Diagrams in students' proving activity in secondary school geometry

Keith Jones[1] and Kotaro Komatsu[2]

[1]University of Southampton, UK; d.k.jones@soton.ac.uk

[2]Shinshu University, Japan / University of Southampton, UK; kkomatsu@shinshu-u.ac.jp

Keywords: Geometry, proof, proving, diagrams, DGS.

Introduction

Diagrams appear in many different forms in mathematics and in its teaching and learning. While the use of diagrams is pervasive, research on learners' activity with diagrams is somewhat limited (for partial reviews, see Jones, 2013; Sinclair et al., 2016). In this short paper, our focus is on the question of how secondary school students work with diagrams during proving activity in secondary school geometry. We use data from a project investigating the design of dynamic geometry software (DGS) tasks that facilitate students' proving activity (see Komatsu & Jones, 2017).

Existing research and theoretical background

Samkoff et al (2012, p. 49) argue that "diagrams are viewed by mathematicians and mathematics educators alike as an integral component of doing and understanding mathematics". Even so, existing research indicates that, amongst other things, learners' beginning identification and interpretation of diagrams tends to be based on spatio-graphical properties represented in the diagram (Laborde, 2005) and that learners can have difficulties distinguishing within the configurations of a geometric diagram the visual characteristics that are relevant from those that are not (Gal & Linchevski, 2010). How secondary school students work with diagrams during proving activity in secondary school geometry is likely to vary depending on whether the geometrical diagram is 'static' (as in physical books and worksheets) or 'dynamic' (via digital technologies; for example Yerushalmy & Naftaliev, 2011).

The study

Data (in the form of transcribed student talk, student written work, and digital files) come from a task-based interview using the tasks in Figure 1 with a triad of 11th grade students (16-17 years old), Kakeru, Sakura, and Yuka (pseudonyms), from an upper secondary school in Japan. The students had previously learnt geometric proofs, including using the conditions for congruent and similar triangles. As such, they were familiar with the inscribed angle theorem, the inscribed quadrilateral theorem, and the alternate segment theorem. Prior to the task-based interview, they had four hours using DGS.

For Q1, and only using paper and pencil, the students conjectured that $\Delta PAB \sim \Delta PDC$ and wrote a suitable proof based on the inscribed angle theorem. For our analysis for this paper, we focus on what happened as the students worked on Q2 after they had used the DGS to construct the figure.

Findings

Our analysis found that during the time that the students worked on Q2, they moved ('dragged') points A, B, C, and D to various places on circle O. In working with this 'dynamic' diagram, we found that their discussion settled on various versions of the diagram that we could categorise into the six types of diagram shown in Figure 2.

Q1. (1) As shown in the diagram given, there are four points A, B, C, and D on circle O. Draw lines AC and BD, and let point P be the intersection point of the lines. What relationship holds between ΔPAB and ΔPDC? Write your conjecture. (2) Prove your conjecture.

Q2. Construct the diagram shown in Q1 with GeoGebra. Move points A, B, C, and D to various places on circle O to examine the following questions. (1) Is your conjecture in Q1 always true? (2) Is your proof in Q1 always valid?

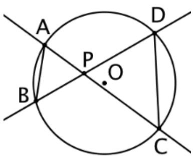

Figure 1: Tasks used in the interview

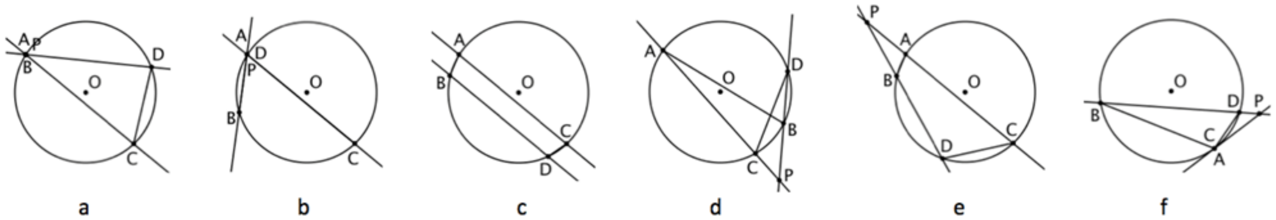

Figure 2: Types of diagrams the students produced

For Figure 2a, which the students created by dragging points so that points A and B (and consequently point P) coincided, they commented that "if (the points) overlap, (our conjecture is) impossible" because "(triangle PAB) disappears". For Figure 2c, where they considered that line AC was parallel to line BD, they concluded that "(our conjecture is) impossible" because "intersection point (P) disappears when (lines AC and BD are) parallel". Eventually, they concluded that their conjecture was not true for these two cases (for more on the other types of diagram, see Komatsu & Jones, 2017).

Concluding comment

The diagrams we found in the students' proving activity underlines the observation by Samkoff et al. (ibid) that the processes involved in using diagrams in mathematics are "surprisingly complex", especially the extent to which students are aware of the general result or a specific diagram.

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