Introduction to the papers of TWG04: Geometry education

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Introduction

Around 25 researchers of various geographical origins (from across Europe and also from North America, the Middle East, East Asia) participated in Working Group 4 on Geometry Education. Some 20 contributions (15 papers and 5 posters) informed five discussion sessions and two further sessions dedicated to debates and the preparation of a final report that was presented at the end of the conference. Each discussion session was structured around a selection of contributions, each of which was briefly introduced and followed a reaction from a pre-arranged reactor to inform the collective discussion.

The name of this group was previously *Geometrical Thinking*, and this was modified for this CERME to emphasize the focus on the teaching and learning of geometry. During the previous CERME, four competencies were used to describe geometrical thinking: reasoning, figural, operational and figural. The group took these dimensions as a background that was very helpful to understand each other and to compare our approaches to the issue of what is at stake in the teaching and learning of geometry.

This choice was all the more crucial given that many approaches and issues were discussed during the group sessions. Three main issues were addressed during the working group:

- The role of material activity in the construction of mathematical concepts, including using instruments, manipulation, investigation, modelling…
- Visualization and spatial skills;
- Language, proof and argumentation.

In comparison to the previous CERME, this time psychological points of view, among others, were represented. This raised new questions, often with very different theoretical and methodological backgrounds. As rich as the discussions were, mutual understanding was a great issue. Consequently, we did not focus, during the discussions, especially on one single topic at a given school level. In each of the three issues aforementioned, we tried to identify the interest of various theoretical or methodological approaches, of different cultural or institutional contexts, and the ruptures or continuity during the education process.

It is important to note that almost all the papers addressed ‘classical’ issues in this WG: this means teaching geometry to young children, the impact of specific contexts, geometrical activities in pre-service teacher training, moving from practical to theoretical geometry, using Digital Geometry Environments, and so on. Nevertheless, the main part of the discussions were about confronting,

sometimes in passionate ways, the theoretical and methodological approaches (for instance, didactical engineering was a 'classical' element for this WG) of the phenomena being studied. We try here to give an overview of these debates.

Topics of rich debate in the group

Role of manipulation and thoughtful experiment

This very broad topic has been a great field of study and experiment during the previous CERMEs. The discussed papers concerned students at all level, from kindergarten to university and included the use of instruments for investigation, manipulation and modeling. As these subtopics were strongly linked in the papers, we decided not to split the topic.

The use of two kinds of instruments was evident. One kind of instrument was in the form of ‘material manipulatives’ used as ways to enhance the didactical potentialities of the manipulation by pupils: such manipulatives include the protractor, paper-pin, mathematical machine, 2D and 3D shapes, miniature, compasses, and so on. The other kind of instrument the participants studied comprised various technological tools, including DGEs, videos, IWB, tutorial system, touch-screen tablet. Some papers described the use of only one kind of instruments, while other ones proposed educational environments in which the two kinds of tools were used by students and teachers within complementary and synergistic approaches.

Those papers had different approaches and theoretical backgrounds. For instance, there was discussions about papers that aimed at fostering the use of tools to mediate mathematical meanings (e.g. geometric reflection, Pythagorean theorem), with explicit reference to the theory of semiotic mediation. In papers that used DGEs or manipulation (of shapes, 3D models, geometric miniatures) to pass from the global spatial perception (iconic visualization) to an analytic visualization and to identify proprieties (non-iconic visualization), the main references were to the instrumental approach, the works of Duval and Van Hiele's levels.

Some of the papers examined how the use of tools give opportunities for new experiments that can be useful in teaching. These tools included images used as a way to stimulate dialogic talk amongst student, or technological tools used to change the way of teaching. In this last case, the double approach (didactic and ergonomic) was used.

Two papers focused on teacher education (pre-service and in-service) and reported on the use of DGE to improve generalization and geometrical construction (with their justifications within Euclidean geometry). Here it seems that DGEs are no longer ‘new’ and specific in the classes but nevertheless remain somewhat complicated within teachers’ education.

The group noted, as detailed later, that there is a true need for improving the ‘networking’ between the didactical approach and the psychological approach concerning the use of tools.

Visualization and spatial skills

Some 8 contributions mentioned visualization or spatial skills as a keyword. This topic has been raised over the three previous CERMEs and continues to be an important and autonomous subject in our discussions. We chose to use the word *skills* rather than abilities, capacity or capability, as it can be that these latter terms induce pejorative interpretations, seeing it as something innate that cannot

be changed or trained. The research questions were multiple and intertwined: What are the children spatial skills? How can we evaluate or train it? What is the role of spatial skills in the teaching and learning of mathematics? Visualization: what are we talking about? How to train visualization in geometry? What for? What are the links with language issues? We first had to clarify the relations between visualization and spatial skills: are these referring to the same thing?

In terms of spatial skills, these are related to a psychological point of view. They are linked to the perception, representation, (mental) manipulation of objects, orientation (following a path...), spatial knowledge, location in space. Spatial skills have many facets, and from a psychological point of view visualization is one of these (but it is not very precisely defined in the literature). Spatial skills are very important in mathematics education and has various meanings: sometimes it is not specific to geometry (STEM education), and sometimes it is linked to spatial problems and spatial knowledge (Berthelot & Salin). We pointed some mutual understanding issues between the two fields: for instance, micro/macro space (Berthelot & Salin) are similar to small/large scale (Montello).

What we called visualization is more specific to geometry, and involves combination of perception, interpretation and reasoning. It links perception to reasoning, and helps back and forth between practical and theoretical matters in geometry, so that it depends on spatial skills, mathematical constraints and language. Then, the precise meaning of visualization depends on the topics: visualization is not the same when drawing plane projections of 3D models or when trying to prove a result. From a didactical perspective it has a double nature – psychological and mathematical – and, in this case, spatial skills are a part of visualization. We will keep this acceptation in this text.

Spatial skills are very important for early geometry, as most of the tasks are related to the perception of the space: role in the learning of geometric characteristics of the shapes (Douaire & Emprin), need for the coordination of small/large scale, micro/macro space, local/holistic perception (Vendeira, Papadaki, Klaren)… A psychological point of view is crucial to make more clear general cognitive difficulties of the tasks, and a didactical one links it to the teaching of mathematics. Visualization is more a mathematics education issue, so it is related both to spatial skills and to mathematical knowledge. In a general way, the question is "*How to get enough information using a drawing to solve a given problem?*". It is declined, with very different aims, in every context: as an obstacle (prototypical shapes or too obvious results), using DGS, differences with Autistic Syndrom Disorder students, identification of geometrical properties or characteristics on a drawing… It is a great issue for early geometry, but it is often neglected when students get older, and we suggest this should be studied.

Language, proof and argumentation

The former topics are linked to proof and argumentation by langage. Argumentation and formal proof are linguistic activities about abstract objects, but they involve working on material objects (and then manipulation and visualization). Many works pointed this out. For instance Fujita's dialogic process involves both visualization and social interaction, Klaren's work on ASD students suggests that not seeing a square as a rectangle could be lined to the dutch word for rectangle, and we worked on Duval's dimensional deconstruction which is a discursive process and visualization at the same time.

Some five contributions addressed proof and argumentation, not necessarily about proof itself but about ways of motivating proofs or argumentation. One topic for discussion was the influence of prototypical images on the reasoning process. Another topic was how teachers can have different concept images of a geometric figure (such as a rhombus) and different conceptions of a valid geometric construction of the shape. A third topic for discussion was the design of tasks that can provoke surprise, uncertainty or cognitive conflict, and tasks that can provoke the reconsideration of conjectures or proofs. This last consideration was strongly linked to the visualization issues, as for instance using non-euclidean geometry was seen as a way to give less visual information and to provoke the use of mathematical proof.

A particular focus for discussion was the digital environment *QED Tutrix* which is being designed to provide hints to the student user, while taking into account a judgment of the student's cognitive state based on the way they are using the system.

Perspectives and conclusion

As might be discerned from the introduction, and as can be found in the papers that follow, there are a number of topics that continue to be of great interest to this topic working group. These include the role of instruments, manipulation, representations, proof and argumentation, and initial geometrical knowledge, in geometry education. We also note that the variety of the teaching and learning contexts increased: young children, secondary school, pre and in-service teachers training, but also university, specific education (ASD students), clinical studies… The synthesis of this numerous points of view required intense and rich debates. At this CERME, in the continuity to the former ones, a number of topics became more important. These include visualization and spatial skills which had already been discussed in the last CERMEs, and language in doing geometry, whose role has increased during this session.

In conclusion, the working group continues to feature great diversity: in cultural backgrounds (curricula, school culture, teaching culture, research culture …), research questions, theoretical backgrounds and methodology. This continues to present some challenges in people understanding each other, sometimes linked to language and sometimes to what can be implicit meanings due to different research backgrounds.

A very visible benefit of the great diversity is that it invariably leads to very fruitful discussions and to attempts (and success) to clarify participants' points of view. In taking forward the work of the group, there is an increasingly important need for combining the frameworks, both theoretical and methodological.