Supplementary Information
Lattice induced strong coupling and line narrowing of split resonances in metamaterials

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1. Metamaterial illumination with THz waves

The THz beam spot is measured to be 5 mm in diameter and the 1 cm x 1 cm metamaterials are held by a sample holder with a circular aperture of 1 cm diameter. Therefore, 25% of the exposed sample is being excited by the incident THz beam. Depending on the metamaterial's periodicity, this corresponds to 1400 (P = 120 μm) to 4000 (P = 70 μm) resonators within the beam spot area, which is sufficient to approximate the illumination of an infinite array by a plane wave[1].

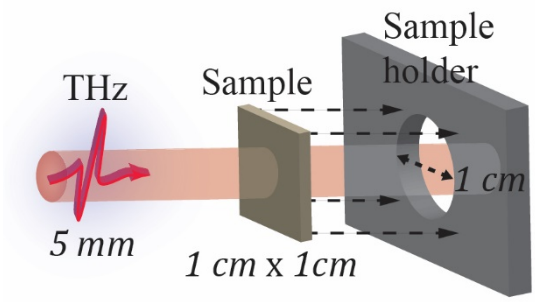

Figure S1: THz beam, metamaterial sample and sample holder with their respective dimensions.

2. Experimental and simulated results of symmetric split ring resonator arrays

Symmetric resonators of the same dimensions as the TASR structure but with symmetric gaps, d = 0 were used to investigate the coupling effect of the fundamental lattice mode to the metamaterial resonance. This investigation serves as a control measurement to find the factors necessary and sufficient for mode splitting/hybridization. These are the excitation of the metamaterial's bright mode which persist in both symmetric and asymmetric resonators, the lattice mode which is due to the periodicity of the structures and the dark mode that results from the asymmetry in the structure. In Figure 2 of the main paper, all three factors contribute to the observed mode splitting. In contrast, in Figure S2 only two factors remain, the bright mode and the lattice mode, whereas the dark mode that requires asymmetry is absent. Starting from high lattice mode frequencies (small periodicity), as the lattice mode frequency is tuned across the broad dipole metamaterial resonance, the resonance line width changes, and the resonance transmission increases but no mode splitting is observed. From these results we can conclude that the lattice mode and bright mode alone are not sufficient for mode splitting, indicating that the dark mode of an asymmetric resonator is also required.

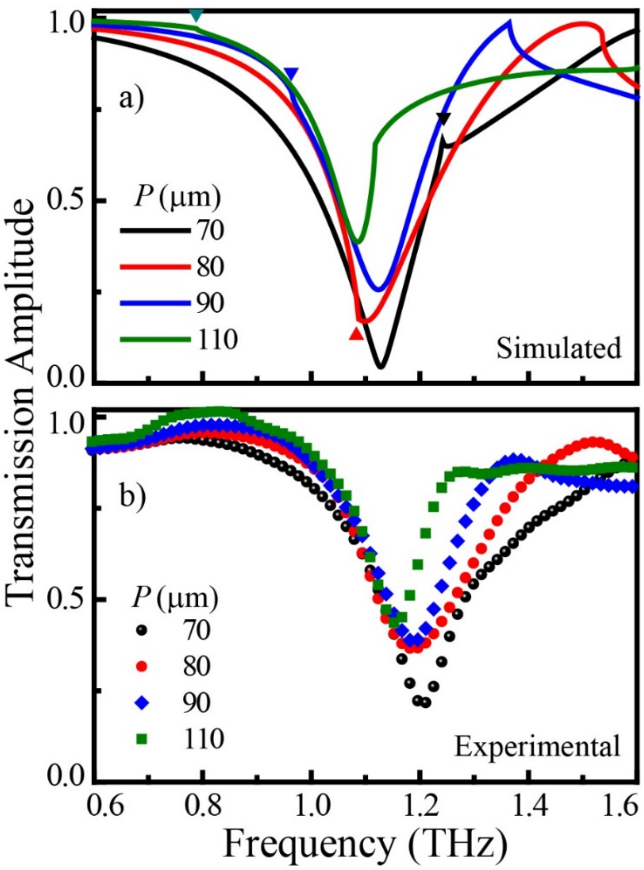

Figure S2: Transmission amplitude of the symmetric metamaterial array (same unit cell dimensions in Figure 1 of the main manuscript but with asymmetry d = 0 μm) for lattice parameters P of 70, 80, 90, and 110 μm. (a) Simulated and (b) experimental transmission spectra, with the arrow heads denoting the first order lattice mode.

3. Three-coupled oscillator model

To theoretically investigate the strong coupling between the resonator modes and the lattice mode, we model the resonances with a three-oscillator model. This model was chosen to interpret both the simulations and experimental results. It could be seen for larger periodicities (low lattice mode frequencies) of our asymmetric TASR metamaterial that the lattice mode decouples from the metamaterial resonance, leading to a broad resonance, where the transparency peak vanishes (the EIT peak vanishes for $P \geq 90$ μm, Figure 2(g, h) of the main paper). From previous studies, metamaterial EIT has been known to result from coupling between bright and dark modes[2-6]. As shown in Figure S2, coupling of our symmetric split-ring resonator with the lattice mode does not lead to any mode splitting, as the dark mode was not excited in this case. The observed dependence of EIT on the lattice period in this study indicates that coupling between the bright and dark modes may be mediated by the lattice mode (Figure S3). Therefore, we model the lattice mode as the mediator between the bright and dark mode, see Equation (S1) with the coupling strengths Ω_1 and Ω_2.

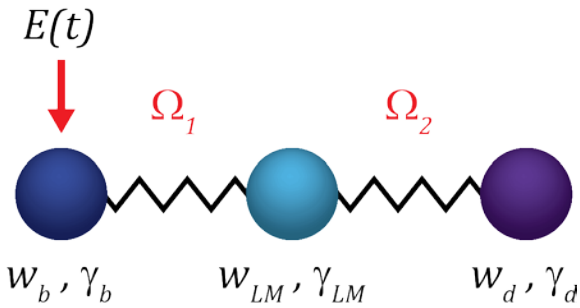

Figure S3: Pictorial representation of the three-oscillator model, showing the coupling between the bright (*b*) and dark (*d*) modes with the lattice mode (*LM*) as the mediator.

The equations of motion for the three coupled oscillators are given as

$$\ddot{x}_b + \gamma_b \dot{x}_b + \omega_b^2 x_b + \Omega_1^2 x_{LM} = E(t) \tag{S1a}$$

$$\ddot{x}_{LM} + \gamma_{LM} \dot{x}_{LM} + \omega_{LM}^2 x_{LM} + \Omega_1^2 x_1 - \Omega_2^2 x_d = 0 \tag{S1b}$$

$$\ddot{x}_d + \gamma_d \dot{x}_d + \omega_d^2 x_d + \Omega_2^2 x_{LM} = 0 \tag{S1c}$$

where $x_{i=b,LM,d} = \tilde{x}_i e^{-i\omega t}$, $(\omega_b, \omega_{LM}, \omega_d)$ and $(\gamma_b, \gamma_{LM}, \gamma_d)$ are the displacement vectors, resonance angular frequencies, and damping rates of the bright mode (*b*), lattice mode (*LM*) and dark mode (*d*) respectively. Ω_1 and Ω_2 are the bright-lattice and dark-lattice mode coupling strengths, respectively. The incident field, $E(t) = \tilde{E}(\omega) e^{-i\omega t}$, directly couples to the metamaterial resonance, resulting in a bright dipolar mode. Whereas the dark mode arises from the structural asymmetric coupling and the lattice mode from the periodicity, and both do not directly couple to the incident field. Solving the derivatives of the displacement vectors and transforming Equation (S1) from the time domain to Equation (S2) in the frequency domain with the corresponding amplitudes $(\tilde{x}_b, \tilde{x}_{LM}, \tilde{x}_d, \tilde{E}(w))$ yields

$$(-\omega^2 - i\omega\gamma_b + \omega_b^2)\tilde{x}_b + \Omega_1^2 \tilde{x}_{LM} = \tilde{E}(w) \tag{S2a}$$

$$(-\omega^2 - i\omega\gamma_{LM} + \omega_{LM}^2)\tilde{x}_{LM} + \Omega_1^2 \tilde{x}_b - \Omega_2^2 \tilde{x}_d = 0 \tag{S2b}$$

$$(-\omega^2 - i\omega\gamma_d + \omega_d^2)\tilde{x}_d + \Omega_2^2 \tilde{x}_{LM} = 0 \tag{S2c}$$

The parameters $(\omega_b, \omega_{LM}, \omega_d)$ and $(\gamma_b, \gamma_{LM}, \gamma_d)$ are taken from the experiments to match the model results. The bright mode frequency $\omega_b = 1.1$ THz is the resonant frequency of the symmetric split-ring resonator from Figure S2a, whereas ω_{LM} and ω_d are obtained from the simplified expression of Equation (1) of the paper. ω_d is taken to be equal ω_{LM} as the lattice mode is assumed to mediate the coupling between dark to the bright mode. Based on the linewidths of the resonances, the damping rates were taken as $\gamma_b = 10^{12}$ rad/s and $\gamma_{LM} = \gamma_d = 0.15 \times 10^{12}$ rad/s while Ω_1 and Ω_2 are taken as free parameters for the numerical fitting to solve Equation (2). The eigenvalues of the coupled equations can be solved by representing Equation (2) with a matrix as shown below

$$\begin{pmatrix} \Lambda_b & \Omega_1^2 & 0 \\ \Omega_1^2 & \Lambda_{LM} & -\Omega_2^2 \\ 0 & \Omega_2^2 & \Lambda_d \end{pmatrix} \begin{pmatrix} \tilde{x}_b \\ \tilde{x}_{LM} \\ \tilde{x}_d \end{pmatrix} = \begin{pmatrix} \tilde{E}(w) \\ 0 \\ 0 \end{pmatrix} \tag{S3}$$

After that the eigenvalues are obtained from diagonalizing the matrix M by finding $Det(M) = 0$ and solving for ω.

$$M = \begin{pmatrix} \Lambda_b & \Omega_1^2 & 0 \\ \Omega_1^2 & \Lambda_{LM} & -\Omega_2^2 \\ 0 & \Omega_2^2 & \Lambda_d \end{pmatrix} \tag{S4}$$

where $\Lambda_i = -\omega^2 - i\omega\gamma_i + \omega_i^2$ and $i = b, LM, d$. The eigenvalues, ω, of the diagonalized matrix correspond to the two hybrid modes of the coupled system and the lattice mode. A numerical fitting was performed to obtain the best values to fit the simulations, resulting in $\Omega_1 = 2.30 \times 10^{12}\ \mathrm{rad/s}$ and $\Omega_2 = 1.84 \times 10^{12}\ \mathrm{rad/s}$. The coupling strength where $\Omega > \gamma$ tells us that the system is strongly coupled, which is physically represented by the anti-crossing plot of Figure 5 of the paper.

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