High Dimensional Codebook Design for the SCMA Down-Link

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***Abstract*— An efficient high-dimensional codebook design is conceived for sparse code multiple access (SCMA) systems. This generalized technique has the compelling benefit that its power-efficiency is monotonically increased with its dimensionality. A striking further practical benefit is that this increased power-efficiency is achieved without increasing the per-symbol detection complexity.**

I. Introduction

In order to support large-scale connectivity in the forthcoming fifth generation mobile communication networks, non-orthogonal multiple access (NOMA) techniques may be advocated. As an important NOMA candidate, the sparse code multiple access (SCMA) technique has attracted remarkable attention in recent years.

In more detail, in order to support more users than the number of chips in a code division multiple access (CDMA) system, we have to sacrifice the spreading code's orthogonality, which results in a rank-deficient system. A compelling compromise was struck between the robustness and complexity in a rank-deficient CDMA system by Hoshyar *et al.* [1] upon combining their novel low-density signature (LDS) scheme with the message passing algorithm (MPA) of [2]. The LDS scheme relies on beneficial user-specific sparse spreading sequences, which is synonymous with designing a good signature matrix[1] in conjunction with the conventional phase-shift keying (PSK) modulation scheme. However, as reported in [3], incorporating a more sophisticated constellation design into the LDS system has the potential of significantly improving the power-efficiency without increasing the detection complexity, which results in the SCMA scheme. Naturally, the specific choice of both the signature matrix and of the modulation constellation become a pair of critical factors that dominate the performance of a SCMA system. The joint optimization of these two factors constitutes part of the "SCMA codebook design" process.

For example, Yang *et al.* [4] focused their attention on the signature matrix design, while Bao *et al.* [5], [6] concentrated their research efforts on the constellation design. As a further advance, Zhou *et al.* [7] and Peng *et al.* [8] aimed for the joint design of these two factors. However, to the best of our knowledge, no publications are available in the open literature on the "high-dimensional" SCMA codebook design.

Against this background and inspired by [9], the above-mentioned pair of design factors are beneficially replaced by a single design factor by designing a "decimal signature matrix", which simplifies our codebook design process. More explicitly, a high-dimensional codebook design concept is proposed for SCMA systems, which constitutes a generalized technique of monotonically increasing the power-efficiency upon increasing the signature-matrix dimensionality by invoking "Latin Rectangular" matrices as our "decimal signature matrix". Finally, this bneficial monotonically increased power-efficiency improvement is achieved without substantially increasing the system's per-symbol detection complexity.

The rest of this paper is organized as follows. Our system model is introduced in Section II, while our SCMA codebook design is discussed in Section III. Finally, several SCMA codebook design examples obeying the proposed design guidelines are provided in Section IV, where their performance is also characterised.

II. System model

As mentioned in Section I, we focus our attention on the down-link of a SCMA system. The entire network is portrayed in Fig. 1. Consequently, during a SCMA down-link transmission block, a binary user-specific signature of $\mathbf{s}_u = \left[s_u^1, s_u^2, \ldots, s_u^{\Omega}\right]^T, s_u^{\omega} \in (0,1)$ is assigned by the Base Station (BS) to the u^{th} active mobile user (MU) for the sake of spreading the modulated symbol of the u^{th} MU over all the available channel-resources. The number of orthogonal channel-resources (CR) assigned to the SCMA down-link, such as the time-slots or sub-carriers, is denoted by Ω. However, owing to the sparsity of $\mathbf{s}_u$, only N of the entire set of Ω CRs are actually occupied by the u^{th} MU, i.e. the number of nonzero elements within the signature $\mathbf{s}_u$ is only N, which may be significantly lower than Ω.

Then, we invoke an N-dimension constellation $\mathbf{c}_u = \left[c_u^1, c_u^2, \ldots, c_u^N\right]^T$ for representing a codeword pertaining to the user-specific SCMA codebook $\mathscr{C}_u$ of the u^{th} MU, i.e. we have $\mathbf{c}_u \in \mathscr{C}_u$.

Accordingly, the above spreading process invoked for the u^{th} MU at the BS can be formulated as

$$\mathbf{x}_u = \Delta_{\cancel{0cols}}\Big[\mathrm{diag}\big(\mathbf{s}_u\big)\Big]\mathbf{c}_u, \tag{1}$$

where the operation $\Delta_{\cancel{0cols}}\big[\mathbf{A}\big]$ omits all the zero-columns[2] of the matrix $\mathbf{A}$, hence we have $\Delta_{\cancel{0cols}}\Big[\mathrm{diag}\big(\mathbf{s}_u\big)\Big] \in \mathbb{C}^{\Omega \times N}$ and $\mathbf{x}_u \in \mathbb{C}^{\Omega \times 1}$. The operation $\mathrm{diag}(\mathbf{B})$ casts the vector $\mathbf{B}$ to a diagonal matrix.

Then, the BS superimposes all the temporary codewords $\big\{\mathbf{x}_u\big\}_{u=1}^{U}$ for generating the final modulated symbol, which will be broadcast from the BS and written as

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[1]The signature matrix is constituted by employing every user-specific sparse spreading sequence as one of its column.

[2]Herein, the column which only has zero-valued elements is termed as a zero-column.

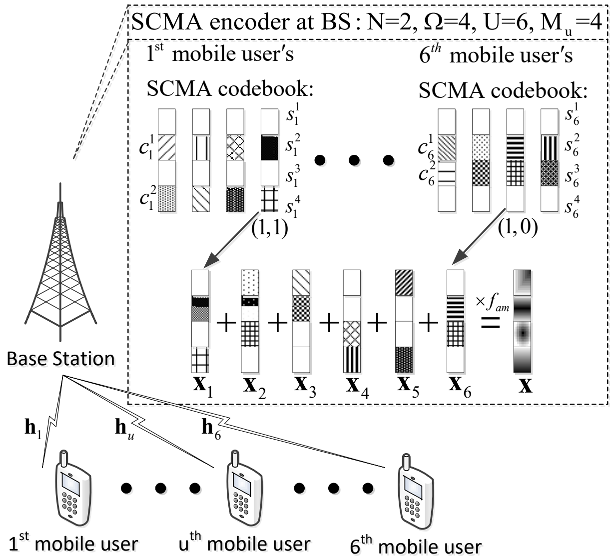

Fig. 1: System model of the SCMA downlink spanning a single transmission block, where the SCMA encoding process implemented at the BS is visualized. A configuration of $N = 2, \Omega = 4, U = 6, M_u = 4$ is adopted as an example.

$$f_{\rm am} = \sqrt{\frac{\sum_{u=1}^{U} P_u}{E\left(\left\|\sum_{u=1}^{U} \mathbf{x}_u\right\|^2\right)}},$$
$$\mathbf{x} = f_{\rm am} \sum_{u=1}^{U} \Delta_{\theta cols}\Big[\mathrm{diag}\big(\mathbf{s}_u\big)\Big]\mathbf{c}_u. \quad (2)$$

In the first row of (2), the amplification factor $f_{\rm am}$ is introduced for constraining the average power of the final modulated symbols to the total available power of all the U MUs, where P_u represents the available power allocated to the u^{th} MS and $E(\cdot)$ represents the expectation operation.

On the other hand, during the same SCMA down-link transmission block, the associated signal vector received by the u^{th} MU is given by

$$\mathbf{y}_u = \mathbf{h}_u \cdot \mathbf{x} + \mathbf{n}_u, \quad (3)$$

where $\mathbf{h}_u = \left[\mathbf{h}_u^1, \mathbf{h}_u^2, \ldots, \mathbf{h}_u^{\Omega}\right]^T$, its element $\mathbf{h}_u^{\omega}$ denotes the channel impulse response experienced by ω^{th} channel resources employed in this down-link transmission. Then, $\mathbf{n}_u = \left[\mathbf{n}_u^1, \mathbf{n}_u^2, \ldots, \mathbf{n}_u^{\Omega}\right]^T$ is the complex-valued additive white Gaussian noise (AWGN) vector imposed on the u^{th} MU, whose elements obey an independent and identical distribution of $\mathcal{CN}(0, \sigma^2)$. Moreover, "$\cdot$" in (3) represents an element-wise multiplication.

III. SCMA Codebook Design

The final modulated Ω-dimensional symbol $\mathbf{x}$ given in (2) carries all the information intended for the down-link MU population. The properties of the SCMA down-link transmission will be substantially affected by the particular construction of $\mathbf{x}$. Hence, we regard the alphabet of $\mathbf{x}$, namely $\mathscr{X}$, as the eventual codebook of the SCMA down-link transmission. With this spirit in mind, we observe in (2) again that, the critical factors determining $\mathscr{X}$ can be intuitively categorized into two aspects: a) the set of user-specific signatures $\left\{\mathbf{s}_u\right\}_{u=1}^{U}$; b) the set of user-specific SCMA codebooks $\left\{\mathbf{c}_u\right\}_{u=1}^{U}$.

In order to holistically characterise the effects of the entire user-specific signature set $\left\{\mathbf{s}_u\right\}_{u=1}^{U}$, as well as to adapt it to the message passing algorithm, the concept of the signature matrix $\mathbf{S} = \left[\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_U\right]$ was introduced in both the LDS scheme [1] and in the SCMA scheme [3]. According to (2), the down-link codebook could be constructed by first distributing the non-zero elements across this signature matrix $\mathbf{S}$. Then, each M-ary user-specific codebook $\mathscr{C}_u$ is simply generated based on a classical M-ary PSK constellation, where the N-fold Cartesian product of the m^{th} PSK symbol itself is directly employed as the m^{th} codeword of $\mathscr{C}_u$. In fact, this is right the strategy employed by LDS schemes to construct their user-specific codebooks. For the sake of convenience, a user-specific codeword of this codebook is represented as $\mathbf{c}_{u\text{-}LDS}$. Then, the classical M-ary PSK constellation is regarded as the "mother constellation". By contrast, if more sophisticated lattice based constellations are employed for generating the user-specific codebooks, we arrive at the SCMA scheme. Hence, in general, a SCMA user-specific codeword could be regarded as a LDS codeword multiplied by a specific N-dimensional generator vector of $\mathbf{g}_u$ as follows

$$\mathbf{c}_u = \mathbf{g}_u \cdot \mathbf{c}_{u-\rm LDS}. \quad (4)$$

Then upon substituting (4) into (2), a codeword of the SCMA codebook is rewritten as

$$\mathbf{x} = f_{\rm am} \sum_{u=1}^{U} \Delta_{\theta cols}\Big[\mathrm{diag}\big(\mathbf{s}_u\big)\Big]\left[\mathbf{g}_u \cdot \mathbf{c}_{u-\rm LDS}\right],$$
$$= f_{\rm am} \sum_{u=1}^{U} \Delta_{\theta cols}\Big[\mathrm{diag}\big(\hat{\mathbf{s}}_u\big)\Big]\mathbf{c}_{u-\rm LDS}, \quad (5)$$

where the new decimal spreading sequence $\hat{\mathbf{s}}_u$ encapsulates the impact of $\mathbf{g}_u$, which is imposed by consecutively replacing the N nonzero elements of $\mathbf{s}_u$ by the N elements of $\mathbf{g}_u$. For example, if $\mathbf{s}_u = [0, 1, 1, 0]^T$, $\mathbf{g}_u = \left[g_u^1, g_u^2\right]^T$, then we have $\hat{\mathbf{s}}_u = \left[0, g_u^1, g_u^2, 0\right]^T$. After defining the decimal spreading sequence $\hat{\mathbf{s}}_u$, we further invoke the decimal signature matrix of $\hat{\mathbf{S}} = \left[\hat{\mathbf{s}}_1, \hat{\mathbf{s}}_2, \ldots, \hat{\mathbf{s}}_U\right]$.

Observe in (5) that the decimal signature matrix $\hat{\mathbf{S}} \in \mathbb{C}^{\Omega \times U}$ is capable of encapsulating all the critical factors[3] that dominate the performance of a SCMA down-link codebook $\mathscr{X}$. As a benefit, we no longer have to simultaneously conceive both the binary signature matrix $\mathbf{S}$ and the user-specific codebooks $\left\{\mathbf{c}_u\right\}_{u=1}^{U}$. Instead, we equivalently transform the SCMA codebook design to the design of a better decimal signature matrix.

[3]Please bear in mind that $\mathbf{c}_{u-\rm LDS}$ is simply a N-fold Cartesian product of a conventional modulation symbol, e.g. a QPSK symbol.

A. High-Dimensional Design

According to lattice theory [10], higher coding and shaping gains become possible by increasing the dimensionality of the lattice based modulation constellation. This tendency was explicitly witnessed in [11, Fig.2-5]. The SCMA down-link codebook $\mathscr{X}$ may also be viewed as a multi-dimensional constellation. Hence, it inspires us to increase the dimensionality of the SCMA down-link codebook $\mathscr{X}$ to be designed. On the other hand, let η be the normalized user-load of a SCMA system, which is characterised[4] by the ratio of $\eta = \frac{U}{\Omega}$ [3]. Hence, at a fixed η, increasing the dimensionality of $\mathscr{X}$ is equivalent to proportionally enlarging the size of the decimal signature matrix $\hat{\mathbf{S}}$.

For example, in the original SCMA codebook designs of [3], [12], a **binary** signature matrix having 4 rows and 6 columns is utilized for illustration. Most of the ensuing literature on the subject of SCMA codebook design [4], [5] inherited this 4×6 configuration as their signature matrix size. By contrast, we proposed to extend the size of the **decimal** signature matrix $\hat{\mathbf{S}}$, e.g. to 6×9, or further to 8×12, and so on for the sake of achieving improved coding and shaping gains. At the receiver, the signature matrix of a SCMA system will be employed as the parity check matrix (PCM) in the associated message passing detection algorithm. This detection process is extremely similar to that of a LDPC code. According to the principles of designing a good LDPC code [13] , a large PCM size will facilitate constructing a good Tanner graph, which results in a high girth and a high minimum Hamming distance. This also implies that extending the size of the SCMA signature matrix improves its performance.

B. Low Complexity Design

The potential benefits introduced in Section III-A motivate us to employ a high-dimensional SCMA codebook design, which is equivalent to employing a large signature matrix. However, recall that the number of columns in the signature matrix is identical to that of the MUs involved in a SCMA down-link transmission block. Hence a larger signature matrix implies that more information bits have to be carried by a SCMA down-link transmission block. Consequently, the detection complexity is increased.

In more detail, if the message passing algorithm proposed in [1], [3] is employed again as our detection algorithm, the detection complexity becomes proportional to

$$\mathcal{C} \propto \mathcal{O}\left(\Omega \cdot d_f \cdot \left[M^{d_f}\right] + U \cdot d_v \cdot [M]\right), \qquad (6)$$

where d_f denotes the weight of a function node involved in the MPA, which equals to the number of nonzero elements in a row of the signature matrix. Furthermore, d_v denotes the weight of a variable node, which equals to the number of nonzero elements in a column of the signature matrix.

The average computational complexity required by detecting an information symbol of a single MU is termed as the "complexity per user". Based on this definition and on the relationship of $\Omega \cdot d_f = U \cdot d_v$, we have

$$\mathcal{C}_{\text{per-user}} \propto \mathcal{O}\left(d_v \cdot \left[M^{d_f}\right] + d_v \cdot [M]\right). \qquad (7)$$

According to (7), it is clear that upon fixing d_f and d_v, as well as satisfying $\frac{d_v}{d_f} = \frac{\Omega}{U}$, the "complexity per user" remains the same, while we simultaneously increase the number of rows and columns of the signature matrix by the same factor.

During our design, we firstly specify a small signature matrix, which has a regular weight distribution, such as that of the 4×6 signature matrix used in [3], [12]. Then, its weight distribution will be adopted again in the dimension-extension procedure introduced in Section III-A for the sake of fixing the detection complexity to make a fair comparison.

C. "Latin Rectangular" Matrix Based Signature Design

In Sections III-A and III-B, we have specified how to configure both the size and weight distribution of the signature matrix. Then, the principle of constructing a good LDPC PCM, which attempts to avoid low-girth cycles in the associated Tanner graph is relied upon again for determining the locations of the nonzero elements within the signature matrix. Hence the last step of completing the design of the decimal signature matrix $\hat{\mathbf{S}}$ is that of assigning the specific value of its every nonzero element. This process is termed as "signature labelling".

Fortunately, we find that the signature labelling method proposed in [9], which was again inherited in [7], [14] adapts very well to our high-dimensional decimal signature matrix. In more detail, the value of each nonzero element in $\hat{\mathbf{S}}$ will always be taken from a finite complex-valued set $\mathcal{A} = \{a_k, k = 0, 1, \ldots, d_f - 1\}$, where a_k is calculated as follows

$$a_k = \exp\left(j\frac{2\pi}{P}k\right), \quad P = M \cdot d_f. \qquad (8)$$

In (8), M is the specific number of symbols included in the "mother constellation" of (4). Then, while assigning the value of $\{a_k\}_0^{d_f-1}$ to the nonzero elements of $\hat{\mathbf{S}}$, it obeys the "Latin Rectangular" principle[5] of [9].

D. Guideline and Example of Our Design

Our high-dimensional SCMA codebook design method introduced before is concisely summarized as follows:

1 Assume that the size of our decimal signature matrix is initialized to $\hat{\mathbf{S}} \in \mathbb{C}^{\Omega \times U}$, where the user-load of the SCMA system and the weight distributions are fixed to η, d_f, d_v, respectively. The relationship of $\eta = \frac{U}{\Omega} = \frac{d_f}{d_v}$ should be satisfied.

2 We construct a length U $(d_v, d_f)-$regular LDPC parity-check matrix, where the low-girth cycles have been omitted. Then, the positions of nonzero elements

[4]It is assumed in this paper that a single layer, i.e. a single column of the signature matrix is assigned to a MU.

[5]If a matrix obeys the properties that a) all of its rows or columns are uniquely distinguishable from each other; b) any nonzero element of this matrix will no longer occur more than once in the same row or column, we say that this matrix has a "Latin Rectangular" form [15].

in this parity-check matrix are copied to our signature matrix $\hat{\mathbf{S}}$.

3 A set of specific values $\{a_k\}_0^{d_f-1}$ is calculated according to (8), which is assigned to the nonzero elements of $\hat{\mathbf{S}}$. This "signature labelling" process should obey the "Latin Rectangular" principle.

4 If a higher power-efficiency is required, we could further extend the size of $\hat{\mathbf{S}}$ by a factor of k, and then repeat the previous Steps 1-3 again for generating a new decimal signature matrix of $\hat{\mathbf{S}}' \in \mathbb{C}^{\Omega \cdot k \times U \cdot k}$. During this construction procedure of $\hat{\mathbf{S}}'$, if the parameters of η, d_f, d_v remain fixed to their previous value, the user-load capability and detection complexity per symbol of the SCMA system will not be impaired.

For instance, according to the above design guidelines, if we want to enhance the power-efficiency of our SCMA system, we can rely on extending the size of $\hat{\mathbf{S}}$ from 4×6 to 6×9, and then further to 8×12. If the normalized user-load is fixed to $\eta = \frac{U}{\Omega} = 150\%$, the resultant decimal signature matrices are given in (9)-(10). Observe in (9)-(10) that according to the "low-complexity design" principle of Section III-B, the weight distribution of $\hat{\mathbf{S}}_{4,6}$ results in $d_f \equiv 3, d_v \equiv 2$, which is adopted again for constructing $\hat{\mathbf{S}}_{6,9}$ and $\hat{\mathbf{S}}_{8,12}$. Then, implanting the appropriately chosen nonzero elements in these positions of $\hat{\mathbf{S}}_{4,6}$, $\hat{\mathbf{S}}_{6,9}$ and $\hat{\mathbf{S}}_{8,12}$ allows us to avoid girth-4 cycles in their Tanner graphs. Finally, according to the "signature labelling" principle introduced in Section III-C, we have $a_0 = 1, a_1 = \exp\left(j\frac{\pi}{6}\right)$, and $a_2 = \exp\left(j\frac{\pi}{3}\right)$, when considering a conventional QPSK constellation as the "mother constellation". Explicitly, the matrices given in (9)-(10) are all "Latin-Rectangular" matrices.

IV. Simulation Results

The SCMA codebook design principles of Section III are now employed in our simulations for the construction of the decimal signature matrices $\hat{\mathbf{S}}$. Then, we expect that the resultant performance would indeed reflect the benefits of our codebook design.

The network of Fig 1 is considered in our simulations. Since we focus our attention on the SCMA codebook design, we will invoke neither channel coding nor sophisticated power allocation. Again, MPA-based SCMA detection similar to that of [1] is used at the receiver by each MU. The transmit power is uniformly shared amongst the U MUs. All other simulation parameters are listed in Table I.

The symbol error probability (SEP) achieved by employing different decimal signature matrices $\hat{\mathbf{S}}$ is quantified in Fig.2, where we firstly consider AWGN channels. Explicitly, increasing the dimensionality of $\hat{\mathbf{S}}$ by obeying the guidelines stated in Section III is capable of attaining a constant performance gain. For example, a round 0.5 dB performance gain is attained at the SEP target of 10^{-4} upon replacing $\hat{\mathbf{S}}_{4,6}$ by $\hat{\mathbf{S}}_{6,9}$, and then, an additional 1 dB performance gain is achieved by further extending $\hat{\mathbf{S}}_{6,9}$ to $\hat{\mathbf{S}}_{8,12}$.

A similar phenomenon is observed again in Fig.3 for uncorrelated Rayleigh fading channels upon assuming perfect channel state information, where the same signature

Number of MUs	$6 \rightarrow 9 \rightarrow 12$
Orthogonal Channel Resources	$4 \rightarrow 6 \rightarrow 8$
Normalized User-Load	$\eta = 1.5$
Weight Distribution	$d_f = 3, d_v = 2$
Signature Matrix	"Latin-Rectangular"
Channel Model	AWGN or Rayleigh
Number of Samples	$2 \cdot 10^6$ Transmission Blocks
SNR	P_u/N_0
Mother Constellation	QPSK
Detection Algorithm	MPA

TABLE I: System Parameters

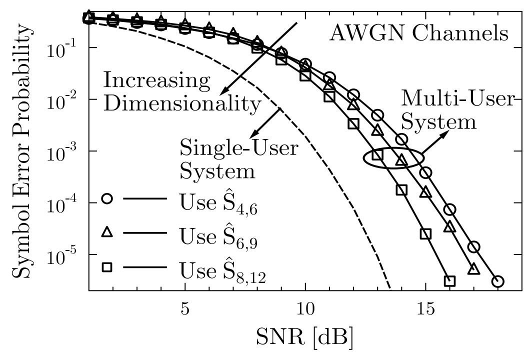

Fig. 2: SER vs SNR performance of employing different dimensionality signature matrices of $\hat{\mathbf{S}}$ over AWGN channels, while using the parameters of Table I. Furthermore, the performance of a single-user system employing classic QPSK modulation is also shown.

matrices and the SCMA detection algorithm used in Fig.2 are adopted.

Finally, the impact of increasing the dimensionality of $\hat{\mathbf{S}}$ on the computational complexity of the detection process is visualized in Fig.4, where the number of multiplications required for detecting a single information symbol is employed as our complexity metric. Observe in Fig.4 that the system complexity associated with a signature matrix $\hat{\mathbf{S}}$ is always reduced upon increasing the SNR value, because the log-likelihood ratio (LLR) values generated during the MPA will exceed the predefined threshold in fewer iterations. Hence the MPA procedure will be completed by invoking fewer iterations. Furthermore the system complexity remains near-constant upon increasing the dimensionality of the associated signature matrix from 4 to 8. This result is in line with our previous complexity analysis formulated in (7). Hence, by jointly assessing the experimental results of Fig.2 and Fig.4, we may claim that the proposed scheme is capable of improving the power-efficiency of SCMA without increasing its detection complexity [6].

[6] In the down-link transmission, the other users' information symbols $\{\mathbf{s}_k\}_{k=1, k\neq u}^{U}$ are discarded at the u^{th} MU. In this context, the power gain of the proposed scheme is obtained at the cost of linearly increasing the system's complexity.

$$\hat{\mathbf{S}}_{4,6} = \begin{pmatrix} 0 & a_0 & a_2 & 0 & a_1 & 0 \\ a_2 & 0 & a_0 & 0 & 0 & a_1 \\ 0 & a_1 & 0 & a_2 & 0 & a_0 \\ a_1 & 0 & 0 & a_0 & a_2 & 0 \end{pmatrix}, \hat{\mathbf{S}}_{6,9} = \begin{pmatrix} 0 & 0 & a_0 & 0 & 0 & 0 & a_1 & 0 & a_2 \\ 0 & a_2 & 0 & a_1 & 0 & 0 & a_0 & 0 & 0 \\ 0 & 0 & a_1 & a_2 & 0 & a_0 & 0 & 0 & 0 \\ a_1 & 0 & 0 & 0 & a_2 & 0 & 0 & 0 & a_0 \\ a_2 & 0 & 0 & 0 & 0 & a_1 & 0 & a_0 & 0 \\ 0 & a_1 & 0 & 0 & a_0 & 0 & 0 & a_2 & 0 \end{pmatrix}, \quad (9)$$

$$\hat{\mathbf{S}}_{8,12} = \begin{pmatrix} 0 & 0 & 0 & a_0 & a_2 & 0 & 0 & 0 & 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 & 0 & 0 & 0 & a_0 & 0 & 0 & a_1 & 0 \\ a_1 & a_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_0 & 0 & 0 \\ 0 & a_1 & 0 & 0 & 0 & 0 & a_2 & 0 & 0 & 0 & 0 & a_0 \\ 0 & 0 & 0 & 0 & 0 & a_0 & 0 & 0 & a_1 & a_2 & 0 & 0 \\ 0 & 0 & a_0 & 0 & a_1 & a_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_0 & a_1 & a_2 & 0 & 0 & 0 \\ a_0 & 0 & a_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_2 & 0 \end{pmatrix}. \quad (10)$$

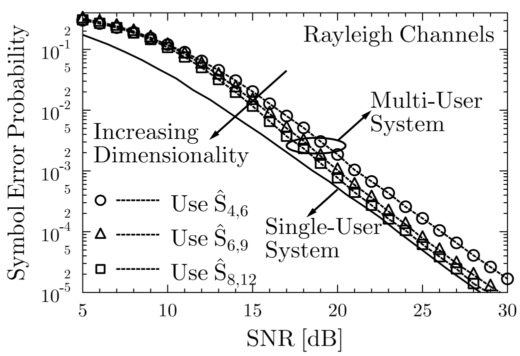

Fig. 3: SER vs SNR performances upon employing different-dimensionality signature matrices of $\hat{\mathbf{S}}$ over uncorrelated Rayleigh fading channels. The perfect channel state information is assumed and the parameters of Table I are used. Furthermore, the performance of a single-user system employing classic QPSK modulation is also shown.

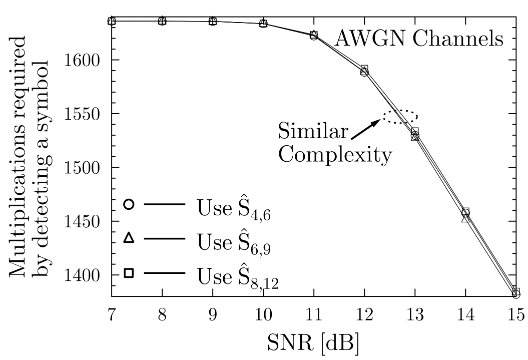

Fig. 4: The computational complexity per detected symbol vs SNR in an AWGN communication scenario using the parameters of Table I. Observe that the complexity remains unaffected by the size of $\hat{\mathbf{S}}$.

V. Conclusion

Recursive decimal signature matrix construction was proposed for SCMA systems. Explicitly, a high-dimensional low-complexity "Latin-Rectangular" matrix based signature design was conceived, which exhibited an SNR-gain upon increasing the size of the signature matrix without increasing the per-symbol detection complexity.

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