Evaluation of a New Version of I^2 with Emphasis on Diagnostic Problems

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Abstract

This paper describes results stemming from an in-depth analysis of Higgins' measure of heterogeneity for a meta-analysis applied in the context of meta-analysis for diagnostic problems. Higgins measure of heterogeneity I^2 has been criticized for being confounded by the study-specific sample size, in the sense that different I^2 –values can be achieved for the same value of across-study variance if only the study-specific variance is varying enough. In particular, I^2 approaches one for any value of the heterogeneity variance (variance across studies) if the within-study variance becomes large. The paper proposes a measure which is unconfounded by sample size. It is essentially a philosophical question which heterogeneity measure is chosen. Nevertheless, a detailed simulation study has been launched and the results indicate that the newly suggest measure of heterogeneity has beneficial statistical properties. Both measures are also exemplified at hand of some meta-analytic case studies.

Key words: diagnostic accuracy, meta-analysis, measure of heterogeneity

Contents

1. Introduction and background

We consider a meta-analytic setting in which *k* studies are available to investigate the diagnostic performance of a given diagnostic procedure. Important performance measures in diagnostic methodology are sensitivity *p*, defined as the probability of classifying a diseased subject correctly, and specificity *u*, defined as the probability of classifying a healthy subject correctly. Now, each study delivers the estimated sensitivity $\hat{p}_i = y_i / m_i$ in study *i*, where y_i is the number of correctly classified diseased individuals among m_i diseased subjects. Also, each study delivers the estimated specificity $1 - \hat{u}_i = x_i / n_i$ in study *i*, where x_i is the number of correctly classified healthy individuals among n_i healthy subjects. It should be pointed out that the results from the *k* component studies, here estimated sensitivity and estimated specificity, are not derived by the meta-analyst. They are found and retrieved from the component studies which are typically found in various publication outlets dealing with the are of interest. The meta-analyst has only choices in including or excluding studies, as per protocol. After the set of derived summary statistics has been compiled from the published evidence, it is the question how to analyze these summary data. This is what we consider here.

It has been considered for convenience to provide two separate meta-analyses, one for sensitivity and one for specificity. However, due to the *cut-off-value problem*, these separate meta-analyses can be largely misleading. The cut-off value problem refers to the fact that a continuous or ordered categorical test is made binary, leading to a positive or negative result, by choosing a cutoff value. Now, if this cutoff value varies across studies, it creates heterogeneity in sensitivity as well as in specificity, despite the fact, that the diagnostic procedure might perform homogeneously across studies. Fig. 1 illustrates the point: if the cut-off value is varied, as we would expect across studies, also associated sensitivities and specificities change. We note that cut-off variation might be observed in the meta-analysis, but often it is not recorded. Hence a ROC approach can be considered, whether a cut-off value has been observed or not.

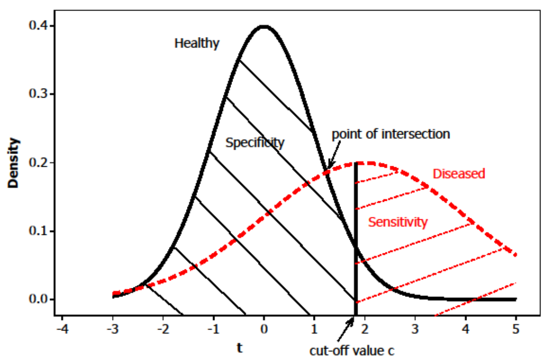

Fig.1 Illustration of the cut-off value problem for a continuous or ordered categorical test: variation of the cut-off value *c* leads to an increased specificity and decreased sensitivity if *c* is moved to the right, and vice versa if *c* is moved to the left

As a consequence, interest has focused on the ROC curve which plots, with varying cut-off value, sensitivity against the false positive rate (1-specificity). Evidently, the ROC curve copes with cut-off value variation as different cut-off values would lead to points on the same ROC curve, representing identical diagnostic accuracy as differences in sensitivity and specificity are caused by cut-off value variation. A test with uniformly better diagnostic performance than a comparative, second diagnostic test would have an ROC curve lying entirely above the second test. Fig. 2 shows some examples of ROC curves, each of these representing a different diagnostic accuracy.

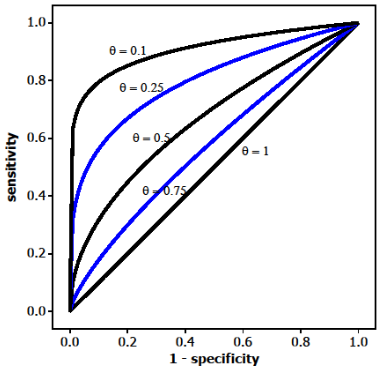

Fig.2 Illustration of various ROC curves in the ROC space spanned by sensitivity and 1-specificity

In the meta-analytic setting of having k studies available with estimated sensitivity $\hat{p}_i = y_i / m_i$ and estimated specificity $1-\hat{u}_i = x_i / n_i$ for each study *i*, the SROC diagram is considered where S stands for summary. As we only have summary data, we consider plotting $\hat{p}_i$ against $\hat{u}_i$. A typical example of an SROC curve is given in Fig. 3.

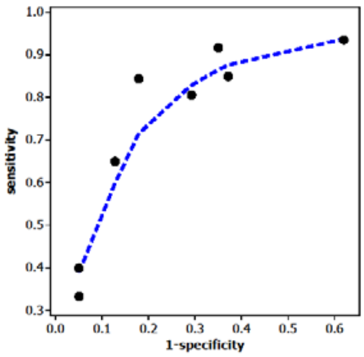

Fig.3 Illustration of a SROC curve for a diagnostic meta-analysis involving seven studies

Note again that the ROC curve catches the global diagnostic performance of the diagnostic test. In a meta-analysis, it is unrealistic to seek and find a simple, parametric ROC model fitting all the points (representing the studies) in the ROC space. However, it is easy to find a model which can fit each point with a single, but study specific parameter.

We consider the Lehmann model for the ROC curve

$$p = u^{\theta} \qquad (1)$$

or, equivalently, $\log p = \theta \log u$, where *p* is the sensitivity and *u* is the false positive rate (1-specificity). This has been discussed previously in detail in Böhning *et al.* (2002, 2008, 2010) and Holling *et al.* (2007, 2011, 2012), in more detail in Doebler *et al.* (2011, 2015) and Charoensawat *et al.* (2014). The Lehmann model is attractive for two reasons: it meets the constraints of the ROC curve and it has one parameter representing the diagnostic accuracy.

Suppose *k* diagnostic studies are available and assume that

$$\log \hat{\theta}_i = \log \theta_i + \varepsilon_i \tag{2}$$

where $\hat{\theta}_i = \log \hat{p}_i / \log \hat{u}_i$ is the observed diagnostic accuracy in study *i* and θ_i is the true diagnostic accuracy of study *i* with $\log \theta_i \sim \mathrm{N}(\log \theta, \tau^2)$. In other words, the model (2) fits its own model, namely a straight line through the origin, for each study. It is further assumed that there is a population model which follows the same pattern.

Furthermore, ε_i is a normal mean-zero random error with variance σ_i^2 and the study-specific variances are assumed to be known. Note that $\log \theta_i \sim \mathrm{N}(\log \theta, \tau^2)$ is a study random-effect accounting for variation between studies and it is assumed that $\log \theta_i \perp \varepsilon_i$ for all *i=1,…,k.* (2) is a form of the classical *random effects model of meta-analysis.*

It follows that

$$\mathrm{var}(\log \hat{\theta}_i) = \mathrm{var}(\log \theta_i) + \mathrm{var}(\varepsilon_i) = \tau^2 + \sigma_i^2 \ . \tag{3}$$

Furthermore, it can be shown (Charoensawat *et al.* 2014) that

$$\sigma_i^2 \approx \frac{1 - p_i}{p_i (\log p_i)^2} \frac{1}{m_i} + \frac{1 - u_i}{u_i (\log u_i)^2} \frac{1}{n_i} \tag{4}$$

where m_i is the size of the disease arm in study *i* and n_i is the size of the control arm in study *i*. Note that (3) shows that the total variance of $\mathrm{var}(\log \hat{\theta}_i)$ is consisting of two parts: the within-study variance and the between-study variance. It is important to include both variance components.

Traditionally, the magnitude of heterogeneity is measured by Higgins's I^2 defined as

$$I^2 = \frac{\tau^2}{\tau^2 + \bar{\sigma}^2} \tag{5}$$

where $\bar{\sigma}^2$ is an average of the study-specific variances. Higgins and Thompson (2002) suggested a form of harmonic mean

$$\bar{\sigma}_1^2 = \frac{\sum_i w_i (k-1)}{(\sum_i w_i)^2 - \sum_i w_i^2} \ , \tag{6}$$

where $w_i = 1/\sigma_i^2$.

We will also look at the harmonic mean

$$\bar{\sigma}_2^2 = \frac{k}{\sum_i w_i} \tag{7}$$

and the arithmetic mean

$$\bar{\sigma}_3^2 = \frac{1}{k}\sum_i \sigma_i^2 \tag{8}$$

leading to three different forms of I^2 : I_1^2 , I_2^2 , I_3^2 defined as

$$I_j^2 = \frac{\tau^2}{\tau^2 + \bar{\sigma}_j^2} \tag{9}$$

for j=1,2,3. The means (6) and (7) are closely related as could be recently shown in Böhning *et al.* (2017).

2. Problems with the definition of I^2

It has been noted in the literature by Rücker, Schwarzer, Carpenter und Schumacher (2008) or Borenstein, Hedges, Higgins, Rothstein (2009) that I^2 crucially depends on $\bar{\sigma}^2$ (whatever average is considered) and that $\bar{\sigma}^2$ involves the sample size of disease and control arm. This leads to the to the artifact that a meta-analysis with larger-sized studies show more heterogeneity in comparison to a meta-analysis with smaller-sized studies, but both having the same τ^2 . Rücker, Schwarzer, Carpenter und Schumacher (2008) write: „as precision increases, while estimates of the heterogeneity variance remain unchanged on average, estimates of I^2 increase rapidly to nearly 100%.“

This suggests to consider a new definition of $\bar{\sigma}^2$ where the effect of the sample size is removed:

$$\sigma_{i0}^2 = \frac{1-p_i}{p_i(\log p_i)^2} + \frac{1-u_i}{u_i(\log u_i)^2} \quad . \tag{10}$$

This leads then to an adjusted I_0^2 with different versions for the different forms of means considered: $I_{10}^2, I_{20}^2, I_{30}^2$. We will describe in detail how these quantities are estimated in the next section.

We would like to mention that the idea of adjusting *I*^{2} for within study sample size has been recently put forward in Sangnawakij *et al.* (2017).

3. Simulation work

To investigate the performance of these measures in more detail a simulation study was undertaken.

3.1 Design

We have chosen $\theta = 0.1, 0.3, 0.5$ and $m=n = 50, 100$. Also, $u = 0.4, 0.6$.

The target values for were $I^2 = 0.1, 0.5, 0.8$.

It follows that $p = u^{\theta}$ and

$$\sigma_i^2 = \frac{1}{m}\left[\frac{1-p}{p(\log p)^2} + \frac{1-u}{u(\log u)^2}\right]$$

$$w_i = 1/\sigma_i^2 .$$

From this we also can deduce the values of the heterogeneity variance $\tau^2 = \frac{I^2\sigma_i^2}{1-I^2}$ which we will use in the simulation to reach the target values of I^2. Note that we use the original definition of the heterogeneity measure I^2.

Then we sample $\log\theta_i$ from $N(\log\theta, \tau^2)$ and ε_i from $N(0, \sigma_i^2)$ for i=1,…k.

This leads to $\log\hat{\theta}_i = \log\theta_i + \varepsilon_i$, hence $\hat{\theta}_i = \exp(\log\hat{\theta}_i)$. Note that

$$\sigma_{i0}^2 = \frac{1-p}{p(\log p)^2} + \frac{1-u}{u(\log u)^2}$$

$$w_{i0} = 1/\sigma_{i0}^2 .$$

The true values for θ, p, u and τ^2 are clear. For the study-specific variances we

compute $\bar{\sigma}_1^2 = \frac{(k-1)\sum w_i}{\left(\sum w_i\right)^2 - \sum w_i^2}$, $\bar{\sigma}_2^2 = \frac{k}{\sum w_i}$, $\bar{\sigma}_3^2 = \frac{\sum \sigma_i^2}{k}$ $\Rightarrow I_j^2 = \frac{\tau^2}{\tau^2 + \bar{\sigma}_j^2}$

and $\bar{\sigma}_{10}^2 = \frac{(k-1)\sum w_{i0}}{\left(\sum w_{i0}\right)^2 - \sum w_{i0}^2}$, $\bar{\sigma}_{20}^2 = \frac{k}{\sum w_{i0}}$, $\bar{\sigma}_{30}^2 = \frac{\sum \sigma_{i0}^2}{k}$ $\Rightarrow I_{j0}^2 = \frac{\tau^2}{\tau^2 + \bar{\sigma}_{j0}^2}$.

Note that the design of the simulation allows to specify true values of the heterogeneity measures, in the adjusted or original version.

3.2 Estimation

With $\hat{\theta}_i = \exp(\log \hat{\theta}_i)$ generated we compute readily $\hat{\mathrm{p}}_\mathrm{i} = \mathrm{u}^{\hat{\theta}_i}$

as well as

$$\hat{\sigma}_\mathrm{i}^2 = \frac{1}{\mathrm{m}}\left[\frac{1-\hat{\mathrm{p}}_\mathrm{i}}{\hat{\mathrm{p}}_\mathrm{i}(\log \hat{\mathrm{p}}_\mathrm{i})^2} + \frac{1-\mathrm{u}}{\mathrm{u}(\log \mathrm{u})^2}\right]$$

$$\hat{\sigma}_\mathrm{i0}^2 = \frac{1-\hat{\mathrm{p}}_\mathrm{i}}{\hat{\mathrm{p}}_\mathrm{i}(\log \hat{\mathrm{p}}_\mathrm{i})^2} + \frac{1-\mathrm{u}}{\mathrm{u}(\log \mathrm{u})^2}$$

$$\hat{\mathrm{w}}_\mathrm{i} = 1/\hat{\sigma}_\mathrm{i}^2$$

$$\hat{\mathrm{w}}_\mathrm{i0} = 1/\hat{\sigma}_\mathrm{i0}^2 .$$

With these we are able to compute the overall estimate of diagnostic accuracy

$$\widehat{\log \theta} = \frac{\sum \hat{w}_i \log \hat{\theta}_i}{\sum \hat{w}_i}$$

and $$\hat{\theta} = \exp(\widehat{\log \theta}) \quad . \tag{11}$$

We also have $$\chi^2 = \sum \hat{w}_i \left(\log \hat{\theta}_i - \widehat{\log \theta}\right)^2$$

and $$\hat{\tau}^2 = \frac{\chi^2 - (k-1)}{\sum \hat{w}_i - \sum \hat{w}_i^2 / \sum \hat{w}_i} \quad . \tag{12}$$

This is the DerSimonian-Laird estimator of heterogeneity variance which is a standard estimator used in in the connection with Higgins's I^2. Other estimators of heterogeneity variance are possible (see Bender *et al.* 2017 for an overview) but this is not a critical aspect for this investigation.

Furthermore, we have mean estimated study-specific variances

$$\bar{\bar{\sigma}}_1^2 = \frac{(\mathrm{k}-1)\sum \hat{\mathrm{w}}_\mathrm{i}}{\left(\sum \hat{\mathrm{w}}_\mathrm{i}\right)^2 - \sum \hat{\mathrm{w}}_\mathrm{i}^2}, \quad \bar{\bar{\sigma}}_2^2 = \frac{\mathrm{k}}{\sum \hat{\mathrm{w}}_\mathrm{i}}, \quad \bar{\bar{\sigma}}_3^2 = \frac{\sum \hat{\sigma}_\mathrm{i}^2}{\mathrm{k}}$$

leading to

$$\hat{I}_j^2 = \frac{\hat{\tau}^2}{\hat{\tau}^2 + \bar{\bar{\sigma}}_j^2} \tag{13}$$

as well as mean estimated size-adjusted study-specific variances

$$\bar{\bar{\sigma}}_{10}^2 = \frac{(\mathrm{k}-1)\sum \hat{\mathrm{w}}_\mathrm{i0}}{\left(\sum \hat{\mathrm{w}}_\mathrm{i0}\right)^2 - \sum \hat{\mathrm{w}}_\mathrm{i0}^2}, \quad \bar{\sigma}_{20}^2 = \frac{\mathrm{k}}{\sum \hat{\mathrm{w}}_\mathrm{i0}}, \quad \bar{\sigma}_{30}^2 = \frac{\sum \hat{\sigma}_\mathrm{i0}^2}{\mathrm{k}}$$

leading to

$$\hat{I}^2_{j0} = \frac{\hat{\tau}^2}{\hat{\tau}^2 + \overline{\overline{\sigma}}^2_{j0}} \quad . \tag{14}$$

. .

3.3 Performance measures for the estimators

We consider the mean, the bias and the standard deviation in their estimated versions for a replication size of 10000 to eliminate almost any simulation error. We have the following:

Mean:

$$\hat{\mathrm{E}}(\hat{\theta}) = \sum \hat{\theta}^{(\mathrm{b})} / 10000, \quad \hat{\mathrm{E}}(\hat{\tau}^2) = \sum \hat{\tau}^{2(\mathrm{b})} / 10000,$$

$$\hat{\mathrm{E}}(\hat{\mathrm{I}}^2_{\mathrm{j}}) = \sum \hat{\mathrm{I}}^{2(\mathrm{b})}_{\mathrm{j}} / 10000, \quad \hat{\mathrm{E}}(\hat{\mathrm{I}}^2_{\mathrm{j0}}) = \sum \hat{\mathrm{I}}^{2(\mathrm{b})}_{\mathrm{j0}} / 10000, \text{ j = 1, 2, 3}$$

Bias:

$$\mathrm{Bias}(\hat{\theta}) = \hat{\mathrm{E}}(\hat{\theta}) - \theta, \quad \mathrm{Bias}(\tau^2) = \hat{\mathrm{E}}(\hat{\tau}^2) - \tau^2$$

$$\mathrm{Bias}(\hat{\mathrm{I}}^2_{\mathrm{j}}) = \hat{\mathrm{E}}(\hat{\mathrm{I}}^2_{\mathrm{j}}) - \mathrm{I}^2_{\mathrm{j}}, \quad \mathrm{Bias}(\hat{\mathrm{I}}^2_{\mathrm{j0}}) = \hat{\mathrm{E}}(\hat{\mathrm{I}}^2_{\mathrm{j0}}) - \mathrm{I}^2_{\mathrm{j0}}$$

SD:

$$SD(\hat{\theta}) = \sqrt{\sum \left(\hat{\theta}^{(b)} - \hat{E}(\hat{\theta})\right)^2 / 10000}, \quad SD(\hat{\tau}^2) = \sqrt{\sum \left(\hat{\tau}^{2(b)} - \hat{E}(\hat{\tau}^2)\right)^2 / 10000}$$

$$SD(\hat{I}^2_j) = \sqrt{\sum \left(\hat{I}^{2(b)}_j - \hat{E}(\hat{I}^2_j)\right)^2 / 10000}, \quad SD(\hat{I}^2_{j0}) = \sqrt{\sum \left(\hat{I}^{2(b)}_{j0} - \hat{E}(\hat{I}^2_{j0})\right)^2 / 10000}$$

4. Results

In the following we describe the major results of the simulation work. Details can be found in the Figures 4 – 11 and in the tables in Appendix II. Some key points are as follows:

- The bias of $\hat{\theta}$ and $\hat{\tau}^2$ converge to zero with increasing within study sample size m, faster for low heterogeneity (Fig. 4 and Fig. 5)
- The bias for unadjusted $\hat{I}^2_j$ remains stable for increasing within study sample size, but is smaller for $\hat{I}^2_3$. The explanation here is that bias defined in reference to a population version of I^2 which involves the within study sample size. So, if this increases, also the population parameter of I^2 increases and bias stays the same. (Fig. 6).
- The bias for the new $\hat{I}^2_{j0}$ converges to zero if the within study sample size increases. The faster the smaller the heterogeneity and the ranking seems to be that $\hat{I}^2_{30}$ is

better than $\hat{I}^2_{20}$ which seems slightly better than $\hat{I}^2_{10}$. The number studies seems to play no major role. (Fig. 7).

- The remaining graphs show the associated estimated standard deviations and basically show what is expected to see. Note that for $\hat{I}^2_j$ the standard deviations do not go down with increasing within study sample size, an artefact described already above.

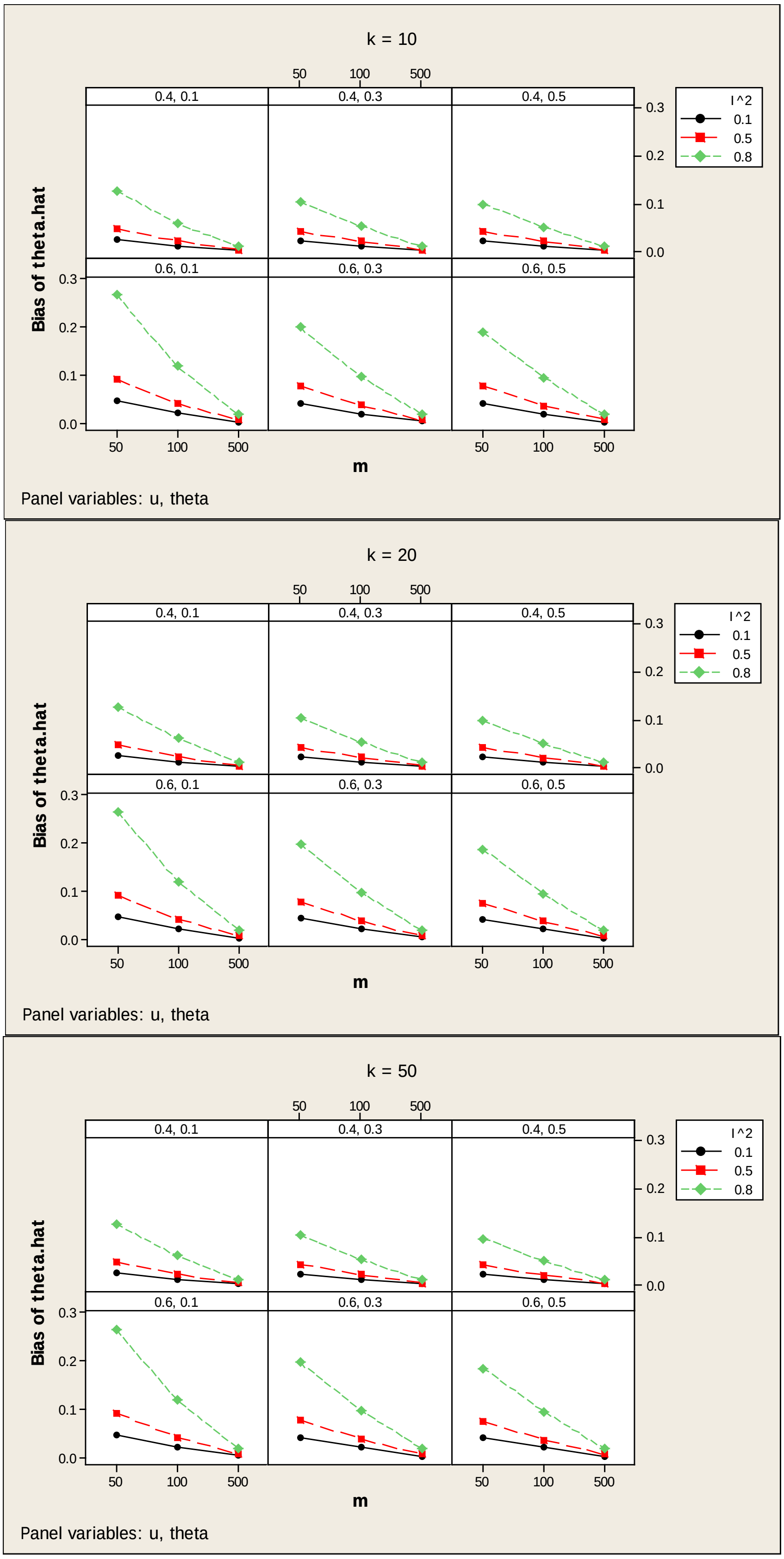

Fig.4 The graphs of bias for $\hat{\theta}$ against sample size for various values of the heterogeneity measure I^2

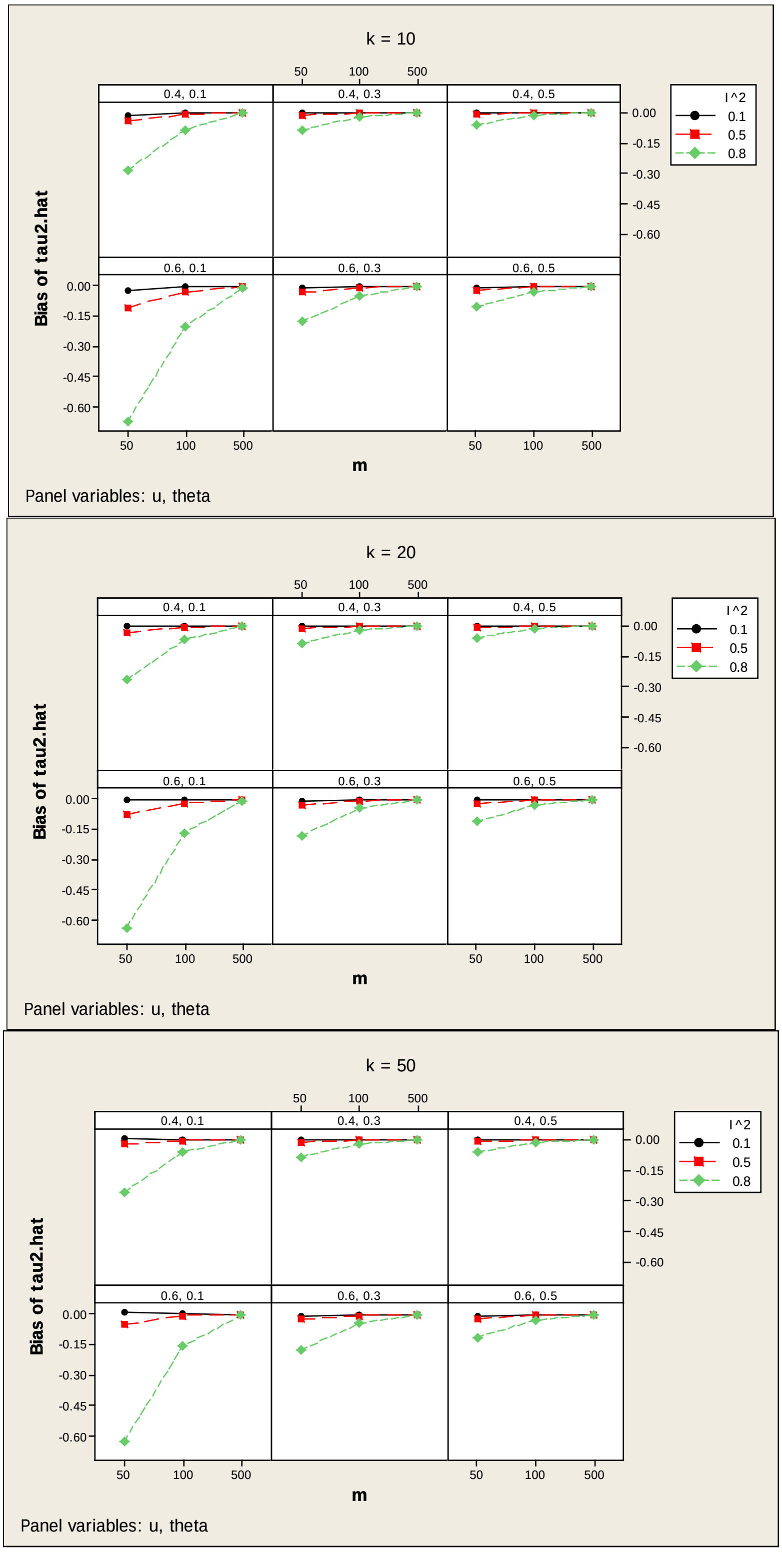

Fig.5 The graphs of bias for $\hat{\tau}^2$ against sample size for various values of the heterogeneity measure I^2

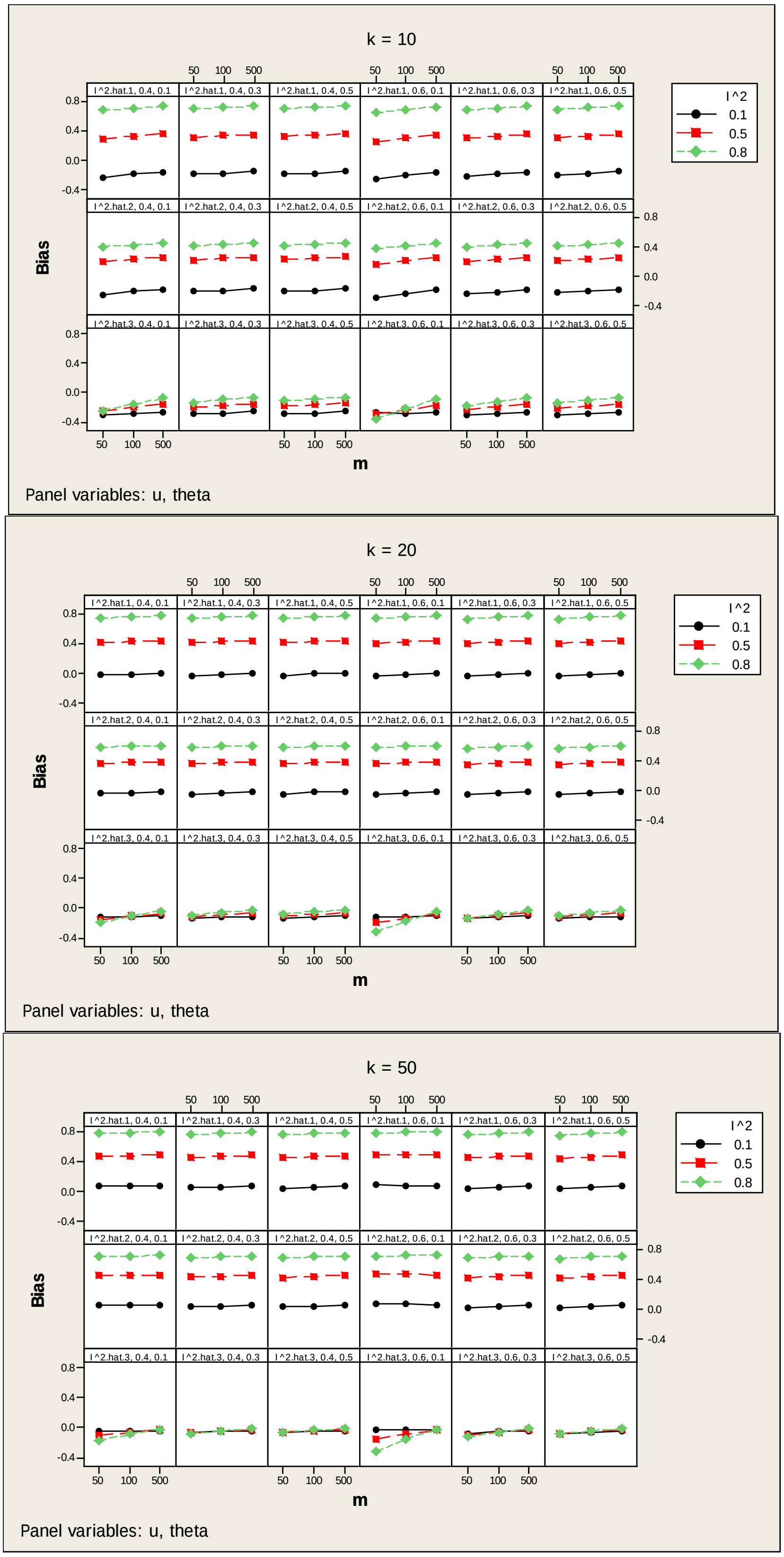

Fig.6 The graphs of bias for $\hat{I}_1^2$ (the first row), $\hat{I}_2^2$ (the second row) and $\hat{I}_3^2$ (the third row) against sample size for various values of the heterogeneity measure I^2

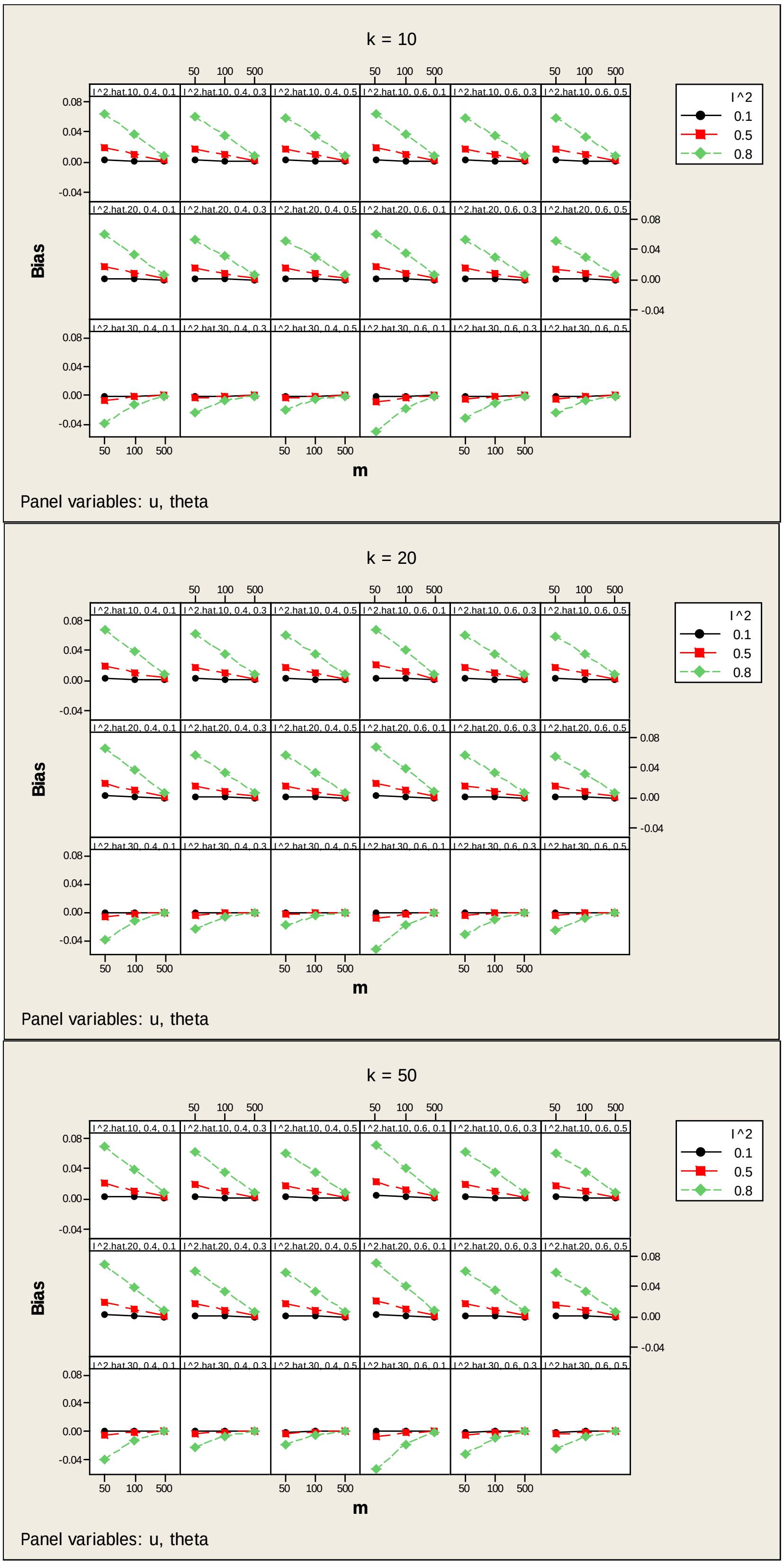

Fig.7 The graphs of bias for $\hat{I}^2_{10}$ (the first row), $\hat{I}^2_{20}$ (the second row) and $\hat{I}^2_{30}$ (the third row) against sample size for various values of the heterogeneity measure I^2

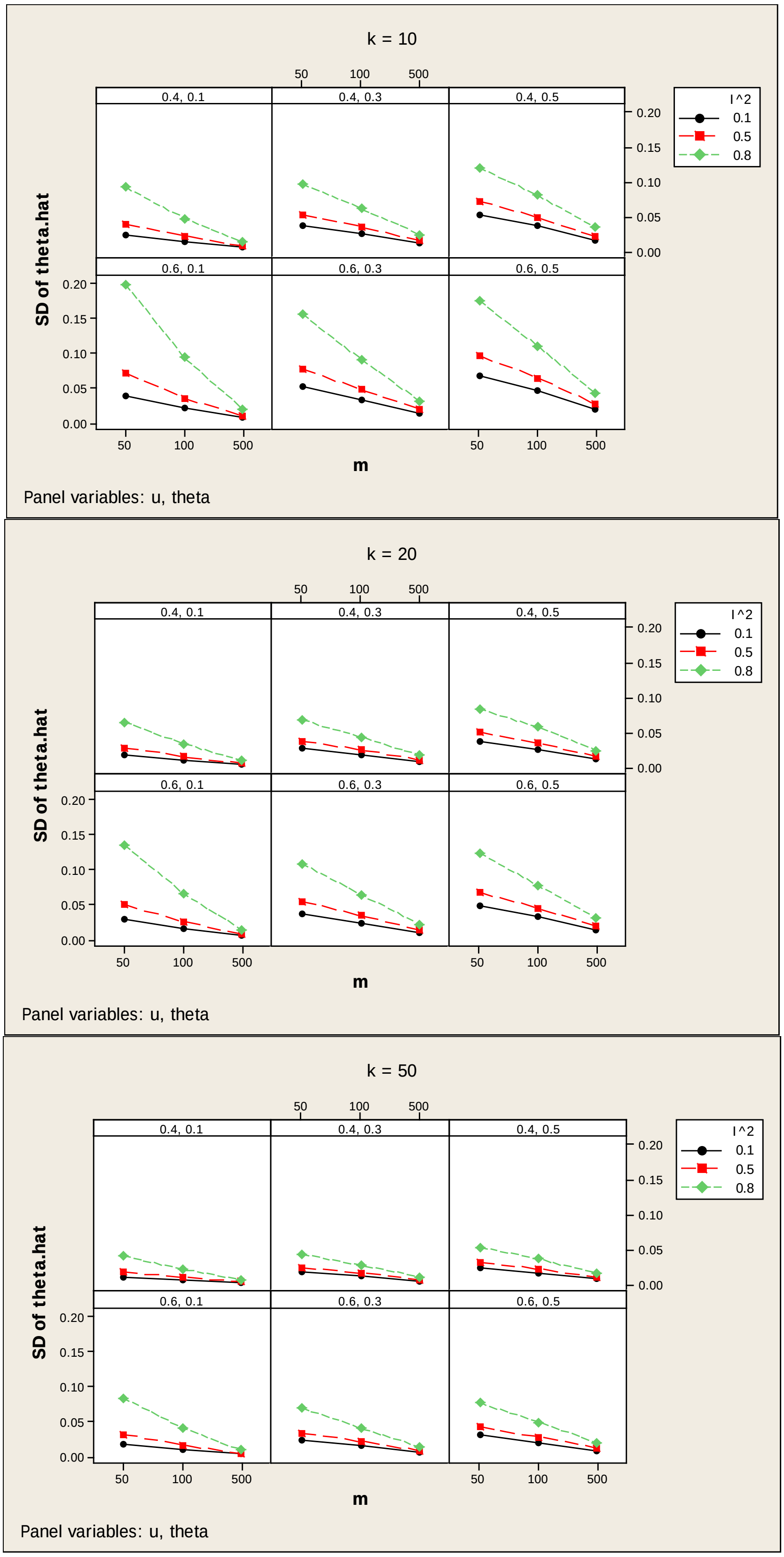

Fig.8 The graphs of SD for $\hat{\theta}$ against sample size for various values of the heterogeneity measure I^2

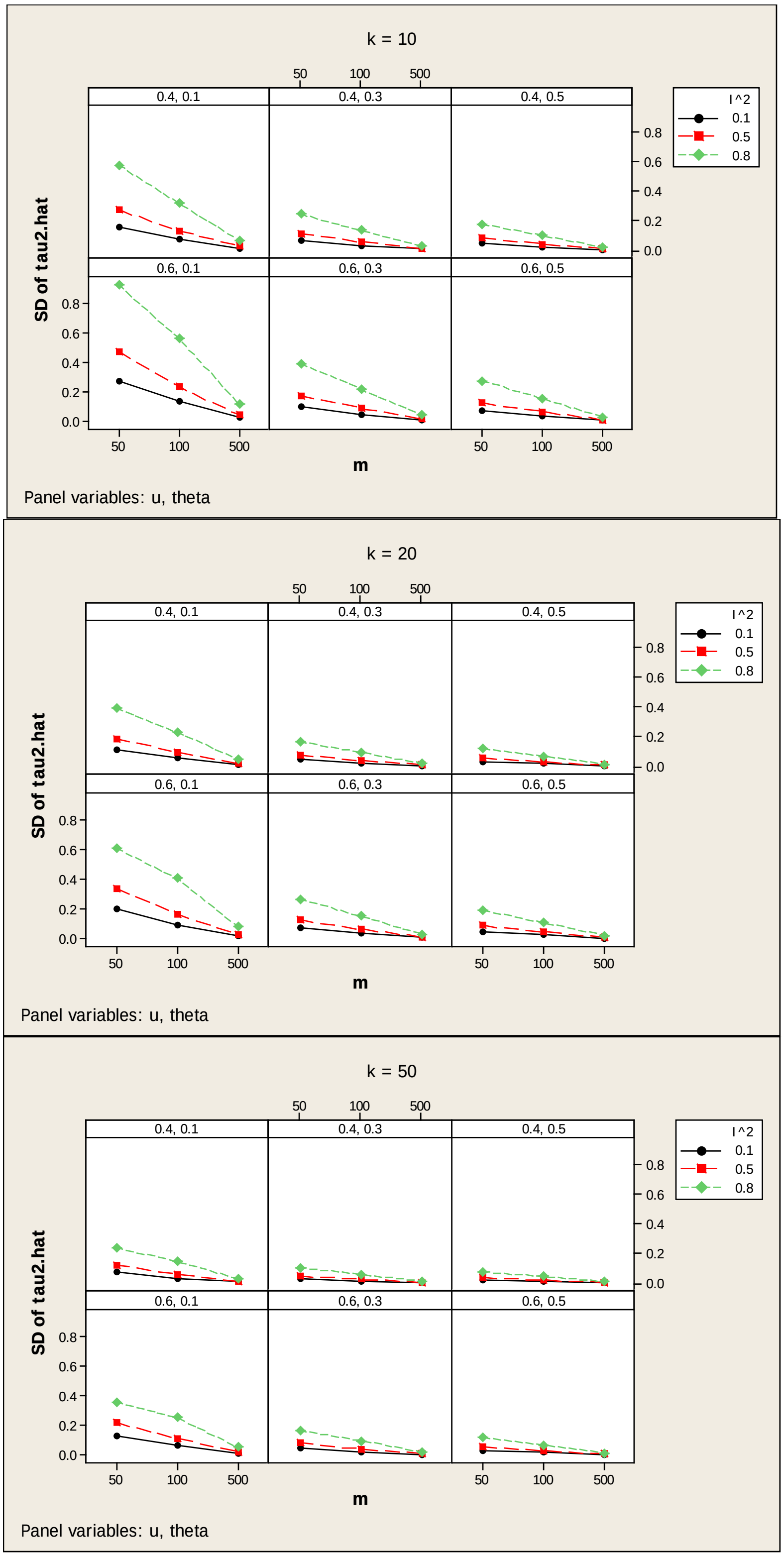

Fig.9 The graphs of SD for $\hat{\tau}^2$ against sample size for various values of the heterogeneity measure I^2

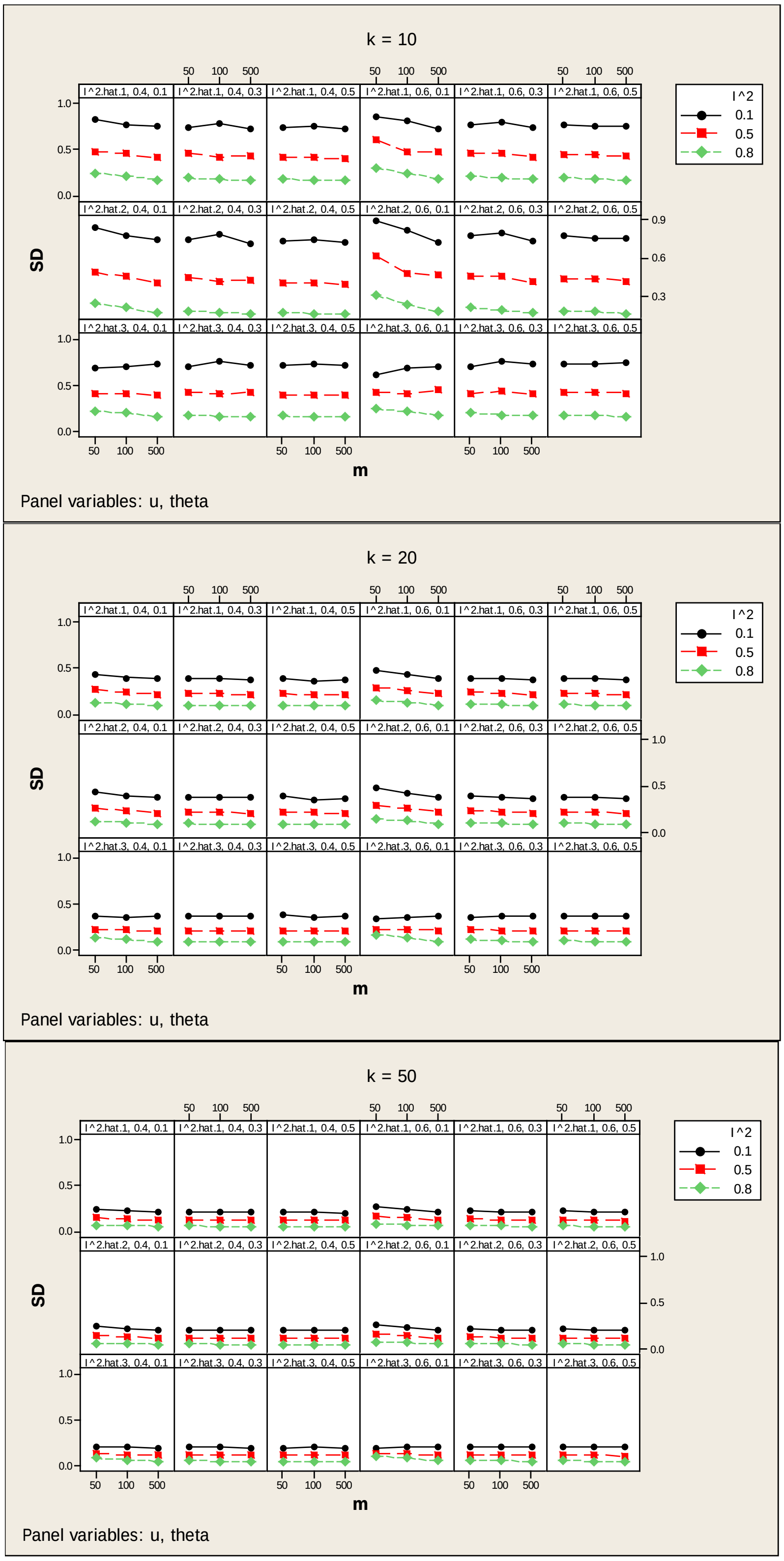

Fig.10 The graphs of SD for $\hat{I}_1^2$ (the first row), $\hat{I}_2^2$ (the second row) and $\hat{I}_3^2$ (the third row) against sample size for various values of the heterogeneity measure I^2

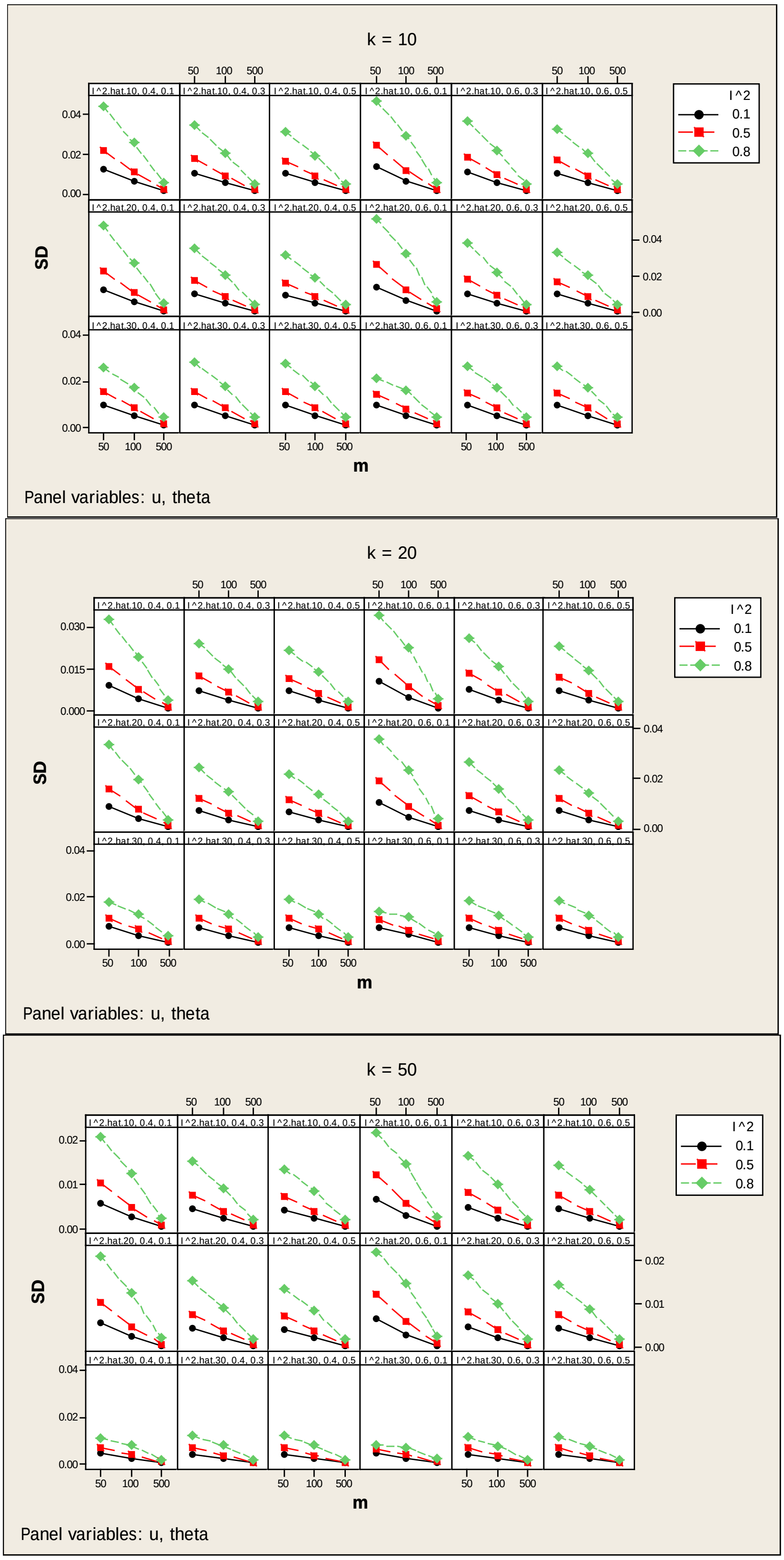

Fig.11 The graphs of SD for $\hat{I}^2_{10}$ (the first row), $\hat{I}^2_{20}$ (the second row) and $\hat{I}^2_{30}$ (the third row) against aginst sample size for various values of the heterogeneity measure I^2

5. Application to case studies

In Charoensawat et al. (2014) a number of meta-analytic diagnostic studies were discussed including a meta-analysis by Doust *et al.* (2004) on the diagnostic accuracy of the brain natriuretic peptides (BNP) procedure as a diagnostic test for heart failure (heart failure), a meta–analysis by Mitchell *et al.* (2009) on the diagnostic accuracy of the mini-mental state examination (MMSE) as a diagnostic test for the detection of dementia (dementia), and a meta–analysis of the diagnostic accuracy of a PH questionnaire for diagnosing depressive disorders (Mood) by Wittkampf *et al.* (2007). In the example 1, Table 1, we reproduce here the data of the heart failure meta-analysis for illustration.

Example 1: Heart failure

Table 1: Meta-analytic data of the brain natriuretic peptides (BNP) procedure as a diagnostic test for heart failure

Study i	TP (y_i)	FN (m_i-y_i)	TN (x_i)	FP (n_i-x_i)	m_i+n_i	m_i	n_i	$\hat{p}_i$	$\hat{u}_i$
1	29	7	46	19	101	36	65	0.8056	0.2923
2	34	6	22	13	75	40	35	0.8500	0.3714
3	49	9	78	17	153	58	95	0.8448	0.1789
4	4	6	1612	85	1707	10	1697	0.4000	0.0501
5	20	40	1339	71	1470	60	1410	0.3333	0.0504
6	29	2	102	166	299	31	268	0.9355	0.6194
7	26	14	75	11	126	40	86	0.6500	0.1279
8	11	1	93	50	155	12	143	0.9167	0.3497

For computation we use the formulae: $\hat{p}_i = y_i / m_i$, $\hat{u}_i = 1 - x_i / n_i$ and $\hat{\theta}_i = \log \hat{p}_i / \log \hat{u}_i$

$$\hat{\sigma}_i^2 = \frac{1}{m_i} \frac{1-\hat{p}_i}{\hat{p}_i (\log \hat{p}_i)^2} + \frac{1}{n_i} \frac{1-\hat{u}_i}{\hat{u}_i (\log \hat{u}_i)^2} \qquad \hat{\sigma}_{i0}^2 = \frac{1-\hat{p}_i}{\hat{p}_i (\log \hat{p}_i)^2} + \frac{1-\hat{u}_i}{\hat{u}_i (\log \hat{u}_i)^2},$$

and for all others the formulae as defined before in section 3.2

The results of the analysis for all three meta-analyses are provided below in Table 2. It is clearly seen that the conventional heterogeneity measure of Higgins produces large values for the magnitude of heterogeneity. This is also the case for the meta-analysis of heart failure which is considered to be a case of low heterogeneity. These large values for Higgins I^2 are likely due to the fact that in diagnostic studies we often have large within study samples sizes. In contrast, we see that with the new heterogeneity measure we have much lower values of magnitude in heterogeneity. The heterogeneity ranking based upon the new measure also corresponds much more with other informative measures on heterogeneity, for example the SROC graphs as developed in Charoensawat *et al.* (2014). Hence this underlines that the adjusted measure of Higgins could be an interesting alternative to the conventional measure, but clearly further experience needs to be collected and brought into perspective.

Table 2: The results for the meta-analytic case studies heart failure, dementia, and mood

Data	heart failure	dementia	mood
$\hat{\theta}$	0.246	0.130	0.143
$\hat{\tau}^2$	0.172	0.388	0.507
$\overline{\overline{\sigma}}_1^2$	0.134	0.050	0.074
$\overline{\overline{\sigma}}_2^2$	0.111	0.048	0.062
$\overline{\overline{\sigma}}_2^2$	0.292	0.158	0.230
$\overline{\overline{\sigma}}_{10}^2$	6.550	7.300	6.389
$\overline{\overline{\sigma}}_{20}^2$	6.335	7.260	6.273
$\overline{\overline{\sigma}}_{30}^2$	8.405	11.725	7.830
$\hat{I}_1^2$	0.561	0.887	0.873
$\hat{I}_2^2$	0.608	0.889	0.891
$\hat{I}_3^2$	0.371	0.711	0.688
$\hat{I}_{10}^2$	0.026	0.050	0.073
$\hat{I}_{20}^2$	0.026	0.051	0.075
$\hat{I}_{30}^2$	0.020	0.032	0.061

6. Discussion

This work has pointed out some difficulties with Higgins' I^2 measure: given the same amount of heterogeneity (variance between studies), it can deliver varying values according to the sample sizes of the component studies. The problem is immanent in all meta-analytic settings which involve sample size dependent component study variances. A typical example would be the difference of two means. Variances would involve the sample size of the two groups to be compared. The larger the groups in the component studies, the larger I^2. Consequently, it was suggested here to define a new version of I^2 which uses, similar to variance component analysis, variances of the component studies with the effect of the study sample size removed. The difference of the conventional I^2 and the newly suggested one is large when the component studies have large sample sizes. This is typically the case for diagnostic studies so that the diagnostic accuracy measure made the point well.

Acknowledgements

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Appendix I: R-code

The simulation work has used the following R-code which we list here for completeness and reproducibility.

File name: ma_hetero2_estimator method II different I2__USE

```
#-------------------------------------------------------------------------------------------------
#        Start program
#-------------------------------------------------------------------------------------------------
function(B,k,m,theta,u,i2) {
theta.hat <- rep(0,B)
tau2.hat <- rep(0,B)
sigma2.bb1 <- rep(0,B)
sigma2.bb2 <- rep(0,B)
sigma2.bb3 <- rep(0,B)
i2.hat1 <- rep(0,B)
i2.hat2 <- rep(0,B)
i2.hat3 <- rep(0,B)
sigma2.bb10 <- rep(0,B)
sigma2.bb20 <- rep(0,B)
sigma2.bb30 <- rep(0,B)
i2.hat10 <- rep(0,B)
i2.hat20 <- rep(0,B)
i2.hat30 <- rep(0,B)

for (h in 1:B) {
p <- u^theta
sigma2.i <- ((((1-p)/(p*(log(p))^2)))+(((1-u)/(u*(log(u))^2))))/m
w.i <- 1/sigma2.i
tau2 <- (sigma2.i*i2)/(1-i2)
log.theta.i <- rnorm(k,log(theta),sqrt(tau2))
e.i <- rnorm(k,0,sqrt(sigma2.i))
log.thetahat.i <- log.theta.i+e.i
thetahat.i <- exp(log.thetahat.i)

sigma2.i0 <- (((1-p)/(p*(log(p))^2)))+(((1-u)/(u*(log(u))^2)))
w.i0 <- 1/sigma2.i0
#-------------------------------------------------------------------------------------------------
#        The true values
#-------------------------------------------------------------------------------------------------
sigma2.1 <- (sum(w.i*(k-1)))/((sum(w.i))^2-sum(w.i^2))
sigma2.2 <- k/(sum(w.i))
sigma2.3 <- mean(sigma2.i)
i2.1 <- tau2/(tau2+sigma2.1)                     #---I2.1
i2.2 <- tau2/(tau2+sigma2.2)                     #---I2.2
i2.3 <- tau2/(tau2+sigma2.3)                     #---I2.3
sigma2.10 <- (sum(w.i0*(k-1)))/((sum(w.i0))^2-sum(w.i0^2))
sigma2.20 <- k/(sum(w.i0))
sigma2.30 <- mean(sigma2.i0)
i2.10 <- tau2/(tau2+sigma2.10)                        #---I2.10
i2.20 <- tau2/(tau2+sigma2.20)                        #---I2.20
i2.30 <- tau2/(tau2+sigma2.30)                        #---I2.30
#-------------------------------------------------------------------------------------------------
#        The estimators
```

```
#-----------------------------------------------------------------------------------------------------
phat.i <- u^thetahat.i
sigma2hat.i <- ((1/m)*((1-phat.i)/(phat.i*(log(phat.i))^2)))+((1/m)*((1-u)/(u*(log(u))^2)))
sigma2hat.i0 <- ((1-phat.i)/(phat.i*(log(phat.i))^2))+((1-u)/(u*(log(u))^2))
w.hat.i <- 1/sigma2hat.i
w.hat.i0 <- 1/sigma2hat.i0

logtheta.hat <- (sum(w.hat.i*log.thetahat.i))/(sum(w.hat.i))
theta.hat[h] <- exp(logtheta.hat)
#---estimator: theta.hat
chi2 <- sum(w.hat.i*(log.thetahat.i-logtheta.hat)^2)
tau2.hat[h] <- (chi2-(k-1))/(sum(w.hat.i)-(sum(w.hat.i^2))/(sum(w.hat.i)))
#---estimator: tau2.hat

sigma2.bb1[h] <- (sum(w.hat.i*(k-1)))/((sum(w.hat.i))^2-(sum(w.hat.i^2)))
sigma2.bb2[h] <- k/(sum(w.hat.i))
sigma2.bb3[h] <- mean(sigma2hat.i)
i2.hat1[h] <- tau2.hat[h]/(tau2.hat[h]+sigma2.bb1[h])
#---estimator: I2.hat1
i2.hat2[h] <- tau2.hat[h]/(tau2.hat[h]+sigma2.bb2[h])
#---estimator: I2.hat2
i2.hat3[h] <- tau2.hat[h]/(tau2.hat[h]+sigma2.bb3[h])
#---estimator: I2.hat3

sigma2.bb10[h] <- (sum(w.hat.i0*(k-1)))/((sum(w.hat.i0))^2-sum(w.hat.i0^2))
sigma2.bb20[h] <- k/(sum(w.hat.i0))
sigma2.bb30[h] <- mean(sigma2hat.i0)
i2.hat10[h] <- tau2.hat[h]/(tau2.hat[h]+sigma2.bb10[h])
#---estimator: I2.hat10
i2.hat20[h] <- tau2.hat[h]/(tau2.hat[h]+sigma2.bb20[h])
#---estimator: I2.hat20
i2.hat30[h] <- tau2.hat[h]/(tau2.hat[h]+sigma2.bb30[h])
#---estimator: I2.hat30
} #end loop B

ave.theta.hat <- mean(theta.hat)
#---mean of estimators
ave.tau2.hat <- mean(tau2.hat)
ave.i2.hat1 <- mean(i2.hat1)
ave.i2.hat2 <- mean(i2.hat2)
ave.i2.hat3 <- mean(i2.hat3)
ave.i2.hat10 <- mean(i2.hat10)
ave.i2.hat20 <- mean(i2.hat20)
ave.i2.hat30 <- mean(i2.hat30)

bias.theta.hat <- ave.theta.hat-theta
#---bias of estimators
bias.tau2.hat <- ave.tau2.hat-tau2
bias.i2.hat1 <- ave.i2.hat1-i2.1
bias.i2.hat2 <- ave.i2.hat2-i2.2
bias.i2.hat3 <- ave.i2.hat3-i2.3
bias.i2.hat10 <- ave.i2.hat10-i2.10
bias.i2.hat20 <- ave.i2.hat20-i2.20
bias.i2.hat30 <- ave.i2.hat30-i2.30
```

```
# calculation of estimated standard errors
mse.theta.hat <- sqrt(mean((theta.hat-ave.theta.hat)^2))
mse.tau2.hat <- sqrt(mean((tau2.hat-ave.tau2.hat)^2))
mse.i2.hat1 <- sqrt(mean((i2.hat1-ave.i2.hat1)^2))
mse.i2.hat2 <- sqrt(mean((i2.hat2-ave.i2.hat2)^2))
mse.i2.hat3 <- sqrt(mean((i2.hat3-ave.i2.hat3)^2))
mse.i2.hat10 <- sqrt(mean((i2.hat10-ave.i2.hat10)^2))
mse.i2.hat20 <- sqrt(mean((i2.hat20-ave.i2.hat20)^2))
mse.i2.hat30 <- sqrt(mean((i2.hat30-ave.i2.hat30)^2))

cat(tau2,'\t',ave.theta.hat,'\t', ave.tau2.hat,'\t',ave.i2.hat1,'\t', ave.i2.hat2,'\t',
ave.i2.hat3,'\t',ave.i2.hat10,'\t', ave.i2.hat20,'\t', ave.i2.hat30,'\n')
cat(bias.theta.hat,'\t', bias.tau2.hat,'\t',bias.i2.hat1,'\t', bias.i2.hat2,'\t',
bias.i2.hat3,'\t',bias.i2.hat10,'\t', bias.i2.hat20,'\t', bias.i2.hat30,'\n')
cat(mse.theta.hat,'\t', mse.tau2.hat,'\t',mse.i2.hat1,'\t', mse.i2.hat2,'\t',
mse.i2.hat3,'\t',mse.i2.hat10,'\t', mse.i2.hat20,'\t', mse.i2.hat30,'\n')

} #end program
```

Appendix II: Detailed numerical results of the simulation

Table A.1 The mean of estimators ($\hat{\theta}$, $\hat{\tau}^2$, $\hat{I}_j^2$, $\hat{I}_{j0}^2$, j = 1, 2 , 3) for various settings

					Estimators								
k	**m**	θ	**u**	I^2	$\hat{\theta}$	$\hat{\tau}^2$	$\hat{I}_1^2$	$\hat{I}_2^2$	$\hat{I}_3^2$	$\hat{I}_{10}^2$	$\hat{I}_{20}^2$	$\hat{I}_{30}^2$	τ^2
10	**50**	**0.1**	**0.4**	**0.1**	0.1240	0.0169	-0.2416	-0.2471	-0.1982	0.0017	0.0019	0.0010	**0.03**
				0.5	0.1466	0.2214	0.2910	0.2948	0.2564	0.0184	0.0192	0.0133	**0.26**
				0.8	0.2269	0.7716	0.6815	0.6909	0.5602	0.0638	0.0679	0.0351	**1.06**
			0.6	**0.1**	0.1469	0.0289	-0.2673	-0.2797	-0.1756	0.0022	0.0025	0.0010	**0.05**
				0.5	0.1917	0.3467	0.2519	0.2561	0.2094	0.0188	0.0203	0.0108	**0.45**
				0.8	0.3681	1.1362	0.6501	0.6643	0.4463	0.0630	0.0694	0.0237	**1.81**
		0.3	**0.4**	**0.1**	0.3239	0.0091	-0.1921	-0.1930	-0.1844	0.0015	0.0015	0.0013	**0.01**
				0.5	0.3415	0.1049	0.3087	0.3098	0.2977	0.0171	0.0172	0.0156	**0.12**
				0.8	0.4037	0.3911	0.6989	0.7016	0.6675	0.0600	0.0610	0.0504	**0.48**
			0.6	**0.1**	0.3434	0.0106	-0.2264	-0.2288	-0.2070	0.0012	0.0013	0.0009	**0.02**
				0.5	0.3777	0.1599	0.2946	0.2967	0.2745	0.0167	0.0170	0.0140	**0.19**
				0.8	0.5004	0.5917	0.6833	0.6883	0.6189	0.0591	0.0609	0.0425	**0.77**
		0.5	**0.4**	**0.1**	0.5230	0.0067	-0.1883	-0.1887	-0.1852	0.0014	0.0014	0.0013	**0.01**
				0.5	0.5413	0.0813	0.3254	0.3260	0.3200	0.0170	0.0171	0.0163	**0.09**
				0.8	0.5990	0.3036	0.7042	0.7055	0.6902	0.0591	0.0596	0.0544	**0.36**
			0.6	**0.1**	0.5417	0.0090	-0.2096	-0.2106	-0.2013	0.0012	0.0013	0.0011	**0.02**
				0.5	0.5785	0.1192	0.3061	0.3073	0.2951	0.0165	0.0166	0.0151	**0.14**
				0.8	0.6892	0.4552	0.6968	0.6995	0.6641	0.0587	0.0597	0.0491	**0.56**
	100	**0.1**	**0.4**	**0.1**	0.1122	0.0118	-0.1939	-0.1959	-0.1774	0.0010	0.0011	0.0008	**0.01**
				0.5	0.1220	0.1220	0.3240	0.3266	0.3022	0.0098	0.0100	0.0081	**0.13**
				0.8	0.1584	0.4432	0.7057	0.7113	0.6462	0.0359	0.0375	0.0252	**0.53**
			0.6	**0.1**	0.1222	0.0201	-0.2198	-0.2243	-0.1832	0.0012	0.0012	0.0008	**0.03**
				0.5	0.1419	0.1970	0.3000	0.3041	0.2658	0.0099	0.0103	0.0071	**0.23**
				0.8	0.2197	0.7027	0.6927	0.7023	0.5809	0.0369	0.0398	0.0201	**0.91**
		0.3	**0.4**	**0.1**	0.3118	0.0053	-0.1845	-0.1850	-0.1811	0.0009	0.0009	0.0008	**0.01**
				0.5	0.3214	0.0561	0.3370	0.3377	0.3311	0.0093	0.0093	0.0089	**0.06**
				0.8	0.3534	0.2176	0.7232	0.7247	0.7088	0.0348	0.0352	0.0316	**0.24**
			0.6	**0.1**	0.3211	0.0080	-0.1974	-0.1983	-0.1896	0.0009	0.0009	0.0007	**0.01**
				0.5	0.3382	0.0873	0.3243	0.3256	0.3129	0.0091	0.0092	0.0083	**0.10**
				0.8	0.3981	0.3334	0.7102	0.7130	0.6803	0.0341	0.0348	0.0283	**0.38**
		0.5	**0.4**	**0.1**	0.5114	0.0037	-0.1849	-0.1851	-0.1834	0.0008	0.0008	0.0008	**0.01**
				0.5	0.5213	0.0430	0.3409	0.3412	0.3381	0.0092	0.0092	0.0090	**0.05**
				0.8	0.5508	0.1685	0.7267	0.7274	0.7203	0.0346	0.0348	0.0331	**0.18**
			0.6	**0.1**	0.5202	0.0057	-0.1884	-0.1889	-0.1849	0.0008	0.0008	0.0007	**0.01**
				0.5	0.5370	0.0648	0.3276	0.3283	0.3218	0.0091	0.0091	0.0087	**0.07**
				0.8	0.5967	0.2502	0.7156	0.7171	0.7010	0.0339	0.0342	0.0307	**0.28**
	500	**0.1**	**0.4**	**0.1**	0.1021	0.0027	-0.1730	-0.1733	-0.1704	0.0002	0.0002	0.0002	**0.00**
				0.5	0.1043	0.0261	0.3531	0.3538	0.3481	0.0020	0.0020	0.0019	**0.03**
				0.8	0.1109	0.1026	0.7376	0.7389	0.7265	0.0079	0.0080	0.0073	**0.11**
			0.6	**0.1**	0.1042	0.0047	-0.1705	-0.1711	-0.1655	0.0002	0.0002	0.0002	**0.01**
				0.5	0.1077	0.0435	0.3412	0.3423	0.3322	0.0020	0.0020	0.0018	**0.05**
				0.8	0.1198	0.1717	0.7304	0.7328	0.7094	0.0080	0.0082	0.0068	**0.18**
		0.3	**0.4**	**0.1**	0.3024	0.0013	-0.1544	-0.1544	-0.1538	0.0002	0.0002	0.0002	**0.00**
				0.5	0.3039	0.0118	0.3470	0.3471	0.3458	0.0020	0.0020	0.0020	**0.01**
				0.8	0.3104	0.0466	0.7384	0.7387	0.7358	0.0077	0.0077	0.0076	**0.05**
			0.6	**0.1**	0.3044	0.0020	-0.1700	-0.1702	-0.1688	0.0002	0.0002	0.0002	**0.00**
				0.5	0.3074	0.0189	0.3511	0.3514	0.3486	0.0020	0.0020	0.0019	**0.02**
				0.8	0.3194	0.0746	0.7358	0.7364	0.7304	0.0078	0.0078	0.0074	**0.08**
		0.5	**0.4**	**0.1**	0.5023	0.0010	-0.1565	-0.1566	-0.1563	0.0002	0.0002	0.0002	**0.00**
				0.5	0.5042	0.0090	0.3573	0.3573	0.3567	0.0020	0.0020	0.0020	**0.01**
				0.8	0.5109	0.0354	0.7370	0.7371	0.7358	0.0077	0.0077	0.0076	**0.04**
			0.6	**0.1**	0.5040	0.0015	-0.1658	-0.1659	-0.1652	0.0002	0.0002	0.0002	**0.00**
				0.5	0.5082	0.0137	0.3492	0.3493	0.3480	0.0019	0.0019	0.0019	**0.01**
				0.8	0.5189	0.0552	0.7403	0.7406	0.7377	0.0078	0.0078	0.0076	**0.06**

					Estimators								
k	**m**	θ	**u**	I^2	$\hat{\theta}$	$\hat{\tau}^2$	$\hat{\mathrm{I}}_1^2$	$\hat{\mathrm{I}}_2^2$	$\hat{\mathrm{I}}_3^2$	$\hat{\mathrm{I}}_{10}^2$	$\hat{\mathrm{I}}_{20}^2$	$\hat{\mathrm{I}}_{30}^2$	τ^2
20	**50**	**0.1**	**0.4**	**0.1**	0.1254	0.0273	-0.0359	-0.0365	-0.0241	0.0024	0.0025	0.0018	**0.03**
				0.5	0.1472	0.2310	0.4083	0.4114	0.3483	0.0193	0.0197	0.0139	**0.26**
				0.8	0.2266	0.7885	0.7436	0.7484	0.6002	0.0674	0.0694	0.0345	**1.06**
			0.6	**0.1**	0.1478	0.0493	-0.0433	-0.0448	-0.0164	0.0029	0.0030	0.0017	**0.05**
				0.5	0.1923	0.3755	0.4014	0.4064	0.3010	0.0203	0.0210	0.0113	**0.45**
				0.8	0.3651	1.1704	0.7375	0.7453	0.4749	0.0680	0.0711	0.0217	**1.81**
		0.3	**0.4**	**0.1**	0.3234	0.0086	-0.0433	-0.0434	-0.0410	0.0014	0.0015	0.0013	**0.01**
				0.5	0.3422	0.1054	0.4051	0.4058	0.3887	0.0173	0.0174	0.0159	**0.12**
				0.8	0.4031	0.3918	0.7400	0.7413	0.7040	0.0612	0.0617	0.0508	**0.48**
			0.6	**0.1**	0.3438	0.0133	-0.0496	-0.0499	-0.0435	0.0015	0.0015	0.0012	**0.02**
				0.5	0.3792	0.1648	0.3968	0.3983	0.3651	0.0174	0.0175	0.0146	**0.19**
				0.8	0.4985	0.5880	0.7305	0.7331	0.6552	0.0601	0.0610	0.0419	**0.77**
		0.5	**0.4**	**0.1**	0.5235	0.0065	-0.0448	-0.0448	-0.0438	0.0014	0.0014	0.0013	**0.01**
				0.5	0.5414	0.0806	0.4057	0.4061	0.3982	0.0171	0.0171	0.0164	**0.09**
				0.8	0.5977	0.3057	0.7413	0.7420	0.7250	0.0604	0.0607	0.0553	**0.36**
			0.6	**0.1**	0.5424	0.0098	-0.0443	-0.0445	-0.0420	0.0014	0.0014	0.0012	**0.02**
				0.5	0.5766	0.1203	0.3964	0.3971	0.3801	0.0168	0.0168	0.0154	**0.14**
				0.8	0.6887	0.4494	0.7334	0.7348	0.6959	0.0593	0.0597	0.0490	**0.56**
	100	**0.1**	**0.4**	**0.1**	0.1121	0.0137	-0.0221	-0.0223	-0.0186	0.0011	0.0011	0.0010	**0.01**
				0.5	0.1225	0.1236	0.4230	0.4247	0.3902	0.0099	0.0100	0.0083	**0.13**
				0.8	0.1604	0.4595	0.7609	0.7638	0.6924	0.0380	0.0387	0.0259	**0.53**
			0.6	**0.1**	0.1224	0.0251	-0.0275	-0.0279	-0.0181	0.0013	0.0013	0.0010	**0.03**
				0.5	0.1424	0.2067	0.4198	0.4229	0.3619	0.0103	0.0105	0.0075	**0.23**
				0.8	0.2211	0.7384	0.7595	0.7644	0.6287	0.0397	0.0411	0.0203	**0.91**
		0.3	**0.4**	**0.1**	0.3118	0.0055	-0.0254	-0.0254	-0.0248	0.0009	0.0009	0.0009	**0.01**
				0.5	0.3216	0.0562	0.4228	0.4232	0.4144	0.0093	0.0094	0.0089	**0.06**
				0.8	0.3535	0.2177	0.7593	0.7600	0.7427	0.0352	0.0354	0.0319	**0.24**
			0.6	**0.1**	0.3214	0.0089	-0.0256	-0.0256	-0.0241	0.0009	0.0009	0.0009	**0.01**
				0.5	0.3390	0.0891	0.4190	0.4198	0.4023	0.0093	0.0094	0.0085	**0.10**
				0.8	0.3995	0.3383	0.7551	0.7565	0.7206	0.0351	0.0354	0.0288	**0.38**
		0.5	**0.4**	**0.1**	0.5117	0.0044	-0.0123	-0.0123	-0.0122	0.0010	0.0010	0.0009	**0.01**
				0.5	0.5209	0.0430	0.4229	0.4231	0.4191	0.0093	0.0093	0.0091	**0.05**
				0.8	0.5507	0.1680	0.7596	0.7599	0.7521	0.0349	0.0350	0.0333	**0.18**
			0.6	**0.1**	0.5216	0.0063	-0.0280	-0.0280	-0.0273	0.0009	0.0009	0.0009	**0.01**
				0.5	0.5381	0.0648	0.4177	0.4182	0.4094	0.0091	0.0092	0.0087	**0.07**
				0.8	0.5959	0.2509	0.7540	0.7547	0.7370	0.0343	0.0345	0.0310	**0.28**
	500	**0.1**	**0.4**	**0.1**	0.1024	0.0030	-0.0042	-0.0042	-0.0041	0.0002	0.0002	0.0002	**0.00**
				0.5	0.1042	0.0258	0.4353	0.4356	0.4283	0.0020	0.0020	0.0019	**0.03**
				0.8	0.1111	0.1035	0.7741	0.7748	0.7613	0.0080	0.0081	0.0074	**0.11**
			0.6	**0.1**	0.1043	0.0051	-0.0087	-0.0087	-0.0082	0.0002	0.0002	0.0002	**0.01**
				0.5	0.1078	0.0447	0.4357	0.4364	0.4227	0.0020	0.0020	0.0019	**0.05**
				0.8	0.1202	0.1734	0.7721	0.7733	0.7478	0.0081	0.0082	0.0069	**0.18**
		0.3	**0.4**	**0.1**	0.3024	0.0013	-0.0115	-0.0115	-0.0114	0.0002	0.0002	0.0002	**0.00**
				0.5	0.3041	0.0118	0.4361	0.4362	0.4344	0.0020	0.0020	0.0020	**0.01**
				0.8	0.3105	0.0465	0.7719	0.7721	0.7689	0.0077	0.0077	0.0076	**0.05**
			0.6	**0.1**	0.3044	0.0021	-0.0078	-0.0078	-0.0077	0.0002	0.0002	0.0002	**0.00**
				0.5	0.3079	0.0188	0.4354	0.4356	0.4319	0.0020	0.0020	0.0019	**0.02**
				0.8	0.3193	0.0748	0.7717	0.7720	0.7654	0.0078	0.0078	0.0075	**0.08**
		0.5	**0.4**	**0.1**	0.5022	0.0010	-0.0072	-0.0072	-0.0071	0.0002	0.0002	0.0002	**0.00**
				0.5	0.5042	0.0089	0.4348	0.4348	0.4340	0.0019	0.0019	0.0019	**0.01**
				0.8	0.5100	0.0358	0.7726	0.7726	0.7712	0.0078	0.0078	0.0077	**0.04**
			0.6	**0.1**	0.5041	0.0015	-0.0113	-0.0113	-0.0112	0.0002	0.0002	0.0002	**0.00**
				0.5	0.5076	0.0138	0.4365	0.4366	0.4348	0.0020	0.0020	0.0019	**0.01**
				0.8	0.5187	0.0547	0.7719	0.7721	0.7689	0.0077	0.0077	0.0076	**0.06**

					Estimators								
k	m	θ	u	I^2	$\hat{\theta}$	$\hat{\tau}^2$	$\hat{I}_1^2$	$\hat{I}_2^2$	$\hat{I}_3^2$	$\hat{I}_{10}^2$	$\hat{I}_{20}^2$	$\hat{I}_{30}^2$	τ^2
50	**50**	**0.1**	**0.4**	**0.1**	0.1255	0.0312	0.0602	0.0604	0.0537	0.0026	0.0026	0.0021	**0.03**
				0.5	0.1473	0.2394	0.4708	0.4722	0.3956	0.0200	0.0201	0.0143	**0.26**
				0.8	0.2267	0.7971	0.7752	0.7771	0.6197	0.0696	0.0704	0.0337	**1.06**
			0.6	**0.1**	0.1485	0.0625	0.0767	0.0770	0.0631	0.0033	0.0034	0.0022	**0.05**
				0.5	0.1933	0.4009	0.4829	0.4854	0.3500	0.0217	0.0219	0.0119	**0.45**
				0.8	0.3661	1.1809	0.7779	0.7808	0.4810	0.0711	0.0723	0.0198	**1.81**
		0.3	**0.4**	**0.1**	0.3239	0.0102	0.0383	0.0383	0.0370	0.0017	0.0017	0.0016	**0.01**
				0.5	0.3427	0.1062	0.4489	0.4492	0.4296	0.0176	0.0176	0.0161	**0.12**
				0.8	0.4037	0.3928	0.7587	0.7593	0.7205	0.0621	0.0623	0.0511	**0.48**
			0.6	**0.1**	0.3434	0.0142	0.0260	0.0260	0.0248	0.0015	0.0015	0.0013	**0.02**
				0.5	0.3791	0.1662	0.4454	0.4461	0.4070	0.0176	0.0176	0.0148	**0.19**
				0.8	0.4977	0.5938	0.7549	0.7559	0.6745	0.0615	0.0618	0.0419	**0.77**
		0.5	**0.4**	**0.1**	0.5229	0.0069	0.0326	0.0326	0.0321	0.0015	0.0015	0.0015	**0.01**
				0.5	0.5410	0.0809	0.4458	0.4460	0.4370	0.0172	0.0173	0.0166	**0.09**
				0.8	0.5964	0.3029	0.7555	0.7557	0.7378	0.0605	0.0606	0.0552	**0.36**
			0.6	**0.1**	0.5425	0.0094	0.0208	0.0208	0.0203	0.0013	0.0013	0.0013	**0.02**
				0.5	0.5757	0.1201	0.4377	0.4381	0.4185	0.0168	0.0168	0.0154	**0.14**
				0.8	0.6861	0.4476	0.7513	0.7518	0.7110	0.0597	0.0598	0.0489	**0.56**
	100	**0.1**	**0.4**	**0.1**	0.1123	0.0153	0.0619	0.0620	0.0577	0.0012	0.0012	0.0011	**0.01**
				0.5	0.1227	0.1261	0.4753	0.4761	0.4360	0.0101	0.0101	0.0085	**0.13**
				0.8	0.1605	0.4660	0.7875	0.7887	0.7143	0.0388	0.0391	0.0260	**0.53**
			0.6	**0.1**	0.1228	0.0293	0.0704	0.0706	0.0626	0.0014	0.0014	0.0011	**0.03**
				0.5	0.1434	0.2197	0.4881	0.4896	0.4160	0.0109	0.0110	0.0079	**0.23**
				0.8	0.2208	0.7475	0.7924	0.7944	0.6519	0.0408	0.0413	0.0200	**0.91**
		0.3	**0.4**	**0.1**	0.3117	0.0056	0.0456	0.0457	0.0448	0.0009	0.0009	0.0009	**0.01**
				0.5	0.3215	0.0559	0.4620	0.4622	0.4522	0.0093	0.0093	0.0089	**0.06**
				0.8	0.3531	0.2169	0.7758	0.7761	0.7581	0.0353	0.0354	0.0319	**0.24**
			0.6	**0.1**	0.3218	0.0091	0.0466	0.0466	0.0449	0.0010	0.0010	0.0009	**0.01**
				0.5	0.3397	0.0895	0.4628	0.4632	0.4431	0.0094	0.0094	0.0086	**0.10**
				0.8	0.3994	0.3383	0.7745	0.7750	0.7378	0.0353	0.0355	0.0289	**0.38**
		0.5	**0.4**	**0.1**	0.5116	0.0043	0.0467	0.0467	0.0463	0.0009	0.0009	0.0009	**0.01**
				0.5	0.5204	0.0429	0.4622	0.4623	0.4578	0.0093	0.0093	0.0091	**0.05**
				0.8	0.5498	0.1671	0.7747	0.7748	0.7667	0.0349	0.0349	0.0333	**0.18**
			0.6	**0.1**	0.5213	0.0063	0.0426	0.0427	0.0419	0.0009	0.0009	0.0009	**0.01**
				0.5	0.5376	0.0646	0.4571	0.4573	0.4474	0.0091	0.0091	0.0087	**0.07**
				0.8	0.5964	0.2517	0.7728	0.7731	0.7547	0.0347	0.0347	0.0313	**0.28**
	500	**0.1**	**0.4**	**0.1**	0.1024	0.0029	0.0620	0.0620	0.0610	0.0002	0.0002	0.0002	**0.00**
				0.5	0.1044	0.0262	0.4783	0.4785	0.4702	0.0020	0.0020	0.0019	**0.03**
				0.8	0.1112	0.1038	0.7922	0.7924	0.7783	0.0081	0.0081	0.0074	**0.11**
			0.6	**0.1**	0.1044	0.0054	0.0662	0.0662	0.0642	0.0002	0.0002	0.0002	**0.01**
				0.5	0.1080	0.0449	0.4803	0.4806	0.4651	0.0020	0.0020	0.0019	**0.05**
				0.8	0.1204	0.1754	0.7931	0.7936	0.7669	0.0082	0.0082	0.0070	**0.18**
		0.3	**0.4**	**0.1**	0.3024	0.0013	0.0616	0.0616	0.0614	0.0002	0.0002	0.0002	**0.00**
				0.5	0.3043	0.0119	0.4766	0.4767	0.4747	0.0020	0.0020	0.0020	**0.01**
				0.8	0.3108	0.0469	0.7884	0.7885	0.7851	0.0078	0.0078	0.0076	**0.05**
			0.6	**0.1**	0.3043	0.0021	0.0589	0.0589	0.0584	0.0002	0.0002	0.0002	**0.00**
				0.5	0.3076	0.0189	0.4740	0.4740	0.4699	0.0020	0.0020	0.0019	**0.02**
				0.8	0.3198	0.0753	0.7892	0.7893	0.7824	0.0079	0.0079	0.0075	**0.08**
		0.5	**0.4**	**0.1**	0.5024	0.0010	0.0581	0.0581	0.0580	0.0002	0.0002	0.0002	**0.00**
				0.5	0.5041	0.0090	0.4755	0.4755	0.4747	0.0020	0.0020	0.0020	**0.01**
				0.8	0.5103	0.0357	0.7878	0.7879	0.7864	0.0078	0.0078	0.0077	**0.04**
			0.6	**0.1**	0.5042	0.0015	0.0547	0.0547	0.0545	0.0002	0.0002	0.0002	**0.00**
				0.5	0.5077	0.0139	0.4764	0.4765	0.4745	0.0020	0.0020	0.0020	**0.01**
				0.8	0.5196	0.0552	0.7887	0.7887	0.7854	0.0078	0.0078	0.0076	**0.06**

Table A.2 The bias of estimators

					Estimators								
k	**m**	θ	**u**	I^2	$\hat{\theta}$	$\hat{\tau}^2$	$\hat{I}_1^2$	$\hat{I}_2^2$	$\hat{I}_3^2$	$\hat{I}_{10}^2$	$\hat{I}_{20}^2$	$\hat{I}_{30}^2$	τ^2
10	**50**	**0.1**	**0.4**	**0.1**	0.0240	-0.0125	-0.2416	-0.2581	**-0.2982**	0.0017	0.0017	**-0.0012**	**0.03**
				0.5	0.0466	-0.0429	0.2910	0.2039	**-0.2436**	0.0184	0.0172	**-0.0063**	**0.26**
				0.8	0.1269	-0.2856	0.6815	0.4052	**-0.2398**	0.0638	0.0599	**-0.0390**	**1.06**
			0.6	**0.1**	0.0469	-0.0214	**-0.2673**	-0.2907	-0.2756	0.0022	0.0022	**-0.0012**	**0.05**
				0.5	0.0917	-0.1061	0.2519	0.1652	**-0.2906**	0.0188	0.0183	**-0.0088**	**0.45**
				0.8	0.2681	-0.6750	0.6501	0.3786	**-0.3537**	0.0630	0.0615	**-0.0504**	**1.81**
		0.3	**0.4**	**0.1**	0.0239	-0.0042	-0.1921	-0.2040	**-0.2844**	0.0015	0.0013	**-0.0009**	**0.01**
				0.5	0.0415	-0.0145	0.3087	0.2189	**-0.2023**	0.0171	0.0152	**-0.0040**	**0.12**
				0.8	0.1037	-0.0868	0.6989	0.4159	**-0.1325**	0.0600	0.0530	**-0.0237**	**0.48**
			0.6	**0.1**	0.0434	-0.0108	-0.2264	-0.2398	**-0.3070**	0.0012	0.0010	**-0.0014**	**0.02**
				0.5	0.0777	-0.0323	0.2946	0.2058	**-0.2255**	0.0167	0.0150	**-0.0056**	**0.19**
				0.8	0.2004	-0.1769	0.6833	0.4026	**-0.1811**	0.0591	0.0530	**-0.0315**	**0.77**
		0.5	**0.4**	**0.1**	0.0230	-0.0034	-0.1883	-0.1997	**-0.2852**	0.0014	0.0012	**-0.0009**	**0.01**
				0.5	0.0413	-0.0098	0.3254	0.2351	**-0.1800**	0.0170	0.0151	**-0.0033**	**0.09**
				0.8	0.0990	-0.0608	0.7042	0.4198	**-0.1098**	0.0591	0.0516	**-0.0196**	**0.36**
			0.6	**0.1**	0.0417	-0.0066	-0.2096	-0.2216	**-0.3013**	0.0012	0.0010	**-0.0012**	**0.02**
				0.5	0.0785	-0.0211	0.3061	0.2164	**-0.2049**	0.0165	0.0146	**-0.0045**	**0.14**
				0.8	0.1892	-0.1060	0.6968	0.4138	**-0.1359**	0.0587	0.0517	**-0.0249**	**0.56**
	100	**0.1**	**0.4**	**0.1**	0.0122	-0.0028	-0.1939	-0.2069	**-0.2774**	0.0010	0.0010	**-0.0003**	**0.01**
				0.5	0.0220	-0.0101	0.3240	0.2356	**-0.1978**	0.0098	0.0090	**-0.0018**	**0.13**
				0.8	0.0584	-0.0854	0.7057	0.4256	**-0.1538**	0.0359	0.0335	**-0.0132**	**0.53**
			0.6	**0.1**	0.0222	-0.0051	-0.2198	-0.2353	**-0.2832**	0.0012	0.0011	**-0.0004**	**0.03**
				0.5	0.0419	-0.0294	0.3000	0.2132	**-0.2342**	0.0099	0.0093	**-0.0028**	**0.23**
				0.8	0.1197	-0.2029	0.6927	0.4165	**-0.2191**	0.0369	0.0358	**-0.0184**	**0.91**
		0.3	**0.4**	**0.1**	0.0118	-0.0014	-0.1845	-0.1959	**-0.2811**	0.0009	0.0008	**-0.0003**	**0.01**
				0.5	0.0214	-0.0036	0.3370	0.2468	**-0.1689**	0.0093	0.0083	**-0.0010**	**0.06**
				0.8	0.0534	-0.0214	0.7232	0.4389	**-0.0912**	0.0348	0.0312	**-0.0068**	**0.24**
			0.6	**0.1**	0.0211	-0.0027	-0.1974	-0.2093	**-0.2896**	0.0009	0.0008	**-0.0004**	**0.01**
				0.5	0.0382	-0.0088	0.3243	0.2347	**-0.1871**	0.0091	0.0082	**-0.0016**	**0.10**
				0.8	0.0981	-0.0508	0.7102	0.4273	**-0.1197**	0.0341	0.0308	**-0.0101**	**0.38**
		0.5	**0.4**	**0.1**	0.0114	-0.0013	-0.1849	-0.1961	**-0.2834**	0.0008	0.0007	**-0.0003**	**0.01**
				0.5	0.0213	-0.0025	0.3409	0.2503	**-0.1619**	0.0092	0.0082	**-0.0009**	**0.05**
				0.8	0.0508	-0.0137	0.7267	0.4417	**-0.0797**	0.0346	0.0308	**-0.0053**	**0.18**
			0.6	**0.1**	0.0202	-0.0021	-0.1884	-0.1999	**-0.2849**	0.0008	0.0007	**-0.0004**	**0.01**
				0.5	0.0370	-0.0054	0.3276	0.2374	**-0.1782**	0.0091	0.0081	**-0.0012**	**0.07**
				0.8	0.0967	-0.0304	0.7156	0.4314	**-0.0990**	0.0339	0.0302	**-0.0077**	**0.28**
	500	**0.1**	**0.4**	**0.1**	0.0021	-0.0002	-0.1730	-0.1843	**-0.2704**	0.0002	0.0002	**0.0000**	**0.00**
				0.5	0.0043	-0.0003	0.3531	0.2629	**-0.1519**	0.0020	0.0018	**-0.0001**	**0.03**
				0.8	0.0109	-0.0032	0.7376	0.4532	**-0.0735**	0.0079	0.0072	**-0.0007**	**0.11**
			0.6	**0.1**	0.0042	-0.0003	-0.1705	-0.1821	**-0.2655**	0.0002	0.0002	**0.0000**	**0.01**
				0.5	0.0077	-0.0018	0.3412	0.2514	**-0.1678**	0.0020	0.0018	**-0.0002**	**0.05**
				0.8	0.0198	-0.0094	0.7304	0.4471	**-0.0906**	0.0080	0.0074	**-0.0011**	**0.18**
		0.3	**0.4**	**0.1**	0.0024	0.0000	-0.1544	-0.1654	**-0.2538**	0.0002	0.0002	**0.0000**	**0.00**
				0.5	0.0039	-0.0001	0.3470	0.2562	**-0.1542**	0.0020	0.0018	**0.0000**	**0.01**
				0.8	0.0104	-0.0012	0.7384	0.4530	**-0.0642**	0.0077	0.0069	**-0.0004**	**0.05**
			0.6	**0.1**	0.0044	-0.0002	-0.1700	-0.1812	**-0.2688**	0.0002	0.0002	**0.0000**	**0.00**
				0.5	0.0074	-0.0004	0.3511	0.2605	**-0.1514**	0.0020	0.0018	**-0.0001**	**0.02**
				0.8	0.0194	-0.0022	0.7358	0.4507	**-0.0696**	0.0078	0.0070	**-0.0005**	**0.08**
		0.5	**0.4**	**0.1**	0.0023	0.0000	-0.1565	-0.1676	**-0.2563**	0.0002	0.0002	**0.0000**	**0.00**
				0.5	0.0042	-0.0001	0.3573	0.2664	**-0.1433**	0.0020	0.0018	**0.0000**	**0.01**
				0.8	0.0109	-0.0011	0.7370	0.4514	**-0.0642**	0.0077	0.0069	**-0.0003**	**0.04**
			0.6	**0.1**	0.0040	-0.0001	-0.1658	-0.1768	**-0.2652**	0.0002	0.0002	**0.0000**	**0.00**
				0.5	0.0082	-0.0004	0.3492	0.2584	**-0.1520**	0.0019	0.0017	**-0.0001**	**0.01**
				0.8	0.0189	-0.0010	0.7403	0.4549	**-0.0623**	0.0078	0.0070	**-0.0003**	**0.06**

k	m	θ	u	I^2	Estimators								τ^2
					$\hat{\theta}$	$\hat{\tau}^2$	$\hat{I}_1^2$	$\hat{I}_2^2$	$\hat{I}_3^2$	$\hat{I}_{10}^2$	$\hat{I}_{20}^2$	$\hat{I}_{30}^2$	
20	50	0.1	0.4	0.1	0.0254	-0.0021	-0.0359	-0.0420	**-0.1241**	0.0024	0.0023	**-0.0004**	**0.03**
				0.5	0.0472	-0.0333	0.4083	0.3638	**-0.1517**	0.0193	0.0187	**-0.0057**	**0.26**
				0.8	0.1266	-0.2688	0.7436	0.5818	**-0.1998**	0.0674	0.0654	**-0.0396**	**1.06**
			0.6	0.1	0.0478	-0.0010	-0.0433	-0.0503	**-0.1164**	0.0029	0.0029	**-0.0005**	**0.05**
				0.5	0.0923	-0.0773	0.4014	0.3588	**-0.1990**	0.0203	0.0200	**-0.0083**	**0.45**
				0.8	0.2651	-0.6408	0.7375	0.5786	**-0.3251**	0.0680	0.0671	**-0.0524**	**1.81**
		0.3	0.4	0.1	0.0234	-0.0046	-0.0433	-0.0489	**-0.1410**	0.0014	0.0013	**-0.0009**	**0.01**
				0.5	0.0422	-0.0141	0.4051	0.3582	**-0.1113**	0.0173	0.0164	**-0.0037**	**0.12**
				0.8	0.1031	-0.0861	0.7400	0.5747	**-0.0960**	0.0612	0.0577	**-0.0233**	**0.48**
			0.6	0.1	0.0438	-0.0081	-0.0496	-0.0554	**-0.1435**	0.0015	0.0014	**-0.0010**	**0.02**
				0.5	0.0792	-0.0274	0.3968	0.3507	**-0.1349**	0.0174	0.0165	**-0.0050**	**0.19**
				0.8	0.1985	-0.1805	0.7305	0.5664	**-0.1448**	0.0601	0.0570	**-0.0322**	**0.77**
		0.5	0.4	0.1	0.0235	-0.0036	-0.0448	-0.0503	**-0.1438**	0.0014	0.0013	**-0.0009**	**0.01**
				0.5	0.0414	-0.0105	0.4057	0.3584	**-0.1018**	0.0171	0.0161	**-0.0032**	**0.09**
				0.8	0.0977	-0.0587	0.7413	0.5753	**-0.0750**	0.0604	0.0567	**-0.0187**	**0.36**
			0.6	0.1	0.0424	-0.0058	-0.0443	-0.0500	**-0.1420**	0.0014	0.0013	**-0.0010**	**0.02**
				0.5	0.0766	-0.0200	0.3964	0.3495	**-0.1199**	0.0168	0.0158	**-0.0042**	**0.14**
				0.8	0.1887	-0.1119	0.7334	0.5681	**-0.1041**	0.0593	0.0558	**-0.0250**	**0.56**
	100	0.1	0.4	0.1	0.0121	-0.0010	-0.0221	-0.0278	**-0.1186**	0.0011	0.0011	**-0.0002**	**0.01**
				0.5	0.0225	-0.0086	0.4230	0.3771	**-0.1098**	0.0099	0.0095	**-0.0016**	**0.13**
				0.8	0.0604	-0.0691	0.7609	0.5971	**-0.1076**	0.0380	0.0367	**-0.0126**	**0.53**
			0.6	0.1	0.0224	0.0000	-0.0275	-0.0334	**-0.1181**	0.0013	0.0013	**-0.0001**	**0.03**
				0.5	0.0424	-0.0197	0.4198	0.3753	**-0.1381**	0.0103	0.0100	**-0.0024**	**0.23**
				0.8	0.1211	-0.1672	0.7595	0.5978	**-0.1713**	0.0397	0.0391	**-0.0182**	**0.91**
		0.3	0.4	0.1	0.0118	-0.0011	-0.0254	-0.0309	**-0.1248**	0.0009	0.0009	**-0.0002**	**0.01**
				0.5	0.0216	-0.0035	0.4228	0.3756	**-0.0856**	0.0093	0.0089	**-0.0010**	**0.06**
				0.8	0.0535	-0.0212	0.7593	0.5934	**-0.0573**	0.0352	0.0334	**-0.0066**	**0.24**
			0.6	0.1	0.0214	-0.0018	-0.0256	-0.0312	**-0.1241**	0.0009	0.0009	**-0.0002**	**0.01**
				0.5	0.0390	-0.0070	0.4190	0.3722	**-0.0977**	0.0093	0.0089	**-0.0014**	**0.10**
				0.8	0.0995	-0.0460	0.7551	0.5899	**-0.0794**	0.0351	0.0334	**-0.0096**	**0.38**
		0.5	0.4	0.1	0.0117	-0.0006	-0.0123	-0.0178	**-0.1122**	0.0010	0.0009	**-0.0002**	**0.01**
				0.5	0.0209	-0.0026	0.4229	0.3754	**-0.0809**	0.0093	0.0088	**-0.0008**	**0.05**
				0.8	0.0507	-0.0142	0.7596	0.5933	**-0.0479**	0.0349	0.0330	**-0.0052**	**0.18**
			0.6	0.1	0.0216	-0.0015	-0.0280	-0.0336	**-0.1273**	0.0009	0.0008	**-0.0003**	**0.01**
				0.5	0.0381	-0.0053	0.4177	0.3705	**-0.0906**	0.0091	0.0087	**-0.0012**	**0.07**
				0.8	0.0959	-0.0297	0.7540	0.5880	**-0.0630**	0.0343	0.0325	**-0.0074**	**0.28**
	500	0.1	0.4	0.1	0.0024	0.0001	-0.0042	-0.0098	**-0.1041**	0.0002	0.0002	**0.0000**	**0.00**
				0.5	0.0042	-0.0006	0.4353	0.3880	**-0.0717**	0.0020	0.0019	**-0.0001**	**0.03**
				0.8	0.0111	-0.0022	0.7741	0.6081	**-0.0387**	0.0080	0.0077	**-0.0006**	**0.11**
			0.6	0.1	0.0043	0.0000	-0.0087	-0.0142	**-0.1082**	0.0002	0.0002	**0.0000**	**0.01**
				0.5	0.0078	-0.0006	0.4357	0.3888	**-0.0773**	0.0020	0.0019	**-0.0001**	**0.05**
				0.8	0.0202	-0.0077	0.7721	0.6067	**-0.0522**	0.0081	0.0078	**-0.0010**	**0.18**
		0.3	0.4	0.1	0.0024	-0.0001	-0.0115	-0.0170	**-0.1114**	0.0002	0.0002	**0.0000**	**0.00**
				0.5	0.0041	-0.0002	0.4361	0.3886	**-0.0656**	0.0020	0.0019	**0.0000**	**0.01**
				0.8	0.0105	-0.0013	0.7719	0.6054	**-0.0311**	0.0077	0.0073	**-0.0004**	**0.05**
			0.6	0.1	0.0044	0.0000	-0.0078	-0.0133	**-0.1077**	0.0002	0.0002	**0.0000**	**0.00**
				0.5	0.0079	-0.0004	0.4354	0.3880	**-0.0681**	0.0020	0.0019	**-0.0001**	**0.02**
				0.8	0.0193	-0.0020	0.7717	0.6053	**-0.0346**	0.0078	0.0074	**-0.0005**	**0.08**
		0.5	0.4	0.1	0.0022	0.0000	-0.0072	-0.0127	**-0.1071**	0.0002	0.0002	**0.0000**	**0.00**
				0.5	0.0042	-0.0002	0.4348	0.3872	**-0.0660**	0.0019	0.0018	**-0.0001**	**0.01**
				0.8	0.0100	-0.0006	0.7726	0.6060	**-0.0288**	0.0078	0.0074	**-0.0002**	**0.04**
			0.6	0.1	0.0041	-0.0001	-0.0113	-0.0168	**-0.1112**	0.0002	0.0002	**0.0000**	**0.00**
				0.5	0.0076	-0.0002	0.4365	0.3890	**-0.0652**	0.0020	0.0019	**0.0000**	**0.01**
				0.8	0.0187	-0.0014	0.7719	0.6054	**-0.0311**	0.0077	0.0073	**-0.0004**	**0.06**

					Estimators								
k	**m**	θ	**u**	I^2	$\hat{\theta}$	$\hat{\tau}^2$	$\hat{I}_1^2$	$\hat{I}_2^2$	$\hat{I}_3^2$	$\hat{I}_{10}^2$	$\hat{I}_{20}^2$	$\hat{I}_{30}^2$	τ^2
50	**50**	**0.1**	**0.4**	**0.1**	0.0255	0.0019	0.0602	0.0582	**-0.0463**	0.0026	0.0026	**-0.0002**	**0.03**
				0.5	0.0473	-0.0249	0.4708	0.4526	**-0.1044**	0.0200	0.0197	**-0.0053**	**0.26**
				0.8	0.1267	-0.2601	0.7752	0.7031	**-0.1803**	0.0696	0.0688	**-0.0404**	**1.06**
			0.6	**0.1**	0.0485	0.0121	0.0767	0.0748	**-0.0369**	0.0033	0.0033	**0.0000**	**0.05**
				0.5	0.0933	-0.0519	0.4829	0.4658	**-0.1500**	0.0217	0.0215	**-0.0077**	**0.45**
				0.8	0.2661	-0.6303	0.7779	0.7068	**-0.3190**	0.0711	0.0707	**-0.0543**	**1.81**
		0.3	**0.4**	**0.1**	0.0239	-0.0031	0.0383	0.0361	**-0.0630**	0.0017	0.0017	**-0.0006**	**0.01**
				0.5	0.0427	-0.0133	0.4489	0.4296	**-0.0704**	0.0176	0.0172	**-0.0035**	**0.12**
				0.8	0.1037	-0.0851	0.7587	0.6852	**-0.0795**	0.0621	0.0607	**-0.0229**	**0.48**
			0.6	**0.1**	0.0434	-0.0072	0.0260	0.0238	**-0.0752**	0.0015	0.0015	**-0.0009**	**0.02**
				0.5	0.0791	-0.0259	0.4454	0.4265	**-0.0930**	0.0176	0.0172	**-0.0048**	**0.19**
				0.8	0.1977	-0.1748	0.7549	0.6819	**-0.1255**	0.0615	0.0602	**-0.0322**	**0.77**
		0.5	**0.4**	**0.1**	0.0229	-0.0032	0.0326	0.0304	**-0.0679**	0.0015	0.0015	**-0.0008**	**0.01**
				0.5	0.0410	-0.0102	0.4458	0.4263	**-0.0630**	0.0172	0.0169	**-0.0030**	**0.09**
				0.8	0.0964	-0.0615	0.7555	0.6817	**-0.0622**	0.0605	0.0590	**-0.0188**	**0.36**
			0.6	**0.1**	0.0425	-0.0062	0.0208	0.0186	**-0.0797**	0.0013	0.0013	**-0.0010**	**0.02**
				0.5	0.0757	-0.0202	0.4377	0.4185	**-0.0815**	0.0168	0.0164	**-0.0042**	**0.14**
				0.8	0.1861	-0.1137	0.7513	0.6777	**-0.0890**	0.0597	0.0583	**-0.0252**	**0.56**
	100	**0.1**	**0.4**	**0.1**	0.0123	0.0007	0.0619	0.0598	**-0.0423**	0.0012	0.0012	**0.0000**	**0.01**
				0.5	0.0227	-0.0061	0.4753	0.4565	**-0.0640**	0.0101	0.0099	**-0.0014**	**0.13**
				0.8	0.0605	-0.0627	0.7875	0.7146	**-0.0857**	0.0388	0.0383	**-0.0124**	**0.53**
			0.6	**0.1**	0.0228	0.0041	0.0704	0.0684	**-0.0374**	0.0014	0.0014	**0.0000**	**0.03**
				0.5	0.0434	-0.0067	0.4881	0.4700	**-0.0840**	0.0109	0.0108	**-0.0020**	**0.23**
				0.8	0.1208	-0.1581	0.7924	0.7203	**-0.1481**	0.0408	0.0405	**-0.0185**	**0.91**
		0.3	**0.4**	**0.1**	0.0117	-0.0011	0.0456	0.0434	**-0.0552**	0.0009	0.0009	**-0.0002**	**0.01**
				0.5	0.0215	-0.0038	0.4620	0.4426	**-0.0478**	0.0093	0.0091	**-0.0010**	**0.06**
				0.8	0.0531	-0.0221	0.7758	0.7020	**-0.0419**	0.0353	0.0346	**-0.0066**	**0.24**
			0.6	**0.1**	0.0218	-0.0016	0.0466	0.0444	**-0.0551**	0.0010	0.0009	**-0.0002**	**0.01**
				0.5	0.0397	-0.0065	0.4628	0.4436	**-0.0569**	0.0094	0.0092	**-0.0013**	**0.10**
				0.8	0.0994	-0.0460	0.7745	0.7010	**-0.0622**	0.0353	0.0347	**-0.0096**	**0.38**
		0.5	**0.4**	**0.1**	0.0116	-0.0008	0.0467	0.0445	**-0.0537**	0.0009	0.0009	**-0.0002**	**0.01**
				0.5	0.0204	-0.0026	0.4622	0.4427	**-0.0422**	0.0093	0.0091	**-0.0008**	**0.05**
				0.8	0.0498	-0.0151	0.7747	0.7007	**-0.0333**	0.0349	0.0341	**-0.0052**	**0.18**
			0.6	**0.1**	0.0213	-0.0015	0.0426	0.0404	**-0.0581**	0.0009	0.0009	**-0.0002**	**0.01**
				0.5	0.0376	-0.0056	0.4571	0.4377	**-0.0526**	0.0091	0.0089	**-0.0012**	**0.07**
				0.8	0.0964	-0.0289	0.7728	0.6990	**-0.0453**	0.0347	0.0339	**-0.0072**	**0.28**
	500	**0.1**	**0.4**	**0.1**	0.0024	0.0000	0.0620	0.0598	**-0.0390**	0.0002	0.0002	**0.0000**	**0.00**
				0.5	0.0044	-0.0002	0.4783	0.4589	**-0.0298**	0.0020	0.0020	**-0.0001**	**0.03**
				0.8	0.0112	-0.0019	0.7922	0.7184	**-0.0217**	0.0081	0.0079	**-0.0006**	**0.11**
			0.6	**0.1**	0.0044	0.0004	0.0662	0.0640	**-0.0358**	0.0002	0.0002	**0.0000**	**0.01**
				0.5	0.0080	-0.0004	0.4803	0.4610	**-0.0349**	0.0020	0.0020	**-0.0001**	**0.05**
				0.8	0.0204	-0.0057	0.7931	0.7195	**-0.0331**	0.0082	0.0081	**-0.0010**	**0.18**
		0.3	**0.4**	**0.1**	0.0024	0.0000	0.0616	0.0594	**-0.0386**	0.0002	0.0002	**0.0000**	**0.00**
				0.5	0.0043	-0.0001	0.4766	0.4571	**-0.0253**	0.0020	0.0019	**0.0000**	**0.01**
				0.8	0.0108	-0.0009	0.7884	0.7144	**-0.0149**	0.0078	0.0077	**-0.0003**	**0.05**
			0.6	**0.1**	0.0043	0.0000	0.0589	0.0566	**-0.0416**	0.0002	0.0002	**0.0000**	**0.00**
				0.5	0.0076	-0.0003	0.4740	0.4544	**-0.0301**	0.0020	0.0019	**-0.0001**	**0.02**
				0.8	0.0198	-0.0016	0.7892	0.7152	**-0.0176**	0.0079	0.0077	**-0.0004**	**0.08**
		0.5	**0.4**	**0.1**	0.0024	0.0000	0.0581	0.0559	**-0.0420**	0.0002	0.0002	**0.0000**	**0.00**
				0.5	0.0041	-0.0001	0.4755	0.4559	**-0.0253**	0.0020	0.0019	**0.0000**	**0.01**
				0.8	0.0103	-0.0007	0.7878	0.7138	**-0.0136**	0.0078	0.0076	**-0.0003**	**0.04**
			0.6	**0.1**	0.0042	-0.0001	0.0547	0.0525	**-0.0455**	0.0002	0.0002	**0.0000**	**0.00**
				0.5	0.0077	-0.0001	0.4764	0.4569	**-0.0255**	0.0020	0.0019	**0.0000**	**0.01**
				0.8	0.0196	-0.0009	0.7887	0.7147	**-0.0146**	0.0078	0.0077	**-0.0003**	**0.06**

Table A.3 The standard deviations of estimators

k	m	θ	u	I^2	$\hat{\theta}$	$\hat{\tau}^2$	$\hat{I}_1^2$	$\hat{I}_2^2$	$\hat{I}_3^2$	$\hat{I}_{10}^2$	$\hat{I}_{20}^2$	$\hat{I}_{30}^2$
					Estimators							
10	**50**	**0.1**	**0.4**	**0.1**	0.0241	0.1545	0.8208	0.8370	**0.6995**	0.0122	0.0126	**0.0101**
				0.5	0.0394	0.2709	0.4783	0.4874	**0.4058**	0.0220	0.0231	**0.0158**
				0.8	0.0926	0.5767	0.2398	0.2427	**0.2219**	0.0444	0.0477	**0.0261**
			0.6	**0.1**	0.0398	0.2785	0.8550	0.8895	**0.6213**	0.0136	0.0144	**0.0099**
				0.5	0.0726	0.4735	0.5969	0.6210	**0.4298**	0.0243	0.0265	**0.0145**
				0.8	0.2002	0.9290	0.2971	0.3054	**0.2442**	0.0468	0.0519	**0.0215**
		0.3	**0.4**	**0.1**	0.0378	0.0625	0.7380	0.7414	**0.7112**	0.0103	0.0104	**0.0098**
				0.5	0.0534	0.1100	0.4489	0.4508	**0.4323**	0.0176	0.0178	**0.0159**
				0.8	0.0976	0.2430	0.1859	0.1863	**0.1820**	0.0345	0.0352	**0.0283**
			0.6	**0.1**	0.0519	0.1019	0.7727	0.7799	**0.7148**	0.0106	0.0108	**0.0097**
				0.5	0.0774	0.1783	0.4536	0.4579	**0.4179**	0.0183	0.0187	**0.0153**
				0.8	0.1572	0.3937	0.2114	0.2124	**0.2043**	0.0368	0.0381	**0.0269**
		0.5	**0.4**	**0.1**	0.0523	0.0460	0.7301	0.7315	**0.7183**	0.0099	0.0099	**0.0097**
				0.5	0.0723	0.0802	0.4055	0.4062	**0.3988**	0.0165	0.0165	**0.0157**
				0.8	0.1207	0.1766	0.1711	0.1713	**0.1689**	0.0313	0.0316	**0.0282**
			0.6	**0.1**	0.0673	0.0722	0.7696	0.7729	**0.7423**	0.0101	0.0102	**0.0096**
				0.5	0.0970	0.1252	0.4381	0.4400	**0.4221**	0.0169	0.0171	**0.0154**
				0.8	0.1761	0.2775	0.1867	0.1873	**0.1821**	0.0329	0.0336	**0.0270**
	100	**0.1**	**0.4**	**0.1**	0.0147	0.0743	0.7695	0.7767	**0.7134**	0.0058	0.0059	**0.0052**
				0.5	0.0222	0.1329	0.4505	0.4546	**0.4172**	0.0107	0.0110	**0.0087**
				0.8	0.0476	0.3166	0.2098	0.2110	**0.2012**	0.0256	0.0272	**0.0173**
			0.6	**0.1**	0.0216	0.1337	0.8058	0.8206	**0.6952**	0.0063	0.0065	**0.0052**
				0.5	0.0356	0.2360	0.4747	0.4829	**0.4093**	0.0117	0.0124	**0.0083**
				0.8	0.0944	0.5660	0.2366	0.2392	**0.2207**	0.0294	0.0325	**0.0163**
		0.3	**0.4**	**0.1**	0.0260	0.0311	0.7876	0.7893	**0.7738**	0.0052	0.0052	**0.0050**
				0.5	0.0354	0.0554	0.4154	0.4163	**0.4081**	0.0091	0.0092	**0.0086**
				0.8	0.0614	0.1340	0.1699	0.1702	**0.1675**	0.0206	0.0209	**0.0181**
			0.6	**0.1**	0.0333	0.0510	0.7896	0.7933	**0.7605**	0.0053	0.0053	**0.0050**
				0.5	0.0488	0.0906	0.4519	0.4540	**0.4348**	0.0095	0.0096	**0.0085**
				0.8	0.0915	0.2197	0.1870	0.1876	**0.1823**	0.0219	0.0226	**0.0175**
		0.5	**0.4**	**0.1**	0.0366	0.0233	0.7463	0.7470	**0.7405**	0.0051	0.0051	**0.0050**
				0.5	0.0495	0.0418	0.4057	0.4061	**0.4026**	0.0089	0.0089	**0.0086**
				0.8	0.0811	0.0982	0.1652	0.1653	**0.1640**	0.0192	0.0194	**0.0180**
			0.6	**0.1**	0.0463	0.0364	0.7557	0.7573	**0.7422**	0.0051	0.0052	**0.0050**
				0.5	0.0649	0.0658	0.4386	0.4396	**0.4307**	0.0091	0.0092	**0.0086**
				0.8	0.1114	0.1551	0.1790	0.1792	**0.1763**	0.0202	0.0205	**0.0177**
	500	**0.1**	**0.4**	**0.1**	0.0057	0.0143	0.7480	0.7494	**0.7372**	0.0011	0.0011	**0.0011**
				0.5	0.0078	0.0257	0.4060	0.4067	**0.4002**	0.0020	0.0020	**0.0019**
				0.8	0.0133	0.0653	0.1684	0.1687	**0.1663**	0.0051	0.0052	**0.0045**
			0.6	**0.1**	0.0076	0.0247	0.7198	0.7223	**0.7003**	0.0011	0.0011	**0.0011**
				0.5	0.0106	0.0442	0.4652	0.4668	**0.4525**	0.0020	0.0021	**0.0019**
				0.8	0.0203	0.1163	0.1794	0.1799	**0.1758**	0.0056	0.0058	**0.0045**
		0.3	**0.4**	**0.1**	0.0111	0.0063	0.7176	0.7179	**0.7151**	0.0011	0.0011	**0.0010**
				0.5	0.0149	0.0114	0.4297	0.4299	**0.4282**	0.0019	0.0019	**0.0019**
				0.8	0.0243	0.0278	0.1637	0.1638	**0.1632**	0.0046	0.0046	**0.0044**
			0.6	**0.1**	0.0141	0.0101	0.7365	0.7372	**0.7313**	0.0011	0.0011	**0.0010**
				0.5	0.0193	0.0181	0.4124	0.4128	**0.4095**	0.0019	0.0019	**0.0018**
				0.8	0.0319	0.0453	0.1721	0.1723	**0.1710**	0.0047	0.0048	**0.0044**
		0.5	**0.4**	**0.1**	0.0160	0.0048	0.7215	0.7217	**0.7204**	0.0010	0.0010	**0.0010**
				0.5	0.0216	0.0085	0.3973	0.3974	**0.3967**	0.0018	0.0019	**0.0018**
				0.8	0.0347	0.0210	0.1619	0.1619	**0.1616**	0.0045	0.0045	**0.0044**
			0.6	**0.1**	0.0197	0.0073	0.7560	0.7563	**0.7534**	0.0010	0.0010	**0.0010**
				0.5	0.0269	0.0131	0.4207	0.4209	**0.4192**	0.0019	0.0019	**0.0018**
				0.8	0.0436	0.0327	0.1624	0.1624	**0.1618**	0.0046	0.0046	**0.0044**

					Estimators							
k	**m**	θ	**u**	I^2	$\hat{\theta}$	$\hat{\tau}^2$	$\hat{I}_1^2$	$\hat{I}_2^2$	$\hat{I}_3^2$	$\hat{I}_{10}^2$	$\hat{I}_{20}^2$	$\hat{I}_{30}^2$
20	**50**	**0.1**	**0.4**	**0.1**	0.0171	0.1099	0.4294	0.4333	**0.3612**	0.0089	0.0090	**0.0073**
				0.5	0.0281	0.1866	0.2597	0.2616	**0.2237**	0.0158	0.0161	**0.0110**
				0.8	0.0650	0.3919	0.1224	**0.1222**	0.1367	0.0327	0.0338	**0.0179**
			0.6	**0.1**	0.0285	0.2027	0.4653	0.4733	**0.3350**	0.0102	0.0104	**0.0071**
				0.5	0.0505	0.3368	0.2854	0.2891	**0.2176**	0.0182	0.0189	**0.0101**
				0.8	0.1360	0.6098	0.1427	**0.1423**	0.1649	0.0343	0.0359	**0.0139**
		0.3	**0.4**	**0.1**	0.0266	0.0428	0.3803	0.3811	**0.3649**	0.0071	0.0072	**0.0068**
				0.5	0.0372	0.0753	0.2204	0.2208	**0.2122**	0.0122	0.0123	**0.0110**
				0.8	0.0679	0.1642	**0.0939**	**0.0939**	0.0946	0.0241	0.0243	**0.0193**
			0.6	**0.1**	0.0363	0.0705	0.3895	0.3912	**0.3575**	0.0074	0.0075	**0.0067**
				0.5	0.0553	0.1259	0.2356	0.2364	**0.2183**	0.0132	0.0134	**0.0109**
				0.8	0.1096	0.2667	**0.1073**	**0.1072**	0.1133	0.0261	0.0266	**0.0183**
		0.5	**0.4**	**0.1**	0.0374	0.0319	0.3867	0.3871	**0.3797**	0.0069	0.0069	**0.0068**
				0.5	0.0505	0.0556	0.2139	0.2141	**0.2102**	0.0116	0.0116	**0.0110**
				0.8	0.0834	0.1205	0.0867	0.0867	**0.0863**	0.0218	0.0219	**0.0194**
			0.6	**0.1**	0.0480	0.0498	0.3787	0.3795	**0.3634**	0.0070	0.0071	**0.0067**
				0.5	0.0681	0.0876	0.2201	0.2205	**0.2116**	0.0120	0.0121	**0.0108**
				0.8	0.1241	0.1889	0.0965	**0.0964**	0.0980	0.0233	0.0235	**0.0188**
	100	**0.1**	**0.4**	**0.1**	0.0102	0.0516	0.3898	0.3916	**0.3587**	0.0041	0.0041	**0.0036**
				0.5	0.0155	0.0934	0.2339	0.2348	**0.2174**	0.0076	0.0077	**0.0061**
				0.8	0.0341	0.2307	**0.1075**	**0.1075**	0.1112	0.0194	0.0199	**0.0125**
			0.6	**0.1**	0.0154	0.0955	0.4209	0.4245	**0.3594**	0.0045	0.0046	**0.0037**
				0.5	0.0257	0.1679	0.2552	0.2571	**0.2231**	0.0087	0.0089	**0.0060**
				0.8	0.0663	0.4067	**0.1236**	**0.1235**	0.1363	0.0224	0.0235	**0.0115**
		0.3	**0.4**	**0.1**	0.0180	0.0218	0.3786	0.3791	**0.3712**	0.0036	0.0036	**0.0035**
				0.5	0.0253	0.0388	0.2142	0.2144	**0.2101**	0.0064	0.0064	**0.0061**
				0.8	0.0434	0.0925	0.0890	0.0890	**0.0885**	0.0146	0.0147	**0.0127**
			0.6	**0.1**	0.0238	0.0353	0.3769	0.3778	**0.3618**	0.0037	0.0037	**0.0035**
				0.5	0.0345	0.0637	0.2195	0.2200	**0.2114**	0.0067	0.0067	**0.0060**
				0.8	0.0649	0.1518	**0.0957**	**0.0957**	0.0961	0.0156	0.0158	**0.0122**
		0.5	**0.4**	**0.1**	0.0255	0.0159	0.3548	0.3550	**0.3517**	0.0035	0.0035	**0.0034**
				0.5	0.0345	0.0291	0.2103	0.2104	**0.2084**	0.0062	0.0062	**0.0060**
				0.8	0.0576	0.0689	0.0865	0.0865	**0.0861**	0.0137	0.0137	**0.0128**
			0.6	**0.1**	0.0327	0.0254	0.3780	0.3784	**0.3706**	0.0036	0.0036	**0.0035**
				0.5	0.0457	0.0446	0.2148	0.2150	**0.2108**	0.0062	0.0063	**0.0059**
				0.8	0.0780	0.1082	0.0919	0.0920	**0.0913**	0.0143	0.0144	**0.0124**
	500	**0.1**	**0.4**	**0.1**	0.0041	0.0096	0.3752	0.3755	**0.3691**	0.0007	0.0007	**0.0007**
				0.5	0.0054	0.0174	0.2093	0.2095	**0.2061**	0.0013	0.0013	**0.0013**
				0.8	0.0096	0.0459	0.0870	0.0871	**0.0864**	0.0037	0.0037	**0.0032**
			0.6	**0.1**	0.0054	0.0170	0.3756	0.3762	**0.3642**	0.0008	0.0008	**0.0007**
				0.5	0.0076	0.0314	0.2182	0.2185	**0.2120**	0.0014	0.0015	**0.0013**
				0.8	0.0146	0.0809	0.0931	0.0932	**0.0925**	0.0040	0.0041	**0.0032**
		0.3	**0.4**	**0.1**	0.0078	0.0043	0.3750	0.3751	**0.3736**	0.0007	0.0007	**0.0007**
				0.5	0.0105	0.0078	0.2067	0.2068	**0.2060**	0.0013	0.0013	**0.0013**
				0.8	0.0171	0.0190	0.0835	0.0835	**0.0832**	0.0031	0.0032	**0.0030**
			0.6	**0.1**	0.0101	0.0070	0.3683	0.3685	**0.3654**	0.0007	0.0007	**0.0007**
				0.5	0.0139	0.0125	0.2084	0.2085	**0.2068**	0.0013	0.0013	**0.0013**
				0.8	0.0223	0.0316	0.0852	0.0852	**0.0847**	0.0033	0.0033	**0.0031**
		0.5	**0.4**	**0.1**	0.0114	0.0033	0.3654	0.3655	**0.3648**	0.0007	0.0007	**0.0007**
				0.5	0.0152	0.0058	0.2040	0.2040	**0.2037**	0.0013	0.0013	**0.0013**
				0.8	0.0243	0.0146	0.0840	0.0840	**0.0839**	0.0031	0.0031	**0.0031**
			0.6	**0.1**	0.0141	0.0051	0.3641	0.3642	**0.3627**	0.0007	0.0007	**0.0007**
				0.5	0.0188	0.0091	0.2065	0.2065	**0.2057**	0.0013	0.0013	**0.0013**
				0.8	0.0307	0.0224	0.0822	0.0823	**0.0820**	0.0032	0.0032	**0.0030**

k	m	θ	u	I^2	$\hat{\theta}$	$\hat{\tau}^2$	$\hat{I}_1^2$	$\hat{I}_2^2$	$\hat{I}_3^2$	$\hat{I}_{10}^2$	$\hat{I}_{20}^2$	$\hat{I}_{30}^2$
					Estimators							
50	**50**	**0.1**	**0.4**	**0.1**	0.0107	0.0694	0.2407	0.2415	**0.2029**	0.0056	0.0057	**0.0046**
				0.5	0.0174	0.1199	0.1427	0.1430	**0.1273**	0.0102	0.0103	**0.0070**
				0.8	0.0407	0.2381	0.0627	0.0626	**0.0821**	0.0209	0.0212	**0.0109**
			0.6	**0.1**	0.0178	0.1280	0.2585	0.2601	**0.1899**	0.0065	0.0066	**0.0045**
				0.5	0.0321	0.2179	0.1591	0.1597	**0.1310**	0.0122	0.0123	**0.0065**
				0.8	0.0844	0.3591	0.0668	**0.0665**	0.1090	0.0217	0.0221	**0.0082**
		0.3	**0.4**	**0.1**	0.0167	0.0271	0.2077	0.2079	**0.1991**	0.0045	0.0045	**0.0043**
				0.5	0.0240	0.0468	0.1200	0.1201	**0.1158**	0.0077	0.0077	**0.0069**
				0.8	0.0432	0.1025	**0.0517**	**0.0517**	0.0534	0.0153	0.0154	**0.0121**
			0.6	**0.1**	0.0235	0.0447	0.2196	0.2200	**0.2012**	0.0047	0.0047	**0.0043**
				0.5	0.0340	0.0786	0.1293	0.1295	**0.1210**	0.0083	0.0083	**0.0068**
				0.8	0.0700	0.1656	0.0578	**0.0577**	0.0654	0.0167	0.0168	**0.0114**
		0.5	**0.4**	**0.1**	0.0230	0.0196	0.1984	0.1984	**0.1946**	0.0043	0.0043	**0.0042**
				0.5	0.0319	0.0347	0.1154	0.1155	**0.1134**	0.0073	0.0073	**0.0069**
				0.8	0.0532	0.0737	**0.0469**	**0.0469**	0.0475	0.0135	0.0136	**0.0120**
			0.6	**0.1**	0.0306	0.0314	0.2141	0.2143	**0.2050**	0.0045	0.0045	**0.0043**
				0.5	0.0425	0.0540	0.1207	0.1208	**0.1163**	0.0075	0.0075	**0.0067**
				0.8	0.0767	0.1155	**0.0519**	**0.0519**	0.0542	0.0145	0.0146	**0.0116**
	100	**0.1**	**0.4**	**0.1**	0.0065	0.0329	0.2194	0.2198	**0.2015**	0.0026	0.0026	**0.0023**
				0.5	0.0099	0.0593	0.1295	0.1297	**0.1215**	0.0049	0.0049	**0.0039**
				0.8	0.0216	0.1448	0.0586	**0.0585**	0.0646	0.0125	0.0127	**0.0079**
			0.6	**0.1**	0.0097	0.0605	0.2359	0.2367	**0.2015**	0.0029	0.0029	**0.0024**
				0.5	0.0163	0.1126	0.1435	0.1438	**0.1293**	0.0058	0.0059	**0.0040**
				0.8	0.0413	0.2530	0.0653	**0.0651**	0.0832	0.0146	0.0148	**0.0071**
		0.3	**0.4**	**0.1**	0.0114	0.0134	0.2039	0.2040	**0.1997**	0.0022	0.0022	**0.0022**
				0.5	0.0159	0.0238	0.1160	0.1160	**0.1138**	0.0040	0.0040	**0.0037**
				0.8	0.0275	0.0576	0.0491	0.0491	**0.0492**	0.0092	0.0092	**0.0080**
			0.6	**0.1**	0.0152	0.0220	0.2078	0.2079	**0.1991**	0.0023	0.0023	**0.0022**
				0.5	0.0218	0.0395	0.1209	0.1210	**0.1167**	0.0042	0.0042	**0.0037**
				0.8	0.0408	0.0956	**0.0534**	**0.0534**	0.0550	0.0100	0.0100	**0.0077**
		0.5	**0.4**	**0.1**	0.0161	0.0101	0.2008	0.2008	**0.1989**	0.0022	0.0022	**0.0022**
				0.5	0.0221	0.0178	0.1126	0.1126	**0.1116**	0.0038	0.0038	**0.0037**
				0.8	0.0361	0.0422	0.0468	0.0468	**0.0467**	0.0085	0.0085	**0.0079**
			0.6	**0.1**	0.0205	0.0158	0.2058	0.2059	**0.2016**	0.0023	0.0023	**0.0022**
				0.5	0.0282	0.0275	0.1155	0.1155	**0.1133**	0.0039	0.0039	**0.0037**
				0.8	0.0495	0.0669	**0.0490**	**0.0490**	0.0492	0.0090	0.0090	**0.0078**
	500	**0.1**	**0.4**	**0.1**	0.0025	0.0060	0.1992	0.1993	**0.1958**	0.0005	0.0005	**0.0004**
				0.5	0.0035	0.0111	0.1151	0.1151	**0.1133**	0.0009	0.0009	**0.0008**
				0.8	0.0061	0.0283	0.0479	0.0479	**0.0477**	0.0023	0.0023	**0.0020**
			0.6	**0.1**	0.0034	0.0109	0.2074	0.2075	**0.2008**	0.0005	0.0005	**0.0005**
				0.5	0.0048	0.0195	0.1183	0.1184	**0.1150**	0.0009	0.0009	**0.0008**
				0.8	0.0091	0.0524	**0.0519**	**0.0519**	0.0523	0.0026	0.0026	**0.0020**
		0.3	**0.4**	**0.1**	0.0049	0.0027	0.1981	0.1981	**0.1973**	0.0005	0.0005	**0.0004**
				0.5	0.0067	0.0049	0.1122	0.1123	**0.1118**	0.0008	0.0008	**0.0008**
				0.8	0.0108	0.0121	0.0456	0.0456	**0.0454**	0.0020	0.0020	**0.0019**
			0.6	**0.1**	0.0063	0.0044	0.2023	0.2023	**0.2006**	0.0005	0.0005	**0.0005**
				0.5	0.0086	0.0080	0.1138	0.1138	**0.1129**	0.0008	0.0008	**0.0008**
				0.8	0.0141	0.0196	0.0458	0.0458	**0.0457**	0.0021	0.0021	**0.0019**
		0.5	**0.4**	**0.1**	0.0072	0.0020	0.1977	0.1977	**0.1974**	0.0004	0.0004	**0.0004**
				0.5	0.0096	0.0037	0.1101	0.1101	**0.1099**	0.0008	0.0008	**0.0008**
				0.8	0.0152	0.0090	0.0442	0.0442	**0.0441**	0.0019	0.0019	**0.0019**
			0.6	**0.1**	0.0088	0.0032	0.2022	0.2022	**0.2014**	0.0005	0.0005	**0.0004**
				0.5	0.0120	0.0056	0.1095	0.1095	**0.1091**	0.0008	0.0008	**0.0008**
				0.8	0.0195	0.0141	0.0452	0.0452	**0.0451**	0.0020	0.0020	**0.0019**

Appendix III: R-code for the case studies

```
##--------------------------------------------------------
## Function for estimation
##--------------------------------------------------------
f <- function(tp, fp, tn, fn){
#NOTE:  tp=yi, fn=mi-yi, tn=xi, fp=ni-xi
#Note:  mi=tp+fn, ni=tn+fp
tp <- ifelse(tp != 0, tp, tp+0.5)                #if tp=0, tp+0.5
tp <- ifelse(tp != tp+fp, tp, tp-0.5)
tn <- ifelse(tn != 0, tn, tn+0.5)
tn <- ifelse(tn != tn+fn, tn, tn-0.5)
fp <- ifelse(fp != 0, fp, fp+0.5)
fn <- ifelse(fn != 0, fn, fn+0.5)
k <- length(tp)                                  #number of the studies
mi <- tp+fn
ni <- tn+fp
phat.i <- tp/mi                                  #the sensitivity of the study i
uhat.i <- 1-(tn/ni)                              #the false-positive rate of the study i
sigma2hat.i <- ((1/mi)*((1-phat.i)/(phat.i*(log(phat.i))^2)))+((1/ni)*((1-
uhat.i)/(uhat.i*(log(uhat.i))^2)))
sigma2hat.i0 <- ((1-phat.i)/(phat.i*(log(phat.i))^2))+((1-uhat.i)/(uhat.i*(log(uhat.i))^2))
w.hat.i <- 1/sigma2hat.i
w.hat.i0 <- 1/sigma2hat.i0
thetahat.i <- log(phat.i)/log(uhat.i)
log.thetahat.i <- log(thetahat.i)
logtheta.hat <- (sum(w.hat.i*log.thetahat.i))/(sum(w.hat.i))
theta.hat <- exp(logtheta.hat)                                     #---estimator: theta.hat
chi2 <- sum(w.hat.i*(log.thetahat.i-logtheta.hat)^2)
tau2.hat <- (chi2-(k-1))/(sum(w.hat.i)-(sum(w.hat.i^2))/(sum(w.hat.i)))        #---estimator:
tau2.hat

sigma2.bb1 <- (sum(w.hat.i*(k-1)))/((sum(w.hat.i))^2-(sum(w.hat.i^2)))
sigma2.bb2 <- k/(sum(w.hat.i))
sigma2.bb3 <- mean(sigma2hat.i)
sigma2.bb10 <- (sum(w.hat.i0*(k-1)))/((sum(w.hat.i0))^2-sum(w.hat.i0^2))
sigma2.bb20 <- k/(sum(w.hat.i0))
sigma2.bb30 <- mean(sigma2hat.i0)

i2.hat1 <- tau2.hat/(tau2.hat+sigma2.bb1)                          #---estimator: I2.hat1
i2.hat2 <- tau2.hat/(tau2.hat+sigma2.bb2)                          #---estimator: I2.hat2
i2.hat3 <- tau2.hat/(tau2.hat+sigma2.bb3)                          #---estimator: I2.hat3
i2.hat10 <- tau2.hat/(tau2.hat+sigma2.bb10)                        #---estimator: I2.hat10
i2.hat20 <- tau2.hat/(tau2.hat+sigma2.bb20)                        #---estimator: I2.hat20
i2.hat30 <- tau2.hat/(tau2.hat+sigma2.bb30)                        #---estimator: I2.hat30

#cat("tp=",tp,"\n","tn=",tn,"\n","fp=",fp,"\n","fn=",fn,"\n","mi=",mi,"\n","ni=",ni,"\n")
cat("theta.hat=",theta.hat,"\t","tau2.hat=",tau2.hat,"\n")
cat("sigma2.bb1=",sigma2.bb1,"\t","sigma2.bb2=",sigma2.bb2,"sigma2.bb3=",sigma2.bb3,"\n
")
cat("sigma2.bb10=",sigma2.bb10,"\t","sigma2.bb20=",sigma2.bb20,"sigma2.bb3=",sigma2.b
b30,"\n")
cat("i2.hat1=",i2.hat1,"\t","i2.hat2=",i2.hat2,"i2.hat3=",i2.hat3,"\n")
cat("i2.hat10=",i2.hat10,"\t","i2.hat20=",i2.hat20,"i2.hat30=",i2.hat30,"\n")
} #end main program
```

```
##------------------------------------------------------------
## Data: heart failure
##------------------------------------------------------------
tp <- c(29,34,49,4,20,29,26,11)
fp <- c(19,13,17,85,71,166,11,50)
tn <- c(46,22,78,1612,1339,102,75,93)
fn <- c(7,6,9,6,40,2,14,1)
result.heart <- f(tp, fp, tn, fn)
##------------------------------------------------------------
## Data: dementia
##------------------------------------------------------------
tp <- c(65,117,48,134,24,67,64,281,13,262,143,183,22,112,152,29,31,10,707,181,
59,74,27,40,317,387,118,44,123,25,73,37,78,72,106,37,67,17)
fp <- c(240,10,63,28,44,48,1,20,44,29,29,33,152,590,126,26,3,12,1438,17,23,16,26,75,
173,16,1,34,98,3,2,1,45,53,410,22,22,1)
tn <-
c(870,110,989,152,292,153,71,286,286,177,123,51,140,2091,1009,236,247,333,10447,
184,74,143,209,528,578,54,44,396,309,171,225,440,376,214,379,118,75,90)
fn <-
c(3,12,19,8,5,15,17,64,1,20,18,33,1,1,81,26,6,3,88,108,29,23,12,6,52,116,65,7,46,43,32,45,
34,12,23,36,30,77)
result.dementia <- f(tp, fp, tn, fn)


##------------------------------------------------------------
## Data: mood MA
##------------------------------------------------------------
tp <- c(65,70,62,36,55,6,121,11,6,85,15,96)
fp <- c(104,74,27,65,43,12,80,5,0,9,4,23)             #contains zero
tn <- c(1192,846,429,474,392,144,720,76,3,460,42,187)
fn <- c(26,13,10,5,11,8,103,5,5,31,1,10)
result.mood <- f(tp, fp, tn, fn)
```