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AN INVERSE METHOD FOR THE IDENTIFICATION OF THE RADIATION RESISTANCE MATRIX FROM MEASURABLE ACOUSTIC AND STRUCTURAL RESPONSES

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The control of structurally radiated noise is becoming more important as vehicles and machinery must become lighter and quieter. Provided that the radiated sound field is known, an Active Structural Acoustic Control (ASAC) system can be implemented to reduce the level of radiated noise when there is a low weight requirement. However, it is often not possible to measure the radiated sound field directly with acoustic sensors and so indirect sensing of the radiated sound power is required. Previously, the radiation resistance matrix, which can be used to estimate the radiated sound power from structural variables alone, has been calculated theoretically for simple structures such as a flat plate in an infinite baffle. However, it is not straightforward to calculate this matrix for more complex practical structures due to the difficulty in obtaining accurate models. To overcome this limitation, a method that is able to estimate the radiation resistance matrix from measurable responses on practical structures is required. In this paper, a method to calculate the radiation resistance matrix using measurable structural and acoustic responses is presented. The presented method requires a series of measurements to be taken on the structure and in the radiated sound field when the structure is excited by different force distributions and these responses are then used to formulate an inverse problem. The accuracy of the solution to this inverse problem is investigated via comparison with the theoretical radiation resistance matrix for a flat plate in an infinite baffle. Through this comparison it has been shown that the accuracy of the solution to the inverse problem depends on the number of structural and acoustic sensors and structural forces used in the identification process.
Keywords: Radiation resistance matrix, Indirect sensing, Active Structural Acoustic Control (ASAC).

1. Introduction

There is an ever increasing requirement to develop technologies and designs that make vehicles and machinery lighter and, therefore, more fuel efficient. These lightweight structures are often more efficient radiators of sound and, therefore, traditional noise and vibration control treatments become impractical due to their weight requirements. Active Structural Acoustic Control (ASAC) has proven to be an effective means of reducing structurally radiated sound without a significant weight requirement and, in its most straightforward form, can be implemented using structural control actuators to minimise the sound pressure measured at error microphones located in the radiated sound field [1, 2]. However, it is often not practical to position error microphones in the radiated sound field and this has led to the development of more sophisticated indirect ASAC systems. These systems aim to control the radiated sound field using only structural error sensors and a model or operating function that estimates the radiated sound from structural measurements [2, 3, 4].

A number of different approaches to estimating the radiated sound power from a structure using only structural measurements have been proposed for the purposes of ASAC. This includes a number of related methods that are based on calculating the radiation modes of the structure, which can be described by the radiation resistance matrix [3, 5] or the Power Tranfer Matrix [6]. Recent alternative methods have also been developed, including the weighted sum of spatial gradients method [7], which allows a reduced number of structural sensors to be utilised in comparison to other approaches. Previous methods, such as the radiation matrix method described in [5], often rely on certain assumptions about the radiating structure and the use of analytical or numerical models. This, however, leads to potential difficulties for complex practical structures that cannot be readily modelled or approximated in this way.

To overcome this limitation, several attempts have been made to develop methods to experimentally identify the radiation resistance matrix [8] or the power transfer matrix [9]. In [8] the radiation resistance matrix is measured experimentally using a bespoke measurement device, referred to as a resistance probe, which generates a known volume velocity from a calibrated loudspeaker. Due to the need for the resistance probe to be small so as not to significantly modify the structural response, this method has a low frequency limit of around 200 Hz and also requires a tedious measurement process where the impedance is measured for multiple points on the structure sequentially by moving the resistance probe. More recently, a method of identifying the radiation resistance matrix using responses measured between different force excitations and both structural measurements and radiated acoustic pressures using an inverse identification procedure has been proposed [10, 11]. This method has been shown to be effective, however, due to the use of pressures to calculate the radiated sound power, it relies on the assumption that the radiating structure is located in a free-field environment and the pressure measurements are taken in the far-field. To overcome this limitation, a method of experimentally identifying the related power transfer matrix using experimental measurements of the structural and acoustic pressure and particle velocity under different structural force excitations has been presented [9]. This method relies on a parametric modelling method that estimates the linear time invariant model that matches the power transfer matrix identified non-parametrically via the solution of an inverse problem. This inverse identification process is only discussed briefly and the limits are not fully explored. Therefore, focusing on the identification of the radiation resistance matrix from measurable data, this paper explores the requirements in terms of the number of input forces and output structural and acoustic measurements required. In addition, it is shown that rather than measuring the acoustic particle velocity, it is possible to use the structural velocity and the near-field acoustic pressure in the identification process, which avoids the need for a acoustic particle velocity sensor.

The structure of this paper is as follows; Section 2 introduces the theoretical radiation resistance for a flat plate in an infinite baffle using the elemental method. Section 3 introduces the proposed method of identifying the radiation resistance matrix from measurable variables. In Section 4 the radiation resistance matrix identified via the solution of the inverse problem is compared to the theoretical radiation resistance matrix and this enables the practical limits on the identification procedure to be investigated. Finally, Section 5 draws conclusions based on the results obtained in the study.

2. Theoretical formulation of the radiation resistance matrix

In this section, the radiation resistance matrix is defined to provide a reference against which the proposed identification method can be compared. To derive the radiation resistance matrix, the structure is divided into a grid of elemental radiators, as shown in Fig. 1 for a flat rectangular plate in an infinite baffle. Since the radiation resistance matrix can be analytically calculated for this classical structure, as shown in [2, 5], this will be used to investigate the practical requirements and limitations of the proposed identification method.

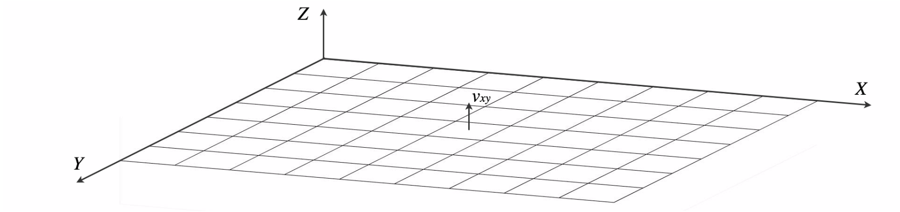


Figure 1: Diagram of the modelled rectangular plate uniformly divided into a grid of elemental radiators.

The radiation resistance matrix can be formulated by assuming that the sound radiation from the structure can be approximated by a number of elemental radiators that each have a complex surface velocity, v, and immediately above each element a corresponding acoustic pressure, p. The vector of pressures above each elemental radiator can then be related to the vector of elemental velocities as

$$\mathbf{p} = \mathbf{Z}\mathbf{v}, \tag{1}$$

where $\mathbf{Z}$ is the matrix of specific acoustic impedances, which incorporates the point and transfer impedance terms over the grid of elements. The acoustic radiation from each structural element can be approximated as a monopole radiating into the half-space defined by the infinite baffle [5] and the impedance between the i-th and j-th elements of the matrix $\mathbf{Z}$ can thus be defined as

$$Z_{ij}(\omega) = \frac{j\omega\rho_0 A_e}{2\pi R_{ij}} e^{-jkR_{ij}}, \tag{2}$$

where ρ_0 is the density of air, k is the wavenumber and R_{ij} is the distance between the centres of the i-th and j-th elements.

The sound power radiated by the structure can be expressed in terms of the vectors of pressures and structural velocities as

$$W = \left(\frac{A_e}{2}\right) \mathbb{R}\text{e}[\mathbf{v}^H\mathbf{p}], \tag{3}$$

where A_e is the area of each element. Assuming that the impedance matrix, $\mathbf{Z}$, is symmetric due to acoustic reciprocity, the radiated sound power can be re-written in terms of the structural velocities alone by substituting eq. 1 into eq. 3 and expanding for the real and imaginary parts, which gives

$$W = \left(\frac{A_e}{2}\right) \mathbb{R}\text{e}\{\mathbf{v}^H\mathbf{Z}\mathbf{v}\} = \left(\frac{A_e}{4}\right) \mathbf{v}^H[\mathbf{Z} + \mathbf{Z}^H]\mathbf{v} = \left(\frac{A_e}{2}\right) \mathbf{v}^H\mathbf{R}\mathbf{v}, \tag{4}$$

where $\mathbf{R}$ is the radiation resistance matrix, defined as $\mathbf{R} = \mathbb{R}\text{e}\{\mathbf{Z}\}$ [2, 5]. From the last part of eq. 4, it can be seen that the radiation resistance matrix can be readily used to estimate the radiated sound power from a structure using only structural velocities. However, it does require the impedance matrix, $\mathbf{Z}$, to be determined and this is potentially not straightforward for complex practical structures where the elemental radiation cannot be accurately approximated by a monopole or a baffled piston. This means that the majority of published work that uses the radiation resistance matrix is based on classical structures for which there are analytical models. That said, it is possible to use a numerical model, such as a finite element model, to model these more complex structures, however, the accuracy of this approach for the purposes of active control of radiated noise is not expected to provide sufficient accuracy.

3. Inverse identification of the radiation resistance matrix

The radiation resistance matrix completely describes the motion of a structure [2], however, previous attempts to identify this matrix experimentally have either required extremely time-consuming measurements [8] or made assumptions about the acoustic environment in which the structure is placed [11]. To overcome these limitations, an identification method is proposed here that builds on the power transfer matrix identification method outlined in [9], but does not require measurements of the acoustic particle velocity and does not make the assumption that the primary force distribution is used in the identification process.

In the proposed method, the radiation resistance matrix is estimated for a practical structure using a series of structural and near-field acoustic pressure measurements that are taken when the structure is excited by a distribution of independent excitation forces. The relationship between the measured structural velocities, and the radiated acoustic pressure can be expressed as

$$\mathbf{p} = \widetilde{\mathbf{H}}_p \mathbf{v} \tag{5}$$

where $\widetilde{\mathbf{H}}_p$ is the matrix of transfer responses that describe the relationship between the vector of structural velocities, $\mathbf{v}$, and the radiated acoustic pressures, $\mathbf{p}$. It is not possible to directly measure these transfer responses, as the elemental structural velocities cannot be driven individually and are only controllable via a transfer response matrix [9]. This is distinct from the method described in [8], where an independent volume velocity source is utilised so that the vibration from individual elements can be approximated. However, this is a time-consuming process and its accuracy is somewhat limited at low resistance levels [8]. Therefore, it is proposed here that these transfer response matrices are instead estimated via the solution of an inverse problem [12].

The structural velocity distribution can be expressed in terms of the force distribution and a transfer response matrix as

$$\mathbf{v} = \mathbf{H}_s \mathbf{f}, \tag{6}$$

where $\mathbf{H}_s$ is a measurable quantity that is defined as the transfer responses between the vector of structural velocities, $\mathbf{v}$, and the force distribution vector, $\mathbf{f}$. By substituting eq. 6 into eq. 5, the vector of acoustic pressures can then be expressed as

$$\mathbf{p} = \widetilde{\mathbf{H}}_p \mathbf{H}_s \mathbf{f}, \tag{7}$$

and the transfer response matrix $\mathbf{H}_p$, which is measurable and relates the acoustic pressures to the force distribution, can be defined as

$$\mathbf{H}_p = \widetilde{\mathbf{H}}_p \mathbf{H}_s. \tag{8}$$

Provided that the structural response matrix, $\mathbf{H}_s$, is square, i.e. when the number of structural velocity measurements is equal to the number of force excitations, and non-singular, $\widetilde{\mathbf{H}}_p$ can be calculated by solving the inverse problem presented by eq. 8 directly as

$$\widetilde{\mathbf{H}}_p = \mathbf{H}_p \mathbf{H}_s^{-1}. \tag{9}$$

However, if the number of force excitations is not equal to the number of sensors, then the inverse problem must be solved in a least-squares sense using the appropriate pseudo-inverse, $\mathbf{H}_s^{\dagger}$.

Assuming that the transfer response matrix $\widetilde{\mathbf{H}}_p$ has now been identified, the radiated sound power can be expressed by substituting eq. 5 into eq. 3 to give

$$W = \left(\frac{A_e}{2}\right) \mathbb{R}\mathrm{e}\{\mathbf{v}^H \widetilde{\mathbf{H}}_p \mathbf{v}\}. \tag{10}$$

This can be expanded in terms of the real and imaginary parts and simplified to

$$W = \left(\frac{A_e}{4}\right) \mathbf{v}^H [\widetilde{\mathbf{H}}_p^H + \widetilde{\mathbf{H}}_p] \mathbf{v} = \left(\frac{A_e}{2}\right) \mathbf{v}^H \hat{\mathbf{R}} \mathbf{v}, \tag{11}$$

where the estimated radiation resistance matrix is defined as

$$\hat{\mathbf{R}} = \mathbb{R}\mathrm{e}\{\widetilde{\mathbf{H}}_p\}. \tag{12}$$

It is important at this point to highlight that, in comparison to the previous related work [9], the need for acoustic particle velocity measurements has been avoided by making the assumption that the surface velocity of the structure is equal to the acoustic particle velocity immediately above the structure, which is consistent with the radiation resistance matrix formulation [2, 5]. In addition, in the previous work [9], the primary disturbance that is to be controlled by the ASAC system was included in the force distribution used during the system identification process. This is potentially rather restricting for practical applications where the primary disturbance may change depending on the operational conditions and, therefore, the power transfer matrix, or radiation resistance matrix derived under these conditions may not provide a complete and general description of the radiation properties of the structure. In the present study, it has not been assumed that prior knowledge of the primary disturbance is available during the system identification and, therefore, the primary disturbance has not been included in the determination of the estimated radiation resistance matrix, $\hat{\mathbf{R}}$.

4. Simulation based investigation of the radiation resistance matrix estimation method

In this section the estimated radiation resistance matrix, $\hat{\mathbf{R}}$, is investigated in simulation and compared to the theoretically derived radiation resistance matrix. To facilitate this comparison, the radiation resistance matrix has been calculated for a rectangular baffled plate using an array of 10×8 elemental radiators to model a 0.414 m $\times$ 0.314 m plate. This arrangement ensures that there are at least 8 structural velocity evaluation points over the shortest acoustic wavelength when the plate is excited up to 1 kHz and, therefore, spatial aliasing is avoided. In order to compare this radiation resistance matrix to the estimated value, it is necessary to define the radiation efficiency, σ, which is given as [5]

$$\sigma = W/\rho_0 c_0 S_T < \bar{v}^2(t) > \tag{13}$$

where c_0 is the speed of sound, S_T is the total surface area of the radiator and $< \bar{v}^2(t) >$ is the space averaged mean-square structural velocity. For each radiating mode of the plate, the relative radiation efficiency can be calculated from the eigenvalues of the radiation resistance matrix [13] and this allows a convenient comparison of the theoretical and estimated matrices under different identification conditions.

In addition to the radiation efficiencies, the directly evaluated sound power is also used to assess the accuracy of the sound power estimated using the estimated radiation resistance matrix. The sound power is calculated directly using the particle velocity, u, and acoustic pressures over a 10×8 grid evaluated 0.1 m above the surface of the plate as,

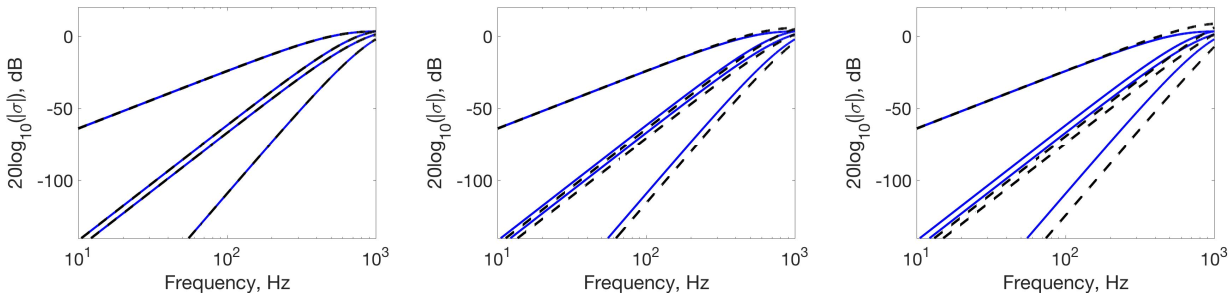
$$W = (A_e/2)\,\mathbb{R}\mathrm{e}[\mathbf{u}^H \mathbf{p}], \tag{14}$$

where $\mathbf{u}$ is the vector of acoustic particle velocities evaluated over the grid.

In the following, the sensitivity of the radiation resistance matrix estimation to the force-sensor arrangement will be investigated in order to demonstrate how the identification process can be carried out in practice. In each case, the radiation resistance matrix has been estimated according to eq. 12.

Initially, the number of forces used in the identification of the radiation resistance matrix has been kept equal to the number of acoustic pressures and structural velocities so that the identification relies on the solution of the inverse problem defined by eq. 9. The number of acoustic pressures, structural velocities and colocated forces used to identify the radiation resistance matrix has then been decreased incrementally in order to assess the resulting effects on the estimation accuracy.

Figure 2 shows the normalised radiation efficiencies of the first four modes calculated according to the radiation resistance matrix, $\mathbf{R}$, and the estimated radiation resistance matrix, $\hat{\mathbf{R}}$. In the first instance, Fig. 2a shows the radiation efficiencies when $\hat{\mathbf{R}}$ has been identified using the 10×8 grid of pressures, structural velocities and forces. From this plot it can be seen that the radiation efficiencies calculated according to the radiation resistance matrix estimated via the proposed inverse method are identical to those for the theoretical radiation resistance matrix under this condition. This confirms in principle that the proposed estimation procedure using near-field acoustic pressures, structural velocities and colocated forces, with a sufficiently high spatial resolution, can provide an accurate estimate of the radiation resistance matrix.

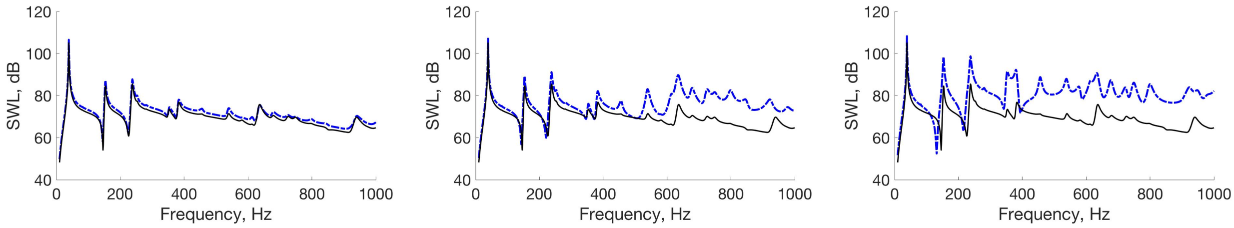


(a) $10 \times 8\ \approx (8/\lambda)$ (b) $4 \times 3\ \approx (3/\lambda)$ (c) $2 \times 2\ \approx (1.5/\lambda)$

Figure 2: The four largest eigenvalues of the radiation resistance matrix, $\mathbf{R}$, (solid blue) plotted along with the four largest eigenvalues of the estimated radiation resistance matrix, $\hat{\mathbf{R}}$, (dashed black) when the number of acoustic pressures, structural velocities and forces used to identify the PTM are reduced.

In practice, it would be beneficial in terms of both cost and complexity to be able to estimate the radiation resistance matrix accurately using a minimum number of sensors and forces. To explore this limit, Figs. 2b and 2c show the radiation efficiencies when the radiation resistance matrix has been estimated using 3 and 1.5 sensors and forces per the shortest acoustic wavelength respectively. From these two results it can be seen that as the number of identification points per wavelength is reduced, the accuracy of the estimated radiation efficiencies also reduces. This is particularly clear at higher frequencies where, for the radiation efficiency corresponding to the first radiation mode, it can be seen that the radiation efficiency is over estimated.

To provide further insight into the accuracy of the radiation resistance estimation procedure, Fig. 3 shows the radiated sound power directly evaluated when the plate is excited by a plane wave incident from $45°$ in the horizontal plane and $45°$ in the vertical plane, along with the sound power calculated using the estimated radiation resistance matrix and an array of structural velocity measurements as the number of evaluation and force points used in the identification process are decreased. From Fig. 3a it can be seen that with 8 forces and evaluation positions per acoustic wavelength at 1 kHz, the sound power estimated using the PTM is accurate over the full presented frequency range. However, as the number of forces and evaluation positions decreases, the accuracy of the sound power level estimation decreases, particularly at higher frequencies where the spatial resolution compared to the acoustic wavelength is more limited. It is important to note that although the accuracy of the sound power level is reduced, the modal frequencies are still identified when there are 3 force and sensors per the shortest acoustic wavelength and, therefore, the estimated radiation resistance matrix may still be sufficient for control applications where it is the reduction in the level that is of interest rather than the absolute level.



(a) $10 \times 8 \ \approx (8/\lambda)$ (b) $4 \times 3 \ \approx (3/\lambda)$ (c) $2 \times 2 \ \approx (1.5/\lambda)$

Figure 3: The sound power estimate calculated using the estimated radiation resistance matrix, $\hat{\mathbf{R}}$, (dashed blue) plotted along with the directly evaluated sound power (solid back) when the number of acoustic pressures, structural velocities and forces used to identify the radiation resistance matrix are reduced.

5. Conclusions

The estimation of the radiated sound power from a structure using only structural measurements is key to the implementation of practical active structural acoustic control systems. Although a variety of methods have been developed to allow the estimation of the radiated sound power from structural measurements, they have not been widely applied to complex practical structures. Therefore, in this paper a method of estimating the radiation resistance matrix from measurable acoustic and structural responses has been proposed.

The presented radiation resistance matrix estimation method is based on the solution of an inverse problem that is used to estimate the matrix of transfer responses between the structural velocities and acoustic pressures. This method requires measurements of the responses between an array of structural forces and both structural velocities and near-field acoustic pressures. The distinction between this identification approach and those previously presented in the literature have been identified and can be summarised as follows: in comparison to [11], there is no requirement for the identification to be conducted in a free-field acoustic environment and far-field pressure measurements are not required; in comparison to [9] there is no requirement for the measurement of the acoustic particle velocity and it has not been assumed that the force distribution due to the primary disturbance is used in the identification process.

To provide an initial assessment of the limits on the accuracy of the proposed estimation method, a series of simulations of a rectangular baffled plate have been conducted and the theoretical radiation resistance matrix has been compared to the radiation resistance matrix estimated using the proposed method with different numbers of identification forces and acoustic pressure and structural velocity measurements per acoustic wavelength. Under these different identification conditions, the accuracy of the estimated radiation resistance matrix has firstly been compared to the theoretical radiation resistance matrix via the modal radiation efficiencies. It has been shown that for 8 identification forces and sensors per wavelength at the upper frequency of interest, the modal radiation efficiencies are accurately estimated. However, as the number of forces and sensors per wavelength at the upper frequency of interest is reduced, the accuracy of the estimate decreases. To provide further insight into the accuracy of the estimated radiation resistance matrix, it has been used to estimate the radiated sound power when the baffled plate is excited by a force distribution that is produced by an incident plane wave. These results again show that the identification procedure is accurate when the number of forces and sensors used in the identification procedure is 8 per wavelength at the upper frequency of interest and the accuracy of the estimated sound power level drops off as the number of forces and sensors is reduced. However, it is highlighted that with 3 forces and sensors per the shortest acoustic wavelength, the modal frequencies are still identified accurately and, therefore, this may provide sufficient accuracy for ASAC applications.

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