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INFORMING THE PEDAGOGY FOR GEOMETRY: LEARNING FROM TEACHING APPROACHES IN CHINA AND JAPAN

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An authoritative report into the teaching and learning of geometry argued, amongst other things, that the most significant contribution to improvements in geometry teaching are to be made by the development of good models of pedagogy, supported by carefully designed activities and resources. This paper considers approaches to the teaching of geometry developed in China and Japan and reviews what research might be able to contribute to developing new pedagogic approaches.

INTRODUCTION

Geometry is recognised as not only one of the most important components of the school mathematics curriculum but also, alongside algebra, as one of most important elements of mathematics itself (Royal Society, 2001; Atiyah, 2001). The reasons for including geometry in the school mathematics curriculum are myriad and encompass providing opportunities for learners not only to develop spatial awareness, geometrical intuition and the ability to visualise, but also to develop knowledge and understanding of, and the ability to use, geometrical properties and theorems (Jones, 2000, 2001, 2002). This being the case, it can be argued that teaching approaches need to encompass the encouragement of the development and use of conjecturing, deductive reasoning and proof, as well as developing skills of applying geometry through modelling and problem solving in a range of contexts (including real world ones), and an awareness of the historical and cultural heritage of geometry in society, and of the contemporary applications of geometry (Clausen-May *et al*, 2000; Jones, 2000). All these considerations tend to make geometry a demanding element of mathematics to teach well, especially when other topics in the mathematics curriculum (such as numeracy and algebra) can dominate curricula considerations (Jones & Mooney, 2003).

In terms of effective pedagogy for geometry, the general situation appears to be that despite many efforts, as Howson (2003) attests, “Euclid-style geometry [is] found extremely difficult (and often uninteresting) by most [school] students”. Nevertheless, in a number of countries there are teachers continuing to work hard at designing lessons that focus on helping learners make the difficult transition to deductive thinking in geometry. This report focuses on how improvements in geometry teaching might result from the development of good models of pedagogy, supported by carefully designed activities and resources, and how a consideration of approaches to the teaching of geometry developed in China and Japan might inform research designed to contribute to developing new pedagogic approaches for geometry.

TEACHING GEOMETRY IN CHINA AND JAPAN

The opportunity to focus on China and Japan is useful as there are interesting similarities and contrasts. For example, both countries have National Curricula for mathematics that cover geometry, amongst other mathematical topics (JSME, 2000; Ministry of Education PRC, 2001). Yet, for teachers in the two countries there are different traditions and different ways in which they have responded to international developments over the years (Jones, Fujita & Ding, 2005).

In this paper we provide examples of geometry teaching designed for Year 9 (called Grade 8 in both China and Japan) from each of these countries.

Teaching Year 9 geometry in China: a theorem about parallelograms

In the following extract from the lesson, T is the teacher, Ss represents more than two students, other letters are individual students, names are pseudonyms.

T: So far we have only known one way to prove a shape is a parallelogram, that is, to use its definition to prove. This means that we need to prove that two pairs of opposite sides are parallel. So, here, we use '{' to show that we need to prove that both pairs of opposite sides are parallel. (The teacher wrote down '{//, //' on the blackboard).

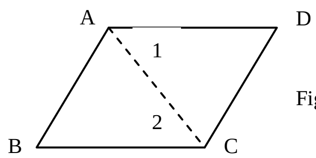

Fig. 1

T: To prove two lines are parallel, what methods did we learn early in grade 7?

Ss: The alternate interior angles.

T: We could use the 'three lines and eight angles'*, couldn't we? (*three lines refers to when two parallel lines are intersected by a transversal; eight angles mean that eight angles, less than 180°, are formed by the three lines). So we need to create the basic figure of three lines and eight angles here. I need to link points A and C. Of course, you could link B and D (The teacher linked A and C in figure 1).

T: Now, I linked A and C. If I want to prove AD//BC, what actually must I prove here?

Ss: Angles are equal (means angle 1 and angle 2).

T: If I want to prove angle 1 is equal to angle 2, what should I prove? (The teacher wrote down // ➔ 1= 2 on the blackboard).

Ss: Congruent triangles.

T: Good. Now we need to prove congruent triangles. Considering about the condition for proving congruent triangles. Here, we have already known that two pairs of sides of triangles are equal. Moreover, the two triangles (means triangles ABC and ADC) share one side. So it is quite easy to prove the congruent triangles, isn't it? (The teacher wrote down 1= 2 ➔ (sss) on the blackboard).

T: So, from now on, as we have learned geometry for over one year, we need to know how to analyze the problem, when we prove a fact. A good student always considers the methods used for proving the problem. An average student still tries to learn how to write logical paragraphs. Here, you should firstly write the proof about congruent triangles. Secondly, you need to write the proof of equal angles. Finally, you write the proof of parallel lines. OK?

T: Now, you know how to prove one pair of opposite sides of the quadrilateral are parallel. In the same way, you could prove another pair of opposite sides of this quadrilateral. But you do not need to write the proof again, as the method is as same as that of the first. You could omit one logical paragraph, and directly provide the fact. Therefore, the quadrilateral is a parallelogram. What is the reason?

Ss: (Very low voice). The definition of parallelogram.

T: Now the method to prove this problem is clear. In the previous lesson, Ning did not fully prove this problem due to the limited time. I pointed out a disadvantage of her proof writing. I still emphasize here that you should write correspondingly the vertices of congruent triangles. In this figure, you need to write that triangle ABC is congruent to triangle CDA. As in triangle ABC, A is opposite to BC.

T: OK, we know that the converse statement of the property of a parallelogram (the property is that 'both pairs of opposite sides of a parallelogram are parallel) is a true statement. This converse statement is that 'if two pairs of opposite sides of a quadrilateral are equal, then this quadrilateral is a parallelogram'. Is this converse statement about the property of a parallelogram? Or about verifying a parallelogram? You see, the conclusion is a parallelogram. So it is used to verify whether a quadrilateral is a parallelogram or not. Therefore, we use this statement as a new theorem to verify parallelogram.

T: So far, except for the definition of parallelogram, we learn a new theorem to verify parallelogram. (The teacher wrote down the lesson title 'The theorem of verifying parallelogram' on the blackboard). Well, who could use words to state this new theorem again? And who could use mathematical language to state the theorem? Lin, could you?

L: If two pairs of opposite sides of a quadrilateral are equal, then this quadrilateral is a parallelogram.

T: Very good. (The teacher repeated the theorem). We could also use mathematical language to state this theorem. In quadrilateral ABCD, what is already known? See, we know that tow pairs of opposite sides are equal. AD=BC, and AB=CD. So I could tell that this is a parallelogram. The reason is the theorem we just learned. (The teacher repeated the theorem). (The teacher wrote down 'In quadrilateral ABCD, AD=BC, AB=CD ( ), Quadrilateral ABCD is a parallelogram. ( )' on the blackboard). We need to use words to state the theorem after we verify that this is a parallelogram. So what is it, if two pairs of opposite sides of a quadrilateral are equal?

Ss: It is a parallelogram.

Teaching Year 9 geometry in Japan

The way teachers structure their lessons in Japan is influenced (as in China and, no doubt, elsewhere) by the specification of the mathematics curriculum, the demands of examinations, and the design of textbooks. Our analysis also suggests that lesson designs in Japan are also influenced by the occurrence of 'Lesson Studies' and by recent Japanese research into the learning and teaching of mathematics. For example, 'Lesson study', practiced by teachers in Japan for the last several decades, is one of the most common forms of professional development and involves teachers working in small teams collaboratively crafting lesson plans through a cycle of planning, teaching and reviewing (Yoshida 1999). Through this process, Japanese teachers appear to have collaboratively developed a view about 'good lessons of mathematics'.

For example, to teach the properties of the parallel lines and ratio in Year 9, teachers would organise a lesson (50 min.) as follows. First, a problem for the day is introduced. To understand the relationship between the parallel lines and ratio, it is useful to know properties of similar triangles. Thus, this lesson could start from a problem 'Let us prove that if PQ//BC in a triangle ABC, then triangles APQ and ABC are similar to each other'. Then, in the development stage, students would undertake to prove this problem, either individually or in groups. Proof of this problem is shared in a whole classroom, and finally, the topic of this lesson is summarised as 'If PQ//BC in a triangle ABC, then triangles APQ and ABC are similar to each other, and therefore AP:AB=AQ:AC=PQ:BC, and if PQ//BC then AP:PB=AQ:QC' (Summary stage).

Kunimune *et al* (2002, p. 69) state that the lesson plan above is a typical one in Japanese secondary schools (we call this 'Typical approach') which is often a result of the lesson study, and could be time efficient, but they speculate that students might be rather passive in this format. As an alternative approach, Kunimune et al (ibid., p. 69-70) propose that a lesson can start from geometrical construction (i.e. construction by only ruler and compass), and we call this approach 'Construction approach'. This lesson starts from a more challenging construction problem 'Let us consider how we can trisect a given straight line AB.' Then, one of ideas from students will be chosen and proof of it will be considered within group work. Finally a theorem is introduced and summarised as 'In a triangle ABC, P and Q are on the line AB and AC respectively. If PQ//BC then AP:AB=AQ:AC=PQ:BC and AP:PB=AQ:QC.' which students would have noticed during the construction activities.

This approach would make the lesson more active, and encourage students to consider why constructions work. Also, students would be able to discover theorems/properties of geometrical figures through construction activities. In fact, the report by the Royal Society/Joint Mathematical Council working group suggest that "the mathematics curriculum should be developed to encourage student to work investigatively, demonstrate creativity and make discoveries in geometrical contexts so that students develop their powers of spatial thinking, visualisation and geometrical reasoning" (Royal Society, 2001, p. 10). A possible disadvantage is, however, that this lesson could be very time consuming in this format (the suggested plan is 3 hours).

It is difficult to conclude ‘Construction approach’ is better than ‘Typical approach’ or vice versa, and rather a task for us is to consider what research would be necessary to establish a good pedagogical model in the teaching of geometry.

CONCLUDING COMMENTS

In both China and Japan, the teaching and learning of deductive reasoning in geometry remains a major objective in Year 9 (Grade 8) but this is not without its problem. For example, research in Japan indicates that while most 14-15 year-old Japanese students can write down a proof, around 70% cannot understand why proofs are needed (Kunimune, 2000). Similar results with a UK student who was educated in Hong Kong are reported in Healy and Hoyles (1998; p. 166).

To counter this, teachers in both countries not only try to maintain students’ interest, but also aim to assist students in creating definitions and conjectures and in gaining insight into new geometrical relationships and inter-relationships.

The recent UK study of geometry teaching (Royal Society, 2001) concludes that “the most significant contribution to improvements in geometry teaching will be made by the development of good models of pedagogy, supported by carefully designed activities and resources” (p19). The analysis of lessons given by mathematics teachers in countries such as China and Japan might inform the development of new pedagogical approaches to teaching geometry. In future research it is necessary to consider questions such as ‘What pedagogical background is underpinned in Chinese and Japanese geoemtry teaching?’, ‘What can we learn from Chinise and Japanese geoemtry teaching?’, ‘What theoretical and methodological models should we develop to examine effective geometry lessons?’ etc.

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BSRLM GEOMETRY WORKING GROUP

The BSRLM geometry working group focuses on the teaching and learning of geometrical ideas in its widest sense. Suggestions of topics for discussion are always welcome. The group is open to all.

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