Energy Efficient Subchannel and Power Allocation for Software Defined Heterogeneous Networks

Haijun Zhang, *Senior Member, IEEE*, Na Liu,
Keping Long, *Senior Member, IEEE*, Julian Cheng, *Senior Member, IEEE*,
Victor C. M. Leung, *Fellow, IEEE*, and Lajos Hanzo, *Fellow, IEEE*

***Abstract*—Visible light communication (VLC) is considered as a promising candidate to improve the performance of indoor communication as the complement of wireless radio frequency (RF) communications due to the scarcity of RF resources. Combining VLC with software-defined small cell networks will substantially improve the user data rates in indoor heterogeneous networks. In this paper, we introduce the software defined philosophy into orthogonal frequency division multiple access (OFDMA) based heterogeneous software-defined and twinned VLC and RF small-cell networks. The pivotal issues of energy efficient subchannel and power allocation are investigated in the context of software-defined VLC and RF small-cell networks. We formulate the energy efficient resource allocation problem as a non-convex optimization problem, and then transform it into a convex one using Dinkelbach's method. Additionally, distributed subchannel and power allocation algorithms for both VLC and RF are proposed for solving the problem based on the powerful alternative direction method of multipliers (ADMM). Simulation results verify the effectiveness of resource allocation algorithms conceived for heterogeneous software-defined twinned VLC and RF small-cell networks in terms of its good convergence and overall performance.**

***Index Terms*—Cooperative Jamming, non-orthogonal multiple access, physical layer security, energy efficiency.**

I. Introduction

The fifth-generation (5G) system is expected to cope with the massive throughput requirements of flawless lip-synchronized multi-media delivery relying on virtualized software-defined network functionalities [1]. The next-generation mobile network will be heterogeneous, integrated and green [2], [3]. To elaborate, in order to support customized services and high-rate data transmission, this heterogeneous network will rely on macro-cells, femto-cells, pico-cells and atto-cells, where several radio access technologies are integrated into a universal network, forming an overlapping coverage area [4]–[6]. However, both the spectral resources and the energy resources are severely limited [7]–[9].

Light emitting diodes (LEDs) have gradually replaced the antiquated incandescent and fluorescent lamps as a benefit of their low power consumption and of their environment-friendly attributes [10]–[12]. Facilitated by the low cost and energy-efficiency of the LEDs, VLC has emerged as an attractive candidate for future indoor communication [13]–[18]. Hence, VLC networks constitute a compelling complement of RF networks in indoor environments with ample spectral resources ranging from 385 Terahertz to 789 Terahertz. Figure 1 gives a typical indoor heterogeneous VLC and RF system deployment scenario. VLC communication is also immune to RF interference. Accordingly, both illumination and communication can be readily realized by VLC systems upon suitably modifying the existing lighting infrastructure [19]–[22].

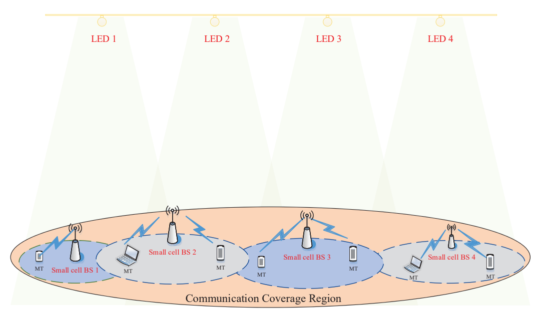

Fig. 1. Heterogeneous VLC and RF system scenario.

Software defined networking (SDN) constitutes a novel paradigm for 5G mobile communication, which is characterized by the separation of the control and user planes [23]. Incorporating the SDN philosophy into wireless mobile networks leads to the concept of software defined wireless networking (SDWN). By elaborately separating the separating the logical control plane and the data transmission plane, SDWNs become capable of efficient data aggregation and centralized network control, whilst simplifying the network configuration, which is achieved by the flexible management of network resources. As a future advance in the field, openflow constitutes a powerful enabling technique for software defined 5G networks [24]. Based on openflow's interface and on the SDWN controllers' reprogrammable nature, the physical infrastructure and the

Haijun Zhang and Keping Long are with Beijing Engineering and Technology Research Center for Convergence Networks and Ubiquitous Services, University of Science and Technology Beijing, Beijing, China (e-mail: haijunzhang@ieee.org, longkeping@ustb.edu.cn).

Na Liu is with College of Information Science and Technology, Beijing University of Chemical Technology, Beijing, 100029, China (e-mail: eceliuna@gmail.com).

Julian Cheng is with the School of Engineering, The University of British Columbia, Kelowna, BC, V1V 1V7 Canada (e-mail: julian.cheng@ubc.ca).

Victor C. M. Leung is with the Department of Electrical and Computer Engineering, The University of British Columbia, Vancouver, BC V6T 1Z4 Canada (e-mail: vleung@ece.ubc.ca).

Lajos Hanzo is with the School of Electronics and Computer Science, University of Southampton, Southampton, SO17 1BJ U.K. (e-mail: lh@ecs.soton.ac.uk).

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radio resources may be readily shared across the SDWN. It is universally acknowledged that next generation networks will no longer rely on operator-centered deployment, but on user- or service-centered deployment. Therefore, there will be diverse operational strategies for SDWNs in accordance with different radio network access methods. In [25], software-defined device-to-device communication was proposed for virtual radio networks, where both the integration of device-to-device communication into SDN and the network function virtualization issue were studied by considering both realistic imperfect and idealized perfect network state information. In [26], a SDN based radio network access mechanism was designed for heterogeneous networks in order to support flexible network control and to guarantee the users' quality of service (QoS).

As a compelling complementary technology to RF communications, VLC has attracted much attention both in academia and in the industry. The authors of [10], [20] conducted an in-depth study of LED-based VLC communications, following a comprehensive survey. Moreover, the design and implementation issues of indoor VLC communications were considered. Additionally, in order to offset the scarcity of RF spectrum, researchers are looking for combining VLC and RF in unified communication systems to solve the data congestion problems. In [21], the area spectrum efficiency(ASE) of RF based small cells and VLC networks was compared, while in [4], [22], delay-guarantee based resource allocation was studied relying on the effective capacity metric of heterogeneous VLC and RF small-cell networks. In [7], the maximization of the energy efficiency problem of OFDMA subchannel allocation and power allocation was formulated and solved in conceived heterogeneous indoor RF and VLC networks.

In order to achieve high-rate transmission, multi-cell deployment relying on a multi-LED enabled OFDMA system was studied in [27]. However, the inter-cell interference imposes severe constraints on the application of VLC. Hence the authors of [28]–[30] analyzed diverse interference infested VLC systems. The authors of [30] pointed out that the interference effects can be ignored, when each LED transmits the same synchronous signals. By contrast, the signal to interference and noise ratio (SINR) will heavily degrade, if the LEDs transmit different signal to increase the area spectrum efficiency. The authors of [31]–[33] studied interference mitigation techniques conceived for VLC systems. However, to the best of the authors' knowledge, subchannel and power allocation in software-defined heterogeneous VLC and RF small-cell system, by jointly considering backhaul constraints, QoS requirements, energy efficiency, and inter-cell interference limits, have not been studied in previous works. Reference [34] is a conference version of this paper. Different from the conference version, we provide the detailed proof for the theorem, Dinkelbach Procedure Algorithm and more simulation results in this paper. The main contributions in this paper can be summarized as follows:

- Development of a novel energy efficient software-defined heterogeneous VLC and RF small-cell network optimization framework: This is a new approach to heterogeneous VLC/RF network optimization design by considering backhaul constraints, QoS requirements, energy efficiency, and inter-cell interference limits in software-defined heterogeneous VLC and RF small-cell network.
- Formulation of a subchannel allocation and power control problem with multiple constraints: We formulate the energy efficient subchannel allocation and power optimization problem in software-defined heterogeneous VLC/RF network as a mixed-integer programming problem. Inter-cell interference limits are used to protect the small cells. A QoS requirement in terms of required minimum capacity is provided to guarantee reliable transmissions.
- Proposal of energy efficient subchannel allocation and power control algorithms for both VLC and RF: A Dinkelbach-style algorithm is utilized for transforming the original fractional form objective function into a subtractive-form function. Then the optimization problem is solved by alternative direction method of multipliers (ADMM) method. The subchannel allocation and power control algorithms are proposed and is shown to converge to the optimal point quickly. The effectiveness of the proposed schemes in twinned VLC/RF system are verified by simulations.

The rest of this paper is organized as follows. The twinned heterogeneous software-defined VLC and RF small-cell network architecture is described in Section II. In Section III, an energy-efficient resource allocation technique relying on subchannel and power allocation considering the inter-cell LED interference and small-cell interference threshold is formulated. In Section IV, both Dinkelbach's procedure and the ADMM technique are used for solving the optimization problems formulated. Our simulation results and the corresponding analysis are presented in Section V. Finally, Section VI concludes this treatise.

II. System Model

A. System Model

As shown in Fig. 1, we consider the energy efficient resource allocation problems of OFDMA based heterogeneous VLC and RF small-cell networks. An indoor downlink-transmission scenario is considered, where M mobile terminals (MTs) are supported by F small-cells of the RF downlink system and by L LED arrays of the VLC downlink system. For convenience, we assume that the term VLC, LEDs and LED arrays represent the same meaning, regarding a VLC network or a VLC access point (AP). We assume that all MTs have multi-homing capability and that all MTs are equipped with both VLC receivers and RF receivers. Again, there are M MTs, L LED arrays and F RF small cell APs. The set of MTs, LED arrays and small cell APs are denoted by $\mathcal{M} = \{1, 2, ..., m, ..., M\}$, $\mathcal{L} = \{1, 2, ..., l, ..., L\}$ and $\mathcal{F} = \{1, 2, ...f, ..., F\}$, where m represents the mth MT, l is the lth LED array and f is the fth small cell in our system model. Each LED array consists of a certain number of LED lights. Let $\mathcal{K}^{VLC} = \{1, 2, ..., k^{VLC}, ..., K^{VLC}\}$ and $\mathcal{K}^{RF} = \{1, 2, ..., k^{RF}, ..., K^{RF}\}$ represent the subchannel sets of the VLC system and RF system, respectively. As shown in Fig. 1, all LED arrays are uniformly spaced on the ceiling, and similarly, all RF APs are uniformly distributed in the room, while all MTs are randomly and uniformly distributed across the coverage area of both the VLC and of the RF APs. Again, each MT has multi-homing capabilities supporting simultaneous association with both the RF and VLC networks. Given the multi-homing capability, the energy efficiency can be substantially improved [7]. We focus our attention on indoor public areas, such as airports, offices, meeting halls and cinemas, etc. Hence, the room size can be expanded from the standard $5.0m \times 5.0m \times 3.0m$ dimensions to larger sizes.

Realistically, there are line-of-sight (LOS) blocking events due to the obstructions in the links between the APs and MTs. The availability of LOS propagation in the RF and VLC systems is defined as the probability that there is no obstacle in the link between MTs and the corresponding APs, which are denoted by ρ_{VLC} and ρ_{RF}, respectively. In the case of the RF communications, the path-loss will increase with the probability of the absence of LOS. For VLC, the received power of light is significantly reduced in the absence of LOS, which may result in low-integrity reception. In this paper, we assume that the non-LOS (NLOS) VLC transmissions are always unsuccessful and we focus our attention on the scenario of direct LOS transmissions in VLC. By contrast, the NLOS transmission in the RF system is still considered to achieve adequate reception quality.

B. Software-Defined VLC and RF Small-Cell Systems

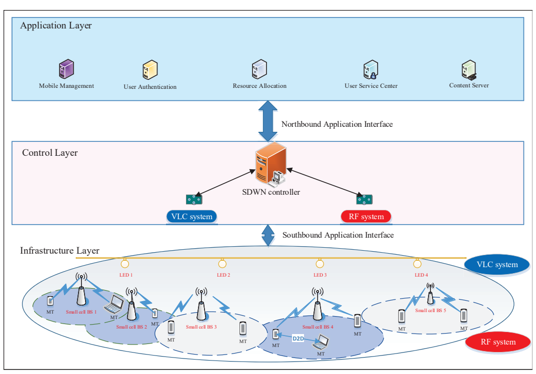

Fig. 2. Software-defined VLC and RF small-cell networks.

The SDN philosophy used for various RF wireless networks has been widely studied in diverse application scenarios, such as, for example, in software-defined device-to-device communication [25]. In this paper, we consider software-defined heterogeneous VLC and RF based small cell systems relying on unified control and data transmission. As shown in Fig. 2, the infrastructure layer illustrates an indoor wireless access network, where again the VLC-aided and RF-based small cell APs are uniformly deployed, and the MTs are randomly distributed in overlapping areas of the VLC cells and RF small-cells. An SDWN controller is used. The SDN controller operates and controls the entire network in centralized or distributed approaches - by connecting the data layer and vertical applications through the APIs - with the knowledge of the entire networks state and that of the radio resources, which depends on the mode of network distribution. In a hierarchical control plane within SDN, it is necessary for SDN controllers to cooperate each other. The APIs are used for communication and control of each parts, interacting with each other through controllers. If the network is more complex than our proposed model, hierarchical controllers can be introduced. The controller has a universal knowledge of the entire network's state and that of the radio resources, hence it handles the management of the entire access network by programming [26], while relies on the allocation of radio network resources and on the allocation of the assignment policy relying on broadcast channels. The controller can adjust both the modulation scheme and the transmission rate by controlling the VLC system. The application layer is intricately connected with the control layer by the application interface. The application layer includes servers from different operators or device providers.

In our proposed software-defined heterogeneous VLC and RF based small cell systems, the resource allocation is regarded as a requested application in application layer. The SDN controller is linked with the application layer to obtain the resource allocation strategy via northbound application interface, and then it operates the configuration of the network resources through southbound application interface to RF and VLC access points in infrastructure layer. Due to the distribution attributes of both network access points and resource allocation algorithm ADMM, our proposed software-defined energy efficient resource allocation algorithm can be readily achieved in heterogeneous VLC and RF based small cell systems.

III. Problem Formulation

A. VLC System

In the VLC system, the information is transmitted within the visible light band. We assume that the LEDs are always on in our proposed system model. As shown in Fig. 1, the LED lights are uniformly distributed. The irradiation angle of LEDs is defined as Ψ_{ir} and Ψ_{in} represents angle incidence of from LED array l to the mth MT. Then we have

$$\Psi_{ir} = \Psi_{in}. \tag{1}$$

Let $g^{VLC}_{l,m,k^{VLC}}$ be the channel's gain of subchannel k^{VLC} in the VLC system spanning from LED array l to MT m. For the VLC system, the channel's gain represents both the LOS and NLOS path-loss for the optical wireless signals. If we assume that P_t is the transmit power of the VLC APs, then the received VLC power of the MTs is denoted as

$$\mathrm{P}_r = g^{VLC}_{l,m,k^{VLC}} P_t \tag{2}$$

where $g^{VLC}_{l,m,k^{VLC}}$ is given by [10]

$$g^{VLC}_{l,m,k^{VLC}} = \begin{cases} \rho_{VLC} \frac{(n+1)A_m T_s(\Psi_{in})}{2\pi {d^{VLC}_{l,m}}^2} \cos^n(\Psi_{ir}) \cos(\Psi_{in}) g(\Psi_{in}), \text{if} \Psi_{in} \le \Psi_c \\ 0, \text{if} \Psi_{in} > \Psi_c \end{cases} \tag{3}$$

and where ρ_{VLC} represents the probability of the VLC LOS scenario. Furthermore, n is the order of the Lambertian emission, as defined by the semi-angle of LEDs at the half illumination power value $\Phi_{1/2}$ via

$$n = \frac{\ln 2}{\ln(\cos \Phi_{1/2})}. \quad (4)$$

In (3), A_m is the physical area of the photo detector at the mth MT; $T_s(\Psi_{in})$ is the gain of an optical filter at a VLC receiver; Ψ_c is denoted as the width of the field of view (FOV) at the VLC receiver of the MT; $d_{l,m}^{VLC}$ is the distance from the LED array l in the VLC system to the light receiver of MT m; $g(\Psi_{in})$ is the gain of an optical concentrator, which is given by

$$g(\Psi_{in}) = \begin{cases} \frac{n^2}{\sin^2 \Psi_c}, & \text{if} \Psi_{in} \leq \Psi_c, \\ 0, & \text{if} \Psi_{in} > \Psi_c. \end{cases} \quad (5)$$

In order to guarantee uniform brightness and the reliability of communication, the LED arrays are usually composed of serval LED lights as described above. These LED arrays have relatively large emission surfaces. Therefore, the probability of blind reception areas caused by obstacles in the FOV of the receivers can be avoided. However, due to the associated multipath propagation, the received signal will suffer from inter-symbol interference (ISI).

Again, the interference arriving from the different light sources can be ignored, when each LED array transmits the same synchronous signal. However, the SINR will seriously degrade, when different light sources transmit different signals. The dominant noise contribution is assumed to be the shot noise due to the ambient light arriving through the windows. We also take the thermal noise into account. Hence, the total variance σ_{VLC}^2 on subchannel k^{VLC} arriving from LED l arriving at MT m becomes:

$$\sigma_{VLC}^2 = \sigma_{shot}^2 + \sigma_{thermal}^2 + \sigma_{ISI}^2. \quad (6)$$

The shot noise variance σ_{shot}^2 is given by [10].

$$\sigma_{shot}^2 = 2q\gamma P_r B + 2qI_{bg}I_1 B, \quad (7)$$

where q is the electronic charge, γ is the photo detector's responsivity, B is the equivalent noise bandwidth, I_{bg} is the background current caused by ambient light and I_1 is an experimentally determined constant. Furthermore, the thermal noise variance is given by [10]

$$\sigma_{thermal}^2 = \frac{8\pi K T_K}{G}\eta A I_1 B^2 + \frac{16\pi^2 K T_K \Gamma}{g_m}\eta^2 A^2 I_2 B^3, \quad (8)$$

where K is Boltzmann's constant, T_K is the absolute temperature, G is the open-loop voltage gain, η is the fixed capacitance of the photo detector per unit area, Γ is the FET's channel noise factor, g_m is FET transconductance and I_2 is also a constant experimental value. Finally, the variance of the ISI is [10]

$$\sigma_{ISI}^2 = \gamma^2 P_{rISI}^2, \quad (9)$$

where γ is the photo detector's responsivity and P_{rISI} denotes the power of the interference.

B. RF Downlink System

Again, we consider the downlink of the twinned VLC and RF systems. Let $g_{f,m,k^{RF}}^{RF}$ denote the channel gain between the RF small cell f and MT m on subchannel k^{RF} of the RF system. The channel's power gain for the RF system captures both the channel fading and path-loss, where $g_{f,m,k^{RF}}^{RF}$ is typically calculated as [35]

$$g_{f,m,k^{RF}}^{RF} = 10^{-PL[dB]/10}, \quad (10)$$

where PL is the RF path-loss in dB, and it is given by [35]:

$$PL[dB] = A\log_{10}(d_{f,m}^{RF}) + B + C\log_{10}\left(\frac{f_c}{5}\right) + X, \quad (11)$$

where f_c is the carrier frequency in GHz and $d_{f,m}^{RF}$ is the distance from the small-cell f to MT m in the RF system. Furthermore, A, B and C are constants depending on the propagation model. For LOS propagaion, $A = 18.7$, $B = 56.8$ and $C = 20$. For NLOS transmission scenarios, we have $A = 36.8$, $B = 43.8$ and $C = 20$. Still referring to (11), X denotes the wall penetration loss in the NLOS transmission scenario, and we assume $X = 5(n_w - 1)$ for thin walls and $X = 12(n_w - 1)$ for heavy walls, where $n_w$is the number of walls between the small-cell APs and MTs.

Therefore, the inter-cell interference channel's gain is $\sum_{i=1, i\neq f}^{F} g_{f,m,k^{RF}}^{RF}$. The variance of the interference emanating from the small-cell f to MT m on subchannel k^{RF} is

$$\sigma_{RF}^2 = \sigma_{AWGN}^2 + \sigma_{\text{inter}}^2, \quad (12)$$

where σ_{AWGN}^2 is the AWGN noise; σ_{inter}^2 is the inter-cell interference and σ_{AWGN}^2 is given by [21]

$$\sigma_{\text{AWGN}}^2 = KTB_{MT}, \quad (13)$$

where K is Boltzmann's constant, T is the ambient temperature and B_{MT} is the bandwidth allocated to user MT.

C. Energy Efficiency Optimization Problem

Let $p_{f,m,k^{RF}}^{RF}$ denote the power transmitted from small-cell f to MT m on the RF subchannel k^{RF}. Similarly, let $p_{l,m,k^{VLC}}^{VLC}$ denote the power of the LED array l transmitted to MT m on the VLC subchannel k^{VLC}. Therefore, the power allocation matrices can be defined as $P^{RF} = \left[p_{f,m,k^{RF}}^{RF}\right]_{F\times M\times K^{RF}}$ and $P^{VLC} = \left[p_{l,m,k^{VLC}}^{VLC}\right]_{L\times M\times K^{VLC}}$. Let us denote the subchannel allocation matrices as $B^{RF} = \left[b_{f,m,k^{RF}}^{RF}\right]_{F\times M\times K^{RF}}$ and $B^{VLC} = \left[b_{l,m,k^{VLC}}^{VLC}\right]_{L\times M\times K^{VLC}}$, where $b_{f,m,k^{RF}}^{RF}$ and $b_{l,m,k^{VLC}}^{VLC}$ are binary variables. For example, $b_{f,m,k^{RF}}^{RF} = 1$ means that the subchannel k^{RF} is assigned to MT m from the small-cell f of the RF system; otherwise, $b_{f,m,k^{RF}}^{RF} = 0$. Similarly, in the VLC system, $b_{l,m,k^{VLC}}^{VLC} = 1$ means that subchannel k^{VLC} is assigned to MT m from the LED array l; otherwise, $b_{l,m,k^{VLC}}^{VLC} = 0$.

We have mentioned that when the LOS path is unavailable due to a blocking event in the transmit link, the receiver will not receive the optical signal. Hence, based on Shannon's capacity formula, the expected values of the achievable downlink capacity on subchannel k^{VLC} from the LED array l can be calculated by taking into acount the probability mass function of the LOS and NLOS availability upon assuming a unit bandwidth for the subchannel as follows:

$$\mathrm{C}_{l,m,k^{VLC}}^{VLC} = \rho_{VLC}\log_2(1+\gamma_{l,m,k^{VLC}}^{LOS,VLC}). \tag{14}$$

By contrast, in the RF system, the NLOS compoment is taken into consideration, hence the capacity is calculated as

$$\begin{aligned}&\mathrm{C}_{f,m,k^{RF}}^{RF} = \rho_{RF}\log_2(1+\gamma_{f,m,k^{RF}}^{LOS,RF})\\ &+(1-\rho_{RF})\log_2(1+\gamma_{f,m,k^{RF}}^{NLOS,RF})\end{aligned} \tag{15}$$

where the SINRs of the VLC and RF systems are respectively defined as

$$\gamma_{l,m,k^{VLC}}^{VLC} = \gamma^2(p_{l,m,k^{VLC}}^{VLC})^2 h_{l,m,k^{VLC}}^{VLC}, \tag{16}$$

$$\gamma_{f,m,k^{RF}}^{RF} = p_{f,m,k^{RF}}^{RF} h_{f,m,k^{RF}}^{RF} \tag{17}$$

with $h_{l,m,k^{VLC}}^{VLC}$ and $h_{f,m,k^{RF}}^{RF}$ respresenting the channel's fading coefficients, respectively

$$h_{l,m,k^{VLC}}^{VLC} = \frac{g_{l,m,k^{VLC}}^{VLC}}{\sigma_{VLC}^2}, \tag{18}$$

$$h_{f,m,k^{RF}}^{RF} = \frac{g_{f,m,k^{RF}}^{RF}}{\sigma_{RF}^2}. \tag{19}$$

Hence, we have $H^{VLC} = \left[h_{l,m,k^{VLC}}^{VLC}\right]_{L\times M\times K^{VLC}}$ and $H^{RF} = \left[h_{f,m,k^{RF}}^{RF}\right]_{F\times M\times K^{RF}}$. The capacity available for MT m from the small-cell f in the RF system is calculated as

$$C_{f,m}^{RF}(P^{RF},B^{RF}) = \sum_{k^{RF}=1}^{K^{RF}} b_{f,m,k^{RF}}^{RF} C_{f,m,k^{RF}}^{RF}. \tag{20}$$

Similarly, the capacity of MT m gleaned from the VLC access point l of the VLC system is

$$C_{l,m}^{VLC}(P^{VLC},B^{VLC}) = \sum_{k^{VLC}=1}^{K^{VLC}}, b_{l,m,k^{VLC}}^{VLC} C_{l,m,k^{VLC}}^{VLC}, \tag{21}$$

while the capacity of each small cell is

$$C_{f}^{RF}(P^{RF},B^{RF}) = \sum_{m=1}^{M}\sum_{k^{RF}=1}^{K^{RF}} b_{f,m,k^{RF}}^{RF} C_{f,m,k^{RF}}^{RF}. \tag{22}$$

The capacity of each VLC access point is

$$C_{l}^{VLC}(P^{VLC},B^{VLC}) = \sum_{m=1}^{M}\sum_{k^{VLC}=1}^{K^{VLC}} b_{l,m,k^{VLC}}^{VLC} C_{l,m,k^{VLC}}^{VLC}. \tag{23}$$

Total capacity of the system hence becomes

$$C_T = \sum_{l=1}^{L} C_{l}^{VLC}(P^{VLC},B^{VLC}) + \sum_{f=1}^{F} C_{f}^{RF}(P^{RF},B^{RF}), \tag{24}$$

while the total power consumed is calculated as [7]

$$P_T = P_{VLC} + P_{RF} + \sum_{f=1}^{F}\sum_{m=1}^{M}\sum_{k^{RF}}^{K^{RF}} p_{f,m,k^{RF}}^{RF}, \tag{25}$$

where the term P_{VLC} is the fixed power consumed by the VLC system for its circuit operation, data processing and data transmission as well as illumination; the second term P_{RF} represents the power consumed by the data processing circuits of the RF system and the third term is the transmit power.

We define the system energy efficiency (EE) as the ratio of the spectrum efficiency and of the total power consumed by both systems [7], [37]. Hence, the objective function can be formulated as

$$\max_{p_{l,m,k^{VLC}}^{VLC}, p_{f,m,k^{RF}}^{RF}, b_{l,m,k^{VLC}}^{VLC}, b_{f,m,k^{RF}}^{RF}} \frac{C_T}{P_T}, \tag{26}$$

$$\begin{aligned}
\text{s.t.}\quad & C1: \sum_{m=1}^{M}\sum_{k^{VLC}=1}^{K^{VLC}} b_{l,m,k^{VLC}}^{VLC} p_{l,m,k^{VLC}}^{VLC} \le p_{l,\max}^{VLC}, \quad \forall l\in\mathcal{L}\\
& C2: \sum_{m=1}^{M}\sum_{k^{RF}=1}^{K^{RF}} b_{f,m,k^{RF}}^{RF} p_{f,m,k^{RF}}^{RF} \le p_{f,\max}^{RF}, \quad \forall f\in\mathcal{F}\\
& C3: 0\le p_{l,m,k^{VLC}}^{VLC} \le p_{l,\max}^{VLC}, \forall l\in L, \forall k^{VLC}\in K^{VLC},\\
& \forall m\in M\\
& C4: 0\le p_{l,m,k^{RF}}^{RF} \le p_{f,\max}^{RF}, \forall f\in F, \forall k^{RF}\in K^{RF},\\
& \forall m\in M\\
& C5: C_{l,m}^{VLC}(P^{VLC},B^{VLC}) + C_{f,m}^{RF}(P^{RF},B^{RF}) \ge C_{m,\min},\\
& \forall m\in M\\
& C6: \sum_{m=1}^{M}\sum_{k^{VLC}=1}^{K^{VLC}} C_{l,m,k^{VLC}}^{VLC} \le C_{l,\max}^{VLC}, \quad \forall l\in\mathcal{L}\\
& C7: \sum_{m=1}^{M}\sum_{k^{RF}=1}^{K^{RF}} C_{f,m,k^{RF}}^{RF} \le C_{f,\max}^{RF}, \quad \forall f\in\mathcal{F}\\
& C8: \sum_{i=1,i\ne f}^{F} g_{i,m,k^{RF}}^{RF} p_{i,m,k^{RF}}^{RF} \le I^{th}, \forall m\in M,\\
& \forall k^{RF}\in K^{RF}\\
& C9: b_{l,m,k^{VLC}}^{VLC}\in\{0,1\}, \forall l\in L, \forall k^{VLC}\in K^{VLC},\\
& \forall m\in M\\
& C10: b_{f,m,k^{RF}}^{RF}\in\{0,1\}, \forall f\in F, \forall k^{RF}\in K^{RF},\\
& \forall m\in M\\
& C11: \sum_{m=1}^{M} b_{l,m,k^{VLC}}^{VLC} \le 1, \quad \forall l\in\mathcal{L}, \quad \forall k^{VLC}\in\mathcal{K}^{VLC}\\
& C12: \sum_{m=1}^{M} b_{f,m,k^{RF}}^{RF} \le 1, \quad \forall f\in\mathcal{F}, \quad \forall k^{RF}\in\mathcal{K}^{RF}
\end{aligned} \tag{27}$$

The above constraints are defined as follows:

- Total power constraints:
  Constraint $C1$ limits the total transmit power of each LED AP to be below the maximum transmit power of each LED AP; Constraint $C2$ limits the total transmit power of

each small- cell AP to be below the maximum transmit power of each small-cell AP; Constraints $C3$ and $C4$ ensure that the transmitters have nonnegative output and maximum optical power;

- User QoS guarantees:
  Constraint $C5$ represents the QoS guarantee, which requires that the capacity (sum of the data rates) of the mth MT is constrained to be higher than or equal to a required minimum capacity;
- Backhaul constraints:
  Constraints $C6$ and $C7$ indicate that the maximum available capacity of the backhaul link of each LED array and of each small-cell AP in each system;
- Interference constraints:
  Constraint $C8$ is the co-tier inter-small-cell interference constraint. We impose an interference temperature limit to constrain the co-tier interference suffered by other small cells. Let I^{th} denote the maximum tolerable interference level on subchannel k^{RF};
- Subchannel scheduling constraints:
  Constraints $C9$ and $C10$ are the user scheduling constraints, requiring that a subchannel is allocated to at most one user in each small-cell and LED-cell at a time instant; Constraints $C11$ and $C12$ (combined with $C9$ and $C10$) are imposed in order to guarantee that each subchannel can only be assigned to at most one user in each cell.

IV. Resource Allocation Algorithm

In this section, the energy efficient resource allocation problem is transformed into a convex optimization problem [36]. Then, we propose an iterative algorithm for calculating the optimal powers and subchannel allocations. As for the computational complexity, we develop a distributed low-complexity subchannel and power allocation algorithm, which relies on three main steps [7], [19], [37]–[39]:

(1) A Dinkelbach-style algorithm is utilized for transforming the original fractional-form objective function into a subtractive-form function;

(2) Suboptimal subchannel allocation takes into account the heterogeneous user QoS guarantee to be satisfied;

(3) ADMM is used for finding the optimal power allocation under co-tier small-cell interference constraints.

The optimal solution of the objective function can be obtained by finding the roots of the equation $F(\lambda)=0$, where the parameter λ is an auxiliary variable. We transform the original fractional-form objective function into a subtractive-form function to find the optimal solution for the optimization problem

$$F(\lambda)=\max_{p^{VLC}_{l,m,k^{VLC}},p^{RF}_{f,m,k^{RF}},b^{VLC}_{l,m,k^{VLC}},b^{RF}_{f,m,k^{RF}}} C_T-\lambda P_T. \tag{28}$$

Our Dinkelbach-style algorithm is summarized as Algorithm 1, which has been shown to be convergent [4]. The optimum solution of (27) can be found within a limited number of iterations.

Then, we solve the resultant problem $F(\lambda)$ with the aid of distributed subchannel and power allocation scheme.

Algorithm 1 Dinkelbach-style Procedure

1: Initialization:
2: Set $p^{VLC}_{l,m,k^{VLC}}(0)$, $p^{RF}_{f,m,k^{RF}}(0)$, $b^{VLC}_{l,m,k^{VLC}}(0)$, $b^{RF}_{f,m,k^{RF}}(0)$, $\forall m$;
$\lambda(1)=\eta(0),\ i=1$;
3: **while** $F(\lambda(i))\neq 0$ **do**
4: Solve $F(\lambda)=\max_{p^{VLC}_{l,m,k^{VLC}},p^{RF}_{f,m,k^{RF}},b^{VLC}_{l,m,k^{VLC}},b^{RF}_{f,m,k^{RF}}} C_T-\lambda P_T$ for optimal $\left\{p^{VLC}_{l,m,k^{VLC}}(i),p^{RF}_{f,m,k^{RF}}(i),b^{VLC}_{l,m,k^{VLC}}(i),b^{RF}_{f,m,k^{RF}}(i)\right\},\forall m$;
5: $\lambda(i+1)=\eta(i)$;
6: $i\leftarrow i+1$
7: **end while**
8: Output: $p^{VLC}_{l,m,k^{VLC}}$, $p^{RF}_{f,m,k^{RF}}$, $b^{VLC}_{l,m,k^{VLC}}$, $b^{RF}_{f,m,k^{RF}}$, $\forall m$.

A. Subchannel Allocation

We optimize the subchannel allocations while fixing the power, which means that given the power allocation of each subchannel from each AP to each MT, the subchannel should be allocated to that particular link, whose power gain is the highest, as presented in (29) and (30). Then the remaining subchannels are allocated in order to guarantee the users' QoS requirements, while maximizing the total system capacity. The resultant subchannel allocation is summarized in Algorithm 2.

$$b^{VLC}_{l^*,m^*,k^{VLC}}=\begin{cases}1, & (l^*,m^*)=\arg\max_{l,m} H^{VLC}_{l,m,k^{VLC}}\\ 0, & otherwise.\end{cases} \tag{29}$$

$$b^{RF}_{f^*,m^*,k^{RF}}=\begin{cases}1, & (f^*,m^*)=\arg\max_{f,m} H^{RF}_{f,m,k^{RF}}\\ 0, & otherwise.\end{cases} \tag{30}$$

The subchannel allocation matrices $\tilde{B}^{RF}$ and $\tilde{B}^{VLC}$ can be obtained using Algorithm 2.

We can rewrite the optimization problem of Equation (28) in a subtractive-form as

$$\begin{aligned}&\min_{p^{VLC}_{l,m,k^{VLC}},p^{RF}_{f,m,k^{RF}}} -f(p^{VLC}_{l,m,k^{VLC}},p^{RF}_{f,m,k^{RF}},\tilde{b}^{VLC}_{l,m,k^{VLC}},\tilde{b}^{RF}_{f,m,k^{RF}})\\ &=\lambda P_T-\tilde{C}_T\end{aligned} \tag{31}$$

where $(\lambda P_T-\tilde{C}_T)$ can be reformulated as

$$\begin{aligned}\lambda P_T-\tilde{C}_T&=\lambda(P_{VLC}+P_{RF}\\&+\sum_{f=1}^{F}\sum_{m=1}^{M}\sum_{k^{RF}}^{K^{RF}}p^{RF}_{f,m,k^{RF}})\\&-\sum_{l=1}^{L}\sum_{m=1}^{M}\sum_{k^{VLC}}^{K^{VLC}}\tilde{b}^{VLC}_{l,m,k^{VLC}}C^{VLC}_{l,m,k^{VLC}}\\&-\sum_{f=1}^{F}\sum_{m=1}^{M}\sum_{k^{RF}}^{K^{RF}}\tilde{b}^{RF}_{f,m,k^{RF}}C^{RF}_{f,m,k^{RF}}\end{aligned} \tag{32}$$

Let us consider the same power constraints, namely (26) in the context of (27). Furthermore, we can use the ADMM technique of [38] in each network. The proof can be found in Subsection A of the Appendix.

B. Power Allocation

We introduce the ADMM for solving the optimal power allocation problem. A brief introduction to the ADMM can be found in Subsection B of the Appendix.

Algorithm 2 Proposed Subchannel Allocation Algorithm

Set $\mathrm{b}^{VLC}_{l,m,k^{VLC}}=0, C^{VLC}_{l,m}=0,\ \forall l,m,k^{VLC}$; $\mathrm{b}^{RF}_{f,m,k^{RF}}=0, C^{RF}_{f,m}=0,\ \forall f,m,k^{RF}$;
Each LED cell has k^{VLC} subchannels; Each RF small-cell has k^{RF} subchannels;
Initialize the power allocation with an equal power distribution: the LED array l shares its power equally among of k^{VLC} subchannels, $p^{VLC}_{l,m,k^{VLC}}=\frac{p^{VLC}_{l,\max}}{K^{VLC}},\ \forall l$; the RF small-cell f divides its power equally among of k^{RF} subchannels $p^{RF}_{f,m,k^{RF}}=\frac{p^{RF}_{f,\max}}{K^{RF}},\ \forall f$;
for $m=1$ to M **do**
  while $C_m = C^{VLC}_{l,m}(P^{VLC},B^{VLC}) + C^{RF}_{f,m}(P^{RF},B^{RF}) < C_{m,\min}$ **do**
    $b^{VLC}_{l,m,k*^{VLC}}=1$, where $k*^{VLC}=\arg\max_{k^{VLC}} H^{VLC}_{l,m,k^{VLC}}$; $b^{RF}_{f,m,k*^{RF}}=1$, where $k*^{RF}=\arg\max_{k^{RF}} H^{RF}_{f,m,k^{RF}}$;
    $\mathcal{K}^{VLC}:=\mathcal{K}^{VLC}\backslash k*^{VLC}$; $\mathcal{K}^{RF}:=\mathcal{K}^{RF}\backslash k*^{RF}$;
    $K^{VLC}:=K^{VLC}-1$, $K^{RF}:=K^{RF}-1$;
    $\mathrm{C}_m := C_m + C^{VLC}_m(P^{VLC},B^{VLC}) + C^{RF}_m(P^{RF},B^{RF})$
  end while
end for
while $\mathcal{K}^{VLC}\neq\emptyset$ **do**
  $b^{VLC}_{l,m*,k^{VLC}}=1$, where $m*=\arg\max_m H^{VLC}_{l,m,k^{VLC}}$
  if $K^{VLC}>0$ **then**
    $K^{VLC}:=K^{VLC}-1$;
    $\mathcal{K}^{VLC}:=\mathcal{K}^{VLC}\backslash k^{VLC}$;
  end if
end while
while $\mathcal{K}^{RF}\neq\emptyset$ **do**
  $b^{RF}_{f,m*,k^{RF}}=1$, where $m*=\arg\max_m H^{RF}_{f,m,k^{RF}}$
  if $K^{RF}>0$ **then**
    $K^{RF}:=K^{RF}-1$;
    $\mathcal{K}^{RF}:=\mathcal{K}^{RF}\backslash k^{RF}$;
  end if
end while

In order to solve the optimization problem using the unscaled-form ADMM, we introduce the auxiliary vectors of $\mathbf{x}_{RF}$ and $\mathbf{z}_{RF}$, while $\mathbf{x}_{RF}$ consist of the elements in the power allocation matrix and $\mathbf{z}_{RF}$ is a global auxiliary vector with each of its element corresponding to one in $\mathbf{x}_{RF}$. We also define the matrix Φ as the set of Constraint $C5$. Let us now introduce the indicator function of

$$g(\mathbf{z}_{RF})=\begin{cases}0, & \mathbf{z}_{RF}\in\Phi\\ +\infty, & otherwise.\end{cases} \tag{33}$$

Then, the optimization problem can be reformulated as

$$\min_{\mathbf{x}_{RF},\mathbf{z}_{RF}} -f(\mathbf{x}_{RF},\tilde{\mathbf{B}}^{RF})+g(\mathbf{z}_{RF}), \tag{34}$$

$$\text{s.t.}\qquad \mathbf{x}_{RF}-\mathbf{z}_{RF}=0. \tag{35}$$

The augmented Lagrangian in scaled-form can be formulated as

$$\begin{aligned}\mathrm{L}_{\rho}{}^{RF}=&-f(\mathbf{x}_{RF},\tilde{\mathbf{B}}^{RF})+g(\mathbf{z}_{RF})\\&-\tfrac{\rho}{2}\left\|\mu_{RF}\right\|_2^2+\tfrac{\rho}{2}\left\|\mathbf{x}_{RF}-\mathbf{z}^t_{RF}+\mu_{RF}\right\|_2^2\end{aligned} \tag{36}$$

where μ_{RF} is the scaled dual variable, while ρ_{RF} is a constant penalty parameter. Our optimization problem can now be solved using the following steps

$$\begin{aligned}&\mathbf{x}^{t+1}_{RF}:=\arg\min_{\mathbf{x}_{RF}}\left\{-f(\mathbf{x}_{RF},\tilde{\mathbf{B}}^{RF})+\tfrac{\rho}{2}\left\|\mathbf{x}_{RF}-\mathbf{z}^t_{RF}+\mu^t_{RF}\right\|_2^2\right\}\\&\mathbf{z}^{t+1}_{RF}:=\arg\min_{\mathbf{z}_{RF}}\left\{\left\|\mathbf{x}^{t+1}_{RF}-\mathbf{z}_{RF}+\mu^t_{RF}\right\|_2^2\right\}\\&\mu^{t+1}_{RF}:=\mu^t_{RF}+\left(\mathbf{x}^{t+1}_{RF}-\mathbf{z}^{t+1}_{RF}.\right).\end{aligned} \tag{37}$$

Similarly, in the VLC system, we also use the ADMM to update the power allocation. Because this procedure is almost identical for both RF and VLC systems, hence we do not repeat it.

The power allocation algorithm using ADMM for our twinned VLC and RF systems is summarized in Algorithms 3 and 4.

Algorithm 3 Power Allocation in the RF system

Initialization;
Each small cell $f\in F$ collects CSI;
Each small cell $f\in F$ decides the subchannel allocation matrix $\tilde{\mathbf{B}}^{RF}$ through Algorithm 2;
small cell initializes $\mathbf{x}^0_{RF}$=0, $\mathbf{z}^0_{RF}\in C5$ and $\mu^0_{RF}>0$, penalty parameter ρ_{RF}=0 , stop criteria $\xi>0$ and iteration index $t=0$;
while $f(p^{RF}_{f,m,k^{RF}},\tilde{b}^{RF}_{f,m,k^{RF}})>\xi$ **do**
  Each small cell $f\in F$ updates $\mathbf{x}^{t+1}_{RF}$;
  Each small cell $f\in F$ updates $\mathbf{z}^{t+1}_{RF}$;
  Each small cell $f\in F$ updates μ^{t+1}_{RF};
  $t:=t+1$;
end while

Algorithm 4 Power Allocation in the VLC system

Initialization:
Each LED cell $l\in L$ collects CSI;
Each LED cell $l\in L$ decides the subchannel allocation matrix $\tilde{\mathbf{B}}^{VLC}$ through Algorithm 2;
LED cell initializes $\mathbf{x}^0_{VLC}$=0, $\mathbf{z}^0_{VLC}\in C5$ and $\mu^0_{VLC}>0$, penalty parameter ρ_{VLC}=0 , stop criteria $\xi>0$ and iteration index $t=0$;
while $f(p^{VLC}_{l,m,k^{VLC}},\tilde{b}^{VLC}_{l,m,k^{VLC}})>\xi$ **do**
  Each LED cell $l\in L$ updates $\mathbf{x}^{t+1}_{VLC}$;
  Each LED cell $l\in L$ updates $\mathbf{z}^{t+1}_{VLC}$;
  Each LED cell $l\in L$ updates μ^{t+1}_{VLC};
  $t:=t+1$;
end while

V. Simulation Results and Analysis

In this section, our simulation results are discussed in order to characterize our resource allocation algorithms. The indoor environment considered in our simulations is the downlink of a heterogeneous twinned VLC and RF network. The users are randomly distributed in the overlapping coverage areas of both VLC APs and RF based small-cell APs. The carrier frequency of the VLC system is 10^{14} Hz; the bandwidth is 20 MHz [7]. The carrier frequency of the RF small-cell is 2 GHz, and the bandwidth is 10 MHz. In our OFDMA downlink transmission model, the subchannel bandwidth is 200 KHz, including a protective guard-band [19]. Therefore, we have $B^{VLC} = 100$ and $B^{RF} = 50$. For the convenience of simulation, we assume that the subchannel bandwidth is 1, hence the overall bandwidth of the VLC system is 100 and that of the RF based small-cell system is 50. The optical transmit power of each LED light is 20 mW. The maximum transmit power of each RF small-cell AP is 20mW.

The main parameters used for the VLC system are listed in Table I [21].

TABLE I
THE PARAMETERS USED FOR SIMULATIONS

Parameters	Notations	Valus
The electronic charge	q	$1.6 \times 10^{-19}C$
The detectors responsivity	γ	$0.54 Amp./Watt.$
The background current	I_{bg}	$5.1 \times 10^{-3}A$
The Boltzmann's constant	K	1.38×10^{-23}
The noise bandwidth factor (constant experimental value)	I_1 I_2	0.562 0.0868
The absolute temperature	T_K	$295K$
The open-loop voltage gain	G	10
The FET channel noise factor	Γ	1.5
The FET transconductance	g_m	$30mS$
Fixed capacitance of the PD per area	η	$112pF/cm^2$
The gain of an optical filter	T_s	1.0

Figure 3 shows the convergence of our proposed energy efficient resource allocation algorithm in terms of the total energy efficiency versus the number of iterations of the heterogeneous VLC and RF small-cell system, of the VLC-only system and of the RF-only system. We assume having 4 LED arrays, each of which consists of 1000 LED lights to satisfy the illumination requirements. The fixed power P_{VLC} of the VLC system is 4 W and the fixed power P_{RF} used for data processing in the RF system is 6.7 W. The transmit power of each RF small-cell AP is 20 mW. All small cell APs are uniformly distributed in the room, and 16 MTs are randomly located in the overlapping areas. The room size is assumed to be $20m \times 20m \times \ 3m$. Let us now consider the energy efficiency of the VLC system and of the RF small-cell system, respectively. Observe from Fig. 3 that our proposed energy efficient resource allocation algorithm has reached convergence after iteration index 20. Quantitatively, the total energy efficiency of the heterogeneous VLC and RF small-cell converges to about 46 $bps/Hz/Joule$ and that of the VLC-only system to about 11 $bps/Hz/Joule$, while that of RF -small-cell only system is above 19 $bps/Hz/Joule$. It can be also observed that the energy efficiency of the RF-only system is higher than that of the VLC-only system. This is because the VLC-only system assigns a substantial fraction of its power to illumination.

Figure 4 shows the total downlink energy efficiency versus the number of MTs in each overlapping VLC and RF coverage area. The number of users ranges from 1 to 7. The semi-angle at half power is assumed to be 60 degree. For ease of comparison, the total energy efficiency of the RF-only system and of the VLC-only system is also included. We can see that the more MTs are supported, the better the performance becomes. This is because the number of subchannels is fixed in both the VLC and RF small-cell system, hence when the number of MTs increases in each overlapping coverage area, each subchannel has more candidate links to select from. Hence, the energy efficiency increases upon increasing the number of MTs. It can also be seen that the energy efficiency does not increase linearly, because it saturates due to the interference encountered both by the VLC system and the RF system.

Figure 5 shows the total energy efficiency of the heterogeneous VLC and RF small-cell system versus the semi-angle at half power. The value of semi-angle at half power in the VLC system changes from 50 to 90 degrees. Observe from Fig. 5 that in the RF small-cell system the energy efficiency changes slowly, because the semi-angle at half power limitation does not influence the RF small-cell system. It can be observed from Fig. 5 that the total energy efficiency of the heterogeneous VLC and RF system and of the VLC-only system increases as this angle changes from 50 to 70 degrees. When the semi-angle at half-power changes from 70 degrees to 90 degrees, the total energy efficiency is slightly reduced. The reason is that the order n of the Lambertian emission will vary with the semi-angle value at half-power. As a result, the total energy efficiency will change with the order of the Lambertian emission.

Figure 6 shows the total energy efficiency of the heterogeneous VLC and RF downlink versus the required minimum capacity, which is varied from 1 bps/Hz to 6 bps/Hz. The total energy efficiency is reduced as the minimum required capacity value is increased for both the heterogeneous VLC and RF system, as well as for the VLC-only and the RF-only system, because the higher capacity requirement shrinks the feasible region of the optimization problem. This leads to the reduction of the total energy efficiency.

Figure 7 shows the total energy efficiency of the heterogeneous VLC and RF small-cell system versus the size of the room, which is varied from $5m \times 5m \times 3m$ to $20m \times 20m \times 3m$. The number of MTs in each overlapping area is assumed to be 4. We can see from Fig. 6 that the total energy efficiency of both the heterogeneous VLC and RF system, as well as of the VLC-only system and of the RF-only system increases with the room size. Upon increasing the room size, the MTs are distributed more sparsely due to having a fixed number of MTs. Therefore, the interference suffered by each MT decays.

Figure 8 shows the total energy efficiency of the heterogeneous VLC and RF system versus the number of LEDs in each LED array. In this figure, we assume that the number of LEDs in each array varies from 100 to 1000. Because this number only affects the power consumed by the VLC system, we can see that the total energy efficiency of the RF network part does not change radically. However, as the number of LEDs increases, the total energy consumption of the heterogeneous VLC and RF system as well as of the VLC-only system is reduced, because a diminishing fraction of the power is required for communications, whilst readily satisfying the illumination requirements.

Finally, Figure 9 shows the total energy efficiency versus the probability of LOS communication. We assume that the probability of LOS communication in the VLC system is equal to that of the RF system. Observe from Fig. 9 that the total energy efficiency of the heterogeneous VLC and RF system as well as of the VLC-only system increases with the LOS probability, whilst that of the RF-only system changes moderately. This is because the NLOS communication in the VLC system is unsuccessful, while in the RF systems, it is successful.

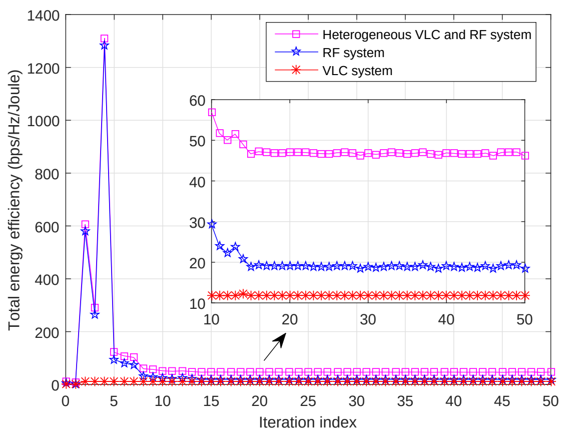

Fig. 3. Convergence in terms of the total energy efficiency.

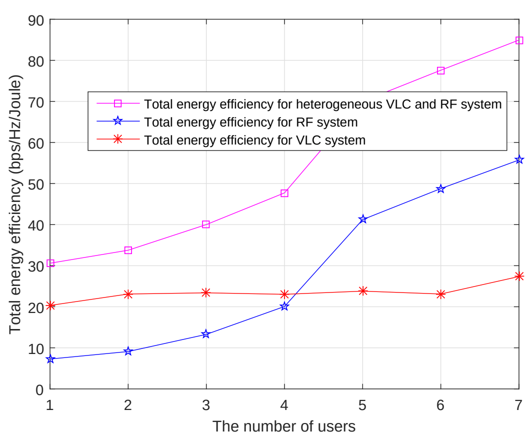

Fig. 4. The total energy efficiency versus the number of users.

VI. Conclusions

In this paper, we investigated the energy-efficient resource allocation of heterogeneous VLC and RF based systems relying on the software-defined philosophy. Explicitly, we investigated the energy efficient resource allocation problem by considering heterogeneous user requirements and inter-cell interference. This problem was first formulated as a non-convex optimization problem and then transformed into a convex one. Distributed OFDMA subchannel and power allocation

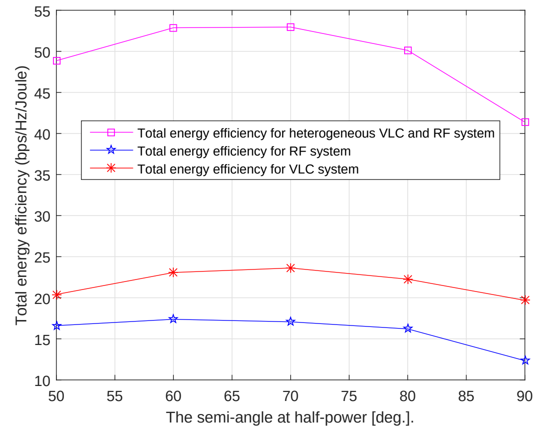

Fig. 5. The total energy efficiency versus the semi-angle at half-power.

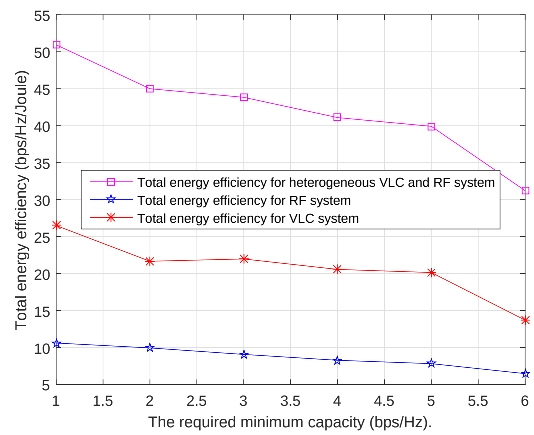

Fig. 6. The total energy efficiency versus the required minimum capacity.

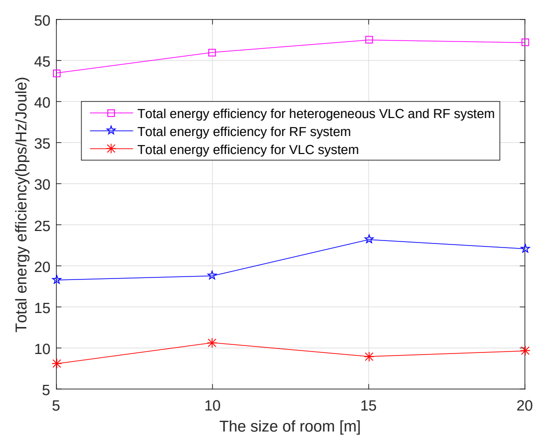

Fig. 7. The total energy efficiency versus the size of the room.

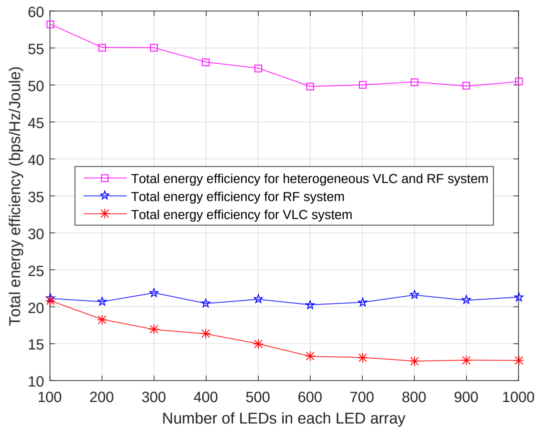

Fig. 8. The total energy efficiency versus the number of LEDs in each LED array.

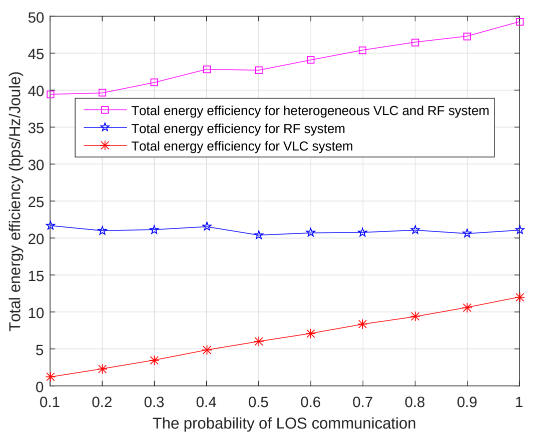

Fig. 9. The total energy efficiency versus the probability of LOS communication.

algorithms were then proposed for solving the convex optimization problem at each VLC and RF AP. We transformed the fractional-form optimization problem into a subtractive one by using a Dinkelbach-style algorithm. Then, we developed our subchannel and power allocation algorithm relying on the ADMM approach. The simulation results demonstrated that our proposed scheme is capable of converging within a few iterations, whist substantially increasing the throughput attained at a certain power consumption, which is a behalf of avoiding the interference between the VLC and RF small-cell.

Appendix

A. Proof of the using of ADMM for the power allocation

In this subsection, we will prove that the proposed power allocation algorithm can be solved by the ADMM algorithm. Moreover, the objective function can be shown to be separable. Hence, we can use the ADMM algorithm for the in both VLC system and RF system, separately. With the aid of Equation (14), (15) and (25), the Equation (31) can be rewritten as

$$\begin{aligned}&\lambda P_T - \tilde{C}_T = \lambda\Big(P_{VLC} + P_{RF} + \sum_{f=1}^{F}\sum_{m=1}^{M}\sum_{k^{RF}}^{K^{RF}} p_{f,m,k^{RF}}^{RF}\Big)\\&- \sum_{l=1}^{L}\sum_{m=1}^{M}\sum_{k^{VLC}=1}^{K^{VLC}} b_{l,m,k^{VLC}}^{VLC}\rho_{VLC}\log_2(1+\gamma_{l,m,k^{VLC}}^{LOS,VLC})\\&- \sum_{f=1}^{F}\sum_{m=1}^{M}\sum_{k^{RF}}^{K^{RF}} \begin{array}{l}\tilde{b}_{f,m,k^{RF}}^{RF}[\rho_{RF}\log_2(1+\gamma_{f,m,k^{RF}}^{LOS,RF})\\ +(1-\rho_{RF})\log_2(1+\gamma_{f,m,k^{RF}}^{NLOS,RF})]\end{array}\end{aligned} \tag{38}$$

Furthermore, Equation (38) can be rewritten as

$$\begin{aligned}&\lambda P_T - \tilde{C}_T = \lambda(P_{VLC} + P_{RF})\\&- \sum_{l=1}^{L}\sum_{m=1}^{M}\sum_{k^{VLC}=1}^{K^{VLC}} b_{l,m,k^{VLC}}^{VLC}\rho_{VLC}\log_2(1+\gamma_{l,m,k^{VLC}}^{LOS,VLC})\\&- \left[\sum_{f=1}^{F}\sum_{m=1}^{M}\sum_{k^{RF}}^{K^{RF}} \begin{array}{l}\tilde{b}_{f,m,k^{RF}}^{RF}(\rho_{RF}\log_2(1+\gamma_{f,m,k^{RF}}^{LOS,RF})+\\ (1-\rho_{RF})\log_2(1+\gamma_{f,m,k^{RF}}^{NLOS,RF}))\end{array}\right]\\&+\lambda\sum_{f=1}^{F}\sum_{m=1}^{M}\sum_{k^{RF}}^{K^{RF}} p_{f,m,k^{RF}}^{RF}\end{aligned} \tag{39}$$

Equation (39) is a separable objective function. The first term $\lambda(P_{VLC}+P_{RF})$ is a fixed term. The second term can be solved by using the ADMM algorithm for the VLC system, as seen in Algorithm 4. The second and the third term can be solved for the RF system, as seen in Algorithm 3.

B. A Short Introduction to the ADMM Algorithm

ADMM has been widely used for solving the distributed convex optimization problems, since it is capable of rapidly converging to optimum [38], [40]. Generally, the ADMM algorithm is used to solve the following type of problems:

$$\min_{\mathbf{x},\mathbf{z}} \quad f(\mathbf{x})+g(\mathbf{z}) \tag{40}$$

$$s.t. \quad \mathbf{Ax}+\mathbf{Bz}=\mathbf{c} \tag{41}$$

where we have $\mathbf{x}\in\mathbb{R}^{n\times 1}$, $\mathbf{z}\in\mathbb{R}^{m\times 1}$, $\mathbf{A}\in\mathbb{R}^{p\times n}$, $\mathbf{B}\in\mathbb{R}^{p\times m}$, and $\mathbf{c}\in\mathbb{R}^{p\times 1}$. The function f and g represent convex optimization problems.

Therefore, the optimization of the problem in (40) can be formulated as

$$p^* = \inf\{f(\mathbf{x}) + g(\mathbf{z}) | A\mathbf{x} + B\mathbf{z} = \mathbf{c}, \tag{42}$$

where $\inf x$ denotes the infimum of x.

We consider the unscaled form ADMM algorithm, while the corresponding augmented Lagrangian is given by

$$\begin{aligned}L_\rho(\mathbf{x},\mathbf{y},\mathbf{z}) = f(\mathbf{x}) + g(\mathbf{z}) + \mathbf{y}^T(\mathbf{Ax}+\mathbf{Bz}-\mathbf{c})\\ +(\rho/2)||\mathbf{Ax}+\mathbf{Bz}-\mathbf{c}||_2^2\end{aligned} \tag{43}$$

where $\mathbf{y}\in\mathbb{R}^{p\times 1}$ is an unscaled dual variable vector and $\rho>0$ is a penalty parameters, while $||x||$ denotes the Euclidean norm operator. As a result, the unscaled ADMM problem can be solved by following iterations until it satisfies the convergency conditions or cycle times.

$$\mathbf{x}^{t+1} := \arg\min_{\mathbf{x}} L_\rho(\mathbf{x},\mathbf{y}^t,\mathbf{z}^t) \tag{44}$$

$$\mathbf{z}^{t+1} := \arg\min_{\mathbf{z}} L_\rho(\mathbf{x}^{t+1},\mathbf{y}^t,\mathbf{z}) \tag{45}$$

$$\mathbf{y}^{t+1} := \mathbf{y}^t + \rho(\mathbf{Ax}^{t+1}+\mathbf{Bz}^{t+1}-\mathbf{c}) \tag{46}$$

where t is the iteration index.

ACKNOWLEDGMENT

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Haijun Zhang (M'13, SM'17) is currently a Full Professor in University of Science and Technology Beijing, China. He was a Postdoctoral Research Fellow in Department of Electrical and Computer Engineering, the University of British Columbia (UBC), Vancouver Campus, Canada. From 2011 to 2012, he visited Centre for Telecommunications Research, King's College London, London, UK, as a Visiting Research Associate. He serves as Editor of IEEE Transactions on Communications, IEEE 5G Tech Focus, EURASIP Journal on Wireless Communications and Networking, and Journal of Network and Computer Applications, and serves/served as a Leading Guest Editor for IEEE Communications Magazine, and IEEE Transactions on Emerging Topics in Computing. He serves/served as General Co-Chair of GameNets'16, Symposium Chair of Globecom'19 and GameNets'14, Track Chair of ScalCom2015, TPC Co-Chair of INFOCOM 2018 Workshop on Integrating Edge Computing, Caching, and Offloading in Next Generation Networks, General Co-Chair of ICC 2018 (ICC 2017, Globecom 2017) Workshop on 5G Ultra Dense Networks, and General Co-Chair of Globecom 2017 Workshop on LTE-U. He received the IEEE ComSoc Young Author Best Paper Award in 2017.

Na Liu received the BS degree in electronic information engineering from Beijing University of Chemical Technology, Beijing, China, in 2016. She is currently pursuing the M.S. degree at the College of Information Science and Technology, Beijing University of Chemical Technology, Beijing, China. Her research interests include resource allocation, power control, energy efficiency in wireless communications, software-defined wireless networks, and visible light communications.

Keping Long (SM'06) received the M.S. and Ph.D. degrees from the University of Electronic Science and Technology of China, Chengdu, in 1995 and 1998, respectively.

From September 1998 to August 2000, he was a Postdoctoral Research Fellow at the National Laboratory of Switching Technology and Telecommunication Networks, Beijing University of Posts and Telecommunications (BUPT), China. From September 2000 to June 2001, he was an Associate Professor at BUPT. From July 2001 to November 2002, he was a Research Fellow with the ARC Special Research Centre for Ultra Broadband Information Networks (CUBIN), University of Melbourne, Australia. He is currently a professor and Dean at the School of Computer and Communication Engineering, University of Science and Technology Beijing. He has published more than 200 papers, 20 keynote speeches, and invited talks at international and local conferences. His research interests are optical Internet technology, new generation network technology, wireless information networks, value-added services, and secure technology of networks.

Dr. Long has been a TPC or ISC member of COIN 2003/04/05/06/07/08/09/10,IEEE IWCN2010, ICON2004/06, APOC2004/06/08, Co-Chair of the organization Committee for IWCMC2006,TPC Chair of COIN 2005/08, and TPC Co-Chair of COIN 2008/10. He was awarded by the National Science Fund for Distinguished Young Scholars of China in 2007 and selected as the Chang Jiang Scholars Program Professor of China in 2008. He is a member of the Editorial Committees of Sciences in China Series F and China Communications.

Julian Cheng (S96M04SM13) received the B.Eng. degree (Hons.) in electrical engineering from the University of Victoria, Victoria, BC, Canada, in 1995, the M.Sc.(Eng.) degree in mathematics and engineering from Queens University, Kingston, ON, Canada, in 1997, and the Ph.D. degree in electrical engineering from the University of Alberta, Edmonton, AB, Canada, in 2003. He is currently a Full Professor in the School of Engineering, Faculty of Applied Science, The University of British Columbia, Kelowna, BC, Canada. He was with Bell Northern Research and NORTEL Networks. His current research interests include digital communications over fading channels, statistical signal processing for wireless applications, optical wireless communications, and 5G wireless networks. He was the Co-Chair of the 12th Canadian Workshop on Information Theory in 2011, the 28th Biennial Symposium on Communications in 2016, and the 6th EAI International Conference on Game Theory for Networks (GameNets 216). He currently serves as an Area Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS, and he was a past Associate Editor of the IEEE TRSACTIONS ON COMMUNICATIONS, the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, the IEEE COMMUNICATIONS LETTERS, and the IEEE ACCESS. Dr. Cheng served as a Guest Editor for a Special Issue of the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS on Optical Wireless Communications. He is also a Registered Professional Engineer with the Province of British Columbia, Canada. Currently he serves as the President of the Canadian Society of Information Theory.

Victor C. M. Leung (S75, M89, SM97, F03) received the B.A.Sc. (Hons.) degree in electrical engineering from the University of British Columbia (UBC) in 1977, and was awarded the APEBC Gold Medal as the head of the graduating class in the Faculty of Applied Science. He attended graduate school at UBC on a Canadian Natural Sciences and Engineering Research Council Postgraduate Scholarship and received the Ph.D. degree in electrical engineering in 1982.

From 1981 to 1987, Dr. Leung was a Senior Member of Technical Staff and satellite system specialist at MPR Teltech Ltd., Canada. In 1988, he was a Lecturer in the Department of Electronics at the Chinese University of Hong Kong. He returned to UBC as a faculty member in 1989, and currently holds the positions of Professor and TELUS Mobility Research Chair in Advanced Telecommunications Engineering in the Department of Electrical and Computer Engineering. Dr. Leung has co-authored more than 1000 journal/conference papers, 38 book chapters, and co-edited 14 book titles. Several of his papers had been selected for best paper awards. His research interests are in the broad areas of wireless networks and mobile systems.

Dr. Leung is a registered Professional Engineer in the Province of British Columbia, Canada. He is a Fellow of IEEE, the Royal Society of Canada, the Engineering Institute of Canada, and the Canadian Academy of Engineering. He was a Distinguished Lecturer of the IEEE Communications Society. He is serving on the editorial boards of the IEEE Transactions on Green Communications and Networking, IEEE Transactions on Cloud Computing, IEEE Access, Computer Communications, and several other journals, and has previously served on the editorial boards of the IEEE Journal on Selected Areas in Communications Wireless Communications Series and Series on Green Communications and Networking, IEEE Transactions on Wireless Communications, IEEE Transactions on Vehicular Technology, IEEE Transactions on Computers, IEEE Wireless Communications Letters, and Journal of Communications and Networks. He has guest-edited many journal special issues, and provided leadership to the organizing committees and technical program committees of numerous conferences and workshops. He received the IEEE Vancouver Section Centennial Award, the 2011 UBC Killam Research Prize, and the 2017 Canadian Award for Telecommunications Research. He co-authored papers that won the 2017 IEEE ComSoc Fred W. Ellersick Prize and the 2017 IEEE Systems Journal Best Paper Award.

Lajos Hanzo (http://www-mobile.ecs.soton.ac.uk) FREng, FIEEE, FIET, Fellow of EURASIP, DSc received his degree in electronics in 1976 and his doctorate in 1983. In 2009 he was awarded an honorary doctorate by the Technical University of Budapest and in 2015 by the University of Edinburgh. In 2016 he was admitted to the Hungarian Academy of Science. During his 40-year career in telecommunications he has held various research and academic posts in Hungary, Germany and the UK. Since 1986 he has been with the School of Electronics and Computer Science, University of Southampton, UK, where he holds the chair in telecommunications. He has successfully supervised 111 PhD students, co-authored 18 John Wiley/IEEE Press books on mobile radio communications totalling in excess of 10 000 pages, published 1700+ research contributions at IEEE Xplore, acted both as TPC and General Chair of IEEE conferences, presented keynote lectures and has been awarded a number of distinctions. Currently he is directing a 60-strong academic research team, working on a range of research projects in the field of wireless multimedia communications sponsored by industry, the Engineering and Physical Sciences Research Council (EPSRC) UK, the European Research Council's Advanced Fellow Grant and the Royal Society's Wolfson Research Merit Award. He is an enthusiastic supporter of industrial and academic liaison and he offers a range of industrial courses. He is also a Governor of the IEEE VTS. During 2008 - 2012 he was the Editor-in-Chief of the IEEE Press and a Chaired Professor also at Tsinghua University, Beijing. For further information on research in progress and associated publications please refer to http://www-mobile.ecs.soton.ac.uk Lajos has 33 000+ citations and an H-index of 73.