

Estimation of the pressure at a listener's ears in an active headrest system using the remote microphone technique

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Abstract

The remote microphone technique is considered in this paper as a way of estimating the error signals at a listener's ears in an active headrest system, using remotely installed monitoring microphones. A least-squares formulation for the optimal observation filter is presented, including a regularization factor that is chosen to satisfy both the estimation accuracy and robustness to uncertainties. The accuracy of the nearfield estimation is first investigated for a diffuse field via simulations. Additionally, simulations of a free field are also used to investigate the effect of the spatial directivity of the primary field. Finally, experiments in an anechoic chamber are conducted with 24 monitoring microphones around a dummy head positioned in an active headrest system. When six loudspeakers, driven by uncorrelated random disturbances, are used to generate the primary field, the best arrangement of monitoring microphones is considered, taking into account both accuracy and robustness.

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1 I. INTRODUCTION

2 The objective of local active sound control in enclosures is generally to reduce the acous-
3 tic potential energy at particular locations, which produces localized zones of quiet around
4 controlled error sensors. Previous research on active headrest systems, as a practical con-
5 figuration of local control, has shown that this system potentially extends the controllable
6 frequency range and the attenuation performance at desired positions, such as at the ears of
7 a listener, compared to global active sound control [1] [2] [3] [4]. The advantages of a local
8 active headrest system, over a global active control system, may be listed as [2] [5]:

9 - The secondary loudspeakers and error sensors are acoustically well-coupled and so the
10 loudspeakers do not have to drive too hard to control the error signals. As a result, noise
11 enhancement at other positions can be low.

12 - The proximity between the secondary loudspeakers and error sensors can reduce the
13 acoustic propagation delay in the control path, so that is shorter than the propagation time
14 associated with the primary disturbance signals and this allows active control process to
15 remain causal despite electrical delays in the control system.

16 - The plant response, which is the acoustic transfer response between the secondary
17 loudspeakers and error sensors, is relatively simple and largely determined by the local
18 geometry since the direct field of the secondary source is dominant over the reverberant
19 field. Consequently, ill-conditioning can be avoided and the control system can be more
20 robust to small perturbations in the response of the enclosure.

21 Despite these advantages, the direct installation of error sensors in the desired positions
22 has limited the practical application of the active headrest system. For example, passengers
23 in a car are unlikely to accept the need to wear in-ear microphones over long periods of time
24 for local active control. In addition, the zone of quiet around a fixed error sensor, within
25 which more than 10 dB attenuation is achieved, is limited to a diameter of about 1/10 of
26 an acoustic wavelength in a diffuse field, which is about 3.4 cm at 1 kHz [6]. Therefore, any
27 separation between the error sensors and the targeted control regions can lead to the zone
28 of quiet being outside of the desired region of control, and noise enhancement may even be
29 generated due to the constructive interference between the disturbance and control signals.
30 A wide variety of virtual sensing algorithms have been proposed to overcome this problem,
31 by estimating the pressures that would be measured at microphones located in the control-

32 targeted regions, which are referred to as virtual error sensors, from remotely installed
33 physical sensors, which are referred to as monitoring sensors. The virtual microphone ar-
34 rangement, [3] [7] [8], assumes that the disturbance signals at the virtual error sensors are
35 almost equal to those at the monitoring sensors, although the plant responses from the
36 secondary sources to the monitoring and virtual sensors are significantly different due to
37 the nearfield of the secondary sources. With this assumption, Garcia-Bonito *et al.* [8] have
38 shown, in the context of the attenuation performance of an active headrest system in a diffuse
39 sound field, that at frequencies, below about 500 Hz, the attenuation performance achieved
40 using the virtual microphone arrangement is comparable to that achieved when error sig-
41 nals at the ears of a dummy head are used directly, since the primary acoustic fields at the
42 virtual and monitoring sensors are similar. At frequencies from 500 Hz to 1 kHz, however,
43 the substantial difference between the primary acoustic fields at the virtual and monitoring
44 sensors limits the attenuation performance with the virtual microphone arrangement.

45 The remote microphone technique was initially suggested by Roure and Albarrazin [9] and
46 has also been widely investigated in the literature [4] [10] [11]. To overcome the limitation of
47 the virtual microphone arrangement at higher frequencies, the remote microphone technique
48 applies a specific filter, called the observation filter here, to estimate the disturbance signals
49 at the virtual error sensors by optimally weighting the disturbance signals at the monitoring
50 sensors. In a preliminary identification stage, physical error sensors are typically installed
51 at the positions of the virtual error sensors and the disturbance signals are simultaneously
52 measured at both the error sensors and monitoring sensors. From these signals, the obser-
53 vation filter can be designed in either the frequency or time domain [10] [11] [12]. After this
54 initial training, the physical error sensors can be removed and the disturbance signals at the
55 virtual error sensors are estimated by filtering the disturbance signals at the monitoring sen-
56 sors with the observation filter. The attenuation performance of an active headrest system
57 using either the virtual microphone arrangement or the remote microphone technique has
58 been compared by Das *et al.* [4] who showed that about 20 dB more reduction is achieved
59 with the remote microphone technique, compared with the virtual microphone arrangement,
60 with a tonal primary noise of frequency of around 196 Hz and a feedback active headrest
61 system. Although a relatively low frequency tonal noise was controlled in this study, it
62 was clear that a more accurate estimation of the virtual error signals, using the remote
63 microphone technique, can significantly improve the attenuation performance because the

64 feedback control system is particularly affected by inaccurate error signals. The accuracy of
65 the remote microphone technique depends on the spatial properties of the primary acoustic
66 field, however, and this needs to be investigated to determine the attenuation performance
67 and the convergence and stability of the active control system in different applications.

68 Other virtual sensing algorithms have been suggested, including the forward difference
69 extrapolation technique [13] [14], and the Kalman filtering virtual sensing method [15] [16].
70 In the forward difference extrapolation technique, pressure and pressure gradient at the vir-
71 tual error sensor are estimated by fitting a spatial polynomial to the signals measured at
72 a number of monitoring sensors, but the estimation accuracy is limited to low frequencies,
73 for which the distance between the virtual and monitoring sensors is smaller than the wave-
74 length. In the Kalman filtering virtual sensing method, the active control responses are
75 modelled using a state-space system and estimates of the plant states are calculated using
76 measured error signals at the monitoring sensors to estimate signals at the virtual error
77 sensors. These alternative methods were compared with the remote microphone technique
78 in [10].

79 This paper considers the remote microphone technique, with an observation filter cal-
80 culated using the least-squares method, when different arrays of monitoring microphones
81 are used in different sound fields, since the accuracy of the nearfield estimation is mainly
82 determined by the spatial properties of the primary acoustic field. It is shown that an ap-
83 propriate regularization factor is necessary to achieve nearfield estimation that is robust to
84 both acoustic and experimental uncertainties and this can be obtained from the trade-off
85 between the accuracy of the nearfield estimation and the robustness to uncertainties. **An**
86 **understanding of the performance of the remote microphone technique in different environ-**
87 **ments is important, since this is one of the main limitation on the alternative achieved using**
88 **local active control without physical error microphones.**

89 In Section II, the optimal observation filter for the remote microphone technique is formu-
90 lated in the frequency domain. The disturbances are assumed to be random and stationary
91 and can thus be described by their power and cross spectral densities. If the disturbances
92 are broadband random, the causality of an observation filter needs to be considered and the
93 corresponding formulation can be obtained in the time domain as a matrix of finite impulse
94 response filters, as suggested by [11] [17]. However, if the disturbance signals have a nar-
95 rowband spectrum, the frequency domain formulation can be applied without the causality

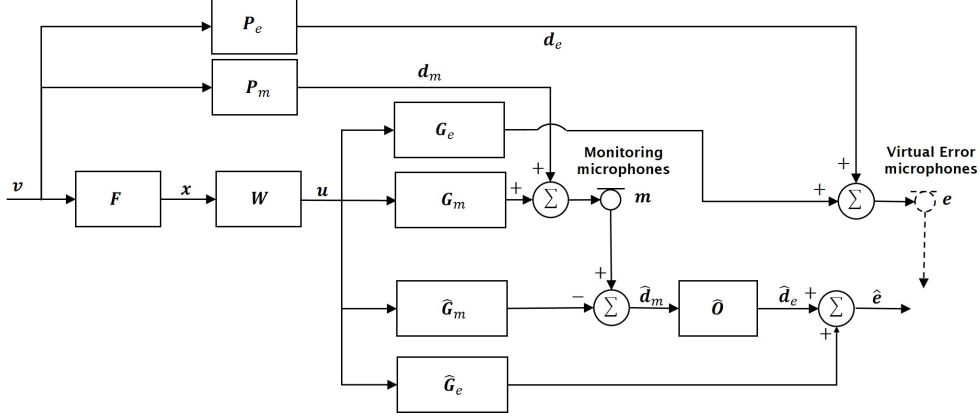


FIG. 1. Block diagram of a feedforward control system using the remote microphone technique.

96 constraint. The formulation of the observation filter in the frequency domain can provide
 97 an efficient tool to investigate the nearfield estimation performance of different sensor ge-
 98 ometries for various acoustic fields using either a numerical model of the acoustic field or
 99 measurements in a practical arrangement. Section III describes numerical simulation results
 100 of nearfield estimation with either a single or multiple monitoring sensors in a diffuse field.
 101 Section IV presents both simulation and experimental results for the remote microphone
 102 technique in an anechoic chamber. Finally, Section V presents the conclusions of this work.

103 II. THE REMOTE MICROPHONE TECHNIQUE

104 The remote microphone technique, which estimates virtual error signals from the signals
 105 measured at the remote monitoring microphones, can be combined with a feedforward active
 106 control system, as shown in Fig. 1. The N_v primary sources, which for random disturbances
 107 have a vector of complex source strengths in a single frequency bin of $\mathbf{v} = [v_1, v_2 \dots v_{N_v}]^T$,
 108 produce a vector of N_e complex disturbance signals, $\mathbf{d}_e = [d_{e1}, d_{e2} \dots d_{eN_e}]^T$ at the virtual
 109 error microphones and a vector of N_x reference signals, $\mathbf{x} = [x_1, x_2 \dots x_{N_x}]^T$ at the reference
 110 microphones via the matrices of transfer responses \mathbf{P}_e and \mathbf{R} , respectively. The reference
 111 signals drive the matrix of control filters, \mathbf{W} , to produce a vector of N_u control signals,
 112 $\mathbf{u} = [u_1, u_2 \dots u_{N_u}]^T$, which are transmitted to the virtual error microphones, via the
 113 matrix of complex plant responses, \mathbf{G}_e , to minimize these disturbance signals. The vector of
 114 complex error signals, \mathbf{e} , at the virtual error microphones after control can thus be defined

115 as

$$\mathbf{e} = \mathbf{d}_e + \mathbf{G}_e \mathbf{u} = \mathbf{d}_e + \mathbf{G}_e \mathbf{W} \mathbf{x} = \mathbf{P}_e \mathbf{v} + \mathbf{G}_e \mathbf{W} \mathbf{R} \mathbf{v}, \quad (1)$$

116 where the dependence on frequency of all of these signals and responses has been suppressed
 117 for notational convenience. Any feedback from the secondary sources to the reference sen-
 118 sors is assumed to be negligible, which can be ensured through the use of feedback can-
 119 cellation filters for example. When the direct measurement of the virtual error signals, \mathbf{e} ,
 120 is infeasible, they can be estimated from the N_m complex monitoring microphone signals,
 121 $\mathbf{m} = [m_1, m_2 \dots m_{N_m}]^T$, which are equal to

$$\mathbf{m} = \mathbf{d}_m + \mathbf{G}_m \mathbf{u} = \mathbf{d}_m + \mathbf{G}_m \mathbf{W} \mathbf{x} = \mathbf{P}_m \mathbf{v} + \mathbf{G}_m \mathbf{W} \mathbf{R} \mathbf{v}, \quad (2)$$

122 where $\mathbf{d}_m = [d_{m1}, d_{m2} \dots d_{mN_m}]^T$ is a vector of disturbance signals measured at the moni-
 123 toring microphones and \mathbf{G}_m is the matrix of plant responses between the secondary sources
 124 and the monitoring microphones. In practice, true matrices of plant responses, \mathbf{G}_e and
 125 \mathbf{G}_m are not usually available and therefore estimates of the plant responses are used in the
 126 controller, as denoted by $\hat{\mathbf{G}}_e$ and $\hat{\mathbf{G}}_m$. Similarly \mathbf{d}_m and \mathbf{d}_e are measured in practice as $\hat{\mathbf{d}}_m$
 127 and $\hat{\mathbf{d}}_e$. If $\hat{\mathbf{O}}$ is the estimated observation filter used to calculate $\hat{\mathbf{d}}_e$ from $\hat{\mathbf{d}}_m$, the estimated
 128 error signals, $\hat{\mathbf{e}}$, at the virtual error microphones can be written as

$$\hat{\mathbf{e}} = \hat{\mathbf{d}}_e + \hat{\mathbf{G}}_e \mathbf{u} = \hat{\mathbf{O}} \hat{\mathbf{d}}_m + \hat{\mathbf{G}}_e \mathbf{u} = \hat{\mathbf{O}} (\mathbf{m} - \hat{\mathbf{G}}_m \mathbf{u}) + \hat{\mathbf{G}}_e \mathbf{u}, \quad (3)$$

129 where in all cases the superscript $\hat{\cdot}$ represents an estimate of an actual value.

130 The observation filter in Eq. (3) will influence how accurately $\hat{\mathbf{d}}_e$ is estimated, and the
 131 estimation error between $\hat{\mathbf{d}}_e$ and \mathbf{d}_e is an important factor in determining the performance of
 132 the control system. The optimal observation filter, \mathbf{O}_{opt} , can be derived by minimizing the
 133 mean squared estimation error between \mathbf{d}_e and $\hat{\mathbf{d}}_e$. In practice, however, it is also necessary
 134 to include a term in the cost function that is proportional to the mean squared magnitude
 135 of all the coefficients in the observation filter, to improve the robustness of this filter. If \mathbf{d}_e
 136 and \mathbf{d}_m are measured in preliminary experiments before active control, so that in this case
 137 \mathbf{d}_m is known, \mathbf{O}_{opt} can be obtained by minimizing the cost function [11],

$$J_1 = \text{trace} \left\{ E [(\mathbf{d}_e - \mathbf{O}\mathbf{d}_m)(\mathbf{d}_e - \mathbf{O}\mathbf{d}_m)^H + \beta \mathbf{O}\mathbf{O}^H] \right\} \quad (4a)$$

$$= \text{trace} \left\{ \mathbf{S}_{d_e d_e} - \mathbf{S}_{d_m d_e} \mathbf{O}^H - \mathbf{O} \mathbf{S}_{d_m d_e}^H + \mathbf{O} (\mathbf{S}_{d_m d_m} + \beta \mathbf{I}) \mathbf{O}^H \right\}, \quad (4b)$$

138 where $E[\]$ is the expectation operator, H is the Hermitian, complex conjugate, transpose
 139 and β is a positive real regularizing parameter, \mathbf{I} is the identity matrix, $\mathbf{S}_{d_e d_e} = E[\mathbf{d}_e \mathbf{d}_e^H]$ and
 140 $\mathbf{S}_{d_m d_m} = E[\mathbf{d}_m \mathbf{d}_m^H]$ are the power spectral density matrices for \mathbf{d}_e and \mathbf{d}_m respectively, and
 141 $\mathbf{S}_{d_m d_e} = E[\mathbf{d}_e \mathbf{d}_m^H]$ is the cross spectral density matrix between \mathbf{d}_m and \mathbf{d}_e . In a stationary
 142 random sound field, when the number of monitoring microphones (N_m) is larger than the
 143 number of error microphones (N_e), the problem of minimizing Eq. (4) is mathematically
 144 overdetermined. The term, $\beta \mathbf{O}\mathbf{O}^H$, in Eq. (4a) always ensures that the matrix, $\mathbf{S}_{d_m d_m} + \beta \mathbf{I}$,
 145 in Eq. (4b) is positive definite, and hence invertable. As a result, the cost function J_1 , which
 146 is a quadratic function of the real and imaginary parts of each element in \mathbf{O} , has a unique
 147 minimum value. The optimal value, \mathbf{O}_{opt} , can be calculated using similar methods to those
 148 described in Ref. [18] to give

$$\mathbf{O}_{\text{opt}} = \mathbf{S}_{d_m d_e} (\mathbf{S}_{d_m d_m} + \beta \mathbf{I})^{-1} = \mathbf{P}_e \mathbf{S}_{vv} \mathbf{P}_m^H (\mathbf{P}_m \mathbf{S}_{vv} \mathbf{P}_m^H + \beta \mathbf{I})^{-1}, \quad (5a,b)$$

149 where \mathbf{S}_{vv} is the power spectral density matrix of the primary source strengths, \mathbf{v} , and \mathbf{P}_e
 150 and \mathbf{P}_m are the matrices of acoustical transfer responses from the primary sources to the
 151 error microphones and monitoring microphones respectively, as shown in Fig. 1. When prac-
 152 tical estimates of \mathbf{P}_e and \mathbf{P}_m are used to calculate the observation filter in Eq. (5b), this is
 153 denoted $\hat{\mathbf{O}}_{\text{opt}}$, as used in Eq. (3).

154 Although the term, $\mathbf{S}_{d_m d_m} + \beta \mathbf{I}$ in Eq. (5a) is positive definite, its inverse can be ill-
 155 conditioned, so that \mathbf{O}_{opt} is sensitive to both physical and numerical uncertainties. A
 156 reduction in the condition number can be obtained by the appropriate selection of the
 157 regularization factor, β . As β increases, the solution for the optimal observation filter be-
 158 comes more robust to practical uncertainties [19], but an excessively large regularization
 159 factor can produce a biased solution, with a higher estimation error. Therefore, to select
 160 an appropriate regularization factor, it is necessary to consider the trade-off between the
 161 robustness and the accuracy with which $\hat{\mathbf{d}}_e$ estimates \mathbf{d}_e .

162 The normalized mean squared estimation error level at the virtual microphone, L_ϵ , can be
 163 defined as

$$L_\epsilon = 10 \log_{10} \left| \frac{S_{\epsilon\epsilon}}{S_{d_e d_e}} \right|, \quad (6)$$

164 where ϵ is defined as $d_e - \hat{d}_e$ at a single virtual microphone location.

165 III. NUMERICAL SIMULATIONS IN **A DIFFUSE FIELD**

166 A. Estimating the disturbance at a single virtual error microphone with a single 167 monitoring microphone

168 When the complex disturbance signal, $d_m(\mathbf{x}_1)$, at a single monitoring microphone at a
 169 point described by the coordinate vector, \mathbf{x}_1 , is used to estimate the complex disturbance
 170 signal, $d_e(\mathbf{x}_2)$, at a single error microphone at a point, \mathbf{x}_2 , the accuracy depends on the
 171 spatial cross-correlation function between the two signals, which for a diffuse primary field,
 172 as derived in [2] [20], can be written as

$$\langle d_e(\mathbf{x}_2) d_m^*(\mathbf{x}_1) \rangle = \langle |p|^2 \rangle [\sin(k\Delta x)] / (k\Delta x) = \langle |p|^2 \rangle \text{sinc } k\Delta x. \quad (7)$$

173 where $\langle \ \rangle$ denotes the operation of spatial averaging by taking samples of the pressure
 174 field over a volume of space with dimensions much larger than an acoustic wavelength, *
 175 indicates complex conjugation, $\langle |p|^2 \rangle$ is the space-averaged mean squared pressure in the
 176 diffuse field, k is the wavenumber and Δx is the Euclidean distance between \mathbf{x}_1 and \mathbf{x}_2 .

177 If only a single monitoring microphone is used, $\mathbf{S}_{d_m d_m}$ is a positive scalar and thus Eq. (5)
 178 can be used with β set to zero, with Eq. (7), to give the space-averaged optimal observation
 179 filter, $O_{\text{opt}}(\Delta x)$ in a diffuse field as

$$O_{\text{opt}}(\Delta x) = \langle d_e(\mathbf{x}_2) d_m^*(\mathbf{x}_1) \rangle \langle d_m(\mathbf{x}_1) d_m^*(\mathbf{x}_1) \rangle^{-1} = \text{sinc } k\Delta x. \quad (8)$$

180 Using Eq. (8) for the nearfield estimation, the space-averaged estimation error between
 181 $d_e(\mathbf{x}_2)$ and $\hat{d}_e(\mathbf{x}_2)$ is given by

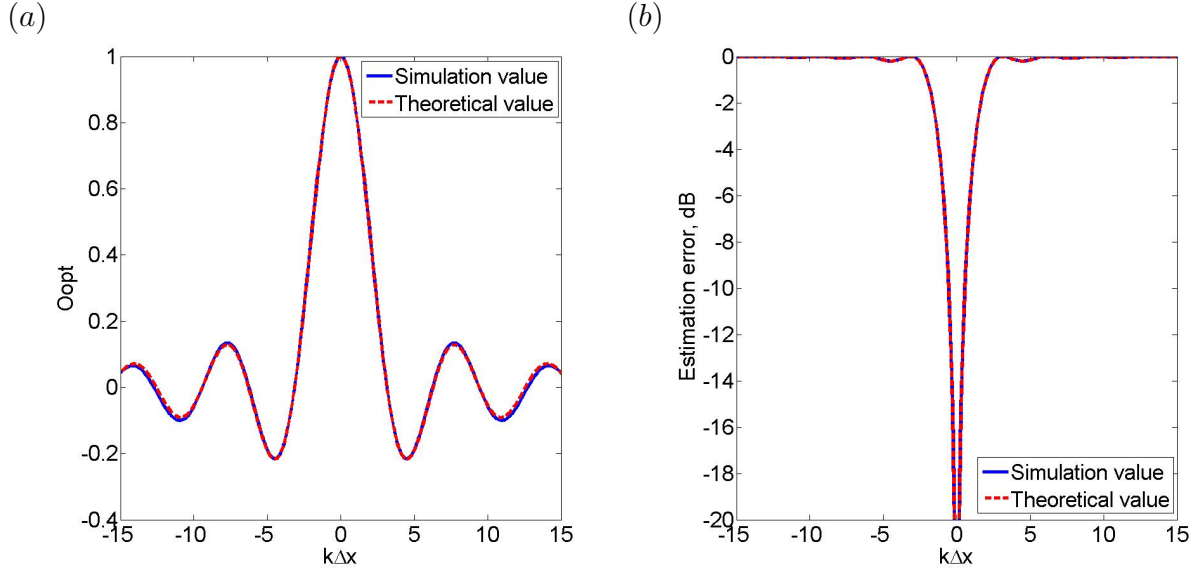


FIG. 2. (a) The response of the observation filter and (b) the estimation error when a single monitoring microphone is used to estimate the disturbance signals at a single error sensor **in a diffuse field**. Simulation (solid line), Theoretical value (dashed line)

$$\begin{aligned}
 L_\epsilon &= 10 \log_{10} \left(\frac{\langle |d_e(\mathbf{x}_2) - O_{\text{opt}}(\Delta x) d_m(\mathbf{x}_1)|^2 \rangle}{\langle |d_e(\mathbf{x}_2)|^2 \rangle} \right) \\
 &= 10 \log_{10}(1 - \text{sinc}^2 k\Delta x).
 \end{aligned} \tag{9}$$

182 To confirm these predictions, a series of simulations have been implemented in a synthetic
 183 diffuse field, as in [11], using 642 uncorrelated monopole sources uniformly distributed over
 184 a sphere of radius 3 m around the monitoring and error microphones. The observation
 185 filter and estimation error are calculated from the disturbance signals of a single monitoring
 186 microphone at the center of the sphere and a 51×51 grid of potential virtual error microphone
 187 locations, in the $1 \text{ m} \times 1 \text{ m}$ region. The results of this simulation are presented in Fig. 2
 188 and show that there is a good agreement between the analytical and simulation results.
 189 The estimation error is significantly greater than -10 dB as $k\Delta x$ is increased beyond about
 190 ± 0.55 , so that Δx is greater than about $1/11.4$ of a wavelength, which occurs when either the
 191 frequency is increased or the distance between the monitoring and virtual error microphones
 192 is increased.

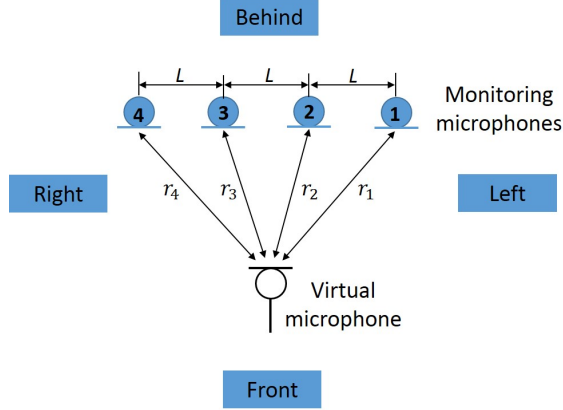


FIG. 3. Geometry of a linear array with four monitoring microphones used to estimate the pressure at a given virtual microphone location.

193 **B. Estimating the disturbance at a single virtual error microphone using multiple**
 194 **monitoring microphones**

195 To demonstrate the performance with multiple monitoring microphones, simulations were
 196 performed using four monitoring microphones to estimate the signal at a single virtual error
 197 microphone in a diffuse field. Assuming that there are four monitoring microphones arranged
 198 in a linear array, as in Fig 3, the elements of $\mathbf{S}_{d_m d_e}$ and $\mathbf{S}_{d_m d_m}$ can be calculated to show
 199 that

$$\mathbf{S}_{d_m d_e} = \langle |p|^2 \rangle [\text{sinc } kr_1 \quad \text{sinc } kr_2 \quad \text{sinc } kr_3 \quad \text{sinc } kr_4], \quad (10)$$

200 where r_1 , r_2 , r_3 and r_4 are the distances from the monitoring microphones to the virtual
 201 microphone, and

$$\mathbf{S}_{d_m d_m} = \langle |p|^2 \rangle \begin{pmatrix} 1 & \text{sinc } kL & \text{sinc } 2kL & \text{sinc } 3kL \\ \text{sinc } kL & 1 & \text{sinc } kL & \text{sinc } 2kL \\ \text{sinc } 2kL & \text{sinc } kL & 1 & \text{sinc } kL \\ \text{sinc } 3kL & \text{sinc } 2kL & \text{sinc } kL & 1 \end{pmatrix}, \quad (11)$$

202 where $\langle |p|^2 \rangle$ is again the space-averaged mean squared pressure in the diffuse field and L
 203 is the separation distance between the monitoring microphones. So that with regularization,

$$\mathbf{O}_{\text{opt}} = [\text{sinc } kr_1 \quad \text{sinc } kr_2 \quad \text{sinc } kr_3 \quad \text{sinc } kr_4] \begin{pmatrix} 1 + \beta & \text{sinc } kL & \text{sinc } 2kL & \text{sinc } 3kL \\ \text{sinc } kL & 1 + \beta & \text{sinc } kL & \text{sinc } 2kL \\ \text{sinc } 2kL & \text{sinc } kL & 1 + \beta & \text{sinc } kL \\ \text{sinc } 3kL & \text{sinc } 2kL & \text{sinc } kL & 1 + \beta \end{pmatrix}^{-1}. \quad (12)$$

204 In this overdetermined case it is important that the regularization factor, β is carefully
 205 selected such that both the estimation error and condition number are acceptable. Fig. 4(a)
 206 shows the condition number and estimation error, as defined by Eq. (6), for different regular-
 207 ization factors when a line array of four monitoring microphones, with a separation distance
 208 along the x-axis of $L = 0.1$ m, is used to estimate the disturbance signal at a single virtual
 209 error microphone at $x = 0$, $y = -2L$, i.e. in front of the microphone array in Fig. 3, in a
 210 diffuse field at a single frequency of around 135 Hz, which corresponds to $kL = 0.25$. From
 211 these results, it can be seen that the condition number is significantly reduced for values of
 212 β above $\beta = 10^2$ and good nearfield estimation is maintained below $\beta = 10^4$. A good choice
 213 of the regularization factor in this case is thus $\beta = 10^3$. It is clear from Eq. (12) that higher
 214 values of regularization will be required at low frequencies, when $kL \ll 1$, whereas when
 215 $kL \gg 1$, the matrix that is being inverted tends to the identity matrix and no regularization
 216 is required. The observation filter responses in the case considered here are shown in Fig. 4(c)
 217 as the location of the virtual error microphone is varied along the x-axis at $y = -2L$. For
 218 comparison, the observation filter responses with $\beta = 0$ are shown in Fig. 4(b). From these
 219 results, it can be seen that with regularization, the amplitudes of the observation filter re-
 220 sponses are significantly reduced, and also the amplitude and phase of the responses change
 221 less with the position of the virtual microphone, which indicates that the estimation will be
 222 more robust to spatial uncertainties in the position of the virtual microphone.

223 Fig. 5 shows contour plots of the estimation error area for different positions of the virtual
 224 error microphone on a $\pm 5L$ grid, at different frequencies, with four monitoring microphones
 225 positioned as in Fig. 3. For these results, the regularization factors for different frequencies
 226 are determined via the process illustrated in Fig. 4, giving $\beta = 10^3$ at $kL = 0.25$, $\beta = 6.6 \times 10^3$
 227 at $kL = 0.5$, $\beta = 6.6 \times 10^4$ at $kL = 1$ and $\beta = 0$ at $kL = 2$. In these plots, the black area(-20
 228 dB error zone) indicates the zone in which -20 dB or less estimation error is achieved and

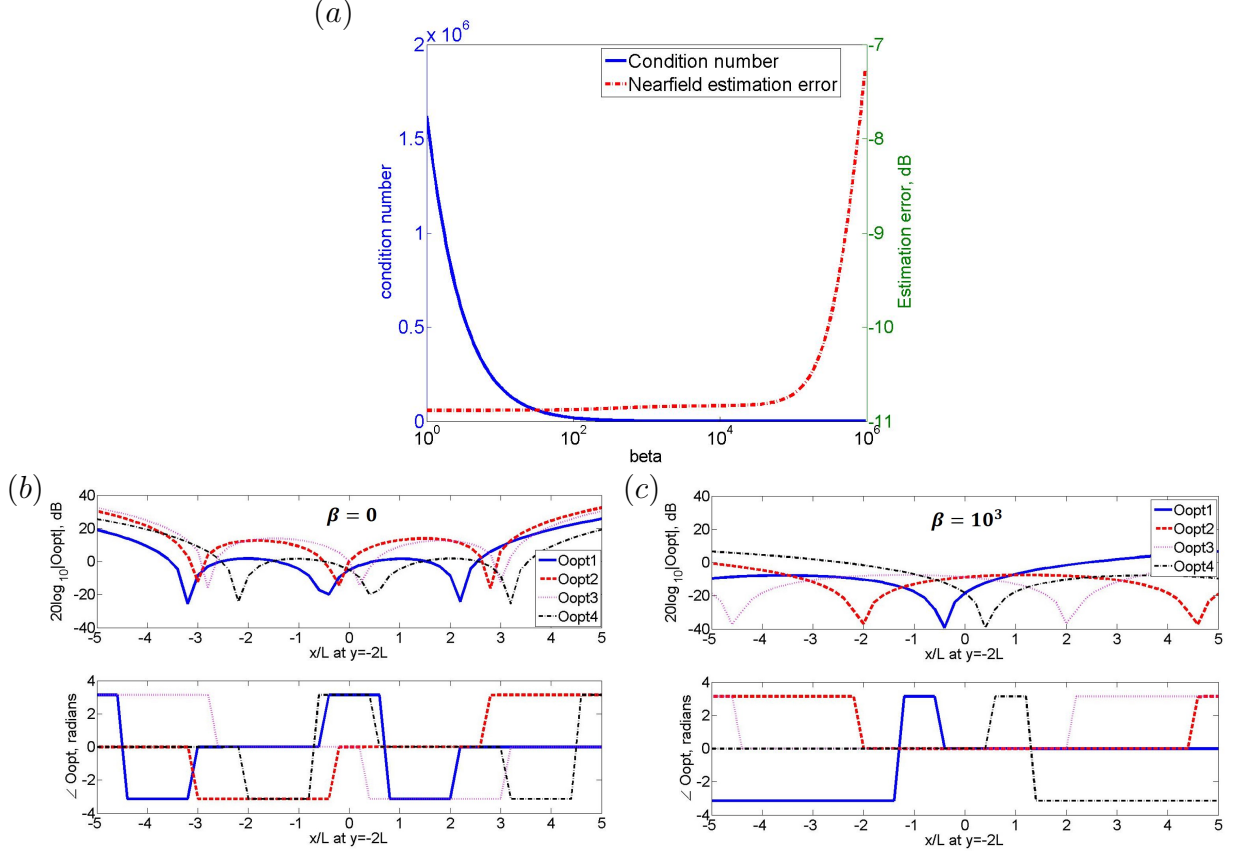


FIG. 4. (a) Condition number of the inverse term in Eq. (5) (Solid line) and the nearfield estimation error (Dash-dot line) with different regularization factors in the observation filter used to estimate disturbance signals of a virtual error microphone at $x = 0$, $y = -2L$ from an array of four monitoring microphones in a diffuse field. The responses of the observation filter matrix, as the location of the single virtual error microphone is varied along x-axis, is also shown (b) without and (c) with the regularization factor ($\beta = 10^3$).

229 the gray area(-10 dB error zone) indicates the zone in which the estimation error is between
 230 -20 dB and -10 dB. Comparing the results in Fig. 5 to the results presented for the single
 231 monitoring microphone case, it can be seen that the shapes of the zones depend on the
 232 geometry of the multiple monitoring microphones. Additionally, although the size of the
 233 zones reduces as the frequency increases, the use of the multiple monitoring microphones
 234 can clearly extend the size of the zones within which the virtual microphone signal can
 235 be accurately estimated, compared to a single monitoring microphone. For example, at
 236 $kL = 0.5$, with a single monitoring microphone, the -10 dB error zone is generated as a
 237 sphere of diameter $\lambda/5.7(= 2.2L)$, from Eq. (9), but with four monitoring microphones, the
 238 length of the -10 dB error zone is around $\lambda/1.3(= 9.6L)$ in the x direction, as shown in

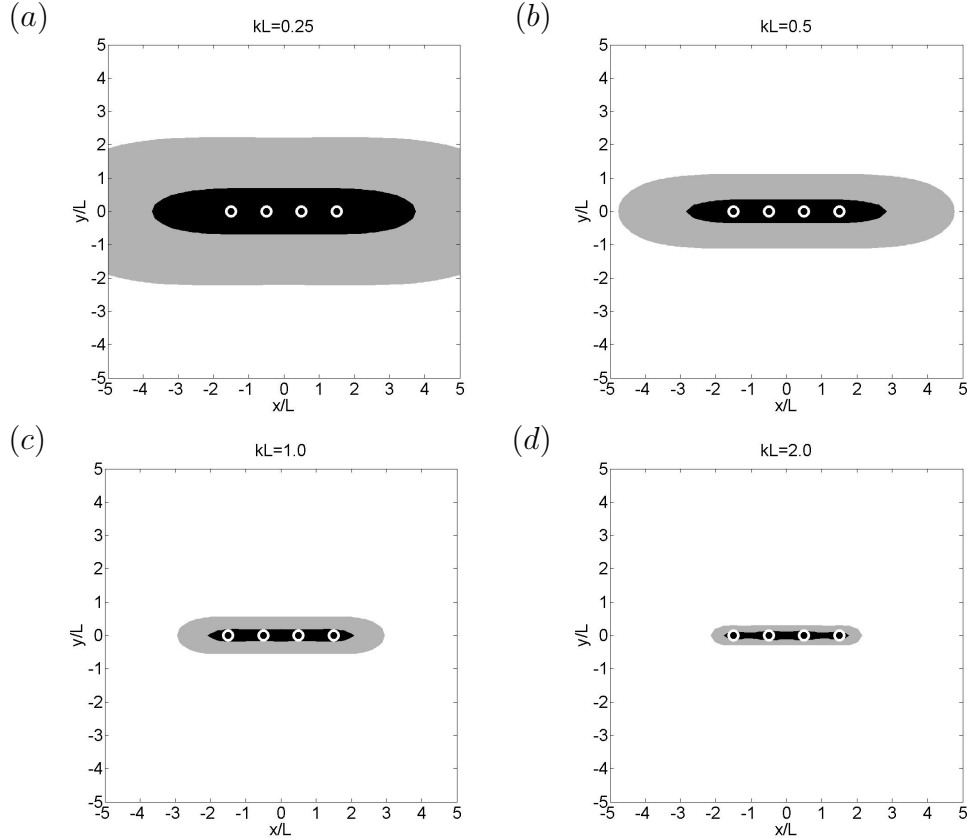


FIG. 5. The -20 dB error zone (black area) in which -20 dB or less estimation error is achieved and the -10dB error zone (gray area) in which an estimation error between -20 dB and -10 dB is achieved when four monitoring microphones at $y = 0$ (white circles) are used to estimate the disturbance at different virtual error microphone locations **at a single frequency in a diffuse field:** (a) $kL = 0.25$ with $\beta = 10^3$, (b) $kL = 0.5$ with $\beta = 6.6 \times 10^3$, (c) $kL = 1$ with $\beta = 6.6 \times 10^4$ and (d) $kL = 2$ with $\beta = 0$.

239 Fig. 5(b).

240 IV. SIMULATIONS AND EXPERIMENTS IN THE FREE FIELD

241 In the previous section, the performance of the remote microphone technique in an ide-
 242 alized diffuse field has been investigated via a number of numerical simulations. It is also
 243 interesting to investigate the influence of the spatial directivity of the primary field on the
 244 accuracy of the nearfield estimation. Therefore, this section first presents simulation results
 245 of nearfield estimation obtained with the same array of four monitoring microphones as used
 246 in the previous section, with a line array of primary sources operating at a specific loca-
 247 tion in a free field environment. Additionally, we investigate the accuracy of the nearfield

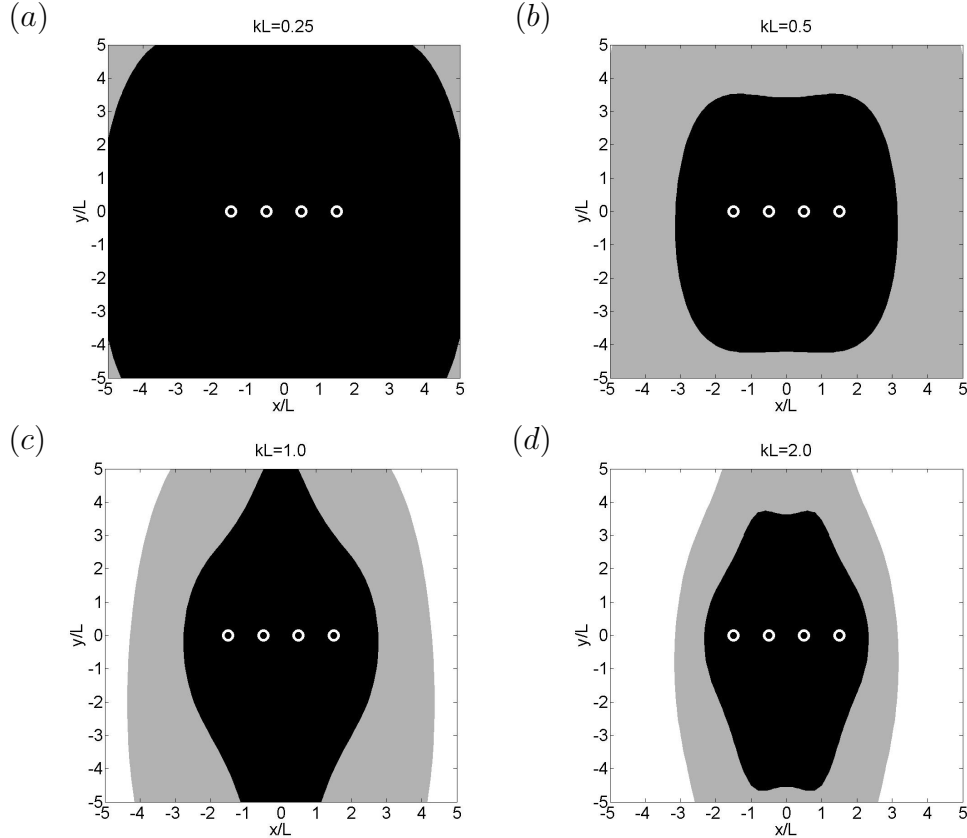


FIG. 6. The -20 dB error zone (black area) in which -20 dB or less estimation error is achieved and the -10dB error zone (gray area) in which an estimation error between -20 dB and -10 dB is achieved when four monitoring microphones at $y = 0$ (white circles) are used to estimate the disturbance at different virtual error microphone locations, with 6 primary sources separated by $8L$ at $y = 30L$, *behind* the microphone array in Fig. 3, producing disturbances in the free field: (a) $kL = 0.25$ with $\beta = 15$, (b) $kL = 0.5$ with $\beta = 60$, (c) $kL = 1$ with $\beta = 200$ and (d) $kL = 2$ with $\beta = 0$.

248 estimation that can be obtained with an experimental installation consisting of arrays of
 249 monitoring microphones around an active headrest system and a number of loudspeakers
 250 that generate disturbances in an anechoic chamber.

251 **A. Simulation study: Estimating disturbance signals due to primary sources lo-** 252 **cated on one side of the monitoring microphones**

253 The same geometry of monitoring microphones and virtual microphones are used for
 254 the following simulations in the free field as used in Fig. 5. The locations of the primary
 255 sources, however, are different. Fig. 6 shows contour plots of the estimation error at different

256 virtual error microphone locations when 6 primary sources with a inter-source spacing of
 257 $8L$ are located at $y = 30L$, i.e. behind the microphone array in Fig. 3 if this is a plan view,
 258 and driven by uncorrelated signals. If only a single primary source is assumed for these
 259 simulations, the spatial correlation is unity at all points in the sound field and so perfect
 260 estimation can, in principle, be achieved. The regularization factors for different frequencies
 261 are again obtained through consideration of a trade-off between the nearfield estimation
 262 and the robustness to uncertainties, and the regularization factors used are: $\beta = 15$ at
 263 $kL = 0.25$, $\beta = 60$ at $kL = 0.5$, $\beta = 200$ at $kL = 1$, $\beta = 0$ at $kL = 2$. From the results
 264 shown in Fig. 6, it can be seen that both the -20 dB error zone and -10 dB error zone in the
 265 free field are significantly larger than those in the diffuse field, since the spatial correlation
 266 between the monitoring microphones and the virtual error microphone in the free field is
 267 improved. Even at the highest frequency of about 1,082 Hz which corresponds to $kL = 2$,
 268 the -10 dB error zone extends to more than $10L$ in the y-direction and this result suggests
 269 that when fixed monitoring microphones are installed on a headrest of a seat, for example,
 270 the pressure at the ears of a listener, which may be up to around 0.5 m away from the
 271 monitoring microphones, could be accurately estimated for primary fields propagating from
 272 one direction.

273 Fig. 7 shows the results of the nearfield estimation using the same monitoring micro-
 274 phones, but when the line array of primary sources is located at $x = -30L$, i.e. on the right
 275 of the microphone array in Fig. 3 if this is a plan view. From these results, it can be seen
 276 that the -20 dB error zone and the -10 dB error zone in the y direction in this case are
 277 smaller than those in Fig. 6 since the relative geometry between the primary sources and
 278 the monitoring microphones in Fig. 6 maintains a higher spatial correlation between the
 279 virtual error microphone and monitoring microphones over a broader area, compared to the
 280 results in Fig. 7. It is interesting that the two error zones in the diffuse field in Fig. 5 can be
 281 approximately generated by the intersection of the corresponding zones in Fig. 6 and Fig. 7,
 282 even though the diffuse field consists of an infinite number of plane waves emanating from
 283 all possible propagation directions.

284 For comparison, the error zones achieved with a single monitoring microphone with the 6
 285 primary sources to the right of the monitoring microphone are shown in Fig. 8. The spatial
 286 extent of the zones in the y direction is similar to those of the four monitoring microphone
 287 array shown in Fig. 7, but the size of the zones is rather less in the x direction, as may

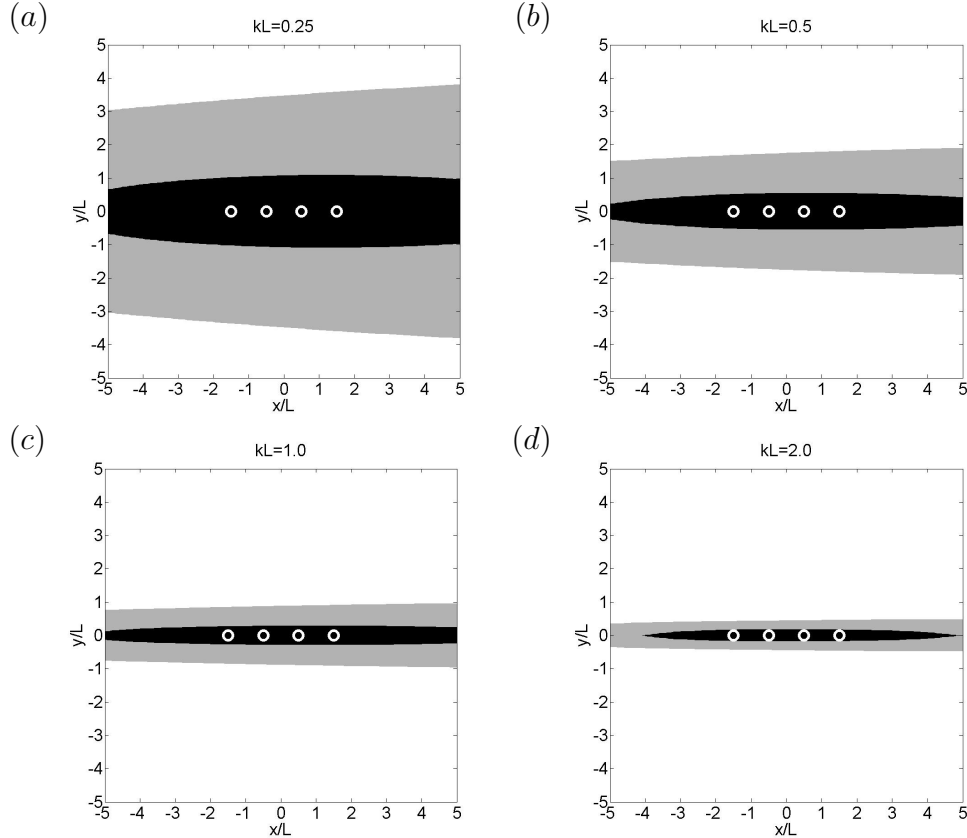


FIG. 7. The -20 dB error zone (black area) in which -20 dB or less estimation error is achieved and the -10dB error zone (gray area) in which an estimation error between -20 dB and -10 dB is achieved when four monitoring microphones at $y = 0$ (white circles) are used to estimate the disturbance at different virtual error microphone locations with 6 primary sources separated by $8L$ at $x = -30L$, on the *right* of the microphone array in Fig. 3, producing disturbances in the free field: (a) $kL = 0.25$ with $\beta = 15$, (b) $kL = 0.5$ with $\beta = 60$, (c) $kL = 1$ with $\beta = 240$ and (d) $kL = 2$ with $\beta = 955$.

288 be expected. Note that very similar results to those in Fig. 8 are obtained with a single
 289 monitoring microphone if the 6 primary sources are behind the monitoring microphone,
 290 except that the plots are rotated by interchanging the x and y axes.

291 An alternative way of looking at these results is to plot the normalized estimation error at
 292 a particular point as a function of frequency, as in Fig. 9. The virtual microphone in this case
 293 is located at position $x = -L$, $y = -2L$ with $L = 0.1$ m, which corresponds approximately
 294 to the location of the right ear of the dummy head used in the following experiments. The
 295 results in Fig. 9 are shown for the numerical simulations with the array of four monitoring
 296 microphones in a diffuse field and with 6 uncorrelated primary sources located to the rear
 297 of the monitoring microphone array. Results when using a single monitoring microphone at

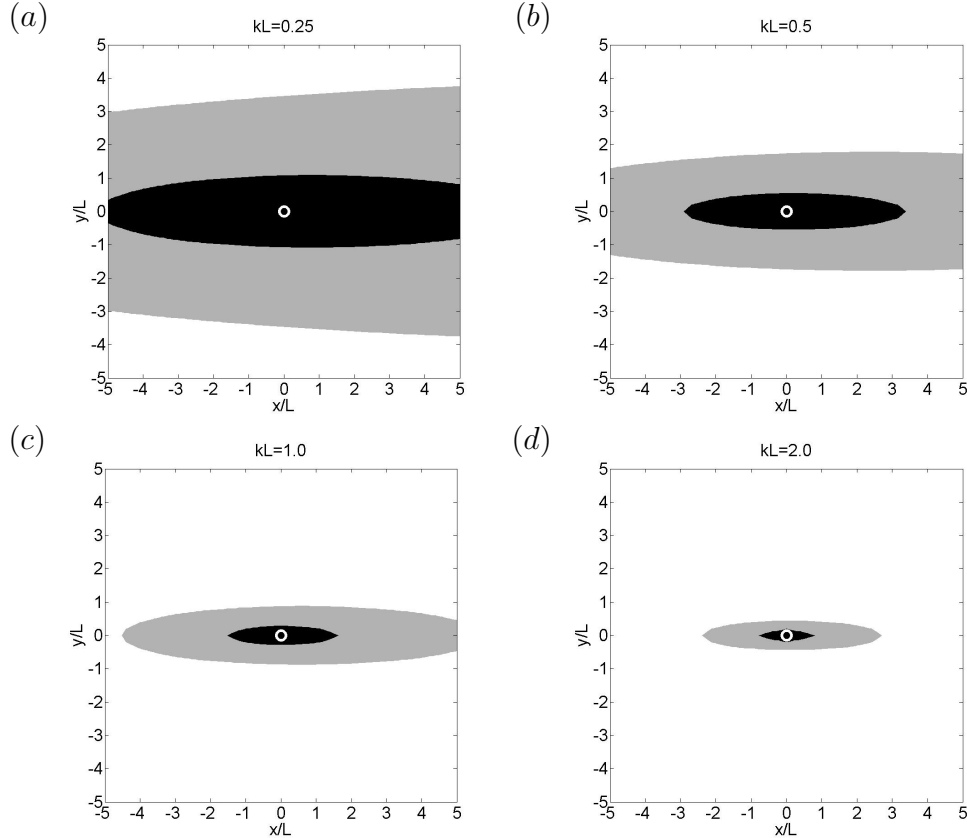


FIG. 8. The -20 dB error zone (black area) in which -20 dB or less estimation error is achieved and the -10dB error zone (gray area) in which an estimation error between -20 dB and -10 dB is achieved when a single monitoring microphones at $x = 0$, $y = 0$ (white circle) is used to estimate the disturbance at different virtual error microphone locations with 6 primary sources separated by $8L$ at $x = -30L$, on the *right* of the single microphone, producing disturbances in the free field: (a) $kL = 0.25$, (b) $kL = 0.5$, (c) $kL = 1$ and (d) $kL = 2$.

298 the origin to estimate the disturbance at the virtual error microphone are also shown. From
 299 these results it can be seen that in general, the estimation error increases with frequency,
 300 as expected. Although the results with the four monitoring microphone array are only
 301 slightly better than with the single monitoring microphone in the diffuse field, with the
 302 normalized error being below -10 dB up to around 200 Hz in both cases, the estimation
 303 error with the monitoring microphone array is much less if the primary sources are behind
 304 of the array, in which case the error is below -20 dB up to 1 kHz. In the latter case, the
 305 incident wave from the primary sources located to the rear is accurately detected by the
 306 monitoring microphone array before it reaches the virtual microphone position, so that its
 307 waveform can be accurately estimated. The results for the primary sources on the right of
 308 the monitoring microphones are similar to those for the diffuse field in this case and so are

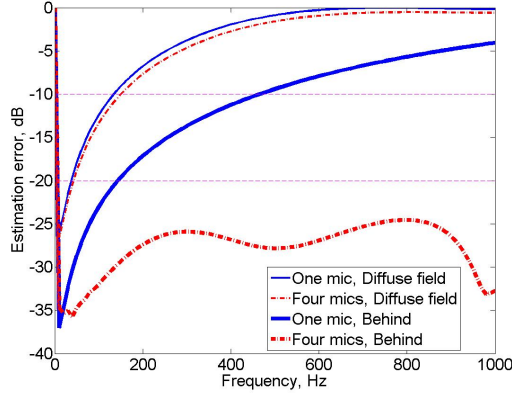


FIG. 9. The estimation error as a function of frequency when disturbance signals at a single virtual error microphone located at $x = -L$, $y = -2L$ with $L = 0.1$ m, which is an approximate location of the right ear of the dummy head in the following experiment, are estimated using: a single monitoring microphone (thin solid line) and four monitoring microphones (thin dashed dot line) in the diffuse field, and a single monitoring microphone (bold solid line) and four monitoring microphones (bold dashed dot line) in the free field with an array of six primary sources behind the monitoring microphone array.

309 not shown.

310 **B. Experimental study: Estimating disturbance signals due to primary sources in** 311 **an anechoic chamber using arrays of monitoring microphones**

312 To estimate the effect of using the remote microphone technique in practical acoustic
 313 fields, six loudspeakers, which function as primary sources, were installed behind an active
 314 headrest system with a dummy head in an anechoic chamber, together with 24 monitoring
 315 microphones, as shown in Fig. 10. Fig. 10(b) and (c) show how these 24 monitoring mi-
 316 crophones, located at different positions on a mounting structure and the headrest, were
 317 numbered. The aim of the experimental study was to select four monitoring microphones
 318 for the nearfield estimation, in order to investigate the relationship between the geometry
 319 of the monitoring microphone array and the accuracy of the nearfield estimation. The mon-
 320 itoring microphones were originally installed at many possible locations around the head.
 321 To compare the estimated disturbance signals at the virtual error microphone with the ac-
 322 tual disturbance signals, two microphones in the ears of the dummy head were used as the
 323 physical error sensors, in order to identify the observation filter. To be consistent with the

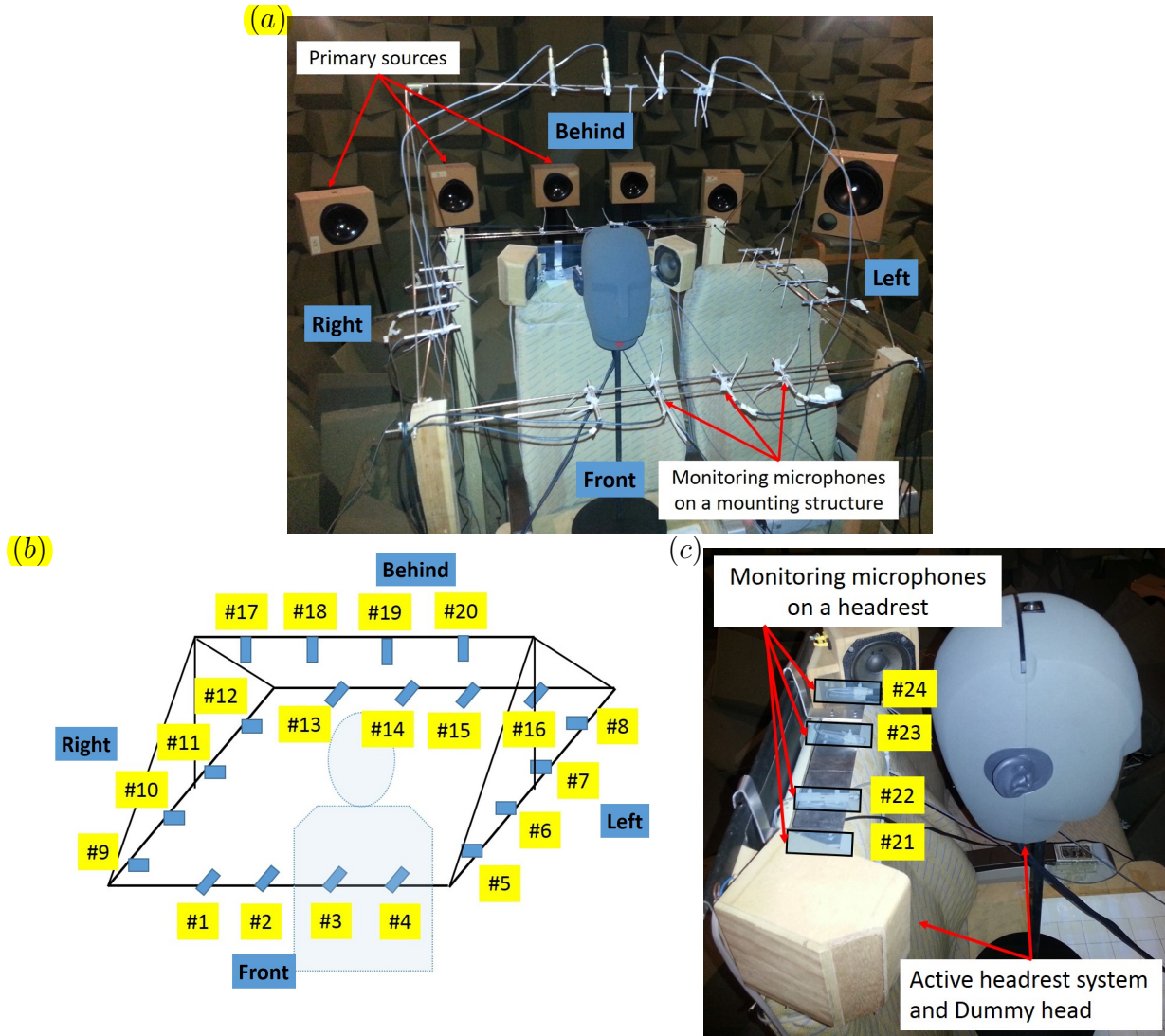


FIG. 10. (a) The overall installation for the nearfield estimation experiments when six primary sources produce uncorrelated random disturbance signals and with monitoring microphones installed around an active headrest system to estimate the disturbance signals at two error microphones in the ears of a dummy head in an anechoic chamber. (b) 20 potential monitoring microphones on a mounting structure. (c) four monitoring microphones on the headrest and two error microphones in the dummy head.

324 above simulations, the six loudspeakers used as the primary sources were driven with un-
 325 correlated white noise signals. Measurements of the various acoustic transfer responses were
 326 conducted and used to calculate optimum observation filters for the various combinations
 327 of monitoring microphones, using Eq. (5), and hence the potential accuracy of the remote
 328 microphone technique has been estimated. Regularization factors for different combinations
 329 of monitoring microphones were also obtained via consideration of the trade-off between the

330 estimation accuracy and the robustness, as in the previous sections.

331 Fig. 11(a) shows, for example, the estimation error at the ears of the dummy head, as a
332 function of frequency, when using monitoring microphones #21, #22, #23 and #24, which
333 are located on the headrest. This result may be compared with the simulation results in
334 Fig. 9. The difference between the experimental results and the simulation results in the
335 free field is mainly due to reflections from the seat, the headrest system and the dummy
336 head. A -10 dB estimation error can be considered to be the limit for a sufficiently accurate
337 practical estimation, because the active headrest system can then achieve 10 dB attenuation
338 with this estimation error if \mathbf{G}_e and \mathbf{G}_m in Fig. 1 are almost identical to $\hat{\mathbf{G}}_e$ and $\hat{\mathbf{G}}_m$ [22].
339 Therefore, the experimental results in Fig. 11(a) show that this monitoring microphone array
340 can accurately estimate the disturbance up to about 250 Hz.

341 To investigate the effect of different selections of monitoring microphone positions on
342 the accuracy of the nearfield estimation, the estimation error for arrays with other sets
343 of four monitoring microphones was investigated. When monitoring microphones #3, #7,
344 #10, #14 were selected, which form a ring around the headrest, the results are shown in
345 Fig. 11(b). It can be seen that the estimation error with these monitoring microphones,
346 at frequencies below 700 Hz, is better than those in Fig. 11(a) but the estimation error in
347 the frequency range above 700 Hz is degraded compared to the result in Fig. 11(a). It is
348 found through an exhaustive search of all sets of four monitoring microphones, however,
349 that with monitoring microphones #13, #22, #23, #16, with two microphones on the
350 headrest and two on the rear supporting structure, the best nearfield estimation is achieved,
351 as shown in Fig. 11(c). It is clear from the results in Fig. 11 that selection of the monitoring
352 microphone positions should be considered with regard to both the spatial correlation of
353 the primary field and the condition number of the inverted term in Eq. (5). For instance,
354 in the case shown in Fig. 11(a), although the monitoring microphones are located to match
355 with the spatial characteristics of the primary acoustic field, the small distance between the
356 monitoring microphones results in a large condition number corresponding to the inverted
357 term with $\beta = 0$, which suggests that the observation filter will be strongly influenced by the
358 physical and numerical uncertainties in the inverted term. To improve the robustness of this
359 microphone array, a larger regularization factor is required up to 700 Hz, which degrades the
360 nearfield estimation error in this frequency range. Conversely, in Fig. 11(b), although the
361 condition number is lower due to the larger spacing between the microphones, the formation

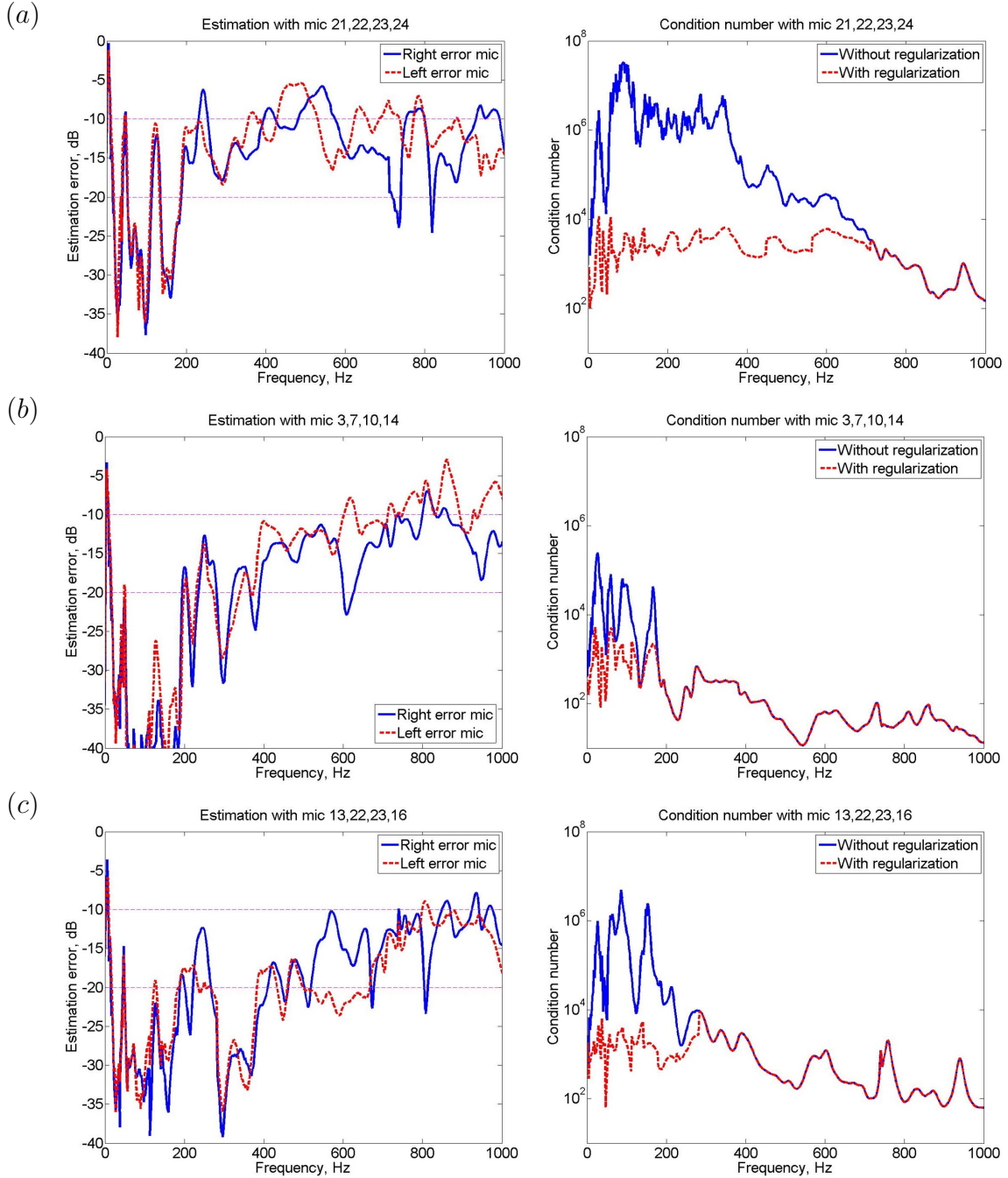


FIG. 11. The nearfield estimation error (left) and condition number (right) calculated from the measured data when different arrays of four monitoring microphones are selected from the monitoring array in Fig. 10 to estimate the disturbance signals at the ears of the dummy head when 6 primary sources located at the rear of the headrest are driven with uncorrelated white noise: (a) monitoring microphones #21, #22, #23, #24, (b) monitoring microphones #3, #7, #10, #14 and (c) monitoring microphones #13, #22, #23, #16.

362 of the microphones is not well-matched with the spatial distribution of the sound field and
363 therefore the nearfield estimation accuracy in the frequency range above 700 Hz is not
364 improved. In conclusion, the monitoring microphone array in Fig. 11(c) achieves a good
365 trade-off between spatial matching of the acoustic field and the condition number associated
366 with the inversion. Slight asymmetries in the geometric arrays give rise to differences in the
367 nearfield estimation error at the both ears of the dummy in Fig. 11. **If the primary sources
368 are located at different locations, different combinations of monitoring microphones need to
369 be considered to achieve the best trade-off between the spatial properties of the primary
370 acoustic field and the condition number.**

371 V. CONCLUSION

372 This paper has investigated the potential for using the remote microphone technique in
373 an active headrest system, to avoid the installation of physical error sensors at the desired
374 cancellation positions such as the ears of a listener. A general least-squares formulation
375 has been presented for the optimal observation filter in the frequency domain, using power
376 and cross spectral densities of random and stationary disturbances. In this formulation, a
377 regularization factor has been included to improve the conditioning of the matrix that needs
378 to be inverted, which is chosen using the trade-off between the accuracy of the nearfield
379 estimation and the conditioning of the inversion. This formulation has been used to assess
380 the accuracy of the nearfield estimation using both numerical and experimental investigation.
381 Although a time domain formulation for the least-squares observation filter has been pre-
382 sented [11], in which the constraint of causality can be imposed, in practice it is possible to
383 use a delayed virtual error signal in an adaptive controller [21] and so the lack of causality
384 in the frequency domain approach is not necessarily a limitation, in practice.

385 Simulations using this optimal observation filter have been implemented in a diffuse
386 field. When a single monitoring microphone was used for the nearfield estimation, the
387 observation filter is equal to the sinc $k\Delta x$ function, where k is the wavenumber and Δx is the
388 distance from the monitoring microphone to the virtual error microphone. For simulations,
389 an approximation to a diffuse field was synthesized using 642 uncorrelated monopole sources
390 uniformly distributed over a sphere, and there was good agreement between the analytic
391 results and those from simulations. An array of four monitoring microphones was also

392 investigated to estimate the pressure at a single virtual error microphone in a diffuse field
393 and the estimation regions were calculated within which a virtual microphone would have a
394 mean-squared error of -10 dB or -20 dB. The shapes of these zones depend on the geometry
395 of the monitoring microphone array and the primary source locations, although the area of
396 the zones is generally reduced as the frequency increases, as expected.

397 Finally, the performance of the remote microphone technique on an active headrest has
398 been investigated via experiments in an anechoic chamber, using multiple uncorrelated pri-
399 mary sources to investigate the error at virtual microphones located at the ears of a dummy
400 head. 24 monitoring microphones were installed on a mounting structure and on the head-
401 rest, and the dummy head was located in the seat. Using measured acoustical transfer
402 responses, the results show that the choice of the monitoring microphone positions should
403 consider both the spatial correlation of the primary field and the condition number of the
404 inverted term of the observation filter. This paper has thus investigated the performance of
405 the remote microphone technique under ideal and practical conditions, using both numerical
406 models and measurements in practical arrangements. The results provide an insight into
407 the use of the remote microphone technique in several potential applications, particularly in
408 an active headrest system.

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