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Control of propagation of spatially localized polariton wave packets in a Bragg mirror with embedded quantum wells

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Abstract. We considered the nonlinear dynamics of Bragg polaritons in a specially designed stratified semiconductor structure with embedded quantum wells, which possesses a convex dispersion. The model for the ensemble of single periodically arranged quantum wells coupled with the Bragg photon fields has been developed. In particular, the generalized Gross-Pitaevskii equation with the non-parabolic dispersion has been obtained for the Bragg polariton wave function. We revealed a number of dynamical regimes for polariton wave packets resulting from competition of the convex dispersion and the repulsive nonlinearity effects. Among the regimes are spreading, breathing and soliton propagation. When the control parameters including the exciton-photon detuning, the matter-field coupling and the nonlinearity are manipulated, the dynamical regimes switch between themselves.

1. Introduction

Our nowadays' life is largely built on optoelectronic devices. The most of already existing devices operate separately with their optical and electronic parts. We believe that joining together the photonic and electronic parts will substantially change the face of modern optoelectronics, make new devices more efficient, targeted and successful. To answer this technological challenge, we resort to methods and approaches of Polaritonics, the interdisciplinary research area which is at the heart of modern optoelectronics and quantum light wave technologies.

Among the most demanded and promising optoelectronic devices are the devices for information storage and processing based on macroscopic manipulation by coherent coupled matter-light states [1]. Cavity quantum electrodynamics, the study of strong coherent interactions of quantized cavity electromagnetic field with matter, provides a versatile platform for this goal. Cavity-QED arrays composed of engineered optical cavity modes and few-level oscillators, e.g. excitons in quantum wells (QWs), may serve as many-body systems for the light manipulation [2]. However, the strength of light-matter coupling in microcavities is limited by the number of QWs embedded in each cavity. To overcome this technological limitation, the concept of Bragg polaritons has been proposed [3,4].



The Bragg polaritons are eigenmodes of the structure created by incorporation of QWs that are holders of excitons periodically throughout a Bragg mirror representing, in fact, a 1D binary photonic crystal.

Recently it has been shown that the modified Bragg mirror structures can act as hyperbolic dispersion materials for Bragg polaritons [5–7]. The effects of reduction of the group velocity and the strong negative refraction have been predicted in them. Control of the dispersion properties of the structure through manipulation by the radiative properties of excitons has been shown in the linear regime. However, the considered structure can demonstrate strong nonlinear properties in comparison with pure optical nonlinearities achieved [8]. The nonlinearity originated from the repulsive Coulomb interaction of excitons has a defocusing character. The combination of the defocusing nonlinearity and the anomalous dispersion allows one to expect formation of spatially localized Bragg polariton wave packets in the considered structure. Furthermore, the tunability of the dispersion and nonlinear properties of the structure opens the way to the control of dynamics of the polariton wave packets. These issues are addresses in our work.

2. The model of the resonant Bragg polaritons structure

The structure under consideration has been first proposed in [5,6]. It represents a 1D binary photonic crystal composed of alternating semiconductor layers of two types, A and B. The layers differ from each other by their widths and refractive indices, d_A, n_A and d_B, n_B . The structure is periodic with the period $d = d_A + d_B$. Single QWs that are holders of excitons are embedded in the centers of the layers of type A. Schematically variation of the refractive index in the structure growth direction is shown in figure 1. In the mean-field approximations the evolution of the coupled exciton-photon system can be described by the coupled Schrödinger and Gross-Pitaevskii equations for the photon, $\psi \equiv \psi(t, z)$, and the exciton, $\chi \equiv \chi(t, z)$, wave functions

$$i\hbar\partial_t\psi = \hat{H}_C\psi + \hbar\Omega_R\chi, \quad (1)$$

$$i\hbar\partial_t\chi = \left[\hat{H}_X + \alpha n_X |\chi|^2 \right] \chi + \hbar\Omega_R\psi \quad (2)$$

with the linear operators \hat{H}_C and \hat{H}_X given by

$$\hat{H}_{C,X} = -\frac{\hbar^2}{2m_{C,X}} \frac{\partial^2}{\partial z^2} + V_{C,X}(z), \quad (3)$$

where m_C and m_X are the effective masses of photons and bulk excitons, respectively. $V_{C,X}(z)$ describe potentials for photons and excitons that are periodic in the growth direction, $V_{C,X}(z) = V_{C,X}(z + d)$. The constant α is the strength of the exciton-exciton interactions that are responsible for nonlinear processes, n_X is the line density of excitons in the growth direction. The parameter Ω_R is the exciton-photon coupling constant.

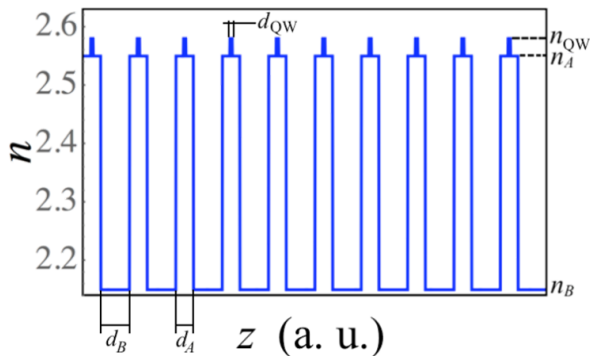


Figure 1. Schematic picture of the refractive index variation along the growth axis (labelled as z axis) of the structure of the Bragg mirror with embedded QWs.

To study the 1D macroscopic dynamics of light under the coupling with the QW excitons in the considered structure, we use one of the perturbation methods that is the method of multiple scales [9]. Following [10–12], we reduce the coupled photon and exciton equations (1) and (2) to the 1D equation for the slowly varying Bragg polariton envelope function. To this end, we introduce different time and space coordinate scales, $t_j = \varepsilon^j t$ and $z_j = \varepsilon^j z$, where ε is a dimensionless fictitious small parameter, $\varepsilon \ll 1$. Importantly, the new variables, t_j and z_j , are now considered independently.

We expand the solution of Eqs. (1) and (2) in the series

$$\psi = \sum_{j=1} \varepsilon^j \psi_j, \quad \chi = \sum_{j=1} \varepsilon^j \chi_j. \quad (4)$$

We now substitute the expansions (4) to Eqs. (1) and (2) provided with the introduction of the new variables, t_j and z_j . The further computational routine is connected with the subsequent analysis of the equations appearing at different orders of the perturbation parameter ε .

We search the first-order photon and exciton WFs in the following form:

$$\psi_1 = \zeta_1 \Xi(\mathbf{T}, \mathbf{Z}) e^{-i\Omega_{\nu n}(k)t} \phi_{\nu k}(z), \quad (5)$$

$$\chi_1 = \zeta_2 \Xi(\mathbf{T}, \mathbf{Z}) e^{-i\Omega_{\nu n}(k)t} f_{nk}(z), \quad (6)$$

where $\phi_{\nu k}(z)$ and $f_{nk}(z)$ are the eigenfunctions of the linear eigenproblems

$$\hat{H}_C \phi_{\nu k}(z) = \hbar \omega_{C\nu}(k) \phi_{\nu k}(z), \quad (7)$$

$$\hat{H}_X f_{nk}(z) = \hbar \omega_{Xn}(k) f_{nk}(z). \quad (8)$$

The variable k is the quasimomentum of particles that is actually the Bloch wave vector. The functions $\phi_{\nu k}(z)$ and $f_{nk}(z)$ obey the orthonormality conditions

$$\int_d \phi_{\nu k}^* \phi_{\nu' k'} dz = \frac{d}{2\pi} \delta_{\nu\nu'} \delta(k - k'), \quad \int_d \phi_{\nu k}^* \phi_{\nu' k'} dz = \frac{d}{2\pi} \delta_{\nu\nu'} \delta(k - k'). \quad (9)$$

In (5)–(8), ν and n numerate photonic and excitonic energy bands, $\omega_{C\nu}(k)$ and $\omega_{Xn}(k)$ describe dispersions for photons and excitons in the corresponding bands, respectively. $\Omega_{\nu n}(k)$ describes the dispersion of the eigenmodes of the modified Bragg mirror appearing as a result of coupling of photons in the band ν and excitons in the band n . Two modes are supported possessing the dispersion in the form:

$$\Omega_{\nu n}(k) = 0.5 \left(\omega_{C\nu}(k) + \omega_{Xn}(k) \pm \sqrt{\delta_{\nu n}^2(k) + 4\omega_R^2(k)} \right), \quad (10)$$

where $\delta_{\nu n}(k) = \omega_{C\nu}(k) - \omega_{Xn}(k)$ is the exciton-photon detuning, $\omega_R(k)$ is the k -dependent splitting of the structure eigenmodes, found as

$$\omega_R(k) = \frac{2\pi\Omega_R}{d} \int_d \phi_{\nu k}^* f_{nk} dz. \quad (11)$$

The function $\omega_R(k)$ is convex and symmetric with respect to $k=0$ that means $\omega_R(0) = \text{Max}[\omega_R(k)]$. For the simplicity, we make the notation $\omega_R = \omega_R(0)$.

The eigenmodes of the structure referred to as Bragg polaritons [4] belonging to different dispersion branches (11) can be considered independently of each other in different experimental conditions, e.g. in the case of resonant excitation of the lower branch. Below we pay our attention only to the lower-branch polaritons characterized by the eigenfrequency (10) with “−” at the last term.

The amplitude $\Xi(\mathbf{T}, \mathbf{Z})$ is a function of slow variables $\mathbf{T} = (\varepsilon t, \varepsilon^2 t)$ and $\mathbf{Z} = (\varepsilon z, \varepsilon^2 z)$. It describes the envelope function of the Bragg polariton wave packet. For the lower-branch polaritons the weighting coefficients $\zeta_{1,2}$ in (5) and (6) are given by

$$\zeta_{1,2} = \mp \frac{1}{\sqrt{2}} \left(1 \mp \frac{\delta_{vn}(k)}{\sqrt{\delta_{vn}^2(k) + 4\omega_R^2(k)}} \right)^{1/2}. \quad (12)$$

As a result of applying the method of multiple scales, we finally arrive at the nonlinear equation for the Bragg polariton envelope function

$$i\hbar \left(\frac{\partial \Psi}{\partial t} + v_g \frac{\partial \Psi}{\partial z} \right) = \hbar \hat{\Omega} \Psi + g |\Psi|^2 \Psi, \quad (13)$$

where we made the redefinition $\Psi = \varepsilon \Xi$, $\int_{-\infty}^{\infty} |\Psi|^2 dz = 1$. The operator $\hat{\Omega}$ describes the non-parabolic dispersion of polaritons. It is obtained by the change $k \rightarrow \hat{k}$ in (10), where $\hat{k} = -i\partial_z$ is the momentum operator. g is the nonlinearity coefficient

$$g = \frac{2\pi\alpha n_x |\zeta_2|^4}{d} \int_d |f_{nk}(z)|^4 dz. \quad (14)$$

3. Bragg polaritons with convex dispersion

We start our consideration with the QW-free structure which represents a conventional 1D photonic crystal. To reveal its dispersion properties, we follow the references [5–7] and use the convenient transfer matrix method [13].

Figure 2a demonstrates dispersions of the three lower photon eigenmodes of the infinite 1D photonic crystal. As a model structure, we consider GaN/Al_{0.3}Ga_{0.7}N structure. We choose the following parameters of the layers: $n_A = 2.55$, $d_A = 64.8$ nm and $n_B = 2.15$, $d_B = 115.3$ nm. Since the Bragg reflection condition is not fulfilled for our structure, i.e. $n_A d_A \neq n_B d_B$, the second photonic band gap opens. The centre, ω_{2Br} , and the half-width, $\Delta\omega_{2Br}$, of the gap are estimated as $\omega_{2Br} = 2\pi c/(n_A d_A + n_B d_B)$ and $\Delta\omega_{2Br} \approx 0.5(n_B - n_A)(1 - n_A d_A/n_B d_B)/(n_A + n_B)$, respectively — see Supplementary material to reference [5]. The second photonic band is convex that provides anomalous dispersion even in the center of the 1st BZ. Keeping the convex shape of the dispersion of eigenmodes of the structure in combination with the defocusing nonlinearity one possible to expect that the structure supports propagation of spatially localized optical wave packets.

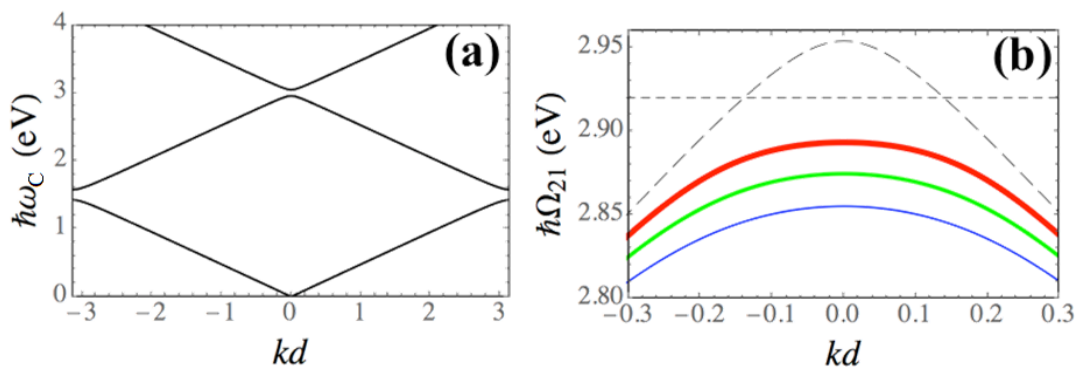


Figure 2. (a) Energy band gap structure of the QW-free photonic crystal. In the considered structure (parameters are given in the text) the second photonic gap opens. (b) Dispersion of the lower branch Bragg polaritons for different values of the exciton-photon coupling strength (from the thickest to the thinnest curves): $\hbar\omega_R = 40$ meV, 60 meV and 80 meV. The dashed curve and the dashed horizontal line correspond to the dispersions of non-interacting photons and excitons, respectively. The exciton-photon detuning is taken $\Delta = 35$ meV.

Now let us turn to the considered structure with embedded QWs. In order to obtain the structure possessing the convex dispersion of polariton eigenmodes, we choose QWs such that the ground-state ($n=1$) exciton energy is tuned to the vicinity of the top of the second photonic band, $\omega_{x1} = \omega_{c2}(0) - \Delta$, where Δ is the small k -independent exciton-photon detuning, $|\Delta| \ll \omega_{x1}, \omega_{c2}(0)$. Due to a strong confinement of excitons in QWs, ω_{x1} can be considered constant. In our model structure we consider $\text{In}_{0.12}\text{Ga}_{0.88}\text{As}$ QWs. Figure 2b demonstrates dispersion of the lower Bragg polariton mode resulting from coupling of the ϕ_{2k} photon mode and the f_{1k} exciton mode at different exciton-photon coupling strength ω_R .

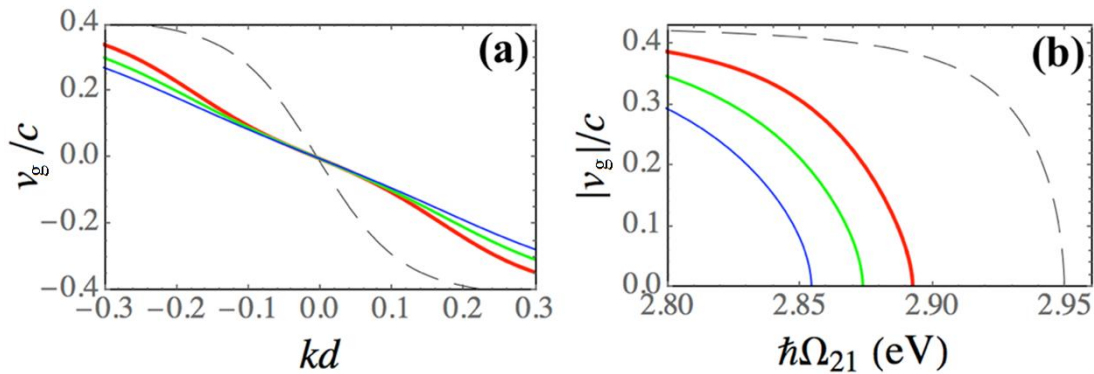


Figure 3. (a) The group velocity of the lower-branch Bragg polaritons in dependence of the polariton quasimomentum and (b) the parametric dependence of the group velocity on the polariton energy for different values of the exciton-photon coupling strength. Notations and values of the parameters are the same as those in figure 2b.

According to the definition, the group velocity of the Bragg polaritons can be found as $v_g = \partial\Omega_{21}/\partial k$ that gives

$$v_g = -i \frac{2\pi\hbar}{d m_c m_x} \left(m_x \zeta_1^2 \int_d \phi_{2k}^* \partial_z \phi_{2k} dz + m_c \zeta_2^2 \int_d f_{1k}^* \partial_z f_{1k} dz \right). \quad (15)$$

Noteworthy, the expression (15) obtained from the definition coincides with that found at the second order of the method of multiple scales. Figure 3a shows the group velocity of the lower-branch Bragg polaritons in dependence of their wave vector for different exciton-photon coupling strengths. Even at a small change of the wave vector the group velocity can change by tens of percent. It is noteworthy that the eigenwaves with positive wave vectors possess a negative group velocity while the eigenwaves with the negative wave vector propagate with the positive group velocity, i.e. the group and phase velocities in the considered structure are anti-parallel [14–15]. The optical waves with parallel group and phase velocities entering the structure from vacuum excite polariton Bloch waves with negative wave vectors that propagate with the positive group velocity. Figure 3b demonstrates the parametric dependence of the absolute value of the group velocity on the polariton energy for different coupling strengths. The variables v_g and Ω_{21} are linked through the polariton quasimomentum k .

4. Nonlinear dynamics of polariton wave packets in the convex-dispersion structure

The dynamics of Bragg polariton wave packets in the considered structure is determined by competition of the dispersion and nonlinearity effects. Summarizing the discussion above, the following options allow one to control the dispersion of Bragg polaritons (see (10)): the exciton-photon detuning Δ and the matter-field coupling strength ω_R . Herewith, if the former can be chosen only at the stage of growth of the structure, the latter can be tuned at any stage by external impact through the tuning of the radiation properties of excitons, see Appendix to reference [7].

The nonlinear effects result in the considered structure from the Coulomb repulsion of excitons within each QW. According to (14), the nonlinearity is k -dependent. To estimate this dependence, we use the assumption given above: we consider excitons perfectly localized within each QW and neglect interactions between QWs because of large distance between them. Neglecting the k -dependence of the exciton WF, we combine the coefficients in (14) in a fitting constant $G = 2\pi\alpha n_x d^{-1} \int_d |f_1(z)|^4 dz$ and re-write (14) in the reduced form

$$g = G |\zeta_2|^4. \quad (16)$$

We also have two options to control the nonlinearity in the considered structure. The first one is the exciton-photon coupling enclosed in $|\zeta_2|^4$ together with the dependence on k . The second option is the intensity of the wave packet proportional to the line density of excitons in the coefficient G .

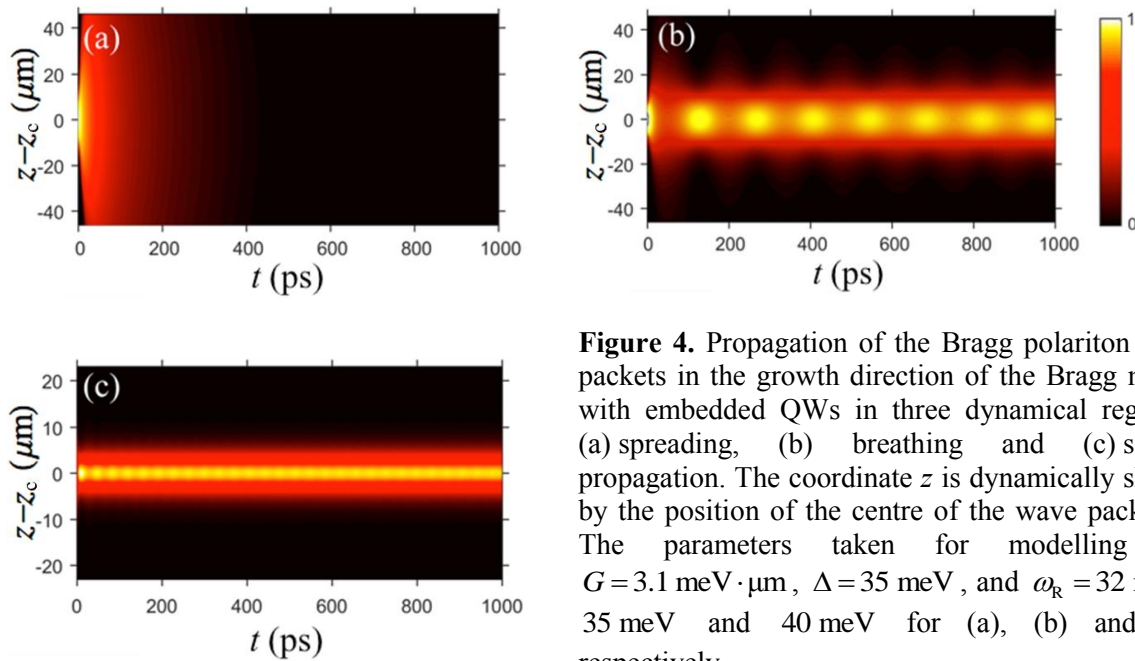


Figure 4. Propagation of the Bragg polariton wave packets in the growth direction of the Bragg mirror with embedded QWs in three dynamical regimes: (a) spreading, (b) breathing and (c) soliton propagation. The coordinate z is dynamically shifted by the position of the centre of the wave packet z_c . The parameters taken for modelling are $G = 3.1 \text{ meV} \cdot \mu\text{m}$, $\Delta = 35 \text{ meV}$, and $\omega_r = 32 \text{ meV}$, 35 meV and 40 meV for (a), (b) and (c), respectively.

In the considered structure, we revealed three dynamical regimes for Bragg polariton wave packets that are spreading, breathing and soliton propagation. Such a set of regimes is typical for dynamical systems. For example, similar regimes have been found in the discrete systems of ultracold atoms [16,17] and conventional atomic polaritons [18,19] with normal dispersion and attractive nonlinearity. Examples of the Bragg polariton wave packet evolution in the considered structure are presented in figure 4. The first regime is one of spreading, when the polariton wave packet blurs in time, see figure 4a. Two other regimes are spatially localized that means that the wave packet width changes in finite limits during propagation. In the breathing regime presented in figure 4b the wave packet moves with an oscillating width. In the soliton regime, the wave packet propagates with unchanged shape. Propagation of the polariton wave packet in the regime close to solitonic is presented in figure 4c. All pictures were obtained for the fixed detuning Δ and the nonlinearity coefficient G . The matter-field coupling constant ω_r is the only control parameter different for all pictures.

5. Conclusions

We have considered the nonlinear dynamics of the Bragg exciton-polaritons in the modified Bragg mirror with embedded periodically arranged QWs. Starting with the Schrödinger and Gross-Pitaevskii equations for photon and exciton wave functions in a spatially periodic potential, we have obtained the

generalized nonlinear Gross-Pitaevskii equation with the convex non-parabolic dispersion for Bragg polaritons. We have shown the possibility of formation of localized polariton wave packets under the competition of the effects of the convex dispersion and defocusing nonlinearity. We have revealed three dynamical regimes for polaritonic wave packets including spreading, breathing and soliton propagation. Switching between the regimes can be carried out using the control parameters that are the exciton-photon detuning, the matter-field coupling strength and the intensity of the polariton wave packet. Our results provide a platform for the studies of collective properties of light in stratified nonlinear media.

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