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EFFICIENT ROBUST DESIGN FOR MANUFACTURING PROCESS CAPABILITY

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ABSTRACT

The presence of process variations in manufacturing any product is inevitable. Manufacturing variations can result in performance loss, high scrap, redesign and product failure. This paper proposes a methodology for robust design against manufacturing process variations. The proposed method is employed to seek compressor blade designs which have less sensitive aerodynamic performance in presence of manufacturing uncertainties. A novel geometry modeling technique is presented to model the manufacturing uncertainty in compressor blades. A Gaussian Stochastic Process Model is employed as a surrogate to the expensive CFD simulations. The probabilistic performance of each design is evaluated using Bayesian Monte Carlo Simulation. This is combined with a Multiobjective Optimization process to allow explicit trade-off between the mean and standard deviation of the performance. The aim is to provide the designer with a Pareto-Optimal robust design set to choose the design which meets the performance specifications in presence of manufacturing uncertainty. *Keywords*: Robust Design; Bayesian Monte Carlo; Manufacturing Variations; Process Capability; Compressor Blades

INTRODUCTION

Manufacturing variations can lead to loss in quality due to performance degradation, non-conformance to specifications, high cost of redesign or scrap and failure. Although the manufacturing process is stochastic in nature, the traditional methods assume deterministic conditions during the design activity. As a result, the manufactured product may differ from the proposed ideal design and its performance may be sensitive to manufacturing variations. Further to assure that a product meets the design specification, tight tolerance or a higher precision manufacturing process is selected, which can lead to considerable increase in manufacturing cost. Hence, it is important to consider the effect of manufacturing variations on a product during the design phase and select a design which is less sensitive to such variations. More often than not the design engineers do not have enough information about the downstream manufacturing process capability [1]. This can lead to designs with infeasible manufacturability or tolerances which are costly or impossible to achieve using existing manufacturing processes. In recent years there have been many attempts to provide the designer with process capability data [2]. Process Capability is the expected probability distribution of the manufactured products using a manufacturing process in ideal conditions [3], see figure 1. Numerous methods for measuring process capability exist in the manufacturing literature [3–6]. In the presence of prior knowledge about the process capability of the manufacturing processes one can model the manufacturing uncertainty and propagate it through the design system.

A robust design problem is one in which a design is sought that is relatively insensitive to small changes in uncertain parameters. Robust design primarily deals with minimizing the effect of uncertainty on the system without reducing the sources of uncertainty [7]. In the 1970s, Taguchi emphasized the need to reduce variation in products and processes to improve their quality [8, 9]. Welch et al [10, 11] proposed a system for quality improvement via computer experiments as an alternative to Taguchi's method. Statistical decision theory has also been used to formulate robust design as an optimization problem. The minimax strategy [12] can be used to find a design with optimal worst case performance [13]. Huyse et al [14, 15] used the idea of Bayes risk minimization to achieve consistent improvement of the performance over a given range of uncertainty parameters.

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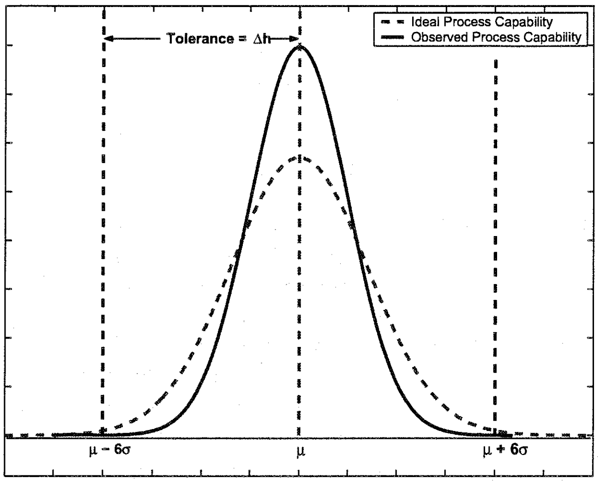

Figure 1. Ideal and observed process capability

The main limitation of these methods is that the uncertainty analysis is very computationally expensive which makes it infeasible for applications to industrial problems. The aim of this paper is to present a methodology for fast uncertainty analysis with applications to robust design problems. The proposed method is applied to aerodynamic design of compressor blades. Bayesian Monte Carlo Simulation is employed for evaluating the statistics of aerodynamic performance in the presence of manufacturing uncertainty. A multi-objective optimizer is employed to explicitly trade-off between the mean and standard deviation of the performance. Finally, the performance of a selected robust design is compared with an optimal design obtained using a standard deterministic formulation.

ROBUST DESIGN METHOD

The presence of some degree of uncertainty in characterizing any real engineering system is inevitable. In aerodynamic design, the most common geometric uncertainties observed are due to erosion, icing, damage or manufacturing variations. Traditional design methods tend to optimize designs for their nominal performance. These deterministic methods, when used for product design, either tend to over optimize or produce solutions that perform well at design points but may have poor off-design characteristics. Robust design methods take into account these uncertainties and seek designs which are less sensitive in performance.

Taguchi Method

The most popular robust design method is Taguchi's method which aims at exploiting the interactions between the parameters of the system to reduce variability. In this method, the inputs are classified as (1) control factors ($\mathbf{x} = \{x_1, x_2, ..., x_p\} \in \chi$) and (2) noise factors ($\xi = \{\xi_1, \xi_2, ..., \xi_q\} \in \mathcal{E}$). Control factors are the parameters which can be easily controlled by the designer whereas noise factors are the parameters that are difficult or expensive to control. To obtain the objective function Taguchi methods employ a Signal to Noise Ratio (SNR) to define a quantitative measure of the variability. The objective is to maximize SNR which leads to a reduction in variations. An overview of Taguchi's experimentation strategy and parameter design method can be found in [7]. The use of SNR for robust design may lead to non-optimal solutions [16] and loss of information in data [17]. Another major criticism of the Taguchi method is the use of a cross product array of control and noise factors to evaluate SNR. This requires a large dataset and is computationally expensive [10, 18]. For a detailed critical overview of Taguchi methods the reader is referred to [19, 20].

Combined Array Based Methods

Welch et al. [10] and Box et al. [21] suggested the use of combined array and metamodeling techniques to alleviate some of the limitations of Taguchi's method. In the combined array approach both the control and noise factors are varied together using DOE techniques. This saves computational effort as compared to the cross product array method. In the combined array (CA) based metamodel strategy for robust design a DOE (Design of Experiments) is performed over the $\{\mathbf{x}, \xi\} \in \mathcal{R}^{p+q}$. Let us denote this new variable as $\tilde{\mathbf{x}}$, then the response at the DOE points using the analysis code can be represented as $[\tilde{\mathbf{x}}^{(i)}, y(\tilde{\mathbf{x}}^{(i)})]$. This dataset can be utilized for finding the robust design by minimizing some loss function. The training dataset, obtained using the CA method, can also be used to train a metamodel $\hat{y}(\tilde{\mathbf{x}})$ for predicting the response quantity $y(\tilde{\mathbf{x}})$ [11]. Using the metamodel, the robust design problem could be formulated as

$$\mathbf{x}^* = arg\ \underset{\mathbf{x}}{min} \int_{\mathcal{E}} \hat{y}(\mathbf{x}, \xi) P(\xi) d\xi. \qquad (1)$$

Trosset et al. [22] proposed a four step procedure to improve the metamodel based approach. The steps are (1) choose initial set of points by varying the design and noise variables $\tilde{\mathbf{x}}$, (2) compute the function $y(\tilde{\mathbf{x}})$ at these points, (3) construct a surrogate objective function $\hat{y}$, (4) evaluate the integral in equation 1 and minimize. The minimax method has also been employed to seek a design with the optimal worst case performance [12]. However, the minimax principle is conservative as it seeks to protect the decision maker against the worst case scenario [22].

Multi-Objective Robust Design

Many researchers have proposed to minimize a single objective to achieve robust design. These methods can be classified into two groups where the aim is to minimize (1) the expectation of the objective function in its neighborhood [15,23–25], (2) the variance of the objective function [26]. [27] discussed the drawbacks of minimizing the expectation of the objective function. He argued that positive and negative deviations in the function value in the neighborhood of a target may cancel each other and mislead to a non-robust optimal design. Minimizing the variability function alone can lead to designs that are robust but not optimal and hence not desirable [27,28]. Therefore, it is desirable to optimize both the expectation and variance of the objective function. Robust design can be presented as a multiobjective problem with the goals of minimizing the (1) mean μ value of the performance, and (2) the variance σ^2 of the performance. This method allows explicit tradeoff between the mean and variability of the performance. The multiobjective formulation can be expressed as:

$$\mathbf{x}^* = arg\ \underset{\mathbf{x}}{min}\{\mu,\sigma\} \tag{2}$$

where

$$\mu = \int_{\Xi} \hat{y}(\mathbf{x},\xi)P(\xi)d\xi \tag{3}$$

and

$$\sigma^2 = \int_{\Xi} (\hat{y}-\mu)^2 P(\xi)d\xi. \tag{4}$$

Conventional methods simplify the equations 3 and 4 to obtain a combined objective function of the form $\mu(\mathbf{x}) + W\sigma(\mathbf{x})$ [29]. These methods are referred to as Weighted Sum (WS) methods. The WS methods can only be used if the Pareto front is convex and fails to produce an even distribution of points from all parts of the Pareto set [30]. Genetic Algorithms (GA) are inherently well suited for Multi-objective problems like robust design, as they have the ability to find multiple Pareto-optimal solutions in one single simulation run. In this study we employ the NSGA-II algorithm [31] to solve the multi-objective problem. The NSGA-II method is fast as it has a computational complexity of $O(MN^2)$ (where M is the number of objectives and N is the population size) when compared to other non-dominated GA with computational complexity $O(MN^3)$. NSGA-II method also uses elitism to enhance the performance of the GA and prevent the loss of good solutions once they are found.

All the above mentioned methods require statistics obtained from solving equations 3 and 4. More often than not, the above integrals are impossible or too expensive to evaluate. In the next section we propose a fast uncertainty analysis method

BAYESIAN MONTE CARLO METHOD

Monte Carlo Simulation (MCS) can be applied to compute the statistics of the response quantities of interest provided sufficient number of samples is used. MCS [32,33] employs a random number generator to select points say $\mathbf{X}_n = [\mathbf{x}^{(1)}, \mathbf{x}^{(2)},...,\mathbf{x}^{(n)}]$, in the design space where the response quantity $y^{(i)} = y(\mathbf{x}^{(i)})$ is evaluated using a computational model. The statistics can be expressed as:

$$\langle y(\mathbf{x})\rangle \approx \tilde{y} = \frac{1}{n}\sum_{i=1}^{n} y(\mathbf{x}^{(i)}). \tag{5}$$

where $\tilde{y}$ is referred to as the Monte Carlo estimate. The variance of the Monte Carlo estimate is:

$$Var(y) = \frac{1}{n(n-1)}\sum_{i=1}^{n}\left(y(\mathbf{x}^{(i)}) - \tilde{y}\right)^2 = \frac{\sigma_y{}^2}{n}, \tag{6}$$

where σ_y is the sample estimate of the variance of $y(\mathbf{x})$. The variance computed using equation 6 can be used to evaluate the accuracy of the Monte Carlo estimate. It can be deduced from equation 6 that the standard error of $\tilde{y}$ is independent of the dimension of the design space and is given by $\sigma_y/\sqrt{n}$. Hence, MCS estimate has a convergence rate of $O(1/\sqrt{n})$.

The MCS method has many limitations and is often employed as the method of last resort. Firstly, a large sample size is required for accurate inferencing, which rules out its application to computationally expensive high fidelity simulations models. Another drawback of MCS is that the random samples generated for evaluation are not essentially space filling. Unlike field and laboratory experiments, which have randomness and non-repeability, computational models are deterministic. Therefore, to extract the most information it is important to choose the training points which fill the design space in an optimal sense [34]. Finally, the major drawback is that MCS ignores the information in the existing dataset. MSC only uses the observations of response quantity $[y^{(1)}, y^{(2)},...,y^{(n)}]$ for estimating the statistics and does not take into account the points $\mathbf{x}^{(1)}, \mathbf{x}^{(2)},...,\mathbf{x}^{(n)}$ at which they were observed [35].

Several pseudo MCS methods have been proposed to address the deficiency of basic MCS sampling. Some of the widely used pseudo MCS methods are Stratified MCS, Latin Hypercube Sampling (LHS) [36] and orthogonal array (OA) sampling. The underlying idea of Stratified MCS [37] and LHS is to divide the design space into regions of equal probability (bins) and generate pseudo random points, such that no two points lie in the same bin. Many researchers have also used interpolation and regression response models to make MCS computationally less expensive. However, these methods again do not fully utilize the information available in the observed dataset.

Bayesian Monte Carlo

A method is sought which alleviates the above mentioned concerns. The problem at hand is to predict the response at a new point using the information available in the existing dataset. By using Bayes Theorem the conditional probability for $y^{(n+1)}$ at any new point $\mathbf{x}^{(\mathbf{n+1})}$ given the observed data $[\mathbf{X}_n, \mathcal{Y}_n]$ is

$$P(y^{(n+1)}|\mathcal{Y}_n) = \frac{P(\phi^{(n+1)}, \mathcal{Y}_n)}{P(\mathcal{Y}_n)} \tag{7}$$

Equation 7 needs prior knowledge of the probability distribution of the response $\mathcal{Y}$. The observed response outputs, $\mathcal{Y}_n = [y^{(1)}...y^{(n)}]^T$ can be thought to be the realization of a stochastic process. In absence of any prior knowledge of the properties of these random variables, we assume that they are realizations of a Gaussian stochastic process with mean β and covariance Γ. Once the joint probability distribution function is known the information available in the existing dataset can be used for making predictions at any new data point. Having assumed a Gaussian prior over the response outputs, the output at any new point and, the joint probability $P(y^{(n+1)}, \mathcal{Y}_n)$ is Gaussian. Hence, the conditional probability $P(y^{(n+1)}|\mathcal{Y}_n)$ given by equation 7 is also Gaussian. The posterior distribution is given by

$$P(y^{(n+1)}|\mathcal{Y}_n) \propto exp\left[-\frac{1}{2}\mathcal{Y}_{n+1}\Gamma_{n+1}^{-1}\mathcal{Y}_{n+1}^T\right] \tag{8}$$

where Γ_{n+1} is the $(n+1)\times(n+1)$ covariance matrix for the vector $\mathcal{Y}_{n+1} = (y^{(1)}, y^{(2)}, ..., y^{(n+1)})^T$. Ideally the mean β and covariance Γ should depend on the training dataset. The mean β is usually taken as zero and the covariance Γ is a parametrized function. The parametrized covariance function can be tuned to dataset using Maximum Likelihood Estimation techniques; see for example [38–42].

For the case, when the input vector is uncertain the output response statistics can be efficiently estimated analytically or using simulation techniques via equation 8 [38, 43]. It may be noted that this method alleviates the drawbacks of MCS discussed above. At the same time it provides us with a emulator (Gaussian Process Model), which can be used as a cheap surrogate for expensive analysis tools [44]. In the next section we present its application to robust design for compressor blades in the presence of manufacturing process capability information.

MANUFACTURING UNCERTAINTY MODELING

Process capability is a measure of how a manufacturing process will perform. The most common quantitative definition of process capability is the process spread, or 6σ. The performance of a manufacturing process can be measured using the dimensions of parts produced by the process. In the limiting case, the process is assumed to have a normal distribution with standard deviation σ. In practice, the observed variations in the process will be greater than that predicted by process capability due to temperature variations, tool wear, material properties etc, see figure 1. Once a manufacturing process for manufacturing compressor blade is fixed (flank milling or point milling) and the process capability is known, the manufacturing variations can be modeled. Figure 2 shows the manufacturing uncertainty band around the nominal compressor blade. The task at hand is to simulate

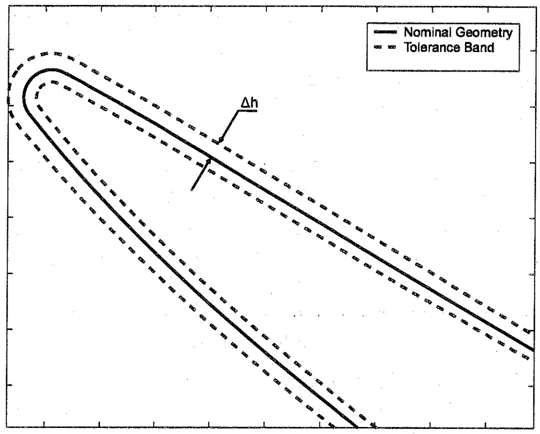

Figure 2. Manufacturing uncertainty band in compressor blade

a manufacturing process such that the observed manufactured blades have a normal distribution with $6\sigma = \Delta h$.

Parametrization and Analysis

To model manufacturing uncertainty we need to find a method which can describe geometry variations in a given tolerance band around the nominal geometry. These could be variations in chord, camber and thickness. Here we present an efficient method using combination of Hick-Hennes functions and splines for modeling manufacturing variations. To parametrize the blade section geometry we use a linear combination of Hicks-Henne functions super-imposed on a baseline shape. For the problem under consideration, we have used 10 Hicks-Henne functions, five each for the upper and lower airfoil section, to parametrize the compressor fan blade. The Hicks-Henne shape functions can be expressed as

$$b_i(x) = sin^4(\pi x^{m_i}) \ , \ \ m_i = ln(0.5)/ln(x_{M_i}) \ \ i = 1,2,...n \quad (9)$$

where x is the normalized chord-wise coordinate starting from the trailing edge encompassing the whole airfoil and back to the trailing edge. $(0 \leq x \leq 1)$, x_{M_i} are preselected-selected values corresponding to the location of the maxima and n is the number of Hicks-Henne functions used. In the present study, the locations of x_{M_i} for $i = 1,2,....,5$ are chosen in a manner to ensure clustering near the leading edge. This ensures more points where the curvature is higher and thus more variety in shapes near the leading edge. Some typical shapes are shown in figure 3 Once a parametrized geometry model is available, they

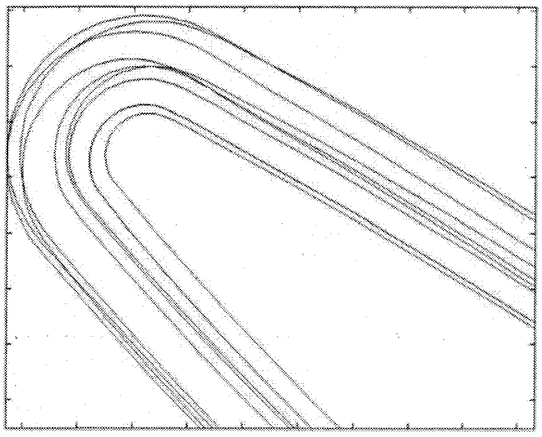

Figure 3. Small perturbations in the design variables within the tolerance band is used to model the manufacturing variations at each design point

parameters can be varied to produce a normal distribution for simulating the manufacturing process. This is combined with the Rolls-Royce propriety code PADRAM, a parametric design and meshing routine employed for automating the geometry creation and grid generation process [45]. PADRAM makes use of both transfinite interpolation and elliptic grid generation to generate hybrid C-O-H meshes. An orthogonal body fitted O mesh is used to capture the viscous region of the airfoil whilst an H mesh is used near the boundary where stretched cells are required, for example in the wake region. After grid refinement studies we select a mesh of the order of 28,000 cells in two dimensions.

A non-linear, unstructured viscous flow solver HYDRA is used for the CFD simulation [46]. It solves the Reynolds Averaged steady Navier-Stokes (RANS) equations with the Spalart-Allmaras turbulence model. To accelerate convergence to steady-state HYDRA employs a multigrid algorithm with preconditioning [47]. A four level multigrid is used for the present simulations. The inlet boundary conditions for the CFD analysis are Total temperature = 290 Kelvin, Total Pressure = 63400 Pascal, Whirl Angle = -37.28 Degrees and the outlet boundary condition is Static Pressure = 52000 Pascal. An initial uniform flow condition with Density = 0.7675 kg/m^3, Velocity = 0 and Pressure = 66932 Pascal is considered. The converged CFD solution is used to calculate the pressure loss at the nominal geometry and typically takes 1200 seconds (0.5 hour) using a Intel(R) Xeon(TM) CPU 3.06GHz dual processor machine.

Probabilistic Analysis

Bayesian Monte Carlo Simulation (BMCS) is employed in conjunction with the above mentioned parametric model and CFD analysis tool, to simulate the manufacturing process for the baseline geometry. A normal distribution for the noise variables is assumed with the tolerance limits assigned to lie at $[-6\sigma, +6\sigma]$. A 100,000 point BMCS is run using the surrogate model to generate the probability distribution of the pressure loss. The histogram for the pressure loss is shown in Fig. 4. The BMCS using the surrogate model takes less than 8 min-

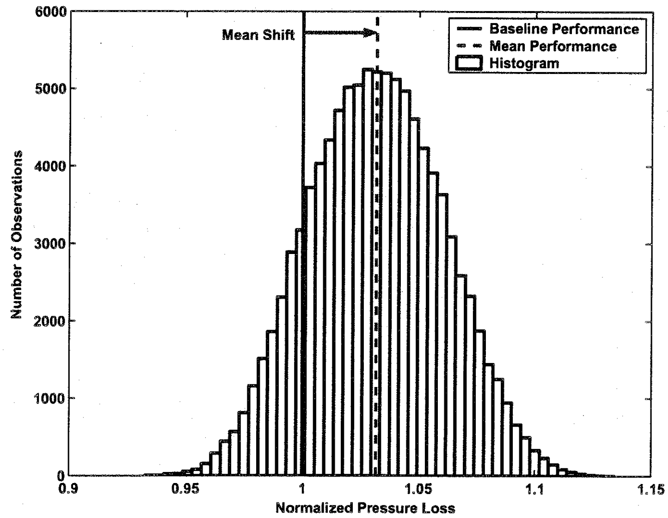

Figure 4. Histogram of performance of baseline geometry with manufacturing uncertainty

utes to carry out 100,000 evaluations on an Intel(R) Xeon(TM) CPU 3.06GHz dual processor machine. This shows substantial savings in computational time, as compared to using the high-fidelity CFD model for probabilistic studies which would have taken 2000000 minutes (33333.34 hours) for the same analysis. The results obtained suggest that manufacturing variations can deteriorate the aerodynamic performance of a blade significantly. In the worst case there can be 14% degradation in pressure loss coefficient. It also suggests almost 4% shift in mean performance from the nominal performance.

NUMERICAL STUDIES

In the robust design method employed we first select the design space and use DOE techniques to rationally choose a set of compressor blade sections as m initial candidate points. These blades are modeled using the parameterization method discussed earlier in this paper. PADRAM is used to produce high quality hybrid meshes and the multigrid RANS solver HYDRA is used for CFD simulations to calculate the total pressure loss over the compressor blade sections. The resulting dataset is used to train the covariance parameters for the Gaussian prior using maximum likelihood estimation. The mean and standard deviation of the total pressure loss at each design point (over the noise variables) is evaluated using a 10,000 point BMCS. The steps involved in the proposed robust design methodology are shown in figure 5.

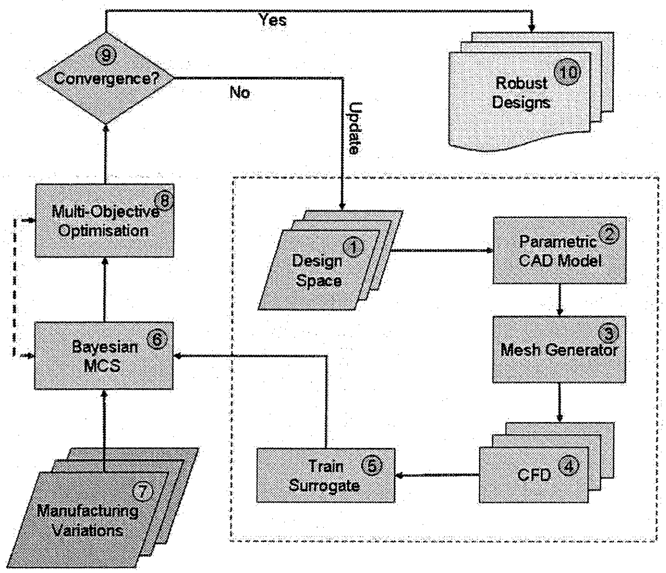

Figure 5. Flowchart for BMCS based Robust Design

NSGA-II is used in conjunction with the surrogate model to search the entire design space to obtain Pareto-optimal solutions. A low-crowding algorithm, which maximizes the euclidean distance between the Pareto points, is used to select points which are then verified by running full scale CFD simulations. Figure 6 shows the initial dataset, subsequent updates points and the final Pareto Front after ten updates. Note that exact aerodynamic analysis using the high fidelity code was conducted only at 300 points.

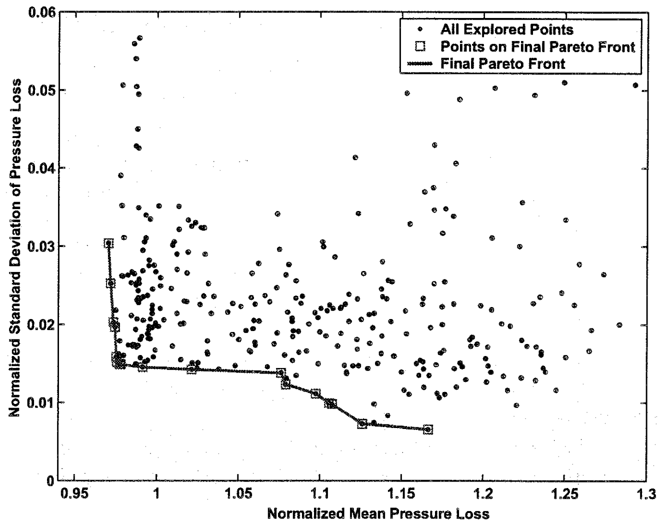

Figure 6. Final Pareto front with all explored points

Significant improvement in Pareto front as compared to the initial Pareto front is obtained, see figure 7.

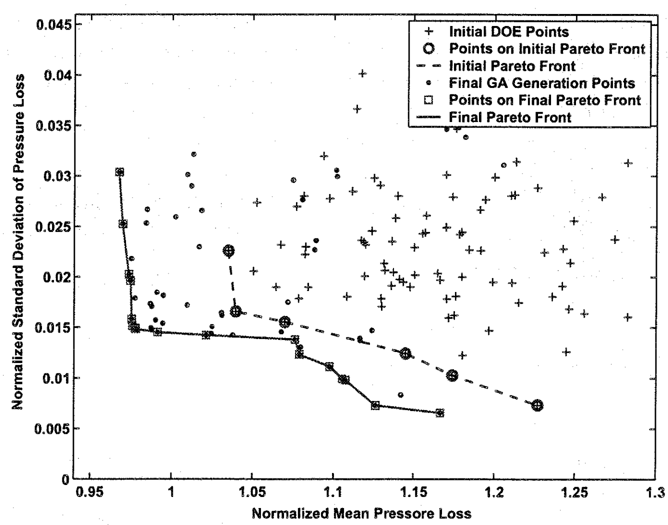

Figure 7. Improvement in Pareto Front from initial Pareto Front

Comparison with Deterministic Design

It is sensible to compare the robust blades obtained with a deterministically optimized blade to understand the trade-offs obtained. To provide a benchmark against which the results of a multiobjective robust design search can be compared, we perform a traditional deterministic optimization study. Deterministic design methods seek to optimize the nominal performance of the system, i.e., optimize blade geometries for low pressure loss coefficients. A simulated annealing algorithm, with direct search employed for initial guess, is employed. This search is performed in conjunction with the surrogate model and the initial dataset. A robust design from the final Pareto Set is selected for comparison. The total pressure loss for the deterministically optimal blade and robust blade is verified using CFD solution. The nominal performance of the deterministically optimal blade is better as compared to the robust blade. BMCS of 100,000 points over the manufacturing variations (noise space) is performed for both the designs. Figure 8 shows the comparison in the performance of the two blades. As expected, the optimal blade obtained from the deterministic optimization has better nominal performance but the performance deteriorates significantly in the presence of manufacturing variations. The histogram of the robust geometry

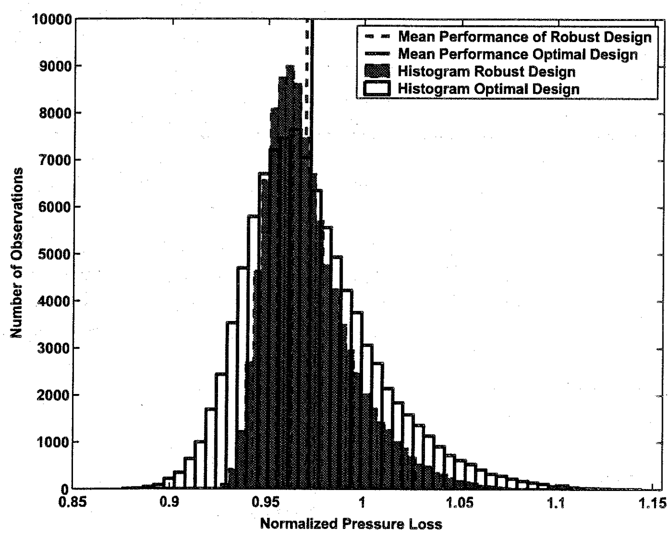

Figure 8. Comparison between probabilistic performance of Robust blade and Deterministic Optimal blade

shows less variability in pressure loss coefficient as compared to the design obtained using a deterministic approach. The comparison of the statistics of the baseline, deterministic and robust design is shown in table 1. There is also a considerably lower shift in the mean performance of 1.05% from the nominal performance for the robust blade geometry as compared to almost

Blade Designs	Nominal Performance	Mean Performance	Standard Deviation	Mean Shift
Baseline	1.0	1.0317	0.0295	3.17%
Deterministic	0.9387	0.9724	0.0335	3.59%
Robust	0.9610	0.9711	0.0231	1.05%

Table 1. Comparison between the Baseline, Deterministically Optimal and Robust Design

3.59% for the deterministic optimal blade geometry, see Table 1. The standard deviation of the robust blade [$\sigma = 0.0231$] is lower than the standard deviation of the deterministically optimal blade [$\sigma = 0.0335$]. It can be observed that low variability has been achieved in the robust design at the expense of a marginal loss in the nominal performance. However, the mean performance of the robust blade [$\mu = 0.9711$] is better than the mean performance for the deterministically optimal blade [$\mu = 0.9724$].

CONCLUSION

An efficient robust design methodology was proposed and its application to the design of compressor blades for a given manufacturing process capability was demonstrated. A robust design from the obtained Pareto set was selected for comparison with a deterministically optimal blade design. A BMCS based robust design method was executed and the robust design was found to be considerably less sensitive to manufacturing variations as compared to the deterministic optimal design. The robust design was also found to have better performance than deterministic optimal design in all respects, [Mean, Standard Deviation, Mean Shift], except the nominal performance. Significant computational savings for the robust design studies was also reported.

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