased displacement was found to result in the development of tensile membrane forces and load resistance. However, behaviour of the flat-slab specimens demonstrated distinctive flexural response. The specimens were found to behave as a pair of cantilevers tied by the critical T2 reinforcement at D. This was a significant feature in the response of specimens C02, C03 and M02. Each of these specimens sustained tensile rupture of the T2 midspan reinforcement layers at displacements of 291mm (7.3°), 445mm (11.6°) and 404mm (9.7°), resulting in the loss of all tension membrane force. However, inspection of the load-rotation graphs provided in Appendix D shows that these specimens did not collapse. Rather, C02, C03 and M02 demonstrate a nominal load resistance attributed to the hogging strength of cantilevers BD and FD. Although this residual frame action would not prevent incipient collapse under load-controlled support loss conditions, this is a significant observation as all current analytical methods for the assessment of catenary response assume pure plastic membrane behaviour, with no residual flexural resistance.

Specimen M01 sustained failure of the T2 hogging reinforcement at hinge locations B and F, in bending, prior to tensile rupture of the T2 reinforcement at the midspan. It follows that between displacements of 208mm (6.1°) and 277mm (8.1°) load resistance of the specimen was attributed to pure TMA response, sustained by the critical reinforcement T2, at the midspan, and B2, at the supports. Furthermore, failure of the T2 reinforcement layer at the midspan resulted in complete loss of load resistance and collapse of the specimen.
Response observed in specimen C01 was not consistent with the other flat-slab specimens. Inspection of the test chronology for C01 showed that the specimen failed by rebar rupture at support F. Fracture of the T2 reinforcement was observed at a displacement of 221mm (5.1°), under combined bending and tension. The specimen then collapsed following failure of the B2 reinforcement layer, at the same location, at a displacement of 249mm (5.7°). Specimen C01 is the only test where maximum TMA resistance was sustained in primary catenary response, prior to the fracture of the extreme tension rebar at point F.

Inspection of the vertical restraint reaction data (see Appendix D) provides evidence that the difference in response noted in specimen C01 can be attributed to an asymmetric load arrangement. It can be seen that, at a chord rotation of approximately 0.2°, the support reactions $R_1$ and $R_2$ (corresponding to points B and F, respectively) diverge and the $R_2$ to record shows a reaction consistently higher than $R_1$. This divergence is likely attributed to mechanical slip in the $R_1$ hanger, due to the coarse tread tolerance of the vertical DiwiDag tie bars, resulting in minor downward movement at support B and increased bending stress across support F. However, the results for this test have not been discarded as catenary behaviour was evident.

The load resistance records of the six flat-slab specimens are provided in Figure 5-20 and Figure 5-21, for comparison. Inspection of the load-rotation behaviour of the column and middle-strip specimens shows a distinct relationship between the performances of the specimens. It can be seen that catenary resistance is directly proportional to the area of reinforcement. Furthermore, specimens with lower span-depth ratios appear to demonstrate a larger chord rotation at maximum TMA resistance and collapse. This latter observation contradicts observations made by previous researchers and TMA design guidance (DoD, 1998; GSA, 2003) that suggest more restrictive characteristic ultimate displacements in catenary action for RC slab systems of low span-depth ratio. This subject is investigated in greater detail in the following sections.

![Figure 5-20 – Load-rotation record of column-strip flat-slab specimens C01, C02, C03 and C04.](image-url)
5.4 Rotation Limits & Specimen Extensibility

One of the objectives of the experimental investigation was to obtain ultimate displacement data for primary and secondary TMA response. The displacements and chord rotations corresponding to key discontinuities in the load-displacement plots of each test specimen have been identified (summarised for individual tests in Appendix D).

This section discusses trends in the displacement/rotation at which the specimens were found to change from CMA to TMA response ($\theta_0$), at which failure of the extreme tension reinforcement occurred ($\theta'_u$) and at which failure of catenary response occurred ($\theta''_u$). Furthermore, in support of observations made in Section 5.3, the influence of reinforcement arrangement is investigated by examining the results recorded for each test group. The results are provided below.

5.4.1 CMA-TMA transition, $\theta_0$

CMA-TMA transition was taken to occur at $\Delta_0$, the midspan displacement at which the compressive membrane force was found to reduce to zero prior to the development of the tensile axial force associated with catenary action (see Figure 5-2 and Figure 5-30). Values of $\Delta_0$ are identified in Table 5-1 and shown below normalised against section height ($h$), see Figure 5-22.

Inspection shows that transition between membrane responses is directly related to the ratio $\Delta_0/h$. The average transition displacement found in the tests was $\Delta_0 = 1.34h$ (with standard deviation of 0.23 and $R^2$ value of 77.6%); translating to a transition chord rotation of $\theta_0 = \tan^{-1} 1.34h/L$. This result indicates transition to TMA at displacements higher than suggested by Park and Gamble (2000), who found this corresponded ‘approximately’ to 1.0$h$ for the small scale two-way slabs tested.
Further investigation of the test results showed $\Delta_0$ to be inversely proportional to span-thickness ratio ($2L/h$). This trend is in general agreement with observations made by Keenan (1969) and is shown below. Based on a linear trend line (of $\Delta_0 = -8.3L/h + 335.7$), the test results were found to give a $R^2$ value of 79.4%.

These results were examined to investigate additional trends between $\Delta_0$, $2L/h$ and area of reinforcement. However, no correlations were evident. The results were also examined in sub-groups based on specimen type, physical condition at transition and ultimate failure mode (as differentiated
in Figure 5-22 and Figure 5-23). However, no correlations were evident. Average results obtained for edge, slab and flat-slab test groups were 1.32, 1.21 and 1.40\(h\) respectively. This showed that reinforcement detailing had no significant influence on \(\Delta_0\). Furthermore, from inspection of Figure 5-22 and Figure 5-23 it can be seen that reinforcement failure prior to transition had no significant influence on transition displacement.

Given that the results obtained were typically greater than \(\Delta_0/h \approx 1.0\), suggested by Park and Gamble (2000), an investigation was undertaken to establish the validity of this observation. Results obtained by other authors, for similar beam and slab strip test specimens, were investigated to allow comparison. Examination of the eight and eleven specimen test series reported by Yu and Tan (2013) and Su et al. (2008) provided average values of approximately 1.04 and 0.74\(h\), respectively. This supports an interpretation that transition in testing occurred at relatively large displacements. However, the tests reported by Gouverneur et al. (2013b) give results of approximately 1.62 and 1.74\(h\) for the two test specimens and therefore support the finding that \(\Delta_0\) can occur at displacements exceeding \(h\). A common feature of the test series reported here and those of Gouverneur et al. is that specimens were continuous across interior supports (B and F, Figure 5-1), whereas Yu and Tan and Su et al. specimens were fully encastre. This suggests that the higher transition displacements observed might be attributed to rotation across interior supports and that \(\Delta_0\) will increase as rotational restraint is reduced. This hypothesis is supported by results obtained by Yi et al. (2008) in testing a four bay, three floor, planar frame specimen (see Chapter 3). This test arrangement featured semi-rigid boundary constraints and inspection of the lateral translation record of the exterior columns, \(\Delta_0\) was reported at between 1.2 and 1.4\(h\).

5.4.2 Ultimate plastic chord & joint rotation, \(\theta_u'\) & \(\phi_u'\)

The ultimate plastic rotation \((\theta_u')\) is defined here as the chord rotation \((\theta)\) at which the extreme tension reinforcement was found to fracture. Thus, \(\theta_u'\) defines the transition from primary and secondary TMA response.

Figure 5-24 provides a summary of the \(\theta_u'\) recorded for each test specimen. Values shown are the midspan chord rotations recorded at the principal/first extreme tension reinforcement fracture event of each test – subsequent fracture events are not shown. The results shown are associated with rupture of the B2 reinforcement layer at the midspan, with the exception of those values given for specimens E02 and C01 for which first fracture was found to occur at in the T2 reinforcement layer at or near support locations. As indicated in Section 5.2, half of the test specimens (E01, E03, C02, C04, M01 and M02) were found to reach their ultimate plastic rotation during snap-through response \((\theta_u' < \theta_0)\), prior to the development of any tensile membrane force. These results are identified and shown in Figure 5-24.
By inspection of the full population of $\theta_u'$ – including those recorded during snap-through and TMA response – the data gives an average $\theta_u'$ of 5.56°. The maximum ultimate plastic rotations were recorded for test specimens S02 and S03, with an average $\theta_u'$ of 10.33° – 1.86 times the average recorded for the full population. Test specimen E01 was found to sustain the minimum ultimate plastic rotation, at 1.99°.

Park and Gamble (2000), DoD (2005), GSA (2003) and Stevens (2008) have suggested the use of a characteristic ultimate displacement of 12° for the analysis and design of primary catenary response (see Chapter 3). This value of $\theta_u'$ is used for overload and robustness applications and can be seen to be the basis of the tie-force requirements stipulated by BSI (2005b; 2008a) and DoD (2013). However, the proposed rotation limit is 2.16 times the average $\theta_u'$ recorded in testing and, by inspection of Figure 5-24, it can be seen all test specimens sustained fracture of the extreme tension reinforcement prior to reaching a 12° chord rotation. With an average recorded $\theta_u'$ of 5.56°, the test results show a closer correlation with the 6° rotation limit recommended by DoD (1990; 2008; 2014) for conventional unrestrained RC elements, with shear reinforcement. However, only 25% of test specimens (S02, S03 and C03) exceeded 6° before sustaining fracture in the extreme reinforcement layers at hinge locations.

By excluding results obtained at $\theta < \theta_0$, the average ultimate chord rotation in primary TMA response ($\theta'_\text{TMA}$) was found to be 7.05°, with a standard deviation of 2.51°. Thus, the 12° ultimate end-rotation criterion was found to over-predict the average recorded primary TMA response limit by 41.3%.

By inspection of the results obtained for individual test groups, the average recorded $\theta_u'$ for the edge-beam, slab-strip and flat-slab specimens were found to be 4.31°, 8.25° and 4.84°, respectively. These results demonstrate the significant performance advantage found in the response of the slab-strip specimens.
Section 5.3.2 describes the three-hinge mechanism formed by test specimen S01 and the four-hinge mechanisms developed by S02 and S03. In each case $\theta_u'$ was attained upon fracture of the B2 reinforcement layer at the midspan. However, the $\theta_u'$ recorded for specimens S02 and S03 is 1.38 and 1.16 times that recorded for S01, respectively. The difference between these results demonstrates the importance of accounting for the mechanism and number of plastic hinges formed in large-displacement response. Figure 5-25 shows sketches of three and four-hinge mechanisms – schematic representations of those formed by slab-strip specimens. Inspection shows the rotation sustained at each joint, defined here as joint rotation angle ($\phi$), to be approximately equal to the chord rotation ($\theta$) for all interior support locations. However, $\phi \approx 2\theta$ at the midspan of the three-hinge mechanism where the midspan hinge locations of the four-hinge system sustain $\phi \approx \theta$.

Figure 5-25 – Sketch showing derivation of chord and joint rotation angles in (a) three-pin and (b) four-pin mechanisms.

Figure 5-26 provides a summary of the principal ultimate plastic joint rotation ($\phi_u'$) of each test specimen. Values of $\phi_u'$ were calculated using the convention shown in Figure 5-25. Thus $\phi_u' = \theta_u'$ for specimens S02 and S03, which sustained four-hinge mechanisms, and specimens E02 and C01, which sustained principal fracture of the extreme tension T2 reinforcement near support locations. The remaining specimens sustained three-hinge mechanisms and ultimate plastic rotations at midspan hinge locations, therefore $\phi_u' = 2\theta_u'$.

Figure 5-26 – Ultimate joint rotation against reinforcement ratio of the local extreme tension rebar.
By resolving the ultimate plastic chord rotation data for ultimate plastic joint rotation ($\phi_u'$), the results are found to provide improved unity. Inspection of the data provided shows that specimen E01 was found to feature the lowest ultimate joint rotation, the midspan B2 reinforcement fracturing at 4.0°. The highest recorded value of $\phi_u'$ was 13.17° and was observed in specimen C03, again attributed to midspan B2 reinforcement fracture. Assessment of the recorded data provides a mean $\phi_u'$ of 8.49° with a standard deviation of 2.67°.

An investigation was conducted in an attempt to ascertain a relationship between the results obtained and the properties of the test specimens. The closest correlation established was based upon the reinforcement ratio ($\rho$) of the extreme tension steel found at each principal hinge location. This relationship is shown in Figure 5-26. A regression analysis demonstrates only a weak relation between the two factors, giving an $R^2$ value of 0.335 based on the total test population and a best fit straight line.

### 5.4.3 Secondary ultimate chord rotation, $\theta''_u$

The ultimate load capacity of a given catenary mechanism is dependent upon its ability to sustain significant chord rotations without loss of in-plane force (Chapter 3). Figure 5-27 identifies the ultimate chord rotation recorded in secondary catenary response ($\theta''_u$). These values correspond to the peak TMA load resistance at chord rotations $\theta > \theta''_u$. With the exception of specimen C01, where $P'_{TMA} > P''_{TMA}$, all values of $\theta''_u$ correspond to the peak TMA load resistance ($P_{TMA}$) of each test specimen (see Section 5.2).

![Figure 5-27 – Experimental record of failure chord rotation in TMA response.](image)

As indicated, the chord rotations specified for specimens E01, E02, S02 and S03 correspond with the midspan deflection at $P''_{TMA}$ and termination of the test, as the displacement limit of the test rig precluded collapse. These values are therefore considered lower-bound results of $\theta''_u$ for the respective
specimens. Conversely, the large-displacement behaviour of specimen C01 was identified as anomalous, due to asymmetric restraint conditions across the interior supports. These results are identified in Figure 5-27 (and Figure 5-28).

Assessment of the test data (inclusive of the lower-bound results recorded for specimens E01, E02, S02 and S03 and the anomalous result of C01) gives an average $\theta_u''$ of 10.91°, with a standard deviation of 3.10°. With the exception of C01, the minimum $\theta_u''$ was 6.97°, recorded in specimen C02 at incipient collapse.

The USACE provides guidance for the analysis and design of RC elements in secondary catenary action (see Chapter 3). Recommendations are based on empirical observations made by Woodson and Garner (1985), Guice (1986) and Woodson (1990, 1992 and 1994) and point to incipient collapse at 20° chord rotation (thus, $\theta_u'' \geq 20°$). This recommendation is only used in overload applications (DoD, 1998; USACE, 2008a), not double-span or robustness conditions. However, this rotation limit is 1.83 times the average $\theta_u''$ recorded in testing. Tests S02 and S03 were terminated prior to collapse and sustained the maximum recorded chord rotations of 15.01° and 14.98°, without failure. However, the test data provides no conclusive evidence to suggest that these specimens could have sustained 20° before outright failure.

The test results can be seen to have a better correlation with the 12° ultimate rotation limit recommended made by Park and Gamble (2000), intended to characterise ultimate rotation under primary catenary response conditions (see Chapter 3). However, this recommendation exceeds the average recorded $\theta_u''$ by 10%. Furthermore, it can be seen that half of the test specimens sustained outright collapse prior to a chord rotation of 12°.

By inspection of the individual test groups, the results show a distinct difference in performance. The flat-slab specimens, which featured continuous reinforcing, were found to give an average $\theta_u''$ of 8.99°, with a standard deviation of 2.45°. By comparison, the average $\theta_u''$ recorded for the edge beam and slab-strip specimens were 12.86° and 12.80°, respectively. Given that 50% of the edge beam and slab specimens did not sustain collapse, this shows the specimens with conventional reinforcement detailing to have sustained average ultimate secondary rotations more than 42% greater than the control specimens. With reference to Sections 5.2 and 5.3, this result provides evidence that the greater performance recorded in the edge beam and slab specimens was attributed to their capacity for greater ultimate chord rotations than counterpart flat-slab specimens. Furthermore, it suggests that conventional reinforcement arrangements are favourable, provided that they facilitate the development of the catenary mechanisms observed in testing.

The test data was used to investigate trends between the ultimate chord rotation ($\theta_u''$) and the properties of individual test specimens. Given the difference in performance observed between test series, the results were examined by groups of specimen type and thus catenary mechanism developed at incipient
collapse (see Section 5.3). As results obtained for two thirds of the edge beam and slab-strip specimens were lower-bound, interpretation of incomplete and four-hinge catenary mechanisms could not be supported. Thus, the flat-slab strip specimens (C&M Group) were considered in isolation, with S01, to investigate results obtained of the three-hinge catenaries. Note; the result obtained for C01 was excluded as anomalous (see Section 5.3.3).

Figure 5-28a shows a plot of $\theta_u''$ results against area of critical reinforcement. It can be seen that the results follow a general trend, where recorded values of $\theta_u''$ increase with area of critical reinforcement [mm²]. Inspection of specimens C02, C03, C04, M01, M02 and S01 gives an R² value of 71.9%. By considering the percentage area of reinforcement [%] and reinforcement ratio ($\rho$), see Figure 5-28b, this relationship is weakened – giving R² values of 55.4% and 51.6%, respectively. All R² values were calculated based on best-fit linear trend lines.

![Figure 5-28](image)

Figure 5-28 – Relationship of recorded ultimate secondary chord rotation against (a) area of critical reinforcement and (b) ratio of critical reinforcement.
Although the trends identified in Figure 5-28a and b are too weak to provide any conclusive evidence in the prediction or interpretation of \( \theta''_u \), it can be seen that the area of reinforcement and section properties were found to influence ultimate chord rotation of a three-hinge catenary mechanism.

### 5.4.4 Elongation at ultimate secondary displacement

An investigation of ultimate elongation (\( \delta L/L \)) was undertaken to assess the strain smearing sustained by each of the test specimens. Given incipient system collapse was sustained at \( \theta''_u \) (except for test specimens E01, E03, S02 and S03), ultimate elongation was calculated as follows:

\[
\delta L/L = \sqrt{\left(\frac{L^2 + \Delta^2}{L}\right) - L} \cong \frac{1}{2} \left(\frac{\Delta}{L}\right)^2
\]

Equation 5-3 is derived for the elongation sustained by the bilinear catenary profile shown in Figure 5-31 for a given midspan displacement (\( \Delta \)). This is a simplification of the deflection profiles and mechanisms observed in Section 5.3 but provides a basis for comparison, and is consistent with the approach taken by Regan (1975) and Thodi et al. (2014) in assessment of catenary extensibility.

Figure 5-29 provides a graphical summary of the results obtained for each test specimen.

![Figure 5-29](image-url)

**Figure 5-29** – Specimen percentage elongation, recorded at incipient collapse.
5.5 Load Resistance & Membrane Forces in Catenary Action

Figure 5-30 shows the force-rotation records obtained for slab-strip specimen S03 and serves as an example of the relationship observed between the applied load \((P)\) and the horizontal restraint force \((R_H)\) with displacement. The raw data for each test is provided in Appendix D but in Figure 5-30, values of \(P\) and \(R_H\) have been shown as a percentage of the maximum recorded values, for clarity.

![Figure 5-30 – Force-rotation plot for test specimen S03 showing applied load and restraint force.](image)

Three phases of large displacement laterally restrained behaviour are identified in Figure 5-30; CMA and Snap-through, primary TMA and secondary TMA response. During primary and secondary catenary (TMA) response it can be seen that \(R_H\) exhibits characteristic elastic-plastic tensile behaviour due to rebar strains induced by displacement of the specimen. Both modes of catenary response were thus found to be composed of two key stages; an elastic tensile membrane phase \((\theta_0 < \theta < \theta'_y; \theta'_u < \theta < \theta''_y)\) and the plastic tensile membrane phase \((\theta'_y < \theta < \theta'_u; \theta''_u < \theta < \theta''_y)\) whereby the horizontal restraint force was found to remain almost constant under subsequent displacement but showed some evidence of strain hardening.

Inspection of Figure 5-30 shows both horizontal restraint force and rotation increase with applied load. The applied load increased linearly between \(\theta_0\) and \(\theta'_y\) and continued to increase linearly from \(\theta'_y\) until \(\theta'_u\) but with a reduced slope. Similar elastic-plastic load-resistance behaviour is observed in secondary response. Furthermore, for a given horizontal restraint capacity, the ultimate load resistance of the system \((P_{TMA})\) was found to be dependent upon the ultimate rotation capacity of the mechanism.

The test rig was designed and instrumented so as to provide values of all relevant loads and forces to allow direct scrutiny of the relationships between the applied load \((P)\), horizontal restraint force \((R_H)\), axial or membrane forces \((N)\) and chord rotation \((\theta = \tan^{-1} \Delta/L)\) in catenary response. Figure 5-31a shows a sketch of the test arrangement, geometry and reactions as recorded in testing of the specimens. Existing TMA theory (Park and Gamble, 2000 and Regan, 1975) assumes a simple plastic membrane – i.e. with no flexural resistance – forming a bilinear or parabolic deflected profile as the basis of
analysis. Figure 5-31b shows the resolution of the test arrangement to form a simple three-hinge catenary mechanism between points B, D and F, consistent with the existing analytical approaches.

![Diagram showing specimen geometry and reaction diagram](image)

**Figure 5-31a** – Specimen geometry and reaction diagram – in-test arrangement and convention.

![Diagram showing simple catenary free-body diagram](image)

**Figure 5-31b** – Simple catenary (BDF) free-body diagram.

Note; in accordance with the sign convention the horizontal restraint reaction is negative in TMA response and results in a positive tensile in-plane membrane force.

The following sections detail an investigation of the relationship between $P$, $R_H$, $N$ and $\theta$ in primary and secondary catenary response. The mechanism shown by Figure 5-31b is used as the basis of analysis. It is noted that the bilinear profile BDF is less complex than the catenary mechanisms developed by some of the test specimens (see Section 5.3). Inaccuracies that result from the simplification of the theoretical catenary mechanism are also investigated and discussed.

### 5.5.1 Horizontal restraint & membrane force, $R_H$ & $N$

As shown in Figure 5-30, the load resistance curve of a catenary system was typically nonlinear – a function of the chord rotation and the elastic-plastic response of the reinforcement membrane. However, the theoretical studies of catenary performance investigated in Chapter 3, such as those conducted by Regan (1975), Merola (2000) and Li et al. (2011), entail the use of a simplified linear load-displacement relationship whereby the horizontal restraint force was assumed to be constant. This approach is supported by guidance for the direct design and analysis of catenary action in RC elements, as seen in DoD (2005; 2014), USACE (2008a; 2008b) and GSA (2003). However, the investigation in Chapter 3 showed that sources conflicted regarding the basis of an appropriate membrane or horizontal restraint force for the prediction of ultimate load resistance. Specifically, there is uncertainty regarding what area of reinforcement to be considered and whether to account for strain hardening effects.
Previous investigations have assumed in-plane membrane force ($N$) to be approximately equal to the horizontal restraint force ($R_H$), $R_H \approx N$. However, this provides an underestimate of membrane force and thus an inaccurate interpretation of tensile stress in the membrane reinforcement. From the free body diagrams in Figure 5-31, the tensile membrane force is greatest at the supports, points B and F, because of the distributed weight of the specimen ($g_k$). As the resultant of the vertical ($R_{BD}$; $R_{DF}$) and horizontal restraint reactions, the membrane force exceeds the horizontal restraint force at this location and can be expressed as:

$$N = \sqrt{R_{BD}^2 + R_H^2} = \sqrt{(-g_kL - 0.5P)^2 + R_H^2}$$

Equation 5-4

Appendix D provides a full account of horizontal restraint force ($R_H$) data for each of the test specimens. This section details an investigation of the horizontal and in-plane membrane ($N$) force sustained in testing and identifies tension stress and reinforcement area criteria sustained at ultimate load resistance, for primary and secondary catenary response ($P'_{TMA}$ and $P''_{TMA}$).

**5.5.1.1 Membrane force at ultimate secondary TMA response, $N''$**

Table 5-3 shows the values of $R_H$ recorded for each test specimen at secondary ultimate load, $P''_{TMA}$. Values given for $R_{BD}/DF$ and $N''$ were calculated using Equation 5-4. By evaluating membrane force ($N''$) at ultimate load ($P''_{TMA}$), this allowed the investigation of the ultimate in-plane tensile force and stress appropriate to catenary analysis and design in secondary TMA response.

In secondary TMA response, two areas of reinforcement were considered for investigation of membrane force: $A_{s-0.5}$ and $A_{s-crit}$. $A_{s-crit}$ was the actual area of critical reinforcement identified during testing to sustain catenary action in secondary response – thus the smaller of the B2 layer at hinge locations D/F or C/E and the T2 layer at the midspan hinge, location D. The area of reinforcement identified $A_{s-0.5}$ was taken as 50% of the total area of continuous longitudinal reinforcement (T2 and B2 layers). This is consistent with guidance provided by DoD (1998; 2002; 2008; 2014) and Woodson (1994) for the analysis of RC elements in secondary catenary response – which means assuming fracture of the extreme tension reinforcement at critical sections has already taken place and the catenary is sustained by the remaining reinforcement layers. Note that $A_{s-0.5}$ was equal to $A_{s-crit}$ for the flat-slab specimens due to the continuous and equal B2 and T2 reinforcement layers. Results associated with this prescribed area of reinforcement were investigated in order to assess the validity of this approach to quantifying $N''$.

The ultimate in-plane membrane force $N''$ was assessed using both areas of reinforcement to give results for ultimate tensile membrane stress ($N''/A_k$) and ratios $N''/R_e$ and $N''/R_m$ – where $R_e$ and $R_m$ are the measured yield and tensile strengths of the corresponding reinforcement, respectively (see Appendix C for full set of test results). It should be noted that the average strain hardening, recorded
during reinforcement testing, was found to constitute an 18.5% increase on yield strength, thus \( R_m = 1.185 R_e \).

\[
R_m = 1.185 R_e.
\]

Table 5-3 – Horizontal restraint and tension membrane force data, recorded at ultimate secondary TMA load.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Reaction Data at ( P^{''}_{TMA} )</th>
<th>50% Continuous Reinforcement</th>
<th>Critical Reinforcement at Locations B, D or F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R_{nt} ) ( R_n ) ( N^{''} )</td>
<td>( A_{s,0.5} ) ( N^{''}/A_{s,0.5} ) ( N^{''}/R_e ) ( N^{''}/R_m )</td>
<td>( A_{s-crit} ) ( N^{''}/A_{s-crit} ) ( N^{''}/R_e ) ( N^{''}/R_m ) ( N^{''}/R_{0.5(e+m)} )</td>
</tr>
<tr>
<td></td>
<td>[kN] [kN] [kN]</td>
<td>[mm²]</td>
<td>[MPa]</td>
</tr>
<tr>
<td>Edge beam specimens – 2L/d = 35.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E01*</td>
<td>28.4 -81.4 86.2</td>
<td>129 669 1.26 1.06</td>
<td>157 549 1.07 0.91 0.98</td>
</tr>
<tr>
<td>E02</td>
<td>37.3 -128.0 133.3</td>
<td>193 690 1.30 1.09</td>
<td>236 566 1.10 0.94 1.01</td>
</tr>
<tr>
<td>E03*</td>
<td>51.2 -195.3 201.9</td>
<td>287 702 1.35 1.15</td>
<td>339 595 1.13 0.97 1.04</td>
</tr>
<tr>
<td>Mean</td>
<td>- - -</td>
<td>-</td>
<td>687 1.31 1.10</td>
</tr>
<tr>
<td>Slab-strip specimens – 2L/d = 65.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S01</td>
<td>5.5 -28.4 29.0</td>
<td>25 1152 2.08 1.71</td>
<td>50 576 1.04 0.85 0.94</td>
</tr>
<tr>
<td>S02*</td>
<td>21.0 -55.8 59.7</td>
<td>50 1187 2.14 1.76</td>
<td>101 593 1.07 0.88 0.97</td>
</tr>
<tr>
<td>S03*</td>
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<td>75 1164 2.10 1.73</td>
<td>151 582 1.05 0.86 0.95</td>
</tr>
<tr>
<td>Mean</td>
<td>- - -</td>
<td>-</td>
<td>1168 2.10 1.73</td>
</tr>
<tr>
<td>Column-strip specimens – 31.6 ≤ 2L/d ≤ 60.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C01</td>
<td>8.8 -49.4 50.2</td>
<td>101 499 0.90 0.74</td>
<td>101 499 0.90 0.74 0.81</td>
</tr>
<tr>
<td>C02</td>
<td>10.2 -57.0 57.9</td>
<td>101 576 1.04 0.85</td>
<td>101 576 1.04 0.85 0.94</td>
</tr>
<tr>
<td>C03</td>
<td>23.9 -90.0 93.1</td>
<td>151 618 1.11 0.92</td>
<td>151 618 1.11 0.92 1.00</td>
</tr>
<tr>
<td>C04</td>
<td>36.1 -119.7 125.0</td>
<td>201 622 1.12 0.92</td>
<td>201 622 1.12 0.92 1.01</td>
</tr>
<tr>
<td>Mean</td>
<td>- - -</td>
<td>-</td>
<td>578 1.04 0.86</td>
</tr>
<tr>
<td>Middle-strip specimens – 40.0 ≤ 2L/d ≤ 50.0</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>M01</td>
<td>3.9 -28.7 28.9</td>
<td>50 576 1.04 0.85</td>
<td>50 576 1.04 0.85 0.94</td>
</tr>
<tr>
<td>M02</td>
<td>11.0 -47.4 48.6</td>
<td>79 619 1.21 1.03</td>
<td>79 619 1.21 1.03 1.11</td>
</tr>
<tr>
<td>Mean</td>
<td>- - -</td>
<td>-</td>
<td>597 1.12 0.94</td>
</tr>
</tbody>
</table>

*Collapse not achieved – test terminated prior to out-right failure.

Results obtained for \( A_{s-crit} \) were found to provide good agreement between \( N^{''}/R_e \) and \( R_m \). By inspection of the results it can be seen that membrane force at ultimate load (\( P^{''}_{TMA} \)) was reliably greater than the yield strength of the critical reinforcement. The results give an average \( N^{''}/R_e \) of 1.07. Although this does not constitute the full strength enhancement of 18.5%, this is evidence of some strain hardening.

Full strain hardening was found to occur in test specimen M02, only. An average \( N^{''}/R_m \) of 0.89 was obtained. However, it is important to note that tests S02, S03, E01 and E03, which did not reach \( N^{''}/R_m \) of 1.0, were terminated prior to collapse. Thus, \( N^{''} \) results for these specimens are not necessarily representative of conditions at ultimate load and may be conservative. Furthermore, the
lowest ultimate membrane force was recorded for specimen C01. As discussed in Section 5.3 the response of this particular specimen was abnormal compared with the other test specimens – the system failure by fracture of the T2 and B2 reinforcement layers at support location F, due to a misalignment of hanger height. This result is therefore considered unrepresentative as failure of this specimen was found to occur at a significantly smaller displacement than the other specimens and observed response distinctly flexural in nature. By discounting these results, an average \( N''/R_m \) of 0.91 was obtained, with a standard deviation of 6%.

Column 13, Table 5-3 provides values of \( N''/R_{0.5(e+m)} \), where \( R_{0.5(e+m)} \) is the average of the measured yield and ultimate strength of the reinforcement, corresponding to \( A_{s-crit} \). This is the design tensile stress recommended by DoD (1990; 2002; 2008; 2014) for the analysis and design of laterally restrained slab elements, sustaining chord rotations of 6-12° (see Chapter 3). Note; dynamic increase factors, referenced by DoD (1990; 2002; 2008; 2014), were neglected as testing was quasi-static. Considering the full test population, it can be seen that \( R_{0.5(e+m)} \) provides the closest relationship with the recorded in-plane membrane force, giving an average \( N''/R_{0.5(e+m)} \) of 0.98. By disregarding the anomalous results of S02, S03, E01, E03 and C01, this gives an average \( N''/R_{0.5(e+m)} \) of 0.99, with a standard deviation of 6%.

Figure 5-32 provides a graphical illustration of values \( N''/R_e, N''/R_m \) and \( N''/R_{0.5(e+m)} \) calculated using \( A_{s-crit} \).

![Figure 5-32 - Ratios of recorded membrane force upon measured yield and tensile strength of the critical reinforcement.](image)

The critical area of reinforcement identified for the edge beam specimens was not the smallest area of reinforcement available at the supports. Rather, \( A_{s-crit} \) was identified in testing to be the intermediate B2 reinforcement layer, lapped between points C1 and C2, due to the development of incomplete
catenary mechanisms. The compression reinforcement at the supports is identified in Table 5-4 and Appendix C, denoted $A_{s3}$. The reinforcement test data, corresponding to $A_{s3}$, show $R_e$ to be 56kN, 84kN and 121kN for specimens E01, E02 and E03, respectively. Furthermore, $R_m$ results were 68kN, 102kN and 142kN. This gives average $N''/R_e$ and $N''/R_m$ ratios of 1.60 and 1.33. Thus, by assessing secondary catenary response for $A_{s3}$, ultimate in-plane membrane force would be underestimated by 37.5% or 24.8%, when assessed with the yield and tensile strengths of the reinforcement. This observation demonstrates the importance of accurately identifying the catenary mechanism and critical reinforcement formed in secondary response.

Inspection of the results obtained for $A_{s=0.5}$ shows that the area of reinforcement prescribed by DoD (1998; 2002; 2008; 2014) and Woodson (1994) did not accurately represent $N''$. It can be seen that $A_{s=0.5}$ effectively identified the critical area of reinforcement for the flat-slab specimens. However, this was due to the use of continuous and equal B2 and T2 reinforcement layers. Whereas $A_{s=0.5}$ was not an accurate prediction of area of reinforcement for the edge-beam and slab-strip specimens. Results obtained for the edge beam specimens show that the use of $A_{s=0.5}$ underestimates $N''$, with average $N''/R_e$ and $N''/R_m$ ratios of 1.31 and 1.10. Similarly, average results obtained for the slab-strip specimens given $N''/R_e$ and $N''/R_m$ ratios of 2.10 and 1.73.

The underestimation of $N''$ is related to the quantification of $A_{s=0.5}$, assuming 50% of all continuous reinforcement (B2 and T2 layers). Due to the curtailment of the T2 reinforcement layer, this dictates that $A_{s=0.5}$ be 50% the area of the B2 reinforcement, which is irrational. Similarly, given the unequal B2 and T2 reinforcement layers in the edge beams specimens, the resulting $A_{s=0.5}$ is an arbitrary area of reinforcement which is indefensible for secondary catenary conditions. Whilst the use of $A_{s=0.5}$ was found to be conservative for these test specimen, it is apparent that this method would have resulted in an overestimate of reinforcement area and $N''$ had the B2 reinforcement at hinge locations B and F been less than the T2 layer. Given that this is a common detail practice and hinge locations were frequently found to form at B and F, a more accurate and technically defensible identification of area of reinforcement is needed.

With reference to Chapter 3, several sources (Keenan, 1969; Black, 1975; Park and Gamble, 2000; DoD, 1990; GSA, 2003; DoD, 2002, 2008 and 2014) have recommended the use of the total area of principal reinforcement for calculation of the tension membrane force at ultimate load resistance ($P_{TMA}$). As noted in Section 5.2 test results indicate $P_{TMA} = P'_{TMA}$ for all specimens, except C01. Thus, by inspection, it can be seen that the use of $2A_{s=0.5}$ would result in a significant overestimate of ultimate membrane force, with the exception of the slab specimens. Further, the use of $2A_{s=0.5}$ is not justified for secondary response, where a number of the reinforcement layers have failed.
5.5.1.2 Membrane force at ultimate primary TMA response, $N'$

As indicated in Section 5.2, ultimate load resistance ($P_{TMA}$) was typically sustained in secondary TMA response. However, one of the objectives of the experimental investigation was to establish the in-plane membrane force associated with ultimate primary TMA response. Thus, the values of $R_H$ shown in Table 5-4 were recorded for each test specimen at primary ultimate load, $P'_{TMA}$. Values given for $R_{BD/DF}$ and $N'$ were calculated using Equation 5-4, as in the previous section. By evaluating membrane force ($N'$) at ultimate load ($P'_{TMA}$), this allowed the investigation of the ultimate in-plane tensile force and stress appropriate to catenary analysis and design in primary TMA response.

As noted in Section 5.2, test specimens E01, E03, C02, C04, M01 and M02 were found to sustain fracture of the extreme tension reinforcement at hinge locations prior to developing TMA response. Therefore, no appropriate primary TMA data was available for these specimens.

Three areas of reinforcement were considered for investigation of membrane force in primary TMA response; $A_{s-1.0}$, $A_{s-0.5}$ and $A_{s3}$. The area of reinforcement denoted $A_{s-1.0}$ was taken as 100% of the total area of continuous longitudinal reinforcement (T2 and B2 layers). This is consistent with guidance provided by DoD (1990), GSA (2003) and DoD (2002, 2008 and 2014) and recommendations by Keenan (1969), Black (1975), Park and Gamble (2000) for the analysis of RC elements in ultimate catenary response. $A_{s-0.5}$ was taken as $0.5A_{s-1.0}$, consistent with guidance provided by DoD (1998; 2002; 2008; 2014) and Woodson (1994) and as identified in the previous section. The third area of reinforcement considered, $A_{s3}$, was taken to be the area of the B2 layer reinforcement at support locations, B, D and F. This was identified for assessment as it represents the

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Reaction Data at $P_{TMA}$</th>
<th>Total Continuous Reinforcement</th>
<th>50% Continuous Reinforcement</th>
<th>B2 Reinforcement at B, D &amp; F</th>
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<tr>
<td></td>
<td>$R_{BD/DF}$</td>
<td>$R_H$</td>
<td>$N'$</td>
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<tr>
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<td>[kN]</td>
<td>[kN]</td>
<td>[kN]</td>
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<td>Edge beam specimens – 2L/d = 35.0</td>
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<td>S03</td>
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<td>Column-strip specimens – 40.0 ≤ 2L/d ≤ 60.2</td>
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Table 5-4 – Horizontal restraint and tension membrane force data, recorded at ultimate primary TMA load.
smallest area of longitudinal reinforcement found in the edge beam specimens, given their non-
symmetrical reinforcement arrangement (see Section 5.3 and Appendix C).

The ultimate in-plane membrane force \( N' \) was assessed using the areas of reinforcement identified to
establish the ratios \( N'/R_e \) and \( N'/R_m \) at ultimate load, \( P'_TMA \). As in the previous section, \( R_e \) and \( R_m \) are the measured yield and tensile strengths of the corresponding reinforcement, respectively (see Appendix C for full set of test results). This allowed direct interpretation of strain hardening at ultimate load in primary catenary response. As noted in the previous section; the average strain hardening recorded during reinforcement testing, was found to constitute an 18.5% increase on yield strength, i.e. \( R_m = 1.185R_e \).

Inspection of Table 5-4 shows a significant difference between results obtained for the areas of reinforcement investigated. However, results obtained for \( A_{s3} \) were found to provide the best relation with \( N' \). Average \( N'/R_e \) and \( N'/R_m \) ratios of 0.79 and 0.65 were obtained, with standard deviations of 33 and 28% respectively. The membrane force of 67% of test specimens did not reach yield strength. This suggests that \( P'_TMA \) was not achieved in pure TMA response. This observation is confirmed by closer inspection of the test results based on an assumed area of reinforcement \( A_{s3} \).

Inspection of results obtained for specimens S02 and S03 show that the specimens sustained membrane
force 1.03 and 0.94\( R_e \) and 0.85 and 0.78\( R_m \). Thus, specimens S02 and S03 demonstrated the highest
level of TMA behaviour in primary response and an average tension membrane force of 0.99\( R_e \). The higher membrane force sustained by S02 and S03 at \( P'_TMA \) can be traced to the significant chord rotations achieved (see Sections 5.4.2) and characteristic TMA behaviour observed in testing, before fracture of the B2 reinforcement layer at midspan (see Sections 5.3). However, these results indicate that the yield strength of extreme tension reinforcement was barely achieved despite the evidence of strain hardening in Figure 5-30. This suggests that the tension force in the reinforcement was not attributed to pure membrane response alone.

The behaviour observed in all other specimens at \( P'_TMA \) demonstrated some level flexural response (see Section 5.3). Therefore, the minor membrane force recorded in the remaining specimens might be explained by the specimens exhibiting some level of flexural resistance at \( P'_TMA \).

Edge beam specimen E02 was found to have the lowest results of \( N'/R_e \) and \( N'/R_m \). Ultimate
membrane force was found to be 29% of the yield strength and 24% the tensile strength of the smallest
available area of reinforcement, \( A_{s3} \). Given that these results were obtained at \( P'_TMA \), immediately
prior to fracture of the extreme tension reinforcement, this points to a combination of bending and
TMA response. Figure 5-33 shows the moment-axial force interaction diagram for specimen E02.
The M-N envelope was calculated for ultimate resistance at locations B, D and F, using the
methodology given in Appendix D. The plot shows an overlay of the moment and membrane force
recorded during testing, at midspan (point D) and support (point B) locations.
For E02, $P'_{TMA}$ corresponded with first failure of the extreme tension reinforcement, found to occur near the interior support in the T2 layer (at point C1). This event is shown on the M-N record above. From inspection it can be seen that $P'_{TMA}$ was attained when force effects were predominantly flexural. This finding is of significance given that all current analytical models assume pure TMA response (see Chapter 3). Furthermore, the investigation of specimen E02 shows that by using the yield strength of the smallest available layer of reinforcement $A_{s-0.5}$, this can result in a significant over prediction of ultimate primary membrane force when flexural response is still a factor.

The results provided in Table 5-4 show significant confusion for the ratios $N'/R_e$ and $N'/R_m$ corresponding to $A_{s-1.0}$ and $A_{s-0.5}$. Inspection shows the results were not consistent with observed behaviour and cannot be consistently validated by the test data. The basis of this inconsistency is the arbitrary means of identifying reinforcement area, as discussed in Section 5.5.1.1. For example, inspection of results obtained for specimens S02 and S03, which were found to develop the most advance TMA response in primary conditions, it can be seen that $A_{s-0.5}$ was a significant underestimation of the area of reinforcement engaged in TMA response. Conversely, inspection of the results obtained for $A_{s-1.0}$ show that by assuming that all reinforcement achieves yield at $P'_{TMA}$, the membrane force of the edge and flat-slab specimens would have been overestimated by an average of 62.7%.

The results demonstrate that the total test population did not conform to the recommendations made by DoD (1990), GSA (2003) and DoD (2002, 2008 and 2014) and recommendations by Keenan (1969), Black (1975), Park and Gamble (2000), whereby:

$$R_H \approx N = f_g A_{s-1.0}$$

**Equation 5-5**
The recommendation by Woodson (1994) and DoD (1998; 2002; 2008; 2014) to substitute $A_{s-1.0}$ with $A_{s-0.5}$ in Equation 5-5 was found to provide an unrealistic interpretation of $N'$.

5.5.2 System equilibrium at ultimate load carrying capacity, $P_{TMA}$

As detailed in Chapter 2, the DoD, USACE, GSA and authors such as Regan (1975), Park and Gamble (200), Merola (2009) and Li et al. (2011) suggest that the load-displacement history of a two-dimensional catenary mechanism can be evaluated by considering the equilibrium of a laterally restrained three-hinged mechanism. Figure 5-31 shows a schematic elevation of such a mechanism formed between the hangers of the test rig, denoted as system BDF. Previously, authors have provided no account of geometric changes incurred due to the fracture of the extreme tension reinforcement at the midspan or supports or the development of more complex catenary mechanisms, such as the incomplete catenaries observed in the edge beam specimens or the four-hinged mechanisms develop in test specimens S02 and S03 (see Section 5.3.2). It follows that the global displacement ($\Delta$) and resulting chord rotation ($\theta$) are used to express the geometric nonlinearity of the mechanism.

Considering the half catenary BD (Figure 5-31b), taking moments about B or D gives the following relationship:

$$R_H = \frac{L}{2\Delta}(P + g_kL) = \frac{1}{2\tan\theta}(P + g_kL)$$

Equation 5-6

Where chord rotation, $\theta$, is:

$$\theta = \tan\frac{\Delta}{L}$$

Equation 5-7

It should be noted that $g_k$ is the measured self-weight of the specimen, therefore constant and assumed to act as a uniformly distributed load for the length of the specimen.

The following sections detail an investigation of the relationship between $P$, $R_H$ and $\theta$ at ultimate load resistance in primary and secondary catenary response. The test data recorded at ultimate load ($P_{TMA}'$ and $P_{TMA}''$) was used to verify the adequacy of Equation 5-6 in the evaluation of catenary response.

5.5.2.1 Load resistance in secondary TMA response

With the exception of specimen C01, $P_{TMA}''$ was the maximum load resistance ($P_{TMA}$) sustained by all specimens during TMA response (see Section 5.2). Table 5-5 shows the geometric and reaction data recorded at ultimate secondary load resistance, $P_{TMA}''$, for each test specimen.

Equation 5-6, rearranged for applied load gives:
Thus, ultimate load in secondary TMA response can be evaluated by the term:

\[
P''_{\text{TMA-theory}} = \frac{2R_H \Delta''_{\text{TMA}}}{L} - g_k L
\]

Column 8 of Table 5-5 shows theoretical loads \(P''_{\text{TMA-theory}}\) calculated using Equation 5-9 and the test data \(R_H; \Delta''_{\text{TMA}}\) recorded at \(P''_{\text{TMA}}\). These values have been used to investigate the consistency between the experimental results and the evaluation of catenary response at first principles.

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<tbody>
<tr>
<td></td>
<td>L</td>
<td>g (_\alpha)</td>
<td>(\Delta''_{\text{TMA}})</td>
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<td></td>
<td>[mm]</td>
<td>[kN/m]</td>
<td>[mm]</td>
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<td><strong>Edge beam specimens – 2L/d = 35.0</strong></td>
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<td><strong>Slab-strip specimens – 2L/d = 65.8</strong></td>
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<td></td>
</tr>
<tr>
<td>S01</td>
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<tr>
<td>C01</td>
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<td><strong>Middle-strip specimens – 40.0 ≤ 2L/d ≤ 50.0</strong></td>
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Table 5-5 – Summary table of experimental and theoretical values of ultimate load resistance in secondary TMA response.
Comparison of the experimental and theoretical values shows that Equation 5-6 results in the consistent under prediction of $P''_{TMA}$. The average ratio of theoretical upon experimental $P''_{TMA}$ results were established to be 0.78, 0.79, 0.71 and 1.06 for the edge beam, slab and middle and column-strip flat-slab specimens, respectively. Thus, giving an average underestimation of 19.6% for the full test population, with a standard deviation of 17.5%. The highest error found was found to occur for the theoretical results calculated in specimens C01 and M01. Theoretical values for these specimens were 0.57 and 1.34 times the recorded ultimate load resistance. This shows significant over and under estimation.

The condition of specimen C01 at secondary ultimate TMA load was irregular compared to the other test specimens. As described in Section 5.2, 5.3.3 and Appendix D, $P''_{TMA}$ was sustained following fracture of the hogging T2 reinforcement at location F. Thus, load resistance in secondary response was sustained by a combination of TMA action and bending – the specimen acted as a propped cantilever, restrained at locations A and B, loaded at the midspan (location D) and propped at point F via the critical B2 reinforcement. This is therefore inconsistent with the mechanism used as the basis of analysis (see Figure 5-31b). Hence, the significant under prediction of $P''_{TMA}$ can be attributed to residual flexural response, not accounted for in the pure plastic membrane response assumed in theoretical evaluation.

Inspection of the condition of M01 at ultimate secondary load shows that the specimen was acting as a simple one-way catenary (see Appendix D). At $P''_{TMA}$ the specimen had sustained fracture of the B2 reinforcement at midspan (point D) and T2 reinforcement at both supports (points B and F). Thus, the specimen was sustained by tension in the critical reinforcement, with no bending capacity at the supports or midspan, consistent with the mechanism shown in Figure 5-31b and used in the derivation of Equation 5-6. Therefore $P''_{TMA-theory}$ should have been consistent with other predictions in underestimating ultimate load. By inspection of the test data, it can be seen that S01 was found to sustain a similar chord rotation at $P''_{TMA} - \theta''_{TMA}$ values of 8.3° and 7.9° obtained for S01 and M01, respectively. Furthermore, the critical area of reinforcement ($A_{S-Crit}$) for S01 and M01 were identical for both test specimens (see Section 5.2). It follows that the reaction data for the two specimens should be similar. However, whilst inspection of Table 5-5 and Table 5-3 shows $R_H$ and $N''$ to be almost identical for the two specimens, the recorded value $P''_{TMA}$ was 68.8% greater in specimen S01. The author undertook an investigation to establish the source of this inconsistency but was unable to identify a definitive cause. However, given the compliance of S01 with results obtained for the remaining test population, this suggests that the $P''_{TMA}$ value for specimen M01 was anomalous.

By discarding results related to C01 and M01 (justified above) the average ratio of predicted and experimental results becomes 77.3%, thus indicating an underestimation of 22.7%, but with a standard deviation of just 3.8%.

5-43
Figure 5-34 is graphical plot of theoretical verses experimental $P''_{TMA}$ results. Note that the experimental results lay on a straight line. Thus, the accuracy of the theoretical results can be seen from their deviation from the straight line.

As detailed in Section 5.3.1, the *incomplete catenary* mechanisms formed by the edge beam specimens (E Group) were characterised by hinge formation inset from the interior supports, corresponding to the curtailment location of the hogging reinforcement (points C and E, Figure 5-1). This had the effect of shortening the effective span of the catenaries and thus increasing chord rotation for a given midspan displacement. Equation 5-6 shows that this condition results in an increase of $P'_{TMA}$. By resolving for $P''_{TMA-theory}/P''_{TMA}$ of 1.0, it was found that a 450 to 630mm reduction of $L$ accounts for the discrepancy illustrated in Figure 5-34. This reduction in effective span roughly corresponds with the progression of scabbing recorded along the soffit of the specimens at maximum load. This therefore demonstrates that, in the case of *incomplete catenaries*, $P''_{TMA-theory}$ calculated based on the total span may be conservative and accurate prediction of hinge locations and catenary mechanism development is an important factor in the estimation of $P_{TMA}$.

An interesting observation was made when evaluating the catenaries using chord rotation ($\theta$), where Equation 5-6 becomes:

$$P''_{TMA-theory} = 2R_{H}\tan\theta''_{u} - g_{k}L$$

Equation 5-10

When calculated using the chord rotation, rather than measured midspan displacement, values of $P''_{TMA-theory}$ are greater. The chord rotation based calculation gives 3-25% higher prediction of $P''_{TMA}$. This was found to increase the mean accuracy against experimental from 80.4% to 89.8% at
ultimate secondary load. This discrepancy between theoretical values was found to be most severe at small displacements and chord rotations.

5.5.2.2 Load resistance in primary TMA response

Table 5-6 shows the geometric and reaction data recorded at ultimate primary load resistance $P'_{TMA}$ for each specimen. It should be noted that TMA response was not found to develop in specimens E01, E03, C02, C04, M01 and M02 prior to first fracture of the extreme tension reinforcement, at midspan or support locations. Thus, test data for these specimens has not been provided.

Theoretical values $P'_{TMA-theory}$ were calculated using Equation 5-6 and test data ($R_H$; $\Delta'_{TMA}$) recorded at $P'_{TMA}$, thus:

$$P'_{TMA-theory} = \frac{2R_H\Delta'_{TMA}}{L} - g_kL$$

Equation 5-11

The results are provided in Table 5-6, together with the ratio of $P'_{TMA-theory}/P'_{TMA}$ for each specimen. Inspection shows that $P'_{TMA-theory}$ was found to significantly underestimate $P'_{TMA}$. The results show an average theoretical value $P'_{TMA-theory}$ of only 0.39 $P'_{TMA}$.

<table>
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<tr>
<th>Specimen Data</th>
<th>Ultimate Primary TMA Response Data</th>
<th>Ultimate Primary TMA Response Prediction</th>
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</thead>
<tbody>
<tr>
<td>L [mm]</td>
<td>$g_k$ [kN/m]</td>
<td>$\Delta'_{TMA}$ [mm]</td>
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Table 5-6 – Summary table of experimental and theoretical values of ultimate load resistance in primary TMA response.

Specimens S02 and S03 were found to demonstrate the highest consistency between theoretical and experimental results. These specimens were found to sustain primary catenary response at the highest recorded chord rotation, $\theta'_{TMA}$, and demonstrated the most pronounced characteristic membrane
behaviour, prior to first failure of the B2 extreme tension reinforcement at the midspans. Furthermore, inspection of results obtained for S02 show that $P'_{TMA}$ was attained at a chord rotation of 11.24°. Thus, response of this particular specimen was consistent with characteristic ultimate displacement limit recommended by Park and Gamble (2000) of $0.1L_{EM}$ or 11.3°. However, despite Equation 5-11 being consistent with the analytical approach recommended by Park and Gamble (2000), it can be seen that the method results in a 37% underestimate of $P'_{TMA}$.

The greatest difference between theoretical and experimental values of $P'_{TMA}$ correspond with specimen E02. Section 5.5.2.1 demonstrates that the effective shortening of catenary span attributed to incomplete catenary response was found to be a factor in under estimating load resistance at peak load. However, the deflection profile of specimen E02 at $\Delta'_{TMA}$ was relatively linear and thus effective shortening was found to have a nominal effect on $P'_{TMA}$. The greatest difference between theoretical and experimental values of $P'_{TMA}$ correspond with specimen E02. Section 5.5.2.1 demonstrates that the effective shortening of catenary span attributed to incomplete catenary response was found to be a factor in under estimating load resistance at peak load. However, the deflection profile of specimen E02 at $\Delta'_{TMA}$ was relatively linear and thus effective shortening was found to have a nominal effect on $P'_{TMA}$. The greatest difference between theoretical and experimental values of $P'_{TMA}$ correspond with specimen E02. Section 5.5.2.1 demonstrates that the effective shortening of catenary span attributed to incomplete catenary response was found to be a factor in under estimating load resistance at peak load. However, the deflection profile of specimen E02 at $\Delta'_{TMA}$ was relatively linear and thus effective shortening was found to have a nominal effect on $P'_{TMA}$.

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An investigation was undertaken to establish the cause of the discrepancy between the theoretical and experimental results obtained for $P'_{TMA}$. Park and Gamble (2000), and supporting subsequent studies (see Chapter 3), suggest that the prediction of $P'_{TMA}$ should be undertaken assuming $R_H$ and $N'$ as approximately equal to the yield strength of total area of reinforcement. Inspection of Table 5-4 shows that $N'$ was found to be only 51% of the yield strength of the total area of longitudinal reinforcement in the specimen. Rather, the investigation of ultimate membrane force in Section 5.5.1.2 showed that $N'$ at ultimate load was equal to 1.03 times the yield strength of the B2 joint reinforcement, only. This suggests that Equation 5-11 and the terms currently used in analysis of primary TMA response severely underestimate load resistance when actual membrane force is considered rather than the inaccurate assumed yield of all reinforcement. This topic is investigated and discussed in further detail in Chapter 6.

The assumption of pure plastic membrane response is inconsistent with all test observations made at $P'_{TMA}$. Load resistance at this point was sustained by the test specimens prior to first fracture of any extreme tension reinforcement. Thus, by definition, each of the specimens were capable of sustaining some degree of flexural resistance in addition to TMA response. Further, physical characteristics of flexural response were most pronounced at low chord rotations, decreasing as displacement was increased. With this in mind, the discrepancy of predicted and experimental $P'_{TMA}$ results in Table 5-6 was found to be inversely proportional to chord rotation. This suggests that the assumption of pure plastic membrane response might be unsuitable for primary catenary response. To explore this idea, the principal of supposition was used to derive Equation 5-12.

$$P'_{TMA} + P_{FA} = \frac{2(M'_{RES} + M_{RES})}{L} + \frac{2R_H\Delta'_{TMA}}{L} - 2g_kL \quad \text{Equation 5-12}$$

Results obtained using Equation 5-12 are provided in Table 5-4 and Figure 5-35b. Inspection shows that the predicted load resistance provides a closer correlation with the experimental results than those
established with plastic membrane theory (see Figure 5.35a). The predicted results can be seen to be conservative, under estimating load resistance by an average of 24.8%, with a standard deviation of 10.3%.

![Figure 5.35 – Experimental vs. predicted values of ultimate secondary catenary load capacity.]

### 5.6 Discussion & Conclusions

Testing was conducted for twelve laterally restrained half-scale beam and slab strip specimens. The specimens were subject to an applied midspan load until collapse was achieved or the displacement limit of the test rig was reached, by which point a failure mechanism was identified. The objective was to obtain experimental data related to primary and secondary TMA response in reinforced concrete beam and slab specimens under double-span, support loss conditions. The force-displacement record obtained for each test specimen showed characteristic membrane response. Peak loads were recorded for both CMA and TMA phases and the lateral restraint reaction was found to show the development of compressive and tensile membrane force with each mode of response.

The test arrangement was instrumented such that the relationship of applied load ($P$), horizontal restraint ($R_H$) and in-plane membrane force ($N$), vertical restrained reactions ($R_1; R_2; R_{v1}; R_{v2}$) and midspan displacement ($\Delta$) could be investigated for each specimen. Consistent with the objectives of the experimental study, this allowed for direct investigation of the current analytical method used in the calculation of ultimate TMA load resistance ($P_{TMA}$) and a review of the existing ultimate limit parameters used in the design and direct assessment of catenary action. This section details the main findings and conclusions made by investigation of test data recorded for TMA response.

Transition of CMA-TMA response was found to occur at a displacement ($\Delta_0$) directly proportional to the thickness ($h$) of the each specimen and inversely proportional to span-thickness ratio ($2L/h$). The latter trend is consistent with observations made by Keenan (1969) and gave an $R^2$ value of 79.4%.
based on a linear trend line, of best fit. Similarly, inspection of the test results showed transition to occur at an average displacement of $\Delta_0 = 1.34h$, with a standard deviation of 0.23h from the mean. This showed reasonable agreement with the finding made by Park and Gamble, who suggested that transition occurred at approximately $1.0h$, but that transition was observed at relatively large displacements. This potential anomaly was investigated by researching existing data sets reported for similar double-span catenary arrangements. Gouverneur et al. (2013b) and Yi et al. (2008) were also found to have reported transitions of $\Delta_0 \gg h$. Rotation across the interior supports was found to be a common feature of the tests reported herein and those undertaken by Gouverneur et al. and Yi et al. Tests conducted with fully encastré catenary arrangements (Yu and Tan, 2013; Su et al. 2008) found transition to typically occur at $\Delta_0 \leq h$. This suggests that $\Delta_0$ can be dependent on rotational constraint across the interior support of a catenary.

The test results at transition allowed comparison with the findings of Keenan (1969) and Park and Gamble (2000), in terms of load resistance ($P_0$). Test observations made by the authors, using small-scale slab specimens, demonstrated $P_0$ to typically be less than the theoretical ultimate flexural strength ($P_{FA}$) of the specimens, due to concrete crushing during CMA response. By inspection of the test results obtained for all twelve test specimens, an average $P_0/P_{FA}$ of 0.88 was calculated with minimum and maximum results of 0.02$P_{FA}$ and 1.31$P_{FA}$. However, in this experimental investigation, results of $P_0/P_{FA} \leq 1.0$ were typically found to correspond with test specimens that sustained failure of the extreme tension reinforcement at supports or midspan at $\Delta \leq \Delta_0$, which occurred in 50% of test specimens. For these specimens an average $P_0/P_{FA}$ of 0.56 was calculated. Test specimens that achieved transition prior to any reinforcement failure were all found to sustain $P_0/P_{FA} > 1.0$, giving an average value of 1.19$P_{FA}$. Thus, it was found that load resistance at transition was not less than the theoretical ultimate flexural resistance of the specimen unless the ultimate joint rotation capacity of the section was inadequate to support $\Delta_0$ without failure of reinforcement. In this case, resistance at $\Delta_0$ was found to be dependent on residual flexural strength and the onset of secondary TMA.

The development of primary TMA response was found to be dependent upon the ductility of the reinforcement and joint rotation capacity at critical sections. This chapter identifies the ultimate plastic chord rotation ($\theta'_u$) and joint rotation ($\phi'_u$) recorded at first fracture of either the B2 reinforcement layer at midspan or T2 reinforcement layer at hinge locations, associated with hogging moment. Investigation of the test results shows significant variance in $\theta'_u$. An average chord rotation of 5.56° was recorded, with a standard deviation of 2.47°. Results obtained for $\phi'_u$ were calculated based on the mechanism observed to develop in each test specimen. The results showed a closer correlation, with an average 8.49° and standard deviation of 2.67° hinge rotation at reinforcement fracture. An investigation was conducted to identify trends between the specimen properties and results obtained for $\theta'_u$ and $\phi'_u$. The closest correlation was obtained for $\phi'_u$ and the reinforcement ratio of the extreme tension reinforcement at the hinge location.
The literature review (Chapter 3) identifies a 12° plastic chord rotation as the current ultimate limit criterion for the design and assessment of ultimate primary TMA load resistance, $P'_{TMA}$ – in accordance with Park and Gamble (2000), DoD (2005), GSA (2003) and Stevens (2008). However, by comparison with recorded values of $\theta'_{u}$ the experimental investigation found that all of the test specimens sustained reinforcement failure before reaching this chord rotation. Further, test specimens E01, E03, C02, C04, M01 and M02 were all found to sustain fracture of the T2 reinforcement at support locations or the B2 reinforcement layer at the midspan at chord rotations $\theta \leq \theta_0$ – during snap-through response, prior to tensile membrane development. Thus, primary TMA response was only observed in half of the total test population. The average ultimate chord rotation in primary TMA response ($\theta'_{TMA}$) was 7.05°. Therefore, by considering the average results of $\theta'_{u}$ and $\theta'_{TMA}$, the 12° ultimate chord rotation limit recommended by Park and Gamble (2000), DoD (2005), GSA (2003) and Stevens (2008) was found to over-predict average recorded failure rotation by 53.7% and 41.3%, respectively. It follows that $\theta'_{u}$ and $\theta'_{TMA}$ are better represented by the end-rotation limit of 6° as recommended by DoD (1990; 2008; 2014) for unrestrained RC elements.

The failure of half the test specimens to sustain primary TMA response supports the findings of Regan (1975) and Su et al. (2009), who also found primary TMA response to be unreliable in double-span conditions. The tests by Su et al. were conducted at small span-depth ratios ($2L/d$), not considered representative of double-span conditions. However, by obtaining results for test specimens of $66 \geq 2L/d \geq 35$, this experimental investigation strongly points to primary TMA as an unreliable mechanism of emergency load redistribution due to insufficient rotation capacity ($\theta'_{u}$) at critical sections. This therefore suggests that criteria for robust design, based on primary TMA response, recommended by Park and Gamble (2000), DoD (2005; 2013), GSA (2003) and Stevens (2008) may be unsafe.

Primary TMA response was observed in specimens E02, S01, S02, S03, C01 and C03. The results showed enhanced load resistance in each specimen at peak primary TMA load ($P'_{TMA}$). Inspection of the test results gave an average enhancement ($P'_{TMA}/P_{FA}$) of 2.90 and showed that $P'_{TMA}$ and enhancement was directly proportional to the area of reinforcement and ultimate chord rotation ($\theta'_{u}$). The lowest enhancement was recorded for specimen S01, which achieved 1.65$P_{FA}$. This specimen possessed the lowest area of reinforcement of those specimens that achieved primary response and sustained fracture of the B2 tension reinforcement at midspan at a relatively minor chord rotation.

An investigation was undertaken to investigate the current analytical approach to modelling the relationship between recorded forces $P$ and $R_H$ as a function of $\Delta$. Thus, the equilibrium of a simple perfectly plastic membrane, forming a bilinear three-hinge mechanism, was assumed and implemented. This was consistent with first-principle theory used in investigations by Regan (1975) and Merola (2009) and allowed direct implementation of reaction data recorded at ultimate primary and secondary TMA load, to assess analytical accuracy.
The investigation showed that this analytical approach was inadequate for the prediction of $P'_{TMA}$. The results showed theoretical values of $P'_{TMA}$, calculated assuming pure TMA response, underestimated load resistance by as much as 97% (as observed in the analysis of specimen E02). On average, theoretical values of load resistance were found to under-estimate $P'_{TMA}$ by 60.7%. Inspection of the test results showed the accuracy of prediction to be dependent on the displacement at $P'_{TMA}$. Thus, results for specimens S02 and S03, which obtained the highest ultimate chord rotation ($\theta'_{TMA}$) and behaviour most characteristic of plastic membrane response, were found to provide the best estimate of response (0.63 and 0.63$P'_{TMA}$, respectively).

An overlay of the ultimate M-N interaction envelope, calculated for critical sections, and the moment and axial force record obtained for each test specimen was used to investigate specimen response at $P'_{TMA}$. The results demonstrated significant flexural characteristics at $\theta'_{TMA}$. This was most evident in the specimen E02 and least pronounced in specimens S02 and S03. Thus, the investigation found the assumption of pure membrane response to be unfounded for the theoretical evaluation of $P'_{TMA}$. Rather, the test results demonstrate that accurate prediction of response, especially at low values of $\theta'_{TMA}$, requires a term that accounts for the combined effects of flexural and TMA resistance. To emphasise this observation an attempt was made to account for the combined TMA and bending resistance of each specimen. By adding the theoretical ultimate flexural capacity $P_{FA}$ to the theoretical TMA resistance ($P'_{TMA-theory}$), an average of 0.85$P'_{TMA}$ was obtained, with standard deviation of 10.3%.

One objective of the experimental study was to ascertain the in-plane membrane force ($N$) at ultimate load. The force-displacement records obtained in testing showed TMA response to be composed of an elastic tensile phase, followed by a plastic tensile phase. This was evident for both primary and secondary TMA response and resulted in a change in slope for both the $P \cdot \theta$ and $R_h \cdot \theta$ histories. Inspection of the force-displacement plots show plastic regions of response demonstrated some degree of strain-hardening. An investigation was undertaken to identify the area of reinforcement and tensile stress corresponding to $N$ at ultimate primary and secondary response (N' and N'').

The results obtained for primary ultimate membrane force at (N') provide further evidence of combined flexural and TMA response at $P'_{TMA}$. $N'$ recorded for specimens E02, S01, S02, S03, C01 and C02 were found to be, on average, 21% less than the yield strength ($R_e$) of the smallest available reinforcement layer ($A_{s3}$). In general $N'/f_yA_{s3}$ was found to tend to unity as chord rotation increased. Thus, $N'$ recorded for slab specimens S02 and S03, which sustained $P'_{TMA}$ at chord rotations of 11.24° and 9.43°, were found to be approximately equal to the yield strength of $A_{s3}$. However, E01 was found to sustain $P'_{TMA}$ at a chord rotation of 5.88°. The specimen was found to sustain the lowest ratio of $N'/R_e$ at 24%, assuming the yield strength of reinforcement $A_{s3}$. These findings provide additional evidence that pure plastic membrane response was unfounded at primary limits. Thus, the accurate prediction of primary catenary response requires an account of combined TMA and flexural effects.
Secondary TMA response was observed in all test specimens. The ultimate load resistance ($P''_{TMA}$) was found to constitute the ultimate TMA resistance ($P_{TMA}$) for all test specimens, except C01 for which $P''_{TMA} < P'_{TMA}$. Results recorded for $P''_{TMA}$ correspond to incipient collapse for all specimens except E01, E02, S02 and S03, whereby the displacement limit of the rig preceded outright failure. However, results show $P''_{TMA}$ to be directly proportional to the ultimate chord rotation ($\theta''_u$) and area of reinforcement. Load enhancement calculated for secondary response show an average ratio $P''_{TMA}/P_{FA}$ of 3.07. The highest enhancement was recorded for the slab-strip specimens, S02 and S03, which achieved 6.86 and 6.77$P_{FA}$. This is attributed to their area of reinforcement and significant extensibility, which supported TMA response until the tests were terminated at chord rotations of 15.1° and 15.0°. Conversely, M01 sustained the lowest in-test $\theta''_u$ and enhancement, at 1.14$P_{FA}$.

The investigation identified the chord rotation at $P''_{TMA}$ for each test specimen ($\theta''_u$). From the test results an average $\theta''_u$ of 10.91° was obtained. This result was conservative (lower-bound) as results recorded for specimens E01, E02, S02 and S03 were obtained when the tests were terminated, rather than at incipient collapse.

Chapter 3 showed that a 20° end-rotation limit was recommended by DoD (1998) and USACE (2008a) for the prediction of ultimate secondary TMA response in overload conditions. Specimens S02 and S03 sustained $\theta''_u$ of 75.7 and 75.0% rotation limit without incipient collapse. However, it can be seen that the limit recommended by DoD (1998) and USACE (2008a) is 83.3% higher than the average $\theta''_u$ recorded in testing. Thus, the test results demonstrate significantly inferior joint rotation and extensibility when compared with the experimental findings of Woodson and Garner (1985), Guice (1986) and Woodson (1990, 1992 and 1994), which are the basis of the 20° end-rotation limit. This suggests an inconsistency between TMA testing for overload and support-loss conditions and demonstrates the importance of developing response limit criteria specific to double-span research.

Results obtained for $\theta''_u$ were found to have a standard deviation of 3.10° on the mean, thus demonstrating significant variance. An attempt was made to establish trends between the results and properties of each specimen. A relationship was established that suggested $\theta''_u$ to be proportional to the reinforcement ratio of the critical reinforcement. However, a regression analysis showed the data to provide too weak a relation to support any conclusion.

Catenary response at $P''_{TMA}$ was found to be more consistent with plastic membrane theory than at $P'_{TMA}$. An attempt was made to calculate ultimate load resistance using $P, R_H$ and $\Delta$ test data recorded at $P''_{TMA}$. Equilibrium of a plastic TMA three-hinge mechanism was assumed, consistent with first-principle theory implemented by Regan (1975) and Merola (2009). The results showed theoretical values of $P''_{TMA}$ to be generally conservative, underestimating the recorded load by an average of 19.6%, with a standard deviation of 17.5%. By identifying outlying results (obtained for specimens C01 and M01 in particular, which did not form typical secondary TMA mechanisms) the investigation showed the analytical approach to be susceptible to gross under-prediction of load at minor
displacements and where residual bending resistance was evident in the test specimens. Additional attention was paid to results obtained for specimens E01, E02 and E03 that formed incomplete catenaries at $P''_{TMA}$. It was found that the discrepancy between theoretical and experimental results at $\Delta''_{TMA}$ could be accounted for by consideration of the detailed geometry of the mechanism; which resulted in an effective shortening of $L$ and consequential increase of $\theta$, at a given displacement, and thus increase in theoretical value of $P''_{TMA}$.

Results recorded at $P''_{TMA}$ were found to provide a good basis for the investigation of $N''$ parameters and further evidence of pure membrane response in secondary conditions. Plastic tensile response and evidence of strain hardening was observed in the $R_H- \theta$ histories recorded in testing. Thus, the most defensible correlation was obtained for the area of critical reinforcement ($A_{s-crit}$) of each specimen – identified during testing as the smallest reinforcement layer available at hinge locations, sustaining the specimen in secondary response. Average results obtained for $N''$ were found to correspond with 1.07 times the measured yield strength ($R_e$) and 0.89 times the measured ultimate tensile strength ($R_m$) of $A_{s-crit}$. This shows that full strain hardening was not achieved at ultimate load. The closest prediction of $N''$ was obtained by using the empirical term $0.5(R_e + R_m)$, consistent with DoD (1990; 2002; 2008; 2014) guidance. This gave an average tensile force 2% greater than recorded values and a standard deviation of 7%.

In order to ascertain whether detailing influenced catenary performance the edge beam and slab-strip specimens were detailed with reinforcement arrangements consistent with EC2 guidance. Thus, these specimens featured reinforcement curtailments and lapping regions. Furthermore, the specimens featured areas of reinforcement that were biased to in-service bending conditions; slab-strip specimens were singly reinforced along the intermediate length, and; the T2 reinforcement of the edge beam specimens was larger at support locations than the B2 reinforcement. This allowed for direct comparison of large-displacement behaviour and TMA performance against the flat-slab strip specimens, which featured equal B2 and T2 reinforcement layers that were continuous for the full double-span. The test results demonstrated that the catenary mechanisms, behaviour and TMA performance of each test group was significantly different. This was found to be directly related to the reinforcement detailing and span-depth ratio. Slender specimens featuring discontinuities in the longitudinal reinforcement, which promoted extension along regions of reduced reinforcement area, were found to sustain greater ultimate extensibility and chord rotation in primary and secondary TMA response than specimens of low span-depth ration and whose in-plane extension was confined to hinge locations.

The flat-slab strip specimens were typically found to develop plastic hinges at the supports and midspan, with displacement. Reinforcement fracture was generally found to occur in the B2 layer at the midspan hinge. Continued displacement, in secondary TMA response resulted in the fracture of the T2 reinforcement at the midspan hinge location. This resulted in collapse of the TMA mechanism.
but several specimens were found to sustain continued nominal load resistance, attributed to the flexural resistance of the cantilevering segments of the specimens. Thus, flexural response was observed throughout testing.

The edge beam specimens were also found to develop three-hinge mechanisms. However, the plastic hinge locations were found to correspond with curtailment locations for the hogging reinforcement and at the midspan. Fracture of the extreme tension reinforcement generally occurred first at the midspan hinge and continued displacement resulted in fracture of the T2 reinforcement at curtailment locations, inset from the supports. The resulting secondary catenary mechanism is described as an ‘incomplete catenary’, suspended between the two cantilevers formed for the length of the hogging reinforcement at the supports. Increased displacement was found to result in progressive spalling of the concrete cover, from the soffit of the cantilevers. Outright collapse by degradation of the B2 lap at the supports was avoided due to the presence of shear links, which were effective in limiting the progress of cover failure and bar pull-out. This type of response was consistent with test observations made by Regan (1975) and demonstrates the importance of providing effective containment reinforcement (in the form of links or stirrups) that is lapped at the top face, where degradation of the soffit cover does not compromise anchorage. This form of response also demonstrated that the deflection profile and ultimate catenary mechanism is influenced by discontinuities in flexural strength and stiffness along the specimen.

The characteristic mechanisms formed by the edge beam specimens were found to support secondary TMA response to an average recorded $\theta''_u$ of 12.86°, which was 43% greater than the average recorded for the flat-slab specimens. Moreover, results obtained for two of the three test specimens constitute lower-bound results, as incipient collapse was not achieved.

The catenary mechanisms developed by the slab-specimens were also closely related to reinforcement detailing. By lapping the B2 reinforcement across the midspan joint, specimens S02 and S03 were found to develop four-hinge mechanisms. The plastic hinge locations were found to correspond with the end of the B2 lapping region, at midspan and both supports. Furthermore, the lengths of the specimens that featured only B2 reinforcement (a 50% reduction in reinforcement area) were found to sustain extensive traverse tension cracking.

Collapse was not achieved in specimens S02 and S03 and the average $\theta''_u$ recorded for the group was 12.80°, 43% greater than the average recorded for the flat-slab specimens. Furthermore, the investigation showed that the development of four-hinge catenary mechanisms resulted in $\theta'_u$ of approximately twice that recorded for the edge-beam and flat-slab specimens.

As a product of the superior extensibility and rotation capacity observed in the edge beam and slab-strip groups, these specimens were found to out-perform counterpart flat-slab specimens. Specimen C03 featured the same area of critical reinforcement ($A_{S-crit}$) at $P_{TMA}$ as specimen E01 and S03. However, the total work done ($W$), work done in TMA response ($W_{TMA}$), ultimate load enhancement
\(P_{TMA}/P_{FA}\) and \(P_{TMA}\) recorded for C03 was found to be 0.65, 0.76, 0.91 and 0.83 of those results obtained for specimen E01. Similarly, comparison with results recorded for S03 show ratios of 0.42, 0.60, 0.36 and 0.78 against C03. This is a significant observation given that the ultimate flexural resistance \(P_{FA}\) of C03 was only 9.0% less than E01 and 216% that of specimen S03. Similar results are obtained by direct comparison of results recorded for specimens C04 and E02 and specimens C02 and S02. In each case the flat-slab specimens featured equal or larger \(A_{s-crit}\) and larger \(P_{FA}\) than the comparable edge and slab specimens, but results show significantly inferior collapse resistance and ultimate performance. Therefore, the experimental investigation demonstrated that RC elements featuring conventional reinforcement arrangements sustained the highest in-test performance.

A significant finding of the experimental study was established by investigation of reinforcement area and stress parameters for \(N'\) and \(N''\). The investigation implements the recommendations of DoD (1998; 2002; 2008; 2014) and Woodson (1994), whereby the effective area of reinforcement assumed for the assessment of ultimate TMA response is assumed to be 50% of all continuous longitudinal reinforcement (B2 and T2 layers). This area of reinforcement was identified as \(A_{s-0.5}\). The recommendation made by DoD (1990), GSA (2003), DoD (2002, 2008 and 2014), Keenan (1969), Black (1975) and Park and Gamble (2000), for the use of 100% of all continuous longitudinal reinforcement \(A_{s-1.0}\) was also investigated. By comparison of the results obtained for \(N'\) and \(N''\) and the measured yield and tensile strength of the appropriate reinforcement, the experimental investigation identified that \(A_{s-0.5}\) and \(A_{s-1.0}\) were inadequate for the prediction ultimate membrane force. By inspection of the results it was established that the methods of identifying \(A_{s-0.5}\) and \(A_{s-1.0}\) resulted in arbitrary predictions of reinforcement area that were indefensible and could lead to the significant under or over-prediction of \(N'\) and \(N''\). This observation was pronounced for the edge and slab-strip specimens, due to the use of un-equal B2 and T2 reinforcement layer and discontinuous detailing. This points to a need for an effective and technically defensible means of identifying the area of reinforcement engaged in primary and secondary TMA response at ultimate load.
6 Emergency Load Redistribution by Catenary Action

The results obtained in Chapter 5 demonstrate that the ultimate load resistance of conventional RC slab-strip and edge beam specimens was typically the maximum load resistance in secondary catenary response, immediately prior to total collapse. This chapter implements the results and observations made in testing in an analytical investigation of the ultimate resistance of conventional RC framed structures subject to the loss of a mid-elevation peripheral column.

The assessment methodology is simplistic and considers secondary catenary response only. However, the analysis has been implemented to investigate the sensitivity of ultimate catenary resistance of double bay floor systems to different design, detailing and structural configuration parameters.

6.1 Theoretical Estimation of Ultimate Resistance in Catenary Response

Primary catenary response, which has been advocated as a method of direct robust design (GSA, 2003; DoD, 2005), has been found to be an unreliable mechanism of emergency load redistribution as 50% of test specimens were found to achieve their rotation capacity prior to the development of any tensile membrane force (see Chapter 5). Rather, the results obtained in testing demonstrated that the maximum load resistance of the double span test specimens was typically achieved in secondary catenary response. Thus, this analytical investigation is concerned only with secondary catenary response, whereby the mechanism is sustained by the top reinforcement at mid-span and/or bottom reinforcement at interior support locations following failure of the extreme tension layers.

By considering equilibrium of the test specimens in their ultimate condition, it was found that ultimate load ($P_{TMA}$) sustained across the double span ($L_{ALS} = 2L$) could be predicted by the equation:

$$P_{TMA} = \frac{2R_H \Delta}{L} - g_k L$$

This approach was assessed by implementing the horizontal restraint force ($R_H$) and mid-span displacement ($\Delta$) recorded at maximum load resistance, for each specimen of self-weight ($g_k$). However, for the evaluation of conventional RC slab and beam assemblies, for which direct test data is not available on a case-by-case basis, $R_H$ and $\Delta$ at ultimate load need to be defined.

It is typically assumed that membrane force is approximately equal to $R_H$. Where;

$$R_H \equiv N = f_d A_{s-TMA}$$
The experimental investigation (see Chapter 5) provided evidence to support the quantification of tension membrane force \( (N) \) at ultimate catenary load in secondary response. \( A_{s-TMA} \) was found to correspond with the area of critical reinforcement \( (A_{s-crit}) \). The experimental investigation allowed for the physical identification of \( A_{s-crit} \). However, in secondary catenary response this can be identified as the lesser of the area of bottom or top reinforcement layers at joint locations or critical sections. The design tension stress \( (f_d) \) was also identified. The reinforcement was consistently found to exceed yield stress \( (f_y) \) and exhibit minor strain hardening but ultimate tensile stress \( (f_u) \) was not reached. As such, \( f_d \) was found to correspond with the empirical value recommended by the DoD (2002; 2008; 2014), of \( 0.5(f_y + f_u) \).

It follows that:

\[
R_H \cong N = f_d A_{s-TMA} = 0.5(f_y + f_u)A_{s-crit}
\]  
\hspace{10cm} \text{Equation 6-3}

Figure 6-1 and Figure 6-2 provide a summary of the load and chord rotation \( (\theta = \tan^{-1} \Delta/L) \) response predicted using Equation 5-9 and Equation 6-3 for the edge-beam and slab-strip specimens. Comparison with the experimental records demonstrate a good representation of observed linear behaviour.

![Graph showing load vs. chord rotation for experimental and theoretical data](image-url)

**Figure 6-1** – Comparison of experimental and theoretical load-rotation relationship for Edge Beam test specimens.
Figure 6-2 – Comparison of experimental and theoretical load-rotation relationship for Slab Strip test specimens.

To verify the validity of this approach, Appendix F provides a summary of experimental and theoretical load-rotation relationships ($P_{TMA}/\theta$) as recorded in testing and predicted based on assumed values of effective area of tension reinforcement $A_{S-TMA}$ and design tensile stress $f_d$. Seven theories for the quantification of $R_H$ are considered and shown, corresponding to existing design guidance (GSA, 2003; DoD, 1998; 2002; 2008; 2014) and observations made during testing. Comparison of the experimental and theoretical results demonstrates that Equation 6-3 provides an improved representation of $P_{TMA}/\theta$ over other methods. The only approach found to provide a better representation was obtained for $A_{S-TMA} = A_{S-crit}$ and $f_d = f_u$. However, this is attributed to conservatism in Equation 5-9 that has been shown to result in a consistent 19.6% underestimation of applied load (see Chapter 5). Thus, Equation 5-9 and Equation 6-3 are proposed as these parameters support a proven factor of safety that can be quantified or designed out, if required.

The prediction of ultimate displacement ($\Delta_{TMA}$) was investigated using approaches proposed by Regan (1975) and Merola (2009). However, the investigation proved inconclusive, suggesting that Merola’s recent method for the prediction of reinforcement failure was optimistic compared with the experimental results. Given the significant difference in performance observed between the edge-beam (12.86°), slab-strip (12.80°) and Control (8.99°) test specimens, characteristic ultimate displacements were assumed corresponding to average test results (of 12.86°, 12.80° and 8.99° respectively), as follows.

- Standard discontinuous detailing  $\theta_{TMA} \leq 12.8^\circ$
- Continuous detailing  $\theta_{TMA} \leq 9.0^\circ$

Thus, the ultimate displacement is defined by the arrangement and detailing of the reinforcement.
6.2 Sensitivity Study of Collapse Resistance by Catenary Action

An analytical study was conducted to investigate the robustness of various floor systems. A series of floor system exemplars were designed and evaluated by prediction of a demand-capacity ratio (FOS) in the emergency condition.

\[ FOS = \frac{System\ Load\ Capacity}{Emergency\ Load} = \frac{P_{TMA}}{Q_{ALS}} \]

The premise of the study was to establish the displacement required for redistribution of emergency load across a double bay, such that \( FOS \geq 1.0 \). This type of investigation has previously been undertaken by Merola (2009) but by considering the collapse resistance offered by two dimensional beam elements only. The investigation undertaken here implements a simple approach to quantify the FOS ratio of three dimensional double bay floor systems, by accounting for catenary response along the bay depth.

Assumptions, analytical approach, design of the case study floor systems, results and key findings are detailed in the following sections.

6.2.1 Simple analysis of catenary action in a monolithic double structural bay

Figure 6-3 provides a schematic illustration of the catenary mechanism assumed to form across a double bay floor system following external column loss and large mid-span displacement. A bilinear catenary mechanism was assumed, consistent with the specimen response observed in testing (Chapter 5) and secondary catenary response (Figure 6-3b). By neglecting self-weight, Equation 5-9 can be used to determine load resistance \( P_{TMA,i} \) at a given offset \( l_{y,i} \) from the inset column line:

\[ P_{TMA,i} = \frac{2R_H \Delta_i}{L_x} \quad \text{Equation 6-4} \]

Displacement is assumed to occur linearly across the width of the structural bay \( (L_y) \), such that the displacement at distance \( l_{y,i} \) from the interior column line is:

\[ \Delta_i = 0 \quad \text{at} \quad l_{y,i} = 0 \]

\[ \Delta_i = \Delta \quad \text{at} \quad l_{y,i} = L_y \]

Where, \( \Delta \) is the maximum mid-span displacement assumed at the edge of the floor system (Figure 6-3d). Therefore, the total load resistance of the floor system acting in secondary catenary response \( (P_{TMA}) \) can be expressed as:
Figure 6-3 – Free-body diagrams and notations supporting analysis of a double bay bilinear catenary mechanism.

Figure 6-3

1. Isometric sketch of displaced double bay.

2. Elevation and free body diagram of discrete bilinear catenary element.

3. Plan view of displaced double bay, showing horizontal restraint force distribution at slab and beam boundary.

4. Slab/beam section along length Ly, showing resultant catenary resistance at mid-span.
Figure 6-3c indicates the development of tension membrane and horizontal restraint forces as $\Delta_i(\ell_{y,i})$. Whilst the linear increase of force is representative of actual response, the horizontal restraint force is assumed constant for analysis of the system, consistent with Equation 6-3.

$$ R_{H,i} = f_dA_{S-TMA} = 0.5(f_y + f_u)A_{S-crit} $$. 

Where, $A_{S-TMA}$ is the total effective area of reinforcement at $\ell_{y,i}$, per the longitudinal detailing of the slab or beam component.

Appendix F provides a summary of B 500B reinforcement data, obtained from UK reinforcement manufacturers and the Building Research Establishment (BRE). The results presented are a statistical summary of material data obtained from more than 14,000 quality control tension tests, for bar sizes varying between 8mm and 50mm diameter. From the results obtained, the mean yield and ultimate tensile stress of the full population ($f_y = 550.5\text{MPa}$ and $f_u = 657.7\text{MPa}$) was used to calculate ultimate membrane force.

It should be noted that the proposed approach constitutes a simple model of one-way spanning catenary response in an isolated floor system. In practice, a monolithic RC floor system would exhibit complex large-displacement response with additional load resistance mechanisms attributed to the boundary conditions and continuity offered by the interior column line. Mechanisms could include two-way flexural, CMA and TMA responses as well as potential resistance from an intermediate main beam, cantilevering from the adjacent interior framing. Moreover, by assuming a constant span across for breadth of the structural bay (Figure 6-3), the proposed model does not account for the potential development of catenary response between yield-lines formed in the slab, which could potentially result in a shortened catenary span with proximity to the interior column line. These factors therefore suggest that the proposed model represents a conservative means of estimating emergency load redistribution by catenary action. The main exception to this conclusion would be potential detrimental effects of torsion found in the edge beam at large-displacements.

### 6.2.2 Definition of emergency load

To evaluate the demand-capacity ratio of the floor systems, it was necessary to establish an emergency unit floor load ($q_{ALS}$). Chapter 2 provides a review of a number of alternative accidental limit state (ALS) load cases that are recommended by current guidance (ASCE, 2006; BSI, 2000, 2005c and 2009; DoD, 2009; GSA, 2003 and 2013). The ALS load cases were found to differ between guidance documents but the recently revised ASCE, GSA and DoD recommendations are consistent. Moreover,
the latter documents provide an allowance for a dynamic load factor (DLF), in support of static analyses. Thus, the following emergency load case was implemented (GSA, 2013; DoD, 2009):

$$q_{ALS} = DLF(1.2G_k + 0.5Q_k)$$

Equation 6-7

It should be noted that the analysis undertaken was confined to the assessment of local catenary response across the double bay. Therefore the above load case assumes load factors for maximum gravity load in emergency response and was implemented with the unit floor loads, identified in section 6.2.3.1.

Figure 6-4 provides a plan view of an exemplar double bay floor system in emergency response and identifies the tributary area assumed to be dependent on catenary action for emergency load redistribution. It can be seen that the area identified is approximately consistent with the yield-line pattern that commonly forms under this form of displacement (Astaneh-Asl et al. 2001; Tan and Astaneh-Asl 2003; Dat and Hai, 2011; Yi et al. 2014). The load attributed to the adjacent slab regions was assumed to be sustained by the torsional and flexural strength in these regions. Moreover, observations by Dat and Hai (2011) indicate that the excluded areas typically develop a compression ring mechanism, which acts to restrain the inset membrane.

![Figure 6-4 – Tributary area assumed for emergency load redistribution by catenary action.](image)

Although in multi-storey configurations the structural bays immediately above a removed column are typically tied to the floors above via an intermediate column, the proposed assessment assumes each storey to act independently. Thus, no additional load or resistance is assumed.

By consideration of the given tributary area, the total emergency load demand ($Q_{ALS}$) was calculated as follows:
\[ Q_{ALS} = q_{ALS}L_xL_y \]  

**Equation 6-8**

### 6.2.3 Theoretical case study floor systems

Figure 6-5 identifies a typical RC floor system formed of three key components; the edge beam (01), slab (02) and main beam (03). Emergency load redistribution following column loss is facilitated by these elements. The design, detailing and specification of these elements is dependent upon the building configuration, bay dimensions in the x and y directions, structural loading, material properties and standard requirements.

A series of exemplar floor systems were designed, in order to identify the reinforcement detailing and properties of the Bay XX and Bay YY floor components for assessment of catenary performance. Given their importance to catenary action (see Chapter 5), the principal output of the structural designs was the reinforcement area and arrangement. Therefore, the floor systems were designed with strict observation of minimum required area of reinforcement in order to establish baseline parameters for the investigation of the sensitivity of collapse resistance by catenary action.

Appendix E provides example design calculations for the floor components and a summary of input parameters. Key aspects of the designs are described in the following sections.
6.2.3.1 Design basis

As this investigation is concerned with resilience of high occupancy multi-storey buildings, all exemplar floor systems were designed for loads typical of office buildings, defined by the Eurocode 1 (BSI, 2009) as Category B use. Accounting for self-weights, finishes, ceiling fixtures, services and live loads associated with ‘offices and work areas’ (BSI, 1999; 2002; 2009):

Permanent Unit Floor Load, \( g_k = (24h + 2.4) \) kN/m², where \( h \) is the floor slab thickness.

Variable Unit Floor Load, \( q_k = 2.5 \) kN/m²

Slab, edge beam and main beam components (01, 02 and 03) were designed for the load effects (design moment and shear) associated with boundary conditions and requirements of intermediate structural bays, rather than penultimate and end bays. All design was conducted in compliance with ultimate and service limit state (ULS and SLS) requirements of EC2 (BSI, 2014).

It should be noted that a building height of five storeys was assumed, with storey heights of 3.5m, but only to facilitate preliminary sizing of columns and thus beam widths. All preliminary sizing was conducted assuming lateral resistance of the buildings was accommodated by shear cores, rather than frame resistance. The design of the floor components was otherwise assumed independent of building height.

Material properties were assumed for conventional medium-rise RC constructions. Thus, all design was based on the specification of C30/35 \((f_{ck} = 32 \text{ MPa}; f_{cu} = 40 \text{ MPa})\) structural concrete and B 500B reinforcement \((BS 4449:2005 \text{ and } BS EN 10080 \text{ specification}; f_{yk} = 500 \text{ MPa})\).

6.2.3.2 Structural configurations

In Figure 6-5, Bay X-X and Bay Y-Y identify adjacent floor bays located at the perimeter of the floor plan for which external column loss would result in emergency load redistribution across an emergency span of \( L_{ALS} = 2L_y \) and \( L_{ALS} = 2L_x \), for columns CXX and CYY respectively. Span \( L_y \) was assumed to be greater than \( L_x \), giving an aspect ratio of \( k = L_y/L_x \). Framed slabs are commonly designed as one-way spanning for arrangements of \( k \geq 1.5 \) and two-way spanning for configurations of \( k \leq 1.5 \) (Reynolds, 2008). This threshold was implemented in the design of the exemplar floor systems, therefore allowing for the investigation of both configurations for various grid spacings.

6.2.3.3 Detailing & area of reinforcement

Span-thickness ratios, and the effective depth of reinforcement, are known to influence the area of reinforcement required in design. Thus, these parameters were regulated between floor designs to provide some level of consistency and an economical allocation of reinforcement. Preliminary span-thickness ratios of 12-15 were implemented for all beam sections. Similarly, slab components were designed to a span-thickness ratio of 30-36.
Figure 6-6 is a schematic section showing the potential reinforcement arrangement of a conventional RC element across two adjacent structural bays – consistent with the detailing of the test specimens featured in Chapters 4 and 5. By designing each floor system to Eurocode 2 (EC2; BSI, 2014), the minimum required area of reinforcement attributed to bar groups 01 to 04 were calculated for slab, edge beam and main beam components.

The area of bar groups 01 and 04 is dictated by the ULS load effects; the moment calculated across intermediate supports and at the mid-span. Whereas, for the floor configurations considered, the area of reinforcement required in bar groups 02 and 03 was found to be governed by minimum detailing and curtailment requirements, as stipulated by cl.9.2 and cl.9.3 (BSI, 2014), rather than any need for compression reinforcement at mid-span and support locations.

Bar groups 02 and 03 were identified in testing to constitute the critical area of reinforcement in secondary catenary response. Thus, the baseline exemplar floor system design was conducted to strictly provide minimum required area of reinforcement for these reinforcement allocations.

6.2.4 Sensitivity parameters

The floor components of twenty five exemplar RC framed buildings were designed and detailed in order to investigate the sensitivity of emergency load redistribution by catenary action to various changes in structural configuration and design specification. In order to assess sensitivity to differing grid arrangements and column spacings, floor systems of the range $1.2 \leq k \leq 2.0$ and $5m \leq L \leq 12m$ were designed and detailed.

The results obtained in testing indicated that ultimate membrane force was governed by the area of B1 bottom reinforcement (i.e. groups 03 and 04, Figure 6-6). The redistribution of interior support moments is a design parameter known to influence the area of bar groups 03 and 04. Therefore, 30% and 0% redistribution was assumed for similar one-way floor systems ($k \geq 1.5; \ 5 \leq L \leq 12$). In either case, the area of reinforcement allocated to all bar groups was engineered to be as close to minimum requirements as practicably achievable, given the geometry of the section and typical rebar sizes.

The tie force requirements are another aspect of conventional design that influences the area of reinforcement (see Chapter 2 and Appendix A). It was typically found that minimum tie force
requirements of EC2 (BSI, 2014; cl. 9.10) were satisfied or exceeded by the area of reinforcement specified for bar groups 01 and 03. However, to investigate the influence of specifying $A_{s-crit}$ (bar groups 03 and/or 04) to meet minimum tie requirements, the area required for 5, 10 and 15 storey configurations were calculated for assessment.

A summary of the reinforcement areas specified for each bar group is provided in Appendix E.

### 6.2.5 Results and discussion

The exemplar floor systems were assessed to establish the chord rotation ($\theta$) required to effectively redistribute emergency load (i.e. $FOS = 1.0$). A summary of the results obtained for each floor system and reinforcement configuration considered is provided in Appendix F. Graphical plots of the chord rotations required to arrest collapse in Bays XX and Bay YY of individual floor systems are provided, showing the effects of bay area and area of reinforcement on performance.

The following sections provide commentary on the key factors found to influence structural robustness by catenary action.

#### 6.2.5.1 Influence of bay area and orientation

Figure 6-7 provides a graphical plot of results obtained in the study of the exemplar floor systems. The graphs show the FOS ratio calculated for floor systems of different emergency bay area ($2L_xL_y$ and $L_x2L_y$). The FOS ratio of each floor system was calculated as that corresponding to a maximum chord rotation of 12.8° at the free edge of the structural bay (Section 6.1). This was assumed an appropriate empirical ultimate end-rotation for the assessment of the floor system as the slab and edge beam reinforcement arrangements were designed to be consistent with those used in testing.

![Graphical plots showing FOS results](image)

**Figure 6-7** – FOS results obtained for floor systems where $A_{s-crit} = A_{s-b}$.

The results shown above are specified for one-way and two-way spanning floor systems (denoted OW and TW, respectively). The maximum percentage redistribution used in design is referenced by the suffix indicated. These results correspond with baseline exemplar floor designs, whereby the area of
critical reinforcement \( (A_{s-crit}) \) was assumed equal to the area of bar group 03 \( (A_{s3}) \), the minimum area of bottom reinforcement permissible at joint locations in accordance with EC2.

Inspection shows a general trend whereby the collapse resistance of the floor systems reduces as the area of the structural bay increases. This was observed in the two and one-way spanning floor designs of Bay XX, for which the emergency span corresponded with the longest column spacing \( (L_{ALS} = 2L_y) \). The strongest trend between bay area and fall in FOS was found in the two-way spanning floor systems. By analysis of a linear best-fit trend line, it was found that the FOS fell by 28% in Bay XX and 39% in Bay YY between floor areas of 50 and 250m². However, results obtained for the one-way spanning Bay YY indicate that for bay areas greater than 100m² the FOS was found to increase with area. This was traced to a combination of the aspect ratio, whereby the catenary developed across the shorter span \( (L_{ALS} = 2L_x) \), and a corresponding increase in the strength of the slab elements.

For a given floor area, one-way spanning systems of \( k \geq 1.5 \) were found to provide a higher level of progressive collapse resistance by secondary catenary response. By direct comparison of the Bay XX results, it was found that one-way systems of bay area 100, 115 and 157m² were calculated to have 36, 35 and 28% higher FOS than two-way spanning counterparts of the same total floor area. Similar results were obtained for the Bay YY scenarios; 21, 48 and 25%, respectively.

### 6.2.5.2 Sensitivity to area of reinforcement and detailing requirements

The area of critical reinforcement \( A_{s-crit} \) was varied to establish the influence of different minimum reinforcement requirements and curtailment rules on the collapse resistance of the exemplar floor systems. This was found to be the most significant factor in catenary performance.

As stated above, Figure 6-7 provides the results for the baseline minimum area of reinforcement considered, \( A_{s3} \). This arrangement assumed maximum permissible curtailment of the group 03 reinforcement across interior supports. Inspection of the FOS results shown in Figure 6-7 indicate that by detailing beam and slab elements to the minimum requirements of EC2, the reinforcement area specified for group 03 was inadequate to sustain emergency load redistribution by catenary response alone. The minimum FOS predicted was 0.47, obtained for a two-way spanning Bay XX of 180m². Inspection shows that two of the one-way spanning floor systems were predicted to sustain the specified emergency load. Although the resistance of the floor systems was found to differ significantly with bay size and orientation, the average FOS was found to be 0.58 and 0.85, for the two-way and one-way systems respectively.

Figure 6-8 provides a summary of results obtained for \( A_{s-crit} = A_{s4} \), whereby the area of the mid-span tension reinforcement (group 04) was assumed to be constant across the supports. Inspection of the results show a significant improvement over the \( A_{s3} \) scenario. In this case all floor systems were predicted to sustain the specified emergency load by secondary catenary response. The average FOS predicted for two-way and one-way examples was calculated as 1.17 and 1.64 respectively.
Exemplar floor system performance was investigated for a 50% curtailment of the group 04 reinforcement specified, across supports and joint areas, $A_{\text{crit}} = 0.5A_{s4}$. However, this was found to provide no significant advantage over the baseline reinforcement arrangement, $A_{\text{crit}} = A_{s3}$ (Figure 6-7). The results obtained for this arrangement are provided in Appendix F and denoted $A_{s4.5}$.

Inspection of Figure 6-7 and Figure 6-8 demonstrates that redistribution of elastic moments at supports, resulted in increased FOS predictions. This is evident from the results provided for the one-way systems, OW-30% and OW-0%. It can be seen that the increase in area of the bottom reinforcement attributed to 30% redistribution provided an advantage in ultimate catenary resistance for both the $A_{s3}$ and $A_{s4}$ scenarios. Average FOS results of 0.77 and 0.85 for the $A_{s3}$ scenario and 1.44 and 1.64 for the $A_{s4}$ demonstrate a 10% and 12% increase in collapse resistance for the OW-30% designs.

Inspection of Figure 6-9 and Figure 6-10 shows that by assuming secondary catenary response, the allocation of tie force requirements to the bottom reinforcement layers results in improved collapse resistance, compared with the $A_{s3}$. However, Figure 6-9 demonstrates that the requirements for a 5 storey structure were predicted to be inadequate in arresting building collapse. Figure 6-10a shows that by assuming the requirements of a 10 storey structure, the results indicate effective emergency load redistribution for the Bay XX arrangement, provided that the bay area is less than 150m² but that larger configurations are predicted to collapse. However, inspection of Figure 6-10b demonstrates that the tie force requirements across the short bay span ($L_x$) result in collapse predictions of Bay YY areas in excess of 100m².
For the 15 storey configuration, FOS predictions were found to be greater than 1.0 for all floor systems less than 200m² (see Figure 6-11).

Figure 6-9 – FOS results obtained for floor systems where $A_{\text{crit}} = A_{\text{t5}}$.

Figure 6-10 – FOS results obtained for floor systems where $A_{\text{crit}} = A_{\text{t10}}$.

Figure 6-11 – FOS results obtained for floor systems where $A_{\text{crit}} = A_{\text{t15}}$. 
6.2.6 Dynamic response

Due to the sudden application of load and potential motion of the floor system under gravitational acceleration, large-displacement response and progressive collapse associated with instantaneous column loss is in practice a dynamic event. When using linear static analytical procedures, such as that implemented in the existing sensitivity study, current codes of practice (GSA, 2013; DoD 2009) require the use of a dynamic load increase factor (DLF, see Equation 6-7) to account for dynamic effects. The DoD (2009) specifies a maximum DLF of 2.0. This constitutes a worst case condition derived from the dynamic response of an undamped linear elastic system, for which displacement under dynamic conditions is approximately twice the static displacement (Biggs, 1964). However, analytical investigations by authors such as Izzuddin et al. (2007) and Ruth et al. (2006) suggested that this is overly conservative for progressive collapse procedures, in particular when concerned with RC systems whereby large-displacement response exhibits significant inelastic behaviour that effectively damps the system by work done in plastic response.

Appendix F provides tabulated results obtained using the proposed equivalent linear static procedure, for DLF values of 1.0 (assuming static conditions) and 1.5. Given the linear load-displacement model used in estimating secondary TMA resistance and collapse resistance, comparison of the results demonstrated that a 50% increase in the DLF resulted in a 50% increase in the displacement required to arrest collapse and corresponding reduction of the FOS.

The results presented in the previous sections were established by assuming a DLF of 1.5. This was consistent with recommendations made by Ruth et al. (2006) and thus, the findings of the sensitivity study account for the influence of dynamic effects in emergency load redistribution by catenary action, to some extent. However, Yu et al. (2014) has reported an experimental study that examines dynamic catenary response. The study documents the behaviour of three preloaded large-scale laterally restrained RC specimens, tested under instantaneous column loss conditions. A novel aspect of the study is that column failure was induced by the detonation of a contact charge placed against the intermediate column. Thus, the experimental results recorded dynamic response attributed to both gravity loading but also prior uplift in the test specimen. By comparison with statically tested counterparts, the resulting downward response was found to correspond with a DLF of 1.86. This was stated to be an upper-bound value, due to primary blast damage, but this suggests that the results presented here may be under-conservative where concerned with potential collapse resistance related to similar malicious actions.

Investigations of the dynamic behaviour of RC catenary systems under extreme displacements or in incipient collapse conditions, as is the case considered by this thesis, are limited and represent an area of research in need of attention. Although the study by Yu et al. (2014) provides a rare insight into dynamic response of RC catenary systems, the clearance provided beneath test specimens did not support the investigation of secondary TMA response. Therefore, a recommendation of this research is for the experimental investigation of dynamic secondary TMA response to establish suitable
dynamic load and strain rate material strength increase factors in support of emergency load redistribution analyses.

6.3 Summary

This chapter presents a simplified equivalent linear static procedure for the analysis of emergency load redistribution in double-span RC floor systems, following intermediate column loss. The results and conclusions drawn from the experimental investigation of catenary response in one-way strip specimens (Chapter 5) have been used to establish characteristic limit state criterion for the assessment of collapse resistance by secondary TMA response. This model was applied to a number of exemplar floor systems in order to investigate the influence of practical factors in current limit state design and their impact on the potential collapse resistance.

This study identifies the area of critical reinforcement (the bottom reinforcement allocation at interior support locations) to be fundamental to the ultimate load resistance of catenary action. Factors such as structural bay size, the aspect ratio of the floor bay, hypothetical building height and the percentage redistribution of elastic moment used in design were therefore found to influence the area of critical reinforcement specified and thus impact structural robustness. The results will be used in Chapter 8 to draw conclusions about structural requirements.
7 Conclusions & Recommendations

Catenary or tensile membrane action (TMA) is a potential mechanism of second-order load resistance and emergency load redistribution in flexural systems featuring effective lateral resistant. The mechanism has been implemented prescriptively or by direct analysis in the design of structural robustness since the 1960’s and is fundamental to inelastic analyses and engineering practices concerned with ultimate collapse resistance. However, a number of inconsistencies and inadequacies have been identified by this research, associated with the characteristic limit criteria and theoretical approach currently used for the prediction of catenary response in reinforced concrete (RC) systems. This points to an incomplete understanding of emergency load redistribution by membrane response following intermediate support loss and thus a need for further research.

This thesis delivers three original contributions to the field of structural robustness research that investigate the areas of uncertainty identified:

An experimental investigation of catenary action behaviour and incipient collapse conditions in half-scale double-span RC test specimens.

An analytical assessment of the characteristic limit criteria and first-principle theoretical approach currently used in estimating ultimate catenary response.

An analytical sensitivity study that investigates practical factors in statutory design that affect the performance of emergency load redistribution by catenary action in conventional RC floor systems.

The following sections provide a summary of the conclusions drawn from each aspect of the investigation, highlight how the research has contributed to the current understanding of emergency load redistribution by catenary action and makes recommendations for the advancement of this work.

7.1 Experimental investigation of large-displacement response in laterally restrained RC strip specimens

The current understanding of large displacement response in double-span RC elements can be broken into successive phases. Initial displacement is governed by the moment and shear resistance of the element, whereby the system exhibits elastic and plastic flexural response. If the element features effective lateral restraint, compressive membrane action (CMA) is known to develop, providing an enhanced load resistance the magnitude of which is dependent on factors such as the area of reinforcement, span-depth ratio and restraint stiffness. When displacement exceeds approximately half the section depth, CMA response and load resistance decays during snap-through, falling roughly to that of the theoretical ultimate flexural strength. Tensile membrane action (TMA) is assumed to
develop at displacements approximately equal to the section depth. This mechanism is commonly idealised as a perfectly plastic load carrying net or chain, strung between points of vertical and lateral restraint, for which load resistance is directly proportional to displacement and the area of longitudinal reinforcement sustaining the tensile membrane force. Failure of the catenary mechanism results in total collapse and is therefore considered the ultimate mode of second-order load resistance.

This thesis identifies two potential modes of catenary response exclusive to RC systems that are successive and defined by the physical condition of reinforcement at critical sections. Primary TMA response is currently characterised by an ultimate displacement limit of 0.10 times the span, or a 12.8° degree chord rotation (or end rotation). Failure is defined by the rupture of extreme tension reinforcement at mid-span or interior support hinge locations. Secondary TMA response is contingent on a doubly reinforced section. The mechanism is sustained following primary TMA failure by tension in the remaining top and bottom reinforcement layers (the lesser of which is defined here as the critical reinforcement) at midspan and support locations, respectively. This mode is currently characterised by an ultimate displacement limit of 0.18 times the span or a 20.0° chord rotation and failure of the critical reinforcement results in total collapse.

Only primary TMA response is recognised by current and historic progressive collapse design and assessment guidance. As a result investigations of progressive collapse resistance by catenary response have commonly been limited to behaviour prior to primary material failure. Only two investigations document secondary TMA response in double-span conditions and of the three test specimens reported, total collapse was only achieved by two test specimens. However, these investigations demonstrated that ultimate load resistance could occur in secondary TMA response. The significance of this finding was therefore twofold; the limited data available contradicted current guidance, which suggests that ultimate load resistance is achieved in primary not secondary response; and, it identified that secondary catenary action behaviour in emergency load redistribution is significantly under researched but critical to the understanding of total collapse resistance. An experimental programme was therefore designed and implemented to investigate the large-displacement behaviour of laterally restrained RC beam and slab-strip specimens, with the objective of achieving total collapse or a displacement whereby the ultimate failure mechanism could be clearly identified. In doing so, experimental data has been obtained that supports the investigation of primary and secondary TMA mechanisms.

Twelve laterally restrained RC strip specimens were tested in a double-span condition to simulate intermediate support loss and subsequent emergency redistribution of a load applied at the midspan (location of support removal). This constitutes the largest single experimental investigation of its type since that documented by Regan (1975).

Total collapse was achieved in eight of the twelve specimens. The four specimens that did not collapse achieved chord rotations of up to 16° at which point the ultimate load carrying mechanism and failure
mode were evident. The force-displacement record obtained for each test specimen showed that the investigation was successful in its objective; characteristic membrane responses were recorded – an initial peak load associated with CMA, followed by snap-through and subsequent peak loads recorded for primary and secondary TMA. The lateral restraint reaction was recorded and verified the development and decay of compressive and tensile membrane force with each successive mode of response.

The load-displacement histories recorded in testing were used to quantify the peak loads, load enhancement (the ratio of peak recorded load to theoretical ultimate flexural strength) and work done in CMA and primary and secondary TMA responses and therefore investigate the relative performance of each mechanism of emergency load redistribution. The peak loads recorded in CMA and TMA responses were found to exceed the theoretical ultimate flexural strength for all specimens, demonstrating the capability of both forms of membrane response to provide a reserve of strength in emergency load redistribution. However, TMA response was typically found to provide the highest ultimate resistance and load enhancement, with results of between 1.1 and 6.9 times the ultimate flexural strength. Moreover, work done in TMA response was found to constitute an average of 68% of the total energy absorption measured in emergency response. This points to TMA as the dominant collapse resistance mechanism observed in testing.

Ultimate load in primary TMA response was identified as the peak load recorded prior to or at first failure of an extreme tension reinforcement layer. However, this resulted in a significant finding. Six of the twelve specimens tested sustained fracture of the extreme tension reinforcement during snap-through, prior to the development of tension membrane force. Thus, half the test specimens failed to develop primary TMA response. This mechanism of emergency load redistribution was therefore found to be unreliable.

Secondary TMA response was found to occur in all test specimens. Ultimate load resistance was identified as the peak load recorded prior to or at incipient collapse, following first failure of an extreme reinforcement layer. Results of between 1.1 and 6.9 times the theoretical ultimate flexural resistance were recorded, giving an average enhancement of 3.1. This was compared with load enhancement calculated for those specimens that achieved primary TMA response for which ultimate loads between 1.3 and 5.6 times the ultimate flexural strength were recorded, with an average enhancement of 1.9. Thus, ultimate collapse resistance was found to be governed by secondary TMA response, which offered greater load resistance and reliability over primary TMA response. Moreover, these findings challenge the current approach adapted by progressive collapse design guidance, which neglects collapse resistance by secondary TMA response.

Current TMA theory and progressive collapse design guidance might be described as basic and takes no account of reinforcement detailing or arrangement. Therefore the influence of reinforcement detailing on collapse resistance and catenary action performance was a key aspect of the experimental
investigation. Thus, the twelve test specimens were designed and fabricated with three distinctly
different reinforcement arrangements to form three test series. Three *E-series* specimens were
fabricated with a reinforcement arrangement (rebar curtailment and lap lengths) typical of edge beam
elements designed in accordance with *BS 8110* requirements. Curtailment and lapping resulted in
discontinuous longitudinal reinforcement; a reduction in the area of bottom reinforcement across
intermediate supports and the top reinforcement between curtailment points of the hogging rebar.
Three *S-series* specimens were also fabricated, with detailing typical of conventional one-way RC slab
elements. The reinforcement arrangement differed from the *E-series* specimens as *BS 8110*
requirements allowed a singly reinforced section between the curtailment points of the top or hogging
reinforcement and lapping of the bottom longitudinal reinforcement was made across interior supports.
The six *C&M-Series* or flat-slab strip specimens featured continuous reinforcement, with no
curtailments or lapping within the double-bay. It should be noted that minimum shear links were
specified for all specimens to facilitate fixing of the rebar cages.

Whilst all test specimens exhibited similar behaviour at small displacements, in-test observations
showed the physical response and mechanisms formed by each test group at large displacements to
differ significantly. All flat slab-strip (*C&M-Series*) specimens formed basic bilinear three-hinge
catenary mechanisms. That is; displacement was facilitated by rotation and in-plane extension at
plastic hinge locations that formed at mid-span and interior support locations during initial flexural
and CMA response. Failure in *primary* and *secondary* TMA response was by rupture of the extreme
tension and remaining reinforcement at hinge locations.

The edge beam (*E-Series*) specimens also formed three-hinge bilinear catenary mechanisms but plastic
hinge development was found to correspond with points of curtailment in the top and bottom
reinforcement layers. Failure of *primary* TMA response occurred by rupture of the extreme tension
reinforcement at either location. Failure of the top reinforcement layer at an inset from the interior
supports led to the formation of *incomplete catenary mechanisms* in *secondary* response; a bilinear
catenary profile effectively suspended between cantilevers formed by the length of hogging
reinforcement, projected from the supports. The *incomplete catenary mechanisms* were found to
sustain increased load resistance and progressive scabbing of the concrete cover along the soffit of the
supporting cantilevers as displacement was increased. This mode of response demonstrated the
importance of specifying links and lapping them within the top portion of a beam section, as the shear
links were found to limit the progress of cover failure and prevented terminal degradation of laps in
the bottom reinforcement.

Response observed in the slab-strip (*S-Series*) specimens was also found to be influenced by
reinforcement curtailment and lapping. Two key features defined the catenary mechanisms developed.
By lapping the bottom reinforcement across interior supports, the specimens typically formed four-
hinge bilinear catenary mechanisms, with plastic hinges forming at both interior supports and two at
the mid-span; one on each side of the midspan lap length. Also, the curtailment of the top
reinforcement to provide two intermediate lengths of singly reinforced section resulted in pronounced and frequent transverse tension cracking along these regions, as displacement was increased.

The test results demonstrated the importance of the respective catenary mechanisms. By comparison of edge beam and flat slab-strip specimens, with similar ultimate in-plane properties, the edge beam specimens were found to exhibit superior performance by a significant margin. Total work done was 54% greater, work done in TMA response was approximately 32% greater, load enhancement was 10% greater and the ultimate load resistance was 21% greater than results recorded in the equivalent flat slab-strip test specimen. Similarly, a slab-strip specimen of the same area of critical reinforcement sustained 138%, 67%, 178% and 28% higher results for each respective measure performance. By examining the results obtained for edge beam, slab-strip and flat slab-strip specimens of similar theoretical ultimate flexural resistance, the specimens detailed with conventional curtailment and lapping were again found to sustain superior ultimate performance and collapse resistance. This demonstrates that whilst each test group exhibited characteristic CMA and TMA responses, load redistribution by catenary action was strongly influenced by reinforcement curtailment and lapping and the resultant mechanism formed at large displacement.

Load resistance in catenary action and work done are directly proportional to displacement. By assuming rotation and in-plane extension were confined in the main to strain in the reinforcement at crack and hinge locations, the relative performance observed between the three test series has been traced to superior rotational ductility and extensibility properties in the specimens featuring conventional detailing, thereby resulting in greater ultimate displacements in secondary TMA response. The average displacement at peak load recorded in E-Series and S-Series strip specimens was 143% of that recorded for the C&M-Series specimens. Moreover, it should also be noted that all of the C&M-Series specimens sustained total collapse in testing whereas only one in three of the E-Series and S-Series specimens collapsed. It follows that the performance and results recorded for the specimens with conventional detailing can be identified as lower-bound.

These findings and the observed responses suggest that the use of regions of reduced area of longitudinal reinforcement and arrangements encouraging four-hinge mechanism development could improve ultimate collapse resistance in catenary response. This indicates that ultimate limit criteria require consideration of detailing or prediction of hinge locations and the in-plane extension properties of an RC element in order to accurately predict ultimate displacement and collapse resistance.

7.2 Theoretical interpretation of catenary response

The current theoretical interpretation of catenary action is that the relationship between load, tension membrane force and displacement can be examined from first principles by considering equilibrium of a perfectly plastic parabolic or three-hinge bilinear catenary mechanism. The experimental rig and test arrangement was designed and instrumented to provide a statically determinate system and thereby investigated the validity of current plastic membrane theory.
Conditions at ultimate secondary TMA response were found to be consistent with current theory. The span, midspan displacement and horizontal restraint reaction recorded at ultimate secondary TMA load were used to calculate the ultimate theoretical load of each test specimen. Theoretical values were found to be conservative, underestimating load resistance by an average of 19.6%. However, this discrepancy was found to be reduced when assessing secondary TMA mechanisms at large displacements. Also, by examination of the incomplete catenary mechanisms formed in the E-Series specimens, it was found that an accurate account of effective catenary span (rather than the basic total span) could provide better theoretical ultimate load predictions. This therefore demonstrates that current theory can be an adequate but conservative model of catenary response but an accurate prediction of the relative geometry of a catenary mechanism is required for accurate rather than safe estimates of ultimate collapse resistance and restraint force.

The same approach was taken in evaluating primary TMA response. However, by applying the results recorded at ultimate primary TMA load this investigation found the current theoretical approach to be inadequate; underestimating experimental results by as much as 97%. Again, the discrepancy was found to be most pronounced when concerned with ultimate load resistances recorded at relatively small displacements. Given that primary TMA is associated with response prior to first material failure, the corresponding reaction data was used to evaluate the moment-axial force interaction at ultimate load. All specimens were found to exhibit a combination of membrane action and flexural resistance, which is neglected by current plastic membrane theory. Therefore the results suggest that the theoretical interpretation of primary TMA response requires an account of combined flexural and TMA resistance.

The current theory results in a linear increase in load resistance with displacement, as constant membrane force is typically assumed. Therefore, conditions corresponding to CMA-TMA transition and ultimate primary and secondary TMA response are of particular importance in characterising typical nonlinear large-displacement and catenary behaviour. By designing and fabricating all specimens with conventional structural materials (a minimum C40 micro-concrete and B 500B reinforcement) and testing at large-scale, the results obtained provided no scale effects and featured material properties and ductility consistent with full-scale conventional constructions. Moreover, the geometry, span-depth ratio, reinforcement area and reinforcement detailing of the test specimens were designed in accordance with BS 8110 requirements for edge beam, slab and flat-slab elements consistent with light-commercial or office use. Thus, the experimental results were used to identify and assess the validity of load resistance, displacement and membrane force characteristic limit criteria of RC specimens consistent with modern RC constructions.

CMA-TMA transition was identified by the attainment of zero lateral restraint force, after attenuation of CMA peak load and compressive membrane force, and is of significance as it marks the initiation of tension membrane development. This was found to occur at displacements related to the specimen thickness and span-thickness ratio. Current recommendations suggest that transition occurs at a
displacement approximately equal to the thickness of the section. This agreed to some extent with the results obtained in testing for which an average transition displacement of 1.34 times section depth was obtained. However, a discrepancy of 34% is considered appreciable and, by investigating alternative experimental data resources, indicates that reduced rotational restraint at the interior supports may have resulted in higher transition displacements. Moreover, the relationship established with span-thickness ratio was found to provide a stronger (but inverse) linear trend which suggests that this may be a more appropriate means of predicting displacement at transition.

Further inconsistencies were found between current characteristic displacement criteria and conditions recorded in testing. The most significant was associated with the ultimate plastic chord rotation; defined as the displacement or end rotation at failure of extreme tension reinforcement at either mid span or interior support hinge locations and critical sections. Current guidance suggests an ultimate chord rotation of 6.0° for RC elements with shear reinforcement but no tension membrane force and 12.8° at incipient failure of primary TMA. However, an average ultimate chord rotation of 5.56° was recorded in testing. Moreover, with a standard deviation of 2.47° the ultimate chord rotation recorded was found to be highly variable and whilst the experimental results were examined for trends with the section properties and span-depth ratios of the test specimens, none were identified. Given that the test specimens were laterally restrained, thus defined as capable of TMA response, the average ultimate chord rotation attained was therefore only 43% that recommended by current guidance. A more appropriate characteristic ultimate chord rotation would be the 6.0° specified for unrestrained counterparts. However, this would have resulted in an overestimate of ultimate chord rotation in nine of the twelve test specimens.

Chord or end rotation is effectively a measure of deflection and is insensitive to the mechanism formed in catenary response and number of plastic hinges that facilitate displacement. This research found that chord rotation was therefore an inadequate criterion for specifying ultimate displacement as it provided no measure of the rotation and ductility demand at critical sections or the plastic hinge rotation. Thus, the E-Series and C&M-Series formed three-hinge systems and obtained average ultimate chord rotation results of 4.31° and 4.84°, respectively, whereas the four-hinge mechanisms formed by the S-Series specimens resulted in an average result of 8.25° – almost twice that sustained by the other test groups. This discrepancy was traced to the relative ductility demand on the reinforcement at the midspan plastic hinges, as the rotational demand at this location in the S-Series specimens was accommodated by twice the number of hinges. It follows that future recommendations for characteristic ultimate displacement should be defined by ultimate joint rotation capacity and explicit consideration of the number of hinges forming the catenary mechanism, rather than chord rotation. By examining the experimental results using this approach, an average ultimate joint rotation capacity of 8.49° was obtained for the test population, with standard deviation of 2.67°.

By specifying a characteristic ultimate chord rotation of 12.8° for laterally restrained RC systems, current guidance implies that primary TMA response can be reliably achieved and ultimate load
resistance is attained at this displacement. However, it has been shown that none of the test specimens sustained this displacement prior to failure of the extreme tension reinforcement layer. Moreover, inspection of the horizontal restraint force recorded at ultimate chord rotation showed that 50% of specimens suffered reinforcement fracture prior to CMA-TMA transition. This demonstrates that the current theoretical interpretation of primary TMA response is unsafe for design purposes. Furthermore, the current interpretation of CMA-TMA transition is that minor concrete crushing during inelastic bending and CMA response results in a load resistance marginally less than the theoretical ultimate flexural strength. However, due to prior failure of extreme tension reinforcement, this was not consistent with results obtained in testing. Although an average recorded load resistance at transition of 0.88 times the ultimate flexural resistance was obtained for the full test population, the six specimens that sustained failure of the extreme tension reinforcement prior to transition were found to demonstrate only marginal load resistance at transition – an average of 0.56 times the ultimate flexural strength. This further demonstrates the importance of accurately identifying the ultimate joint rotation capacity of modern RC systems and under-conservatism in the current theoretical interpretation of nonlinear large displacement behaviour of laterally restrained RC systems.

This research found that ultimate collapse resistance was governed by secondary TMA response. Therefore, identification of appropriate ultimate limit criteria for this mode of response is of great significance. Current guidance related to secondary TMA response is confined to blast resistance applications and not recognised in progressive collapse resistant design. The former suggests that ultimate secondary response and incipient collapse corresponds with a characteristic displacement limit of 20°. However, this was not achieved in testing. Rather, an average ultimate chord rotation of 10.91° was recorded for the full test population. Whilst this could be considered a lower-bound result, as the testing of four specimens was terminated at the displacement limit of the test rig (between 15° and 16°) before outright collapse was achieved, visual inspection of the catenary mechanisms at test termination suggest that a 20° rotation was not realistically attainable before collapse.

As with the plastic rotation capacity of the specimens, ultimate chord rotation in secondary TMA response was found to be highly variable, with a standard deviation of 3.10° against the mean. Although a weak relationship was observed with reinforcement ratio, the experimental data did not provide sufficient evidence to accurately determine it. The strongest influence on ultimate displacement and the catenary mechanism formed was found to be the reinforcement detailing; the average result obtained for the C&M-Series specimens was 8.99° compared with the 12.86° and 12.80° recorded for the test groups with conventional detailing (E-Series and S-Series, respectively). This research points to a need for further research to define a characteristic ultimate displacement in terms of the in-plane elongation properties attributed to individual secondary catenary mechanisms.

Ultimate membrane force is another limiting criterion specified by current guidance. That is; the current theoretical approach to estimating ultimate load resistance of catenary response entails the identification of a corresponding tensile membrane force. The experimental rig was therefore
instrumented such that the horizontal restraint force was recorded, thereby allowing the assessment of membrane force at key stages of the load-displacement histories.

Horizontal restraint force histories recorded characteristic elastic and inelastic tension response during primary and secondary TMA, with some evidence of strain hardening at ultimate load resistance. Moreover, failure associated with both modes of catenary response was always by tensile failure or rupture in the reinforcement, rather than anchorage failure or pull-out. This indicated the adequacy of standard reinforcement lapping requirements for catenary response but also demonstrated that membrane force was governed by the ultimate tensile properties of specific reinforcement allocations.

The study identified that membrane force and thus secondary catenary resistance was dependent on the area of critical reinforcement. This was typically the bottom reinforcement layer at critical sections. By analysis of the catenary geometry and horizontal restraint force recorded at ultimate secondary TMA load, it was found that membrane force typically exceeded the yield strength of this reinforcement layer but did not reach the ultimate tensile strength. Rather, average results indicated an ultimate membrane stress in the reinforcement of 1.07 times yield strength and 0.89 times the ultimate tensile strength. DoD (1998) suggests a tensile stress equal to the average of the yield and ultimate tensile strength of the effective reinforcement. When applied to the area of critical reinforcement, this approach was found to give the closest prediction of ultimate membrane force in secondary TMA response, with an average tensile force only 2% greater than recorded in testing.

The current theoretical interpretation of ultimate primary TMA response stipulates a membrane force equal to the yield strength of the total area of longitudinal reinforcement (top and bottom rebar allocations). However, analysis of test results recorded at ultimate primary TMA load demonstrated that the average membrane force was 21% less than the yield strength of the smallest available reinforcement layer. Membrane force was found to approach or exceed yield strength of the reinforcement only at larger rotations. However, this was only observed in two slab-strip specimens which attained 11.4° and 9.43° and in which tensile membrane response was more pronounced. This demonstrated that the current approach is not representative of actual primary catenary behaviour and results in a significant over estimate of in-plane membrane force and load resistance.

### 7.3 Catenary action in practical applications

This thesis presents an analytical sensitivity study that implements the results and conclusions drawn from the experimental investigation to examine practical design factors that influence emergency load redistribution by catenary response. Structural bay size, the aspect ratio of a floor system and the height (number of storeys) of a building are all key factors that govern the design and detailing of RC floor systems, in accordance with current limit state practice. Thus the perimeter floor bays of twenty five exemplar RC framed buildings have been designed and detailed in accordance with current
Eurocode 2 (EC2; BSI, 2014) requirements, assuming Category B use (i.e. for loads consistent with light commercial or office use). Grid arrangements were varied between aspect ratios of 1.2 and 2.0, for service spans of between 5m and 12m. This facilitated the design and analysis of one and two-way spanning beam-slab floor configurations for which the edge-beam and slab elements could be designed and detailed for the same basic reinforcement arrangement used in the E-Series and S-Series test specimens. Moreover, by assuming different building heights, the influence of corresponding minimum tie force requirements could be investigated for five, ten and fifteen storey variants of the twenty five exemplar systems.

The exemplar floor systems was modelled by assuming the hypothetical removal of an intermediate perimeter column. A bilinear catenary profile was assumed to evaluate load resistance in one-way secondary TMA response across the double structural bay. A linear load-displacement relationship was implemented with a membrane force of the average yield and ultimate tension force offered by the critical reinforcement – consistent with experimental findings. No account of flexural and arching actions was taken. The performance of each exemplar floor system was assessed by its factor of safety (FOS) against collapse – the ratio of load resistance to emergency load – corresponding to a maximum characteristic ultimate displacement limit of 12.8°, as recorded in testing for the E-Series and S-Series specimens. In each case, the surrounding structure was assumed to provide adequate lateral restraint to sustain catenary action.

Given that the experimental investigation demonstrated the critical reinforcement to be fundamental to ultimate resistance in secondary TMA response, the study examined the sensitivity of collapse resistance given various design philosophies that influenced the area of this rebar allocation. By designing the exemplar floor systems for economy of reinforcement, whereby the minimum area of reinforcement stipulated by EC2 curtailment requirements was specified for the bottom allocation across interior supports, it was found that collapse could not be prevented by catenary action. Various design alternatives were considered which resulted in an increased area of the critical reinforcement. A 30% redistribution of hogging moment was found to increase collapse resistance appreciably, whilst maintaining maximum curtailment. However, the only effective variation, for which the resistance of the floor system was found to prevent collapse, was to avoid curtailment of the bottom longitudinal reinforcement.

The tie force requirements stipulated by EC2 dictate that edge beams and or internal slabs must be tied to the adjacent structural bay by a minimum area of reinforcement. In accordance with the UK National Annex, the minimum required area of tying reinforcement is prescribed as a factor of the number of storeys – a tie force of (20 + 4 times the number of storeys)kN must be achieved. It was found that exemplar floor systems detailed in accordance with maximum permissible curtailment requirements met the minimum tie force requirements corresponding to EC2 guidance for buildings of five storeys. This is because EC2 states that the tie force provision can be fulfilled by either the top or bottom reinforcement allocations at interior supports, or both. Therefore, the influence of specifying
the critical reinforcement in accordance with minimum tie force requirements was investigated for five, ten and fifteen storey structures. The results demonstrated an improvement in collapse resistance compared with aggressively curtailed counterparts. However, in order to provide effective collapse resistance for floor systems of double bay area up to 200m², it was found that the critical reinforcement had to be specified for a tie forces corresponding with a fifteen storey configuration.

It was found that the aspect-ratio and subsequent design philosophy of respective floor systems had an influence on collapse resistance when detailed with maximum curtailment. The most pronounced difference was found to be between floor systems of the same bay area but designed as one and two-way spanning arrangements. Typically the one-way spanning counterparts were found to provide a higher level of structural robustness. However, it should be noted that by specifying the critical reinforcement to meet minimum tie force requirements, the difference in performance observed between these two forms of floor system was significantly reduced – both one and two-way counterparts were found to provide a comparable level of collapse resistance.

This research demonstrates that the design of structural robustness and progressive collapse resistance by catenary action should entail the direct analysis of catenary response and the area of reinforcement specified to support adequate ultimate resistance must be allocated to the bottom reinforcement passing through the interior supports; the critical reinforcement.

7.4 Recommendations for further work

This research has indicated the following areas for further work:

- The experimental programme involved twelve half-scale specimens. These represent the majority of testing on RC beam and slab systems at the high span-depth ratios resulting from single column loss. Further testing is needed to confirm and strengthen the outputs from the experimental programme.
- The specimens tested were of constant depth. The loss of a column would result in a deeper section local to the lost column. Testing of specimens with a midspan downstand is thus recommended.
- The double-span conditions implemented in testing provided a symmetrical arrangement, with each service span being of identical length and reinforcement arrangement. Given the importance of secondary response to ultimate collapse resistance, this mechanism should be investigated for asymmetrical arrangements.
- The results of the experimental programme were obtained using a single test rig with a relatively small horizontal stiffness. Numerical studies suggest that catenary action is relatively insensitive to lateral restraint stiffness. However, this has not been experimentally verified for strip test specimens. It is therefore recommended that further testing should be
carried out in test rigs of different horizontal stiffness to investigate the influence of this factor on the development of catenary action.

- The testing and analytical studies were conducted assuming static conditions. However, large-displacement emergency load redistribution is frequently considered a dynamic response. It is therefore recommended that dynamic testing be undertaken to investigate the influence of rate of displacement on the development of catenary action.

- The analytical study conducted was simplistic and neglected the influence of flexural response in emergency load redistribution. This was considered conservative. Therefore, it is recommended that a more detailed analytical investigation be undertaken to further investigate the robustness of RC floor systems. Of particular interest are the trends observed between exemplar floor systems, which suggest that robustness requirements might be standardised by design parameters, reinforcement detailing and structural bay size.
8 References


Appendices
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of Figures</td>
<td>IV</td>
</tr>
<tr>
<td>List of Tables</td>
<td>VII</td>
</tr>
<tr>
<td>Appendix A Supporting Literature Investigations</td>
<td>1</td>
</tr>
<tr>
<td>A.1 Masonry Infill Panelling as Emergency Bracing</td>
<td>1</td>
</tr>
<tr>
<td>A.1.1 Robust Design and the Need for Further Alternatives</td>
<td>1</td>
</tr>
<tr>
<td>A.1.2 Prediction of the Seismic Response of Infill Walls Subject to In-Plane Loads</td>
<td>3</td>
</tr>
<tr>
<td>A.1.3 Modification of Seismic Equivalent Strut Model (ESM) for Robustness Design</td>
<td>5</td>
</tr>
<tr>
<td>A.1.4 Discussion &amp; Conclusions</td>
<td>10</td>
</tr>
<tr>
<td>A.2 A Review of Current Tie Force Requirements in Indirect Robust Design</td>
<td>12</td>
</tr>
<tr>
<td>A.3 Ultimate Load Criteria &amp; Failure Limits in Catenary Action</td>
<td>16</td>
</tr>
<tr>
<td>A.3.1 Characteristic Ultimate Displacement ($\Delta u$) &amp; Rotation Capacity ($\theta u$)</td>
<td>16</td>
</tr>
<tr>
<td>A.3.2 Ultimate Tension Membrane Force ($N_u$)</td>
<td>23</td>
</tr>
<tr>
<td>Appendix B Test Rig Photographs</td>
<td>29</td>
</tr>
<tr>
<td>B.1 Test Rig – Image Gallery</td>
<td>29</td>
</tr>
<tr>
<td>B.1.1 Frame Elevations</td>
<td>29</td>
</tr>
<tr>
<td>B.1.2 H-frame Buttressing</td>
<td>30</td>
</tr>
<tr>
<td>B.1.3 Holding down System</td>
<td>30</td>
</tr>
<tr>
<td>B.1.4 Hanger &amp; Load Actuator</td>
<td>31</td>
</tr>
<tr>
<td>B.1.5 End Details</td>
<td>32</td>
</tr>
<tr>
<td>B.1.6 Potentiometers</td>
<td>32</td>
</tr>
<tr>
<td>Appendix C Test Specimens &amp; Material Test Data</td>
<td>33</td>
</tr>
<tr>
<td>C.1 Material Test Data</td>
<td>33</td>
</tr>
<tr>
<td>C.1.1 Reinforcement test data</td>
<td>33</td>
</tr>
<tr>
<td>C.1.2 Concrete cube test data</td>
<td>36</td>
</tr>
<tr>
<td>C.1.3 Interpretation of cube test results</td>
<td>37</td>
</tr>
<tr>
<td>C.2 Test Specimens – As Built Drawings</td>
<td>40</td>
</tr>
<tr>
<td>Appendix D Test Results &amp; Observations</td>
<td>52</td>
</tr>
<tr>
<td>D.1 Calculation of Moment Resistance</td>
<td>52</td>
</tr>
<tr>
<td>D.2 Test Data – Starting Reactions</td>
<td>55</td>
</tr>
<tr>
<td>D.3 Test Data – Specimen E01 Results &amp; Observations</td>
<td>56</td>
</tr>
<tr>
<td>D.3.1 Specimen response notes</td>
<td>56</td>
</tr>
<tr>
<td>D.3.2 Test data</td>
<td>57</td>
</tr>
<tr>
<td>D.4 Test Data – Specimen E02 Results &amp; Observations</td>
<td>59</td>
</tr>
<tr>
<td>D.4.1 Specimen response notes</td>
<td>59</td>
</tr>
<tr>
<td>D.4.2 Test data</td>
<td>60</td>
</tr>
<tr>
<td>D.5 Test Data – Specimen E03 Results &amp; Observations</td>
<td>62</td>
</tr>
</tbody>
</table>
List of Figures

Figure A-1 – MIF structure featuring heavy bomb damage to ground floor perimeter columns........ 1
Figure A-2 – Recognised forms of frame response under column loss. ........................................ 2
Figure A-3 – Three predominant modes of infill panel failure under base shear/ lateral load. ........ 4
Figure A-4 – Schematic illustrations of infill response to seismic and collapse stimulus. ............... 6
Figure A-5 – Stress variation between seismic and column-loss ESM.......................................... 7
Figure A-6 – Geometric nomenclature for numerical analysis.......................................................... 8
Figure A-7 – Schematic elevation of mechanism used in the derivation of the basic tie requirements for reinforced concrete (BSI, 1972; Burnett, 1975). ................................................................. 13
Figure A-8 – Horizontal tie arrangements endorsed by the British Standards................................. 14
Figure A-9 – Characteristic resistance-deflection behaviour of conventional slab elements, without shear reinforcement, per DoD (1990; 2008; 2014). ................................................................. 19
Figure A-10 – Load-displacement curves documented by Guice (1986), showing predicted vs experimental results with different membrane force parameters.............................................. 26
Figure A-11 – Load-displacement curves documented by Woodson (1994), showing predicted vs experimental results with different membrane force parameters.............................................. 27
Figure B-1 – Elevation of test rig showing specimen C04 prior to loading..................................... 29
Figure B-2 – H-frame elevations showing specimen E03 at the beginning of Phase 02 (a) and Phase 03 (b). .................................................................................................................................................. 29
Figure B-3 – Lateral restraint buttressing introduced to outside-face of H-frame columns. .......... 30
Figure B-4 – Holding down system devised to enable packing of actuator – specimen E02 shown restrained at 340mm displacement...................................................................................... 30
Figure B-5 – Hanger assembly (a) shown together with detail of load cell (b) and strain gauge arrangements (c)......................................................................................................................... 31
Figure B-6 – Central hanger arrangement (a) and load actuator (b). ................................................ 31
Figure B-7 – East End Detail (at location A) shown on elevation (a) and plan (b) – note vertical and horizontal orientation of load cells.................................................................................. 32
Figure B-8 – West End Detail (at location G). .................................................................................. 32
Figure B-9 – Potentiometers and LVDTs located along specimen span (a) and at column face (b) – note all instruments are secured to an independent frame to ensure accuracy................................................. 32
Figure C-1 – Stress-elongation data recorded in reinforcement testing (ref Table B1 through B5 for values). ................................................................................................................................................ 35
Figure C-2 – Comparison of experimental and theoretical mean cube strength development with age (calculated in accordance with EC2, 2004) for structural concretes, cement class-R ....................................... 38
Figure C-3 – General arrangement and reinforcement detailing – Edge beam strip specimen E01... 40
Figure C-4 – General arrangement and reinforcement detailing – Edge beam strip specimen E02... 41
Figure C-5 – General arrangement and reinforcement detailing – Edge beam strip specimen E03. 42
Figure C-6 – General arrangement and reinforcement detailing – Slab strip specimen S01. 43
Figure C-7 – General arrangement and reinforcement detailing – Slab strip specimen S02. 44
Figure C-8 – General arrangement and reinforcement detailing – Slab strip specimen S03. 45
Figure C-9 – General arrangement and reinforcement detailing – Column strip specimen C01. 46
Figure C-10 – General arrangement and reinforcement detailing – Column strip specimen C02. 47
Figure C-11 – General arrangement and reinforcement detailing – Column strip specimen C03. 48
Figure C-12 – General arrangement and reinforcement detailing – Column strip specimen C04. 49
Figure C-13 – General arrangement and reinforcement detailing – Middle strip specimen M01. 50
Figure C-14 – General arrangement and reinforcement detailing – Middle strip specimen M02. 51
Figure D-1 – RC cross-section stress strain diagram. .................................................. 52
Figure D-2 – Material stress-strain functions used in section analysis. ............................ 52
Figure D-3 – Force-rotation data recorded for E01 test. ............................................. 57
Figure D-4 – Moment-rotation and moment-axial-force results, test E01. ...................... 58
Figure D-5 – Deflection profile record, test E01. .......................................................... 58
Figure D-6 – Force-rotation data recorded for E02 test. ............................................. 60
Figure D-7 – Moment-rotation and moment-axial-force results, test E02. ...................... 61
Figure D-8 – Deflection profile record, test E02. .......................................................... 61
Figure D-9 – Force-rotation data recorded for E03 test. ............................................. 63
Figure D-10 – Moment-rotation and moment-axial-force results, test E03. ................. 64
Figure D-11 – Deflection profile record, test E03. ......................................................... 64
Figure D-12 – Force-rotation data recorded for S01 test. ........................................... 66
Figure D-13 – Moment-rotation and moment-axial-force results, test S01. ................... 67
Figure D-14 – Deflection profile record, test S01. ......................................................... 67
Figure D-15 – Force-rotation data recorded for S02 test. ........................................... 69
Figure D-16 – Moment-rotation and moment-axial-force results, test S02. ................... 70
Figure D-17 – Deflection profile record, test S02. ......................................................... 70
Figure D-18 – Force-rotation data recorded for S03 test. ........................................... 72
Figure D-19 – Moment-rotation and moment-axial-force results, test S03. ................... 73
Figure D-20 – Deflection profile record, test S03. ......................................................... 73
Figure D-21 – Force-rotation data recorded for C01 test. ........................................... 75
Figure D-22 – Moment-rotation and moment-axial-force results, test C01. ................... 76
Figure D-23 – Deflection profile record, test C01. ......................................................... 76
Figure D-24 – Force-rotation data recorded for C02 test. ........................................... 78
Figure D-25 – Moment-rotation and moment-axial-force results, test C02. ................... 79
Figure D-26 – Deflection profile record, test C02. ......................................................... 79
Figure D-27 – Force-displacement data recorded for C03 test. ................................... 81
Figure D-28 – Moment-rotation and moment-axial-force results, test C03................................. 82
Figure D-29 – Deflection profile record, test C03................................................................. 82
Figure D-30 – Force-displacement data recorded for C04 test........................................... 84
Figure D-31 – Moment-rotation and moment-axial-force results, test C04.......................... 85
Figure D-32 – Deflection profile record, test C04................................................................. 85
Figure D-33 – Force-displacement data recorded for M01 test.......................................... 87
Figure D-34 – Moment-rotation and moment-axial-force results, test M01......................... 88
Figure D-35 – Deflection profile record, test M01................................................................. 88
Figure D-36 – Force-displacement data recorded for M02 test.......................................... 90
Figure D-37 – Moment-rotation and moment-axial-force results, test M02......................... 91
Figure D-38 – Deflection profile record, test M02................................................................. 91
Figure E-1 – Bar group references....................................................................................... 95
Figure F-1 – Load-displacement plots for theory#1............................................................. 99
Figure F-2 – Load-displacement plots for theory#2............................................................. 100
Figure F-3 – Load-displacement plots for theory#3............................................................. 101
Figure F-4 – Load-displacement plots for theory#4............................................................. 102
Figure F-5 – Load-displacement plots for theory#5............................................................. 103
Figure F-6 – Load-displacement plots for theory#6............................................................. 104
Figure F-7 – Load-displacement plots for theory#7............................................................. 105
Figure F-8 – Required chord rotation at FOS = 1.0 (DLF = 1.0), calculated for two-way spanning floor systems (moment redistribution 10-30%). ................................................................. 108
Figure F-9 – Required chord rotation at FOS = 1.0 (DLF = 1.0), calculated for one-way spanning floor systems (moment redistribution 10-30%). ................................................................. 109
Figure F-10 – Required chord rotation at FOS = 1.0 (DLF = 1.0), calculated for two-way spanning floor systems (moment redistribution 0-10%). ................................................................. 110
List of Tables

Table A-1 – Horizontal tie force ($F_t$) required in current UK regulated design. ........................................ 12
Table A-2 – End-rotation limits recommended by DoD (1990; 2008b; 2014) for dynamic nonlinear analysis of conventional RC beam and slab elements. ................................................................. 17
Table A-3 – End-rotation limits recommended by DoD (1998) for dynamic nonlinear analysis of conventional RC beam and slab elements. .................................................................................. 17
Table A-4 – Response limits recommended by USACE (2008a) for dynamic nonlinear analysis of conventional RC beam and slab elements................................................................. 18
Table A-5 – Maximum allowable response limits, per GSA (2003). .............................................................. 22
Table A-6 – Maximum allowable response limits, per DoD (2005)............................................................... 22
Table C-1 – Reinforcement yield strength – test sample results................................................................. 33
Table C-2 – Reinforcement tensile strength at maximum force – test sample results. ............................ 33
Table C-3 – Reinforcement tensile strength at fracture – test sample results ......................................... 34
Table C-4 – Reinforcement elongation at maximum force – test sample results. ................................ 34
Table C-5 – Reinforcement elongation at fracture – test sample results. ............................................. 34
Table C-6 – Cube density test data. ............................................................................................................. 36
Table C-7 – Cube strength test data. ............................................................................................................ 36
Table C-8 – Target long-term mean cube strengths (in accordance with EC2, BSI 2008). ....................... 37
Table D-1 – Recorded versus target support reactions taken at the end of Phase 01 & 02, prior to loading. ............................................................................................................................................ 55
Table E-1 – Example design of exemplar structural floor system components........................................... 94
Table E-2 – Reinforcement data for two-way floor systems, designed with 10-30% moment redistribution........................................................................................................................................ 95
Table E-3 – Reinforcement data for one-way floor systems, designed with 10-30% moment redistribution....................................................................................................................................... 96
Table E-4 – Reinforcement data for one-way floor systems, designed with 0-10% moment redistribution........................................................................................................................................ 96
Table F-1 – Summary of reinforcement tension test data (courtesy of BRE and UK rebar manufacturers, 2010). .......................................................................................................................... 97
Table F-2 – Summary of differing theories for the quantification of ultimate membrane force, N.... 98
Table F-3 – Summary table – Chord rotation required at FOS = 1 (DLF=1.0). .......................................... 106
Table F-4 – Summary table – Chord rotation required at FOS = 1 (DLF=1.5). ...................................... 107
Appendix A  Supporting Literature Investigations

A.1  Masonry Infill Panelling as Emergency Bracing

The ability of masonry infill framed (MIF) buildings to resist the onset of disproportionate collapse is well documented, their survivability being noted throughout the bombing campaigns of the Second World War (Baker et al. 1948; Walley, 2001), the troubles in Northern Ireland (Rhodes, 1974) and in numerous incidents thereafter (Faella & Nigro, 2005) typical illustrated by the photographs provided in Figure A-1 showing two views of a MIF structure that had suffered the loss of two columns at the ground floor. This marked resilience of MIF structures is attributed predominantly to a residual strength that is inherent to their design; any enhancement associated with the presence of the masonry being neglected in design and the infill panels being assumed non-structural and purely architectural. The result, in the case of well designed MIF structures, is a significant ‘over strength’ and high level of robustness that is manifest as an increase in lateral strength, global stiffness, energy dissipation and propensity to adopt alternative load paths to a degree far surpassing that of a bare frame (Christopherson, 1945). This appendix assesses the capacity of infill panels in providing an anti-collapse measure, presenting a viable alternative amongst existing robust design methods intended to accommodate column loss.

![Figure A-1](image1.png)  

Figure A-1 – MIF structure featuring heavy bomb damage to ground floor perimeter columns.

A.1.1  Robust Design and the Need for Further Alternatives

The resilience afforded a structure against collapse following column loss comes from the ability of the building to redistribute load, through the formation of a load bearing emergency mechanism. Some of these mechanisms are shown schematically in Figure A-2a to A-2d.

Veirendeel action shown in Figure H-2a is found in reinforced concrete (RC) and steel moment resisting frames; the structure undergoing moment redistribution – adjacent joints and elements taking up the emergency load, induced by column loss, in flexure – in an effort to avoid disproportionate
collapse. For this mechanism to be effective the design of the structure must be carried out with column loss in mind, adopting the ‘double span’ methodology as required by the GSA (2003; 2013) code of practice. Catenary action, see Figure H-2c, is observed in simple (non-continuous) framed buildings, the phenomenon not being dependent on the moment capacity of the joints but rather on their ability to absorb combined tension and rotation, as the emergency load is sustained by the ability of the structure to form a catenary around the damaged area. To achieve this form of emergency mechanism the designer is required to implement the ‘tying force’ methods in accordance with the British Standards (BSI, 2000; 2008a; 2008b). The implementation of alternative load paths is another form of robust design set out in the British standards and commonly entails the integration of outrigger trusses at intervals throughout the height of a framed structure as shown in Figure H-2d.

Each of the types of emergency mechanism described above are developed by means of robust design. Arching action, Figure H-2b, is the only mechanism that cannot be consciously implemented by the designer. It occurs more as a phenomenon inherent within RC structures whereby internal compression struts may be induced under the initial sagging displacement of unsupported beam-column assemblies. This form of structural response precludes catenary action.

![Figure A-2 – Recognised forms of frame response under column loss.](image)

Whether due to direct or indirect robust design, each of these emergency mechanisms has characteristic limitations. To effectively achieve veirendeel action the designer must compromise the economy of the structure, increasing section sizes and joint capacities throughout the building to account for singular column loss at any given location. Furthermore, in the event of multiple column loss, such response raises concerns with regard to the anomaly of structural ‘drag down’ – the weight of the unsupported area overwhelming the subsidiary structure leading to further progressive collapse. The tying force (catenary) method, as presented by the British Standards, was derived following the Ronan
Point collapse of 1968, with little evidence gathered thereafter as to its performance, the method takes no account of the reduction in the tension capacity of a connection with rotation (Byfield and Paramasivam, 2007). As a result it has been subjected to much criticism and reduced confidence in its effectiveness. Arching action is, thus far, an incidental method of robust design that cannot constitute a primary defence against the onset of collapse until it has been given a firm design basis. In addition to the specificity of material and geometric requirements, this form of response suffers further limitations from its very nature, the most important of which is the need for a high level of in-plane restraint. The provision of alternative load paths using outrigger trusses presents a feasible alternative as a form of direct robust design that is both economical and trustworthy. However, it may be considered intrusive by the architect as it requires substantial space in order to conceal the trusses. It is clear from scrutiny of the existing measures for robust design that there is demand for further alternatives in robustness enhancement. With significant compressive capacity, permitting arching and corbelling over a damaged area, the implementation of masonry infill to provide an alternative emergency load path is a clear contender in robust design. Such an approach would be opportune where the building has potential for external masonry cladding and regular internal partitions, allowing emergency load to be resisted by the in-plane stiffness of the infill elements. Figure A-2e is a schematic representation of such an arrangement and, whilst this particular illustration demonstrates a fully MIF structure in load redistribution, it may well be feasible, so long as a clear alternate load path is still discernible, to activate fewer infill panels. Whilst authors such as Christopherson (1945) have concluded that regular masonry panels of 200mm thickness are sufficient to prevent racking and redistribute emergency load, it is necessary to quantify the potential of such a system so as to provide proper design guidance. In order to assess the capabilities and minimum requirements of such a system the complex response of the masonry infill and frame in combination under in-plane loading must be considered and quantified.

A.1.2 Prediction of the Seismic Response of Infill Walls Subject to In-Plane Loads

In order to predict the in-plane behaviour and bracing strength of an infill-frame assembly it is necessary to regard the structure as a form of composite; the relative stiffness and characteristic mechanical properties for the individual frame elements and infill panel must be considered together with its geometric arrangement. There must be an allowance for the variability of both material properties and standard of construction and further consideration of the location and dimensions of structural openings and the frame-infill interaction, the latter of which is known to vary as the system is loaded. These input requirements add to the complication of predicting the behaviour of the anisotropic masonry. An accepted method for quantifying the added resilience afforded a frame structure by infill panelling under column loss is not available at present; however with the enhanced lateral strength and stiffness attributed to infilled frames significantly altering the response of bare framed buildings to seismic loading – reducing global ductility and the natural frequency whilst
significantly increasing the capacity for energy dissipation – methods for modelling such systems have become increasingly important in seismic design. A significant amount of research has been conducted into the seismic response of MIF structures and the findings may be important to the assessment of MIF structural robustness under column loss.

The past five decades have seen extensive research and experimentation to derive a methodology for the analytical modelling of infill systems under seismic action. There is, as yet, no consensus as to the best approach for behavioural prediction or design but information gathered provides valuable insights into their response. Failure of an infill frame will manifest as one of the failure modes shown in Figure A-3, occurring at relatively small lateral displacements with the frame still acting elastically. Each mode of failure is related to the relative properties of the panel and frame and may be detailed as follows; corner crushing, common in infill systems with relatively weak masonry components; diagonal tension cracking, in the case of a weak frame surrounding a strong infill unit; shear sliding, induced in systems with poor mortar/block strength ratio and a strong frame; out-of-plane strut buckling, which is common in infill panels of slender construction; and frame failure, where premature plastic hinges are induced in a weak frame due to the presence of infill or the alteration of the frame mechanical response following initial infill failure. Quantification of the ultimate load capacity of each failure mode has been made possible by the work of Man Muller (1982), Crisafulli (1997) and others who pioneered theories based upon the principal stress distribution, whereby a masonry unit of a laterally (in-plane) loaded infill panel is assumed subject to a vertical principal stress and combined couple, the latter being induced by shear in the bed joint. Complete collapse of such a system results from combinations of the above mentioned failure modes, and whilst each is of significance, the last two can be eliminated by competent design leaving the smallest of the ultimate strength of corner crushing, diagonal tension cracking and shear sliding to give the lower bound lateral load capacity of a masonry infill element. Of these, shear sliding is generally the most common form of failure in masonry infill walls but the ultimate load capacity of each failure mode needs to be found.
The interaction of the infill with the surrounding frame plays a significant part in the combined system behaviour. At low load levels, when interaction is sound, the masonry acts compositely with the frame, however, as the load is increased, separation of the panel from the frame becomes apparent and the two components tend to independent behaviour as the composite action breaks down. There are currently two alternatives in modelling this process, the macro and micro-models. The latter constitutes a detailed and comprehensive prediction of the masonry and surrounding framework, accounting for both global and local effects, using finite element analysis and modelling individual masonry units. For the purposes of the typical practicing engineer, this method is too laborious for application. The macro-model however, is a simplified account of the infill behaviour that may be applied, with general rules (such as those laid down by the documents *FEMA 273, 306 and 356*), to a framework to an acceptable degree of accuracy.

The macro-model has been developed by researchers from Polyakov (1956) and Holmes (1961), to the more recent Panagiotakos and Fardis (1994) and Crisafulli (1997). As outlined by Smyrou (2006) it is based upon the tendency of an infill element to resist in-plane lateral load by forming a diagonal compression strut; a concept similar to gravity induced compression arching and corbelling. Whilst this form of analysis cannot accurately predict the local effects within the framework of the masonry under in-plane loading, the prediction of the infill is sufficient for general analysis and will serve the purposes of robustness assessment.

### A.1.3 Modification of Seismic Equivalent Strut Model (ESM) for Robustness Design

The problem of redistribution of column loads is simpler than that encountered in seismic design. Analysis of an infill frame under earthquake action must account for cyclic lateral loading, taking into consideration degradation of the strength and stiffness of the system upon re-loading, whereas, in the case of robustness analysis, the emergency load induced under column loss is a singular monolithic event. However, the differences in loading between applied base shear associated with seismic analysis and the gravity-induced emergency loading under column loss, dictates that the seismic equivalent strut modelling techniques require some manipulation before being applied to robustness analysis.

Structural response under earthquake action is by ‘horizontal storey drift’ and is experienced by a bracing infill panel as an in-plane lateral load impinging upon the opposing corners of the frame at a line parallel to that of the bed joint (see Figure A-4a). The subsequent response of the infill panel is dictated by the mechanical distortion of the bounding frame, the orientation of the compression strut being reliant upon the contact length resulting between column and infill and beam and infill. In the event of storey drift this contact length is mainly at the column face, so that the equivalent strut tends toward the horizontal which encourages failure of the panel by shear sliding along the bed joint. With regard to robustness assessment in the event of column loss, an infill system is loaded differently to
that of a building subject to lateral thrust as the surrounding frame is subjected to vertical rather than horizontal in-plane loads at opposite corners (see Figure A-4b). The resultant structural response is by sagging of the unsupported lengths of beam and the orientation of the diagonal compression strut is altered, tending toward the vertical and promoting primary failure by means of shear perpendicular to the bed joint of the masonry, where resistance is greater. Figure A-4a and H-4b illustrate schematically the differences in strut development. The illustrations demonstrate a need for changes to the seismic form of ESM before accurate assessment of the masonry infill contribution under column loss can be established.

![Diagram](image)

**Figure A-4 – Schematic illustrations of infill response to seismic and collapse stimulus.**

The principal stress theories for the seismic macro-models, such the Man Muller theory, were devised purely for ‘horizontal storey drift’, that is the external load must act horizontally with the bed joint assumed parallel to the line of external loading, and the majority of frame-interaction is assumed to occur at the masonry headers along the column face. Hence the stress distribution used to determine the ultimate load for each failure mode is based upon the shear capacity parallel to the bed joint and compressive/tensile capacity perpendicular to the bed joint. As shown in Figure A-5, the principal
stress arrangement is inverted in the case of column-loss. Under ‘vertical storey sagging’, the shear capacity and characteristic compressive strength become effective perpendicular and parallel to the mortar joint, respectively. Because of this difference, the ESM derived for seismic assessment cannot be directly applied to column-loss analysis.

For accurate prediction under loss of support an equivalent-compression-strut modelling method must be developed from first principles for this specific form of structural response; the assessment of corner crushing will require little alteration from that derived for seismic excitation; both vertical and horizontal (shear-sliding) shear failure will require assessment, and the potential for tension cracking will have to be quantified using a method newly derived from the amended principal stress arrangement shown in Figure A-5.

For the purposes of this investigation an attempt has been made to quantify the failure load of each failure mechanism, for the column loss scenario, using simple manipulation of established seismic
methods (refer to Figure A-6 for all geometric symbols). The methods and problems encountered are described below.

![Figure A-6 – Geometric nomenclature for numerical analysis.](image)

A.1.3.1 Corner crushing failure

The general method of evaluating corner crushing under lateral loading, as presented by *FEMA 306* (FEMA, 1998), relies upon the calculation of an effective compression strut width (a) as a function of the relative stiffness of the infill panel and surrounding frame. Equation A-1 was developed by Mainstone (1970) and is used to estimate the equivalent compression strut width.

\[
a = 0.175(\lambda h)^{-0.4} b
\]

Equation A-1

\[
\lambda = \left[ \frac{E_m t_m \sin 2\theta}{4E_{\text{conc}} I_c h} \right]
\]

Equation A-2

Where \( t_m \) is the thickness of the masonry (mm), \( I_c \) is the second moment of area of the column and \( E_m \) and \( E_{\text{conc}} \) are the expected moduli of elasticity (N/mm²) for the masonry and concrete respectively. The lateral crushing Load \( P_c \) is then calculated using Equation A-3, as developed by Stafford-Smith and Carter (1969).
\[ P_c = at_m f_{m90} \cos \theta \]

Equation A-3

where \( f_{m90} \) is the characteristic compressive strength of the masonry parallel to the bed joint (N/mm\(^2\)) (approximately 0.5\( f_m \)).

Whilst it is possible to change the above equations to make them more representative of the column loss scenario, the contact length \( Y = a \cos \theta \) was derived empirically and does not yield reliable values for the contact length \( X \), which is of more interest in this case. Thus, for a more accurate interpretation, it is necessary to assess the beam-panel contact length by means of the following.

\[ X = \alpha_b l \]

Equation A-4

\[ \alpha_b = \sqrt[2]{\frac{2(M_{pj} + 0.2M_{pb})}{t_m f_m}} \]

Equation A-5

where \( M_{pj} \) and \( M_{pb} \) are the plastic moment capacities of the joint and beam respectively. The vertical crushing load may then be estimated as.

\[ P_c = X t_m f_m \]

Equation A-6

**A.1.3.2 Tension cracking**

With reference to Figure A-5 it can be seen that the tension cracking load is a function both the shear and the tensile strength of the infill. In the seismic case, the shear contribution is effective along the bed joint of the masonry whilst the tensile strength is mobilised at the header of each unit. In the case of column loss this relationship is inverted, the shear strength being effective perpendicular to the line of the bed joint (through the units) and the tensile strength along the bed joint. It is uncertain whether this alternative arrangement will yield greater resistance as, although the tensile strength is far less than the shear resistance provided at the bed joint, the vertical shear resistance of a masonry unit is significantly larger than that in the horizontal direction (assuming no pre-loading).

Provisionally this mode of failure has been assessed using the following equation.
where $\tau_m$ is the vertical shear strength (N/mm²) of the infill unit.

Given the differences between the two modes of loading, it is clear that the term for tension cracking load (Equation A-7) is unlikely to provide an accurate prediction of this mode of failure in the event of a lost column. An alternative derivation could be devised using similar principles and elastic tension cracking theory (such as that developed by Chen, 1982, for structural concrete) but extensive testing would be needed to confirm the validity of the approach.

A.1.3.3  **Vertical Shear & Shear sliding**

With the alteration of the aspect ratio, with regard to the load orientation, vertical shear failure of the panel is likely to precede any shear sliding at the bed joint (see Figure A-5). Both must be assessed, and a consolidated method for their analysis is currently being finalised. The existing methodology, with regard to the vertical shear failure, is to scrutinise the contact length between the infill panel and beam. If the opposing contact lengths are within a 30degree arc of each other, as dictated by the geometry of the panel, then the infill will form a compression arch and vertical shear failure will not occur before corner crushing. However, should the geometry not permit direct compression arching, it is possible that the vertical shear resistance can be calculated in accordance with *BS 5628-1:2005* (BSI, 2005a) and resolved to give the corresponding load capacity whilst accounting for preload attributed to distortion in the frame.

A.1.4  **Discussion & Conclusions**

The benefits of extensive masonry infill panelling to the robustness of structural frames is well established. Authors such as Christopherson (1945) and Baker *et al.* (1948) advocated the use of masonry infill, citing extensive evidence of the robustness of 1940’s bomb damaged buildings because of the extensive inclusion of 100mm (4’’) and 240mm (9.5’’) brickwork. However, in order to provide guidance as to the effective use of modern brick/blockwork panelling as a robustness measure in current design and construction, the arrangement, material specification and placement of panels needs to be investigated. This would be best accomplished by establishing numerical modelling that could be used to assess the variations in panel construction and placement in terms of the response to the emergency load that would follow column loss.

Extensive research has been conducted in the field of seismic engineering in order to establish the behaviour of masonry infill panels subject to in-plane loads from earthquakes. This chapter has detailed an attempt to adopt the macro-models devised from this research for application to the scenario of redistributing emergency load associated with a lost column, by similar response in the infill panel.
However, due to the differing orientation of principal loads, this attempt was subject to various complications.

As with seismic loading it was found that the response of framed panels in carrying the emergency load associated with loss of a local support induced a diagonal compression strut in the masonry infill. Three principal failure modes were identified and macro models were devised to quantify the maximum in-plane load associated with each one and model their behaviour and in-plane strength. However, direct application of these numerical techniques has not been possible. Minor modification of the terms derived for corner crushing were required to provide reasonable estimates for this form of failure. However the expression for failure by tensile cracking was found to be explicit to seismic loading – the model accounts for shear in the bed joint and principal stresses at the headers of the masonry unit making the approach inflexible – as was the case with the calculation of shear bed-joint failure. Whilst the shear failure mode can probably be represented by the adaptation of current assessment procedures for vertical shear, to establish an accurate model for each failure mode would require extensive experimentation which falls beyond the scope of this thesis.
A.2 A Review of Current Tie Force Requirements in Indirect Robust Design

Effective horizontal ties are endorsed for use in steel, reinforced concrete and masonry construction of a given size and use and regulated by documents; The British Building Regulations (ODPM, 2013), British Standards (BSI, 2005a; 2005b; 2008b), Eurocodes (BSI, 2008a), and US Department of Defense (DoD, 2005; 2009). Vertical tie force requirements are facilitated by walls and columns and detailed by both the British Standards and Eurocodes as fulfilled if tied from foundation to roof such that a floor above damaged column can be suspended under accidental load. The accidental floor loading associated with vertical tying requirements varies between the specified codes of practice. Table A-1 below details the current horizontal tie force ($F_t$) advocated for UK steel and concrete construction.

<table>
<thead>
<tr>
<th>Code</th>
<th>Peripheral Ties</th>
<th>Internal Ties</th>
</tr>
</thead>
<tbody>
<tr>
<td>BS 5950</td>
<td>Greater of [kN]: $0.25(1.4g_k + 1.6q_k)s_lnl$ or $75kN$</td>
<td>Greater of [kN]: $0.5(1.4g_k + 1.6q_k)s_lnl$ or $75kN$</td>
</tr>
<tr>
<td>BS 8110</td>
<td>$1.0F_t$ [kN]</td>
<td>Greater of [kN/m width]: $rac{(g_k + q_k)}{7.5}F_t$ or $1.0F_t$</td>
</tr>
<tr>
<td>Where, $F_t = (20 + 4n_0) \leq 60kN$</td>
<td>Where, $F_t = (20 + 4n_0) \leq 60kN$</td>
<td></td>
</tr>
<tr>
<td>Eurocode2</td>
<td>Greater of [kN]: $l_1q_1$ or $Q_2$</td>
<td>Greater of [kN/m width]: $0.5(l_1 + l_2)q_3$ or $q_4$</td>
</tr>
<tr>
<td>Where $q_1 = 10kN/m$ and $Q_2 = 70kN$</td>
<td>Where $q_3 = 20kN/m$ and $q_4 = 70kN$</td>
<td></td>
</tr>
<tr>
<td>Eurocode2 + UKNA</td>
<td>Greater of [kN]: $l_1q_1$ or $Q_2$</td>
<td>Greater of [kN/m width]: $rac{(g_k + q_k)}{7.5}F_t$ or $1.0F_t$</td>
</tr>
<tr>
<td>Where, $q_1 = (20 + 4n_0)kN/m$ and $Q_2 = 60kN$</td>
<td>Where, $F_t = (20 + 4n_0) \leq 60kN$</td>
<td></td>
</tr>
<tr>
<td>UFC 4-023-03 (2009)</td>
<td>$F_t = 6(1.2g_k + 0.5q_k)l_p$</td>
<td>$F_t = 3(1.2g_k + 0.5q_k)l_p$</td>
</tr>
<tr>
<td>Where, $l_p = 0.91m (3ft)$, the maximum offset permitted from the building edge.</td>
<td></td>
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<td>BS 8110</td>
<td>$1.0F_t$ [kN]</td>
<td>Greater of [kN/m width]: $rac{(g_k + q_k)}{7.5}F_t$ or $1.0F_t$</td>
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</tr>
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<td>Where, $l_p = 0.91m (3ft)$, the maximum offset permitted from the building edge.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A-1 – Horizontal tie force ($F_t$) required in current UK regulated design.

The origins of the tie force requirements provided above may be traced back to their first inclusion to regulatory design in the wake of the collapse of Ronan Point, 1968. Where concerned with the tying of reinforced concrete construction (as in BS 8110 and EC2), it can be seen that a $F_t$ value of ‘the lesser of $(20 + 4n_0)$ or $60kN$’ is imposed – a requirement that has remained unchanged since its first
appearance in *CP110* (BSI, 1972). According to Burnett (1975), the upper-limit of 60kN is derived from the equilibrium of a catenary system sustained across a double span following the loss of the intermediate support. The scenario is that shown schematically below (see Figure A-7). The service span (*l*) of 5m (17ft) is suggested to be common to the time. Similarly, the floor load (*w*) of 4.8kN/m is devolved from a typical service floor load of 150psf whereby characteristic dead and live loads are each taken at 3.6kN/m² (75p.s.f), under an accidental load case of \((1.0g_k + 0.33q_k)\). It can be seen that for equilibrium of the system it is assumed that the 4.8kN/m floor load is sustained at a sag equal to 10% of the emergency span, in this case 0.2*l*.

![Figure A-7 – Schematic elevation of mechanism used in the derivation of the basic tie requirements for reinforced concrete (BSI, 1972; Burnett, 1975).](image)

The alternative term used in determining the basic tie requirement, \((20 + 4n_o)\), can be seen to provide an increased tie force with number of storeys. Burnett (1975) states that this term accounts for the increasing probability of accidental support loss with building size and, as detailed by DoD (2005) Appendix B, was intended to smooth the grading of tying between structures of less than 5 storeys, for which the provision was originally not required. Authors such as Li *et al.* (2011) claim that this requirement is empirical and has no theoretical justification and thus suggest that the term be discarded as unsuitable.

Whether designing to *EC2+UKNA* (BSI, 2008a; 2015) or *BS8110* (BSI, 2005b), it can be seen that, for internal ties the minimum tying requirement is determined as the greater of the basic tie force \(1.0F_t\) or \(\left({\frac{g_k + q_k}{7.5}}\right)\frac{l_r}{5}F_t\). This latter term can be seen to facilitate an adjustment for floor systems of exceeding the parameters used in the derivation of the basic tie force; i.e. systems whereby \(l > 5m\) and \((g_k + q_k) > 7.5\)kN/m². However, in practice the use or enhancement of the upper-limit 60kN tie requirement is rare for buildings less than 10 storeys in height, which suggests that the tie force requirement is inadequate to sustain catenary action.

Further inspection demonstrates that, in accordance with *BS 8110*, peripheral ties are implemented at the basic tie strength (*F_t*) with no account for floor loads or grid dimension that differ from those shown in Figure A-7. Furthermore, given that *F_t* is taken at a maximum of 60kN, this suggests that the peripheral ties are not intended to sustain catenary action. In fact, the detailing suggested for the peripheral ties by *CP 110* (BSI, 1972) and Burnett (1975) suggested that the peripheral ties serve only
to provide anchorage to perpendicular internal tying reinforcement. Where concerned with the accidental or malicious loss of an external load bearing element, this has significant implications with regards the ability of the tie for requirements to prevent collapse by catenary action, especially when the building is heavily loaded or features large grid spacing. Concerns centred upon the peripheral tying capacities are exasperated by the required position of the ties. Peripheral ties are required to be located within 1.2m of the building edge, however, internal ties may either be distributed within the floor slab or grouped at the framing. Figure A-8 provides a plan of two potential tying schemes. It can be seen that should the internal ties running parallel with the building periphery are grouped within the framing (see Figure A-8 b), in the event of external column removal, catenary response of the double bay is dependent upon the intrinsic reinforcement of the slab and the peripheral tie alone.

![Figure A-8](image_url)

**Figure A-8** – Horizontal tie arrangements endorsed by the British Standards.

Upon first inspection the EC2+UKNA requirement for peripheral ties it appears an attempt has been made to address these inconsistencies – the 60kN is used as a lower limit and the empirical factor of 
\[
(20 + 4n_0)
\]
is implemented as a multiple of the end span length, \(l_i\). However, given the compliance the UK National Annex (BSI, 2015) with the original tie force requirements of *BS 8110*, it may be the case that this is erroneous. This observation is supported by design guidance documents, such as Vinci (2005), which provide design examples consistent with the original British Standard regulations.

By inspection of Table A-1 it can be seen that the minimum tie force requirements for structural steel (see *BS 5950*; BSI, 200) are different from those specified in RC construction. The primary difference is the use of a lower-limit, \(F_t \geq 75kN\). Although this investigation yielded no published literature regarding the origins of this lower-limit of the basic tie force, Cosgrove and Way (2007) state explicitly that the \(F_t = 75kN\) is derived from equilibrium of the same geometric system depicted in Figure A-7 – a double span catenary system subject to a 10% sag across the total emergency span. Although speculation, to yield the tie force of 75kN the floor load assumed in it derivation is 6kN/m which, under a ULS load case of \(1.4g_k + 1.6q_k\), suggests an assumed service floor load of approximately
3.8-4.0kN/m² (85p.s.f). The use of the term \((1.4g_k + 1.6q_k)s_{tnl}\) sets the tying requirements for steel apart from the basic tie force used in RC construction as it takes account of the total ULS load adopted by each tie. It should be noted that factor \(n\) is a recent addition to the requirements and only made such that structures of less than 5 storeys still adhere to tying regulations where previously none were required.

The 2005 publication *UFC 4-023-03* (DoD, 2005) states; ‘the British Tie Force requirements are adopted almost verbatim in this UFC…the Tie Force requirements presented in this UFC have been effective for the British over the last three decades and are the most prescriptive measure available for Indirect Design. In lieu of additional research and analysis, they are deemed to be sufficient for DoD construction’. However, this approach has since been discarded.

The revised 2009 document (DoD, 2009) advocates the requirements shown in Table A-1. \(F_t\) is implemented as \(3.0wl\) [kN/m] for internal ties and \(0.91(6.0)wl\) [kN] for the peripheral, with no upper or lower-limit imposed. Assuming it dimensionless, equilibrium of the geometry shown in Figure A-7 gives the term \(2.5wl\). It follows that the revised tie requirements can be numerically justified based upon a catenary system of similar central sag and the minimum tie force directly related to the load and grid dimension, eradicating much of the confusion surrounding the British Standards system. Furthermore, it is an interesting observation that the peripheral ties are introduced at a greater strength than found in the internal system, which is contrary to that found in *BS 8110* and suggests the for the peripheral ties to support catenary action following loss of peripheral support.

The revised requirements (DoD, 2009) include detailed spatial recommendations that are said to have been established following extensive numerical and computational modelling of structural frames under a series of column loss scenarios – assuming the instantaneous removal of individual corner, internal and external load bearing elements and accounting for dynamic effects (Stevens, 2008). Subsequent to this investigation, an additional requirement has been added whereby all horizontal ties are located within the floor slab or, alternatively, if grouped in the frame, the assembly must be shown capable of sustaining the specified minimum tie force whilst subject to a joint rotation of \(11.3°\) \((0.2l)\). It follows that this is the only code that attempts to address concerns regarding the rotational ductility of tied systems.
A.3 Ultimate Load Criteria & Failure Limits in Catenary Action

This section provides an account of the ultimate limit criteria provided for TMA response in conventional RC elements, as documented by blast resistant design and progressive collapse design documents. The origins of these criteria have been investigated in order to establish whether these criteria correspond to primary or secondary TMA response and support a review of their suitability to modern structural design.

A.3.1 Characteristic Ultimate Displacement ($\Delta_u$) & Rotation Capacity ($\theta_u$)

Response limits are typically defined by joint rotation, deformation and ductility demand-capacity criteria. Thus, they are expressed in terms of end-rotation ($\theta$) and ductility ratio ($\mu$).

\[
\theta = \tan^{-1} \frac{\Delta}{L} = 2\tan^{-1} \frac{\Delta}{L_{ALS}} \quad \text{Equation A-8}
\]
\[
\mu = \frac{\Delta}{\Delta_E} \quad \text{Equation A-9}
\]

Where, $\Delta_E$ is the elastic deflection limit and $\Delta$ is the predicted or measured displacement. Ultimate limit criteria ($\theta_u$ and $\mu_u$) correspond with the ultimate displacement, associated with maximum load resistance prior to incipient failure, $\Delta = \Delta_u$.

A.3.1.1 Blast resistant design & overload conditions

Table A-2, Table A-3 and Table A-4 provide a summary of characteristic response limits specified by DoD and USACE guidance for ductile flexural response in conventional RC beam and slab elements subject to far field blast loads ($Z > 1.2\text{m/kg}^{1/3}$) – thus subject to a uniformly distributed load. The response criteria essentially categorise the extent of component damage at given end-rotations, thus supporting the prediction of characteristic component damage using nonlinear analyses. Although the terminology and definitions vary between sources, inspection of the criteria demonstrates a basic level of agreement with DoD (1990; 2008b; 2014), which suggested incipient failure at:

- $\theta \geq 2^\circ$ in conventional slab elements without shear reinforcement or tension membrane.
- $\theta \geq 6^\circ$ in conventional slab elements with shear reinforcement but no tension membrane (previously specified as $4^\circ$ by DoD, 1990).
- $\theta \geq 12^\circ$ in conventional slab elements with adequate lateral restraint and tension membrane.

This suggests that elements capable of developing TMA response can sustain displacements more than twice that of unrestrained counterparts, before incipient failure.
<table>
<thead>
<tr>
<th>Component Type</th>
<th>Adequate Lateral Restraint</th>
<th>Rotation Limit (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slabs (framed &amp; flat slab)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without shear reinforcement.</td>
<td>No</td>
<td>2</td>
</tr>
<tr>
<td>With min. shear reinforcement requirement.</td>
<td>No</td>
<td>6</td>
</tr>
<tr>
<td>With or without min. shear reinforcement requirement.</td>
<td>Yes</td>
<td>12</td>
</tr>
<tr>
<td>Beams2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Singly reinforced.</td>
<td>No</td>
<td>2</td>
</tr>
<tr>
<td>Symmetrical top and bottom reinforcement.</td>
<td>No</td>
<td>4</td>
</tr>
<tr>
<td>With adequate lateral restraint.</td>
<td>Yes</td>
<td>8</td>
</tr>
</tbody>
</table>

1 Previously specified as 4° (DoD, 1990).
2 Minimum shear reinforcement assumed.

Table A-2 – End-rotation limits recommended by DoD (1990; 2008b; 2014) for dynamic nonlinear analysis of conventional RC beam and slab elements.

<table>
<thead>
<tr>
<th>Component Type</th>
<th>Adequate Lateral Restraint</th>
<th>Light Damage1</th>
<th>Moderate Damage2</th>
<th>Heavy Damage3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slabs and Beams</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(L/h ≥ 5)</td>
<td>No</td>
<td>6</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>6</td>
<td>12</td>
<td>20</td>
</tr>
<tr>
<td>Slabs and Beams</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(L/h &lt; 5)</td>
<td>No</td>
<td>2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>6</td>
<td>12</td>
<td>-</td>
</tr>
</tbody>
</table>

1 Light Damage – Moderate cracking and no scabbing, recommended for protection of personnel, sensitive equipment and high value assets when minimal risk of injury or damage is desired. It should be noted that this is the maximum allowable end rotation for unrestrained elements.

2 Medium Damage – Some scabbing, no significant reinforcement rupture and moderate dust and debris hazard to the leeward side. Recommended for protection of personnel and equipment when risk of injury and damage is acceptable.

3 Heavy Damage – At or near incipient failure. Significant rupture of reinforcement, severely cracked (near rubble) concrete within confinement reinforcement, recommended as maximum allowable damage. Suitable only for protection of insensitive assets.

Table A-3 – End-rotation limits recommended by DoD (1998) for dynamic nonlinear analysis of conventional RC beam and slab elements.
Component Type | Adequate Lateral Restraint | Response Limits
--- | --- | ---
|  |  | B1 | B1 | B3 | B4
|  |  | μ | θ (°) | μ | θ (°) | μ | θ (°) | μ | θ (°)

| Slabs and Beams without shear reinforcement. | No | 1 | - | - | 2 | - | 5 | - | 10 |
| Slabs and Beams with compression and shear reinforcement. | No | 1 | - | - | 4 | - | 6 | - | 10 |
| Slabs and Beams (L/h ≥ 5) | Yes | 1 | - | - | 6 | - | 12 | - | 20 |
| Slabs and Beams (L/h < 5) | Yes | 1 | - | - | 6 | - | 7.2 | - | 12 |

1 Damage categories:

- B1-B2 – Moderate Damage – Permanent deflection but is generally repairable.
- B2-B3 – Heavy Damage – Component has failed but has significant permanent deflections.
- B3-B4 – Hazardous Failure – Component has failed and debris velocities range from insignificant to very significant.
- B4 – Blowout – Component overwhelmed by the blast load causing debris with significant velocities.

Table A-4 – Response limits recommended by USACE (2008a) for dynamic nonlinear analysis of conventional RC beam and slab elements.

Figure A-9 shows the resistance-displacement function used by DoD (1990; 2008b; 2014) to allow interpretation of the damage criteria and in support of nonlinear analyses. The resistance enhancement effects of CMA response are ignored (as shown by Figure A-9). Rather, ultimate flexural resistance is assumed fully developed at the elastic displacement, at μ = 1, and to continue until an end-rotation limit of 2° (0.0034rad) is reached. The rotation limit of 2° marks the onset of concrete crushing at the extreme compression fibre, resulting in loss of flexural resistance and the incipient failure of unrestrained elements with no shear reinforcement. If adequate shear reinforcement is present, which mobilises the compression reinforcement and provides effective containment, loss of flexural resistance and incipient failure occurs at a higher end-rotation of 6° degrees (previously 4° degrees, in accordance with DoD, 1990). USACE (2008a) suggests that 6° degrees constitutes only ‘medium damage’ when the element has adequate lateral restraint – indicating that deformation is permanent but damage is repairable. If effective lateral restraint is provided slab elements will ‘continue to develop substantial resistance up to maximum support rotations of approximately 12°’ (DoD, 2008b; 2014).
The 12° end-rotation limit for TMA response is recommended by DoD (1990; 2008b; 2014) as the ultimate displacement threshold before incipient failure. However, this is inconsistent with the damage definitions provided by DoD (1998) and USACE (2008a; 2008b); for conventional RC elements of flexural span-thickness ratio, $L_{ALS}/h \geq 5$, damage is described as ‘no significant reinforcement rupture’ and ‘component has not failed but has significant permanent deflections’. This suggests that a 12° end-rotation corresponds with the ultimate displacement and failure of primary TMA response – I.E. the displacement at peak load, prior to first failure of the extreme tension reinforcement at critical sections and not necessarily representative of incipient collapse.

This observation has been confirmed by investigating the origin of the 12° end-rotation limit, which can be traced to small-scale experimental investigations of TMA response in two-way spanning slabs by Powell (1956), Park (1964b), Keenan (1969), Black (1975) and Brotchie and Holley (1971). Powell (1956) reported displacements of approximately $0.13L_{ALS}$ attained by seven small-scale two-way rectangular slabs ($L_{ALS}/H = 16$, span-thickness ratio), acting in TMA response. These tests were terminated prior to reinforcement failure. Park’s specimens (Park, 1964b) were larger and more slender ($L_{ALS}/H = 20$) two-way spanning rectangular slab specimens. Two of the four specimens tested were reported to sustain similar displacements (0.12 and $0.15L_{ALS}$) without failure but the remaining two specimens were observed to fail at 0.095$L_{ALS}$ and 0.12$L_{ALS}$ by rupture of the bottom reinforcement layers at mid-span. Ultimate displacements in primary catenary response were also recorded in the static and dynamic tests conducted by Keenan (1969) and Black (1975), which were predominantly concerned with blast loading applications and collapse in small-scale two-way square RC panels. The authors reported deflections of 0.08-0.12$L_{ALS}$ and 0.14-0.16$L_{ALS}$ at maximum TMA load, respectively. Black reports subsequent displacement, with decaying load resistance, before failure of extreme tension reinforcement at displacements as large as 0.21$L_{ALS}$.
Park (1964b) provided the first study to recommend the use of a characteristic ultimate displacement of $0.1L_{ALS}$ (an end-rotation of 11.3°) for which maximum TMA resistance can be ‘safely’ predicted. His recommendation was based on the test observations made by Powell (1956) and Park (1964b). This recommendation remained unchanged by Park and Gamble (2000), who point to the subsequent testing by Keenan (1969), Black (1975) and Brotchie and Holley (1971) as verification that this assumption was ‘conservative’. The 12° end-rotation limit has therefore been adopted by industry as an estimate of ultimate displacement in primary catenary response.

Guidance documents DoD (1998) and USACE (2008a; 2008b) specify an additional end-rotation limit of 20°. This is level of response is specified for slab and beam elements of $L_{ALS}/h \geq 5$, with adequate lateral restraint and TMA response. USACE (2008a; 2008b) describes this level of response as ‘blowout’, indicating complete collapse, whilst the DoD (1998) describes less severe response; ‘significant rupture of reinforcement, severely cracked (near rubble) concrete within confinement reinforcement, recommended as maximum allowable damage – suitable only for protection of insensitive assets’.

DoD (1998) specifies intense shear reinforcement requirements in design, excessive when compared to conventional design. This may account for the less severe response indicated by DoD (1998). However, the higher end-rotation limit of 20° can be traced to investigations by the USACE [Woodson and Garner (1985), Guice (1986) and Woodson (1990, 1992 and 1994)]. Woodson and Garner (1985) documented an experimental investigation of fifteen laterally restrained one-way slab specimens subject to large inelastic displacements. The study featured small-scale test specimens, of span-thickness ratio $L_{ALS}/h = 10$, with different longitudinal and traverse reinforcement arrangements. The slab specimens were statically loaded and found to sustain ultimate displacements, at peak load resistance, of between $0.17L_{ALS}$ (19°) and $0.24L_{ALS}$ (25°). Inspection of the experimental data shows that at these displacements the majority of test specimens had sustained either complete or partial failure of the extreme tension reinforcement layers, at the mid-span and/or interior supports. This observation is supported by the subsequent studies of Woodson (1990, 1992 and 1994) who reported on the testing of an additional sixteen laterally restrained one-way slab specimens all of which attained end-rotations exceeding 20°, without collapse. Furthermore, Woodson (1990 and 1994) examined the results of two hundred and fifty eight large displacement tests conducted on RC plate elements for comparison. Fifty four were static tests on one-way slabs. The remaining were dynamically loaded and of mixed configuration. Test specimens were of varied span-thickness ratios, reinforcement configurations, area of longitudinal and shear reinforcement and restraint conditions. From the data available Woodson concluded that laterally restrained slab specimens could sustain 12° end-rotation without significant concrete scabbing or reinforcement rupture. Incipient failure, following significant reinforcement rupture, was found to occur reliably at end-rotations of 20° or greater. This therefore forms the basis of response limits recommended by DoD (1998) and USACE (2008a; 2008b) for
laterally restrained elements of \( L_{ALS}/h \geq 5 \) and provides evidence that the 20° response limit was an ultimate criterion for secondary TMA response.

The ultimate end-rotation limits recommended for beam and deep elements \((L_{ALS}/h < 5)\) appear to be drawn from results obtained by Guice (1986). The author documented the testing of sixteen \( \frac{1}{4} \) scale slabs specimens and demonstrated that different rotational freedom at the boundary of one-way slabs increased the displacement at which maximum resistance occurred and noted significantly greater TMA response in more slender samples. The author found thick group specimens \((L_{ALS}/h = 10.4)\) to fail at 0.125\( L_{ALS} \) (14°) but slender group \((L_{ALS}/h = 14.8)\) to sustain significantly greater ultimate displacements and demonstrated that as thickness increases, rotational freedom promotes TMA performance. This may be the basis for the more restrictive rotation limits imposed by the guidance sources for beam elements and ‘deep elements’ of \( L_{ALS}/h < 5 \). However, recommendations can be seen to be relatively inconsistent between guidance documents – DoD (1990; 2008b; 2014) suggesting ultimate end-rotations be limited to 8° whilst USACE (2008a; 2008b) and DoD (1998) suggest that incipient failure may occur at a higher rotation of 12°.

A.3.1.2 Progressive collapse design guidance & support loss conditions

Response limits used in progressive collapse design guidance are based on those used for blast resistant design. In this instance, their use is specifically to support nonlinear static and nonlinear dynamic analyses for direct robust design and progressive collapse assessment by notional element removal (see Chapter 2).

Progressive collapse design guidance provided by GSA (2003) suggested that response limits either be; established by experimental testing of structural connections or be consistent with the empirically derived DoD (1998) recommendations. Table A-5 is an extract from GSA (2003), which specifies maximum allowable end-rotations for a variety of structural RC components, with and without tension membrane response capability. Inspection shows that the specified response limits are consistent with those specified by DoD (1998). However, the 20° ultimate end-rotation limit provided by DoD (1998) for secondary TMA response is not listed. This suggests that secondary TMA response is not acknowledged by GSA (2003) for support loss conditions. Furthermore, the ultimate end-rotation tolerance advocated for beam elements is more restrictive – specifying a 6° ultimate end-rotation, regardless of lateral restraint condition. DoD (1998) allows up to 12° end-rotation for laterally restrained deep elements. This therefore demonstrates conservatism in the GSA approach.

DoD (2005) provides an alternative set of response limits for the assessment of ultimate TMA resistance under support loss conditions. Unlike GSA (2003) the document does not specify response limits for beams and slabs. Rather, components capable of TMA response are distinguished by span-thickness ratio (as seen in DoD, 1998). For Medium and High Level of Protection (LOP) design, end-
rotation is limited to 12° when of ‘normal proportions’ (L/h ≥ 5) and 8° when of ‘deep proportions’ (L/h < 5), which is generally consistent with requirements imposed by GSA (2003). For Low LOP design, response limits are significantly greater; limited to 20° when of ‘normal proportions’ (L/h ≥ 5) and 12° when of ‘deep proportions’ (L/h < 5). This guidance therefore limits catenary action to primary response for buildings that require a Medium or High LOP and secondary response for those buildings where Low LOP is adequate.

<table>
<thead>
<tr>
<th>Component Type</th>
<th>Adequate Lateral Restraint</th>
<th>Rotation Limit (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RC Beams.</td>
<td>Yes/No</td>
<td>6</td>
</tr>
<tr>
<td>RC Slabs: One-way and two-way spanning.</td>
<td>No</td>
<td>6</td>
</tr>
<tr>
<td>RC Slabs: Two-way spanning.</td>
<td>Yes</td>
<td>12</td>
</tr>
</tbody>
</table>

Table A-5 – Maximum allowable response limits, per GSA (2003).

<table>
<thead>
<tr>
<th>Component</th>
<th>AP for Low LOP</th>
<th>AP for Medium and High LOP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ductility (µ)</td>
<td>Rotation, Degrees (θ)</td>
</tr>
<tr>
<td>Slab and Beam With Tension Membrane³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal Proportions (L/h ≥ 5)</td>
<td>-</td>
<td>20</td>
</tr>
<tr>
<td>Deep Proportions (L/h &lt; 5)</td>
<td>-</td>
<td>12</td>
</tr>
</tbody>
</table>

Table A-6 – Maximum allowable response limits, per DoD (2005).

It should be noted that in recent years progressive collapse design/assessment guidance has omitted guidance for the direct analysis of TMA and catenary mechanisms, due to concerns regarding the ductility of connections (Stevens, 2008). GSA (2013) currently only supports direct robust design by flexural action and rotation limits are confined to plastic response, as found in seismic design practice (per ACI 46-01). DoD (2013) now commits all load redistribution by catenary action to indirect design and tie force requirements, exclusive to slab systems. However, Stevens (2008) and DoD (2009; 2013) provide a narrative, explaining the derivation of the tie force requirements. This states than an ultimate displacement of 0.10L_{ALS} or 11.3° was used. This is consistent with the rotation limit used in the derivation of the tie force requirements specified by the Eurocodes, British Standards and UK Building Regulations (Burnett, 1975).

A.3.1.3 Summary

By consolidating the information gathered from available guidance documents, it can be seen that TMA response limits are similar whether used in nonlinear blast or progressive collapse analyses.
Furthermore, both disciplines rely on the same test data, recorded for TMA response in overload conditions.

Typical TMA response limits can be summarised as follows;

\[ \theta = 12^\circ \quad \text{primary TMA response in conventional elements of } L_{ALS}/h \geq 5 \text{ with/without shear reinforcement.} \]

\[ \theta = 20^\circ \quad \text{secondary TMA response in conventional elements of } L_{ALS}/h \geq 5 \text{ with shear reinforcement.} \]

\[ \theta = 8^\circ \quad \text{primary TMA response in conventional elements of } L_{ALS}/h < 5 \text{ with/without shear reinforcement.} \]

\[ \theta = 12^\circ \quad \text{secondary TMA response in conventional elements of } L_{ALS}/h < 5 \text{ with shear reinforcement.} \]

### A.3.2 Ultimate Tension Membrane Force (\(N_u\))

The theoretical macro-models specified in Chapter 3, Section 3.3, assume RC catenary systems to behave as plastic membranes. Combined bending with tensile membrane action is not considered. TMA performance is therefore assumed to be dependent upon three main factors; displacement, reinforcement ductility and in-plane membrane force. Ultimate displacement criteria have been investigated in Section A.3.1. This section details an investigation of appropriate in-plane membrane force (\(N\)).

All available guidance for the direct computation of TMA load resistance assumes that all concrete is cracked through and incapable of carrying any tensile load and that all reinforcement has reached yield. However, guidance regarding an appropriate design stress for the reinforcement (\(f_d\)) and the area of reinforcement (\(A_{s-TMA}\)) that should be considered in the prediction of TMA load resistance appear to differ significantly.

GSA (2003) and DoD (2005) provide guidance supporting the direct assessment of TMA performance for progressive collapse applications. However, both documents refer to directly to Park and Gamble (2000) and UFC 3-340-01 (DoD, 2002) for design and computational methodologies.

Guidance provided by TM5-1300 (DoD, 1990) and UFC 3-340-01 (DoD, 2002; 2008a; 2014) is used for blast resistant design and is based on theoretical and empirical conclusions drawn for overload conditions, which can be traced to Park and Gamble (2000). It follows that the sources are generally
consistent. Recommendations made by Park and Gamble suggest that the design membrane force \( (N_d) \) be taken as the yield strength of the total area of continuous reinforcement.

\[
N_d = f_y (A_{s-TMA}) = f_y (A_s' + A_s)
\]

Equation A-10

Where \( A_s' \) and \( A_s \) are the area of the top and bottom reinforcement layers [mm^2], respectively. \( f_y \) is the yield stress of the reinforcement.

Park (1964b) implements this approach to compare theoretical predictions made using Equation 3-3 (Chapter 3) with test results obtained for a series of two-way RC slab specimens in primary response. One set of specimens featured discontinuous reinforcement at the top face. The prediction for these specimens was therefore based on the yield strength of the bottom reinforcement only, \( N_d = f_y A_s \). Comparison with the experimental results showed this approach to give conservative predictions, under estimating TMA response by 32-60%. This conservatism was attributed to a combination of strain hardening in the reinforcement and residual flexural response caused by the curtailed top reinforcement. Comparison with experimental results obtained by Powell (1956) showed improved accuracy – 71-83% of experimental values. In this instance top and bottom reinforcement were accounted for (i.e. \( A_{s-TMA} = A_s' + A_s \)) as both reinforcement layers were continuous for the full span. For these test specimens tensile strength (\( f_u \)) of the reinforcement was found to be 1.6-1.8 times yield strength (\( f_y \)), thus use of tensile strength was not advocated for design as this would have resulted in an unsafe prediction of ultimate TMA load resistance.

This area of investigation was taken up by Keenan (1969) and Black (1975) who undertook investigations based on subsequent experimental testing of small-scale two-way slab specimens. Tests were again based on primary TMA response. By implementing Park’s load-displacement function (Equation 3-3), Black and Keenan concluded that yield strength was most appropriate for the prediction of ultimate TMA response in statically loaded slabs – the use of full tensile strength for all continuous reinforcement, \( N_d = f_u (A_s' + A_s) \), led to overestimate of TMA performance by as much as 70%. However, for dynamically loaded slabs, it was found that the best estimate of resistance was attained assuming either partial plastic membrane (thus implementing the constant, \( k = 20 \)) and ultimate tensile stress, or fully plastic membrane (\( k = 13.4 \)) but yield stress. It should be noted that the dynamic tests by Keenan and Black were conducted under impulsive regimes, inducing high strain rates applicable to blast loading not progressive collapse.

No additional information is provided by TM5-1300 (DoD, 1990) and UFC 3-340-01 (DoD, 2002; 2008a; 2014), which simply state that tensile membrane force should be calculated for all continuous reinforcement. However, the design tensile stress of the reinforcement of beams and slabs should be taken as the average of the dynamic yield and ultimate tensile strength.
\[ f_d = 0.5(DIF f_y + DIF f_u) \quad \text{Equation A-11} \]

Where \( DIF \) is the dynamic increase factor for reinforcement, appropriate to blast resistant design practice. However, at lower strain rates, as likely in progressive collapse conditions, dynamic increase factors are likely to be non-conservative.

The definition of continuous reinforcement, as provided in \( \text{TM5-1300} \) (DoD, 1990) and \( \text{UFC 3-340-01} \) (DoD, 2002; 2008a; 2014), is ambiguous. Inspection of the lapping and anchorage requirements imposed for TMA response suggest that continuous reinforcement may not be achievable in progressive collapse conditions, for conventional constructions. Firstly, lapping and anchorage requirements for elements designed with effective lateral restraint are superior to those in conventional design. \( \text{TM5-1300} \) (DoD, 1990) specifies that laps should be made not less than 1.3 times the minimum lap requirement of 40 bar diameters or 600mm, for #11 reinforcement (36mm diameter) or smaller. This has been retained in the recent revisions of the guidance (DoD, 2002; 2008a; 2014), which now refers to the full tensile lap requirement stipulated by \( \text{ACI 318} \), with a factor of 1.3, for all reinforcement. Secondly; these guidance documents also stipulate that laps should not be made near critical sections but located in areas of low stress, where the area of reinforcement is to be twice that required, preferably outside the span of the element under consideration. These requirements suggest conventional design lap lengths are inadequate. Compression lapping of the bottom reinforcement at interior supports would be a significant concern in double span conditions as the reduced anchorage would promote anchorage failure in the bottom reinforcement. Therefore, unless detailed with mechanical couplers, compliance with DoD (2002; 2008a; 2014) detailing requirements makes determination of \( A_{S-TMA} \) inconclusive. At best, this guidance would suggest that \( A_{S-TMA} \) be limited to the area of top reinforcement, where conventional lap lengths are largest.

DoD (1998) is not referenced by progressive collapse design guidance. This document recognises the potential for secondary response and stated that rupture of some principal reinforcement was anticipated when designing for ultimate TMA response. Therefore, for design purposes, 50% of the total principal reinforcement should be considered effective in TMA and full strain hardening is assumed (per the below).

\[ N_d = f_u(A_{S-TMA}) = f_u(A_s + A_z)/2 \quad \text{Equation A-12} \]

Principal reinforcement was defined as longitudinal reinforcement covering the entire span. As with \( \text{TM5-1300} \) (DoD, 1990) and \( \text{UFC 3-340-01} \) (DoD, 2002; 2008a; 2014), conventional lapping practices conflict with this requirement, especially for double span conditions.

The recommendation provided by DoD (1998) can be traced to USACE research by Woodson and Garner (1985), Guice (1986) and Woodson (1994). Guice (1986) investigated the use of different membrane force parameters and compared the results with the performance of sixteen one-way slab
test specimens. The author concluded that the slope of the resistance-deflection function (predicted using the load-displacement term derived by Park, 1964b) would generally bound the experimental TMA load-displacement curve when assessed using the yield \( f_d = f_y \) and tensile \( f_d = f_u \) strength of the total area of reinforcement \( A_{S-TMA} = A'_s + A_s \), as shown in Figure A-10. However, it is unclear whether the test results used by Guice (1986) correspond to primary or secondary TMA response.

![Figure A-10](image)

**Figure A-10** – Load-displacement curves documented by Guice (1986), showing predicted vs experimental results with different membrane force parameters.

The study by Woodson and Garner (1985) was based on test results obtained in secondary TMA response observed in small-scale roof slab specimens. The authors showed that by assuming 50% of the principal reinforcement area and ignoring strain hardening, reasonable predictions of TMA resistance were attained at ultimate displacement.

\[
N_d = f_y(A_{S-TMA}) = f_y(A'_s + A_s)/2
\]  

**Equation A-13**

The subsequent study by Woodson (1994) consolidates this observation. The author stated that by assuming only 50% of the total principal reinforcement for the design membrane force, this was representative of secondary response; ‘the condition where all principal reinforcement in the bottom face at midspan has ruptured while all principal reinforcement in the top face at the supports has ruptured’. The slabs were therefore dependent on integrity of the concrete to avoid pull-out or slip of the ruptured reinforcement and a 50% \( N_d = f_y(A'_s + A_s) \) design membrane force was concluded to be more applicable.
It is important to note that the slab specimens tested by Woodson (1994) appear to be reinforced with equal top and bottom reinforcement allocations. This suggests that a more appropriate approach to quantification of $A_{s-TMA}$ would be to identify the area of the critical reinforcement – the smaller of the top reinforcement layer at support locations or the bottom reinforcement layer at mid-span. However, this is not acknowledged by Woodson or supported by the subsequent design guidance DoD (1998).

![Figure 5.68. Experimental and Analytical Comparisons for Slab No. 11](image)

Figure A-11 – Load-displacement curves documented by Woodson (1994), showing predicted vs experimental results with different membrane force parameters.

Inspection of load-deflection curves presented by Woodson and Garner (1985) and Woodson (1994) raises another important finding. The studies show that the test specimens could not attain the 100% $N_d = f_y (A_s + A_t)$ slope before rupture of the extreme tension reinforcement at midspan and/or supports. Thus, computation of the tensile membrane strength based on 100% of the top and bottom reinforcement resulted in an overestimated of TMA resistance. This is illustrated by Figure A-11 and is a significant observation that suggests Equation A-10 presents an unsafe design approach and directly contradicts the recommendations of Park and Gamble (2000) and progressive collapse guidance GSA (2003) and DoD (2005).

By inspection of Figure A-11 it can be seen that Equation A-13 therefore provided an estimate of secondary performance ($12^\circ \leq \theta \leq 20^\circ$) but a conservative estimate in primary response ($\theta \leq 12^\circ$), where reinforcement rupture is less pronounced or has not yet occurred.
Empirical support for the use of $f_d = f_u$, as advocated by DoD (1998) and Equation A-12, could not be found by the author. However, Woodson (1994) states that the use of tensile strength was found to provide an 8% increase in predicted load resistance, which would lessen the conservatism found in assuming a membrane force equal to Equation A-13.
Appendix B  Test Rig Photographs

B.1  Test Rig – Image Gallery

Figure B-1 – Elevation of test rig showing specimen C04 prior to loading.

B.1.1  Frame Elevations

Figure B-2 – H-frame elevations showing specimen E03 at the beginning of Phase 02 (a) and Phase 03 (b).
B.1.2 H-frame Buttressing

Figure B-3 – Lateral restraint buttressing introduced to outside-face of H-frame columns.

B.1.3 Holding down System

Figure B-4 – Holding down system devised to enable packing of actuator – specimen E02 shown restrained at 340mm displacement.
B.1.4 Hanger & Load Actuator

Figure B-5 – Hanger assembly (a) shown together with detail of load cell (b) and strain gauge arrangements (c).

Figure B-6 – Central hanger arrangement (a) and load actuator (b).
B.1.5 End Details

Figure B-7 – East End Detail (at location A) shown on elevation (a) and plan (b) – note vertical and horizontal orientation of load cells.

Figure B-8 – West End Detail (at location G).

B.1.6 Potentiometers

Figure B-9 – Potentiometers and LVDTs located along specimen span (a) and at column face (b) – note all instruments are secured to an independent frame to ensure accuracy.
Appendix C  Test Specimens & Material Test Data

C.1  Material Test Data

The following section provides a commentary and summary of material test data obtained as component of the experimental investigation.

C.1.1  Reinforcement test data

Refer to Chapter 4 for a full commentary on reinforcement testing. The results below were obtained by testing the reinforcement used during the test programme, using a gauge length of 200mm and maximum strain rate of 5mm/min (in accordance with BS 15630 and BS 6892).

C.1.1.1  Tabulated test data

<table>
<thead>
<tr>
<th>Bar Size</th>
<th>Sample Yield Strength, $R_y$ [MPa]</th>
<th>Mean $R_y$ [MPa]</th>
<th>Standard Deviation [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>B6</td>
<td>504.1 499.3 - - - -</td>
<td>501.7</td>
<td>3.4</td>
</tr>
<tr>
<td>B8</td>
<td>557.5 557.5 552.7 549.6 556.9 554.8</td>
<td>554.8</td>
<td>3.2</td>
</tr>
<tr>
<td>B10</td>
<td>512.6 517.1 512.0 512.2 508.8 514.0</td>
<td>512.8</td>
<td>2.7</td>
</tr>
<tr>
<td>B12</td>
<td>537.7 542.8 525.9 510.2 512.5 534.5</td>
<td>527.3</td>
<td>13.5</td>
</tr>
<tr>
<td>B16</td>
<td>556.7 528.7 558.5 555.3 527.8 527.1</td>
<td>542.4</td>
<td>15.9</td>
</tr>
</tbody>
</table>

* All samples tested with a 200mm gauge length and in accordance with BS 15630 and BS 6892.

<table>
<thead>
<tr>
<th>Bar Size</th>
<th>Sample Tensile Strength at Max Force, $R_m$ [MPa]</th>
<th>Mean $R_m$ [MPa]</th>
<th>Standard Deviation [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>B6</td>
<td>556.4 555.5 - - - -</td>
<td>555.9</td>
<td>0.7</td>
</tr>
<tr>
<td>B8</td>
<td>671.2 671.9 682.1 671.3 676.3 672.6</td>
<td>674.2</td>
<td>4.3</td>
</tr>
<tr>
<td>B10</td>
<td>600.7 607.8 599.2 611.8 599.8 603.4</td>
<td>603.8</td>
<td>5.0</td>
</tr>
<tr>
<td>B12</td>
<td>621.2 621.1 616.9 604.2 606.6 603.3</td>
<td>612.2</td>
<td>8.4</td>
</tr>
<tr>
<td>B16</td>
<td>656.8 629.0 657.9 655.1 630.4 627.9</td>
<td>642.8</td>
<td>15.1</td>
</tr>
</tbody>
</table>

* All samples tested with a 200mm gauge length and in accordance with BS 15630 and BS 6892.

Table C-1 – Reinforcement yield strength – test sample results.

Table C-2 – Reinforcement tensile strength at maximum force – test sample results.
### Table C-3 – Reinforcement tensile strength at fracture – test sample results.

<table>
<thead>
<tr>
<th>Bar Size</th>
<th>Sample Tensile Strength at Fracture, $R_t$ [MPa]</th>
<th>Mean $R_t$ [MPa]</th>
<th>Standard Deviation [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>B6</td>
<td>370.9 351.9 - - - -</td>
<td>361.4</td>
<td>13.4</td>
</tr>
<tr>
<td>B8</td>
<td>526.0 538.1 503.5 542.4 541.2 500.6</td>
<td>525.3</td>
<td>18.9</td>
</tr>
<tr>
<td>B10</td>
<td>455.4 483.9 454.5 486.4 497.1 462.8</td>
<td>473.4</td>
<td>18.1</td>
</tr>
<tr>
<td>B12</td>
<td>463.7 493.7 459.4 479.6 446.0 427.6</td>
<td>461.7</td>
<td>23.5</td>
</tr>
<tr>
<td>B16</td>
<td>425.8 492.1 514.2 452.3 448.7 482.0</td>
<td>469.2</td>
<td>32.5</td>
</tr>
</tbody>
</table>

* All samples tested with a 200mm gauge length and in accordance with BS 15630 and BS 6892.

### Table C-4 – Reinforcement elongation at maximum force – test sample results.

<table>
<thead>
<tr>
<th>Bar Size</th>
<th>Sample Elongation at Max Force, $A_{gt}$ [%]</th>
<th>Mean $A_{gt}$ [%]</th>
<th>Standard Deviation [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>B6</td>
<td>3.8 4.1 - - - -</td>
<td>4.0</td>
<td>0.2</td>
</tr>
<tr>
<td>B8</td>
<td>14.5 16.7 16.8 17.4 15.7 15.1</td>
<td>16.0</td>
<td>1.1</td>
</tr>
<tr>
<td>B10</td>
<td>16.0 15.3 15.4 15.7 15.5 15.4</td>
<td>15.5</td>
<td>0.3</td>
</tr>
<tr>
<td>B12</td>
<td>13.9 13.9 15.1 16.4 15.8 15.0</td>
<td>15.0</td>
<td>1.0</td>
</tr>
<tr>
<td>B16</td>
<td>11.0 11.9 10.9 11.2 12.3 11.2</td>
<td>11.4</td>
<td>0.5</td>
</tr>
</tbody>
</table>

* All samples tested with a 200mm gauge length and in accordance with BS 15630 and BS 6892.

### Table C-5 – Reinforcement elongation at fracture – test sample results.

<table>
<thead>
<tr>
<th>Bar Size</th>
<th>Sample Elongation at Max Fracture, $A_5$ [%]</th>
<th>Mean $A_5$ [%]</th>
<th>Standard Deviation [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>B6</td>
<td>5.3 5.6 - - - -</td>
<td>5.5</td>
<td>0.2</td>
</tr>
<tr>
<td>B8</td>
<td>17.9 19.2 19.5 20.4 19.3 18.7</td>
<td>19.2</td>
<td>0.8</td>
</tr>
<tr>
<td>B10</td>
<td>21.3 19.2 18.9 19.2 19.3 19.1</td>
<td>19.5</td>
<td>0.9</td>
</tr>
<tr>
<td>B12</td>
<td>16.8 16.6 19.3 19.9 20.2 19.6</td>
<td>18.8</td>
<td>1.6</td>
</tr>
<tr>
<td>B16</td>
<td>15.6 15.6 14.6 16.4 16.2 15.0</td>
<td>15.6</td>
<td>0.6</td>
</tr>
</tbody>
</table>

* All samples tested with a 200mm gauge length and in accordance with BS 15630 and BS 6892.
C.1.1.2 Stress-elongation records

Figure C-1 – Stress-elongation data recorded in reinforcement testing (ref Table B1 through B5 for values).
C.1.2 Concrete cube test data

Refer to Chapter 4 for a full discussion on concrete sampling and testing. The results below were obtained by testing the concrete used during the test programme, using 100mm cubes, cured and tested in accordance with BS 12390.

### Table C-6 – Cube density test data.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Cube Mass [kg]</th>
<th>Mean Mass [kg]</th>
<th>Mean Density [kN/m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>E01</td>
<td>2.27 2.27 2.26 2.25 2.26 2.21 2.25 2.21 2.25 22.07</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E02</td>
<td>2.30 2.30 2.29 2.31 2.32 2.28 2.28  - 2.30 22.54</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E03</td>
<td>2.34 2.31 2.32 2.31 2.33 2.31 2.30 2.31 2.32 22.73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S01</td>
<td>2.32 2.32 2.32 2.33 2.35 2.31 2.33  - 2.32 22.81</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S02</td>
<td>2.24 2.24 2.23 2.21 2.23  -  -  - 2.23 21.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S03</td>
<td>2.36 2.33 2.31 2.30 2.33 2.33 2.28  - 2.32 22.76</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C01</td>
<td>2.30 2.30 2.28 2.29 2.30 2.28 2.26  - 2.29 22.43</td>
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<td></td>
</tr>
<tr>
<td>C02</td>
<td>2.23 2.24 2.22 2.26 2.26 2.21  -  - 2.24 21.95</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C03</td>
<td>2.27 2.27 2.26 2.24 2.24 2.22 2.25 2.23 2.25 22.04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C04</td>
<td>2.24 2.24 2.24 2.24 2.24 2.24 2.26  - 2.24 22.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M01</td>
<td>2.35 2.35 2.23 2.33 2.34 2.31 2.34  - 2.32 22.77</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M02</td>
<td>2.32 2.32 2.32 2.31 2.32 2.32 2.32 2.33 2.32 22.76</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* All cubes where 100mm sq. and tested immediately following completion of main experiment.

### Table C-7 – Cube strength test data.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Cube Strength, f_{cu} [MPa]</th>
<th>Mean Cube Strength, f_{cm} [MPa]</th>
<th>Standard Deviation [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>E01</td>
<td>70.1 66.2 62.4 65.8 65.8 63.0 66.0 69.9 66.15 2.78</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E02</td>
<td>44.6 38.5 37.2 41.1 42.4 42.9 41.6  - 41.19 2.56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E03</td>
<td>59.7 52.2 52.1 55.2 56.0 52.9 54.4 51.2 54.21 2.77</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S01</td>
<td>52.8 52.8 56.1 54.4 55.3 55.4 55.4  - 54.60 1.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S02</td>
<td>72.2 66.2 74.7 68.6 70.7  -  -  - 70.48 3.27</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S03</td>
<td>50.4 53.8 51.4 42.1 52.8 48.3 46.5  - 49.33 4.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C01</td>
<td>45.4 47.4 50.6 44.8 55.3 51.1 45.1  - 48.53 3.95</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C02</td>
<td>72.8 66.8 65.3 67.7 69.4 73.0  -  - 69.17 3.18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C03</td>
<td>68.6 73.2 65.9 67.9 69.9 69.2 72.6 73.4 70.09 2.73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C04</td>
<td>58.1 64.1 65.3 64.7 59.8 65.5 62.7  - 62.89 2.88</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M01</td>
<td>49.9 49.1 51.5 51.8 52.0 53.9 52.8  - 51.57 1.64</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M02</td>
<td>60.6 56.3 59.9 52.9 62.3 57.9 58.6 51.5 57.50 3.75</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* All cubes where 100mm sq. and tested immediately following completion of main experiment.
C.1.3 Interpretation of cube test results

Concrete is known to follow a normal strength distribution (Teychenne et al., 1997). To ensure a degree of safety, the concrete strength specified in design represents a minimum value for certain percentage confidence. The characteristic compressive strength \( f_{cu} \), used in British design practice, is regulated by material specifications for concrete such that it represents a 95% confidence interval (BS 8110, BS 5328 and EC2) – defective samples, weaker than the specified characteristic strength, account for no more than 5% of the total number of samples taken (Bramforth et al., 2008). It follows that the characteristic compressive strength indicates the quality of the concrete rather than the true strength as found in a structure, furthermore, the use of the mean compressive strength \( f_{cm} \) is likely to provide a more accurate account of building behaviour as it represents the population average.

It has been shown that the standard deviation (SD) of compressive strength varies for concrete grades less than 20MPa (Teychenne et al., 1997). However, for stronger grades, quality control measures imposed on those factors influential to the degree of variability – consistency of constituent materials, their mixed proportions and curing conditions – sustain a consistent SD. BS 8110 and EC2 suggest the current characteristic-mean cube strength SD at 6MPa but the majority of concrete produced by certified plant are said to reliably produce concretes with a SD of 5MPa (Teychenne et al., 1997). Thus, for a concrete designed to British requirements, the mean compressive strength \( f_{cm} \) may be calculated as \( 1.64(5.0) + f_{cu} \) [MPa] for structural concretes greater than C20. Target mean compressive strengths are shown in Table C-8 for their respective strength class.

Calculation of an appropriate long-term concrete strength was based upon typical constituents as advised by several major UK ready-mix concrete suppliers. Values were determined for a 28-day characteristic concrete strength of 40MPa and in accordance with EC2 (BSI, 2008), clause 3.1.2(06) for a cement of ‘Class R’ (CEM 42.5R/52.5N/52.5R). The mean long-term cube strength \( f_{cm(t)} \) is shown in Table C-8 (together with respective characteristic strength, bracketed) for a series of characteristic concrete strengths, at an age of 1, 5, 10 and 15 yrs.

<table>
<thead>
<tr>
<th>Age, ( t ) (days)</th>
<th>Mean Compressive Strength, ( f_{cm} ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C30</td>
</tr>
<tr>
<td>28</td>
<td>38.2</td>
</tr>
<tr>
<td>365 (1yr)</td>
<td>44.1</td>
</tr>
<tr>
<td>1825 (5yr)</td>
<td>45.5</td>
</tr>
<tr>
<td>3650 (10yr)</td>
<td>45.8</td>
</tr>
<tr>
<td>5475 (15yr)</td>
<td>45.9</td>
</tr>
</tbody>
</table>

Table C-8 – Target long-term mean cube strengths (in accordance with EC2, BSI 2008).
Concrete strength is dependent upon the water-cement and aggregate-cement ratios, volume and type of cement, size and type of aggregate, the volume and type of add-mixtures and the curing conditions. An accurate estimate for cube strength, at any given age, must also account for the environmental and restraint conditions for the life of the concrete and thus the strength-age relation is unique to each case. The EC2 method provides a basic estimation that utilises an exponential relationship between age and concrete strength, based upon cement type and 28-day design strength for a curing temperature of 20 degrees Celsius. Figure C-2 is a representation of this relationship and permits comparison with test data obtained for two concretes mixed using ordinary Portland cement and water-cement ratios of 0.49 and 0.53 (obtained from Neville, 1995). Whilst initial inspection would suggest that the EC2 estimates are conservative beyond 28 days, the test data was found from samples stored in optimal curing conditions and are likely to overestimate in particular situations. Furthermore, the test results are consistent with concretes mixed with normal strength cement. They show a steady increase in strength development with time and a comparatively high long-term strength. A concrete, of the same 28day strength but mixed using rapid curing cement (class-R), will exhibit a higher rate of strength development in the early stages of curing but significantly less beyond the 28day strength – favoured in modern construction as it permits the formwork to be struck earlier.

Figure C-2 – Comparison of experimental and theoretical mean cube strength development with age (calculated in accordance with EC2, 2004) for structural concretes, cement class-R.

Comparison of the EC2 estimates and the test data shown in Figure C-2 is as expected and helps verify their validity.
There is a small discrepancy in the concrete strength when testing with the 100x100mm cubes, as used in laboratory testing, and the industrial standard 150x150mm cubes. Research conducted by Mansur & Islam (2002) demonstrated that 150mm and 100mm cubes of a single concrete, cured under identical conditions, produced different cube strengths as related by Equation C-1. The magnitude of this difference is such that the equivalent 100mm cube strength, $f_{cu100}$, is approximately 0.968, 0.997 and 1.016 times the target 150mm cube strength, $f_{cu150}$, for 30, 40 and 50N concretes respectively. Given the variability of concrete and the level of accuracy in simulating a 1-10yr mean concrete strength, the discrepancy between cube strengths is suitably small to be neglected from consideration.

$$f_{cu100} = \frac{f_{cu150} - 3.62}{0.91}$$  \hspace{1cm} \text{Equation C-1}

Given the influence of concrete maturity, deviation with regards characteristic and mean values and discrepancy in sampling, the long-term concrete strength estimates detailed in Table C-8 are sufficiently accurate and suitable target values for this investigation. Thus, to model a structure of between 1 and 15yrs with a design grade of C30-50 structural concrete, the target experimental mean strength of the micro concrete must fall between 44.1 and 70.1MPa.
C.2 Test Specimens – As Built Drawings

Figure C-3 – General arrangement and reinforcement detailing – Edge beam strip specimen E01.
Figure C.4 – General arrangement and reinforcement detailing – Edge beam strip specimen E02.

**GENERAL ARRANGEMENT - PLAN**

**SCALE 1:20**

**REBAR ARRANGEMENT - PLAN**

**SCALE 1:20**

**SECTION A-A**

**SCALE 1:20**

**NOTE:**
6m. Lifting eyes introduced to sample before pour. Anchorage and position checked to ensure safety and nil damage during lifting.
Figure C.5 – General arrangement and reinforcement detailing – Edge beam strip specimen E03.

GENERAL ARRANGEMENT - PLAN
SCALE 1:20

REBAR ARRANGEMENT - PLAN
SCALE 1:20

SECTION A-A
SCALE 1:20

NOTE:
6. Lifting eyes introduced to sample before pour. Anchorage and position checked to ensure safety and nil damage during lifting.
Figure C-6 – General arrangement and reinforcement detailing – Slab Strip specimen S01.

**GENERAL ARRANGEMENT - PLAN**

**SCALE 1:20**

**REBAR ARRANGEMENT - PLAN**

**SCALE 1:20**

**SECTION A-A**

**SCALE 1:20**

NOTE:
Free lifting eyes introduced to sample before pour. Anchorage and position checked to ensure safety and nil damage during lifting.

**NOTES:**
1. Sample to be cast in relevant formwork (see SNW-06-015).
2. Formwork surface to be fully cleaned and coated with even thin coats of mould release before pour.
3. Screeds area to be level and true.

**Note:**
1. Rebar distributed evenly from the sample centre line.
2. Max/min bar spacing in accordance with BS8110-1 specifications.
3. End bars are fixed to inside face of plate via Maccouplers, threaded and secured at specified torque (see relevant drawings). SNW-06-015 for anti-plate details and coupler weld spec.
4. Open and spaces introduced at 750mm to ensure specified cover.
Figure C.7 – General arrangement and reinforcement detailing – Slab Strip specimen S02.

**GENERAL ARRANGEMENT - PLAN**

**SCALE 1:20**

**REBAR ARRANGEMENT - PLAN**

**SCALE 1:20**

**SECTION A-A**

**SCALE 1:20**

**NOTES:**

1. Sample to be cast in relevant formwork (see SENCCH-01).
2. Framework surface to be fully prepared and coated with anti-slip coat of mould oil before pour.
3. Cutting edge to be level and true.

**REBARS**

- B10 – 02
- B08 – 04
- B08 – 01
- B10 – 02

**SLABS**

- 29 B06 – 05
- 150 B1 / T1

**LINKS**

- 20 B10 – 02

**COVER**

- 10 mm (to main bar)

**SECTION A-A**

**NOTES:**

6. Lifting eyes introduced to sample for anchorage and position checked to ensure safety and nil damage during lifting.
Figure C.10 – General arrangement and reinforcement detailing – Column strip specimen C02.
Figure C-11 – General arrangement and reinforcement detailing – Column strip specimen C03.
Figure C.12 – General arrangement and reinforcement detailing – Column strip specimen C01
Figure C.13 – General arrangement and reinforcement detailing – Middle strip specimen M01.
Figure C-14 – General arrangement and reinforcement detailing – Middle strip specimen M02.

GENERAL ARRANGEMENT - PLAN

REBAR ARRANGEMENT - PLAN

SECTION A-A

NOTE:
6. Lifting eyes introduced to sample before pour. Anchorage and position checked to ensure safety and nil damage during lifting.
Appendix D  Test Results & Observations

D.1 Calculation of Moment Resistance

The calculation of moment resistance was based upon equilibrium of a doubly reinforced section, thus:

Equilibrium for the resultant axial force \( N \), gives:

\[
N = C_c + C_s + T \tag{Equation D-1}
\]

Moment equilibrium, about the mid-section, gives:

\[
M = C_c \left(\frac{h}{2} - d_c\right) + C_s \left(\frac{h}{2} - d'\right) + T\left(d - \frac{h}{2}\right) \tag{Equation D-2}
\]

Where \( d_c \), \( d' \) and \( d \) are the depths to the centroid of the concrete stress block, top reinforcement area and bottom reinforcement area, respectively.

Figure D-2 shows the stress-strain functions implemented to define material response.

![Cross Section](image)

![Section Strain](image)

![Concrete Stress](image)

![Steel Stress](image)

![Internal Actions](image)

Figure D-1 – RC cross-section stress strain diagram.

![Material Stress](image)

![Material Strain](image)

Figure D-2 – Material stress-strain functions used in section analysis.
The bi-linear elastic-plastic stress-strain function (Figure D-2a) was used to define reinforcement tension and compressive force:

\[ C_s = A'_s f'_{sc} \]  
\[ T = A_s f'_{st} \]  

Where stresses \( (f_s) \) of the reinforcement, for given strain \( (\varepsilon_s) \), was therefore determined for \( \varepsilon_s \leq \varepsilon_y \):

\[ f_s = \varepsilon_s E_s \]  

and \( \varepsilon_y \leq \varepsilon_s \leq \varepsilon_u \):

\[ f_s = f_y + \left( \frac{f_u - f_y}{\varepsilon_u - \varepsilon_y} \right) (\varepsilon_s - \varepsilon_u) \]  

and \( \varepsilon_s > \varepsilon_u \):

\[ f_s = 0 \]

Where:

\[ f'_{s} = \frac{f_s}{\gamma_s} \]  

By assuming compressive strain at the top face will not exceed the ultimate compressive strain \( \varepsilon_{cu} \), slope \( k \) for a given neutral axis depth \( (x) \) is:

\[ k = \tan^{-1} \frac{\varepsilon_{cu}}{x} \]

Thus strain in the top and bottom reinforcement is:

\[ \varepsilon_{st} = (x - d)k \quad \text{and} \quad \varepsilon_{sc} = (x - d')k \]  

The parabolic stress-strain function (Figure D-2b) was adapted from Todeschini et al. (1964) and Collin et al. (1993) to calculate concrete force:

\[ C_c = b \sum_{d_i=x}^{d_i=0} f'_{ci} \delta d_i \]  

Where,

\[ f'_{ci} = \frac{f_{ci}}{\gamma_c} \]  

and stress \( f_{ci} \) for a given strain \( \varepsilon_i < \varepsilon_{c0} \),

\[ f_{ci} = \frac{4f_{cu}\varepsilon_{c0}\varepsilon_i}{3(\varepsilon_{c0}^2 + \varepsilon_i^2)} = \frac{4f_{cu}\varepsilon_{c0}(x - d_i)k}{3(\varepsilon_{c0}^2 + [(x - d_i)k]^2)} \]

and \( \varepsilon_{c0} \leq \varepsilon_i \leq \varepsilon_{cu} \).
\[ f_{ci} = \frac{2f_{cu}}{3} \quad \text{Equation D-14} \]

and \( \varepsilon_i > \varepsilon_{cu} \) and \( \varepsilon_i \leq 0 \),

\[ f_{ci} = 0 \quad \text{Equation D-15} \]

Where, in accordance with BS 8110:

\[ \varepsilon_{c0} = 2.4 \times 10^{-4} \sqrt{f_{cu}} \quad \text{Equation D-16} \]

and,

\[ \varepsilon_{cu} = 0.0035 \quad \text{Equation D-17} \]

and,

\[ \varepsilon_i = (x - d_i)k \quad \text{Equation D-18} \]

The moment resistance was computed by iteration of the neutral axis depth for zero resultant axial force (Equation D-1).

For the calculation of elastic moment resistance \((M_y)\), \(f_y = f_y\) and \(f_u = f_y\). For the calculation of ultimate moment resistance \((M_u)\), \(f_y = f_y\) and \(f_u = f_u\). Partial safety factors \((\gamma_c\) and \(\gamma_s\) were ignored in the calculation of moment resistance. In accordance with Eurocode 2 and BS 4449:2005, for standard hot-rolled type B 500B reinforcement, ultimate reinforcement strain \((\varepsilon_{u})\) was limited to 5\%. 

D.2 Test Data – Starting Reactions

Table D-1 gives the reactions recorded at the end of Test Phases 1 and 2. Phase 1 reactions were those achieved with the assembly in its service state, as the middle two bays of a continuous six-bay system. Reactions denoted Phase 2 were recorded following removal of the central hanger and redistribution under self-weight. Recorded values are compared with theoretical target values below.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Recorded Reaction</th>
<th>Target Reaction</th>
<th>Recorded/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P [kN] R1 [kN] R2 [kN] R3 [kN] RH [kN]</td>
<td>P [kN] R1 [kN] R2 [kN] R3 [kN] RV [kN]</td>
<td></td>
</tr>
<tr>
<td>PHASE01 – Service state before removal of central support</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E01</td>
<td>3.108 2.402 2.401 -1.232 0.000</td>
<td>2.214 2.089 2.089 -0.472</td>
<td>1.4 1.1 1.1 2.6</td>
</tr>
<tr>
<td>E02</td>
<td>2.225 2.131 2.132 -0.463 0.000</td>
<td>2.260 2.132 2.132 -0.482</td>
<td>1.0 1.0 1.0 1.0</td>
</tr>
<tr>
<td>E03</td>
<td>3.313 3.109 3.089 -0.773 0.000</td>
<td>3.244 3.060 3.060 -0.691</td>
<td>1.0 1.0 1.0 1.1</td>
</tr>
<tr>
<td>S01</td>
<td>1.542 1.341 1.341 -0.344 0.000</td>
<td>1.437 1.356 1.356 -0.306</td>
<td>1.1 1.0 1.0 1.1</td>
</tr>
<tr>
<td>S02</td>
<td>1.506 1.451 1.448 -0.508 0.000</td>
<td>1.379 1.300 1.300 -0.294</td>
<td>1.1 1.1 1.1 1.7</td>
</tr>
<tr>
<td>S03</td>
<td>1.360 1.347 1.365 -0.264 0.000</td>
<td>1.435 1.353 1.353 -0.306</td>
<td>0.9 1.0 1.0 0.9</td>
</tr>
<tr>
<td>C01</td>
<td>2.733 1.802 2.300 -0.742 0.000</td>
<td>2.013 1.899 1.899 -0.429</td>
<td>1.4 0.9 1.2 1.7</td>
</tr>
<tr>
<td>C02</td>
<td>1.908 1.885 1.882 -0.240 0.000</td>
<td>1.922 1.813 1.813 -0.176</td>
<td>1.0 1.0 1.0 1.4</td>
</tr>
<tr>
<td>C03</td>
<td>2.571 2.569 2.568 -0.760 0.000</td>
<td>2.173 2.050 2.050 -0.042</td>
<td>1.2 1.3 1.3 18.0</td>
</tr>
<tr>
<td>C04</td>
<td>2.496 2.582 2.484 -0.280 0.000</td>
<td>2.326 2.194 2.194 0.176</td>
<td>1.1 1.2 1.1 -1.6</td>
</tr>
<tr>
<td>M01</td>
<td>1.601 1.461 1.484 0.132 0.000</td>
<td>1.520 1.434 1.434 0.204</td>
<td>1.1 1.0 1.0 0.7</td>
</tr>
<tr>
<td>M02</td>
<td>2.628 2.387 2.478 -0.402 0.000</td>
<td>2.540 2.396 2.396 -0.356</td>
<td>1.0 1.0 1.0 1.1</td>
</tr>
</tbody>
</table>

| PHASE02 – Redistributed reactions following central support removal |
| E01  | 0.000 5.866 5.702 -3.188 0.884 | 0.000 5.082 5.082 -2.358 | 1.0 1.2 1.1 1.4 |
| E02  | 0.000 5.371 5.456 -1.964 0.399 | 0.000 5.188 5.188 -2.407 | 1.0 1.0 1.1 0.8 |
| E03  | 0.000 6.581 6.095 -2.400 0.035 | 0.000 7.445 7.445 -3.454 | 1.0 0.9 0.8 0.7 |
| S01  | 0.000 3.785 3.789 -0.867 0.676 | 0.000 3.298 3.298 -1.530 | 1.0 1.1 1.1 0.6 |
| S02  | 0.000 2.947 3.095 -1.402 0.118 | 0.000 3.164 3.164 -1.468 | 1.0 0.9 1.0 1.0 |
| S03  | 0.000 2.830 2.616 -1.063 0.502 | 0.000 3.293 3.293 -1.528 | 1.0 0.9 0.8 0.7 |
| C01  | 0.000 3.530 3.609 -1.763 -0.017 | 0.000 4.620 4.620 -2.144 | 1.0 0.8 0.8 0.8 |
| C02  | 0.000 3.809 3.805 -1.229 0.138 | 0.000 3.729 3.729 -1.130 | 1.0 1.0 1.0 1.1 |
| C03  | 0.000 4.237 4.310 -1.382 0.085 | 0.000 3.997 3.997 -0.903 | 1.0 1.1 1.1 1.5 |
| C04  | 0.000 3.910 3.654 -0.740 0.071 | 0.000 4.105 4.105 -0.572 | 1.0 1.0 0.9 1.3 |
| M01  | 0.000 2.842 2.995 -0.405 -0.156 | 0.000 2.644 2.644 -0.246 | 1.0 1.1 1.1 1.6 |
| M02  | 0.000 5.649 5.552 -1.890 0.173 | 0.000 5.191 5.191 -1.881 | 1.0 1.1 1.1 1.0 |

Table D-1 – Recorded versus target support reactions taken at the end of Phase 01 & 02, prior to loading.
D.3 Test Data – Specimen E01 Results & Observations

Basic dimensions of this specimen and test are provided below. Reinforcement details and material specification are provided in Chapter 4 and Appendix C. Dimensions, location references and reaction name and sign conventions are to be read in conjunction with Chapter 4 and Chapter 5, Figure 5-1.

\[ L_{ab} = 2,500\text{mm} \quad L_{EM} = 5,000\text{mm} \quad L_{ab} = 635\text{mm} \]
\[ A_{C} = 39,000\text{mm}^2 \quad h = 175\text{mm} \quad b = 225\text{mm} \]

D.3.1 Specimen response notes

A summary of observations made during the test are provided below together with the chronology and reaction record taken for key events of the test and specimen response. The location of each event (i.e. rebar rupture) is indicated using the figure below.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Description</th>
<th>( \Delta ) [mm]</th>
<th>0( \Delta ) [Degrees]</th>
<th>( P ) [kN]</th>
<th>( R_{H} ) [kN]</th>
<th>( R_{1} ) [kN]</th>
<th>( R_{2} ) [kN]</th>
<th>( R_{V1} ) [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>CMA maximum load</td>
<td>77.0</td>
<td>1.8</td>
<td>-26.3</td>
<td>57.5</td>
<td>43.1</td>
<td>45.0</td>
<td>-27.9</td>
</tr>
<tr>
<td>B</td>
<td>Fracture (bending) Grp03 @ P</td>
<td>86.9</td>
<td>2.0</td>
<td>-25.5</td>
<td>62.7</td>
<td>44.1</td>
<td>46.1</td>
<td>-29.1</td>
</tr>
<tr>
<td>C</td>
<td>Fracture (bending) Grp02 @ R1+R2</td>
<td>206.8</td>
<td>4.7</td>
<td>-10.9</td>
<td>31.7</td>
<td>33.4</td>
<td>37.6</td>
<td>-26.7</td>
</tr>
<tr>
<td>D</td>
<td>Snap through</td>
<td>237.8</td>
<td>5.4</td>
<td>-0.5</td>
<td>0.2</td>
<td>6.5</td>
<td>8.8</td>
<td>-3.9</td>
</tr>
<tr>
<td>E</td>
<td>TMA maximum load - TEST TERMINATED (deflection limit reached)</td>
<td>637.2</td>
<td>14.3</td>
<td>-52.5</td>
<td>-81.4</td>
<td>36.7</td>
<td>37.9</td>
<td>-12.5</td>
</tr>
</tbody>
</table>

80 \leq \Delta \leq 100\text{mm} \quad \text{Extreme tension reinforcement (B2 bar) rupture at midspan, P.}

200 \leq \Delta \leq 220\text{mm} \quad \text{Tension reinforcement (T2 bar) rupture at end of hogging bars, 725-750mm from \( R_{1} \) and \( R_{2} \) centre lines. Specimen remains symmetric.}

\Delta \geq 220\text{mm} \quad \text{Catenary sustained by 2no. cantilevers. Cover progressively pulled from under-side of cantilevers. B2 reinforcement sustained by links.}

\Delta = 640\text{mm} \quad \text{End of test. Maximum displacement attained before failure is achieved.}
D.3.2 Test data

The force-rotation and moment-rotation records, taken during testing, are provided below. Reference points are provided in conjunction with the observations noted in the previous section.

![Graph of force-rotation data](image)

**Figure D-3** – Force-rotation data recorded for E01 test.
Figure D-4 – Moment-rotation and moment-axial-force results, test E01.

Figure D-5 – Deflection profile record, test E01.
D.4 Test Data – Specimen E02 Results & Observations

Basic dimensions of this specimen and test are provided below. Reinforcement details and material specification are provided in Chapter 4 and Appendix C. Dimensions, location references and reaction name and sign conventions are to be read in conjunction with Chapter 4 and Chapter 5, Figure 5-1.

\[ L_{bd} = 2,500\text{mm} \quad L_{EM} = 5,000\text{mm} \quad L_{ab} = 635\text{mm} \]

\[ h = 175\text{mm} \quad b = 275\text{mm} \]

D.4.1 Specimen response notes

A summary of observations made during the test are provided below together with the chronology and reaction record taken for key events of the test and specimen response. The location of each event (i.e. rebar rupture) is indicated using the figure below.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Description</th>
<th>( \Delta ) [mm]</th>
<th>( \theta_\Delta ) [Degrees]</th>
<th>( P ) [kN]</th>
<th>( R_H ) [kN]</th>
<th>( R_1 ) [kN]</th>
<th>( R_2 ) [kN]</th>
<th>( R_{V1} ) [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>CMA maximum load</td>
<td>89.6</td>
<td>2.1</td>
<td>-33.6</td>
<td>30.3</td>
<td>55.5</td>
<td>52.5</td>
<td>-36.2</td>
</tr>
<tr>
<td>B</td>
<td>Snap through</td>
<td>229.2</td>
<td>5.2</td>
<td>-32.9</td>
<td>0.0</td>
<td>54.9</td>
<td>51.7</td>
<td>-35.7</td>
</tr>
<tr>
<td>C</td>
<td>Fracture (bending) Grp02 @ R1</td>
<td>257.5</td>
<td>5.9</td>
<td>-33.7</td>
<td>-15.6</td>
<td>49.3</td>
<td>51.2</td>
<td>-30.3</td>
</tr>
<tr>
<td>D</td>
<td>Fracture (bending) Grp03 (1bar) @ P</td>
<td>284.4</td>
<td>6.5</td>
<td>-31.2</td>
<td>-42.6</td>
<td>22.9</td>
<td>49.9</td>
<td>-9.2</td>
</tr>
<tr>
<td>E</td>
<td>Fracture (bending) Grp03 @ P</td>
<td>300.3</td>
<td>6.8</td>
<td>-28.9</td>
<td>-65.8</td>
<td>18.4</td>
<td>42.9</td>
<td>-5.9</td>
</tr>
<tr>
<td>F</td>
<td>Fracture (bending) Grp02 @ R2</td>
<td>443.8</td>
<td>10.1</td>
<td>-47.9</td>
<td>-122.9</td>
<td>26.7</td>
<td>35.8</td>
<td>-4.3</td>
</tr>
<tr>
<td>G</td>
<td>Fracture (tension) Grp04 @ R2</td>
<td>584.6</td>
<td>13.2</td>
<td>-70.2</td>
<td>-128.0</td>
<td>42.9</td>
<td>45.4</td>
<td>-9.5</td>
</tr>
<tr>
<td>H</td>
<td>TMA maximum load – SYSTEM FAILURE</td>
<td>584.6</td>
<td>13.2</td>
<td>-70.2</td>
<td>-128.0</td>
<td>42.9</td>
<td>45.4</td>
<td>-9.5</td>
</tr>
</tbody>
</table>

\[ \Delta \leq 150\text{mm} \quad \text{Bending cracks formed at R}_1, \text{ R}_2 \text{ and P. Bending cracks also formed at end of hogging reinforcement.} \]

\[ 240 \leq \Delta \leq 260\text{mm} \quad \text{Reinforcement (T2 bar) rupture at end of hogging bars, 700-750mm from R}_1 \text{ centre line.} \]

\[ 290 \leq \Delta \leq 310\text{mm} \quad \text{Reinforcement (T2 bar) rupture at end of hogging bars, 700-750mm from R}_2 \text{ centre line (estimated to fail at 305mm displacement). Extreme tension reinforcement (B2 bar) rupture at midspan, P.} \]

\[ \Delta \geq 320\text{mm} \quad \text{Catenary sustained by two cantilevers formed by R}_1 \text{ and R}_2 \text{ hogging reinforcement. Cover is progressively pulled from underside of cantilevers} \]
as angle of incidence increases. B2 reinforcement sustained by links (failure of each is evident in graph).

420 ≤ Δ ≤ 440mm
Failure of link located at end of hogging reinforcement.

440 ≤ Δ ≤ 460mm
Remaining tension reinforcement (B2 bar) fractured at midspan, P.

580 ≤ Δ ≤ 600mm
End of test. Remaining R2 reinforcement (B2 bar) fractured at end of hogging bars.

D.4.2 Test data

The force-rotation and moment-rotation records, taken during testing, are provided below. Reference points are provided in conjunction with the observations noted in the previous section.

Figure D-6 – Force-rotation data recorded for E02 test.
Figure D-7 – Moment-rotation and moment-axial-force results, test E02.

Figure D-8 – Deflection profile record, test E02.
D.5  Test Data – Specimen E03 Results & Observations

Basic dimensions of this specimen and test are provided below. Reinforcement details and material specification are provided in Chapter 4 and Appendix C. Dimensions, location references and reaction name and sign conventions are to be read in conjunction with Chapter 4 and Chapter 5, Figure 5-1.

\[ L_{bd} = 2,500\text{mm} \quad L_{EM} = 5,000\text{mm} \quad L_{ab} = 635\text{mm} \]

\[ h = 175\text{mm} \quad b = 320\text{mm} \]

D.5.1  Specimen response notes

A summary of observations made during the test are provided below together with the chronology and reaction record taken for key events of the test and specimen response. The location of each event (i.e. rebar rupture) is indicated using the figure below.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Description</th>
<th>[ \Delta ] [mm]</th>
<th>[ \theta_\Delta ] [Degrees]</th>
<th>[ P ] [kN]</th>
<th>[ R_H ] [kN]</th>
<th>[ R_1 ] [kN]</th>
<th>[ R_2 ] [kN]</th>
<th>[ R_V1 ] [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>CMA maximum load</td>
<td>114.5</td>
<td>2.6</td>
<td>-50.7</td>
<td>72.1</td>
<td>89.7</td>
<td>79.0</td>
<td>-62.6</td>
</tr>
<tr>
<td>B</td>
<td>Fracture (bending) Grp03 (1bar) @ P</td>
<td>188.6</td>
<td>4.3</td>
<td>-42.0</td>
<td>28.4</td>
<td>75.3</td>
<td>68.1</td>
<td>-54.7</td>
</tr>
<tr>
<td>C</td>
<td>Fracture (bending) Grp03 @ P</td>
<td>221.2</td>
<td>5.1</td>
<td>-29.2</td>
<td>0.1</td>
<td>59.5</td>
<td>53.0</td>
<td>-41.6</td>
</tr>
<tr>
<td>D</td>
<td>Snap through</td>
<td>228.2</td>
<td>5.2</td>
<td>-22.9</td>
<td>0.0</td>
<td>53.8</td>
<td>47.6</td>
<td>-38.1</td>
</tr>
<tr>
<td>E</td>
<td>Fracture (bending) Grp02 @ R1</td>
<td>388.3</td>
<td>8.8</td>
<td>-92.3</td>
<td>-232.8</td>
<td>73.4</td>
<td>76.6</td>
<td>-33.0</td>
</tr>
<tr>
<td>F</td>
<td>Fracture (bending) Grp02 @ R2</td>
<td>491.4</td>
<td>11.1</td>
<td>-96.1</td>
<td>-195.3</td>
<td>57.3</td>
<td>85.7</td>
<td>-18.0</td>
</tr>
<tr>
<td>G</td>
<td>TMA maximum load</td>
<td>491.4</td>
<td>11.1</td>
<td>-96.1</td>
<td>-195.3</td>
<td>57.3</td>
<td>85.7</td>
<td>-18.0</td>
</tr>
<tr>
<td>H</td>
<td>Failure (bond) of shear – TEST TERMINATED</td>
<td>548.7</td>
<td>12.4</td>
<td>-92.3</td>
<td>-182.8</td>
<td>64.4</td>
<td>50.0</td>
<td>-20.9</td>
</tr>
</tbody>
</table>

\[ \Delta \leq 100\text{mm} \quad \text{Cracking formed at } R_1, R_2 \text{ and } P. \]
\[ 100 \leq \Delta \leq 120\text{mm} \quad \text{Cracking formed at end of hogging reinforcement.} \]
\[ 190 \leq \Delta \leq 210\text{mm} \quad \text{Extreme tension reinforcement (B2 bar) rupture at midspan, P.} \]
\[ 210 \leq \Delta \leq 230\text{mm} \quad \text{Extreme tension reinforcement (B2 bar) rupture at midspan, P.} \]
\[ 230 \leq \Delta \leq 250\text{mm} \quad \text{Extreme tension reinforcement (B2 bar) rupture at midspan, P.} \]
\[ 270 \leq \Delta \leq 290\text{mm} \quad \text{Cover pulled away from underside of specimen (beneath } R_1 \text{ hogging bar).} \]
\[ 380 \leq \Delta \leq 400\text{mm} \quad \text{Reinforcement rupture (T2 bar) at end of } R_1 \text{ hogging reinforcement.} \]
\[ 480 \leq \Delta \leq 500\text{mm} \quad \text{Reinforcement rupture (T2 bar) at end of } R_2 \text{ hogging reinforcement.} \]
560 ≤ Δ ≤ 580mm    End of test.  Cover pulled from underside of R₁ and R₂ cantilevers.  Links fail at end of R₁ and R₂ hogging reinforcement.  Deflection limit reached.

D.5.2  Test data

The force-rotation and moment-rotation records, taken during testing, are provided below.  Reference points are provided in conjunction with the observations noted in the previous section.

Figure D-9 – Force-rotation data recorded for E03 test.
Figure D-10 – Moment-rotation and moment-axial-force results, test E03.

Figure D-11 – Deflection profile record, test E03.
D.6 Test Data – Specimen S01 Results & Observations

Basic dimensions of this specimen and test are provided below. Reinforcement details and material specification are provided in Chapter 4 and Appendix C. Dimensions, location references and reaction name and sign conventions are to be read in conjunction with Chapter 4 and Chapter 5, Figure 5-1.

\[ L_{bd} = 2,500\text{mm} \quad L_{EM} = 5,000\text{mm} \quad L_{ab} = 635\text{mm} \]

\[ h = 90\text{mm} \quad b = 275\text{mm} \]

D.6.1 Specimen response notes

A summary of observations made during the test are provided below together with the chronology and reaction record taken for key events of the test and specimen response. The location of each event (i.e. rebar rupture) is indicated using the figure below.

### S01 – Key events in response

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Description</th>
<th>( \Delta )</th>
<th>( \theta )</th>
<th>( P )</th>
<th>( R_H )</th>
<th>( R_1 )</th>
<th>( R_2 )</th>
<th>( R_{V1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>CMA maximum load</td>
<td>128.7</td>
<td>2.9</td>
<td>-3.1</td>
<td>0.0</td>
<td>6.9</td>
<td>7.3</td>
<td>-1.8</td>
</tr>
<tr>
<td>B</td>
<td>Snap through</td>
<td>130.7</td>
<td>3.0</td>
<td>-3.1</td>
<td>-0.1</td>
<td>6.9</td>
<td>7.3</td>
<td>-1.8</td>
</tr>
<tr>
<td>C</td>
<td>Fracture (bending) Grp02 @ P</td>
<td>178.3</td>
<td>4.1</td>
<td>-3.9</td>
<td>-12.3</td>
<td>6.4</td>
<td>7.2</td>
<td>-2.8</td>
</tr>
<tr>
<td>D</td>
<td>Fracture (bending) Grp01 @ R1</td>
<td>292.9</td>
<td>6.7</td>
<td>-6.7</td>
<td>-27.6</td>
<td>6.5</td>
<td>7.5</td>
<td>-2.3</td>
</tr>
<tr>
<td>E</td>
<td>TMA maximum load</td>
<td>365.5</td>
<td>8.3</td>
<td>-8.1</td>
<td>-28.4</td>
<td>4.0</td>
<td>9.1</td>
<td>0.1</td>
</tr>
<tr>
<td>F</td>
<td>SYSTEM FAILURE - Fracture (tension) Grp02 @ P</td>
<td>371.5</td>
<td>8.5</td>
<td>-7.3</td>
<td>-25.3</td>
<td>3.8</td>
<td>8.7</td>
<td>0.0</td>
</tr>
</tbody>
</table>

\( \Delta \leq 100\text{mm} \) Bending cracks formed at \( R_1 \), \( R_2 \) and P.

\( 170 \leq \Delta \leq 190\text{mm} \) Cracking formed at end of hogging reinforcement (T2 bar) located across point P. Extreme tension reinforcement (B2 bar) rupture at midspan, P.

\( \Delta \geq 210\text{mm} \) Profile of specimen reshares to adopt a direct tension line between remaining rebar.

\( \Delta \geq 270\text{mm} \) Local crushing initiates each side of and beneath bearing point of actuator cross-head.

\( 290 \leq \Delta \leq 300\text{mm} \) Hogging reinforcement (T2 bar) ruptures at \( R_1 \) centre line. Tension crack forms at end of \( R_2 \) hogging reinforcement.

\( \Delta \geq 340\text{mm} \) Crack opens at each side of P hogging reinforcement.
360 ≤ Δ ≤ 380mm  End of test. B2 reinforcement ruptures to R1 side of P. Rupture is located at crack developed at end of P hogging reinforcement.

D.6.2 Test data

The force-rotation and moment-rotation records, taken during testing, are provided below. Reference points are provided in conjunction with the observations noted in the previous section.

![Graph of test data](image.png)

**Figure D-12 – Force-rotation data recorded for S01 test.**
Figure D-13 – Moment-rotation and moment-axial-force results, test S01.

Figure D-14 – Deflection profile record, test S01.
D.7 Test Data – Specimen S02 Results & Observations

Basic dimensions of this specimen and test are provided below. Reinforcement details and material specification are provided in Chapter 4 and Appendix C. Dimensions, location references and reaction name and sign conventions are to be read in conjunction with Chapter 4 and Chapter 5, Figure 5-1.

\[ L_{bd} = 2,500\text{mm} \quad L_{EM} = 5,000\text{mm} \quad L_{ab} = 635\text{mm} \]

\[ h = 90\text{mm} \quad b = 275\text{mm} \]

D.7.1 Specimen response notes

A summary of observations made during the test are provided below together with the chronology and reaction record taken for key events of the test and specimen response. The location of each event (i.e. rebar rupture) is indicated using the figure below.

### S02 – Key events in response

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Description</th>
<th>( \Delta ) [mm]</th>
<th>( \theta_\Delta ) [Degrees]</th>
<th>( P ) [kN]</th>
<th>( R_H ) [kN]</th>
<th>( R_1 ) [kN]</th>
<th>( R_2 ) [kN]</th>
<th>( R_{V1} ) [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>CMA maximum load</td>
<td>63.3</td>
<td>1.5</td>
<td>-6.2</td>
<td>0.0</td>
<td>10.3</td>
<td>10.2</td>
<td>-5.6</td>
</tr>
<tr>
<td>B</td>
<td>Snap through</td>
<td>69.3</td>
<td>1.6</td>
<td>-6.2</td>
<td>0.0</td>
<td>10.2</td>
<td>10.2</td>
<td>-5.5</td>
</tr>
<tr>
<td>C</td>
<td>Fracture (bending) Grp02 @ P</td>
<td>496.9</td>
<td>11.2</td>
<td>-32.3</td>
<td>-54.7</td>
<td>20.5</td>
<td>18.4</td>
<td>-6.6</td>
</tr>
<tr>
<td>D</td>
<td>TMA maximum load</td>
<td>674.6</td>
<td>15.1</td>
<td>-39.4</td>
<td>-55.8</td>
<td>23.8</td>
<td>21.4</td>
<td>-6.9</td>
</tr>
<tr>
<td>E</td>
<td>TEST TERMINATED (displacement limit reached)</td>
<td>675.8</td>
<td>15.1</td>
<td>-38.9</td>
<td>-55.1</td>
<td>23.5</td>
<td>21.2</td>
<td>-6.9</td>
</tr>
</tbody>
</table>

\( \Delta \leq 100\text{mm} \)  
Bending crack formed at \( R_1 \) and \( R_2 \) centre lines and to either side of jack bearing at point P.

\( \Delta \geq 140\text{mm} \)  
Tension cracks form along the linear region of specimen (length with no T2 bar). Cracks form across full depth of section and reduction of applied load noted as each crack opens. Specimen remains symmetric.

\( \Delta \geq 310\text{mm} \)  
Catenary formed between two cantilevers. Cover is progressively pulled from underside of cantilevers as angle of incidence increases. B2 reinforcement sustained by links (failure of each is evident in graph).

\( 480 \leq \Delta \leq 500\text{mm} \)  
Extreme tension reinforcement (B2 bar) rupture at midspan, P.

\( \Delta = 670\text{mm} \)  
End of test. Maximum displacement attained before failure is achieved.
D.7.2  Test data

The force-rotation and moment-rotation records, taken during testing, are provided below. Reference points are provided in conjunction with the observations noted in the previous section.

Figure D-15 – Force-rotation data recorded for S02 test.
Figure D-16 – Moment-rotation and moment-axial-force results, test S02.

Figure D-17 – Deflection profile record, test S02.
D.8 Test Data – Specimen S03 Results & Observations

Basic dimensions of this specimen and test are provided below. Reinforcement details and material specification are provided in Chapter 4 and Appendix C. Dimensions, location references and reaction name and sign conventions are to be read in conjunction with Chapter 4 and Chapter 5, Figure 5-1.

\[ L_{bd} = 2,500\text{mm} \quad L_{EM} = 5,000\text{mm} \quad L_{ab} = 635\text{mm} \]

\[ h = 90\text{mm} \quad b = 275\text{mm} \]

D.8.1 Specimen response notes

A summary of observations made during the test are provided below together with the chronology and reaction record taken for key events of the test and specimen response. The location of each event (i.e. rebar rupture) is indicated using the figure below.

### S03 – Key events in response

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Description</th>
<th>(\Delta) [mm]</th>
<th>(\theta_A) [Degrees]</th>
<th>(P) [kN]</th>
<th>(R_H) [kN]</th>
<th>(R_1) [kN]</th>
<th>(R_2) [kN]</th>
<th>(R_{V1}) [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>CMA maximum load</td>
<td>123.7</td>
<td>2.8</td>
<td>-9.3</td>
<td>0.9</td>
<td>12.9</td>
<td>12.3</td>
<td>-6.4</td>
</tr>
<tr>
<td>B</td>
<td>Snap through</td>
<td>127.7</td>
<td>2.9</td>
<td>-9.2</td>
<td>0.1</td>
<td>12.6</td>
<td>12.1</td>
<td>-6.5</td>
</tr>
<tr>
<td>C</td>
<td>Fracture (bending) Grp02 @ P</td>
<td>415.1</td>
<td>9.4</td>
<td>-37.7</td>
<td>-76.2</td>
<td>24.1</td>
<td>23.0</td>
<td>-8.5</td>
</tr>
<tr>
<td>D</td>
<td>Fracture (bending) Grp01 @ R2</td>
<td>564.6</td>
<td>12.7</td>
<td>-45.6</td>
<td>-78.9</td>
<td>28.2</td>
<td>26.4</td>
<td>-8.9</td>
</tr>
<tr>
<td>E</td>
<td>Fracture (bending) Grp01 @ R1</td>
<td>669.1</td>
<td>15.0</td>
<td>-55.6</td>
<td>-82.8</td>
<td>32.8</td>
<td>27.9</td>
<td>-9.2</td>
</tr>
<tr>
<td>F</td>
<td>TEST TERMINATED (displacement limit reached)</td>
<td>669.1</td>
<td>15.0</td>
<td>-55.6</td>
<td>-82.8</td>
<td>32.8</td>
<td>27.9</td>
<td>-9.2</td>
</tr>
</tbody>
</table>

\(\Delta = 60\text{mm}\) Cracking developed at end of \(R_1\) and \(R_2\) hogging reinforcement (T2 bar).

\(\Delta = 80\text{mm}\) Minor bending cracking formed at end of \(R_1\) and \(R_2\) hogging reinforcement.

\(\Delta \geq 120\text{mm}\) Tension cracks form along the linear region of specimen (length with no T2 bar). Cracks form across full depth of section and reduction of applied load noted as each crack opens. Specimen remains symmetric.

\(400 \leq \Delta \leq 420\text{mm}\) Extreme tension reinforcement (B2 bar) rupture at midspan, P (estimated to be just before 420mm was achieved).

\(420 \leq \Delta \leq 520\text{mm}\) Profile of specimen reshapes to adopt a direct tension line between remaining rebar.

\(560 \leq \Delta \leq 580\text{mm}\) Rebar fracture (T2 hogging bar at \(R_2\)).
670 ≤ Δ ≤ 695mm  Rebar fracture (T2 hogging bar at R1).

**D.8.2 Test data**

The force-rotation and moment-rotation records, taken during testing, are provided below. Reference points are provided in conjunction with the observations noted in the previous section.

Figure D-18 – Force-rotation data recorded for S03 test.
Figure D-19 – Moment-rotation and moment-axial-force results, test S03.

Figure D-20 – Deflection profile record, test S03.
D.9  Test Data – Specimen C01 Results & Observations

Basic dimensions of this specimen and test are provided below. Reinforcement details and material specification are provided in Chapter 4 and Appendix C. Dimensions, location references and reaction name and sign conventions are to be read in conjunction with Chapter 4 and Chapter 5, Figure 5-1.

\[ L_{bd} = 2,500\text{mm} \quad L_{EM} = 5,000\text{mm} \quad L_{ab} = 635\text{mm} \]

\[ h = 110\text{mm} \quad b = 320\text{mm} \]

D.9.1 Specimen response notes

A summary of observations made during the test are provided below together with the chronology and reaction record taken for key events of the test and specimen response. The location of each event (i.e. rebar rupture) is indicated using the figure below.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Description</th>
<th>( \Delta )</th>
<th>( \theta_\Delta )</th>
<th>( P )</th>
<th>( R_H )</th>
<th>( R_1 )</th>
<th>( R_2 )</th>
<th>( R_{V1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>CMA maximum load</td>
<td>119.4</td>
<td>2.7</td>
<td>-7.7</td>
<td>1.0</td>
<td>13.3</td>
<td>14.9</td>
<td>-8.0</td>
</tr>
<tr>
<td>B</td>
<td>Snap through</td>
<td>123.1</td>
<td>2.8</td>
<td>-7.8</td>
<td>-0.1</td>
<td>13.3</td>
<td>14.9</td>
<td>-8.0</td>
</tr>
<tr>
<td>C</td>
<td>Fracture (bending) Grp01 @ R2</td>
<td>221.0</td>
<td>5.1</td>
<td>-17.3</td>
<td>-70.5</td>
<td>12.9</td>
<td>13.4</td>
<td>-6.3</td>
</tr>
<tr>
<td>D</td>
<td>TMA maximum load</td>
<td>221.0</td>
<td>5.1</td>
<td>-17.3</td>
<td>-70.5</td>
<td>12.9</td>
<td>13.4</td>
<td>-6.3</td>
</tr>
<tr>
<td>E</td>
<td>SYSTEM FAILURE - Fracture (tension) Grp02 @ R2</td>
<td>249.1</td>
<td>5.7</td>
<td>-13.7</td>
<td>-49.4</td>
<td>13.6</td>
<td>7.0</td>
<td>-7.2</td>
</tr>
</tbody>
</table>

210 \leq \Delta \leq 230\text{mm}  \quad \text{Hogging rebar (T2 bar) fractures at } R_2.

230 \leq \Delta \leq 250\text{mm}  \quad \text{End of test. Remaining rebar (B2 bar) fractures at } R_2.
D.9.2 Test data

The force-rotation and moment-rotation records, taken during testing, are provided below. Reference points are provided in conjunction with the observations noted in the previous section.

Figure D-21 – Force-rotation data recorded for C01 test.
Figure D-22 – Moment-rotation and moment-axial-force results, test C01.

Figure D-23 – Deflection profile record, test C01.
D.10 Test Data – Specimen C02 Results & Observations

Basic dimensions of this specimen and test are provided below. Reinforcement details and material specification are provided in Chapter 4 and Appendix C. Dimensions, location references and reaction name and sign conventions are to be read in conjunction with Chapter 4 and Chapter 5, Figure 5-1.

\[ L_{bd} = 2,275\text{mm} \quad L_{EM} = 4,550\text{mm} \quad L_{ab} = 860\text{mm} \]
\[ h = 118\text{mm} \quad b = 320\text{mm} \]

D.10.1 Specimen response notes

A summary of observations made during the test are provided below together with the chronology and reaction record taken for key events of the test and specimen response. The location of each event (i.e. rebar rupture) is indicated using the figure below.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Description</th>
<th>( \Delta ) [mm]</th>
<th>( \theta_\Delta ) [Degrees]</th>
<th>( P ) [kN]</th>
<th>( R_H ) [kN]</th>
<th>( R_1 ) [kN]</th>
<th>( R_2 ) [kN]</th>
<th>( R_{V1} ) [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>CMA maximum load</td>
<td>124.3</td>
<td>3.1</td>
<td>-10.5</td>
<td>2.0</td>
<td>13.6</td>
<td>14.0</td>
<td>-5.0</td>
</tr>
<tr>
<td>B</td>
<td>Fracture (bending) Grp02 @ P</td>
<td>154.2</td>
<td>3.9</td>
<td>-9.6</td>
<td>0.5</td>
<td>13.3</td>
<td>13.6</td>
<td>-5.7</td>
</tr>
<tr>
<td>C</td>
<td>Snap through</td>
<td>160.2</td>
<td>4.0</td>
<td>-4.8</td>
<td>0.1</td>
<td>10.6</td>
<td>11.1</td>
<td>-5.2</td>
</tr>
<tr>
<td>D</td>
<td>TMA maximum load</td>
<td>277.9</td>
<td>7.0</td>
<td>-16.7</td>
<td>-57.0</td>
<td>13.7</td>
<td>13.2</td>
<td>-5.0</td>
</tr>
<tr>
<td>E</td>
<td>SYSTEM FAILURE – Fracture (tension)</td>
<td>290.9</td>
<td>7.3</td>
<td>-13.6</td>
<td>-40.8</td>
<td>13.0</td>
<td>12.1</td>
<td>-5.2</td>
</tr>
</tbody>
</table>

\( \Delta \leq 100\text{mm} \quad \) Distributed cracking developed local to \( R_1 \), \( R_2 \) and P.
\( \Delta = 150\text{mm} \quad \) Extreme tension reinforcement (B2 bar) rupture at midspan, P.
\( 280 \leq \Delta \leq 290\text{mm} \quad \) Remaining reinforcement (T2 bar) rupture at midspan, P.
\( 290 \leq \Delta \leq 295\text{mm} \quad \) End of test. Hogging reinforcement (T2 bar) rupture at \( R_2 \).
D.10.2 Test data

The force-rotation and moment-rotation records, taken during testing, are provided below. Reference points are provided in conjunction with the observations noted in the previous section.

Figure D-24 – Force-rotation data recorded for C02 test.
Figure D-25 – Moment-rotation and moment-axial-force results, test C02.

Figure D-26 – Deflection profile record, test C02.
D.11 Test Data – Specimen C03 Results & Observations

Basic dimensions of this specimen and test are provided below. Reinforcement details and material specification are provided in Chapter 4 and Appendix C. Dimensions, location references and reaction name and sign conventions are to be read in conjunction with Chapter 4 and Chapter 5, Figure 5-1.

\[ L_{bd} = 2,160\text{mm} \quad L_{EM} = 4,320\text{mm} \quad L_{ab} = 975\text{mm} \]
\[ h = 140\text{mm} \quad b = 320\text{mm} \]

D.11.1 Specimen response notes

A summary of observations made during the test are provided below together with the chronology and reaction record taken for key events of the test and specimen response. The location of each event (i.e. rebar rupture) is indicated using the figure below.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Description</th>
<th>(\Delta) [mm]</th>
<th>(\theta) [Degrees]</th>
<th>(P) [kN]</th>
<th>(R_H) [kN]</th>
<th>(R_1) [kN]</th>
<th>(R_2) [kN]</th>
<th>(R_{V1}) [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>CMA maximum load</td>
<td>98.7</td>
<td>2.6</td>
<td>-23.5</td>
<td>44.8</td>
<td>25.4</td>
<td>25.7</td>
<td>-13.0</td>
</tr>
<tr>
<td>B</td>
<td>Snap through</td>
<td>189.0</td>
<td>5.0</td>
<td>-22.0</td>
<td>-0.1</td>
<td>23.8</td>
<td>24.2</td>
<td>-10.4</td>
</tr>
<tr>
<td>C</td>
<td>Fracture (bending) Grp02 @ P</td>
<td>249.4</td>
<td>6.6</td>
<td>-30.5</td>
<td>-57.9</td>
<td>24.5</td>
<td>24.6</td>
<td>-8.8</td>
</tr>
<tr>
<td>D</td>
<td>TMA maximum load</td>
<td>416.9</td>
<td>10.9</td>
<td>-43.5</td>
<td>-90.0</td>
<td>28.8</td>
<td>29.0</td>
<td>-8.2</td>
</tr>
<tr>
<td>E</td>
<td>Fracture (tension) Grp01 @ P</td>
<td>421.9</td>
<td>11.1</td>
<td>-42.4</td>
<td>-85.6</td>
<td>28.5</td>
<td>28.8</td>
<td>-8.3</td>
</tr>
<tr>
<td>F</td>
<td>Fracture (tension) Grp01 @ P</td>
<td>430.9</td>
<td>11.3</td>
<td>-31.8</td>
<td>-55.8</td>
<td>25.3</td>
<td>25.8</td>
<td>-8.9</td>
</tr>
<tr>
<td>G</td>
<td>SYSTEM FAILURE - Fracture (tension) Grp01 @ P</td>
<td>444.8</td>
<td>11.6</td>
<td>-20.5</td>
<td>-26.1</td>
<td>21.0</td>
<td>22.6</td>
<td>-9.4</td>
</tr>
</tbody>
</table>

\[ \Delta \leq 100\text{mm} \quad \text{Distributed cracking developed local to } R_1, R_2 \text{ and } P. \]
\[ 240 \leq \Delta \leq 250\text{mm} \quad \text{Extreme tension reinforcement (B2 bar) rupture at midspan, } P. \]
\[ 400 \leq \Delta \leq 420\text{mm} \quad \text{1no. of remaining reinforcement (T2 bar) rupture at midspan, } P. \]
\[ 420 \leq \Delta \leq 440\text{mm} \quad \text{End of test. Remaining reinforcement (T2 bar) rupture at midspan, } P. \]
D.11.2 Test data

The force-rotation and moment-rotation records, taken during testing, are provided below. Reference points are provided in conjunction with the observations noted in the previous section.

Figure D-27 – Force-displacement data recorded for C03 test.
Figure D-28 – Moment-rotation and moment-axial-force results, test C03.

Figure D-29 – Deflection profile record, test C03.
D.12 Test Data – Specimen C04 Results & Observations

Basic dimensions of this specimen and test are provided below. Reinforcement details and material specification are provided in Chapter 4 and Appendix C. Dimensions, location references and reaction name and sign conventions are to be read in conjunction with Chapter 4 and Chapter 5, Figure 5-1.

\[ L_{bd} = 2,025\text{mm} \quad L_{EM} = 4,050\text{mm} \quad L_{ab} = 1,110\text{mm} \]

\[ h = 160\text{mm} \quad b = 320\text{mm} \]

D.12.1 Specimen response notes

A summary of observations made during the test are provided below together with the chronology and reaction record taken for key events of the test and specimen response. The location of each event (i.e. rebar rupture) is indicated using the figure below.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Description</th>
<th>( \Delta ) [mm]</th>
<th>( \theta_\Delta ) [Degrees]</th>
<th>( P ) [kN]</th>
<th>( R_H ) [kN]</th>
<th>( R_1 ) [kN]</th>
<th>( R_2 ) [kN]</th>
<th>( R_{V1} ) [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>CMA maximum load</td>
<td>87.0</td>
<td>2.5</td>
<td>-35.0</td>
<td>24.7</td>
<td>32.9</td>
<td>32.4</td>
<td>-15.6</td>
</tr>
<tr>
<td>B</td>
<td>Fracture (bending) Grp02 @ P</td>
<td>192.0</td>
<td>5.4</td>
<td>-34.5</td>
<td>0.1</td>
<td>32.9</td>
<td>32.9</td>
<td>-14.8</td>
</tr>
<tr>
<td>C</td>
<td>Snap through</td>
<td>196.0</td>
<td>5.5</td>
<td>-30.6</td>
<td>-0.1</td>
<td>31.0</td>
<td>31.0</td>
<td>-14.2</td>
</tr>
<tr>
<td>D</td>
<td>Fracture (bending) Grp02 @ P</td>
<td>233.0</td>
<td>6.6</td>
<td>-23.4</td>
<td>-5.9</td>
<td>27.5</td>
<td>27.5</td>
<td>-14.8</td>
</tr>
<tr>
<td>E</td>
<td>TMA maximum load</td>
<td>467.0</td>
<td>13.0</td>
<td>-67.6</td>
<td>-119.7</td>
<td>41.1</td>
<td>41.6</td>
<td>-11.2</td>
</tr>
<tr>
<td>F</td>
<td>Fracture (tension) Grp01 @ P</td>
<td>467.0</td>
<td>13.0</td>
<td>-67.6</td>
<td>-119.7</td>
<td>41.1</td>
<td>41.6</td>
<td>-11.2</td>
</tr>
<tr>
<td>G</td>
<td>SYSTEM FAILURE - Fracture (tension) Grp01 @ P</td>
<td>482.0</td>
<td>13.4</td>
<td>-55.3</td>
<td>-86.7</td>
<td>37.2</td>
<td>37.8</td>
<td>-11.8</td>
</tr>
</tbody>
</table>

180 \( \leq \Delta \leq 200\text{mm} \) 2/4 tension reinforcement (B2 bar) rupture at midspan, P.

210 \( \leq \Delta \leq 230\text{mm} \) 2/4 tension reinforcement (B2 bar) rupture at midspan, P.

450 \( \leq \Delta \leq 480\text{mm} \) End of test. Remaining reinforcement (T2 bar) rupture at midspan, P. Note that the rebar were found to rupture individually rather than simultaneously.
D.12.2 Test data

The force-rotation and moment-rotation records, taken during testing, are provided below. Reference points are provided in conjunction with the observations noted in the previous section.

![Force-displacement data recorded for C04 test.](image-url)
a) Moment-rotation curve

b) Recorded M-N interaction (at point B)

c) Theoretical M-N interaction (at point D)

Figure D-31 – Moment-rotation and moment-axial-force results, test C04.

Figure D-32 – Deflection profile record, test C04.
D.13 Test Data – Specimen M01 Results & Observations

Basic dimensions of this specimen and test are provided below. Reinforcement details and material specification are provided in Chapter 4 and Appendix C. Dimensions, location references and reaction name and sign conventions are to be read in conjunction with Chapter 4 and Chapter 5, Figure 5-1.

\[ L_{bd} = 1,950\text{mm} \quad L_{EM} = 3,900\text{mm} \quad L_{ab} = 1,185\text{mm} \]

\[ h = 105\text{mm} \quad b = 320\text{mm} \]

D.13.1 Specimen response notes

A summary of observations made during the test are provided below together with the chronology and reaction record taken for key events of the test and specimen response. The location of each event (i.e. rebar rupture) is indicated using the figure below.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Description</th>
<th>( \Delta )</th>
<th>( \theta_\Delta )</th>
<th>( P )</th>
<th>( R_H )</th>
<th>( R_1 )</th>
<th>( R_2 )</th>
<th>( R_{V1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>CMA max load</td>
<td>55.1</td>
<td>1.6</td>
<td>-8.2</td>
<td>53.6</td>
<td>9.9</td>
<td>9.6</td>
<td>-5.5</td>
</tr>
<tr>
<td>B</td>
<td>Fracture (bending) Grp02 @ P</td>
<td>106.6</td>
<td>3.1</td>
<td>-3.7</td>
<td>62.2</td>
<td>8.6</td>
<td>8.3</td>
<td>-6.9</td>
</tr>
<tr>
<td>C</td>
<td>Snap through</td>
<td>183.5</td>
<td>5.4</td>
<td>-1.6</td>
<td>0.1</td>
<td>5.1</td>
<td>5.2</td>
<td>-2.2</td>
</tr>
<tr>
<td>D</td>
<td>Fracture (bending) Grp01 @ R2</td>
<td>207.6</td>
<td>6.1</td>
<td>-1.7</td>
<td>-2.9</td>
<td>4.9</td>
<td>4.7</td>
<td>-2.2</td>
</tr>
<tr>
<td>E</td>
<td>Fracture (bending) Grp01 @ R1</td>
<td>224.7</td>
<td>6.6</td>
<td>-1.8</td>
<td>-12.0</td>
<td>4.7</td>
<td>2.5</td>
<td>-3.2</td>
</tr>
<tr>
<td>F</td>
<td>TMA maximum load</td>
<td>270.9</td>
<td>7.9</td>
<td>-2.8</td>
<td>-28.7</td>
<td>3.1</td>
<td>3.7</td>
<td>-1.2</td>
</tr>
<tr>
<td>G</td>
<td>SYSTEM FAILURE - Fracture (tension) Grp02 @ P</td>
<td>276.9</td>
<td>8.1</td>
<td>-4.0</td>
<td>-24.4</td>
<td>3.0</td>
<td>3.4</td>
<td>-1.4</td>
</tr>
</tbody>
</table>

\[ \Delta = 40\text{mm} \quad \text{Cracking develops to left-hand-side of } R_1 \text{ hanger (indicating hogging between } R_{V1} \text{ and } R_1). \]

\[ 100 \leq \Delta \leq 120\text{mm} \quad \text{Extreme tension reinforcement (B2 bar) rupture at midspan, } P. \]

\[ 190 \leq \Delta \leq 210\text{mm} \quad \text{Remaining rebar (T2 bar) ruptured at midspan, } P. \quad \text{System is formed of two cantilevers.} \]

\[ 210 \leq \Delta \leq 230\text{mm} \quad \text{Hogging rebar (T2 bar) rupture at } R_1. \]

\[ 270 \leq \Delta \leq 290\text{mm} \quad \text{End of test. Hogging rebar (T2 bar) rupture at } R_2. \]
D.13.2 Test data

The force-rotation and moment-rotation records, taken during testing, are provided below. Reference points are provided in conjunction with the observations noted in the previous section.

Figure D-33 – Force-displacement data recorded for M01 test.
Figure D-34 – Moment-rotation and moment-axial-force results, test M01.

Figure D-35 – Deflection profile record, test M01.
D.14 Test Data – Specimen M02 Results & Observations

Basic dimensions of this specimen and test are provided below. Reinforcement details and material specification are provided in Chapter 4 and Appendix C. Dimensions, location references and reaction name and sign conventions are to be read in conjunction with Chapter 4 and Chapter 5, Figure 5-1.

\[ L_{bd} = 2,360\text{mm} \quad L_{EM} = 4,720\text{mm} \quad L_{ab} = 775\text{mm} \]

\[ h = 145\text{mm} \quad b = 320\text{mm} \]

D.14.1 Specimen response notes

A summary of observations made during the test are provided below together with the chronology and reaction record taken for key events of the test and specimen response. The location of each event (i.e. rebar rupture) is indicated using the figure below.

Displacement at point P measured at 7.5mm (3.5mm initial hog plus 4mm displacement below builders line) following support removal.

\[ \Delta = 40\text{mm} \quad \text{Bending cracks form at } R_1, R_2 \text{ and } P. \]

\[ 190 \leq \Delta \leq 210\text{mm} \quad \text{Extreme tension reinforcement (B2 bar) ruptured at midspan, } P \text{ (estimated to occur at near 210mm displacement).} \]

\[ \Delta \geq 270\text{mm} \quad \text{Minor tensile cracks form in midsection.} \]

\[ 400 \leq \Delta \leq 420\text{mm} \quad \text{End of test. Remaining reinforcement (T2 bar) at } P \text{ ruptured.} \]
D.14.2 Test data

The force-rotation and moment-rotation records, taken during testing, are provided below. Reference points are provided in conjunction with the observations noted in the previous section.

Figure D-36 – Force-displacement data recorded for M02 test.
Figure D-37 – Moment-rotation and moment-axial-force results, test M02.

Figure D-38 – Deflection profile record, test M02.
### Appendix E  Exemplar Floor System Design

#### E.1  Example Floor System Design

**RC Frame Design #01**

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Load Factor</td>
</tr>
<tr>
<td></td>
<td>ALS fact</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>1.95</td>
</tr>
<tr>
<td></td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>2.05</td>
</tr>
<tr>
<td></td>
<td>3.00</td>
</tr>
<tr>
<td></td>
<td>4.00</td>
</tr>
</tbody>
</table>

#### Grid Summary

- **Story story height (ft):** 12 ft
- **Bay (ft):** 12 ft
- **Floor (ft):** 12 ft
- **Total floor area (sq ft):** 1,440 sq ft

#### Assumptions

- All columns same height, effectively bracketed and defined as "short column".
- Roof load is taken equal to floor load.
- **Moment area ratio** = 0.75

#### Load Cases - CCY

- **Total Dead Load:** 10 psf
- **Total Live Load:** 40 psf
- **Total Roof Live Load:** 0 psf

#### Design Charts

- **Ac/Ad - Slab:**
- **Ac/Ad - Beam:**
- **Ac/Ad - Column:**
- **Ac/Ad - Frame:**

#### Element 01 - Edge Beam Design (assumed as L-section)

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective span, L (ft)</td>
<td>10.00</td>
</tr>
<tr>
<td>k = 0.75</td>
<td>5.00</td>
</tr>
<tr>
<td>2.00</td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

#### Element 01 - Edge Beam Design (assumed as L-section)

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective span, L (ft)</td>
<td>10.00</td>
</tr>
<tr>
<td>k = 0.75</td>
<td>5.00</td>
</tr>
<tr>
<td>2.00</td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

#### Load Effect - CSA

- **Total Dead Load:** 10 psf
- **Total Live Load:** 40 psf
- **Total Roof Live Load:** 0 psf
- **Total Roof Live Load:** 0 psf

#### Distribution

- **Distribution factor:** 0.90

#### Accessories

- **Roof:** 12 ft
- **Floor:** 12 ft
- **Wall:** 12 ft

#### Column spacing

- **BAY Y-Y**
- **BAY X-X**

#### Elements

- **Element 01 - Edge Beam Design (assumed as L-section)**
- **Element 01 - Edge Beam Design (assumed as L-section)**

---

92
### Element 01 - Slab Design (WSS) - y-z moment design

**Dimensions**
- Effective span, L (m): 6.000
- Depth, d (mm): 300
- Gross moment, M (kNm): 134.1

**Load Effects**
- Total ULS design load, F (kN/m width): 60.42
- Moment factor: 0.963
- ULS moment, M_{ULS} = 0.963 \times 40.4 = 39.0

**ULS Design - Reinforcement**

<table>
<thead>
<tr>
<th>Reinforcement</th>
<th>423.53</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension steel required, A_t (mm^2)</td>
<td>0.00</td>
</tr>
<tr>
<td>Compression steel required, A_c (mm^2)</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Links (mm)**: 10 mm dia.

### Element 02 - Slab Design (WSS) - x-x distribution design

**Dimensions**
- Effective span, L (m): 10.830
- Depth, d (mm): 209
- Gross moment, M (kNm): 134.1

**Load Effects**
- Effective depth, d (mm): 163
- Depth of compression reinforcement, d (mm): 37

**ULS Design - Reinforcement**

<table>
<thead>
<tr>
<th>Reinforcement</th>
<th>T2 area provided</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00 mm</td>
<td></td>
</tr>
</tbody>
</table>
### Table E-1 – Example design of exemplar structural floor system components.

<table>
<thead>
<tr>
<th>Element 02 - Main Beam Design (assumed as T-section)</th>
<th>Element 03 - Main Beam Design (assumed as T-section)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dimensions</strong></td>
<td><strong>Main beam not required as one-way spanning system</strong></td>
</tr>
<tr>
<td>Effective span, L (m)</td>
<td></td>
</tr>
<tr>
<td>10.80</td>
<td></td>
</tr>
<tr>
<td>L/6 = V (kN)</td>
<td></td>
</tr>
<tr>
<td>7.60</td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>k (mm)</td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td></td>
</tr>
<tr>
<td>Nk (kN/m²)</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td></td>
</tr>
<tr>
<td>k = b - h + 2(l/3)(mm)</td>
<td></td>
</tr>
<tr>
<td>2662</td>
<td></td>
</tr>
<tr>
<td>2.27</td>
<td></td>
</tr>
<tr>
<td>20.80</td>
<td></td>
</tr>
<tr>
<td>17.09</td>
<td></td>
</tr>
<tr>
<td>Cover (mm)</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td></td>
</tr>
<tr>
<td>Effective depth (b1) of beam (mm)</td>
<td></td>
</tr>
<tr>
<td>48</td>
<td></td>
</tr>
<tr>
<td>Load Effects</td>
<td></td>
</tr>
<tr>
<td>Total SLS dist load, qk (kN/m²)</td>
<td>64.80</td>
</tr>
<tr>
<td>Total ULS dist load, qk (kN/m²)</td>
<td>10.30</td>
</tr>
<tr>
<td>Maximum ULS Moments (min):</td>
<td></td>
</tr>
<tr>
<td>B1 (mm)</td>
<td>104.4</td>
</tr>
<tr>
<td>A2 (mm)</td>
<td>81.7</td>
</tr>
<tr>
<td>Design ULS Moments (rounded):</td>
<td>79.1</td>
</tr>
<tr>
<td>SLS (mm)</td>
<td>64.1</td>
</tr>
<tr>
<td>ULS Design</td>
<td></td>
</tr>
<tr>
<td>Reinforcement requirement at mid-span (designed for breaths)</td>
<td></td>
</tr>
<tr>
<td>Distribution factor, β = 1.1</td>
<td>10.30</td>
</tr>
<tr>
<td>Tension steel required, A₁ (mm²)</td>
<td>3.4.4.4</td>
</tr>
<tr>
<td>Compression steel required, A₂ (mm²)</td>
<td>0.08</td>
</tr>
<tr>
<td>Reinforcement requirement at interior support</td>
<td></td>
</tr>
<tr>
<td>Distribution factor, β₁ = 1.1</td>
<td>7.90</td>
</tr>
<tr>
<td>Tension steel required, A₁₁ (mm²)</td>
<td>20.4.8</td>
</tr>
<tr>
<td>Compression steel required, A₂₁ (mm²)</td>
<td>0.08</td>
</tr>
<tr>
<td>Reinforcement summary</td>
<td></td>
</tr>
<tr>
<td>L (mm)</td>
<td>12 mm (dia)</td>
</tr>
<tr>
<td>Mid-span</td>
<td></td>
</tr>
<tr>
<td>Compression reinforcement in layer: T₁</td>
<td></td>
</tr>
<tr>
<td>Ø mm</td>
<td>8</td>
</tr>
<tr>
<td>A₁ (mm²)</td>
<td>804</td>
</tr>
<tr>
<td>Mₐ permissible reinforcement, 100(A₁h₁A₂h₂)%</td>
<td>0.30 Table 3.25</td>
</tr>
<tr>
<td>Mₐ required reinforcement, 100(A₁h₁A₂h₂)%</td>
<td>0.00</td>
</tr>
<tr>
<td>Mₐ required in detailing, 100(A₁h₁A₂h₂)%</td>
<td>0.17 Figure 3.24</td>
</tr>
<tr>
<td>Mₐ permissible reinforcement, 100(A₁h₁A₂h₂)%</td>
<td>4.00</td>
</tr>
<tr>
<td>Tension reinforcement in layer: B₁</td>
<td></td>
</tr>
<tr>
<td>Ø mm</td>
<td>32</td>
</tr>
<tr>
<td>A₁ (mm²)</td>
<td>3820</td>
</tr>
<tr>
<td>100(A₁h₁A₂h₂)%</td>
<td>0.99</td>
</tr>
<tr>
<td>Mₐ permissible reinforcement, 100(A₁h₁A₂h₂)%</td>
<td>0.26 Table 3.25</td>
</tr>
<tr>
<td>Mₐ required reinforcement, 100(A₁h₁A₂h₂)%</td>
<td>0.82</td>
</tr>
<tr>
<td>Mₐ permissible reinforcement, 100(A₁h₁A₂h₂)%</td>
<td>4.00</td>
</tr>
<tr>
<td>Compression reinforcement in layer: B₁</td>
<td></td>
</tr>
<tr>
<td>Ø mm</td>
<td>32</td>
</tr>
<tr>
<td>A₁ (mm²)</td>
<td>3227</td>
</tr>
<tr>
<td>100(A₁h₁A₂h₂)%</td>
<td>0.84</td>
</tr>
<tr>
<td>Mₐ permissible reinforcement, 100(A₁h₁A₂h₂)%</td>
<td>0.26 Table 3.25</td>
</tr>
<tr>
<td>Mₐ required reinforcement, 100(A₁h₁A₂h₂)%</td>
<td>0.82</td>
</tr>
<tr>
<td>Mₐ permissible reinforcement, 100(A₁h₁A₂h₂)%</td>
<td>4.00</td>
</tr>
<tr>
<td>SLS checks - Reflection</td>
<td></td>
</tr>
<tr>
<td>Distribution factor, β ≤ 1.0</td>
<td>10.30</td>
</tr>
<tr>
<td>Allowable SHE /W</td>
<td>35.80</td>
</tr>
<tr>
<td>Allowable SHE /A1</td>
<td>17.09</td>
</tr>
<tr>
<td>SLS check</td>
<td></td>
</tr>
<tr>
<td>Reinforcement requirement at interior support</td>
<td></td>
</tr>
<tr>
<td>F₁ (kN/m²)</td>
<td>118.17</td>
</tr>
<tr>
<td>M₁ (kN.m/m)</td>
<td>133.02</td>
</tr>
<tr>
<td>M₁ permissible reinforcement, 100(A₁h₁A₂h₂)%</td>
<td>667.98</td>
</tr>
<tr>
<td>M₁ required reinforcement, 100(A₁h₁A₂h₂)%</td>
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</tr>
<tr>
<td>M₁ permissible reinforcement, 100(A₁h₁A₂h₂)%</td>
<td>4.00</td>
</tr>
<tr>
<td>100(A₁h₁A₂h₂)%</td>
<td></td>
</tr>
</tbody>
</table>
E.2 Reinforcement Output Summary

The following section provides a summary of the reinforcement area attributed to individual bar groups for slab and edge beam elements of the surrogate floor systems. These parameters constitute the input for the Chapter 6 sensitivity study.

Figure E-1 specifies the reinforcement arrangement assumed in design. Individual bar groups are referenced accordingly.

![Figure E-1 – Bar group references.](image)

**Table E-2 – Reinforcement data for two-way floor systems, designed with 10-30\% moment redistribution.**
Table E-3 – Reinforcement data for one-way floor systems, designed with 10-30% moment redistribution.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>k</th>
<th>L_x</th>
<th>L_y</th>
<th>L_{xy}</th>
<th>A_{s1}</th>
<th>A_{s2}</th>
<th>A_{s3}</th>
<th>A_{s4}</th>
<th>A_{s_{t5}}</th>
<th>A_{s_{t10}}</th>
<th>A_{s_{t15}}</th>
<th>A_{s_{t10}}</th>
<th>A_{s_{t15}}</th>
<th>A_{s_{t20}}</th>
<th>A_{s_{t25}}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[-]</td>
<td>[m]</td>
<td>[m]</td>
<td>[m²]</td>
<td>[mm²]</td>
<td>[mm²]</td>
<td>[mm²]</td>
<td>[mm²]</td>
<td>[mm²/m]</td>
<td>[mm²/m]</td>
<td>[mm²/m]</td>
<td>[mm²/m]</td>
<td>[mm²/m]</td>
<td>[mm²/m]</td>
<td>[mm²/m]</td>
</tr>
<tr>
<td>One-way Bay XX – 10-30% Moment Redistribution</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O01</td>
<td>1.6</td>
<td>5.0</td>
<td>8.0</td>
<td>40.0</td>
<td>942</td>
<td>236</td>
<td>402</td>
<td>1169</td>
<td>400</td>
<td>600</td>
<td>800</td>
<td>263</td>
<td>263</td>
<td>263</td>
<td>263</td>
</tr>
<tr>
<td>O02</td>
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<td>9.6</td>
<td>57.6</td>
<td>1282</td>
<td>339</td>
<td>603</td>
<td>1473</td>
<td>480</td>
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Table E-4 – Reinforcement data for one-way floor systems, designed with 0-10% moment redistribution.
Appendix F Rule Sets & Results of Catenary Action
Sensitivity Study
F.1

Reinforcement – Industry Quality Control Material Test Data

Table F-1 – Summary of reinforcement tension test data (courtesy of BRE and UK rebar manufacturers, 2010).

97


F.2 Sensitivity of TMA Load-Rotation Relationships to Membrane Force

Figure F-1 to Figure F-7 provide the load-displacement records obtained in testing (Chapter 5) and allow comparison with linear load-displacement relationship given the term:

\[ \frac{P_{TMA}}{\Delta} = \frac{2R_H}{L} - g_kL \]  

Equation F-1

Seven load deflection functions are shown, corresponding with differing parameters of horizontal restraint force \((R_H)\), where:

\[ R_H = N = f_dA_{s-TMA} \]  

Equation F-2

A summary of parameters used for the design tension stress and effective reinforcement area is provided below, together with the source references.

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<th>Design Tension Stress, (f_d)</th>
<th>Source Reference</th>
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Table F-2 – Summary of differing theories for the quantification of ultimate membrane force, N.

Where, \((A_s + A_s')\) is considered the total area of principle reinforcement, \(A_{s-crit}\) is the area of critical reinforcement in secondary TMA response and \(A_{s-min}\) is the smallest available reinforcement layer at joint locations. The measured yield and ultimate tension stress of each reinforcement area was used in the computation of predictions shown in Figure F-1 to Figure F-7.
Figure F-1 – Load-displacement plots for theory#1.
Figure F-2 – Load-displacement plots for theory#2.
Figure F.3 – Load-displacement plots for theory#3.
Figure F-4 – Load-displacement plots for theory#4.
Figure F.5 – Load-displacement plots for theory#5.
Figure F-6 – Load-displacement plots for theory#6.
Figure F.7 – Load-displacement plots for theory#7.
### F.3 Sensitivity Study Results

#### Table F.3 – Summary table – Chord rotation required at FOS = 1.0 (F/L=0.5).

| Bay area (m²) | k | h (m) | ly (m) | Rotation (deg) | Bay XX Rotation | Bay YY Rotation | Bay XX Rotation | Bay YY Rotation | Bay XX Rotation | Bay YY Rotation | Bay XX Rotation | Bay YY Rotation | Bay XX Rotation | Bay YY Rotation |
|---------------|---|-------|-------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 30.0          | 1.2| 5.0   | 6.0   | 12.3           | 10.6           | 12.5           | 12.3           | 6.3            | 6.2            | 8.6            | 9.5            | 5.7            | 6.4            | 4.7            | 5.3            |
| 45.2          | 1.2| 6.0   | 7.2   | 14.1           | 13.5           | 14.4           | 14.6           | 7.3            | 7.2            | 10.0           | 11.0           | 6.7            | 7.4            | 5.6            | 6.3            |
| 58.8          | 1.2| 7.0   | 8.4   | 14.1           | 14.0           | 14.6           | 15.2           | 7.4            | 7.2            | 11.4           | 12.3           | 7.7            | 8.3            | 6.5            | 7.2            |
| 76.8          | 1.2| 8.0   | 9.6   | 15.3           | 16.1           | 15.8           | 15.3           | 8.1            | 7.8            | 12.6           | 13.5           | 8.5            | 9.1            | 7.2            | 8.0            |
| 97.3          | 1.2| 9.0   | 10.8  | 15.8           | 15.9           | 17.0           | 14.2           | 8.7            | 7.2            | 13.7           | 14.6           | 9.3            | 9.9            | 8.1            | 8.7            |
| 120.0         | 1.2| 10.0  | 12.0  | 16.1           | 17.0           | 16.7           | 15.0           | 8.5            | 7.7            | 14.7           | 15.6           | 9.9            | 10.5           | 8.8            | 9.4            |
| 35.0          | 1.4| 5.0   | 7.0   | 13.3           | 11.6           | 13.7           | 13.5           | 7.0            | 6.9            | 8.6            | 10.3           | 5.7            | 6.9            | 4.7            | 5.9            |
| 50.4          | 1.4| 6.0   | 8.4   | 14.6           | 15.6           | 14.9           | 14.3           | 7.6            | 7.3            | 10.0           | 11.8           | 6.7            | 7.9            | 5.6            | 6.8            |
| 68.6          | 1.4| 7.0   | 9.8   | 16.7           | 16.0           | 15.5           | 14.8           | 7.9            | 7.5            | 11.4           | 13.2           | 7.7            | 8.9            | 6.5            | 7.8            |
| 89.6          | 1.4| 8.0   | 11.2  | 17.9           | 14.5           | 14.3           | 11.8           | 7.3            | 7.0            | 12.7           | 14.4           | 9.5            | 9.7            | 7.4            | 8.6            |
| **Averages**  |   |       |       | 15.9           | 14.5           | 15.0           | 14.3           | 7.6            | 7.3            | 11.4           | 12.6           | 7.7            | 8.5            | 6.5            | 7.4            |
| 40.0          | 1.6| 5.0   | 8.0   | 8.5            | 11.3           | 9.7            | 10.2           | 4.9            | 5.1            | 8.6            | 11.0           | 5.8            | 5.7            | 4.8            | 6.3            |
| 57.6          | 1.6| 6.0   | 9.6   | 9.1            | 11.1           | 11.3           | 9.8            | 5.7            | 4.9            | 10.1           | 12.6           | 6.8            | 8.5            | 5.7            | 7.3            |
| 78.4          | 1.6| 7.0   | 11.2  | 10.1           | 8.6            | 12.0           | 9.3            | 6.1            | 4.7            | 11.5           | 13.9           | 7.7            | 9.3            | 6.6            | 8.3            |
| 45.0          | 1.8| 5.0   | 9.0   | 9.1            | 11.6           | 10.6           | 10.4           | 5.4            | 5.2            | 8.6            | 11.7           | 5.8            | 7.9            | 4.8            | 6.8            |
| 64.8          | 1.8| 6.0   | 10.8  | 10.1           | 10.2           | 11.3           | 9.9            | 5.7            | 5.0            | 10.2           | 13.2           | 6.8            | 8.9            | 5.7            | 7.8            |
| 50.0          | 2.0| 5.0   | 10.0  | 10.7           | 11.7           | 11.0           | 10.5           | 5.5            | 5.3            | 8.7            | 12.2           | 5.8            | 8.2            | 4.8            | 7.2            |
| 72.0          | 2.0| 6.0   | 12.0  | 11.2           | 10.3           | 11.7           | 10.0           | 5.9            | 5.0            | 10.2           | 13.8           | 6.8            | 9.3            | 5.7            | 8.2            |
| **Averages**  |   |       |       | 9.8            | 10.7           | 11.1           | 10.0           | 5.6            | 5.0            | 9.7            | 12.6           | 6.5            | 8.5            | 5.4            | 7.4            |
| 40.0          | 1.6| 5.0   | 8.0   | 10.3           | 12.2           | 12.6           | 10.8           | 6.4            | 5.5            | 8.6            | 11.0           | 5.8            | 7.4            | 4.8            | 6.3            |
| 57.6          | 1.6| 6.0   | 9.6   | 10.5           | 11.7           | 14.0           | 10.1           | 7.1            | 5.1            | 10.1           | 12.6           | 6.8            | 8.5            | 5.7            | 7.3            |
| 78.4          | 1.6| 7.0   | 11.2  | 12.3           | 8.9            | 14.8           | 9.7            | 7.5            | 4.9            | 11.5           | 13.9           | 7.7            | 9.3            | 6.6            | 8.3            |
| 45.0          | 1.8| 5.0   | 9.0   | 10.2           | 12.4           | 13.6           | 11.0           | 6.9            | 5.5            | 8.6            | 11.7           | 5.8            | 7.9            | 4.8            | 6.8            |
| 64.8          | 1.8| 6.0   | 10.8  | 11.8           | 10.6           | 14.0           | 10.2           | 7.1            | 5.1            | 10.2           | 13.2           | 6.8            | 8.9            | 5.7            | 7.8            |
| 50.0          | 2.0| 5.0   | 10.0  | 11.4           | 12.5           | 13.8           | 11.1           | 7.0            | 5.6            | 8.7            | 12.2           | 5.8            | 8.2            | 4.8            | 7.2            |
| 72.0          | 2.0| 6.0   | 12.0  | 12.6           | 10.6           | 14.7           | 10.3           | 7.5            | 5.2            | 10.2           | 13.8           | 6.8            | 9.3            | 5.7            | 8.2            |
| **Averages**  |   |       |       | 11.3           | 11.3           | 12.9           | 10.4           | 7.1            | 5.3            | 9.7            | 12.6           | 6.5            | 8.5            | 5.4            | 7.4            |
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Table F.4 – Summary table - Chord rotation required at FOS = 1.04 at 1.5.
Figure F-8 – Required chord rotation at FOS = 1.0 (DLF = 1.0), calculated for two-way spanning floor systems (moment redistribution 10-30%).
Figure F-9 – Required chord rotation at FOS = 1.0 (DLF = 1.0), calculated for one-way spanning floor systems (moment redistribution 10-30%).
Figure F-10 – Required chord rotation at FOS = 1.0 (DLF = 1.0), calculated for two-way spanning floor systems (moment redistribution 0-10%).