UNIVERSITY OF SOUTHAMPTON

Three papers in economic growth and inequality

by

Liu Liu

A thesis submitted in partial fulfillment for the
degree of Doctor of Philosophy

in the
Faculty of Social, Human and Mathematical Sciences
School of Social Sciences

October 2017

This Doctoral Thesis is dedicated to my family for their support.

UNIVERSITY OF SOUTHAMPTON

ABSTRACT

FACULTY OF SOCIAL, HUMAN AND MATHEMATICAL SCIENCES
SCHOOL OF SOCIAL SCIENCES

Doctor of Philosophy

by Liu Liu

The aim of this thesis is to deepen the understanding of growth and inequality, especially the issues related the skill premium and education. I analyze the evolution of the distribution of wealth as a new channel, by affecting which exogenous shocks could affect inequality in the long run. With this mechanism, this thesis studies three perspectives of economic growth and inequality.

1, I analyze the effects of different types of technological change on the skill premium. Skill-biased and unskill-biased technological changes have different direct effects on the skill premium, while both of them reduce the skill premium in the long run by transiting the dynamic distribution of wealth into an egalitarian steady state. This result could offer some explanation for the cross-country difference of the skill premium, and for the U-shape evolution of the skill premium in the U.S., throughout the 20th century.

2, Based on the model built in the first paper, I simulate the U.S. economy as benchmark economy and examine the effects of higher education financial policies on the inequality. The model predicts that, since the U.S. economy is already an egalitarian one, offering free higher education by taxing or increasing financial aid to college could not affect inequality in the long run.

3, Furthermore, I endogenize the higher education system and find that the size of market could be an important explanation for the different structures of higher education system. This chapter also analyzes the effect of this structure on the productivity and inequality and implies that the effect on the inequality depends on the initial status of the economy.

Contents

1 Introduction **1**

1.1 Motivation and Literature Review . . . 1

1.1.1 Inequality . . . 1

1.1.1.1 Intergenerational Transfer . . . 2

1.1.2 The Skill Premium . . . 2

1.2 The Structure of the Thesis . . . 3

1.2.1 The Technological change and the Skill Premium . . . 4

1.2.2 The Financial Policies for Higher Education . . . 4

1.2.3 The Structure of Higher Education System . . . 4

2 Inequality, Technological Change, and the Dynamics of the Skill Premium **7**

2.1 Introduction . . . 7

2.1.1 Literature Review . . . 11

2.1.2 Overview . . . 12

2.2 The Model . . . 12

2.2.1 Imperfect Credit Market . . . 13

2.2.2 Households . . . 13

2.2.3 Firms . . . 14

2.3 Static Equilibrium . . . 17

2.4 The Dynamic Model . . . 22

2.4.1 Steady State Cases . . . 22

2.4.2 Technological Changes and the Skill Premium in the the 20th Century . . . 25

2.4.3 Financial Aid in the 1960s-1970s . . . 28

2.5 Conclusion . . . 29

3 Inequality and the U.S. Financial Policies for Higher Education **31**

3.1 Introduction . . . 31

3.1.1 Literature Review . . . 33

3.1.2 Overview . . . 34

3.2 The Theoretical Model . . . 34

3.2.1 Imperfect Credit Market . . . 34

3.2.2 Households . . . 35

3.2.3 Firms . . . 36

3.2.4 The Dynamic Model . . . 37

3.3 Model Parameterization . . . 38

3.3.1 Independently Chosen Parameters . . . 39
3.3.2 The Technology Frontier . . . 39
3.3.3 The Calibrated Parameter . . . 40
3.4 Benchmark Economy . . . 41
3.4.1 The U.S. Economy . . . 41
3.5 Counterfactual Experiments . . . 42
3.5.1 The Role of Cost of Education . . . 42
3.5.2 The Role of The Interest Rate of Student Loan . . . 43
3.5.3 An Example of a Polarized Economy . . . 44
3.6 Conclusion . . . 45

4 The Structure of the Higher Education System and Economic Growth 49
4.1 Introduction . . . 49
4.1.1 Literature Review . . . 53
4.1.2 Overview . . . 55
4.2 The Model . . . 55
4.2.1 Imperfect Credit Market . . . 55
4.2.2 Universities . . . 56
4.2.3 Human Capital Accumulation . . . 56
4.2.4 Households . . . 57
4.3 The Equilibrium of the Higher Education System . . . 60
4.3.1 Higher Education System and Productivity . . . 62
4.3.2 The Effect on the Distribution of Wealth . . . 62
4.4 The Dynamic Model . . . 66
4.4.1 An Economy with Poverty Trap . . . 66
4.4.2 A Rich Economy . . . 69
4.5 Correlation between Ability and Wealth . . . 69
4.5.1 A Rich Economy . . . 70
4.5.2 An Economy with Poverty Trap . . . 70
4.6 Convex Cost of Education . . . 71
4.7 Conclusion . . . 72

5 Conclussion 75

A Appendix 1 77
A.0.1 Proof of Lemma 2.3 . . . 77
A.0.2 Proof of Lemma 2.5 . . . 78
A.0.3 Proof of Proposition 2.6 . . . 78
A.0.4 Proof of Proposition 2.8 . . . 80
A.0.5 Proof of Proposition 2.10 . . . 82

B Appendix 2 83
B.0.1 Proof of Proposition 4.2 . . . 83
B.0.2 Proof of Proposition 4.3 . . . 84
B.0.3 Proof of Proposition 4.4 . . . 85
B.0.4 Proof of Proposition 4.5 . . . 86

C Code 87

C.0.1 Global Parameters . . . 87
C.0.2 Initial Bequest . . . 89
C.0.3 Calibration . . . 90
C.0.4 Static Equilibrium . . . 92
C.0.5 Polarization Parameters . . . 97
C.0.6 Main . . . 99

Bibliography **101**

List of Figures

2.1 Technology Frontier . . . 15
2.2 Equilibrium Labour Ratio and Wages . . . 18
2.3 Supply and Demand of workers and Static Equilibrium . . . 19
2.4 Transition of the Distribution of Wealth . . . 23
2.5 Transition to a Steady State . . . 24
2.6 Unskill-biased Technological Change . . . 27
2.7 Skill-biased Technological Change . . . 27

3.1 The Evolvement of the Bechmark Economy . . . 46
3.2 The Evolvement of A Polarized Economy . . . 47

4.1 The Distribution of Universities in the U.S. and Germany . . . 50
4.2 The CDF of Universities in the U.S. and Germany . . . 51
4.3 Transition of Bequests . . . 63
4.4 Expected Bequests in Different Systems . . . 64
4.5 A Poor Economy with Poverty Trap . . . 67
4.6 The Poor Economy Introducing Diversified System . . . 68
4.7 A Rich Economy . . . 68

Academic Thesis: Declaration of Authorship

I, Liu Liu, declare that this thesis and the work presented in it are my own and has been generated by me as the result of my own original research.

Title of thesis: Three papers in economic growth and inequality.

I confirm that:

1. This work was done wholly or mainly while in candidature for a research degree at this University;
2. Where any part of this thesis has been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated;
3. Where I have consulted the published works of others, this has always been clearly attributed;
4. Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work;
5. I have acknowledged all main sources of help;
6. Where the thesis is based on work done by myself or jointly with others, I have made clear exactly what was done by others and what I have contributed myself;
7. none of this work has been published before submission

Signed: ....

Date:.................................

Acknowledgements

First and foremost, I would like to express my gratitude to my supervisors, Thomas Gall and Arnau Valladares-Esteban, for their guidance and their support. I am grateful to them not only for their ideas, comments, and suggestions, but also for their trust, their encouragement and their patience.

Also, I would love to thank my former supervisor, John Knowles. It was he who introduced me into such a colourful and exciting world of economics. He has also offered very helpful comments for my first chapter.

I want to thank the whole faculty, all the postgraduate students and postdocs of the Division of Economics. All of them are very friendly and generous, and they have offered a lot of help to me. Thank you all.

Thank you to the Division of Economics and the University of Southampton, for offering me a very precious opportunity of this programme and providing such rich resources, which have helped me to conduct my research.

I would also like to thank my friends, Andrea, Chi Wan, Panos, Richard and Stelios, for their very helpful suggestions for both my research and my life in Southampton.

Finally, I am indebted to my family: my father Liu Yongqiang, my mother Sun Jianning, and my sister Liu Qing. They have always encouraged and supported me to pursue my dream. Their love and confidence in me have given me the courage to overcome difficulties and to complete this programme.

Chapter 1

Introduction

In this section, I presents the motivation of this thesis and a general review of the literature related to the major topics discussed in this thesis. Furthermore, I present a brief introduction of the structure and major results of this paper

1.1 Motivation and Literature Review

1.1.1 Inequality

Confucius said, ”it is inequality, but not scarcity, that persecutes governors”. The issue of inequality has drawn attentions of economists from ancient times to the present, and there has been huge disagreement among their views. The essential question of this thesis is that how inequality evolves in the long run and what is the drive behind it.

As stated by Piketty et al. (2014), the classical political economist, Ricardo, believed that the scarcity of land would aggravate the situation of inequality. With population and other output increasing, the scarcity of land will become more and more serious, which increases the rents to landlords, and enables them to claim more share of the output, and therefore, increases inequality. In 19th century, the gap between the profits to capital and labour income kept increasing. By analyzing the industrial capitalism, Marx stated that, to maximize profit and to prevail in the competition, bourgeoisie need to accumulate capital by exploiting workers and transforming their surplus to capital, which accelerates the concentration and increases the inequality.

Both Ricardo and Mark were quite pessimistic about the the issue of inequality. However, in the middle of 20th century, people witnessed continuing technological progress and evidence from available data showing a reduction in inequality in the U.S., which brought some new opinions to the issue of inequality. Kuznets (1955) suggests that economic development could automatically reduce inequality. The reason is that at the beginning of development or industrialization only a small fraction of population can benefit, while in the advanced phase of it, more and more people can access the benefit brought by the development and as a result, inequality decreases by itself, which is known as the Kuznets curve.

Unfortunately, in the next half of the century, even in the U.S., the most developed country in the world, inequality has also increased significantly. Piketty et al. (2014) believe that the fundamental driving force for this divergence in the distribution of wealth is that the rate return to capital is large than the rate of economic growth. In this situation, the inherited wealth grows faster than the income from labour and the capital concentration level will be higher.

1.1.1.1 Intergenerational Transfer

A very critical link on the chain of the work of Piketty et al. (2014) is that wealth is transferred from one generation to the other, to which this thesis is also related. The inequality of one generation is not independent from the previous one. On the individual level, a person's welling being is not only determined by his or her personal ability, choice, effort or luck, but also highly determined by his or her parents' status, as stated by Becker & Tomes (1979). Besides direct bequest, a very important channel through which parents' status could affect their children is the investment in the human capital. Becker (1975) notices that children who have larger inheritance have better access to investment in human capital. Therefore, the distribution of education depends on the distribution of earnings of parents, shown by Loury (1981). On the macroeconomics level, Galor & Zeira (1993) shows how the distribution of wealth is transmitted among generations. The two theoretical models in this thesis are built on the base of this transmitting mechanism.

1.1.2 The Skill Premium

Since the skill premium in the labour market is one of the main sources of income inequality, understand the determinants and the dynamics of the skill premium could offer some implication for the answer to the question raised in this thesis. The literature on the skill premium is vast and views varies.

Technological change, is the most discussed explanation for the change of skill premium, as also in this theis. Acemoglu (1998*a*) states that technological change could increase wage inequality directly and could also increase it indirectly by changing the structure of the labour market.Particularly, Acemoglu (1998*b*) and Galor & Moav (2000) analyze the skill premium with endogenous technological change. However, unlike this paper, most papers focus on the effect of technological change on the demand side of the labour market.

Institutional differences are used to explain the differences of the skill premium. For example, Blau & Kahn (1996) shows that compared with other OECD countries, in the U.S., the rate of unionization is low and the wage-setting mechanisms is decentralized. This leads to a relatively low unskilled wage and high skill premium. Acemoglu (2003*a*) suggests that relatively high minimum wage in European labour markets compresses wage inequality and also encourage the upgrading of unskill-biased technology, which reduces the wage inequality further.

International trade, as discussed in Wood (1995) and Acemoglu (2003*b*) reduces the skill premium. International trade encourages American firms focus on skill-intensive industries, which increases the income of skilled workers in the U.S. Moreover, as suggested by Acemoglu (2003*b*), international trade also encourages American firms to upgrade productivity for skilled workers, which increases their wage further. Similarly, Feenstra & Hanson (1996) provide empirical evidence which shows that outsourcing keeps the skill premium high in the US.

Immigrants could change the share of skilled/unskilled workers in the domestic labour market, and affect the skill premium. Therefore, there are economists, Card (2009) for example, examining the effects of immigration on wage inequality in the U.S. Immigrants affect the skill premium through the supply side of the labour market, which is related to the present paper.

More generally, there also exists vast literature discussing the determinants of income inequality. For example, Piketty & Saez (2003) show that the progressive income tax in the last century has reduced inequality. Guvenen et al. (2014) also provide an explanation for the negative correlation between the progressive labour income taxation and wage inequality, which is that progressive tax compresses the after-wage inequality and reduce the incentives to accumulate human capital, which in turn reduces the before-tax dispersion of wages. Greenwood et al. (2014) show that the assortative mating in marriage market increases income inequality, while the high level of married female labour-force participation reduces income inequality. Besides, data from Piketty & Saez (2003) also shows that inequality dropped during the Great Depression and World War II. A possible reason is that these shocks had a stronger impact on capital income than labour income. Moreover, Fallah et al. (2011) analyze the relation between geography and inequality, and find that those US metropolitan areas with greater market access have stronger outcomes for those at the top of the wage distribution.

1.2 The Structure of the Thesis

The rest part of the thesis is organized as following. In Chapter 2, I analyze how technological change affects the skill premium and inequality in the circumstance with the dynamic distribution of wealth. In Chapter 3, based on the theoretical model built in the previous chapter, I examine the effects of the financial policies for higher education on the skill premium and the In Chapter 4, I endogenize the education sector in the model of the first paper and analyze the determinants of the structure of the higher education system and the effects of this structure on the economic growth and inequality.

1.2.1 The Technological change and the Skill Premium

I study the skill premium with a new channel, the dynamic distribution of wealth, through which exogenous shocks could affect the skill premium. With this mechanism, this paper could offer some explanations for both the cross-country difference in the skill premium and the evolution over time of it. I focus on the effects of technological change. Technological change, skill-biased or unskill-biased, affects the skill premium directly by affecting the productivity and the demand for skilled and unskilled labour. Moreover, the distribution of income affects future cohorts' supply of skilled and unskilled labour. Therefore, technological change can also affect the skill premium and the inequality indirectly in the long-run, since it affects market wages and the transition of the distribution of income. The unskill-biased technological change at the beginning of the 20th century caused the decline in the skill premium in the first half of last century. The sustaining skill-biased technological change has continued to increase the skill premium since the midpoint of last century. However, this paper predicts that in the long-run, skill-biased technological change has an indirect dampening effect on the skill premium, which implies that skill-biased technological change could generate a Kuznets curve of the skill premium.

1.2.2 The Financial Policies for Higher Education

In the second paper, based on the theoretical model built in Chapter 2, I examine the effects of the financial policies for higher education on the skill premium and the inequality. The model is parameterized to the U.S. as the benchmark economy, which has an egalitarian steady state. With the implication from the previous chapter, that the high cost of education and high borrowing interest rate are both necessary but not sufficient conditions for the polarized steady state, the model in this chapter predicts that, firstly, the introducing of free higher education only reduces the level of wealth of everyone in the steady state, but does not affect the inequality; secondly, reducing the interest rate of student loan could not affect the inequality.

1.2.3 The Structure of Higher Education System

In this chapter, I analyze the determinants of the structure of the higher education system and the effects of this structure on the economic growth. The results of this paper indicate that a country with a larger population and a larger proportion of high-ability agents tends to have a diversified system, which consists of a mix of institutions that differ in quality. Otherwise, the country tends to have a unified system, which has very small variance in the quality of universities. Compared with the unified system, the diversified system increases the aggregate productivity. In short run, the diversified system increases inequality, which is, nevertheless, a Pareto improvement. In the long run, by affecting the transition of the distribution of wealth, transforming to a diversified system could eliminate the poverty trap in a relatively poor economy. However, in a relatively rich economy, if the personal ability and wealth are correlated, a diversified system could increase the inequality and reduce the intergenerational mobility at the same time.

Chapter 2

Inequality, Technological Change, and the Dynamics of the Skill Premium

In this paper, I study different types of technological changes as explanations for the U-shape evolution of the skill premium observed in the U.S., throughout the 20th century. Technological change, skill-biased or unskill-biased, affects the skill premium directly by affecting the productivity and the demand for skilled and unskilled labour. Moreover, the distribution of income affects future cohorts' supply of skilled and unskilled labour. Therefore, technological change can also affect the skill premium and the inequality indirectly in the long-run, since it affects market wages and the transition of the distribution of income. The unskill-biased technological change at the beginning of the 20th century caused the decline in the skill premium in the first half of last century. The sustaining skill-biased technological change has continued to increase the skill premium since the midpoint of last century. However, this paper predicts that in the long-run, skill-biased technological change has an indirect dampening effect on the skill premium, which implies that skill-biased technological change could generate a Kuznets curve of the skill premium.

2.1 Introduction

There has been renewed interest in the issue of the inequality of income and its dynamics (e.g. Piketty et al. (2014)). Empirical evidence shows that wage inequality, especially the skill premium in the labour market is one of the main sources of income inequality.[1] Technological change is widely considered to govern the evolution of the skill premium,

[1] See, for example, Kijima (2006), and Lustig et al. (2013).

Table 2.1: Mincerian Mean Rate of Return(Source: Psacharopoulos (1994))

Country	Mincerian coefficient
Low income ($610 or less)	11.2
Lower middle income (to $2,449)	11.7
Upper middle income (to $7,619)	7.8
High income ($7,620 or more)	6.6
World	10.1

but in most of the existing literature, technological change only affects the skill premium directly by shifting the demand for skilled and unskilled workers. In this paper, by taking into consideration credit market imperfections, I allow for the possibility that technological change also affects the skill premium through the supply side of the labour market. With imperfect capital markets, the distribution of wealth affects investments in human capital and thus, the supply of different skills in the labour market. This in turn affects the skill premium and therefore, the distribution of wealth of the next generation. In an OLG model I analyze the long-run interaction between the skill premium and the distribution of wealth. Crucially, when technological change affects the skill premium directly, it also affects the transition of the distribution of wealth, thereby changing the supply of skills in future periods. Considering this subtle distributional effect yields richer dynamics of the skill premium. Moreover, this paper examines the effects of two types of technological changes on the skill premium: unskill-biased and skill-biased technological changes, and predicts that skill-biased technological change increases skill premium directly, but decreases it indirectly in the long-run.

Indeed, this model can provide some explanations for both cross-country and over-time patterns of the skill premium. As shown in Table 2.1, generally, the skill premium is larger in poorer countries. It is well known that this is easily explained by a static version of the model, which predicts that an economy with a higher cost of education, more severe credit market imperfection, and lower average income will have a larger skill premium. However, the skill premium does evolve over time. As in Table 2.1, stated by Goldin & Katz (2007), the skill premium in the U.S. experienced a non-monotonic evolution in the 20th century. The skill premium declined in the first half of the century, and then it increased dramatically in the second half of the century, except the decline which occured in the 1970.[2] The dynamic version of the model provided in this paper can explain this evolution as a result of the unskill-biased technological change in the first half of the 20th century, the skill-biased technological change since the midpoint of the century, and the increased financial aid for college from the government in the 1970s.

[2]A Similar U-shaped pattern in the same period can be found in the data of wage of craftsmen relative to that of labours in England (Clark (2005)).

TABLE 2.2: College/Non-College Log Relative Wages in the U.S.(Source: Goldin & Katz (2007))

Years	100 * Annual Log Changes
1915-1940	-0.56
1940-1950	-1.68
1950-1960	0.83
1960-1970	0.69
1970-1980	-0.74
1980-1990	1.51
1990-2000	0.58
1980-1990	0.50

This paper analyzes a labour market, which has endogenous demand and endogenous supply of two types of workers: skilled and unskilled. Because of credit market imperfection, individuals can only borrow a limited amount in order to finance their investment in education, as in other inequality and growth models (Galor & Zeira (1993); Banerjee & Newman (1993)). Education investment thus depends on the distribution of endowments, which in turn, depends on past wages. Moreover, firms' choice of production technology is endogenous as well, and depends not only on the access to a technology, but also on other factor endowments of the economy. For example, given access to the same technology, firms from a skilled-labour abundant country and firms from an unskilled-labour abundant country will choose different technologies. The choice of technology is modeled using the idea of a technology frontier proposed by Caselli & Coleman (2006). Caselli and Coleman define the choice of technology as the choice of productivities of different types of workers. In this model, firms hire skilled and unskilled workers and choose productivities for them simultaneously. The technology frontier is the set of all non-dominated feasible technology choices, from which a firm in a certain country can choose. Technological change can be viewed as the shifting of the technology frontier. Skill-biased technological change, for example the invention of the computer, makes it cheaper for firms to increase productivity for skilled workers. Unskill-biased technological change, for example the invention of the assembly line, makes it cheaper for firms to increase productivity for unskilled workers. They can be considered as the expansion of the technology frontier in different dimensions.

In each period, individuals decide on their education and become skilled or unskilled workers. Firms individually choose their production technology from a set of all feasible technologies and hire factors given their rental rates. Given the distribution of wealth, the static equilibrium is an allocation of workers, their education choices, and firms' choice of production technology that clears the labour market. The analysis of the static equilibrium generates a first result that an economy tends to have a larger skill premium if the cost of education is higher, the credit market imperfection is more severe, the average wealth is higher, and it is cheaper to increase productivity for skilled workers.

In the long-run, wages affect the wealth and bequests of individuals, and the skill premium interacts with the distribution of wealth. To determine the long-run dynamics of the model, I solve for a steady state which can only be one of two different cases. In the first type of steady state, the egalitarian steady state, the wealth of every individual converges to the same level, the skill premium is small, credit market imperfections play no role, and individuals are indifferent in regards to becoming skilled or not. In the other type of steady state, the polarized steady state, there are two different long-run wealth levels, the skill premium is large and credit market imperfections affect the supply of skills. Which steady state is attained is determined by the initial status of the economy. Generally, an economy with less severe credit market imperfections and cheaper productivity of unskilled workers has an egalitarian steady state.

The exogenous technological change affects the transition to the steady state. When technological change affects the skill premium directly in the current period, it also affects the transition of the distribution of wealth and therefore, the skill premium in the long-run. Unskill-biased technological change decreases the skill premium directly, and if the change is large enough, it also causes the distribution of wealth to transit to an egalitarian steady state. The reason is that unskill-biased technological change makes it cheaper to increase productivity for unskilled workers and expands the technology frontier. As a result, the real wages of both skilled and unskilled workers are increased. Therefore, unskilled workers cannot afford education but they accumulate wealth. Some of their children will gain an education and become skilled. An increase in the supply of skilled workers decreases the skill premium further and the distribution of wealth transit to an egalitarian steady state where every agent has the same wealth. However, the direct and indirect effects of skill-biased technological change on the skill premium are different. Skill-biased technological change increases the skill premium directly because it encourages firms to increase productivity for skilled workers. However, this change also expands the technology frontier and increases the real wages of skilled and unskilled workers. If the skill-biased technological change is large enough, the incomes of unskilled workers converge to a higher level in the next period, which increases the supply of skilled workers as fewer individuals are credit constrained. The distribution of wealth transits to an egalitarian steady state and has a dampening effect on the skill premium. Therefore, a large enough skill-biased technological change generates a Kuznets curve of the skill premium: it increases the skill premium when it happens, but also causes the distribution of wealth to follow an egalitarian transition; when it stops, the egalitarian transition decreases the skill premium.

The previous analysis provides some explanations for the non-monotonic evolution throughout the 20th century. The technological change at the beginning of the 20th century was unskill-biased. Mass production and assembly lines replaced skilled workers and broke down the production process into a series of elementary tasks that could be performed by unskilled workers. This change encouraged firms to increase productivity for unskilled workers, which increased the unskilled wage and decreased the skill premium. According to the previous analysis, both the direct and the indirect effects of unskill-biased technological change decrease the skill premium. This explains the decline in the skill premium in the first half of the 20th century. However, technological change has been skill-biased since the midpoint of last century. The new technology, computers and automatons for example, replaced unskilled workers and encouraged firms to increase productivity for skilled workers. Autor et al. (1998) suggest that the growth of relative demand for skilled workers, which could be largely explained by the spread of computer technology, was still rapid in 1995, which implies that the skill-biased technological change was still in progress at the end of last century. The increase in the skill premium in the second half of the 20th century was a result of the direct effect of the sustaining skill-biased technological change. Moreover, we can predict that if the current skill-biased technological change is large enough, when it stops, the skill premium will decrease since the distribution of wealth will follow an egalitarian transition.

This paper differs from previous literature in the following respects. By introducing the endogenous supply of workers and credit market imperfection into Caselli & Coleman (2006) model, it analyzes the dynamic distribution of wealth as a new channel, through which technological change affects the skill premium. The dynamic model allows the skill premium and the distribution of wealth to interact with each other. Firstly, technological change can not only affect the skill premium through affecting the demand side of the labour market, i.e. affecting the hiring choice of producers, but also affect the supply side of the labour market by affecting the distribution of wealth. Secondly, the model can analyze both the short-run and the long-run effects of technological change on the skill premium. Moreover, compared with Galor & Zeira (1993), the endogenous technology allows us to analyze how the choice of technology in an economy is affected by the distribution of wealth, credit market imperfection and other factors, besides the access to technology.

2.1.1 Literature Review

The existing literature offers several explanations for the difference of the skill premium across countries. Perhaps most significantly, Acemoglu (1998*a*) states that technological change could increase wage inequality directly and could also increase it indirectly by changing the structure of the labour market. Blau & Kahn (1996) and Acemoglu (2003*a*) use institutional differences to explain the differences of the skill premium. Specifically, Acemoglu (2003*a*) finds that European labour market institutions compress wage inequality and also encourage the upgrading of unskill-biased technology, which reduces the wage inequality further. Moreover, since income inequality can be largely explained by the skill premium, this paper is also related to the literature on financial market imperfections, inequality and growth, emphasizing the role of wealth distribution in determining the supply of skilled labour. For example, Beck et al. (2005), find that financial development reduces income inequality. Gall et al. (2014) also discuss the effect of credit market imperfection on FDI and inequality. Gregorio & Lee (2002) indicate that higher educational attainment and more equal distribution of education reduce the income inequality. Furthermore, Guvenen et al. (2014) state that labour income taxation affects wage inequality. This literature tends to abstract from technological change.

Skill-biased technological change is widely used to explain the evolution of the labour market of the U.S. for example, as in Autor & Dorn (2013). Particularly, Acemoglu (1998*b*) and Galor & Moav (2000) analyze the skill premium with endogenous technological change. However, unlike this paper, most papers focus on the effect of technological change on the demand side of the labour market. For example, Acemoglu (1998*b*) argues that an exogenous increase in the supply of college educated students will encourage the inteventions that complement skill and increase the long-run demand for skilled workers and the skill premium. In contrast, this paper analyze how technological shocks affect the dynamics of the distribution of wealth and then affect the lon-run supply of skill and the skill premium. Canidio (2017) also analyzes the effect of technology on the long-run inequality, which is very similar this chapter. However, it considers cost of education as endogenous and generates more complicated steady states. This paper shares a similar argument with Galor & Moav (2000) specifically that the expansion of financial aid for college can explain the fall of the skill premium in the 1970s. There are, of course, other factors that may affect wage premium and inequality: e.g. international trade, as discussed in Wood (1995) and Acemoglu (2003*b*). Other studies, for example, Card (2009) examine the effects of immigration on wage inequality in the U.S.

2.1.2 Overview

The remainder of this paper is organized as follows: in Section 4.2, I set up the basic model. Section 2.3 examines the static equilibrium. Section 4.4 discusses the dynamic version of the model and its steady states. Section 2.5 offers some concluding remarks.

2.2 The Model

We consider a small open economy. The economy is populated by a continuum of agents with a mass of one, which is constant over time.

There is a continuum of competitive firms, which hire three factors to produce: capital, skilled workers and unskilled workers. Skilled and unskilled workers are substitutes to each other. Firms can also choose productivities for both types of workers simultaneously, from the set of all feasible technology choices.

A single good is produced by firms and consumed by agents. In this small open economy, the price of the good is equal to the world market price, which is exogenous and normalized to 1. Agents can save and borrow in order to finance their education, and firms can borrow to finance production in an international credit market. Both the saving and borrowing world market interest rates are also exogenous and constant over time.

2.2.1 Imperfect Credit Market

The credit market is imperfect, in that there is a spread between the risk free saving and borrowing interest rates denoted by r and i, respectively. Set the spread be denoted by $\beta \geq 0$, so that $1 + i = \beta(1 + r)$. This assumption is borrowed from that of Galor & Zeira (1993), which is a tried and tested way in incorporating borrowing constraints. Borrowing constraints are more severe for households than for firms. For simplicity, I assume that firms can borrow at rate r.

2.2.2 Households

Each agent lives for two periods in overlapping generations: young and old. In period t, a young agent receives bequest $x_{a,t}$ from her parent and decides on her education: she has the choice either to invest in education or not. The cost of investing in human capital is h. When old, agents work as skilled or unskilled workers, depending on their education level, and earn skilled or unskilled wages $w_{u,t}$ or $w_{s,t}$. Agents only consume and leave bequests to children in the second period of their life. This framework closely follows Galor & Zeira (1993). Agent a receives lifetime utility $u_{a,t}$ from both consumption $c_{a,t}$ and the bequest $x_{a,t}$:

$$u_{a,t} = \theta \ln c_{a,t} + (1 - \theta) \ln x_{a,t} \tag{2.1}$$

The parameter $\theta \in (0, 1)$ determines the saving rate. The optimal choice of $c_{a,t}$ and $x_{a,t}$ for an individual a in the second period of her life maximizes $u_{a,t}(c_{a,t}, x_{a,t})$ subject to the budget constraint:

$$c_{a,t} + x_{a,t} = \pi_{a,t} \tag{2.2}$$

where $\pi_{a,t}$ is lifetime income. Therefore the individual utility only depends on lifetime income $\pi_{a,t}$ and is strictly increasing in the income. As a result, the agent's problem is to maximize lifetime income by deciding on her education.

An agent working as an unskilled worker without investing has income

$$\pi_{a,t} = w_{u,t} + x_{a,t-1}(1 + r) \tag{2.3}$$

An agent with bequest $x_{i,t-1} \geq h$, who invests in human capital, obtains:

$$\pi_{a,t} = w_{s,t} + (x_{a,t-1} - h)(1 + r) \tag{2.4}$$

An agent, who receives bequest $x_{i,t-1} < h$ and invests, needs to borrow and has income:

$$\pi_{a,t} = w_{s,t} - (h - x_{a,t-1})(1 + i) \tag{2.5}$$

All the agents with bequest $x_{i,t-1} \geq h$ prefer to invest and work as skilled workers if the following assumption holds:

$$w_{s,t} - h(1 + r) \geq w_{u,t} \tag{2.6}$$

If this assumption is violated, all individuals work as unskilled and there is an excess supply of unskilled workers. This drives the unskilled wage down and the skilled wage up until (2.6) is satisfied.

Hence an agent is indifferent between investing and not investing if $\pi_{a,t}(invest) = \pi_{a,t}(not\ invest)$, which pins down an endowment f_t :

$$f_t = \frac{1}{i - r}[w_{u,t} + h(1 + i) - w_{s,t}] \tag{2.7}$$

Therefore, given wages, $w_{s,t}$ and $w_{u,t}$, all agents with endowment greater than f_t will invest in human capital and agents with endowment smaller than f_t will not invest. If the distribution of the bequest in period t is $D_t(x_a)$, then the supply of different workers is:

$$L^S_{u,t}(w_{s,t}, w_{u,t}) = \int_0^{f_t} dD_t(x_a) \tag{2.8}$$

$$L^S_{s,t}(w_{s,t}, w_{u,t}) = \int_{f_t}^{\infty} dD_t(x_a) = 1 - L^S_{u,t}(w_{s,t}, w_{u,t}) \tag{2.9}$$

2.2.3 Firms

In period t, a representative firm generates output using the production function proposed by Caselli & Coleman (2006):

$$y_t = k_t^{\alpha} \left[(A_{u,t}L_{u,t})^{\sigma} + (A_{s,t}L_{s,t})^{\sigma}\right]^{(1-\alpha)/\sigma} \tag{2.10}$$

Three factors are used to produce: capital k_t, the ratio of unskilled workers $L_{u,t}$, and the ratio of skilled workers $L_{s,t}$, with $L_{u,t}$+ $L_{s,t} = 1$. $A_{u,t}$ and $A_{s,t}$ are the productivities

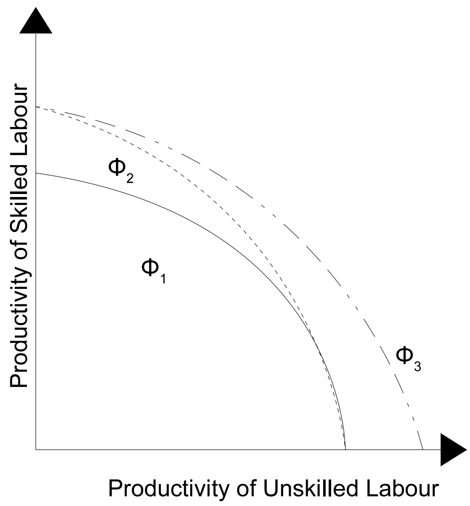


FIGURE 2.1: Technology Frontier

of two types of workers $\alpha \in (0, 1)$. $1/(1-\sigma)$ is the elasticity of substitution between skilled and unskilled workers and $\sigma \in (0, 1)$.

To maximize its profit, taking wages $w_{s,t}$, $w_{u,t}$ and borrowing interest rate r as given, a representative firm optimally chooses factor inputs k_t, $L_{u,t}$ and $L_{s,t}$. A firm also chooses the production technology $(A_{u,t},\ A_{s,t})$ from a set of feasible technology choices in that period. This set is given by:

$$\delta \left(A_{s,t}\right)^{\omega} + \gamma \left(A_{u,t}\right)^{\omega} \leq B \tag{2.11}$$

In Figure 2.1, Φ_1, Φ_2 and Φ_3 illustrate three different technology frontiers, i.e. the sets of all non-dominated $(A_{u,t},\ A_{s,t})$ pairs. Parameters ω, δ, λ and B are exogenous and strictly positive. Parameters ω and γ measure the trade-off between the productivities of skilled and unskilled workers. Both $\mathcal{B}$, δ and γ differ across economies while ω is identical for all the economies.[3]

The parameter B denotes the level of the technology frontier of a country, which represents total factor productivity. Therefore, an increase in B represents an unbiased (balanced) technological change, shown as a shift of the frontier from Φ_1 to Φ_3 in Figure 2.1.

The parameters δ and γ denote the relative prices of productivity of unskilled workers and productivity of skilled workers. A decrease in δ represents skill-biased technological

[3] Caselli & Coleman (2006) prove that the assumption $\omega > \sigma/(1-\sigma)$ needs to hold to rule out the situation that the supply of labour is mixed but some firms always choose to set $A_{u,t} = 0$ and only hire skilled workers and other firms do the opposite.

change. For example, one can argue that the invention of the computer made it less costly to increase the productivity of skilled workers. This can be represented by a decreased δ in the frontier, and a shift from Φ_1 to Φ_2 in Figure 2.1. Then, given this change, firms applied computers in production, which can be shown as firms adjusting their choices of $(A_{u,t},A_{s,t})$. Similarly, a decrease in γ represents unskill-biased technological change. An example of this type of technological change is the invention of the assembly line, which made it less costly to increase the productivity of skilled workers. An unskill-biased technological change can be shown as a shift from Φ_2 to Φ_3 in Figure 2.1.

A firm chooses technology and factor input to solve:

$$\underset{k_t,L_{u,t},L_{s,t},A_{u,t},A_{s,t}}{Max} k_t^{\alpha}\left[(A_{u,t}L_{u,t})^{\sigma}+(A_{s,t}L_{s,t})^{\sigma}\right]^{(1-\alpha)/\sigma}-rk_t-w_{s,t}L_{s,t}-w_{u,t}L_{s,t}$$

(2.12)

subject to:

$$\delta\left(A_{s,t}\right)^{\omega}+\gamma\left(A_{u,t}\right)^{\omega}\leq B$$

The first order conditions for problem (2.2.3) include:

$$\frac{\partial y_t}{\partial k_t} = r \qquad (2.13)$$

$$\frac{\partial y_y}{\partial L_{u,t}} = w_{u,t} \qquad (2.14)$$

$$\frac{\partial y_t}{\partial L_{s,t}} = w_{s,t} \qquad (2.15)$$

$$\frac{\partial y_t}{\partial A_{u,t}} = \lambda\ \gamma\omega(A_{u,t})^{\omega-1} \qquad (2.16)$$

$$\frac{\partial y_t}{\partial A_{s,t}} = \lambda\ \delta\omega(A_{s,t})^{\omega-1} \qquad (2.17)$$

λ is the Lagrangian multiplier. Combining the (2.14) and (2.15) leads to the following equations:

$$\frac{L_{s,t}^D}{L_{u,t}^D} = \left(\frac{A_{s,t}}{A_{u,t}}\right)^{\frac{\sigma}{1-\sigma}} \cdot \left(\frac{w_{s,t}}{w_{u,t}}\right)^{\frac{1}{\sigma-1}} \tag{2.18}$$

Equation (2.18) implies that hiring decision depends on both relative productivity and relative wage. Taken $\frac{A_{s,t}}{A_{u,t}}$ as constant, (2.18) shows firm's relative demand for different workers according to the relative wage, in the situation that technology is not adjustable for firm. Combining the (2.16) (2.17) and (2.18) yields:

$$\frac{A_{s,t}}{A_{u,t}} = \left(\frac{\gamma}{\delta}\right)^{\frac{1-\sigma}{\omega-\sigma-\omega\sigma}} \left(\frac{w_{s,t}}{w_{u,t}}\right)^{\frac{\sigma}{\omega\sigma-(\omega-\sigma)}} \tag{2.19}$$

$$\frac{L_{s,t}^D}{L_{u,t}^D} = \left(\frac{\gamma}{\delta}\right)^{\frac{\sigma}{\omega-\sigma-\omega\sigma}} \cdot \left(\frac{w_{s,t}}{w_{u,t}}\right)^{\frac{\omega-\sigma}{\omega\sigma-(\omega-\sigma)}} \tag{2.20}$$

Equation (2.19) shows firms' relative technology choice for different workers according to the relative wage. It implies that if the wage for skilled workers is relatively higher, the relative productivity of skilled workers will be lower. Equation (2.20) shows firms' relative demand for different workers according to the relative wage, when it can adjust technology along the technology frontier.

The assumption $\omega > \sigma/(1-\sigma)$ implies that $\left|\frac{1}{\sigma-1}\right| < \left|\frac{\omega-\sigma}{\omega\sigma-(\omega-\sigma)}\right|$. By comparing (2.18) with (2.20), we have:

Lemma 2.1. *Optimal technology choice of firms results in a more price elastic relative labour demand.*

Lemma 1 says that if a firm can adjust its technology, relative wage does not only effect firm's relative demand for labours directly, but also effects the relative demand for workers through effecting firm's choice of technology. For example, if the relative wage of skilled workers becomes lower, a firm will increase the relative demand for skilled workers because hiring them costs less. Meanwhile, it will increase the productivity of skilled workers, which also leads the firm to hire more skilled workers because they are more productive.

2.3 Static Equilibrium

In each period, both the price for the good and the interest rates are exogenous, and the static equilibrium is an allocation of workers and investment choices that clears the labour market. Formally,

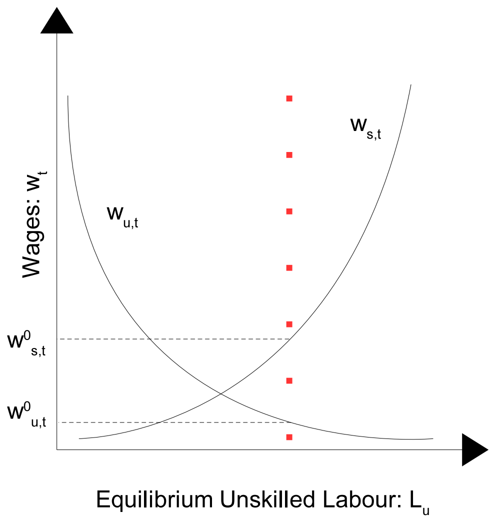


FIGURE 2.2: Equilibrium Labour Ratio and Wages

Definition 2.2. *A static equilibrium is an allocation of factors* $(L_{u,t}, L_{s,t}, k_t)$*, a technology* $(A_{u,t}, A_{s,t})$*, and prices* $(w_{u,t}, w_{s,t})$*, such that in period* t*, for given distribution of endowments* $D_t(x_a)$ *and other parameters* $(\alpha, \sigma, h, i, r, \theta, \mathcal{B}, \delta, \gamma, \omega)$*:*

1. $(A_{u,t}, A_{s,t})$ *satisfies feasibility: the technology frontier (2.11) holds;*

2. (2.8) and (2.9), yield labour supply $L^S_{u,t}$ *and* $L^S_{s,t}$*, so that the utility of each agent is maximized;*

3. $(L^D_{u,t}, L^D_{s,t}, A_{u,t}, A_{s,t}, k_t)$ *solve the problem of the representative firm (2.2.3);*

4. the labour market clears:

$$L^S_{u,t}(w_{u,t}, w_{s,t}) = L^D_{u,t}(w_{u,t}) \tag{2.21}$$

$$L^S_{s,t}(w_{u,t}, w_{s,t}) = L^D_{s,t}(w_{s,t}) \tag{2.22}$$

To examine the static equilibrium, firstly let's imagine that the supply of skills is exogenous. Then, firms maximize their profit by deciding on technology, given the supply of each type of worker. The wage for each type of worker will be equal to the marginal productivity of that type. Hence as illustrated in Figure 2.2, given any possible unskilled labour ratio $L_{u,t}$, there is a corresponding pair of wages.

Lemma 2.3. *As the supply of unskilled labour* $L_{u,t}$ *increases from 0 to 1, unskilled wage* $w_{u,t}$ *decreases from infinity, and skilled wage* $w_{s,t}$ *increases to infinity.*

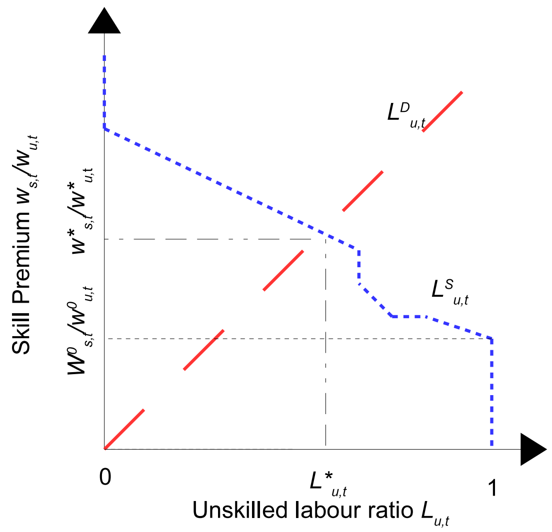


FIGURE 2.3: Supply and Demand of workers and Static Equilibrium

The proof of the lemma is in the appendix. Then I take the endogenous supply of skills into account and we can find that not all the values of $L_{u,t}$ are feasible in the static equilibrium. The reason is that, as illustrated in Figure 2.2, on the left-hand-side of the dotted line, compared with $w_{u,t}$, $w_{s,t}$ is not large enough to ensure that the condition (2.6) holds, which means in this situation the return to the education is very low so that even the agents who do not need to borrow will not invest in education and the supply of skilled workers is equal to zero. As a result, the equilibrium always happens on the right-hand-side of the dotted line. Therefore, we have the following lemma and the proof is in the appendix.

Lemma 2.4. $w_{s,t} - h(1+r) \geq w_{u,t}$ *always holds in the equilibrium.*

If $w_{s,t} - h(1+r) < w_{u,t}$, then it was not profitable for the rich group to invest in education. The supply of unskilled workers would be 1. According to the previous lemma, the unskilled wage $w_{u,t}$ would be low, while the skilled wage $w_{s,t}$ will be infinite, which implies $w_{s,t} - h(1+r) \geq w_{u,t}$. Therefore, Lemma 2.4 is true.

Lemma 2.5. *Only one unique pair* $(w^0_{u,t}, w^0_{s,t})$ *make agents who do not need to borrow indifferent between investing or not, i.e. make*

$$w^0_{s,t} - h(1+r) = w^0_{u,t} \tag{2.23}$$

hold, and the static equilibrium skill premium $\frac{w^*_{s,t}}{w^*_{u,t}} \geq \frac{w^0_{s,t}}{w^0_{u,t}}$.

Figure 2.3 illustrates the static equilibrium. In Figure 2.3, as illustrated by the dashed line, the demand for unskilled workers, $L^D_{u,t}$, is strictly monotonically increasing as the skill premium increases. The supply curve of unskilled workers, illustrated by the dotted line, consists of three parts. The first part corresponds to $\frac{w_{s,t}}{w_{u,t}} < \frac{w^0_{s,t}}{w^0_{u,t}}$, i.e. the skill premium is so low that it is not profitable for anyone to invest, and the supply of unskilled workers is $L^S_{u,t} = 1$. The third part corresponds to a very high skill premium, so that f_t is smaller than the smallest bequest, and every agent is willing to invest, which means that the supply of skilled workers is $L^S_{u,t} = 0$. Between these two parts, $L^S_{u,t}$ decreases in the skill premium. The intersection of $L^D_{u,t}$ and $L^S_{u,t}$ defines the unique static equilibrium labour proportions $(L^*_{u,t}, L^*_{s,t})$ and wages $(w^*_{u,t}, w^*_{s,t})$. The second part of the supply curve is downward sloping but also contains horizontal and vertical segments. This is because if there is no agent with endowment between two levels of the threshold, the supply curve is vertical. Conversely, if a positive mass of agents have the same amount of endowment, the supply curve is horizontal.

According to the previous analysis, the supply of the unskilled workers is a decreasing function of the skill premium on the range of $(1, 0)$, and the demand for unskilled workers is an increasing function of the skill premim on the range of $(0, 1)$. Therefore, there exists a unique equilibrium.

There is a special case of the static equilibrium: every agent has the same bequest: $x_{a,t} = x_t, \forall a$. All individuals in this situation are indifferent between being skilled and unskilled. That is, the equilibrium wages $w^*_{u,t}$ and $w^*_{s,t}$ solve $x_t = f_t$, i.e. the bequest just equals the threshold value that is defined in Equation (2.7).

To derive a closed form solution, let us consider the following assumption:

At the beginning of a period t, the endowments follow a uniform distribution on $[M - \varepsilon, M + \varepsilon]$, with $M \in (0, \infty)$ and $\varepsilon \in (0, M)$.

The parameter M represents the average level of bequests and ε represents endowment inequality. If $M - \varepsilon \leq f_t \leq M + \varepsilon$, then $L^S_{u,t} = (f_t - M + \varepsilon)/2\varepsilon$ and $L^S_{s,t} = (M + \varepsilon - f_t)/2\varepsilon$.

The following proposition states the comparative statics of the static equilibrium. This proposition can offer some explanations for the differences of the skill premium across countries. The proof is in the appendix.

Proposition 2.6. *(Comparative Statics) Consider 2 economies Aand B,*

(i) Suppose $h_A > h_B$, then the equilibrium labour ratio $\frac{L_{s,t}}{L_{u,t}}_A \leq \frac{L_{s,t}}{L_{u,t}}_B$, the equilibrium relative productivity $\frac{A_{s,t}}{A_{u,t}}_A \leq \frac{A_{s,t}}{A_{u,t}}_B$, and the equilibrium skill premium $\frac{w_{s,t}}{w_{u,t}}_A \geq \frac{w_{s,t}}{w_{u,t}}_B$, for all $h \in (0, \infty)$.

(ii) Suppose $\beta_A > \beta_B$, then the equilibrium labour ratio $\frac{L_{s,t}}{L_{u,t}}_A \leq \frac{L_{s,t}}{L_{u,t}}_B$, and the equilibrium relative productivity $\frac{A_{s,t}}{A_{u,t}}_A \leq \frac{A_{s,t}}{A_{u,t}}_B$, and the equilibrium skill premium $\frac{w_{s,t}}{w_{u,t}}_A \geq \frac{w_{s,t}}{w_{u,t}}_B$, for all $i \in (r, \infty)$.

(iii) Suppose $M_A > M_B$, then the equilibrium labour ratio $\frac{L_{s,t}}{L_{u,t}}_A \geq \frac{L_{s,t}}{L_{u,t}}_B$, the equilibrium relative productivity $\frac{A_{s,t}}{A_{u,t}}_A \geq \frac{A_{s,t}}{A_{u,t}}_B$, and the equilibrium skill premium $\frac{w_{s,t}}{w_{u,t}}_A \leq \frac{w_{s,t}}{w_{u,t}}_B$, for all $M \in (0, \infty)$.

(iv) Suppose $\gamma_A \geq \gamma_B$ or $\delta_A \leq \delta_B$, then the equilibrium labour ratio $\frac{L_{s,t}}{L_{u,t}}_A \geq \frac{L_{s,t}}{L_{u,t}}_B$, the equilibrium relative productivity $\frac{A_{s,t}}{A_{u,t}}_A \geq \frac{A_{s,t}}{A_{u,t}}_B$, and the equilibrium skill premium $\frac{w_{s,t}}{w_{u,t}}_A \geq \frac{w_{s,t}}{w_{u,t}}_B$.

Proposition 2.6(i) is straightforward. A higher cost of investing makes education available to fewer agents, which reduces the supply of skilled workers and leads to a larger skill premium.

Proposition 2.6(ii) implies that if a country has more severe credit market imperfections, it is likely to have a larger skill premium. If the credit market imperfection is severe, fewer agents can invest in education by borrowing. This leads to a decrease in the supply of skilled workers. A similar argument is discussed by Banerjee & Newman (1993), and Galor & Moav (2000). This argument is supported empirically by Li et al. (1998), who find that there is a positive relation between income inequality and the imperfections of the credit market.

Proposition 2.6(iii) is best interpreted as that in wealthier countries the proportion and productivity of skilled workers are higher, but the skill premium is lower, which means the income is more equal. This is because with more wealth, more agents can afford to invest in education, which leads to an increase in the supply of skilled workers. This result offers an explanation for the positive relation between the skill premium and GDP per capita in most countries, just as shown in Table 2.1. Caselli & Coleman (2006) also offer estimation results, showing that higher-income countries are skilled labour abundant and use skilled labour more efficiently than lower-income countries. Considering the skill premium as an important reason for income inequality, similar evidence can also be found in the work of Lindert & Williamson (1985), who find that the right portion of the Kuznets Curve is more robust, which means that the inequality falls as the per capita income increases at higher levels of development.

Proposition 2.6(iv) states that the skill premium tends to be larger in countries where the skill-biased technology is relatively cheaper. Firms are therefore more willing to increase the relative productivity of skilled workers and hire more of them. The increased relative productivity also leads to a higher skill premium. This argument is similar to Acemoglu (1998*a*) skilled-biased technology increases the wage inequality, independently of whether it leads to a change in the structure of the labour market or not. Autor et al. (1998) also find that demand for college graduates grew more rapidly on average from 1970 to 1995, which can be explained largely by the spread of computer technology. This proposition also indicates the direct effect of technological change: unskill-biased one decreases the skill premium and skill-biased one increases the skill premium.

2.4 The Dynamic Model

In this section, I develop the model to a dynamic version, by taking into account the transition of the distribution of wealth. Wages determine the income of each agent and then the bequest she gives to her child. As a result, in the long-run, the distribution of wealth becomes endogenous as well. To examine the long-run evolution of the economy and its skill premium, I firstly characterize its steady state, and then analyze how the exogenous shocks affect the skill premium in the long-run, by affecting the transition.

2.4.1 Steady State Cases

From the above section, we know that all the agents can be divided into three groups according to their investment decisions: agents from Group I will not invest because their bequest is lower than the threshold; agents from Group II will invest and borrow because their endowments are higher than the threshold but lower than the cost of education; agents from Group III will invest in education without borrowing, because they receive bequests higher than the cost of education. Bequests of agents evolve as follows:

$$x_{a,t} = \begin{cases} (1-\theta)[w_{u,t} + x_{a,t-1}(1+r)], if\, x_{a,t-1} < f_t & (Group\, I) \\ (1-\theta)[w_{s,t} - (h - x_{a,t-1})(1+i)], if\, f_t \leqslant x_{a,t-1} < h & (Group\, II) \\ (1-\theta)[w_{s,t} + (x_{a,t-1} - h)(1+r)], if\, x_{a,t-1} \geqslant h & (Group\, III) \end{cases} \tag{2.24}$$

I suppose that $(1-\theta)(1+r) < 1$ to focus on interesting dynamics, following Galor & Zeira (1993). This assumption rules out the possibility that the incomes of agents in Group I converge to zero or the incomes of agents in Group III diverge. Formally, I define the steady state of the dynamic model as follows:

Definition 2.7. *The steady state is the static equilibrium as defined in Definition 1 which also satisfies the following conditions: in each period, wages* $(w_{u,t}, w_{s,t})$ *are equal to a constant pair* (w_u, w_s)*; for each agent, the bequest is constant over time, i.e.* $x_{a,t} = x_a$.

In each period, by letting $x_t = x_{t-1}$ for each group of agents, we can solve for the following three possible fixed points for the three groups according to (2.24):

$$\overline{x}_{\mathrm{I,t}} = \frac{(1-\theta)w_{u,t}}{1-(1-\theta)(1+r)} \tag{2.25}$$

$$\overline{x}_{\mathrm{II,t}} = \frac{(1-\theta)\left[h\,(1+i) - w_{s,t}\right]}{(1-\theta)(1+i) - 1} \tag{2.26}$$

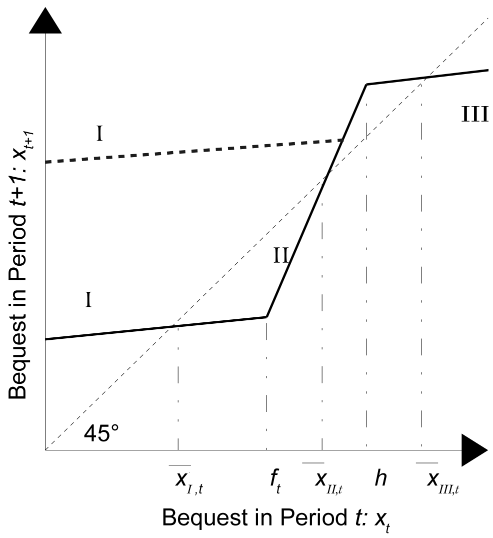


FIGURE 2.4: Transition of the Distribution of Wealth

$$\overline{x}_{\text{III,t}} = \frac{(1-\theta)\left[w_{s,t} - h\,(1+r)\right]}{1-(1-\theta)(1+r)} \tag{2.27}$$

These three possible fixed points determine the dynamics of the distribution of wealth. For example, Figure 2.4 illustrates the dynamics of distribution, i.e. given endowments, how much bequest agents from three group are going to leave to children, when $(1-\theta)(1+i) < 1$. The intercept of the segment representing Group I is governed by the wage of unskilled workers. The intercept of the segment representing Group III is governed by the wage of skilled workers. Thus, the distance between two segments indicates the skill premium. In the dynamic illustrated by the solid line, agents with endowments $x_{\text{t}-1} < f_t$ will be unskilled and their bequests x_t converge to $\overline{x}_{\text{I,t}}$. Agents with endowments $x_{\text{t}-1} \geq \overline{x}_{\text{II}}$ will be skilled and their bequests x_t converge to $\overline{x}_{\text{III,t}}$. Agents with endowments $f_t \leq x_{\text{t}-1} < \overline{x}_{\text{II}}$ will be skilled. But because borrowing is costly, their bequests x_t also converge to the low level $\overline{x}_{\text{I,t}}$. In the next period, the skill premium $\frac{w_{s,t}}{w_{u,t}}$ will be larger since fewer agents can afford education and the supply of skilled worker will be lower. And according to Equation (2.19), in the next period, the relative productivity $\frac{A_{s,t}}{A_{u,t}}$ will decrease, because the supply of unskilled workers is larger and their wage is relatively low. This process is going to be repeated again and again in the following periods, until there exist only two levels of wealth: $\overline{x}_{\text{I}}$ and $\overline{x}_{\text{III}}$. Now no one's wealth is going to evolve anymore. The supply of skilled and unskilled workers will be constant, so is the skill premium. Therefore, the model reaches its steady state, which is not an equal.one.

However, in the dynamic with Group I illustrated by the dash line, the wage of unskilled workers is not very low. Thus, unskilled workers accumulate wealth and the bequests of all the agents converge to $\overline{x}_{\mathrm{I,t}}$. In the next period, the skill premium $\frac{w_{s,t}}{w_{u,t}}$ will be smaller since more agents can afford education and the supply of skilled workers will be larger and the relative productivity $\frac{A_{s,t}}{A_{u,t}}$ will increase. This process is going to be repeated again and again in the following periods, until there exist only one levels of wealth: $\overline{x}_{\mathrm{III}}$. Now no one's wealth is going to evolve any more. The supply of skilled and unskilled workers will be constant, so is the skill premium. Therefore, the model reaches its steady state, which is an equal one. This process could be seen in Figure **?**, which is the results of the simulation of the transition, which shows that as the economy transits to the steady state, the productivity for skilled workers increases, while the productivity for unskilled workers decreases. At the same time, the skilled wage falls and unskilled increases, which leads to a decreasing skill premium.

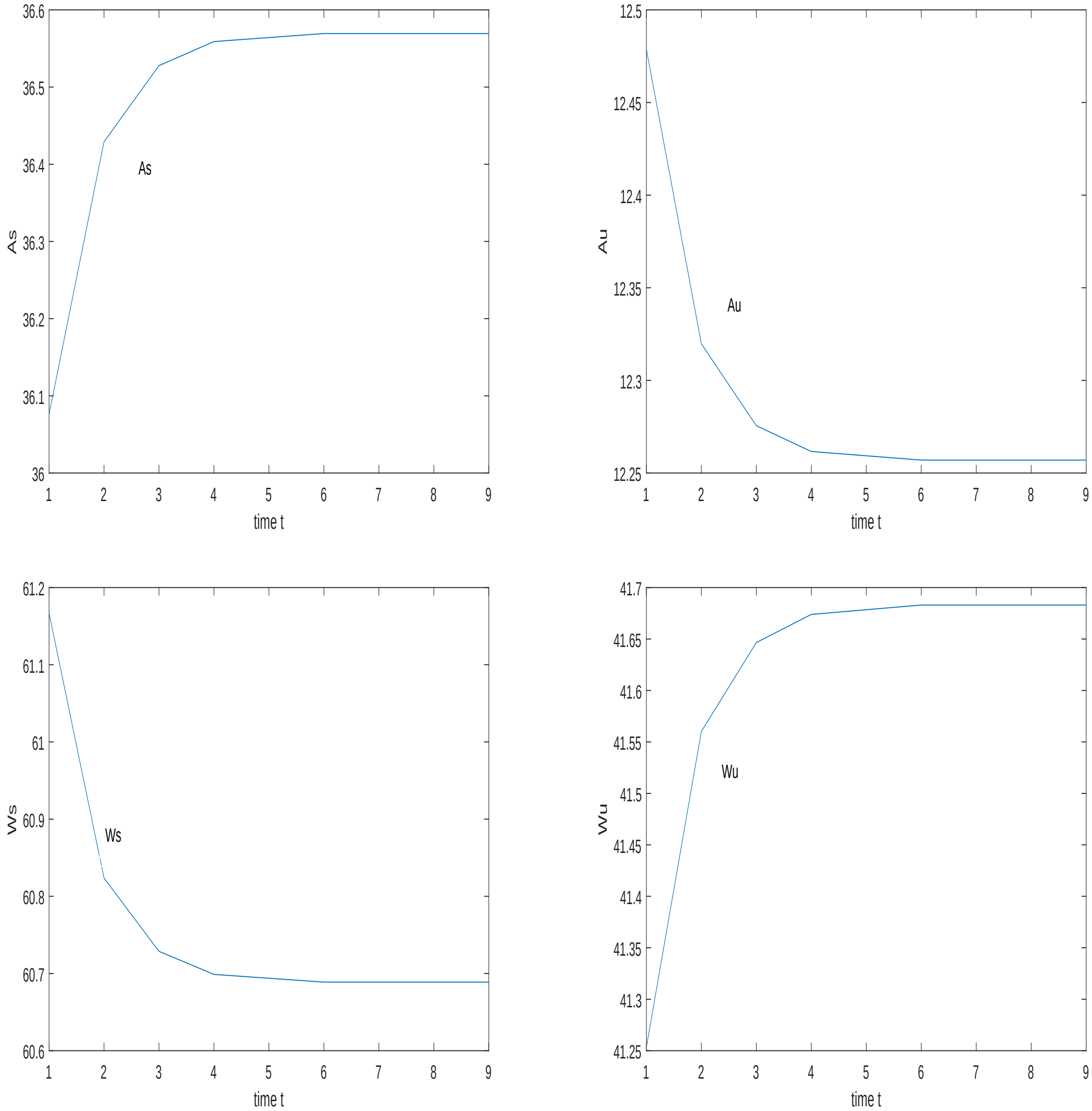


FIGURE 2.5: Transition to a Steady State

From these two examples we can see that the dynamics of this model are very rich. The transition to the steady state is determined by the initial status of the economy, for example, the initial wealth, the technology frontier and the credit market. Therefore, it is difficult to track all the possible transitions. However, the possible types of steady state is limited. We can understand this model by looking at the determinants of the type of the steady state. According to the definition, in a steady state, the bequest of each agent must be equal to $\overline{x}_{\mathrm{I}}$, $\overline{x}_{\mathrm{II}}$ or $\overline{x}_{\mathrm{III}}$, which are the fixed points with the steady state wages.[4] Otherwise, the bequest will still evolve, which violates the definition of the steady state. This logic allows to derive the main result on the dynamics in the following propositions, which do not depend on the initial distribution. The proofs are in the appendix.

Proposition 2.8. *In the steady state,*

if $(1-\theta)(1+i) < 1$, *the model has an egalitarian steady state where all the agents have the same wealth;*

if $(1-\theta)(1+i) > 1$, *the model has two possible cases of steady state: an egalitarian steady state where all the agents have the same wealth and the skill premium is small, or a polarized steady state, where there are two unequal levels of wealth, and the skill premium is large.*

Proposition 2.9. *An economy with less severe credit market imperfection tends to reach an egalitarian steady state; an economy with more severe credit market imperfection tends to reach a polarized steady state.*

Proposition 2.8 states that there are only two possible cases of steady state. One possible case is that each agent has the same wealth and it is indifferent to being skilled or unskilled worker. The other case is that there exist two unequal levels of wealth. The agents in the rich group are skilled while the agents in the poor group are unskilled.

Proposition 2.9 is implied by Proposition 2.8. It states that the credit market imperfection not only affects the static equilibrium skill premium, but also affects the skill premium and the inequality of the economy in the long-run. The economy with a less severe credit market will have a smaller skill premium and an equal distribution of wealth, while the economy with a more severe credit market will have a larger skill premium and an unequal distribution of wealth.

2.4.2 Technological Changes and the Skill Premium in the the 20th Century

In this subsection, I offer an example to show how technological changes affect the skill premium directly and indirectly in the long-run by changing the dynamic distribution

[4] It is not necessary that all these three points exist in the steady state.

of wealth. Furthermore, I argue that the effects of technological changes are possible explanations for the non-monotonic behaviour of the skill premium of the U.S. in the 20th century.

Proposition 2.10. *When the distribution of endowment is continuous and* $(1-\theta)(1+i) > 1$,

there exists a $\overset{\circ}{\gamma}$, *if* $\gamma < \overset{\circ}{\gamma}$, *the model has an egalitarian steady state*;

there exists a $\overset{\circ}{\delta}$, *if* $\delta < \overset{\circ}{\delta}$, *the model has an egalitarian steady state.*

The proof of this proposition is in the appendix. This proposition states how technological change affects the evolution of the skill premium indirectly by affecting the transition of the distribution of wealth. In the long-run, both unskill-biased and skill-biased technological change could cause the distribution of wealth to transit to an egalitarian steady state and decrease the skill premium indirectly, no matter what type of transition the distribution follows before the technological change.[5]

Proposition 2.6 and Proposition 2.10 together state the direct and indirect effects of technological change on the skill premium. Unskill-biased technological change, represented by a decrease in γ, decreases the skill premium immediately, because it encourages firms to increase productivity for unskilled workers. Moreover, if the unskill-biased technological change is large enough, it could also make the distribution of wealth follow an egalitarian transition. In Figure 2.6, this change is indicated by the black arrows. Because the technology frontier is expanded, the real wages of both skilled and unskilled workers are increased. This leads the converging point of the unskilled group $\overline{x}_{\mathrm{I,t}}$ to be higher than the threshold of education f_t, which implies that the agents of the unskilled group will become richer and some of their children will gain an education and become skilled. An increase in the supply of skilled workers decreases the skill premium further and the distribution of wealth transits to an egalitarian steady state where every agent has the same wealth. This indirect effect is indicated by the white arrows in Figure 2.6.

However, the direct and indirect effects of skill-biased technological change on the skill premium are different. Skill-biased technological change, represented by a decrease in δ, increases the skill premium immediately, because it encourages firms to increase productivity for skilled workers. But skill-biased technological change also expands the technology frontier and increases the real wages of skilled and unskilled workers. If the skill-biased technological change is large enough, the income of unskilled workers is high and will converge to a higher level in the next period, which means that the skill-biased technological change makes the distribution of wealth follow an egalitarian transition. In Figure 2.7, this change is indicated by the black arrows. This egalitarian

[5]However, a large γ or δ is not a sufficient condition for a polarized steady state. For example, if the initial endowment of every agent is higher than the cost of education, an egalitarian steady state will be obtained.

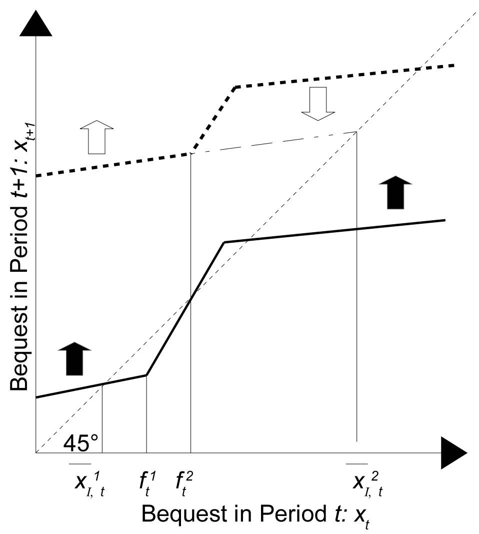


FIGURE 2.6: Unskill-biased Technological Change

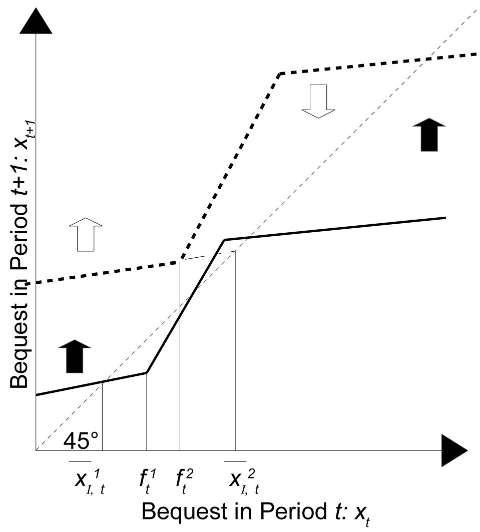


FIGURE 2.7: Skill-biased Technological Change

transition then decreases the skill premium in the long-run, as indicated by the white arrow. Therefore, a large enough skill-biased technological change generates a Kuznets curve in the skill premium: it increases the skill premium when it happens but also makes the distribution of wealth follow an egalitarian transition; when it stops, the egalitarian transition decreases the skill premium, as indicated by the white arrows.

We can offer some explanations for the non-monotonic behaviour of the skill premium of the U.S. in the 20th century. The technological change at the beginning of the 20th century was unskill-biased. Mass production and assembly lines replaced skilled workers and broke down the production process into a series of elementary tasks that could be performed by unskilled workers. This change encouraged firms to increase productivity for unskilled workers, which increased the unskilled wage. According to the previous analysis, both the direct and the indirect effects of unskill-biased technological change decrease the skill premium. As a result, the skill premium kept decreasing in the first half of the 20th century.

The technological change has been skill-biased since the midpoint of the century. The new technology, computers and automatons for example, replaced unskilled workers and encouraged firms to increase productivity for skilled workers. Autor et al. (1998) suggest that the growth of relative demand for skilled workers, which could be largely explained by the spread of computer technology, was still rapid in 1995, which implies that the skill-biased technological change was still in progress at the end of last century. According to the previous analysis, the direct effect of sustaining skill-biased technological change could explain the increase in the skill premium in the second half of the 20th century. Moreover, we can predict that if the current skill-biased technological change is large enough, when it stops, the skill premium will decrease since the distribution of wealth will follow an egalitarian transition.

2.4.3 Financial Aid in the 1960s-1970s

The skill-biased technological change was accelerated in the 1970s, according to Autor et al. (1998). However, there was a decline in the skill premium in the 1970s. A possible reason for this decline is the increased financial aid for college from the government. Government financial aid for higher education increased by a large amount in the U.S. from the late 1960s to the early 1970s (McPherson & Schapiro (1991)). This change reduced the imperfection of the credit market and decreased the gap between saving and borrowing interest rate. As a result, in the short-run the skill premium was decreased, according to Proposition 2.6. Furthermore, according to Proposition 2.9, the reduction in the borrowing interest rate can also cause the transition of the distribution of wealth to follow to an egalitarian one, which decreases the skill premium further. Hence the skill premium kept decreasing in the 1970s. Until the effect of the accelerated skill-biased technological change started to dominate, the skill premium started to increase again.

2.5 Conclusion

In order to analyze the patterns of the skill premium, this paper builds an OLG model, looking into a labour market with endogenous supply and endogenous demand of different types of workers. The paper discusses not only the short-run effects of a series of determinants on the skill premium, but also their long-run effects with dynamic distribution of wealth. Major results include:

The static equilibrium of the model shows that the difference of the skill premium across countries is caused by a variety of reasons, such as the cost of education, the imperfection of the credit market, distribution of wealth, and access to technology. The higher cost of education, the more severe credit market imperfection, the lower average wealth, and more skill-biased technological access lead to a larger skill premium.

In the long-run, the model has two possible cases of steady state: the egalitarian one, where everyone has the same income and the polarized one, where the skill premium is large and the income is unequal. Since the transition of the distribution of wealth interacts with the skill premium, exogenous shocks can affect the skill premium in the long-run by affecting the transition of the distribution of wealth and making it lead to a different steady state. The effects of technological changes could explain the non-monotonic behaviour of the skill premium of the U.S. in the 20th century.

International trade will not affect the result of this chapter, because for a small open economy, the price of good is taken as given. The firms only need to find the optimal way to produce this good, given the endowments of the economy, which will not be affected by international trade. However, a possible direction is to consider more than one types of goods with different skill densities. In this case, international trade could affect firms' choices and individuals' decisions on education.

Chapter 3

Inequality and the U.S. Financial Policies for Higher Education

Based on the theoretical model built in Chapter 2, I examine the effects of the financial policies for higher education on the skill premium and the inequality. The model is parameterized to the U.S. as the benchmark economy, which has an egalitarian steady state. With the implication from the first chapter, that a high cost of education and a high borrowing interest rate are both necessary but not sufficient conditions for the polarized steady state, the model in this chapter predicts that, firstly, the introducing of free higher education only reduces the level of wealth of everyone in the steady state, but does not affect the inequality; secondly, reducing the interest rate of student loan could not affect the inequality.

3.1 Introduction

Financial policies for higher education affect university attendance, the skill premium, and thus the income inequality. Free higher education and lower student loan interest rates have been major proposals of some candidates in the 2016 U.S. presidential election. According to the analysis in the first chapter, the dynamic distribution of income interacts with the skill premium and affects the inequality in the long run. Therefore, policies for higher education not only affect the skill premium in the short run. but also affect it by affecting the transition of the distribution of wealth in the long run. Based on the theoretical model built in the first chapter, in this chapter, I parameterize the model to the U.S. economy, and examine how the policies on the tuition fee and financial aid to higher education will affect the skill premium and the inequality.

Firstly, the result of simulation suggests that the U.S. economy has a trend to transit to an egalitarian steady state. As defined in Chapter 2, it is the steady state where the wealth of every individual converges to the same level, the skill premium is small, credit market imperfections play no role, and individuals are indifferent in regards to becoming skilled or not. Secondly, by comparing private funding and public funding, the model predicts that publicly funded higher education will only increase the proportion of skilled workers and reduce the skill premium, but it will not affect the inequality, since the steady state is still egalitarian. However, publicly funded higher education reduces the level of wealth of everyone in the steady state, because it increases the number of skilled workers but also increases the aggregate social cost of education. Thirdly, with the current level of the tuition fee, reducing the interest rate of student loan could not affect the inequality at all.

This paper analyzes the model built in the second chapter, which looks at a labour market with endogenous demand and endogenous supply of two types of workers: skilled and unskilled. Because of credit market imperfection, individuals can only borrow a limited amount in order to finance their investment in education. Education investment thus depends on the distribution of endowments, which in turn, depends on past wages. In each period, individuals decide on their education and become skilled or unskilled workers. Firms individually choose their production technology from a set of all feasible technologies and hire factors given their rental rates. The choice of technology is modelled using the idea of a technology frontier proposed by Caselli & Coleman (2006). Caselli and Coleman define the choice of technology as the choice of productivities of different types of workers. Given the distribution of wealth, the static equilibrium is an allocation of workers, their education choices, and firms' choice of production technology that clears the labour market.

In the long run, wages affect the wealth and bequests of individuals, and the skill premium interacts with the distribution of wealth. In principle, there can be two types of steady state, as the previous chapter shows. In the first type of steady state, the egalitarian steady state, the wealth of every individual converges to the same level, the skill premium is small, credit market imperfections play no role, and individuals are indifferent in regards to becoming skilled or not. In the other type of steady state, the polarized steady state, there are two different long-run wealth levels, the skill premium is large and credit market imperfections affect the supply of skills.

Which steady state is attained is determined by the initial status of the economy. The most important implication for this paper is that a high cost of education and a high borrowing interest rate are both necessary but not sufficient conditions for the emergency of a polarized steady state. Therefore, if both the cost and the borrowing interest rate are low, keeping one fixed, an increase in the other one will not affect the inequality in the steady state.

The model is calibrated to the U.S. economy as benchmark economy. The result suggests that this economy has an egalitarian steady state. With this economy, some experiments have been conducted to analyze the effects of financial policies on the inequality. Firstly, I examine the effect of tuition fee. With other parameters fixed, I reduce the cost of education to zero and fund the education by taxing all the agents equally. This policy only reduces the skill premium and increased skilled workers, but does not affect the inequality, since the economy still has an egalitarian steady state. With increased tuition fee, the skill premium is increased but the steady state is not changed, so inequality is not affected. Generally, with the borrowing interest rate fixed, the skill premium increases in the cost of education while the steady state remains the same.

Secondly, I adjust the borrowing interest rate, and it turns out that this adjustment has no affect on the inequality neither. And its effects on the skill premium and labour ratio are also very small.

Thirdly, I examine a policy that less the subsidy largely. I assume huge increase happens to both the tuition fee and the borrowing interest rate. In is case, the steady state is altered into a polarized one. This change in policy increases both the skill premium and inequality largely.

3.1.1 Literature Review

The general literature on the skill premium and the inequality has been discussed in the first chapter. To be highlighted, Galor & Zeira (1993), to which the theoretical model of this paper is related, offer a big picture of the dynamics of inequality with imperfect credit market. Similar to the present paper looking at the effect of credit market imperfections on economies with different steady states, Greenwood & Jovanovic (1990) suggest that the effec of financial development on distribution depends on the level of economic development. Gall et al. (2014) show that with credit market imperfection, the economy with large exposure to FDI has large vulnerability to capital shocks and, which reduces potential domestic entrepreneurs.

Also, some empirical evidences could be found in the work by Li et al. (1998), showing that there is a positive relation between income inequality and the imperfections of the credit market. Different from the present chapter, Beck et al. (2007) find evidence showing that financial development reduces income inequality noy only by changing the distribution of income but also by increasing aggregate growth.

Specifically, on the effects of policies for higher education , Galor & Moav (2000) argue that the expanding financial aid to higher education has reduced the college wage premium in the 1970s. Empirical literature on this issue is vast. Generally, there exists a relationship between credit market imperfection and decisions on education. For example, Jacoby (1994) find that in Peru, families that are borrowing constrained are more likely to reduce their children's education. More specifically, Heller (1999) estimates the effects of tuition and state financial aid on public college enrolment, and he also finds that the aggregate effect of the change in tuition fee is not very large, but it affects students from ddifferent racial group differently. Dynarski (2000) analyzes the different effects of two financial aid programmes on the college enrolment. Surpricely, this work finds that the Georgia's scholarship tends to benefit middle and high-income students and increase the gap between low- and high-income families. These two papers offer some implies that loosing the assumptions of homogeneous agents could be a interesting direction to improve the present paper. Heckman (2005) discusses the education funding policies and inequality in China and suggests to introduce privately funded education, which is similar to the result of this chapter.

3.1.2 Overview

The remainder of this paper is organized as follows: in Section 3.2, I provide a brief review of the theoretical model. The parameterization is described in Section 3.3. The results for the U.S. benchmark economy is presented in Section 3.4. In Section 3.5, I conduct some experiments to understand the effects of financial policies for higher education on the U.S. benchmark economy. Section 3.6 offers some concluding remarks.

3.2 The Theoretical Model

We consider a small open economy. The economy is populated by a continuum of agents with a mass of one, which is constant over time.

There is a continuum of competitive firms, which hire three factors to produce: capital, skilled workers and unskilled workers. Skilled and unskilled workers are substitutes to each other. Firms can also choose productivity for both types of workers simultaneously, from the set of all feasible technology choices.

A single good is produced by firms and consumed by agents. In this small open economy, the price of the good is equal to the world market price, which is exogenous and normalized to 1. Agents can save and borrow in order to finance their education, and firms can borrow to finance production in an international credit market. Both the saving and borrowing world market interest rates are also exogenous and constant over time.

3.2.1 Imperfect Credit Market

The credit market is imperfect, in that there is a spread between the risk free saving and borrowing interest rates denoted by r and i, respectively. Set the spread be denoted by $\beta \geq 0$, so that $1 + i = \beta(1 + r)$. This assumption is borrowed from that of Galor & Zeira (1993), which is standard in the literature for incorporating borrowing constraints.

Borrowing constraints are more severe for households than for firms, for it is difficult to track households and avoid their default. For simplicity, I assume that firms can borrow at rate r.

3.2.2 Households

Each agent lives for two periods in overlapping generations: young and old. In period t, a young agent receives bequest $x_{a,t}$ from her parent and decides on her education: she has the choice either to invest in education or not. The cost of investing in human capital is h. When old, agents work as skilled or unskilled workers, depending on their education level, and earn skilled or unskilled wages $w_{u,t}$ or $w_{s,t}$. Agents only consume and leave bequests to children in the second period of their life. This framework closely follows Galor & Zeira (1993). Agent a receives lifetime utility $u_{a,t}$ from both consumption $c_{a,t}$ and the bequest $x_{a,t}$:

$$u_{a,t} = \theta \ln c_{a,t} + (1 - \theta) \ln x_{a,t} \tag{3.1}$$

The parameter $\theta \in (0, 1)$ determines the saving rate. The individual utility only depends on lifetime income $\pi_{a,t}$ and is strictly increasing in the income. As a result, the agent's problem is to maximize lifetime income by deciding on her education.

According to their decisions on education, all agents are divided into three groups: agents working as unskilled without investing; agents with bequest $x_{i,t-1} \geq h$, who invests in human capital; agents who receive bequest $x_{i,t-1} < h$ and invest by borrowing. Hence an agent is indifferent between investing and not investing if $\pi_{a,t}(invest) = \pi_{a,t}(not\ invest)$, which pins down an endowment f_t :

$$f_t = \frac{1}{i - r}[w_{u,t} + h(1 + i) - w_{s,t}] \tag{3.2}$$

Therefore, given wages, $w_{s,t}$ and $w_{u,t}$, all agents with endowment greater than f_t will invest in human capital and agents with endowment smaller than f_t will not invest. If the distribution of the bequest in period t is $D_t(x_a)$, then the supply of different workers is:

$$L^S_{u,t}(w_{s,t}, w_{u,t}) = \int_0^{f_t} dD_t(x_a) \tag{3.3}$$

$$L^S_{s,t}(w_{s,t}, w_{u,t}) = \int_{f_t}^{\infty} dD_t(x_a) = 1 - L^S_{u,t}(w_{s,t}, w_{u,t}) \tag{3.4}$$

3.2.3 Firms

In period t, a representative firm generates output using the production function proposed by Caselli & Coleman (2006)

$$y_t = k_t^{\alpha}\left[(A_{u,t}L_{u,t})^{\sigma} + (A_{s,t}L_{s,t})^{\sigma}\right]^{(1-\alpha)/\sigma} \tag{3.5}$$

Three factors are used to produce: capital k_t, the ratio of unskilled workers $L_{u,t}$, and the ratio of skilled workers $L_{s,t}$, with $L_{u,t}+ L_{s,t} = 1$. $A_{u,t}$ and $A_{s,t}$ are the productivity of two types of workers $\alpha \in (0,1)$. $1/(1-\sigma)$ is the elasticity of substitution between skilled and unskilled workers and $\sigma \in (0,1)$.

To maximize its profit, taking wages $w_{s,t}$, $w_{u,t}$ and borrowing interest rate r as given, a representative firm optimally chooses factor inputs k_t, $L_{u,t}$ and $L_{s,t}$. A firm also chooses the production technology ($A_{u,t}$, $A_{s,t}$) from a set of feasible technology choices in that period. This set is given by:

$$\delta\,(A_{s,t})^{\omega} + \gamma\,(A_{u,t})^{\omega} \leq B \tag{3.6}$$

A firm chooses technology and factor input to solve:

$$\underset{k_t, L_{u,t}, L_{s,t}}{Max} \quad k_t^{\alpha}\left[\left(A^*_{u,t}L_{u,t}\right)^{\sigma} + \left(A^*_{s,t}L_{s,t}\right)^{\sigma}\right]^{(1-\alpha)/\sigma} - rk_t - w_{s,t}L_{s,t} - w_{u,t}L_{u,t} \tag{3.7}$$

subject to:

$$\delta\,(A_{s,t})^{\omega} + \gamma\,(A_{u,t})^{\omega} \leq B$$

The following can be derived from the first order conditions for problem (3.7):

$$\frac{L^D_{s,t}}{L^D_{u,t}} = \left(\frac{\gamma}{\delta}\right)^{\frac{\sigma}{\omega-\sigma-\omega\sigma}} \cdot \left(\frac{w_{s,t}}{w_{u,t}}\right)^{\frac{\omega-\sigma}{\omega\sigma-(\omega-\sigma)}} \tag{3.8}$$

Equation (3.8) shows firms' relative demand for different workers according to the relative wage, when it can adjust technology along the technology frontier.

In each period, both the price for the good and the interest rates are exogenous, and the static equilibrium is an allocation of workers and investment choices that clears the labour market. Formally,

Definition 3.1. *A static equilibrium is an allocation of factors* $(L_{u,t}, L_{s,t}, k_t)$*, a technology* $(A_{u,t}, A_{s,t})$*, and prices* $(w_{u,t}, w_{s,t})$*, such that in period* t*, for given distribution of endowments* $D_t(x_a)$ *and other parameters* $(\alpha, \sigma, h, i, r, \theta, \mathcal{B}, \delta, \gamma, \omega)$*:*

1. $(A_{u,t}, A_{s,t})$ *satisfies feasibility: the technology frontier (3.6) holds;*

2. *(3.3) and (3.4), yield labour supply* $L^S_{u,t}$ *and* $L^S_{s,t}$*, so that the utility of each agent is maximized;*

3. $(L^D_{u,t}, L^D_{s,t}, A_{u,t}, A_{s,t}, k_t)$ *solve the problem of the representative firm (3.7);*

4. *the labour market clears:*

$$L^S_{u,t}(w_{u,t}, w_{s,t}) = L^D_{u,t}(w_{u,t}) \tag{3.9}$$

$$L^S_{s,t}(w_{u,t}, w_{s,t}) = L^D_{s,t}(w_{s,t}) \tag{3.10}$$

According to the analysis in the first chapter, given the distribution of endowments at the beginning of each period, existence of a unique static equilibrium definded in Definition 2.1 is guaranteed.

3.2.4 The Dynamic Model

In this subsection, I develop the model to a dynamic version, by taking into account the transition of the distribution of wealth. Wages determine the income of each agent and then the bequest she gives to her child. As a result, in the long-run, the distribution of wealth becomes endogenous as well. From the above section, we know that all the agents can be divided into three groups according to their investment decisions: agents from Group I will not invest because their bequest is lower than the threshold; agents from Group II will invest and borrow because their endowments are higher than the threshold but lower than the cost of education; agents from Group III will invest in education without borrowing, because they receive bequests higher than the cost of education. Bequests of agents evolve as follows:

$$x_{a,t} = \begin{cases} (1-\theta)[w_{u,t} + x_{a,t-1}(1+r)], if\, x_{a,t-1} < f_t & (Group\, I) \\ (1-\theta)[w_{s,t} - (h - x_{a,t-1})(1+i)], if\, f_t \leqslant x_{a,t-1} < h & (Group\, II) \\ (1-\theta)[w_{s,t} + (x_{a,t-1} - h)(1+r)], if\, x_{a,t-1} \geqslant h & (Group\, III) \end{cases} \tag{3.11}$$

I suppose that $(1-\theta)(1+r) < 1$ to focus on interesting dynamics, following Galor & Zeira (1993). This assumption rules out the possibility that the incomes of agents in Group I converge to zero or the incomes of agents in Group III diverge. Formally, I define the steady state of the dynamic model as follows:

Definition 3.2. *The steady state is the static equilibrium as defined in Definition 1 which also satisfies the following conditions: in each period, wages* $(w_{u,t}, w_{s,t})$ *are equal to a constant pair* (w_u, w_s)*; for each agent, the bequest is constant over time, i.e.* $x_{a,t} = x_a$.

In each period, by letting $x_t = x_{t-1}$ for each group of agents, we can solve for the following three possible fixed points for the three groups according to (3.11):

$$\overline{x}_{\mathrm{I,t}} = \frac{(1-\theta)w_{u,t}}{1-(1-\theta)(1+r)} \tag{3.12}$$

$$\overline{x}_{\mathrm{II,t}} = \frac{(1-\theta)\left[h\,(1+i) - w_{s,t}\right]}{(1-\theta)(1+i)-1} \tag{3.13}$$

$$\overline{x}_{\mathrm{III,t}} = \frac{(1-\theta)\left[w_{s,t} - h\,(1+r)\right]}{1-(1-\theta)(1+r)} \tag{3.14}$$

These three possible fixed points determine the dynamics of the distribution of wealth. In a steady state, the bequest of each agent must be equal to $\overline{x}_{\mathrm{I}}$, $\overline{x}_{\mathrm{II}}$ or $\overline{x}_{\mathrm{III}}$, which are the fixed points with the steady state wages. Otherwise, the bequest will still evolve, which violates the definition of the steady state. According to the previous chapter, there are only two possible cases of a steady state. One possible case is that all agents have the same wealth level and are indifferent between being skilled or unskilled worker. The other case is that there exist two distinct ergodic levels of wealth. The agents in the rich group are skilled, while the agents in the poor group are unskilled, as stated by Proposition 2.6.

3.3 Model Parameterization

In order to analyze the US economy and to conduct counterfactual experiments, the model is parameterized to the U.S. benchmark economy. The model is governed by 12 parameters summarized in Table 1. Six parameters are chosen from a priori information or existing literature. The four parameters governing the technology frontier are determined by running a regression using cross-country data from 1989. The method following to the one proposed by Caselli & Coleman (2006). The remaining two parameters, the proportion of consumption in the income, is determined in the calibration by minimizing the squared distance between the model output and two data moments: the skill premium and the proportion of the skilled workers.

3.3.1 Independently Chosen Parameters

Following a standard convention in the literature, I set the parameter α, which measures the capital share in GDP, equal to 1/3. The empirical labour literature, Katz & Murphy (1992) for example, documents that $1/(1-\sigma)$, the elasticity of substitution between skilled and unskilled labour is between 1 and 2. I follow them to set $1/(1-\sigma) = 1.4$, which implies that $\sigma = 0.286$.

I assume that the initial distribution of bequest follows a log-normal distribution, of which the mean value is $\overline{x}_{t_0}$, and the variation is governed by the initial Gini coefficient (1994) g_{t_0}. According to the data from Bureau of Labor Statistics, the real DPI (disposable personal income) per capita of the U.S. in 1994 is $\$19,900$. Therefore, the mean value of initial bequest $\overline{x}_{t_0}$ is equal to $\$19,900 \cdot (1-\theta)$, which is equal to the mean wealth multiplied by the ratio that is left to children. I normalize the mean value of initial bequest $x_{t_0} = 1$, and adjust other parameters accordingly.

The cost of education is derived by using data from the National Center for Education Statistics (2015), which show that average annual total cost including tuition fees, room and board rates charged for all full-time 4-year undergraduate students in 1994-1995 was \$9.728. Therefore, the tuition fee for the model is $h = 9,728 * 4/[19,900 \cdot (1-\theta)]$, which is adjusted according to the mean initial bequest. According to Delisle (2012), the interest rate for student loans issued between 1994 and 1995 was 7.43%. With the assumption that the loan is paid with the same amount every year and is paid off in 20 years, I calculate the total borrowing interest rate $i = 0.945$.

3.3.2 The Technology Frontier

The next step is to back out the technology frontier $\delta\,(A_{s,t})^{\omega} + \gamma\,(A_{u,t})^{\omega} \leq B$. We use the data of y, k, L_s, L_u and w_s/w_u and method of Caselli and Coleman (2006), who calculate skill premium with mincerian coefficients from countries. Recall that we have the first order condition for the form's problem:

$$\frac{\partial y_y}{\partial L_u} = w_u \tag{3.15}$$

$$\frac{\partial y_t}{\partial L_s} = w_s \tag{3.16}$$

By combine equation (3.15) and (3.16), we have

$$\frac{w_s}{w_u} = \left(\frac{A_s}{A_u}\right)^{\sigma}\left(\frac{L_s}{L_u}\right)^{\sigma-1} \tag{3.17}$$

Combining equation (3.17) with the production function (3.5), I solve for the choice of (A_u, A_s), by following:

$$A_u = \frac{y^{1/(1-\alpha)}k^{-\alpha/(1-\alpha)}}{L_u}\left(\frac{w_u L_u}{w_u L_u + w_s L_s}\right)^{1/\sigma} \tag{3.18}$$

$$A_s = \frac{y^{1/(1-\alpha)}k^{-\alpha/(1-\alpha)}}{L_s}\left(\frac{w_s L_s}{w_u L_u + w_s L_s}\right)^{1/\sigma} \tag{3.19}$$

With the data of y, k, L_s, L_u and w_s/w_u in different countries, we can calculate for (A_u, A_s) for all the countries. Since δ and γ are the prices of skilled and unskilled technology. We can keep one of them a the related price. Therefore, numerically, the technology frontier can be governed by two parameters, δ and B. As a result, we can normalise $\gamma = 1$ for simplicity. Then I rewrite (3.8) as:

$$\frac{1}{\delta^n}\left(\frac{L_s^n}{L_u^n}\right)^{\sigma} = \left(\frac{A_s^n}{A_u^n}\right)^{\omega-\sigma} \tag{3.20}$$

where n represents different countries. Equation (3.20) can be rewritten in logs as:

$$\ln(\frac{A_s^n}{A_u^n}) = \frac{\sigma}{\omega-\sigma}\ln(\frac{L_s^n}{L_u^n}) + \frac{1}{\omega-\sigma}\ln\frac{1}{\delta^n} \tag{3.21}$$

By estimating $\ln(\frac{A_s^n}{A_u^n})$ on $\ln(\frac{L_s^n}{L_u^n})$, the estimate of $\frac{\sigma}{\omega-\sigma}$ is obtained. Since the value of σ is known, the value of ω is obtained. Then the δ^n can be recovered from the regression residual. With the δ, A_u and A_s of the U.S. and with σ I back out the B of the U.S. Therefore, the technology frontier of the U.S. is:

$$0.668 * (A_s)^{0.632} + (A_u)^{0.632} = 11.373 \tag{3.22}$$

3.3.3 The Calibrated Parameter

The calibrated paremeters are θ and r. $1-\theta$ is the proportion of income that parents leave to their chidren. Existing literature reports various results of intergenerationanl transmission. According to Modigliani (1988), most existing literature shows that transfers account for less than 20% of the of total wealth, which implies that θ, is larger than 0.8. The result of calibration shows that $\theta = 0.855$, very much in line with that finding.

The saving interest rate, $r = 0.88$, which is calculated for 20 years. Therefore, the annual rate is $r = 0.44$, which is a little lower than the adjusting the treasury constant maturity rate from Board of Governors of the Federal Reserve System, which is 0.063.

Table 3.1: Benchmark Model Parameters

India

Description	Parameter	Value	Source
Capital share in GDP	α	0.333	Standard
Elasticity of substitution between skilled and unskilled labour	σ	0.286	Katz and Murphy (1992)
Height of technology frontier	B	11.373	Caselli and Coleman (2006)
Curvature of technology frontier	ω	0.632	Caselli and Coleman (2006)
"Price" of productivity of skilled labour	δ	0.668	Caselli and Coleman (2006)
"Price" of productivity of unskilled labour	γ	1	Normalization
Cost of education	h	13.485	NCES (2015)
Saving interest rate	r	0.880	Calibrated
Borrowing interest rate	i	0.945	Calculated from Delisle (2012)
Proportion of consumption in income	θ	0.855	Calibrated
Mean value of initial bequest	x_{t_0}	1	Normalization
Initial Gini coefficient	g_{t_0}	0.400	World Bank

3.4 Benchmark Economy

3.4.1 The U.S. Economy

Given the set of parameters, we have the benchmark economy. According to Table 3.2, compared with the 1990 Census data reported by Autor et al. (1998), the model performs well at predicting the proportions of skilled and unskilled workers and the skill premium.

The benchmark economy has a trend to transit to an egalitarian steady state. The column (a) of Figure 3.1 shows the evolution of the density of agents with different levels of bequests of the benchmark economy. The initial distribution is a log-normal distribution. From period 0 to period 3, all the agents' bequests converge to a same level. The column (b) of Figure 3.1 shows the evolution of the bequest of each agent. From top to the bottom, the periods of three rows are also 0,1, and 3. At period 0, agents' bequests are very different, but the economy becomes more and more equal. We can tell from both two rows that the benchmark economy has an egalitarian steady state, because the bequest of each agent converges to an same level. Moreover, the steady state level of bequest is higher than the initial level of each agent, which means that both skilled and unskilled agents accumulate wealth in the long run.

Table 3.2: Benchmark Economy

Target	Data (1990 Census)	Model
Proportion of skilled workers L_s	0.386	0.409
Proportion of unskilled workers L_u	0.614	0.591
Skill premium w_s/w_u	1.662	1.664
Squared Distance		0.0005

3.5 Counterfactual Experiments

3.5.1 The Role of Cost of Education

In the benchmark economy, the education is costly and purely privately funded. In this section, I compare the benchmark economy with an economy with free education. I assume that in this economy, education is free. However, every agent needs to pay an equal amount of tax to cover the aggregate cost of education in the economy. Since education is free and agents are no longer subjected to their endowments, there is a large increase in the supply of the skilled workers. As a result, the skilled wage decreases while the unskilled wages increases. The equilibrium wages makes agents indifferent between investing in education or not. Just as shown in Table 3.3 it is not a surprise that in the long run, the economy with free education has more skilled workers and lower skilled premium, compared with the benchmark, because education is accessible to more agents. However, the inequality in the steady state is the same in the two economies, since they have the same egalitarian steady state. What is interesting is that the economy with free education even has a lower bequest level in the steady state. The reason of this result is that with more agents investing in education, the aggregate social cost of education is larger. Also, with more skilled and fewer unskilled workers, there will be an increase in the unskilled wage and a decrease in the skilled wage. If the increased part of aggregate income cannot cover the decreased part, in the long run, averagely, agents accumulate less wealth. Therefore, for an egalitarian economy as the benchmark economy, free education cannot make it better off but reduces the total wealth.

I also examine the effect of a higher cost of education. It decreases the number of skilled workers and increases the skill premium, and decreases the income level. This is consistent with the theoretical analysis in the second chapter. Also, since it does not alter the steady state, it has no effect on inequality. I also run the same experiment with different level of saving interest rate r as robustness check, while results are similar.

Table 3.3: Cost of Education

$r = 0.880$

Economies	$h = 13.485$ (Bechmark)	$h = 0$; (with tax)	$h = 20$
L_u	0.591	0.332	0.706
w_s/w_u	1.664	1.000	2.122
Bequest $\overline{x}$	7.716	1.482	6.831

Table 3.4: Cost of Education

$r = 0.92$

Economies	$h = 13.485$	$h = 0$; (with tax)	$h = 20$
L_u	0.600	0.332	0.715
w_s/w_u	1.695	1.000	2.167
Bequest $\overline{x}$	7.536	1.334	6.669

Table 3.5: Cost of Education

$r = 0.86$

Economies	$h = 13.485$	$h = 0$; (with tax)	$h = 20$
L_u	0.587	0.332	0.703
w_s/w_u	1.650	1.000	2.100
Bequest $\overline{x}$	7.812	1.709	6.918

3.5.2 The Role of The Interest Rate of Student Loan

As stated in the introduction of this chapter, there are a amount of existing works showing the effects of financial aid to the college on the inequality. As in Table 3.6, given the cost of education, I examine the effect the borrowing interest rate on the skill premium and inequality. The result shows that a change in the financial aid policy does not affect the proportion of unskilled workers and the skill premium significantly. A lower borrowing interest rate only reduces the skill premium a little and the level of wealth is almost untouched. Similar results come with a higher borrowing rate. The benchmark economy will not be very different with more or less financial aid to college, since the economy stays as an egalitarian one.

Table 3.6 Borrowing Interest Rate

$\theta = 0.855$

Economies	$i = 0.945$ (Bechmark)	$i = 0.9$	$i = 2$
L_u	0.591	0.589	0.655
w_s/w_u	1.664	1.657	1.895
Bequest $\overline{x}$	7.716	7.738	7.218

The result in this and previous subsections show that, the long-run status of inequality is determined by the transition of the distribution of wealth, and the steady state to which it leads. Therefore, higher education policies that only changes tuition fee or borrowing interest rate a little could not affect the inequality in the long-run, unless they are strong enough to alter the steady state.

3.5.3 An Example of a Polarized Economy

In the previous section, I discuss the effects of higher education policies on an egalitarian economy. Next I build a example of a polarized economy and test the effects of policies. I assume that an economy has very high cost of education and high borrowing interest rate. As in Table 3.7, if $i = 12$ and $h = 35$, since it is more costly to invest in education, borrowing or not, the economy has more unskilled workers and much higher skill premium. Also, it leads agents' bequests to converge to different levels, which means this policy creates inequality in the long-run. The transition to a polarized steady state is shown in Figure 3.2. From top to the bottom, the periods of three rows are 0,3, and 10. Column (a) shows that, in the long run, the distribution of agents tends to gather to two different levels. And column (b) also shows that clearly the bequests have two different levels in the long run. In one group agents are rich and skilled while agents in the other group are poor and unskilled.

Then I apply different policies on this economy with polarized steady state. As shown in the second column of Table 3.8, with the same borrowing interest rate, if we reduce the tuition fee, the skill premium drops, as well as the proportion of unskilled workers. And in the long run, The economy transits to an egalitarian steady state. In the third column, we hold the tuition fee and reduce the borrowing interest rate. Similarly, the economy also transits to an egalitarian steady state. This result is consistent with the result from theoretical analysis that high enough tuition fee and borrowing interest rate are necessary for a polarized economy. To transform a polarized economy to an egalitarian one, one policy, reducing tuition fee or reducing borrowing interest rate, is enough.

Table 3.7 A Polarized Economy

$i = 12; h = 35$

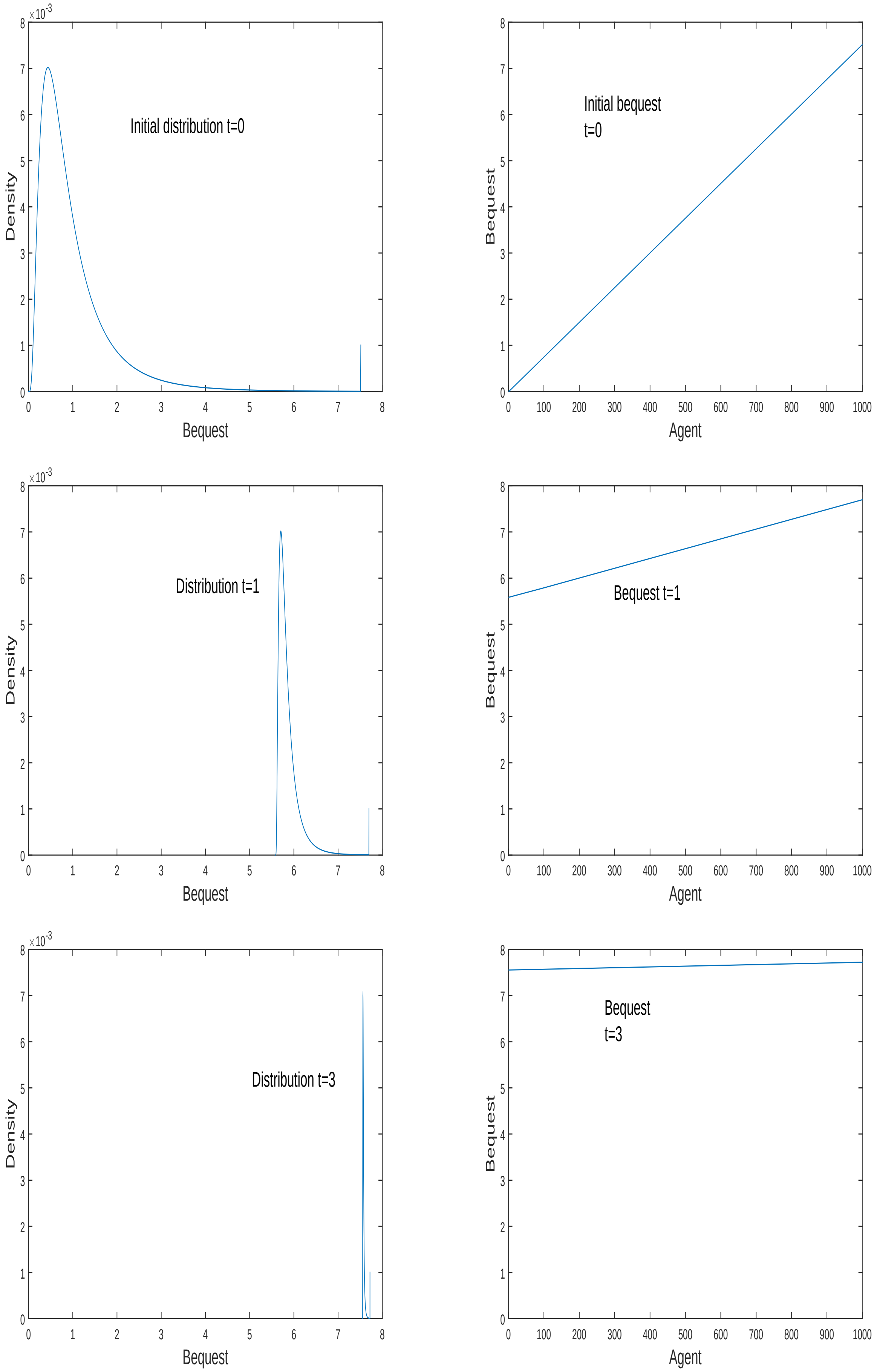
Results	Values
Proportion of unskilled workers L_u	0.997
Skill premium w_s/w_u	20.751
Bequest of the rich group	71.639
Bequest of the poor group	4.085

Table 3.8. Policies on a Polarized Economy

Economies	$i = 12, h = 35$ (Bechmark)	$i = 12, h = 5$	$i = 0.9, h = 35$
L_u	0.997	0.422	0.8613
w_s/w_u	20.751	1.201	.330
Bequest $\overline{x}$	71.639/4.085	9.321	5.684

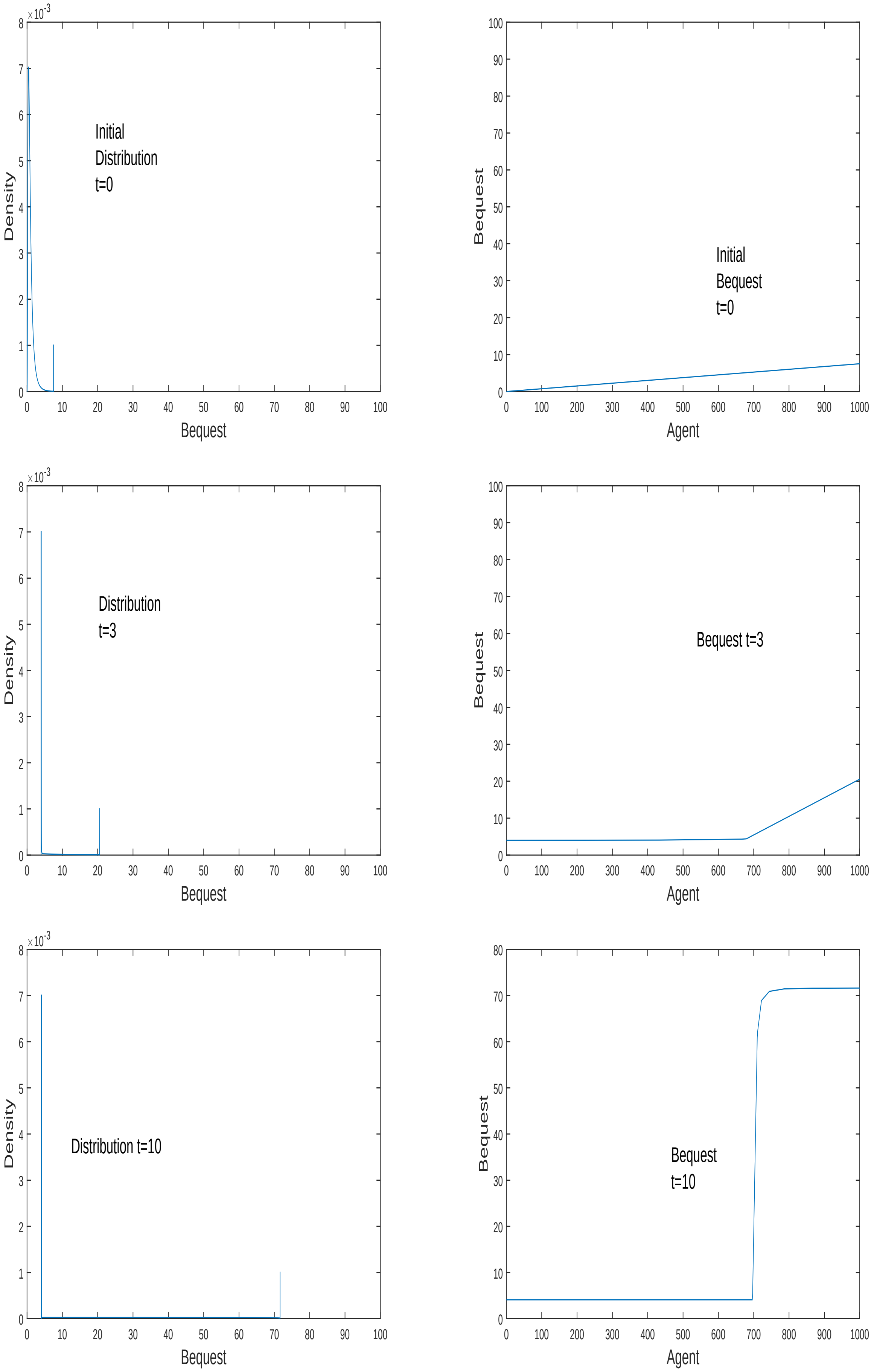
3.6 Conclusion

In this chapter, I replicate the interplay between skill premium and dynamic distribution of wealth with the U.S. as the benchmark economy. With the benchmark, I examine the effects of higher education financial policies on the skill premium and the inequality. The results show that, since the benchmark economy has an egalitarian steady state, publicly funded higher education only reduces the level of wealth of everyone in the steady state. Moreover, increasing financial aid to college does not affect the benchmark economy inequality.



(a) Distribution (b) Bequest

FIGURE 3.1: The Evolvement of the Bechmark Economy



(a) Distribution (b) Bequest

FIGURE 3.2: The Evolvement of A Polarized Economy

Chapter 4

The Structure of the Higher Education System and Economic Growth

In this paper, I analyze the determinants of the structure of the higher education system and the effects of this structure on the economic growth. The results of this paper indicate that a country with a larger population and a larger proportion of high-ability agents tends to have a diversified system, which consists of a mix of institutions that differ in quality. Otherwise, the country tends to have a unified system, which has very small variance in the quality of universities. Compared with the unified system, the diversified system increases the aggregate productivity. In short run, the diversified system increases inequality, which is, nevertheless, a Pareto improvement. In the long run, by affecting the transition of the distribution of wealth, transforming to a diversified system could eliminate the poverty trap in a relatively poor economy. However, in a relatively rich economy, if the personal ability and wealth are correlated, a diversified system could increase the inequality and reduce the intergenerational mobility at the same time.

4.1 Introduction

Quality of education, in particular higher education, is generally perceived as a major determinant in economic growth and development. The structure of the higher education system differs considerably across countries, even within highly developed countries. In some countries, Germany for example, higher education is provided by institutions with very small variance in the quality. This will be referred to as a "unified" system. In other countries, however, the U.S. for example, the higher education system consists of a mix of institutions that differ in prestige, quality, and selectivity of students, and

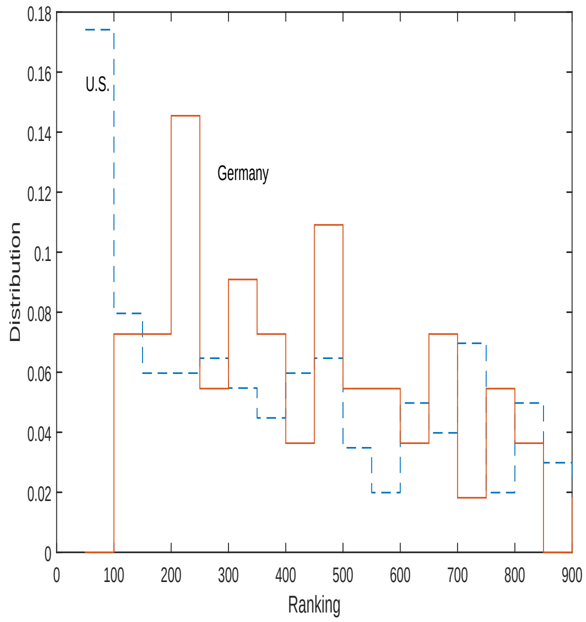


FIGURE 4.1: The Distribution of Universities in the U.S. and Germany

will be referred to as a "diversified" system. This is relevant, because many developing countries are implementing policies that could reshape the structure of the higher education system. For example, China has proposed the 985 Project, which is aimed to promote a selected group of Chinese universities to the world first-rate quality (Zhang et al. (2013)), with an aim of fostering economic growth. In this paper, I analyze how the structure of a country's higher education system affects the investment in human capital, thus income on aggregate and its distribution. I also analyze the effects of the structure on productivity and inequality, especially the effects on the dynamics of inequality in the long-run. Moreover, I allow the structure of higher education to arise endogenously, depending on initial conditions.

The results of this paper show that the size of market is the major reason for the difference in the structure of the higher education system. Compared with Germany, the U.S. higher education has a larger market, simply because the U.S. population is larger and English is more wildly used than German. Because opening a new tier of universities and only offering higher education specially for a group of students with a certain level of ability generates a large fixed cost, following expanding product variety logic, the higher education system in the U.S. is more diversified. Figure 4.1 and 4.2 show the distribution of American and German universities among the top 900 universities in the world (U.S. News (2016)), with the total number of the universities of each country normalized to one. We can find that, compared with the distribution of American universities, which is illustrated by the dashed line, the distribution of German universities, illustrated by the solid line, is more concentrated. Moreover, the U.S. has a clear advantage in the top 100 universities.

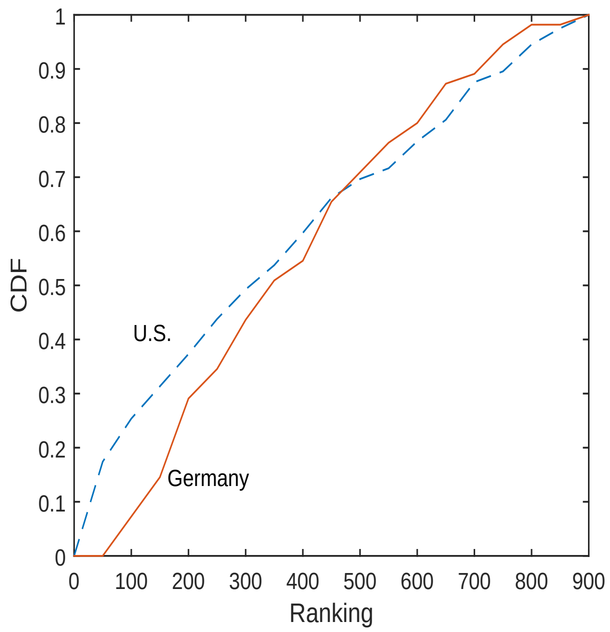


FIGURE 4.2: The CDF of Universities in the U.S. and Germany

Furthermore, the analysis in this paper argues that the diversified system offers higher education that is more suitable to high-ability workers, so it adds more human capital on them and brings higher wage to them. The higher wage also attracts more high-ability people to invest in education. Therefore, the diversified system increases productivity on aggregate, but also increases inequality. This result is consistent with the data shown in Table 4.1, which states that the U.S. has higher college attainment rate and fewer under-matched students, who are smart enough to enter universities but have not. Notice that the U.S. has higher productivity and a higher Gini coefficient than Germany. However, this paper also derives different predictions that, in a relatively poor country, transforming a unified into a diversified system could eliminate a poverty trap in the long-run, since it offers poor people opportunities to invest in education and earn higher wage when they are of high ability. In a relatively rich country, a diversified system increases income and wealth inequality and reduces intergenerational mobility. This result is consistent with the data of the U.S. and Germany and could offer some explanation for the "Great Gatsby Curve" among rich countries, which states the positive correlation between inequality and intergenerational earnings persistence.

Table 4.1: Productivity and Inequality

Country	U.S.	Germany
College Attainment	11.7	11.2
Under-match	0.055	0.104
Labour productivity	64.1	58.3
Gini (before taxes and transfers)	0.513	0.508
Gini (post taxes and transfers)	0.396	0.292

Source: Cooper & Liu (2016) and OECD (2012, 2013)

Notes: Under-match means the individuals with a low level of education (college) but a higher PIACC score than those with high education (College)

This paper analyzes an OLG model in a small open economy, which consists of agents with heterogeneous ability: high or low. Agents supply their labour as educated or uneducated workers. Because of credit market imperfection, individuals can only borrow a limited amount in order to finance their investment in education, as in other inequality and growth models (Galor & Zeira (1993)). Education investment thus depends on the distribution of endowments, which in turn, depends on past wages. In the higher education system, there are two universities and two possible tiers in which the universities could operate. If a university operates in the low tier, it provides low-quality education to both low-ability and high-ability agents. If it operates in the high tier, it provides high-quality education that is only available for high-ability agents. The cost of operating in a tier is fixed but it costs more to operate in the high tier. The human capital that an agent can obtain depends on both her ability and the quality of her education. The wages for different workers are constant and depend on their human capital.

In each period, agents decide on their education and become educated or uneducated workers. The two universities maximize their profit by deciding on the tier to operate in. Given the distribution of wealth, the equilibrium is an allocation of workers, their education choices, and universities' choices of operating tiers. The analysis of the Nash equilibrium generates a first result that an economy with a larger population, a larger proportion of high-ability agents, and a higher return to high-tier education, tends to have a diversified system, which means that one university operates in each tier. The reason is that, for high-ability agents, compared with the low-tier education, high-tier education brings higher wage, thus attracting more students. However, since operating in the high tier also generates higher fixed cost, it is not profitable to specialize in providing high-tier education for high-ability agents, unless the potential market for high-tier education is large enough.

The diversified system adds more human capital on high-ability agents, and brings a higher wage to them, thus attracting more high-ability agents to invest in education and reducing the number of uneducated workers. As a result, the aggregate human capital and productivity is increased. Since the income for educated high-ability workers is higher under the diversified system, the inequality is increased. However, since uneducated and educated low-ability workers are not worse off because the wages for them are not changed, transforming from the unified system to the diversified one is a Pareto improvement.

In the long-run, wages affect the wealth and bequests of agents. Therefore, the distribution of wealth becomes endogenous as well. The transition of the distribution of wealth is determined by the wages for different workers, thus by the structure of the higher education system. When the wages for workers are relatively low, it is possible that there exists a poverty trap, in which the agents stay being uneducated and poor generation after generation, no matter they are high-ability or low-ability. In this case, transforming from the unified system to a diversified system could increase the wage for educated high-ability agents, which makes education profitable for poor but high-ability agents and offers them an opportunity to escape from the trap and converge to the rich groups. Therefore, in each generation, a fraction of poor agents can leave the poverty trap by investing in education. Eventually, all the agents will be educated, and the only difference in income is caused by the difference in ability.

However, in a relatively rich economy without the poverty trap, in the long run all the agents are educated and converge to two rich groups, with different levels of wealth that is only caused by ability. In this case, transforming from the unified system to a diversified system could increase the inequality, since it increases the income for the high-ability group. If we also assume that the possibility to be high-ability increases in the wealth,, when the income for the high-ability group is higher, it is more likely for those agents to stay in the group. Thus, the unified system also reduces the intergenerational mobility.

4.1.1 Literature Review

This paper builds on the work of Galor & Zeira (1993), modelling credit market imperfections and the dynamic distribution of wealth. However, this paper introduces heterogeneous ability and an endogenous higher education system, thus allowing for a much richer set of possible long run outcomes. Jaimovich (2010) also addresses the credit market imperfections and development, but it examines the effect on development through an adverse selection channel in the labour market rather than through market segmentation in investments.

There has been little interest in the determinants of the structure of the higher education in the economic literature. However, there are some illuminating contributions in the literature on education, for example, Fairweather (2000) and Teichler (2008). Fairweather

(2000) finds that both markets and government policies have encouraging and discouraging effects on the diversification in American higher education system. Teichler (2008) offers a review of the analyses and debates on the European higher education system in the last 40 years, but not of how exactly this system has changed. In the 1960s and 1970s, emphasis was on the diversification between types or sectors of higher education institutions (universities or polytechnics, Fachhochschulen, etc.). In the 1980s, attention shifted to vertical diversification, which is the difference among the same type of higher education institutions. Since the 1990s, the vertical diversification has been discussed with international competition globalisation. Market size has been used as explanation for industrialization in growth literature, for example, Murphy et al. (1989*a*) and Murphy et al. (1989*b*). In this two papers, fixed cost is the major reason that market size matters, which is similar to the model in this paper. Particularly, Shaked & Sutton (1987) analyze the relationship between market size and industry structure, but different from the present paper, he addresses both vertical and horizontal differentiations of products, with endogenous fixed costs. Melitz & Ottaviano (2008) also analyzes the affect of market size on the horizontal product variety and productivity. Compared with the literature on the effects of market size, the present paper focuses on the effect on vertical (quality) differentiation in higher education.

This paper also complements the literature on the effects of the structure of education and growth. For example, Allmendinger (1989) categorizes education systems along two dimensions, standardization and stratification, and analyzes their effects on labour market. The empirical analysis shows that in a highly standardized system, which offer education with the same quality nationwide, workers do not change jobs frequently, because they can rely on the information given by their certificates. In a highly stratified system, where a large proportion of a cohort can attain the maximum number of school years provided by the system, workers' labour market outcomes are strongly related to their education attainment. Krueger & Kumar (2004) use the difference in the characteristics of education in the U.S. and Germany to explain the growth differences between the two countries. Compared with "vocational" education in Germany, the U.S. "general" education enables workers to adopt new technology easily and laeds to a fast growth. More generally, Galor et al. (2009) state that because of the adverse effect of the implementation of public education on landowners' income, an economy with more equal distribution of lands tends to implement public education, which accelerates human-capital accumulation and the transition from an agricultural to an industrial economy. Also, Shavit et al. (2007) provide vast empirical studies on the effect of the structure of the higher education on inequality. However, it focuses on the intra-sector relation in the higher education, i.e., the relation between academic education and occupationally oriented programmes, but not the variety of quality among universities as discussed in the present paper.

4.1.2 Overview

The remainder of this paper is organized as follows: in Section 4.2, I set up the basic model. Section 4.3 examines the equilibrium of the higher education system and its short-run effects. Section 4.4 discusses the dynamic version of the model. Section 4.5 releases an assumption of the model and check the robustness of the results. Section 4.7 offers some concluding remarks.

4.2 The Model

I look at the equilibrium of both labour market and higher education system. Different possible structures of higher education system generate different returns to education. Firstly, given different possible structures I analyze the supply of workers and the equilibrium of the labour market. The equilibrium composition of workers with education implies the profit for universities. Secondly, I analyze the equilibrium of higher education system with universities maximizing profit.

We consider a small open economy. The economy is populated by a continuum of agents with a mass of L, with heterogeneous ability a. An agent i has low ability if $a_i = \underline{a}$ and she has high ability if $a_i = \overline{a}$. The proportion of the agents with low ability is λ while the proportion of the agents with high ability is $1 - \lambda$. The distribution of ability is independent from the personal wealth.

There is a continuum of competitive firms, which hire different types of workers to produce a single good. In this small open economy, the price of the good is equal to the world market price, which is exogenous and normalized to 1. Agents can save and borrow to finance their education. Both the saving and borrowing world market interest rates are also exogenous and constant over time.

4.2.1 Imperfect Credit Market

The credit market is imperfect, in that there is a spread between the risk free saving and borrowing interest rates denoted by r and b, respectively. Set the spread be denoted by $\beta \geq 0$, so that $1 + b = \beta(1 + r)$. This assumption is borrowed from the one of Galor & Zeira (1993), which is a tried and tested way to incorporate borrowing constrains. Borrowing constraints are more severe for households than for firms. For simplicity I assume that firms can borrow at rate r.

4.2.2 Universities

There are two universities, $u = 1, 2$, operating in the higher education system. There are two possible tiers of education, $e \in \{\underline{e}, \overline{e}\}$, from which each university could choose to provide. If $e_u = \underline{e}$, university u chooses to provide low tier education, which is available for agents with both low and high ability. If $e_u = \overline{e}$, university u chooses to provide high tier education, which is only available for the agents with high ability. Providing different tiers of education generates different fixed cost $\kappa_{e_u} \in \{\underline{\kappa}, \overline{\kappa}\}$ for each university, and it costs more to provide high tier education than to provide low tier education: $\overline{\kappa} > \underline{\kappa}$. The number of enrolled students that university u could attract, $N_u(e_1, e_2)$, depends on its own choice of which tier education to provide, as well as the choice of the other university. Therefore, y_u, the profit that one university can make by providing education, also depends on both e_1 and e_2:

$$y_u(e_1, e_2) = \tau \cdot N_u(e_1, e_2) - \kappa_{e_u} \tag{4.1}$$

where τ is the tuition fee, and it costs the same for a agent to take high tier or low tier education. The problem of each university is:

$$\underset{e_u}{Max} \quad y_u(e_1, e_2) \tag{4.2}$$

According to the choices of two universities, in principle there are three possible scenarios in the higher education system: both universities provide low tier education; one provides low tier education and the other provides high tier education; both provide high tier education.

4.2.3 Human Capital Accumulation

Higher education adds human capital on agents. The amount of human capital $H(a, e)$ that an agent can obtain by taking education is determined by both her ability a and the tier of education e that she has taken. The human capital of uneducated workers is normalized to 1. Moreover, the human capital $H(a, e)$ of agents with different ability and education follows the following order:

$$1 < H(\underline{a}, \underline{e}) < H(\overline{a}, \underline{e}) < H(\overline{a}, \overline{e}) \tag{4.3}$$

This order implies that the same education adds more human capital on high-ability agents, and for the agents with the same ability, better education adds on more human capital. Wages of agents depend on their human capital. Therefore, wages depend on both individual ability as well as choices of the two universities. The wage of educated workers is $w(a, e_1, e_2) = A_s H(a, e)$, where A_s governs the productivity that is not related to human capital. The wage of uneducated workers is $w_u = A_u$. According to Equation (4.3), the wages of workers with different levels of ability and education follow:

$$w_u < w(\underline{a}, \underline{e}, \underline{e}) < w(\overline{a}, \underline{e}, \underline{e}) < w(\overline{a}, \underline{e}, \overline{e}) \tag{4.4}$$

This means that uneducated wage is lower than educated low-ability wage, which is lower than low-tier educated high-ability wage, which is lower than high-tier educated high-ability wage. Here I assume that the wage, or the marginal productivity of a worker does not depend on the composition of workers. It is plausible with the initial assumption that the economy is small and open, which also allows workers to move.

4.2.4 Households

Each agent lives for two periods in overlapping generations: young and old. In period t, a young agent receives bequest $x_{i,t}$ from her parent and decides on her education: she has the choice either to invest in education or not. The cost of investing in human capital is τ. When old, agents work as educated or uneducated workers, depending on their education level, and earn educated or uneducated wages $w(a, e_u, e_{-u})$ or w_u. Agents only consume and leave bequests to children in the second period of their life. This framework closely follows Galor and Zeira (1993). Agent i receives lifetime utility $v_{i,t}$ from both consumption $c_{i,t}$ and the bequest $x_{i,t}$:

$$v_{i,t} = \theta \ln c_{i,t} + (1 - \theta) \ln x_{i,t} \tag{4.5}$$

The parameter $\theta \in (0, 1)$ determines the saving rate. The optimal choice of $c_{i,t}$ and $x_{i,t}$ for an agent i in the second period of his life maximizes $v_{i,t}(c_{i,t}, x_{i,t})$ subject to the budget constrain:

$$c_{i,t} + x_{i,t} = \pi_{i,t} \tag{4.6}$$

where $\pi_{i,t}$ is lifetime income. Therefore the individual utility only depends on lifetime income $\pi_{i,t}$ and is strictly increasing in the income. As a result, agent's problem is to maximize lifetime income by deciding on her education. An agent working as uneducated without investing gets income:

$$\pi_{i,t} = w_u + x_{i,t-1}(1 + r) \tag{4.7}$$

An agent with bequest $x_{i,t} \geq \tau$, who invests in higher education, gets:

$$\pi_{i,t} = w(a, e_u, e_{-u}) + (x_{i,t-1} - \tau)(1 + r) \tag{4.8}$$

An agent, who receives bequest $x_{i,t} < \tau$ and invests, needs to borrow and gets income:

$$\pi_{i,t} = w(a, e_u, e_{-u}) - (\tau - x_{i,t-1})(1+b) \tag{4.9}$$

As discussed in the previous section, there are three possible scenarios in the higher education system. I look into the supply of different types of workers, given the structure of the higher education system.

If two universities operate in the low tier, $e_1 = e_2 = \underline{e}$, and an agent with $a = \underline{a}$, or a low-ability agent is indifferent between investing and not if bequest $x_{i,t-1}$, is equal to:

$$f_{\underline{a},\underline{e}} = \frac{1}{b-r}[w_u + \tau(1+b) - w(\underline{a}, \underline{e}, \underline{e})] \tag{4.10}$$

An agent with $a = \overline{a}$, or a high-ability agent is indifferent between investing and not if bequest $x_{i,t}$, is equal to:

$$f_{\overline{a},\underline{e}} = \frac{1}{b-r}[w_u + \tau(1+b) - w(\overline{a}, \underline{e}, \underline{e})] \tag{4.11}$$

Therefore, given wages, w_u and $w(\underline{a}, \underline{e}, \underline{e})$, all low-ability agents with endowment greater than $f_{\underline{a},\underline{e}}$ will invest in human capital and agents with endowment smaller than $f_{\underline{a},\underline{e}}$ will not invest. If $D_t(x_i)$ is the distribution of bequest at the beginning of period t, and $F(\cdot)$ is the CDF of the distribution of initial bequest, the supply of educated low-ability workers is:

$$L_{\underline{a},t}\left(w(\underline{a}, \underline{e}, \underline{e}), w_u\right) = \lambda L[1 - F(f_{\underline{a},\underline{e}})] \tag{4.12}$$

Similarly, the supply of educated high-ability workers is:

$$L_{\overline{a},t}\left(w(\overline{a}, \underline{e}, \underline{e}), w_u\right) = (1-\lambda) L[1 - F(f_{\overline{a},\underline{e}})] \tag{4.13}$$

And the supply of uneducated workers is

$$L_{u,t} = L - L_{\underline{a},t} - L_{\overline{a},t}$$

If two universities provide different types of education, $e_1 = \underline{e}, e_2 = \overline{e}$ (or $e_1 = \overline{e}, e_2 = \underline{e}$). The low-ability agents can only take low tier education, and high-ability agents will only choose high tier education, if they can afford that, because education of higher tier brings higher wage and costs the same. An agent with $a = \underline{a}$ is indifferent between investing and not if bequest $x_{i,t-1}$, is equal to:

$$f_{\underline{a},\underline{e}} = \frac{1}{b-r}[w_u + \tau(1+b) - w(\underline{a}, \underline{e}, \overline{e})] \tag{4.14}$$

An agent with $a = \overline{a}$ is indifferent between investing and not if bequest $x_{i,t}$, is equal to:

$$f_{\overline{a},\overline{e}} = \frac{1}{b-r}[w_u + \tau(1+b) - w(\overline{a}, \underline{e}, \overline{e})] \tag{4.15}$$

Therefore, given wages, w_u and $w(\underline{a}, \underline{e}, \overline{e})$, all low-ability agents with endowment greater than $f_{\underline{a},\underline{e}}$ will invest in human capital and agents with endowment smaller than $f_{\underline{a},\underline{e}}$ will not invest. Then the supply of educated low-ability workers is:

$$L_{\underline{a},t}\left(w(\underline{a}, \underline{e}, \overline{e}), w_u\right) = \lambda L[1 - F(f_{\underline{a},\underline{e}})] \tag{4.16}$$

Similarly, the supply of educated high-ability workers is:

$$L_{\overline{a},t}\left(w(\overline{a}, \underline{e}, \overline{e}), w_u\right) = (1-\lambda)L[1 - F(f_{\overline{a},\overline{e}})] \tag{4.17}$$

And the supply of uneducated workers is

$$L_{u,t} = L - L_{\underline{a},t} - L_{\overline{a},t}$$

Since the wage for high-ability workers with high-tier education $w(\overline{a}, \underline{e}, \overline{e})$ is higher than the wage for them with low-tier education $w(\overline{a}, \underline{e}, \underline{e})$, the threshold for them to invest follows: $f_{\overline{a},\overline{e}} < f_{\overline{a},\underline{e}}$. As a result, $L_{\overline{a},t}\left(w(\overline{a}, \underline{e}, \overline{e}), w_u\right)$, the supply of educated high-ability workers when two universities provide different types of education is larger than $L_{\overline{a},t}\left(w(\overline{a}, \underline{e}, \underline{e}), w_u\right)$, the supply of educated high-ability workers when two universities operate in the low tier.

If two universities operate in the high tier, the agents with $a = \underline{a}$ cannot invest in education. An agent with $a = \overline{a}$ is indifferent between investing and not if bequest $x_{i,t-1}$, is equal to:

$$f_{\overline{a},\overline{e}} = \frac{1}{b-r}[w_u + \tau(1+b) - w(\overline{a}, \overline{e}, \overline{e})] \tag{4.18}$$

The supply of educated high-ability workers is:

$$L_{\overline{a},t}\left(w(\overline{a}, \overline{e}, \overline{e}), w_u\right) = (1-\lambda)L[1 - F(f_{\overline{a},\overline{e}})] \tag{4.19}$$

And the supply of uneducated workers is

$$L_{u,t} = L - L_{\overline{a},t}$$

In each period, both the price for the good and the interest rates are exogenous, and given the structure of higher education system, the equilibrium is an allocation of workers that clears the labour market. Notice that the production function is linear and there is no limitation for the number of students that a university could educate. Therefore, every agent who is with proper ability and can afford tuition fee would get a job accordingly. Formally,

Definition 4.1. *A static equilibrium is an allocation of factors (* $L_{u,t}, L_{\underline{a},t}(w(\underline{a}, e_u, e_{-u}), w_u)$, $L_{\overline{a},t}(w(\overline{a}, e_u, e_{-u}), w_u)$*), and prices (* $w_u, w(a, e_u, e_{-u})$*), such that in period t, for given distribution of endowments* $D_t(x_i)$, the structure of higher education system $\{e_1, e_2\}$, *and other parameters (* $\tau, b, r, \theta, A_s, A_u$*), which yields labour supply* $L_{u,t}$ *and* $L_{s,t}$*, so that the utility of each agent is maximized, and clears the labour market.*

Given their different choices of universities, the equilibrium labour composition suggests the profit for universities, which determines the nash equilibrium of the higher education system.

4.3 The Equilibrium of the Higher Education System

In this section, I look at the Nash equilibrium of the higher education system and its effects on productivity and inequality. Given ability, initial wealth and returns to different tiers of education, agents make decision on education. Anticipating the number of enrolled students, universities make decision on tiers to operate in. Now recall that the profit of a university u is $y_u(e_1, e_2) = \tau \cdot N_u(e_1, e_2) - \kappa_{e_u}$, and number of the students it could attract is

$$N_u(e_1, e_2) = \begin{cases} \frac{1}{2}\left\{\lambda L[1 - F(f_{\underline{a},\underline{e}})] + (1-\lambda)L[1 - F(f_{\overline{a},\underline{e}})]\right\}, & if\ e_u = e_{-u} = \underline{e} \\ \lambda L[1 - F(f_{\underline{a},\underline{e}})] & if\ e_u = \underline{e}, e_{-u} = \overline{e} \\ (1-\lambda)L[1 - F(f_{\overline{a},\overline{e}})] & if\ e_u = \overline{e}, e_{-u} = \underline{e} \\ \frac{1}{2}(1-\lambda)L[1 - F(f_{\overline{a},\overline{e}})] & if\ e_u = e_{-u} = \overline{e} \end{cases} \tag{4.20}$$

Now we have the number of enrolled students and the profit for each university, given the choices of two universities. Formally,

Definition 4.2. A Nash equilibrium of the higher education system is a pair of strategies $\{e_1, e_2\}$, that maximize each university's profit y_u, given the other university's choice.

Proposition 4.3. *There always exists a unique Nash equilibrium in the higher education system and there are three possible scenarios : the Nash equilibrium is that both two universities operate in the low tier, which means that* $\{e_1 = \underline{e}, e_2 = \underline{e}\}$*; the Nash equilibrium is that one university operates in each tier, which means that* $\{e_1 = \underline{e}, e_2 = \overline{e}\}$ *or* $\{e_1 = \overline{e}, e_2 = \underline{e}\}$ *; the Nash equilibrium is that both two universities operate in the high tier, which means that* $\{e_1 = \overline{e}, e_2 = \overline{e}\}$ *.*

The proof is in the appendix. Since the scenario that both universities give up low ability market and only operate in the high tier is rare in the real world, this paper focuses on the first two scenarios: the unified system where both universities provide low-tier education to all agents, and the diversified system where one university only provides low-tier education to low-ability agents and the other university only provide high-tier education to the high-ability agents.

As analyzed previously, compared with a higher education system which only offer low tier education, a system that could offer high tier education always attracts more high-ability agents to invest in education, because it adds more human capital on them and brings higher wage. For relatively poor students with high ability, if the return to education is not high enough, they will not invest in education. However, the return to high tier education is higher, which attracts more agents to invest. Therefore, if the total tuition fee paid by the increased fraction of high-ability students is larger than the extra fixed cost generated by offering high tier education rather than low tier, it is profitable to offer high tier education. Otherwise, both universities only offer low tier education. Since the tuition fee is constant, the Nash equilibrium is determined by the number of high-ability agents who would not invest in education if there is only low tier education but are willing to invest if there is high tier education. Therefore, we have the following proposition.

Proposition 4.4. *(i) Compared with the unified higher education system, the diversified system exists when the proportion of high-ability agents* $1 - \lambda$ *is larger;*

(ii) There exists a $\widehat{\lambda}$*, when* $1- > 1 - \widehat{\lambda}$*, compared with the unified higher education system, the diversified system exists when the population L is larger.*

(iii) Compared with the unified higher education system, the diversified system exists when $H(\overline{a}, \overline{e})$*, the human capital that high-tier education can add on high-ability agents is higher.*

Proposition 4.4 (i) and (ii) state the effects of market size on the higher education system. Because the existence of the fixed cost for a university to provide education, it is not profitable for university to operate in high-tier unless high-ability population, i.e. the number of the potential buyers of high-tier education, is large enough. This could explain that the higher education systems in the English-speaking countries, the UK and the US for example, are more diversified.

Proposition 4.4 (iii) states that if the return to high-ability agents with better education is higher, more high-ability agents are willing to invest in education. Thus, it is profitable to provide education specially for high-ability agents.

4.3.1 Higher Education System and Productivity

The productivity of an economy is affected by the aggregate human capital obtained by workers by taking education. Since the population and technology is constant, the productivity is determined by the average human capital $\overline{h}_t$, which can be measured as:

$$\begin{aligned} \overline{h}_t &= (\sum_e \sum_a H(a,e) L_{a,t} + \sum_a L_{u,t})/L \\ &= (H(\underline{a},\underline{e}) L_{\underline{a},t} + H(\overline{a},e) L_{\overline{a},t} + \sum_a L_{u,t})/L \end{aligned} \tag{4.21}$$

By comparing the average human capital with unified and diversified higher education systems, we can have the following proposition. The proof is in the appendix.

Proposition 4.5. *Compared with the unified higher education system, the diversified system brings higher average human capital, thus increases the productivity.*

The diversified higher education system could increase the average human capital, because it could reduce two types of mismatch. The first type is the educated-uneducated mismatch. Because of the credit market imperfection, a fraction of high-ability agents are constrained by the endowments and do not invest in education. High-tier education adds more human capital on high-ability agents than low-tier education does, thus the wage of high-ability agents with high-tier education is higher than the wage of high-ability agents with low-tier education. Therefore, the diversified higher education system could attract more high-ability agents to invest in education. The second type of mismatch is the mismatch between the types of education and ability. Compared with the unified higher education system, the diversified system could support high-ability agents overproportionally and add on more human capital on them. By reducing these two types of mismatch, the diversified higher education system increases productivity of an economy.

4.3.2 The Effect on the Distribution of Wealth

Taking the initial distribution of endowments as given, the distribution of income is determined by the wages of different types of workers, which is affected by the structure

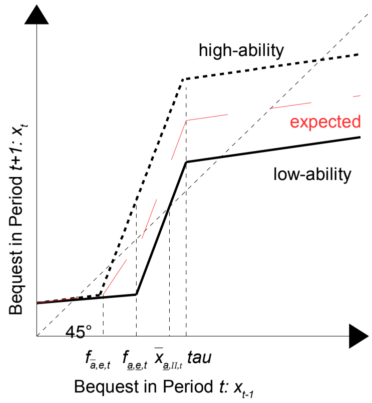


Figure 4.3: Transition of Bequests

of the higher education. In this subsection, I look at the short-run effects of the structure of the higher education in each period. Let us consider the following assumption:

Assumption: *At the beginning of period t, bequests follow a uniform distribution on $[0, M]$, with $M > \tau$. The distribution of bequest is independent from the one of ability.*

From the above section, we know that all the agents with the same ability can be divided into three groups, according to their investment decisions: agents do not invest if their bequest is lower than the threshold; agents invest and borrow if their endowments are higher than the threshold but lower than the cost of education; agents invest in education without borrowing, if they receive bequests higher than the cost of education. Bequests of agents with same ability evolve as follows:

$$x_{i,t} = \begin{cases} (1-\theta)[w_u + x_{i,t-1}(1+r)], if\ x_{i,t-1} < f_{a,e} \\ (1-\theta)[w(a, e_u, e_{-u}) - (\tau - x_{i,t-1})(1+b)], if\ f_{a,e} \leqslant x_{i,t-1} < \tau \\ (1-\theta)[w(a, e_u, e_{-u}) + (x_{i,t-1} - \tau)(1+r)], if\ x_{i,t-1} \geqslant \tau \end{cases} \quad (4.22)$$

I suppose that $(1-\theta)(1+r) < 1$ to focus on interesting dynamics, following Galor & Zeira (1993). This assumption rules out the possibility that the incomes of agents in poor group converge to zero in the long-run or the incomes of agents in rich group diverge. Figure 4.3 illustrated the transition of bequests of agents with different types of ability. The solid line shows how much bequest each low-ability agent is going to leave to her child, given the amount of bequest she receives, following Equation 4.22. Similarly,

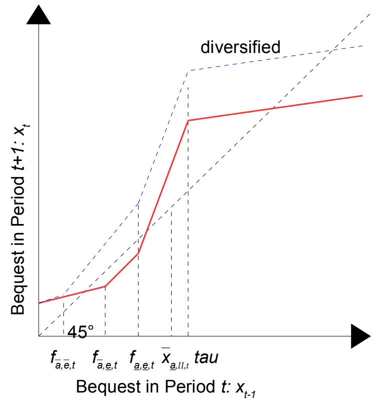


FIGURE 4.4: Expected Bequests in Different Systems

the dotted line illustrated how much bequest each high-ability agent is going to leave to her child, given the amount of bequest she receives. Notice that the threshold for high-ability agents to invest in education is always lower than the threshold for low-ability agents, no matter the higher education system is diversified or unified, i.e., $f_{\overline{a},e} < f_{\overline{a},\underline{e}}$.

Since the distribution of bequest is independent from the one of ability, an agent will be low-ability with the probability of λ. Therefore, given the endowments, before observing their ability, the expected bequests that agents will leave to the next generation, which are illustrated by the dashed line in Figure 4.3, are:

$$x_{i,t} = \begin{cases} (1-\theta)[w_u + x_{i,t-1}(1+r)], & if\ x_{i,t-1} < f_{\overline{a},e} \\ \lambda(1-\theta)[(w_u + x_{i,t-1})(1+r)] \\ \quad + (1-\lambda)(1-\theta)[w(\overline{a}, e_u, e_{-u}) - (\tau - x_{i,t-1})(1+b)], & if\ f_{\overline{a},e} \leqslant x_{i,t-1} < f_{\underline{a},\underline{e}} \\ \lambda(1-\theta)[w(\underline{a}, e_u, e_{-u}) - (\tau - x_{i,t-1})(1+b)] \\ \quad + (1-\lambda)(1-\theta)[w(\overline{a}, e_u, e_{-u}) - (\tau - x_{i,t-1})(1+b)], & if\ f_{\underline{a},\underline{e}} \leqslant x_{i,t-1} < \tau \\ \lambda(1-\theta)[w(\underline{a}, e_u, e_{-u}) + (x_{i,t-1} - \tau)(1+r)] \\ \quad + (1-\lambda)(1-\theta)[w(\overline{a}, e_u, e_{-u}) + (x_{i,t-1} - \tau)(1+r)], & if\ x_{i,t-1} \geqslant \tau \end{cases} \tag{4.23}$$

In Figure 4.4, the solid line illustrated the expected bequests generated by a unified system, and the dotted line illustrated the expected bequests generated by a diversified system. By comparing the unified higher education system with the diversified system, we have the following intuitive results.

Firstly, transforming from a unified system to a diversified one is a Pareto improvement, to both expected bequest and real income. For expected bequest, as in Figure 4.4, expected bequests generated by a diversified system dominate because it increases educated wage for high-ability agents, and it also makes some poor agents who do not invest in a unified system to invest. And the agents of the poorest group who will never invest are not worse off, since the uneducated wage is not changed. For the real income, a diversified system increases the real income of high-ability educated agents, while does not change the real income of low-ability agents and uneducated agents. Since no one is worse off while some are better off, this transforming is a Pareto improvement.

Secondly, transforming from a unified system to a diversified one increases the inequality of the expected income. Since it is complex to analyze the inequality of real income, we analyze the inequality of the expected income, according to Equation (4.23). As in Figure 4.4, the expected bequest of poorest group, also calculated by the first line of Equation (4.23), is not effected by the change of the structure of the higher education. The expected income of all the other groups who are likely to take education is increased, since the return of education to high-ability agents is higher. Therefore, with the diversified system, the inequality is increased, since the income share of the poorest group is smaller, while the share of rich group is larger.

As stated in the previous sections, being educated or not depends both on her ability and the endowment an agent has at the beginning of her life, which is determined by the status of her parent. The more the education decision depends on personal ability but not the endowment, the more intergenerational mobility this economy has. Intuitively, the diversified system has higher educated wage for high-ability agents, which reduces the threshold $f_{\overline{a},e}$ for high-ability agents to invest in education. As a result, more poor agents are constrained by their endowments and are able to invest in education. However, this result is not always consistent with data from the real world. For example, Corak (2013) shows that the U.S. has higher income inequality and less mobility across generations than Germany. There could be several explanations for this inconsistence. Firstly, the short-run and long-run effects of the structure of higher education could be different. In the next section, I will analyze the long-run effects by introducing the dynamic distribution of wealth. Secondly, some assumptions upon which the model is built are not always true in the U.S. and Germany. For example, the model assumes that the distribution of the ability is independent from the distribution of wealth, and that higher education is purely privately funded. In the following section, I will also loose some of these assumptions to check the robustness of the results.

4.4 The Dynamic Model

In this section, I develop the model to a dynamic version, by taking into account the transition of the distribution of wealth. Wages determine the income of each agent and then the bequest she gives to her child. As a result, in the long run, the distribution of wealth becomes endogenous as well. The transition of the distribution of wealth and its steady state depend on the values of parameters, such as the interest rates, the human capital of different workers $H(a, e)$. I restrict attention to certain sets of parameter values and characterize the steady states, which could offer some implications for the effects of the structure of the higher education in the long run. For the reason that credit market imperfection is an important reason of inequality, I only look at the dynamics with $(1-\theta)(1+b) > 1$, which means the scenario that credit market imperfection is severe.

4.4.1 An Economy with Poverty Trap

Consider an economy with a unified higher education system and relatively low wages for high-ability and low-ability educated workers. In this case, the wages of high-ability and low-ability workers are both relatively low. Figure 4.5 illustrates how the distribution of wealth evolves. This figure shows the bequests $x_{i,t}$ that agents will leave to next generation, given their endowments $x_{i,t-1}$. The high-ability and low-ability agents are illustrated by dotted and solid lines. For a low-ability agent i, if her endowment $x_{i,t-1} < f_{\underline{a},\underline{e}}$, she does not invest in education. Otherwise, she invests. However, for agents with endowments between $f_{\underline{a},\underline{e}}$ and $\overline{x}_{\underline{a},\mathrm{II}}$, and

$$\overline{x}_{\underline{a},\mathrm{II}} = \frac{(1-\theta)\left[\tau\left(1+b\right) - w(\underline{a}, \underline{e}, \underline{e})\right]}{(1-\theta)(1+b)-1} \tag{4.24}$$

they can borrow and invest in this period, but the bequests that they leave will be less than their endowments and tend to fall below the threshold $f_{\underline{a},\underline{e}}$, because borrowing is costly. As a result, if their children are also low-ability, they cannot afford investing anymore. Therefore, in the long run, the bequests of low-ability agents tend to converge to two levels: the educated and rich group R_1 with bequest level

$$\overline{x}_{\underline{a},\mathrm{III}} = \frac{(1-\theta)\left[w(\underline{a}, \underline{e}, \underline{e}) - \tau\left(1+r\right)\right]}{1-(1-\theta)(1+r)} \tag{4.25}$$

or the uneducated and poor group P, with bequest

$$\overline{x}_{\underline{a},\mathrm{I}} = \frac{(1-\theta)w_u}{1-(1-\theta)(1+r)} \tag{4.26}$$

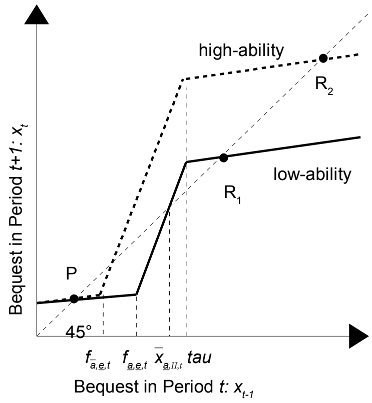


FIGURE 4.5: A Poor Economy with Poverty Trap

Similarly, the bequests of high-ability tend to converge to two levels: the skilled and rich group R_2, with bequest

$$\overline{x}_{\overline{a},\mathrm{III}} = \frac{(1-\theta)\left[w(\overline{a},\underline{e},\underline{e}) - \tau\,(1+r)\right]}{1-(1-\theta)(1+r)} \tag{4.27}$$

or the unskilled and poor group P. Therefore, in the steady state, there exists a poverty trap, which means that there exists a group of agents who stay being poor and uneducated from generation to generation, regardless of their ability, because their initial endowments are low.

However, if the unified higher education system is replaced by a diversified one, and the educated high-ability wage is increased high enough, the dynamic distribution follows the poverty trap could be eliminated.

Proposition 4.6. *(i) In a unified system, when the human capital of high-ability and low-ability skilled workers $H(\overline{a}, e)$ and $H(\underline{a}, \underline{e})$ are low, there exists a poverty trap P, where some agents are always uneducated, regardless of their ability.*

(ii) There exists a $\mathring{H}$, if the human capital of high-ability workers with high tier education $H(\overline{a}, \overline{e}) \geq \mathring{H}$, transforming from a unified higher education system to a diversified one could eliminate the poverty trap and lead to a steady state where all agents are educated.

The proof is in the appendix. The diversified higher education system adds more human capital on high-ability agents and increases their wages. As illustrated in Figure 4.6, higher return to education reduces $f_{\underline{a},\overline{e}}$, the threshold for high-ability agents to invest

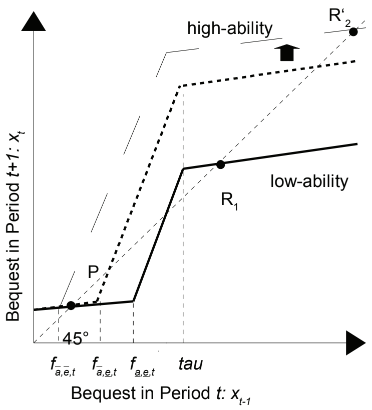


FIGURE 4.6: The Poor Economy Introducing Diversified System

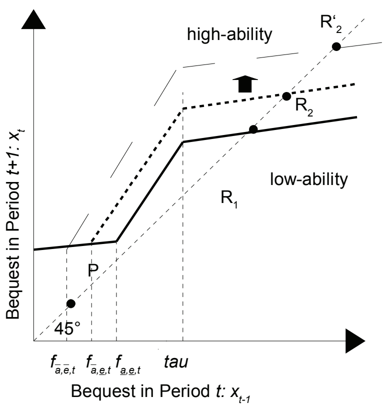


FIGURE 4.7: A Rich Economy

in education. If this threshold is even lower than $\overline{x}_{\underline{a},\mathrm{I}}$, the bequest for agents in the poverty trap, it is profitable for the high-ability agents in the poverty trap to invest in education. Thus, they could leave the trap and move to the rich groups. Gradually, more and more agents escape from the trap, until all the agents are educated, and the difference in the wealth is only caused by the ability. Therefore, transforming from a unified higher education system to a diversified one could reduce the inequality which is caused by the initial endowments and increase the intergenerational mobility. This result could offer an implication for some poor countries that introducing the diversified higher education system and increasing the wage for high-ability educated workers could possibly reduce the poverty.

4.4.2 A Rich Economy

The effect of transforming to a diversified system could be different if the initial status of the economy is different. Consider a rich economy with a unified higher education system and relatively high wages for high-ability and low-ability skilled workers. As illustrated by Figure 4.7, in this economy, $f_{\underline{a},\underline{e}}$ and $f_{\overline{a},\underline{e}}$, the thresholds for high-ability and low-ability agents to invest are very low. Although there are still a fraction of agents cannot invest in education, they accumulate wealth and leave more bequests, since the real wages are high. In the long run, their children can eventually invest in education and all become educated. Agents with low ability converge to group R_1, and agents with high ability converge to group R_2. The only difference in wealth is caused by the ability.

In this case, introducing the diversified higher education system could not affect the transition of the distribution of wealth. All the agents are still going to converge to two rich groups, R_1 and R_2', with bequest levels $\overline{x}_{\underline{a},\mathrm{III}}$ and $\overline{x}_{\overline{a},\mathrm{III}}$. And no one's decision on education is subjected to the bequest she gets. The only change is that, since the diversified higher education system adds more human capital on high-ability agents, it increases $\overline{x}_{\overline{a},\mathrm{III}}$, and therefore, increases the inequality in long run.

4.5 Correlation between Ability and Wealth

The model in the previous sections is built upon the assumption that the distribution of ability is independent from the distribution of wealth. However, one may argue that this is not the case. In the real world, the ability of a pre-college student is affected by background of his or her family, primary and secondary education, and as well as the community where he or she grows up. Therefore, the family wealth could affect the ability of a student. In this section, I include the positive correlation between the ability and wealth and analyze how this change could affect the previous results.

I assume that the possibility for an agent to be high-ability is an increasing function of her bequest, and there exists possibility for the poorest agent to be high-ability: $\lambda_{i,t} = \lambda(x_{i,t})$, $\frac{d\lambda_{i,t}}{dx_{i,t}} < 0$ and $\lambda(0) < 1$.

4.5.1 A Rich Economy

As analyzed in previous section, in a relatively rich economy, transiting to a diversified higher education system increase the inequality but does not affect the intergenerational mobility, since the mobility between two rich groups is only determined by the constant possibility λ. However, the effects are different with the correlation between ability and wealth. Firstly, transiting to a diversified system still increases the inequality, as $\overline{x}_{\overline{a},\mathrm{III}}$, the bequest level for agents in group R_2 increases in the high-ability educated wage $w(\overline{a}, e_u, e_{-u})$, which increases in the human capital. Since $H(\overline{a}, \overline{e}) > H(\overline{a}, \underline{e})$, we have $\overline{x}_{\overline{a},\overline{e},\mathrm{III}} > \overline{x}_{\overline{a},\underline{e},\mathrm{III}}$. Moreover, since $\frac{d\lambda_{i,t}}{dx_{i,t}} < 0$, we have $1 - \lambda(\overline{x}_{\overline{a},\overline{e},\mathrm{III}}) > 1 - \lambda(\overline{x}_{\overline{a},\underline{e},\mathrm{III}})$, which implies that with the diversified higher education system, the children of agents in the high-ability group R_2 are more likely to be also high-ability and stay in the group.

Proposition 4.7. *In a relatively rich economy where there is no poverty trap and the possibility for agents with higher bequest to be high-ability is higher, transforming from a unified higher education system to a diversified one increases the inequality and reduce the intergenerational mobility.*

As shown in Corak (2013), in the U.S., the inequality is higher and the mobility across generations is less than the ones in Germany. Moreover, countries with higher inequality tend to have less mobility across generations. This relation is known as the "Great Gatsby Curve". The analysis in this section could offer some explanations for the relation between inequality and mobility across generations in rich countries.

4.5.2 An Economy with Poverty Trap

The correlation between the ability and wealth could not affect the existence of the poverty trap. As stated previously, the condition for the poverty trap to exist is that the bequest level of the poor group $\overline{x}_{\underline{a},\mathrm{I}}$ is smaller than $f_{\overline{a},\underline{e}}$, the threshold for high-ability to invest in education. Both $\overline{x}_{\underline{a},\mathrm{I}}$ and $f_{\overline{a},\underline{e}}$ are independent from $\lambda_{i,t}$. Thus, the distribution of ability does not affect the existing of the poverty trap. However, it could affect the amount of agents in the poverty trap. For example, as illustrated in Figure 4.5, the agents with bequest $x_{i,t} \in (\overline{x}_{\overline{a},\mathrm{II}}, \overline{x}_{\underline{a},\mathrm{II}})$ tend to fall below the threshold and stay unskilled if they are low-ability, but tend to converge to rich group if that are high-ability. Therefore, if the average possibility for them to be low-ability is high than the constant possibility λ in the previous sections, there would be more agents falling into the poor group.

Moreover, if the possibility for poor agents to be high-ability is relatively low, the diversified higher education system could still eliminate the poverty trap, since $f_{\overline{a},\overline{e}}$, the threshold for high-ability agents to invest into high-tier education is also independent from $\lambda_{i,t}$. Meanwhile, it would take longer time for the poverty trap to disappear. The reason is that the amount of the agents leave the trap in each period is determined by the possibility for them to be high-ability.

4.6 Convex Cost of Education

In the previous analysis, I assume that the cost of education is the same for two tiers. In this section, I will loose this assumption and look at its effects on the growth. However, the discussion is based on given structure of higher education system, which should also be affected by the tuition fee. I will not discuss how the convex tuition fee would effect the outcome of the game of two universities. This issue requires further work.

I assume that tuition fee is a convex function of the human capital it adds on high-ability students, which means that

$$\tau_{\overline{e}} - \tau_{\underline{e}} < H(\overline{a},\overline{e}) - H(\overline{a},\underline{e}) \tag{4.28}$$

where $\tau_{\overline{e}}$ and $\tau_{\underline{e}}$ are the tuition fees of two tiers universities. With different tuition fees,I look at the effects of different structures of higher education on long-run inequality.

How the transformation from a unified system to a diversified one affects the poverty trap is determined by its effect on the threshold for high-ability agents to invest in education:

$$f_{\overline{a},e} = \frac{1}{b-r}[w_u + \tau(1+b) - w(\overline{a},\overline{e},e)] \tag{4.29}$$

Since as discuss previously, the wealth level of the poverty trap $\overline{x}_{a,\mathrm{I}}$ is constant, the existence of the trap is determined by the relative location of $\overline{x}_{a,\mathrm{I}}$ and $f_{\overline{a},e}$. Therefore, we want to know the effect of the transformation on $f_{\overline{a},e}$.

$$\begin{aligned}\Delta f_{\overline{a},e} &= f_{\overline{a},\overline{e}} - f_{\overline{a},\underline{e}} \\ &= \frac{1}{b-r}\{(\tau_{\overline{e}} - \tau_{\underline{e}})(1+b) - [w(\overline{a},\underline{e},\overline{e}) - w(\overline{a},\underline{e},\underline{e})]\} \\ &= \frac{1}{b-r}\{(\tau_{\overline{e}} - \tau_{\underline{e}})(1+b) - A_s[H(\overline{a},\underline{e}) - H(\overline{a},\overline{e})]\}\end{aligned} \tag{4.30}$$

We can see that the transformation from a unified system to a diversified one has two effects on the wealth. Firstly, it has positive effect because it adds on more human capital and bring higher wage. Secondly, it also has negative effect, since it increases the tuition fee. Therefore, $\Delta f_{\overline{a},e}$ could be greater or smaller than 0. If the positive effect is stronger, $\Delta f_{\overline{a},e} < 0$, which means that in short run, more high-ability agents will invest, and if it reduces $f_{\overline{a},\overline{e}}$ lower than the wealth level of the poverty trap $\overline{x}_{a,\mathrm{I}}$, it could eliminate the trap. However, it is also possible that $\Delta f_{\overline{a},e} > 0$, which means that this transformation reduces the number of educated high-ability agents in short run, and in long run, it even could create a poverty trap if $f_{\overline{a},\overline{e}} > \overline{x}_{a,\mathrm{I}}$.

Similarly, the effect on the wealth level of rich high-ability group $\overline{x}_{\overline{a},\mathrm{III}} = \frac{(1-\theta)[w(\overline{a},\underline{e},e)-\tau(1+r)]}{1-(1-\theta)(1+r)}$, could also be negative. This implies that, transforming to a diversified system will hurt both the rich and the poor, if the benefit from increased wage could not cover the loss caused by the higher tuition fee.

4.7 Conclusion

I analyze how socioeconomic composition affects the structure of the higher education system and the effects of the structure on the economic growth.

The analysis indicates that an economy with a larger population, a larger proportion of high-ability agents, and higher return to high-tier education tends to have a diversified system. Compared with the unified system, the diversified system can add more productivity on high-ability agents and reduce the number of uneducated high-ability agents, which increases the aggregate productivity.

In the long run, the effects of the structure of the higher education system depend on the status of the economy. The diversified system could reduce poverty in a poor economy but increase inequality in a rich economy.

However, a major limitation of this chapter is that the analysis and results of this chapter highly depend on the construction of the simple model, especially on some not very realistic assumptions, for example, the uniform tuition fee (which is true in Germany and UK but not in the U.S.), fixed cost of education, only high-ability agents going to high tier university and purely privately funded higher education. To make the results more convincing, some future work could be carried out by releasing these assumptions. Some attempts has been made, for example, the discuss of convex tuition fee. However, it is far from enough.

We can also consider a case that the education is not purely privately funded. If there exists mean-tested financing of the education system, within the structure built in this chapter, its effects on the Nash equilibrium of the higher education system is uncertain. Whether a university is willing to provide better quality education is determined by the additional number of students the better education could attract. Mean-tested financing could lower the threshold for agents to invest in education. Therefore, it could affect the interval of the wealth of the marginal group of students who would be attracted by only high tier education but not low tier. However, the number of students in the marginal group is determined by the distribution of wealth. Therefore, the effects of mean-tested financing on the structure of the higher education system is uncertain. Moreover, since the mean-tested financing could lower the threshold for agents to invest, according to previous analysis, it could possibly eliminate the poor trap in the long run.

Chapter 5

Conclussion

This thesis contributes to the literature on economic growth and inequality, especially to the theoretical literature related to the skill premium and education. By introducing the dynamic distribution of wealth, it studies the effects of technological change on the skill premium and role of the structure of higher education system in growth.

In Chapter 2, both the short-run and long-run effects of technological change on the skill premium have been studied. The most important finding is that both skill-biased and unskill-biased technological change have reducing effect on the skill premium in the long-run, which helps to understand the U-shape evolution of the U.S. skill premium in the last century, and predicts a Kuznets curve of the skill premium in the future.

In Chapter 3, I simulate the model built in Chapter 2, with the U.S. economy as benchmark economy. The experiments suggest that the U.S. economy is already an egalitarian one, offering free higher education by taxing or increasing financial aid to college could not affect inequality in the long run.

In Chapter 4, I argue that the size of market could be an important explanation for the different structures of higher education system. This chapter also implies that the long-run effect on the inequality depends on the initial status of the economy.

In the future, improvement work could be carried out following several directions:

Firstly, in Chapter 4, analysis is based on an assumption that the production function is linear, wages are constant and independent from the supply of workers. By changing this assumption, the model could generate different results. Similarly, as analyzed previously, I assume that the tuition fees are the same for different tiers education and are purely privately funded. These assumption are not necessarily true is all the countries. Some experiments could be carried out by loosing the assumptions.

Secondly, in the three major chapters of the thesis, I assume that all the workers only work in the second period of their life. Therefore, one generation could only affect the other one through the bequest. A more proper OLG model could be built by letting both old and young agents work at the same period.

Appendix A

Appendix 1

A.0.1 Proof of Lemma 2.3

Let labour ratio $l_t = \frac{L_{s,t}}{L_{u,t}}$, and we can rewrite productivities as functions of labour ratio:

$$A_{u,t} = \left(\frac{B/\gamma}{1 + (\gamma/\delta)^{\sigma/(\omega-\sigma)} l_t^{\omega\sigma/(\omega-\sigma)}} \right)^{\frac{1}{\omega}} \tag{A.1}$$

$$A_{s,t} = \left(\frac{B/\delta}{1 + (\gamma/\delta)^{\sigma/(\sigma-\omega)} l_t^{\omega\sigma/(\sigma-\omega)}} \right)^{\frac{1}{\omega}} \tag{A.2}$$

Let $\mathcal{L}_t = (\gamma/\delta)^{\sigma/(\omega-\sigma)} l_t^{\omega\sigma/(\omega-\sigma)}$, and plug (A.1) and (A.2) into the first order conditions for problem 2.2.3, then we can show wages as functions of labour ratio:

$$w_{u,t} = \left(\frac{r}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha)(1+\mathcal{L}_t)^{\frac{1-\sigma}{\sigma}} \left(\frac{B/\gamma}{1+\mathcal{L}_t}\right)^{\frac{1}{\omega}} \tag{A.3}$$

$$w_{s,t} = \left(\frac{r}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha)\left(1+\frac{1}{\mathcal{L}_t}\right)^{\frac{1-\sigma}{\sigma}} \left(\frac{B/\delta}{1+\frac{1}{\mathcal{L}_t}}\right)^{\frac{1}{\omega}} \tag{A.4}$$

then we have

$$\frac{dw_{u,t}}{dl_t} > 0 \tag{A.5}$$

$$\frac{dw_{s,t}}{dl_t} < 0 \tag{A.6}$$

and

$$\lim_{l_t \to +0} w_{s,t} = +\infty \tag{A.7}$$

$$\lim_{l_t \to +\infty} w_{u,t} = +\infty \tag{A.8}$$

A.0.2 Proof of Lemma 2.5

Let

$$G(l_t) = w_{s,t} - h(1+r) - w_{u,t}$$

From (A.5), (A.6), (A.7), and (A.8) we have

$$\lim_{l_t \to +0} G(l_t) = +\infty$$

$$\lim_{l_t \to +\infty} G(l_t) = -\infty$$

$$\frac{dG(l_t)}{dl_t} < 0$$

Therefore, only one unique $(w^0_{u,t}, w^0_{s,t})$ solves $w_{s,t} - h(1+r) = w_{u,t}$, and condition 2.6 only holds when $\frac{w_{s,t}}{w_{u,t}} \geq \frac{w^0_{s,t}}{w^0_{u,t}}$.

A.0.3 Proof of Proposition 2.6

Because $L^S_{u,t} = (f_t - M + \varepsilon)/2\varepsilon$ and $L^S_{s,t} = (M + \varepsilon - f_t)/2\varepsilon$, we have:

$$l_t = \frac{(M + \varepsilon - f_t)/2\varepsilon}{(f_t - M + \varepsilon)/2\varepsilon} \tag{A.9}$$

We can solve for f_t:

$$f_t = \frac{M + \varepsilon + l_t(M - \varepsilon)}{l_t + 1} \tag{A.10}$$

By replacing the f_t, $w_{u,t}$ and $w_{s,t}$ in (2.7) with (A.3) (A.4) and (A.10), we have the reduced equation:

$$
\begin{aligned}
& \frac{M+\varepsilon+l_t(M-\varepsilon)}{l_t+1}(i-r) \\
= \; & h(1+i) \\
& + \left(\frac{r}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)\left[(\mathcal{B}/\gamma)^{\frac{1}{\omega}}(1+\mathcal{L}_t)^{\frac{1-\sigma}{\sigma}-\frac{1}{\omega}}-(\mathcal{B}/\delta)^{\frac{1}{\omega}}(1+\frac{1}{\mathcal{L}_t})^{\frac{1-\sigma}{\sigma}-\frac{1}{\omega}}\right]
\end{aligned}
$$

Let

$$
\begin{aligned}
F(l_t) \; = \; & \frac{M+\varepsilon+l_t(M-\varepsilon)}{l_t+1}(i-r)-h(1+i)- \\
& \left(\frac{r}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)\left[(\mathcal{B}/\gamma)^{\frac{1}{\omega}}(1+\mathcal{L}_t)^{\frac{1-\sigma}{\sigma}-\frac{1}{\omega}}-(\mathcal{B}/\delta)^{\frac{1}{\omega}}(1+\frac{1}{\mathcal{L}_t})^{\frac{1-\sigma}{\sigma}-\frac{1}{\omega}}\right] \\
= \; & 0
\end{aligned}
\tag{A.11}
$$

The $L^*_{u,t}$ and $L^*_{s,t}$ solving (A.11) are the equilibrium proportions of workers.

For $l_t \in (0, +\infty)$, $\mathcal{L}_t = (\gamma/\delta)^{\sigma/(\omega-\sigma)} l_t^{\omega\sigma/(\omega-\sigma)}$ yields:

$$\lim_{l_t \to +0} \mathcal{L}_t = 0 \tag{A.12}$$

$$\lim_{l_t \to +\infty} \mathcal{L}_t = +\infty \tag{A.13}$$

$$\frac{d\mathcal{L}_t}{dl_t} = (\gamma)^{\sigma/(\omega-\sigma)} \omega\sigma/(\omega-\sigma) l_t^{\omega\sigma/(\omega-\sigma)-1} > 0 \tag{A.14}$$

Rewriting the assumption $\omega > \sigma/(1-\sigma)$ yields:

$$\frac{1-\sigma}{\sigma} - \frac{1}{\omega} > 0 \tag{A.15}$$

Then we have:

$$\lim_{l_t \to +0} F(l_t) = (M+\varepsilon)(i-r)-h(1+i)-\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)\left[-(\mathcal{B}/\delta)^{\frac{1}{\omega}}(1+\frac{1}{\lim_{l_t \to +0}\mathcal{L}_t})^{\frac{1-\sigma}{\sigma}-\frac{1}{\omega}}\right] = +\infty \tag{A.16}$$

$$\lim_{l_t \to +\infty} F(l_t) = (M-\varepsilon)(i-r)-h(1+i)-\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)\left[(\mathcal{B}/\gamma)^{\frac{1}{\omega}}(1+\lim_{l_t \to +\infty}\mathcal{L}_t)^{\frac{1-\sigma}{\sigma}-\frac{1}{\omega}}\right] = -\infty \tag{A.17}$$

$$
\begin{aligned}
F_{l_t} \; = \; & \frac{-2\varepsilon(i-r)}{(l_t+1)^2} - \left(\frac{r}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)\left(\frac{1-\sigma}{\sigma}-\frac{1}{\omega}\right) \cdot \\
& \left[(\mathcal{B}/\gamma)^{\frac{1}{\omega}}(1+\mathcal{L}_t)^{\frac{1-\sigma}{\sigma}-\frac{1}{\omega}-1}+(\mathcal{B}/\delta)^{\frac{1}{\omega}}(1+\frac{1}{\mathcal{L}_t})^{\frac{1-\sigma}{\sigma}-\frac{1}{\omega}-1} \cdot \left(\frac{1}{\mathcal{L}_t}\right)^{-2}\right] \cdot \frac{d\mathcal{L}_t}{dl_t} \\
< \; & 0
\end{aligned}
\tag{A.18}
$$

Therefore function $F(l_t)$ has one and only one root in $(0, +\infty)$.

We regard $h, \beta(= i - r), M$ and γ as variables and consider the partial derivatives of F with respect to them:

$$F_h = -(1+i) < 0 \tag{A.19}$$

$$F_M = i - r > 0 \tag{A.20}$$

$$F_\gamma = \left(\frac{r}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha)\left[\mathcal{B}^{\frac{1}{\omega}}\gamma^{-\frac{1}{\omega}-1}(1+\mathcal{L}_t)^{\frac{1-\sigma}{\sigma}-\frac{1}{\omega}}\right]\frac{1}{\omega} > 0 \tag{A.21}$$

$$F_\delta = -\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha)\left[\mathcal{B}^{\frac{1}{\omega}}\delta^{-\frac{1}{\omega}-1}(1+\frac{1}{\mathcal{L}_t})^{\frac{1-\sigma}{\sigma}-\frac{1}{\omega}}\right]\frac{1}{\omega} < 0$$

$$F_\beta = \left[\frac{M+\varepsilon+l_t(M-\varepsilon)}{l_t+1} - h\right](1+r) \tag{A.22}$$

According to condition 2.6 we have

$$w_{s,t} - w_{u,t} > h(1+r)$$

By multiplying both sides with -1 and adding $h(1+i)$ to both sides we have

$$w_{u,t} - w_{s,t} + h(1+i) < h(i-r)$$

which implies

$$\begin{aligned} F_\beta &= \left[\frac{M+\varepsilon+l_t(M-\varepsilon)}{l_t+1} - h\right](1+r) \\ &= \left\{\frac{1}{i-r}[w_{u,t}+h(1+i)-w_{s,t}] - h\right\}(1+r) \\ &< 0 \end{aligned}$$

Then we have $\frac{dl_t}{dh} < 0, \frac{dl_t}{dM} > 0, \frac{dl_t}{d\gamma} > 0, \frac{dl_t}{d\delta} < 0$ and $\frac{dl_t}{d\mathcal{B}} < 0$, which imply the effects of the parameters on the labour ratio. In addition, the effects of the parameters on the skill premium can be derived according to Equation 2.20.

A.0.4 Proof of Proposition 2.8

If $(1-\theta)(1+i) < 1$,

There are only three possible values for the bequests in the steady state: $\overline{x}_{\text{I}}, \overline{x}_{\text{II}}$ and $\overline{x}_{\text{III}}$,because any bequest that is not equal to any one of them will converge to one of them.

$\overline{x}_{\text{I}}$ and $\overline{x}_{\text{II}}$ exist in the steady state at the same time require $\overline{x}_{\text{I}} < f < \overline{x}_{\text{II}}$. $\overline{x}_{\text{I}} < f$ implies that

$$w_u > \frac{\theta + \theta r - r}{(\theta + \theta i - i)} [w_s - h(1+i)] \tag{A.23}$$

while $f \leq \overline{x}_{\text{II}}$ implies that

$$w_u < \frac{\theta + \theta r - r}{(\theta + \theta i - i)} [w_s - h(1+i)] \tag{A.24}$$

There is a contradiction between (A.23) and (A.24), thus $\overline{x}_{\text{I}}$ and $\overline{x}_{\text{II}}$ cannot exist in the steady state at the same time.

$\overline{x}_{\text{II}}$ and $\overline{x}_{\text{III}}$ exist in the steady state at the same time require $\overline{x}_{\text{II}} < h < \overline{x}_{\text{III}}$. $h < \overline{x}_{\text{III}}$ implies that

$$(1-\theta) w_s > h \tag{A.25}$$

$\overline{x}_{\text{II}} < h$ implies that

$$(1-\theta) w_s < h \tag{A.26}$$

There is contraction between (A.25) and (A.26), thus $\overline{x}_{\text{II}}$ and $\overline{x}_{\text{III}}$ cannot exist in the steady state at the same time.

$\overline{x}_{\text{I}}$ and $\overline{x}_{\text{III}}$ exist in the steady state at the same time require $\overline{x}_{\text{I}} < f < h < \overline{x}_{\text{III}}$. From the previous analysis, $h < \overline{x}_{\text{III}}$ implies that $h < \overline{x}_{\text{II}}$; while $\overline{x}_{\text{I}} < f$ implies that $\overline{x}_{\text{II}} < f$. Then there must be $f > h$, which violates $f < h$. Thus $\overline{x}_{\text{I}}$ and $\overline{x}_{\text{III}}$ cannot exist in the steady state at the same time.

Therefore, there could be only one level of bequest in the steady state, which means that the steady state is always an egalitarian one.

If $(1-\theta)(1+i) > 1$,

In this situation, $\overline{x}_{\text{II}}$ is not a stable point any more. So $\overline{x}_{\text{I}}$ and $\overline{x}_{\text{III}}$ exist in the steady state at the same time require $\overline{x}_{\text{I}} < f < h < \overline{x}_{\text{III}}$, which implies

$$w_u < \frac{\theta + \theta r - r}{(\theta + \theta i - i)} [w_s - h(1+i)] \tag{A.27}$$

$$(1-\theta)w_s > h \tag{A.28}$$

When these two conditions hold together, it is a polarized steady state, while if they are violated, it is an egalitarian one.

A.0.5 Proof of Proposition 2.10

According to the proof of Proposition 2.8, the condition for $\overline{x}_{\mathrm{I}} \leq f \leq \overline{x}_{\mathrm{II}}$ to hold is

$$w_{u,t} \leq \frac{\theta+\theta r-r}{(\theta+\theta i-i)}\left[w_{s,t}-h(1+i)\right] \tag{A.29}$$

By rewriting it we have:

$$w_{u,t} + \frac{\theta+\theta r-r}{(i-\theta-\theta i)}w_{s,t} \leq \frac{\theta+\theta r-r}{(i-\theta-\theta i)}h(1+i) \tag{A.30}$$

According to Proposition 2.6 and Equation A.4, the skilled wage $w_{s,t}$ increases when γ decreases. Also, skill premium $\frac{w_{s,t}}{w_{u,t}}$ decreases when γ decreases, which implies that $w_{u,t}$ must increase when γ decreases. Since $\frac{\theta+\theta r-r}{(i-\theta-\theta i)} > 0$, the left-hand side of Equation A.30 increases when γ decreases and the left-hand side of Equation A.30 is infinite when $\gamma \to 0$. Therefore, when γ is small enough, Equation A.29 is violated and the model reaches an egalitarian steady state. A similar conclusion can be proved for parameter δ.

Appendix B

Appendix 2

B.0.1 Proof of Proposition 4.2

Profit for a university is:

$$y_u(e_u, e_{-u}) = \tau \cdot N_u - \kappa_{e_u} \tag{B.1}$$

Thus, we have:

$$y(\underline{e}, \underline{e}) = \frac{1}{2}\tau \left\{\lambda L[1 - F(f_{\underline{a},\underline{e}})] + (1-\lambda)L[1 - F(f_{\overline{a},\underline{e}})]\right\} - \kappa_{\underline{e}} \tag{B.2}$$

$$y(\underline{e}, \overline{e}) = \tau\lambda L[1 - F(f_{\underline{a},\underline{e}})] - \kappa_{\underline{e}} \tag{B.3}$$

$$y(\overline{e}, \underline{e}) = \tau(1-\lambda)L[1 - F(f_{\overline{a},\overline{e}})] - \kappa_{\overline{e}} \tag{B.4}$$

$$y(\overline{e}, \overline{e}) = \frac{1}{2}\tau(1-\lambda)L[1 - F(f_{\overline{a},\overline{e}})] - \kappa_{\overline{e}} \tag{B.5}$$

If $y(\underline{e}, \underline{e}) > y(\overline{e}, \underline{e})$, we have

$$\begin{aligned} & \frac{1}{2}\tau \left\{\lambda L[1 - F(f_{\underline{a},\underline{e}})] + (1-\lambda)L[1 - F(f_{\overline{a},\underline{e}})]\right\} - \kappa_{\underline{e}} \\ > \; & \tau(1-\lambda)L[1 - F(f_{\overline{a},\overline{e}})] - \kappa_{\overline{e}} \end{aligned}$$

Therefore, we must have

$$
\begin{aligned}
& (\kappa_{\overline{e}} - \kappa_{\underline{e}})/L\tau \\
> \; & \frac{1}{2}(1-\lambda)[1-F(f_{\overline{a},\overline{e}})] - \frac{1}{2}(1-\lambda)[1-F(f_{\overline{a},\underline{e}})] \\
& +\frac{1}{2}(1-\lambda)[1-F(f_{\overline{a},\overline{e}})] - \frac{1}{2}\lambda[1-F(f_{\underline{a},\underline{e}})]
\end{aligned}
$$

If $y(\underline{e},\overline{e}) > y(\overline{e},\overline{e})$, we have

$$
\begin{aligned}
& \tau\lambda L[1-F(f_{\underline{a},\underline{e}})] - \kappa_{\underline{e}} \\
> \; & \frac{1}{2}\tau(1-\lambda)L[1-F(f_{\overline{a},\overline{e}})] - \kappa_{\overline{e}}
\end{aligned}
$$

Therefore, we must have

$$
\begin{aligned}
& (\kappa_{\overline{e}} - \kappa_{\underline{e}})/L\tau \\
> \; & \frac{1}{2}(1-\lambda)[1-F(f_{\overline{a},\overline{e}})] - \lambda[1-F(f_{\underline{a},\underline{e}})]
\end{aligned}
$$

Notoce that

$$
\frac{1}{2}(1-\lambda)[1-F(f_{\overline{a},\overline{e}})] - \frac{1}{2}(1-\lambda)[1-F(f_{\overline{a},\underline{e}})] > 0 \tag{B.6}
$$

As a result, if $y(\underline{e},\overline{e}) > y(\overline{e},\overline{e})$,it must be true that $y(\underline{e},\overline{e}) > y(\overline{e},\overline{e})$. But it is not necessarily true reversely. Therefore, there only exist three possible scenarios:

If $y(\underline{e},\underline{e}) > y(\overline{e},\underline{e})$ and $y(\underline{e},\overline{e}) > y(\overline{e},\overline{e})$, two universities operate in Tier 1: $\{e_1 = \underline{e}, e_2 = \underline{e}\}$. If $y(\underline{e},\underline{e}) < y(\overline{e},\underline{e})$ and $y(\underline{e},\overline{e}) > y(\overline{e},\overline{e})$, one university operates in each tier: $\{e_1 = \underline{e}, e_2 = \overline{e}\}$ or $\{e_1 = \overline{e}, e_2 = \underline{e}\}$. If $y(\underline{e},\underline{e}) < y(\overline{e},\underline{e})$ and $y(\underline{e},\overline{e}) < y(\overline{e},\overline{e})$, two universities operate in Tier 2: $\{e_1 = \overline{e}, e_2 = \overline{e}\}$.

B.0.2 Proof of Proposition 4.3

Let

$$
\begin{aligned}
G &= y(\underline{e},\underline{e}) - y(\overline{e},\underline{e}) \\
&= \frac{1}{2}\tau\left\{\lambda L[1-F(f_{\underline{a},\underline{e}})] + (1-\lambda)L[1-F(f_{\overline{a},\underline{e}})]\right\} - \kappa_{\underline{e}} \\
&\quad -\tau(1-\lambda)L[1-F(f_{\overline{a},\overline{e}})] - \kappa_{\overline{e}}
\end{aligned}
$$

Then we have

$$\frac{\partial G}{\partial \lambda} > 0 \tag{B.7}$$

Also

$$\frac{\partial G}{\partial L} = \frac{1}{2}\tau\left\{\lambda[1-F(f_{\underline{a},\underline{e}})] + (1-\lambda)[1-F(f_{\overline{a},\underline{e}})] - 2(1-\lambda)[1-F(f_{\overline{a},\overline{e}})]\right\} - \left(\kappa_{\overline{e}} - \kappa_{\underline{e}}\right) \tag{B.8}$$

Therefore, $\frac{\partial G}{\partial L} < 0$ when

$$\lambda < \mathring{\lambda} = \frac{2\left(\kappa_{\overline{e}} - \kappa_{\underline{e}}\right)/\tau - [1-F(f_{\overline{a},\underline{e}})] + 2[1-F(f_{\overline{a},\overline{e}})]}{[1-F(f_{\underline{a},\underline{e}})] - [1-F(f_{\overline{a},\underline{e}})] + 2[1-F(f_{\overline{a},\overline{e}})]} \tag{B.9}$$

Also

$$\frac{\partial G}{\partial H(\overline{a},\overline{e})} < 0 \tag{B.10}$$

B.0.3 Proof of Proposition 4.4

With a unified system, average human capital is

$$\overline{h}_t^{\upsilon} = (H(\underline{a},\underline{e})L^{\upsilon}_{\underline{a},t} + H(\overline{a},\underline{e})L^{\upsilon}_{\overline{a},t} + L^{\upsilon}_{u,t})/L \tag{B.11}$$

With a diversified system, average human capital is

$$\overline{h}_t^{\delta} = (H(\underline{a},\underline{e})L^{\delta}_{\underline{a},t} + H(\overline{a},\overline{e})L^{\delta}_{\overline{a},t} + L^{\delta}_{u,t})/L \tag{B.12}$$

Then

$$\overline{h}_t^{\delta} - \overline{h}_t^{\upsilon} = \frac{H(\overline{a},\overline{e})L^{\delta}_{\overline{a},t} - H(\overline{a},\underline{e})L^{\upsilon}_{\overline{a},t} - (L^{\delta}_{u,t} - L^{\upsilon}_{u,t})}{L} \tag{B.13}$$

Since $L^{\upsilon}_{u,t} - L^{\delta}_{u,t} = L^{\delta}_{\overline{a},t} - L^{\upsilon}_{\overline{a},t}$, $H(\overline{a},\overline{e}) > H(\overline{a},\underline{e})$ and $L^{\delta}_{\overline{a},t} > L^{\upsilon}_{\overline{a},t}$, we have

$$\overline{h}_t^{\delta} - \overline{h}_t^{\upsilon} = \frac{(H(\overline{a},\overline{e})-1)L^{\delta}_{\overline{a},t} - (H(\overline{a},\underline{e})-1)L^{\upsilon}_{\overline{a},t}}{L} \tag{B.14}$$

$$> 0 \tag{B.15}$$

Therefore, the diversified system could bring higher average human capital.

B.0.4 Proof of Proposition 4.5

Since $(1-\theta)(1+i) > 1$, there are only three possible fixed points: The uneducated fixed point, and fixed points for educated low-ability and high-ability agents.

$$\overline{x}_{\mathrm{I}} = \frac{(1-\theta)w_u\,(1+r)}{1-(1-\theta)(1+r)} \tag{B.16}$$

$$\overline{x}_{\underline{a},\mathrm{III}} = \frac{(1-\theta)\left[w(\underline{a},e_u,e_{-u}) - \tau\,(1+r)\right]}{1-(1-\theta)(1+r)} \tag{B.17}$$

$$\overline{x}_{\overline{a},\mathrm{III}} = \frac{(1-\theta)\left[w(\overline{a},e_u,e_{-u}) - \tau\,(1+r)\right]}{1-(1-\theta)(1+r)} \tag{B.18}$$

For all three points to exist at the same time, the following condition must hold: $\overline{x}_{\mathrm{I}} < f_{\overline{a},e} < f_{\underline{a},\underline{e}} < \overline{x}_{\underline{a},\mathrm{III}} < \overline{x}_{\overline{a},\mathrm{III}}$, which is feasible. The poverty trap exists if $\overline{x}_{\mathrm{I}}$ exists. Specifically, the condition for $\overline{x}_{\mathrm{I}}$ existing is:

$$w_u < \frac{\theta+\theta r - r}{(b-\theta-\theta b)}\left[\tau(1+b) - w(\overline{a},e_u,e_{-u})\right] \tag{B.19}$$

When w_u and $w(\overline{a},e_u,e_{-u})$ are low, this condition holds and there exists a poverty trap.

Given w_u, we can solve for a $\mathring{H}(\overline{a},e)$ that makes

$$w_u = \frac{\theta+\theta r - r}{(b-\theta-\theta b)}\left[\tau(1+b) - \mathring{w}(\overline{a},e_u,e_{-u})\right] \tag{B.20}$$

If the higher education system is transformed from a unified one to a diversified one, $w(\overline{a},\underline{e},\overline{e})$ is increased above $\mathring{w}(\overline{a},e_u,e_{-u})$, which violated the condition above. The poverty trap stops existing.

Appendix C

Code

C.0.1 Global Parameters

```
function globalparameters

global up_bond

theta=0.855;
% 1-theta: warm glow/saving rate
%theta=0.755;
%theta=0.955;

alpha=1/3;
sigma=0.286;
% substitution between skilled and unskilled workers

omega=0.632;
gamma=1;
delta=0.668/gamma;
b=11.37;
% technology frontier

%r_l=0.88;
```

```
% saving rate

r_l=0.88

i=1.0743;

r_b=20*i^20/(i^19+i^18+i^17+i^16+i^15+i^14+i^13+i^12+i^11+i^11+i^9...

+i^8+i^7+i^6+i^5+i^4+i^3+i^2+i+1)-1;

% borrowing rate

%r_b=0.9;

%r_b=2;


h=9728*4/(19900*(1-theta));

% cost of education

%h=20;

%h=0;


global sigma omega gamma b h r_l r_b alpha theta delta


end
```

C.0.2 Initial Bequest

```
function [ support, distribution ] = initial_bequest( initial_gini, dpi )

  standard_de=2^(1/2)*norminv((initial_gini+1)/2,0,1);
  % calculate standard deviation according gini;standard deviation,
  % respectively, of the associated normal distribution.

  mean_wealth=dpi;
  mu=log(mean_wealth)-standard_de^2/2;
  % mean, respectively, of the associated normal distribution.

  highest_wealth=logninv(0.999,mu, standard_de );
  original_wealth=linspace(0.001,highest_wealth,1000);
  % original_weslth cdf=0.999

  for i=1:1000;
   original_d(i)=lognpdf(original_wealth(i),mu,standard_de);
  end;

  sd=sum(original_d);

  for i=1:1000;
   distribution(i)=original_d(i)/sd;
  end
  % nomalized initial distribution

  distribution(1000)=distribution(1000)+0.001;
  support=original_wealth;
 % nomalized initial support

  global number highest_wealth distibution original_wealth mu;
end
```

C.0.3 Calibration

```
alpha=1/3;

sigma=0.286;

% substitution between skilled and unskilled workers

omega=0.632;

gamma=1;

delta=0.668/gamma;

b=11.37;

% technology frontier

% cost of education

% r_l=0.063;

i=1.0743;

r_b=20*i^20/(i^19+i^18+i^17+i^16+i^15+i^14+i^13+i^12+i^11+i^11+i^9...

+i^8+i^7+i^6+i^5+i^4+i^3+i^2+i+1)-1;

% r_b=10*i^10/(i^9+i^8+i^7+i^6+i^5+i^4+i^3+i^2+i+1)-1

% r_b: 10% 1989

thet=linspace(0.01,0.99,30);

for cal_theta=1:30;

theta=thet(cal_theta);

h=9728*4/(19900*(1-theta));

if r_b¡= (theta/(1-theta))

rl=linspace(0.01,r_b,30);

for cal_l=1:30;

r_l=rl(cal_l);

run main;

pre(cal_theta,cal_l)=premium;
```

```
lratio(cal_theta, cal_l)=lu_short;
cal_theta
end
else
rl=linspace(0.01,theta/(1-theta),30);
for cal_l=1:30;
r_l=rl(cal_l);
run main;
pre(cal_theta,cal_l)=premium;
lratio(cal_theta, cal_l)=lu_short;
cal_theta
end
end
end
for cal_theta=1:29;
    for cal_l=1:30;
        v(cal_theta, cal_l)=(pre(cal_theta,cal_l)-1.662)^2+(lratio(cal_theta,cal_l)-
0.614)^2;
    end
end
```

C.0.4 Static Equilibrium

```
global support distribution number up_bond sigma omega gamma b h r_l r_b alpha theta
delta

supportb=support;

% supportb: bequest at the beginning of each period

lu=linspace(0.00001,1,10000);

% lu: initial guess of unskilled labor

% ls: initial guess of skilled labor

for i=1:10000;

ls(i)=1-lu(i);

% ls: initial guess of skilled labor

au(i)=(b/(1+((1/delta)^(sigma/(omega-sigma))*(ls(i)/lu(i))^((sigma*omega)/(omega-sigma)))))^(1/omega);

as(i)=au(i)*(1/delta)^(1/(omega-sigma))*(ls(i)/lu(i))^(sigma/(omega-sigma));

% as: productivity of skilled labor

% au: productivity of unskilled labor

k(i)=(r_l/(((ls(i)*as(i))^sigma+(lu(i)*au(i))^sigma)^((1-alpha)/sigma)*alpha))^(1/(alpha-
1));

% k: capital

ws(i)=k(i)^alpha*(1-alpha)/sigma*((ls(i)*as(i))^sigma+(lu(i)*au(i))^sigma)^(((1-alpha)/sigma)-
1) ...

*sigma*(as(i)*ls(i))^(sigma-1)*as(i);

wu(i)=k(i)^alpha*(1-alpha)/sigma*((ls(i)*as(i))^sigma+(lu(i)*au(i))^sigma)^(((1-alpha)/sigma)-
1) ...

*sigma*(au(i)*lu(i))^(sigma-1)*au(i);

% ws: wage for skilled labor

% wu: wage for unskilled labor

f(i)=1/(r_b-r_l)*(wu(i)+h*(1+r_b)-ws(i));

% thredhold f for every guessed lu;
```

```
edu_profit(i)=-wu(i)-h*(1+r_l)+ws(i);
% edu_profit must ¿0
end
%for i=1:10000;
% diff(i)=abs(edu_profit(i));
%end
%mindiff=min(diff);
%p=find(diff == min(diff));
p=0;
for i=1:10000;
if edu_profit(i)¡0;
p=i+1;
end
end
c=lu(p);
% c is the smallest lu that ensure edu_profit¿0
d=0;
i=0;
while d¡=c;
i=i+1;
d=sum(distribution(1:i));
q=i;
end
% d: lu that
f_even=support(q);
e=f(p);
lu_even=lu(p);
```

```
if support(1)¡=h;

for n=1:1000;

if support(n)¡e;

breakpoint=n;

end

end

br=breakpoint;

l_ush=sum(distribution(1:br)) ;%%%%%5%5555

else

l_ush=0;

end

if l_ush¡=lu_even;

lu_short= lu(p);

ls_short=1-lu_short;

thredhold=f(p);

au_short=au(p);

as_short=as(p);

k_short=k(p);

ws_short=ws(p);

wu_short=wu(p);

else

for i=p:10000;

if support(1)¡=f(i);

for n=1:1000;

if support(n)¡f(i);

break_point=n;

end
```

```
end
b_r=break_point;
l_u(i)=sum(distribution(1:b_r)) ;
else
l_u(i)=0;
end
end
for i=p:10000;
diff(i)=abs(lu(i)-l_u(i));
end
% diff: the differences between guessed values and the returned values of unskilled labor
mindiff=2;
for i=p:10000;
if diff(i)¡=mindiff;
mindiff=diff(i);
a=i;
end
end
%mindiff=min(diff);
% in the short-run equilibrium, diff should be closest to zero
%a=find(diff == min(diff));
% a: the ”right” guess, the short-run equilibrium
lu_short=lu(a+1);
ls_short=1-lu_short;
thredhold=f(a+1);
au_short=au(a+1);
as_short=as(a+1);
```

```
k_short=k(a+1);
ws_short=ws(a+1);
wu_short=wu(a+1);
end

for i=1:1000;
if support(i)¿=h;
supportr(i)=(1-theta)*(ws_short+(support(i)-h)*(1+r_l));
% rich, no need to borrow to take education
elseif (support(i)¡h)&(support(i)¿=thredhold);
supportr(i)=(1-theta)*(ws_short+(support(i)-h)*(1+r_b));
% medium, borrow to take education
else support(i)¡thredhold;
supportr(i)=(1-theta)*(support(i)*(1+r_l)+wu_short);
% poor, no education
end
end
% given the education decisions, the change in bequest
% supportr: bequest for next period

support=supportr;
premium=ws_short/wu_short
lu_short

global lu_short support wu_short ws_short
```

C.0.5 Polarization Parameters

```
function globalparameters_polar

global up_bond

theta=0.855;

alpha=1/3;

sigma=0.286;

% substitution between skilled and unskilled workers

omega=0.632;

gamma=1;

delta=0.668/gamma;

b=11.37;

% technology frontier

% cost of education

r_l=0.88;

% bench

% i=1.0743;

% r_b=20*i^20/(i^19+i^18+i^17+i^16+i^15+i^14+i^13+i^12+i^11+i^11+i^9...

+i^8+i^7+i^6+i^5+i^4+i^3+i^2+i+1)-1;

% count

r_b=12

% r_b=10*i^10/(i^9+i^8+i^7+i^6+i^5+i^4+i^3+i^2+i+1)-1

% r_b: 10% 1989

% bench 1.199

% h=8306/(19900*(1-theta));

% count

h=35;
```

```
% warm glow?saving rate

global sigma omega gamma b h r_l r_b alpha theta delta

end
```

C.0.6 Main

```
run globalparameters_polar.m
% global cn theta
[init_support,distribution]=initial_bequest(0.4,1);

support=init_support;
global support distribution;
% the initial bequest distribution
%

run finalshort;
% a=0
%
% while premium~=a;
%
%
% run finalshort;
% a= premium;
% run finalshort;
% end
```

Bibliography

Acemoglu, D. (1998*a*), Changes in unemployment and wage inequality: an alternative theory and some evidence, Technical report, National Bureau of Economic Research.

Acemoglu, D. (1998*b*), 'Why do new technologies complement skills? directed technical change and wage inequality', *The Quarterly Journal of Economics* **113**(4), 1055–1089.

Acemoglu, D. (2003*a*), 'Cross-country inequality trends', *The Economic Journal* **113**(485).

Acemoglu, D. (2003*b*), 'Patterns of skill premia', *The Review of Economic Studies* **70**(2), 199–230.

Allmendinger, J. (1989), 'Educational systems and labor market outcomes', *European sociological review* pp. 231–250.

Autor, D. H. & Dorn, D. (2013), 'The growth of low-skill service jobs and the polarization of the us labor market', *The American Economic Review* **103**(5), 1553–1597.

Autor, D. H., Katz, L. F. & Krueger, A. B. (1998), 'Computing inequality: have computers changed the labor market?', *The Quarterly journal of economics* **113**(4), 1169–1213.

Banerjee, A. V. & Newman, A. F. (1993), 'Occupational choice and the process of development', *Journal of political economy* **101**(2), 274–298.

Beck, T., Demirguc-Kunt, A. & Levine, R. (2005), 'Smes, growth, and poverty: cross-country evidence', *Journal of economic growth* **10**(3), 199–229.

Beck, T., Demirgüç-Kunt, A. & Levine, R. (2007), 'Finance, inequality and the poor', *Journal of economic growth* **12**(1), 27–49.

Becker, G. S. (1975), *Human capital: a theoretical and empirical analysis, with special refence to education.*

Becker, G. S. & Tomes, N. (1979), 'An equilibrium theory of the distribution of income and intergenerational mobility', *Journal of political Economy* **87**(6), 1153–1189.

Blau, F. D. & Kahn, L. M. (1996), 'International differences in male wage inequality: institutions versus market forces', *Journal of Political Economy* **104**(4), 791–837.

Canidio, A. (2017), 'The technological determinants of long-run inequality', *Journal of Public Economic Theory* .

Card, D. (2009), Immigration and inequality, Technical report, National Bureau of Economic Research.

Caselli, F. & Coleman, W. J. (2006), 'The world technology frontier', *The American Economic Review* **96**(3), 499–522.

Clark, G. (2005), 'The condition of the working class in england, 1209–2004', *Journal of Political Economy* **113**(6), 1307–1340.

Cooper, R. & Liu, H. (2016), Mismatch in human capital accumulation, Technical report, National Bureau of Economic Research.

Corak, M. (2013), 'Income inequality, equality of opportunity, and intergenerational mobility', *The Journal of Economic Perspectives* **27**(3), 79–102.

Delisle, J. (2012), 'Federal student loan interest rates: History, subsidies, and cost. issue brief.', *New America Foundation* .

Dynarski, S. (2000), Hope for whom? financial aid for the middle class and its impact on college attendance, Technical report, National bureau of economic research.

Fairweather, J. S. (2000), 'Diversification or homogenization: How markets and governments combine to shape american higher education', *Higher Education Policy* **13**(1), 79–98.

Fallah, B. N., Partridge, M. D. & Olfert, M. R. (2011), 'New economic geography and us metropolitan wage inequality ', *Journal of Economic Geography* **11**(5), 865–895.

Feenstra, R. C. & Hanson, G. H. (1996), 'Globalization, outsourcing, and wage inequality', *The American Economic Review* **86**(2), 240–245.

Gall, T., Schiffbauer, M. & Kubny, J. (2014), 'Dynamic effects of foreign direct investment when credit markets are imperfect', *Macroeconomic Dynamics* **18**(8), 1797.

Galor, O. & Moav, O. (2000), 'Ability-biased technological transition, wage inequality, and economic growth', *The Quarterly Journal of Economics* **115**(2), 469–497.

Galor, O., Moav, O. & Vollrath, D. (2009), 'Inequality in landownership, the emergence of human-capital promoting institutions, and the great divergence', *The Review of economic studies* **76**(1), 143–179.

Galor, O. & Zeira, J. (1993), 'Income distribution and macroeconomics', *The review of economic studies* **60**(1), 35–52.

Goldin, C. & Katz, L. F. (2007), Long-run changes in the us wage structure: narrowing, widening, polarizing, Technical report, National Bureau of Economic Research.

Greenwood, J., Guner, N., Kocharkov, G. & Santos, C. (2014), 'Marry your like: Assortative mating and income inequality', *American Economic Review* **104**(5), 348–53.

Greenwood, J. & Jovanovic, B. (1990), 'Financial development, growth, and the distribution of income', *Journal of political Economy* **98**(5, Part 1), 1076–1107.

Gregorio, J. D. & Lee, J.-W. (2002), 'Education and income inequality: new evidence from cross-country data', *Review of income and wealth* **48**(3), 395–416.

Guvenen, F., Kuruscu, B. & Ozkan, S. (2014), 'Taxation of human capital and wage inequality: A cross-country analysis', *The Review of Economic Studies* **81**(2), 818–850.

Heckman, J. J. (2005), 'China's human capital investment', *China Economic Review* **16**(1), 50–70.

Heller, D. E. (1999), 'The effects of tuition and state financial aid on public college enrollment', *The Review of Higher Education* **23**(1), 65–89.

Jacoby, H. G. (1994), 'Borrowing constraints and progress through school: evidence from peru', *The Review of Economics and Statistics* pp. 151–160.

Jaimovich, E. (2010), 'Adverse selection and entrepreneurship in a model of development', *The Scandinavian Journal of Economics* **112**(1), 77–100.

Katz, L. F. & Murphy, K. M. (1992), 'Changes in relative wages, 1963–1987: supply and demand factors', *The quarterly journal of economics* **107**(1), 35–78.

Kijima, Y. (2006), 'Why did wage inequality increase? evidence from urban india 1983–99', *Journal of Development Economics* **81**(1), 97–117.

Krueger, D. & Kumar, K. B. (2004), 'Skill-specific rather than general education: A reason for us–europe growth differences?', *Journal of economic growth* **9**(2), 167–207.

Kuznets, S. (1955), 'Economic growth and income inequality', *The American economic review* pp. 1–28.

Li, H., Squire, L. & Zou, H.-f. (1998), 'Explaining international and intertemporal variations in income inequality', *The Economic Journal* **108**(446), 26–43.

Lindert, P. H. & Williamson, J. G. (1985), 'Growth, equality, and history', *Explorations in Economic History* **22**(4), 341–377.

Loury, G. C. (1981), 'Intergenerational transfers and the distribution of earnings', *Econometrica: Journal of the Econometric Society* pp. 843–867.

Lustig, N., Lopez-Calva, L. F. & Ortiz-Juarez, E. (2013), 'Declining inequality in latin america in the 2000s: the cases of argentina, brazil, and mexico', *World Development* **44**, 129–141.

McPherson, M. S. & Schapiro, M. O. (1991), 'Does student aid affect college enrollment? new evidence on a persistent controversy', *The American Economic Review* **81**(1), 309–318.

Melitz, M. J. & Ottaviano, G. I. (2008), 'Market size, trade, and productivity', *The review of economic studies* **75**(1), 295–316.

Murphy, K. M., Shleifer, A. & Vishny, R. (1989*a*), 'Income distribution, market size, and industrialization', *The Quarterly Journal of Economics* **104**(3), 537–564.

Murphy, K. M., Shleifer, A. & Vishny, R. W. (1989*b*), 'Industrialization and the big push', *Journal of political economy* **97**(5), 1003–1026.

Piketty, T., Goldhammer, A. & Ganser, L. (2014), 'Capital in the twenty-first century'.

Piketty, T. & Saez, E. (2003), 'Income inequality in the united states, 19131998*', *The Quarterly Journal of Economics* **118**(1), 1–41.

Psacharopoulos, G. (1994), 'Returns to investment in education: A global update', *World development* **22**(9), 1325–1343.

Shaked, A. & Sutton, J. (1987), 'Product differentiation and industrial structure', *The Journal of Industrial Economics* pp. 131–146.

Shavit, Y. et al. (2007), *Stratification in higher education: A comparative study*, Stanford University Press.

Teichler, U. (2008), 'Diversification? trends and explanations of the shape and size of higher education', *Higher education* **56**(3), 349–379.

Wood, A. (1995), *North-South trade, employment and inequality: changing fortunes in a skill-driven world*, Oxford University Press on Demand.

Zhang, H., Patton, D. & Kenney, M. (2013), 'Building global-class universities: Assessing the impact of the 985 project', *Research Policy* **42**(3), 765–775.