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PRIMARY TRAINEE TEACHERS' KNOWLEDGE OF PARALLELOGRAMS

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Considerable research has indicated that amongst the factors which make the most significant contribution to high student achievement in mathematics is secure subject knowledge on the part of the teacher as this underpins an approach to mathematics in which topics are seen as part of a coherent set of related ideas, with clear progression and links to previous and future learning. This paper reports part of the findings from a study of trainee teachers' knowledge of basic geometrical figures, in particular focusing on what knowledge they have of parallelograms and how they use this knowledge to solve geometrical problems. The findings indicate that only a minority of trainee primary teacher have a fully sophisticated knowledge of parallelograms and of how to use the properties of parallelograms to solve relevant problems.

INTRODUCTION

It is well known that teachers' mathematics knowledge plays a significant role in shaping the quality of their teaching (Ball, Hill & Bass, 2005). Yet as Ball et al (ibid, p16) explain, "although many studies demonstrate that teachers' mathematical knowledge helps support increased student achievement, the actual nature and extent of that knowledge - whether it is simply basic skills at the grades they teach, or complex and professionally-specific mathematical knowledge - is largely unknown". This is not to downplay the studies of teachers' mathematical knowledge that have been, and are being, carried out. More it points to the complexity of the issues involved, especially since the context in which teachers gain their own mathematical knowledge, and the form of teacher training they receive (both pre- and in-service), can be so varied, not only across countries, but also within particular countries.

The data reported in this paper are from one component of a larger study being carried out in the UK. The over-arching focus is on teachers' knowledge of geometry since, at this time in the UK, the nature of the school curriculum is under review (QCA, 2005) and there are recommendations that the geometry component of the mathematics curriculum requires special attention and strengthening (RS/JMC, 2001). The chosen focus for this report is on what knowledge trainee teachers have of parallelograms and how they utilise this knowledge when tackling geometrical problems.

CLASSIFICATION OF PARALLELOGRAMS

Mathematicians prefer a hierarchical classification for quadrilaterals (de Villers, 1994) and school curricula also follow this. One reason for this preference is its economical character; that is, if a statement is true for parallelograms, this means that it is also true for squares, rectangles and rhombuses. While this might seem straightforward to mathematicians, a number of studies have shown that many students have problems with a hierarchical classification of quadrilaterals (de Villers, 1994, p17; Jones, 2000).

Van Hiele's model of the learning of geometry, which suggests that learners advance through levels of thought in geometry (Crowley, 1987, van Hiele, 1999), is generally considered to be a fairly useful model to describe students' behaviours in geometry (Senk, 1989). The model specifies the following four levels: Level 1, visualisation: identifying shapes according to their concrete examples; Level 2, analysis: identifying shapes according to their properties; Level 3, informal deduction: identifying relationships between shapes & producing simple logical deduction; Level 4, deduction: understanding logical deduction.

In terms of this model, students at van Hiele level 3 are, for example, expected to be able to deduce that a rectangle is a special type of parallelogram by considering definitions and properties of these quadrilaterals. Students at level 2 start recognising properties of individual shapes (e.g. that in a square all the sides are the same and that all the angles are the same), while students at the level 1 would recognise a square or rectangle from their shape, and that they are different from a circle. Research evidence suggests that the rate of progress from level 2 to 3 made by many students is slow, or even that many of them remain at level 2 by the end of the secondary (high) schools (Senk, 1989). Thus, the hierarchical classification of quadrilaterals, taken to be van Hiele level 3, can be regarded as a difficult task for many children.

There is some existing evidence that trainee primary teachers have relatively weak knowledge of geometry (see, for example, Jones, Mooney and Harries, 2002). Data from observing such trainee teachers has also indicated that difficulty with the hierarchical classification of quadrilaterals appears to persist with trainee teachers as at least some of them cannot accept, for example, that 'a square is a special type of a rectangle' (Fujita & Jones, 2006). In this paper, we explore this latter issue in particular by considering the questions 'what images of parallelograms do trainee teachers have, and how do they use them to solve a geometrical problem?'

METHODOLOGY

In order to investigate the research questions, trainee primary teachers on a four-year undergraduate teacher training course in Scotland were selected because the curriculum guidelines for Scotland specify that most pupils are expected to be able to define quadrilaterals and classify them in accordance with their properties by the time they are aged 14-15. What is more, the expected level of understanding of mathematics for trainees on the course is that, to be allowed to commence the course, trainee have to have a level of mathematics indicating that they are able to classify quadrilaterals according to their definitions and properties (in Scotland this is called 'Standard Grade Credit level').

We conducted a survey of 105 trainee primary teachers in their second year of University study (most were 19~20 years old) in February 2006. The questionnaire used is adapted from one originally designed to measure the level of understanding of parallelograms held by equivalent trainees in Japan, and it consists of six questions. Because of the limited space of this paper, we report a part of results relating to items 1, 3 and 5 of the questionnaire:

Q1. Choose which are parallelograms from the quadrilaterals 1~15 below.

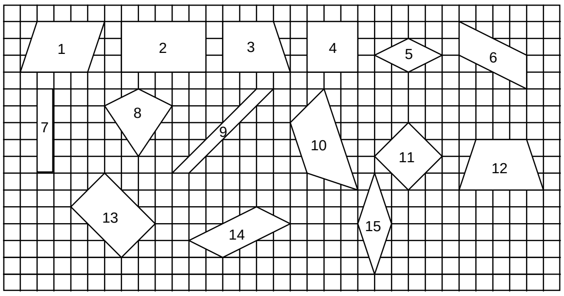


Figure 1. Quadrilaterals in Q1

Q3. Is it possible to draw a parallelogram whose four vertices are on the circumference of a circle? Choose your answer a. or b. If you choose (a), state your opinion why it is not possible. If your answer is (b), draw its shape and name in the circle. (a) No it is not possible, because … or (b) Yes, it is possible.

Q5. What is the quadrilateral described as 'a parallelogram which has a right angle'?.

While Q1 checks students' basic knowledge of parallelograms, Q3 asks students to determine parallelograms which can be inscribed in a circle. The answer to Q3 is a rectangle. This question checks whether students can use a hierarchical relationship to solve a problem, i.e. whether they can understand that it is not only a square, but also any kind of rectangle, of which a square is just one, which can be inscribed in a circle. Similarly, Q5 checks whether they can decide the type of a parallelogram which will satisfies additional information (a right angle). The table below summarises the marking criteria for each question. For example, in Q1, if a student can identify parallelograms correctly (i.e. 1, 2, 3, 4, 5, 6, 7, 9, 11, 13, 14, 15 in fig. 1), he/she receives '3' points for Q1.

Table 1. Marking criteria for Q1, Q3 & Q5

	3 pt.	2 pt.	1 pt.	0 pt.
Q1	1, 2, 4, 5, 6, 7, 9, 11, 13, 14, 15	1, 6, 9, 14, 5, 11, 15 or 1, 2, 4, 6, 7, 9, 11, 13, 14	1, 6, 9, 14	Others
Q3	b. and draws & name a rectangle	b. and draws & name a rectangle and a square	b. and draws a correct image b. and draws & name a rhombus	Others
Q5	Rectangle Rectangle (square)	Rectangle & square Square	Trapezium	Others

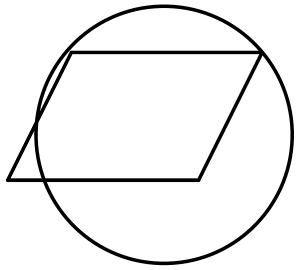
RESULT AND DISCUSSION

Table 2 summarises the results of the Q1, Q3 and Q5.

Table 2. The result of Q1 & Q3

	Q1	Q3	Q5
3 pt	21 (20%)	8 (7.6%)	45 (42.9%)
2 pt	27 (25.7%)	11 (10.5%)	23 (21.9%)
1 pt	47 (44.8%)	10 (9.5%)	0 (0%)
0 pt	10 (9.5%)	76 (72.4%)	37 (35.2%)

Figure 2. Q3 & Prototype image



As table 2 shows, while just 21 of 105 trainees (20%) could identify all correct images of parallelograms, 47 (44.8%) chose the images 1, 6, 9 and 14 (fig. 1) which are likely to be considered to be prototype images of parallelograms. This implies that almost half students still regard parallelograms in terms of prototype images. The performance by the trainees on Q3 shows their weaknesses with this topic. While just 8 trainees (7.6%) could answer that ‘it is a rectangle which satisfies the Q3’, 76 (72.4%) answered ‘(b) No, it is not possible’. Of these 76 trainees, 20 trainees gave no answer, 15 reasoned their answer by stating ‘you will always get a right angle to draw a parallelogram in a circle’ or ‘There will only 2 vertices which touch’, five just drew an image based on a prototype image of a parallelogram (fig. 2), and 20 stated their reasons and drew an image based on the prototype image, i.e. 40 of 76 trainees who scored ‘0’ point in the Q3 used the prototype image of a parallelogram to tackle this question. The performance for Q5 is slightly better than the other questions, in that 45 trainees (42.9%) could answer correctly, i.e. they could answer that a parallelogram which has a right angle is a rectangle.

In Q1, about half of the trainees chose just prototype images of parallelograms, and, again, 40 used the prototype images to solve the Q3. There is a slight correlation between the performances of these questions. The value of the Pearson's correlation coefficient between Q1 and Q3 is $r=0.48$ ($p<0.01$), i.e. the trainees who could choose the correct images of parallelograms are likely to get better scores than those who just chose the prototype images. A reason for why there is not a strong correlation is that 6 of 21 trainees who scored '3' points and 18 of 28 who scored '2' points for Q1 could not answer correctly Q3, i.e. their ability to control their images still does not appear consolidated when they solve geometrical problems. In Q1, 27 trainees scored '2' points, and 17 of them (about 63% of 27) chose the images of rectangles (e.g. 2, 7, 13 in fig. 1) or squares (4 or 11 in fig. 1), and the others chose the prototype images of parallelograms (1, 6, 9, 14 + 5, 9, 15 in fig. 1). Interestingly, 8 of 17 could get either '1', '2', or even '3' points for Q3. This implies that one of important factors in solving Q3 is what images of parallelograms they would utilise.

The results from Q3 and Q5 somehow contradict each other. While only 18 could see 'a rectangle (or a square)' as 'a parallelogram', in Q5 more than half of them could answer 'a parallelogram which has a right angle is a rectangle (or a square)', i.e. they could see 'a rectangle (or a square)' as 'a parallelogram'. A reason for this idiosyncratic result is still uncertain, but a possible reason is that the words 'the quadrilateral' and 'a right angle' in the question might remind them of 'a rectangle (or a square)', rather than them using geometrical reasoning to answer this question.

CONCLUDING COMMENT

The findings indicate that only a minority of trainee primary teacher have a fully sophisticated knowledge of parallelograms and of how to use the properties of parallelograms to solve relevant problems.

In terms of the van Hiele model, on the one hand, the results described above can be explained by the fact that the trainees are still at the level 2 (or below) and therefore they just choose only prototype images of parallelograms, and do not use geometrical reasoning when they solve Q3. On the other hand, to be allowed to commence their training course, the trainees have to have a mathematics qualification at a level that indicates that they should be able to classify quadrilaterals according to their definitions, i.e. the level 3 in the van Hiele model. The results described above suggest that regression has happened after they have achieved Scottish Standard Grade Credit level (at age 16). It is necessary to investigate why this regression might happen, in particular focusing on curriculum design, textbooks, actual teaching of geometry etc. As Ball et al (2005, p. 16) recommend "What is needed are more programs of research that complete the cycle, linking teachers' mathematical preparation and knowledge to their students' achievement".

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