

# Analysis and modelling of flame speed in autoignitive mixtures of n-heptane and methane

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## Abstract

The effects of pre-ignition chemistry on laminar flame speed in methane/n-heptane fuel blends are investigated numerically, leading to flame speed modelling accounting for these effects. The laminar flame speeds of fuel blends are important input parameters for turbulent combustion models needed to support design of dual-fuel engines. At the autoignitive conditions found in engines, pre-ignition reactions cause the speed of the reaction front to increase. Fuels that exhibit two-stage ignition behaviour, such as n-heptane, also exhibit a two-stage increase in the speed of the reaction front as the reactant residence time increases. There is a corresponding reduction in the flame thickness until the residence time approaches the ignition delay time, whereupon the deflagrative scaling of flame thickness breaks down. The analysis shows that the increase in flame speed is due to distinct contributions of heat release, reactant consumption, and enhanced reactivity ahead of the flame. Addition of methane to n-heptane–air mixtures retards and reduces the first-stage increase in flame speed, in part due to dilution of the more-reactive n-heptane fuel, and in part due to consumption of radical species by the methane chemistry. The effect of methane/n-heptane fuel blending on flame speed is described adequately by a linear mixing rule. The effect of pre-ignition chemistry can then be modelled as a linear function of the progress variable ahead of the flame – accounting for heat release, reactant consumption, and enhanced reactivity ahead of the flame.

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The flame speed model accurately describes the variation of flame speed across the full range of methane/n-heptane blends at engine-relevant conditions, up to the deflagration/ignition transition.

*Keywords:* cool-flame, low temperature chemistry, two-stage ignition, residence time, flame speed, dual-fuel

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## **1. Introduction**

High efficiency engines require high temperature and high pressure combustion conditions – conditions at which the reactants may become autoignitive. Even in engine systems nominally characterised by deflagarative combustion, flame propagation may still be affected by the pre-ignition chemical reactions taking place ahead of the flame. Such reactions can affect flame stability and prompt flashback in steady-flow combustors [1], and affect ignitability [2], heat release rates [3] and detonation phenomena [4] in piston engines. Ignition chemistry differs markedly among fuels, with heavier hydrocarbon fuels exhibiting two-stage ignition under some conditions [5], leading to differences in the structure and speed of flame propagation through autoignitive mixtures [6]. These differences may be particularly relevant in dual fuel engines in which fuels with dissimilar ignition behaviours, such as natural gas and diesel, are introduced separately [7]. However the effect that blending fuels with dissimilar ignition chemistry has on flame propagation through autoignitive mixtures is not well characterised.

Hydrogen and light hydrocarbon fuels such as methane exhibit single-stage high-temperature ignition behaviour. Pre-ignition reactions have limited effect on flame propagation through single-stage ignition mixtures until the mixture is close to the point of autoignition [6, 8]. In contrast, heavier hydrocarbons relevant to real transport fuels, such as n-heptane, exhibit low temperature chemistry (LTC) [9] and *cool flames* [10]. The chemical kinetics controlling cool flames arise at high pressure and moderate temperatures, and are closely related to those of two-stage ignition and Negative Temperature Coefficient

25 (NTC) behaviour [5, 11]. Low temperature chemistry can significantly modify  
26 the chemical and transport properties of a mixture [3], thereby affecting the  
27 laminar flame speed well before the occurrence of high-temperature ignition [6,  
28 12]. The response of laminar flame properties to blending of fuels with dissimilar  
29 ignition behaviours, such as methane and n-heptane, has not been characterised  
30 fully at autoignitive conditions. Methane/n-heptane mixtures are of particular  
31 interest because these have been used in research studies as surrogate fuels for  
32 the natural gas/diesel mixtures arising in pilot-ignited dual fuel engines [e.g.  
33 13, 14, 15].

34 The laminar flame speed and laminar flame thickness are centrally important  
35 in characterisation and modelling of turbulent combustion in engines because  
36 they directly affect the turbulent flame speed [16]. The dependence of the  
37 turbulent flame speed on laminar flame properties persists for flames affected by  
38 LTC [1, 3]. Knowledge of the laminar flame speed and laminar flame thickness  
39 are therefore key to understanding and modelling combustion processes under  
40 the autoignitive conditions that predominate in practical combustion systems.  
41 However established empirical models for the variation of flame speeds with  
42 temperature, pressure, equivalence ratio and dilution, such as in Ref. [17], do  
43 not account for the effect of pre-ignition chemical processes on flame speed.

44 *Laminar flame speed.* Pre-ignition chemical reactions ahead of a flame affect the  
45 propagation speed. As such, there is not a unique freely-propagating laminar  
46 flame speed ( $s_l$ ) at autoignitive conditions. Rather, the flame speed depends  
47 on the extent of the pre-ignition reactions ahead of the flame [6, 8, 12], which  
48 can be related to the residence time  $\tau_f$  of the reactants upstream of the flame  
49 front [18]. A non-autoignitive freely-propagating laminar flame corresponds to  
50 the limit where the residence time is much less than the ignition delay time,  
51  $\tau_f \ll \tau_{ign}$ , whereas the limit  $\tau_f \rightarrow \tau_{ign}$  corresponds to the transition from a  
52 deflagrative flame to a pure ignition front [19].

53 The propagation speed of the flame front can be evaluated in a general way  
54 from the density-corrected displacement speed  $s_f$ , given by  $s_f = \rho s_d / \rho_u$ , where

55  $\rho$  and  $s_d$  are the local density and displacement speed, and  $\rho_u$  is the density  
 56 of the unburnt reactants [20]. Evaluating  $s_f$  in any unstrained one-dimensional  
 57 stationary flame configuration yields a well-defined residence time-dependent  
 58 flame speed, provided that diffusive fluxes through the inlet of the flow domain  
 59 are negligible [6].

60 *Laminar flame thickness.* In the case of diffusion-limited flame propagation with  
 61 negligible chemical reaction upstream of and within the preheat layer, the ther-  
 62 mal thickness of the flame front  $l_f$  scales with the thermal diffusivity  $\alpha$  and  
 63 laminar flame speed as [21]

$$l_f \sim \frac{\alpha}{s_f}. \quad (1)$$

64 The thermal thickness is  $l_f = \Delta T / (dT/dx)_{max}$ , where  $\Delta T$  is the temperature  
 65 rise across the front and  $(dT/dx)_{max}$  is the maximum thermal gradient within  
 66 the flame. Since the variation of thermal diffusivity and flame speed with re-  
 67 actant properties are typically well-modelled by established empirical relations  
 68 [e.g. 17], this scaling relationship provides a simple means for estimating how  
 69 the reaction front thickness varies across a range of combustion conditions for  
 70 which the flame front behaviour is deflagarative. This scaling relationship breaks  
 71 down as the flame transitions into an ignition front. The transport equation for  
 72 progress variable  $c$  within a stationary ignition front is

$$u \frac{\partial c}{\partial x} = \frac{\omega_c}{\rho}, \quad (2)$$

73 where  $x$  and  $u$  are the displacement and velocity normal to the flame and  $\omega_c$   
 74 is the reaction source term for progress variable. The thermal thickness of  
 75 the reaction front can be approximated using the progress variable gradient:  
 76  $l_f \sim (\partial c / \partial x)_{max}^{-1}$ , giving the relationship

$$l_f \sim \frac{\rho_u}{\omega_{c,max}} s_f, \quad (3)$$

77 in which the factor  $\rho_u / \omega_{c,max}$  is positive, with magnitude dependent on the  
 78 thermochemical state of the reactants. The relationship between  $l_f$  and  $s_f$

79 therefore provides an indication of whether a flame front is *deflagrative* ( $\sim$  Eq.  
80 1) or *ignitive* ( $\sim$  Eq. 3).

81 The objectives of this study are to investigate and to model the effects of  
82 different methane/n-heptane fuel blends and their pre-ignition chemistry on the  
83 speed of reaction fronts under dual-fuel compression-ignition engine conditions.  
84 The following section sets out the physical modelling and numerical approach  
85 adopted. The results are then analysed in order (1) to assess the effects on flame  
86 speed and structure due to thermal and chemical contributions of the different  
87 fuels; and (2) to develop a new modelling approach for reaction front speeds  
88 under autoignitive conditions.

## 89 2. Methodology

90 Simulations of adiabatic un-stretched one-dimensional laminar flames are  
91 used to investigate the combined effects of methane/n-heptane ratios and pre-  
92 ignition chemistry at engine-relevant temperatures. The effect of the pre-ignition  
93 chemistry is assessed by varying the residence time upstream of the flame front.  
94 The residence time  $\tau_f$  at the flame front is evaluated as

$$95 \quad \tau_f = \int_{x_i}^{x_f} \frac{1}{u(x)} dx, \quad (4)$$

96 where  $x_i$  is the  $x$ -location of the inlet to the solution domain,  $x_f$  is the location  
97 of the upstream edge of the flame, here defined as being half of one thermal  
98 thickness upstream of the maximum temperature gradient location.

99 Reactant mixtures are described in terms of their total-equivalence ratio,  
100  $\phi_{tot}$ , evaluated in the conventional manner by considering the stoichiometric  
101 oxygen-fuel ratio for the fuel mixture, and fuel-equivalence ratios. The fuel  
102 equivalence ratios are defined by  $\phi_{CH_4} = \nu_{CH_4} Y_{(CH_4,u)} / Y_{(O_2,u)}$  and  $\phi_{C_7H_{16}} =$   
103  $\nu_{C_7H_{16}} Y_{(C_7H_{16},u)} / Y_{(O_2,u)}$ , where subscript  $u$  denotes the unburnt composition  
104 and  $\nu_i$  is the stoichiometric oxygen-to-fuel mass ratio for the  $i^{th}$  fuel species.  
105 These definitions of the fuel equivalence ratio can be added to obtain the total  
106 equivalence ratio,  $\phi_{tot} = \phi_{CH_4} + \phi_{C_7H_{16}}$ .

106 The flames are simulated using the COSILAB one-dimensional flame solver  
107 [22] with multi-component molecular transport and semi-perfect gas models  
108 based on seven-coefficient polynomials for the temperature-dependence of ther-  
109 modynamic properties. The methane/n-heptane chemistry is modelled using a  
110 106 species mechanism for n-heptane combustion [23]. There are few experimen-  
111 tal data available for autoignition and flame propagation in methane/n-heptane  
112 fuel blends. Therefore the chemical model has been selected on the basis of sat-  
113 isfactory autoignition and flame propagation predictions of experimental data  
114 for pure methane–air and n-heptane–air mixtures, and satisfactory agreement  
115 with more detailed models for autoignition and flame propagation in methane/n-  
116 heptane fuel blends. Validation data for the 106 species mechanism used are  
117 provided as supplementary material.

118 The COSILAB software employs adaptive grid refinement and a stationary  
119 flame solution is obtained using a modified-Newton method [22]. The grid-  
120 independence of the solution data presented in this paper has been established  
121 by incrementally tightening the adaptive grid error tolerances until numerical  
122 convergence is achieved, requiring between one and four hundred grid points,  
123 depending on the simulation conditions.

### 124 **3. Results and Discussion**

#### 125 *3.1. Effects of pre-ignition chemistry on flame speed*

126 Figure 1 shows the variation of flame speed with residence time for pure  
127 methane, pure n-heptane and two methane/n-heptane fuel blends at 40 bar and  
128 850 or 1000 K. The flame speed of the methane–air mixture remains approx-  
129 imately constant until the residence time approaches the ignition delay time,  
130 when it increases indefinitely, as observed previously by Habisreuther *et al.* [8].  
131 For an unburned temperature of 1000 K where n-heptane–air mixtures exhibit  
132 single-stage ignition, the evolution of the flame speed of the n-heptane–air mix-  
133 ture is qualitatively similar to the pure methane case. However, for an unburned  
134 temperature of 850 K where n-heptane–air mixtures exhibit two-stage ignition,

135 the flame speed also increases in two distinct stages: the flame speed increases  
136 by 15 % when the residence time reaches the first-stage ignition delay time, then  
137 increases gradually during the second-stage ignition-delay, before increasing in-  
138 definitely as the residence time approaches final ignition delay time. The same  
139 two-stage increase in flame speed is observed for methane/n-heptane blends that  
140 show two-stage ignition behaviour, and the two-stage effect is stronger as the  
141 proportion of n-heptane increases.

142 Figure 2 shows that the first-stage increase in flame speed coincides with the  
143 first-stage increase in temperature, occurring between 0.13-0.18 ms residence  
144 time for 40 bar 850 K stoichiometric n-heptane–air flames. The flame speed  
145 is expected to be affected by the temperature increase, however consumption  
146 of major reactants and production of intermediate species by the pre-ignition  
147 chemical reactions ahead of the flame also affect flame speed. We conduct a nu-  
148 mercial experiment in order to quantify the relative influence that the thermal  
149 and chemical changes have on the flame speed during first-stage ignition. The  
150 experiment isolates the effect of the intermediate species from the thermal effects  
151 by performing modified flame simulations in which we remove all intermediate  
152 species a short distance upstream of the flame while keeping the temperature  
153 unchanged following the procedure set out in the Appendix. Since the inter-  
154 mediate species have been removed from the modified flame, the change in the  
155 speed of the *Modified flame* shown in Fig. 2 is entirely due to the temperature  
156 rise and the consumption of major reactants ahead of the flame front. The re-  
157 sults of the modified flame experiment indicate that that the first stage increase  
158 in flame speed is due to both thermal and chemical influences in approximately  
159 equal measure. Modelling for the flame speed should take each of these effects  
160 into account.

161 *3.2. Effects of pre-ignition chemistry on flame thickness*

162 Diffusion-limited (i.e. deflagarative) flame propagation is expected to exhibit  
163 a scaling relationship between flame thickness and flame speed given by Eq. 1.  
164 Figure 3 shows the variation of the high-temperature flame’s speed with  $\alpha/l_f$

165 for premixed combustion of stoichiometric methane–air and n-heptane–air at a  
166 range of reactant temperature ( $T_u = 700$  to  $1000$  K). Data are normalised by  
167 the short-residence time laminar flame speeds of methane ( $s_{l,CH_4} = 0.79ms^{-1}$ )  
168 and n-heptane ( $s_{l,C_7H_{16}} = 1.20ms^{-1}$ ) at  $850$  K and  $40$  bar. Flames with short  
169 residence times (shown as circular symbols) follow the deflagarative scaling relation-  
170 ship as  $T_u$  is increased from  $700$  to  $1000$  K. The gradient of  $s_f$  with respect  
171 to  $\alpha/l_f$  is greater for n-heptane–air flames due to the greater reactivity of n-  
172 heptane compared to methane. As residence time increases (dashed lines), the  
173 flame speed of the n-heptane–air mixture remains largely unchanged until the  
174 mixture undergoes first-stage ignition. Subsequently the reaction front speed  
175 increases, and the reaction front thickness initially reduces in accordance with  
176 the deflagarative scaling given in Eq. 1. Finally, as the temperature-based  
177 progress variable ( $c \equiv (T - T_u)/(T_b - T_u)$ ) ahead of the flame reaches  $0.15$ ,  
178 the deflagarative scaling breaks down and the front thickness starts to increase  
179 ( $\alpha/l_f$  reduces) as the residence time ahead of the flame approaches the ignition  
180 delay time – marking the transition from deflagarative to ignitive scaling of the  
181 front thickness.

182 Figure 4 presents the variation of the cool-flame’s speed and thickness for the  
183  $850$  K  $40$  bar stoichiometric n-heptane–air flame as the residence time increases.  
184 The thickness of the cool-flame follows the linear scaling behaviour given for igni-  
185 tion fronts in Eq. 3. Despite its name, the cool-flame exhibits ignitive rather  
186 than deflagarative behaviour across all of the conditions in this study. The  
187 absence of deflagarative cool-flames is consistent with Ref. [24], where steady  
188 deflagarative cool-flames could only be stabilised when aerodynamic straining  
189 was used to prevent the development of a high-temperature flame. These ob-  
190 servations support a hypothesis that deflagarative cool-flames, if they arise, do  
191 not survive in premixed combustion configurations in which the cool-flame is  
192 chased by a high-temperature flame – since the high-temperature flame speed  
193 would typically be fast enough to overtake a diffusion-limited cool-flame.

194 The transition between deflagarative and ignitive scaling of the flame thick-  
195 ness is accompanied by diminishing importance of diffusive transport within the

196 flame fronts. Figure 5 shows mass fraction transport budgets for stoichiometric  
197 n-heptane flames at 850 K and 40 bar for two residence times  $\tau_f/\tau_{ign} = 0.85$   
198 and 0.95. The reaction (R), convection (C) and diffusion (D) terms [6] are pre-  
199 sented for the alkyl hydroperoxy radical (QOOH) within the cool-flame and for  
200 OH within the high-temperature flame front. The budgets indicate that dif-  
201 fusive transport in the high-temperature reaction front becomes gradually less  
202 important as the residence time increases, and is generally less important in the  
203 cool-flame front. The magnitude of the diffusion term in the high-temperature  
204 flame is lower at  $\tau_f/\tau_{ign} = 0.95$ , at which point the flame thickness is increasing  
205 with flame speed, than at  $\tau_f/\tau_{ign} = 0.85$ , at which point the flame thickness is  
206 reducing in accordance with the deflagarative scaling (Eq. 1). However the grad-  
207 ual reduction of the diffusive transport contribution does not provide as clear  
208 a delineation of the transition between deflagarative and ignitive behaviours as  
209 the changes in the  $s_f - l_f$  dependence shown in Figs. 3 and 4.

210 Analysis of the flame thickness indicates that the deflagarative scaling in Eq.  
211 1 applies to the high-temperature flame front across a wide range of autoignitive  
212 conditions and, given models for  $s_f$  and  $\alpha$ , provides a useful means of modelling  
213 the variation of  $l_f$  in a flow. Eq. 1 is not applicable to cool-flames or to high-  
214 temperature flames burning through mixture that exhibits ignitive behaviour,  
215 however this restriction is not very limiting since the flame thickness was sought  
216 for use in flamelet combustion models, and such models are not expected to be  
217 valid in relation to flames with ignitive behaviour.

218 *3.3. Modelling methane/n-heptane flame speeds for dual-fuel engines.*

219 The laminar flame speed is a key input for a number of turbulent combus-  
220 tion models [16]. Several empirically-derived algebraic models for laminar flame  
221 speed have been developed and used widely for combustion at non-autoignitive  
222 conditions [e.g. 17]. For applications involving flame propagation through inho-  
223 mogeneous mixtures of natural-gas/diesel or methane/n-heptane, the preceding  
224 results suggest that it would be beneficial to employ a laminar flame speed  
225 model that accounts both for the local composition of the fuel blend, and for

226 the effects of pre-ignition chemistry. Algebraic modelling for these effects is  
 227 developed following a three step approach. First, established empirical models  
 228 for the variation of flame speed with temperature, pressure, equivalence ratio  
 229 and dilution are calibrated for pure methane and for pure n-heptane fuels at  
 230 engine-relevant conditions. Second, new modelling is introduced to account for  
 231 the effects of pre-ignition chemistry on the flame speed – accounting for the  
 232 distinct contributions of heat release, reactant consumption, and enhanced re-  
 233 activity ahead of the flame. Third, a mixing rule is employed in order to model  
 234 the flame speed in methane/n-heptane mixtures based on the flame speed of the  
 235 pure fuels.

236 *Step 1: Flame speeds of pure fuels.* The laminar flame speeds for pure methane  
 237 and pure n-heptane fuels are evaluated for a set of engine-relevant conditions,  
 238 recording the converged flame speed in the limit where the flame residence time  
 239 is much less than the ignition delay time. Algebraic functions describing the  
 240 dependence of flame speed on equivalence ratio [25] and unburnt-temperature,  
 241 pressure and dilution [17] are then fitted to this data. Simulation data for  
 242 the variation of methane and n-heptane flame speeds with equivalence ratio at  
 243 reference conditions of  $T_{u,ref} = 850$  K and  $p_{ref} = 40$  bar are presented in Fig.  
 244 6. The variation of flame speed with equivalence ratio at these conditions is  
 245 adequately modelled by a four-parameter Gaussian function [25],

$$s_{l,ref} = A_1 \phi_{tot}^{-A_2} \exp \left[ -A_3 (\phi_{tot}^{-A_4})^2 \right]. \quad (5)$$

246  $\mathbf{A} = \{A_1, A_2, A_3, A_4\}$  is a set of fitted coefficients for a particular fuel. At  
 247  $T_{u,ref} = 850$  K and  $p_{ref} = 40$  bar, we obtain  $\mathbf{A}_{CH_4} = \{11500, -5.44, 1.20, -1.04\}$   
 248 and  $\mathbf{A}_{C_7H_{16}} = \{344000, -4.80, 0.628, -2.56\}$  by least-squares fitting across the  
 249 range  $0.45 < \phi_{tot} < 1.3$ , yielding the close agreement shown in Fig. 6.

250 Metghalchi and Keck [17] provide the following empirical model for the de-  
 251 pendence of flame speed on the unburnt temperature  $T_u$ , pressure  $p$ , equivalence  
 252 ratio  $\phi_{tot}$  and the mass fraction  $\xi$  of diluents such as recirculated combustion

253 products,

$$s_{l,fuel} = s_{l,ref}(\phi_{tot}) \cdot \left(\frac{T_u}{T_{u,ref}}\right)^\alpha \cdot \left(\frac{p}{p_{ref}}\right)^\beta \cdot \gamma. \quad (6)$$

254  $\alpha$ ,  $\beta$  and  $\gamma$  are model parameters given by expressions of the form:  $\alpha =$   
255  $B_1 - B_2(\phi_{tot} - 1)$ ;  $\beta = -B_3 + B_4(\phi_{tot} - 1)$ ;  $\gamma = 1 - B_5\xi$ , in which  $\mathbf{B}_{fuel} =$   
256  $\{B_1, B_2, B_3, B_4, B_5\}$  is a set of fitted constants. Metghalchi and Keck fit-  
257 ted  $\mathbf{B}_{fuel}$  values using flame speed data for unburnt temperatures up to 700  
258 K, however these values are not generally applicable to flames at the higher-  
259 temperatures found in piston engines. We obtain  $\mathbf{B}_{CH_4} = \{3.04, 0.70, 0.40, 0.10, 2.49\}$   
260 and  $\mathbf{B}_{C_7H_{16}} = \{2.79, -0.05, 0.26, 0.02, 3.02\}$  by least-squares fitting to a set of 45  
261 flame speeds for each fuel at conditions comprising  $T_u = \{700, 775, 850, 925, 1000\}$   
262 K,  $p = \{20, 40, 60\}$  bar, and  $\phi_{tot} = \{0.8, 1, 1.2\}$ . In order to evaluate  $B_5$ ,  
263 four additional diluted flames are included with  $\xi = \{0.00, 0.05, 0.10, 0.15\}$  at  
264  $\phi_{tot} = 1.0$ ,  $T_u = 850$  K and  $p = 40$  bar. The rms error of the fits for  $\mathbf{B}_{CH_4}$  and  
265  $\mathbf{B}_{C_7H_{16}}$  are  $0.020ms^{-1}$  and  $0.028ms^{-1}$ , and the maximum relative error of 10%  
266 occurs at the coldest and leanest conditions. The dataset used for parameter  
267 fitting is provided as supplementary material.

268 *Step 2: Flame speeds at autoignitive conditions.* Figure 7 shows that the laminar  
269 flame speed at autoignitive conditions increases approximately linearly with the  
270 value of progress variable just ahead of the flame for the full range of methane/n-  
271 heptane fuel blends investigated. Therefore modelling for the flame speed at  
272 autoignitive conditions will be developed as a function of the progress variable  
273 in the form

$$s_{f,fuel} = s_{l,fuel} (1 + \zeta_{fuel} c) \quad (7)$$

274 where  $\zeta_{fuel}$  is a model coefficient and progress variable is based on temperature.  
275 Figure 7 reports values for progress variable two thermal thicknesses ahead of  
276 the location of the peak temperature gradient in the flame, however results show  
277 low sensitivity to distances between one and five thermal thicknesses ahead of  
278 the flame. The data presented in Fig. 7 suggests that  $\zeta_{fuel} \approx 2.4 \pm 0.5$  for the  
279 blends considered ( $\phi_{CH_4}/\phi_{tot} = 0, 0.5, 0.8, 1.0$ ) for progress variable less than

280 0.1. Super-linear behaviour is expected as the residence time approaches the  
281 ignition delay and the flame transitions into an ignition front, however flamelet-  
282 based turbulent combustion models that require flame speed as an input would  
283 not be applicable to flames with ignitive, rather than deflagrative behaviour.  
284 Therefore the linear modelling approach remains valid for diffusion-limited com-  
285 bustion conditions at which the flamelet approach is applicable.

286 Since the variation of  $\zeta$  among methane/n-heptane fuel blends is relatively  
287 small it is convenient to adopt a single value of  $\zeta$  for the full range of fuel blends.  
288 Figure 1 shows that using  $\zeta = 2.4$  in Eq. 7 provides an adequate prediction of  
289 the variation of flame speed with residence time for all of the fuel blends and  
290 temperatures investigated. In order to apply the flame speed model in engine  
291 simulations, the progress variable in the mixture ahead of the flame then needs  
292 to be modelled, either by simulating the evolution of the chemical composition  
293 during ignition as in Ref. [26], or potentially by modelling the progress variable  
294 as a function of the Livengood-Wu integral [27].

295 Analysis in Section 3.1 shows that the flame speed is affected by the accu-  
296 mulation of intermediate species ahead of the flame, in addition to the effects  
297 of heat release and dilution due to reactant consumption. Equation 6 contains  
298 factors accounting for effects of temperature and dilution of the *unburned mix-*  
299 *ture* on flame speed. Similar factors may be applied to account for effects of the  
300 increase of temperature and dilution of reactants due to pre-ignition reactions  
301 ahead of the flame front:  $(T/T_u)^\alpha$  and  $1 - B_5 \xi_{pr}$ , where  $\xi_{pr}$  is the fraction of  
302 the reactants that have been consumed by pre-ignition reactions. Additional  
303 modelling is then necessary to account for the increased reactivity due to the  
304 accumulation of intermediate species ahead of the flame.

305 The increase in flame speed due to the presence of intermediate species is  
306 shown for stoichiometric n-heptane-air at 850 K by the difference between  $s_f$   
307 and  $s_{f,mod}$  in Fig. 2. Fig. 7 shows that  $s_f/s_{f,mod}$  also has an approximately  
308 linear dependence on progress variable given by  $1 + \delta_{C_7H_{16}} c$ , with  $\delta_{C_7H_{16}} \approx 1.18$ .  
309 Finally, thermal expansion due to pre-ignition heat release changes the velocity  
310 of the flow into which the high-temperature flame propagates. Accounting for

311 the thermal expansion upstream of a one-dimensional planar flame, the flame  
 312 speed is multiplied by the density ratio  $\rho/\rho_u$  where  $\rho$  is the fluid density just  
 313 upstream of the flame, finally giving

$$s_{f,fuel} = s_{l,fuel} \cdot \left( \frac{T}{T_u} \right)^{\alpha_{fuel}} \cdot (1 - B_{5,fuel} \xi_{pr}) \cdot (1 + \delta_{fuel} c) \cdot \left( \frac{\rho}{\rho_u} \right). \quad (8)$$

314 Each of the factors in Eq. 8 can be expressed as a function of progress vari-  
 315 able: rearranging the definition of progress variable gives  $T/T_u = 1 + c(T_b/T_u -$   
 316 1); approximating the fraction of reactants that have been consumed by the  
 317 progress variable gives  $\xi_{pr} \approx c$ ; and, neglecting changes in pressure and molar  
 318 mass, the ideal gas equation gives  $\rho/\rho_u \approx [1 + c(T_b/T_u - 1)]^{-1}$ . Since the model  
 319 is only required to be valid for small values of progress variable, a first-order  
 320 Taylor expansion of Eq. 8 yields

$$s_{f,fuel} = s_{l,fuel} \left( 1 + c \left[ (\alpha_{fuel} - 1) \times \left( \frac{T_u}{T_b} - 1 \right) - B_{5,fuel} + \delta_{fuel} \right] \right). \quad (9)$$

321 Entering the previous values for stoichiometric n-heptane fuel ( $\alpha_{C_7H_{16}} = 2.79$ ,  
 322  $T_u/T_b = 3.18$ ,  $B_{5,C_7H_{16}} = 3.02$ ,  $\delta_{C_7H_{16}} = 1.18$ ) in Eq. 8 corresponds to a value  
 323 of  $\zeta_{C_7H_{16}} = 2.1$  in Eq. 7, which is reasonably close to the value of  $\zeta \approx 2.4$   
 324 obtained from Fig. 7. Equations 8 and 9 therefore provide a breakdown of the  
 325 various contributions to the flame speed increase.

326 *Step 3: Flame speeds of mixtures.* In order to model the laminar flame speed  
 327  $s_{f,mix}$  in methane/n-heptane blends we compare the linear [25], Hirasawa *et*  
 328 *al.* [28], and Di Sarli *et al.* [29] mixing-rules in Fig. 6. The Di Sarli *et al.*  
 329 model tends to over-emphasise the contribution of methane. The linear model  
 330 is simpler and marginally more accurate than the Hirasawa *et al.* model for  
 331 methane/n-heptane blends, and it is adopted hereafter although either model is  
 332 acceptable. The linear model is given by

$$s_{f,mix} = Z_{CH_4} s_{f,CH_4} + (1 - Z_{CH_4}) s_{f,C_7H_{16}}, \quad (10)$$

333 where  $Z_{CH_4} = Y_{CH_4,u}/(Y_{CH_4,u} + Y_{C_7H_{16},u})$  is the local mass fraction of methane  
 334 in the fuel blend. In agreement with Bourque *et al.* [30], adding higher-  
 335 hydrocarbons to methane increases the flame speed disproportionately under

336 fuel-rich conditions. Potentially, a dependence on equivalence ratio could be  
337 incorporated into Eq. 10 in order to achieve even closer agreement. The com-  
338 plete new model is validated for a range of stoichiometric methane/n-heptane  
339 fuel blends in Fig. 1 and for the full range of lean equivalence ratios in Fig.  
340 6 (solid circles), adequately capturing the effects of pre-ignition chemistry on  
341 flame speed across the full range of conditions with deflagarative behaviour.

342 **4. Conclusions**

343 The effects of pre-ignition chemistry on laminar flame speed in autoignitive  
344 methane/n-heptane fuel blends are investigated using premixed laminar flame  
345 simulations. The flame speed and thickness are important input parameters for  
346 turbulent combustion models based on flamelet assumptions.

347 Pre-ignition reactions cause the speed of the flame to increase. Fuels that ex-  
348 hibit two-stage ignition behaviour, such as n-heptane, also exhibit a two-stage  
349 increase in the speed of the reaction front as the reactant residence time in-  
350 creases. The increase in flame speed is due to distinct contributions of heat  
351 release, reactant consumption, and enhanced reactivity ahead of the flame. Ad-  
352 dition of methane to n-heptane–air mixtures retards and reduces the first-stage  
353 increase in flame speed, in part due to dilution of the more-reactive n-heptane  
354 fuel, and in part due to consumption of radical species by the methane chemistry.  
355 As the residence time of the reactants approaches the ignition delay time, the  
356 reaction front transitions into a pure ignition front, in which diffusive transport  
357 is negligible.

358 Prior to transitioning into a pure ignition front, the behaviour of the flame  
359 can be classified as deflagarative or ignitive depending whether the flame thick-  
360 ness and flame speed obey the deflagarative scaling  $l_f \sim \alpha/s_l$ , subject to per-  
361 turbations of the temperature ahead of the flame. The thickness of cool-flames  
362 exhibits ignitive scaling with flame speed,  $l_f \sim s_f$ , for all conditions simulated.  
363 The transition between deflagarative and ignitive scaling for high-temperature  
364 flame fronts occurs in 850 K stoichiometric n-heptane–air at a mixture residence

365 time of around 93% of the ignition delay time, as the progress variable in mix-  
366 ture ahead of the flame passes 0.15. The transition between deflagarative and  
367 ignitive scaling is associated with a reduction in the relative magnitude of dif-  
368 fusive transport within the flame front, however examination of the transport  
369 budget alone does not provide a clear delineation between deflagarative and  
370 ignitive behaviours.

371 Modelling for flame speed in dual-fuel blends at autoignitive conditions  
372 should account for the local composition of the fuel blend, and for the effects of  
373 pre-ignition chemistry. A linear mixing rule is adequate to describe the varia-  
374 tion of flame speed in methane/n-heptane fuel blends. Modelling is introduced  
375 to account for the effects of pre-ignition chemistry on the flame speed. The  
376 model is a simple linear function of the progress variable ahead of the flame –  
377 accounting for the distinct contributions of heat release, reactant consumption,  
378 and enhanced reactivity ahead of the flame. The flame speed model accurately  
379 describes the variation of flame speed and hence flame thickness for the full  
380 range of methane/n-heptane blends at engine-relevant conditions, up to the  
381 deflagration/ignition transition.

## 382 Appendix

383 *Procedure for removing intermediate species ahead of the flame.* The modified  
384 flame simulation involves two steps illustrated in Fig. 8.

385 *Step 1.* The first step involves a single precursor flame simulation (Fig. 8 top)  
386 that is used to determine the variation of the reactant composition with resi-  
387 dence time upstream of the flame front. The composition recorded for each resi-  
388 dence time is then modified by replacing the intermediate species with a mixture  
389 of reactants and major products of stoichiometric combustion. The replacement  
390 mixture consists of  $\text{CH}_4$ ,  $\text{C}_7\text{H}_{16}$ ,  $\text{O}_2$ ,  $\text{N}_2$ ,  $\text{CO}_2$ , and  $\text{H}_2\text{O}$ . The modified mixture  
391 composition is determined as a function of the fuel equivalence ratios ( $\phi_{\text{C}_7\text{H}_{16}}$   
392 and  $\phi_{\text{CH}_4}$ ), the temperature  $T_u$  of the original unburnt mixture, and the local  
393 temperature  $T$  recorded from the precursor flame. The modified mass fraction

<sup>394</sup> vector  $\mathbf{Y}'$  is calculated by weighting the unburnt and burnt composition,  $\mathbf{Y}_u$   
<sup>395</sup> and  $\mathbf{Y}_b$  respectively, with a progress variable  $c_Y$

$$\mathbf{Y}' = \mathbf{Y}_b c_Y + \mathbf{Y}_u (1 - c_Y). \quad (11)$$

<sup>396</sup> The burnt composition is taken as the products of complete stoichiometric com-  
<sup>397</sup> bustion, consisting of N<sub>2</sub>, CO<sub>2</sub> and H<sub>2</sub>O. The progress variable  $c_Y$  is given by

$$c_Y = \frac{\sum_{\alpha=1}^{n_{spec}} [Y_\alpha h_\alpha(T) - Y_{\alpha,u} h_\alpha(T_u)]}{\sum_{\alpha=1}^{n_{spec}} [Y_{\alpha,b} h_\alpha(T_b) - Y_{\alpha,u} h_\alpha(T_u)]}, \quad (12)$$

<sup>398</sup> where  $\mathbf{Y}$  and  $T$  are the original unmodified mass fraction vector and tempera-  
<sup>399</sup> ture, and  $h_\alpha$  is the specific enthalpy of each species.

<sup>400</sup> *Step 2.* The second step produces a flame solution using the modified com-  
<sup>401</sup> position as the inlet condition with the flame positioned 67  $\mu\text{m}$  from the inlet  
<sup>402</sup> (Fig. 8 bottom). The flame residence time for the modified flame simulation  
<sup>403</sup> is sufficiently small that the results are not influenced significantly by chemical  
<sup>404</sup> reaction upstream of the modified flame front, and sufficiently large that the  
<sup>405</sup> flame speed is not influenced significantly by diffusive flux through the domain  
<sup>406</sup> inlet. This is confirmed in Fig. 2 by applying the two-step procedure with-  
<sup>407</sup> out removing intermediate species from the reactant mixture: the flame speeds  
<sup>408</sup> obtained using this ‘reduced’ solution domain closely follow the residence time-  
<sup>409</sup> dependence obtained using a single ‘full’ solution domain. The difference due to  
<sup>410</sup> the use of the reduced domain procedure is negligible compared with the effect  
<sup>411</sup> of replacing the intermediate species. This confirms the validity of the modified  
<sup>412</sup> flame approach for determining the relative influences of intermediate species  
<sup>413</sup> and thermal effects.

<sup>414</sup> **Declaration of interest**

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417 **Data statement**

418 All data supporting this study are openly available from the University of  
419 Southampton repository at (doi to be provided upon acceptance).

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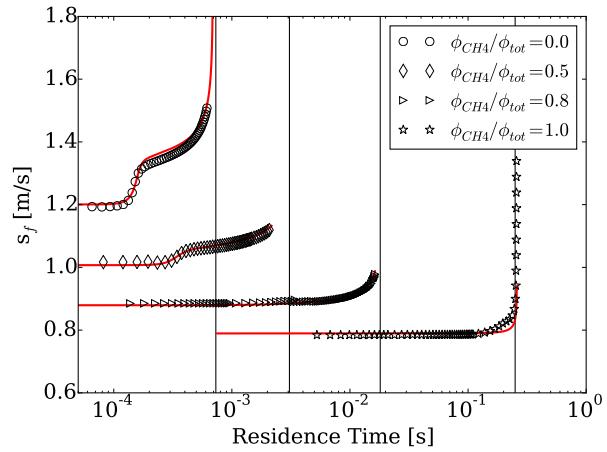
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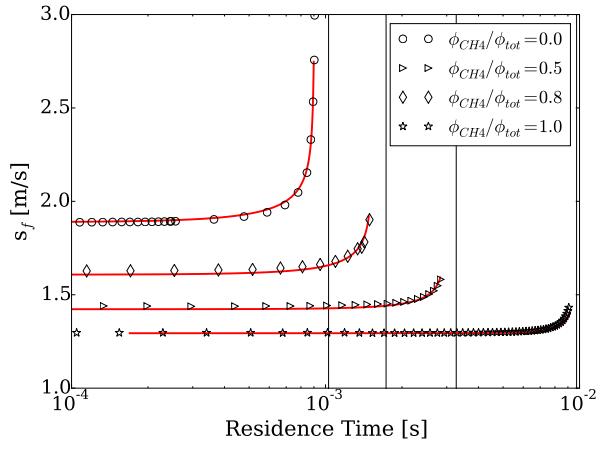
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(a) 850 K



(b) 1000 K

Figure 1: Laminar flame speeds of stoichiometric methane/n-heptane blends versus residence time at 40 bar: simulations (symbols); model Eqs. 7 and 10 (solid).

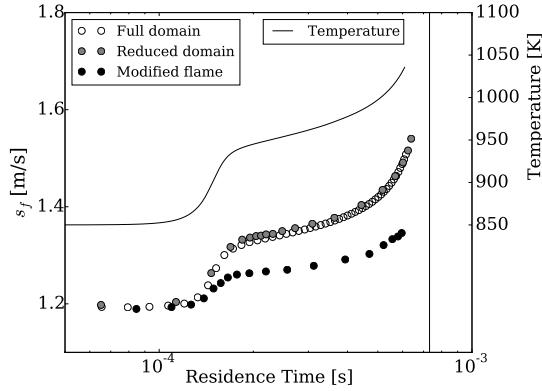


Figure 2: Laminar flame speed and temperature versus residence time for stoichiometric n-heptane-air at 40 bar and 850 K: Unmodified flame in the full domain (white circles); 0.2 mm domain with inlet composition from the unmodified flame (grey circles); 0.2 mm domain with intermediate species removed from the inlet composition (black circles).

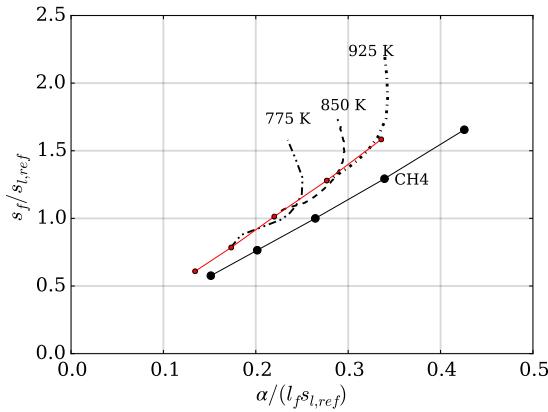


Figure 3: Normalised high-temperature reaction front speed  $s_f/s_{l,ref}$  versus  $\alpha/(l_f s_{l,ref})$  for short residence time stoichiometric methane-air and n-heptane-air flames with unburnt temperatures 700, 775, 850, 925 and 1000 K (symbols). Data for n-heptane-air combustion with residence times up to 98% of the ignition delay time at 775, 850, and 925 K (various dashed lines as labelled).

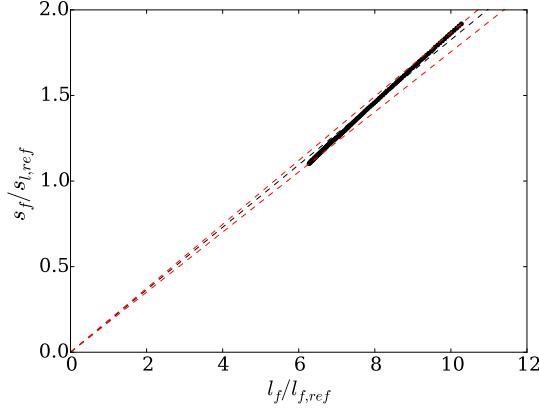


Figure 4: Cool-flame reaction front speed  $s_f$  versus thermal thickness  $l_f$  for stoichiometric n-heptane–air flames at 850 K and 40 bar for a range of residence times (symbols). Dashed lines correspond to Eq. 3.

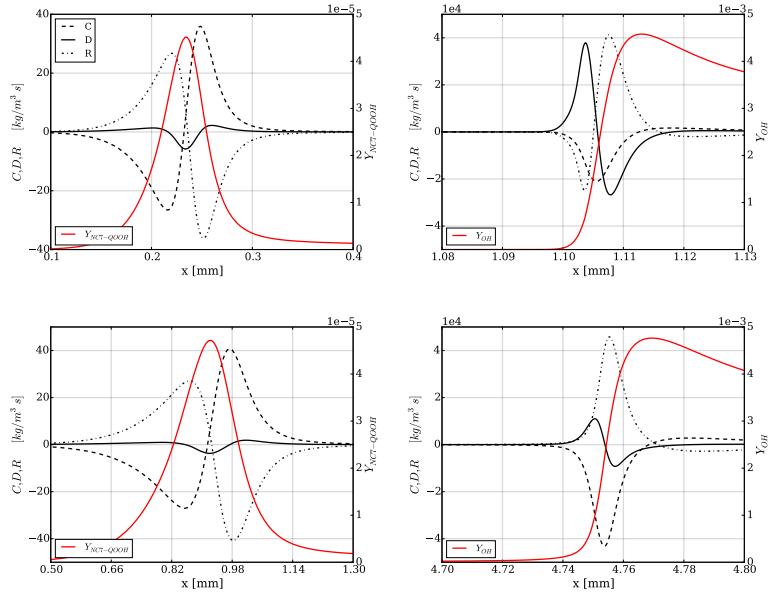


Figure 5: Transport budgets and mass fraction profiles for stoichiometric n-heptane–air at 850 K and 40 bar for  $Y_{QOOH}$  (left column) and  $Y_{OH}$  (right column). The residence times at the flame front are 85% (top row), and 95% (bottom row) of the overall ignition delay time. The data are plotted versus the distance from the inlet.

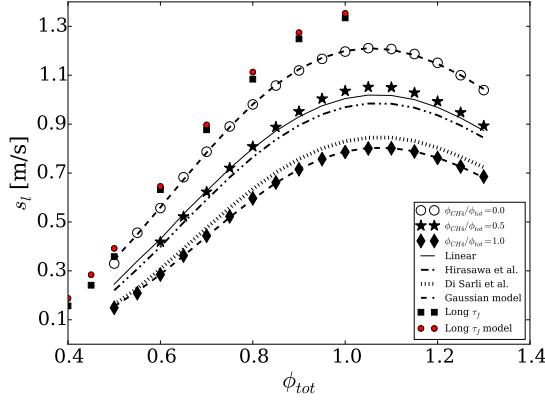


Figure 6: Variation of flame speed with equivalence ratio for methane/n-heptane fuel blends at 850 K and 40 bar for  $\phi_{CH_4}/\phi_{tot} = 0, 0.5$  and  $1.0$ ; Fitted flame speeds from Eq. 5; and linear [25], Hirasawa *et al.* [28], and Di Sarli *et al.* [29] mixing-rules. Speeds for a long residence time flame from simulations and from predictions of the model (Eqs. 7 and 10) are also shown.

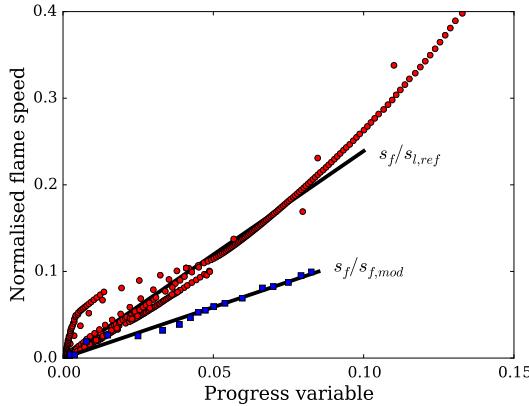


Figure 7: Normalised flame speed versus the progress variable upstream of the flame front: Round symbols show  $s_f/s_{l,ref}$  at  $\phi_{CH_4}/\phi_{tot} = 0, 0.5, 0.8$  and  $1.0$  for  $T_u = 850$  K and  $1000$  K; Square symbols show  $s_f/s_{f,mod}$  based on data from Fig. 2. Solid lines show  $s_f/s_{l,ref} = 2.4c$  and  $s_f/s_{f,mod} = 1.18c$ .

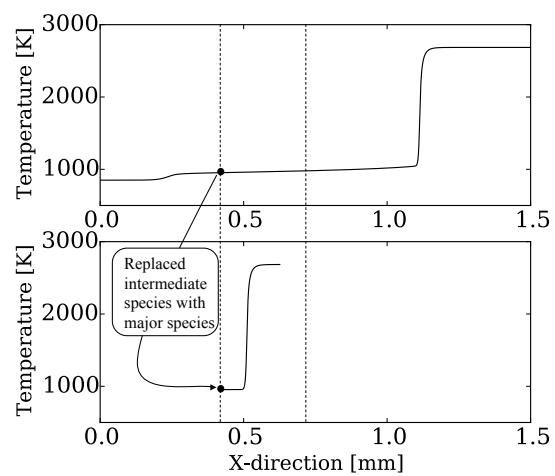


Figure 8: Illustration of the reduced-domain simulation procedure: temperature field from a full-domain simulation of a stoichiometric n-heptane–air flame with  $\tau_f = 0.95\tau_{ign}$  (top); temperature field from the reduced-domain simulation.