Transmit Antenna Combination Optimization for Generalized Spatial Modulation Systems

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Abstract—Generalized Spatial Modulation (GSM), where both the Transmit Antenna Combination (TAC) index and the Amplitude Phase Modulation (APM) symbols convey information, is a novel low-complexity and high efficiency Multiple Input Multiple Output (MIMO) technique. In Conventional GSM (C-GSM), the legitimate TACs are selected randomly to transmit the APM symbols. However, the number of the TACs must be a power of two, hence the excess TACs are discarded, resulting in wasting some resource. To address these issues, in this paper, an optimal TAC set-aided Enhanced GSM (E-GSM) scheme is proposed, where the optimal TAC set is selected with the aid of the Channel State Information (CSI) by maximizing the Minimum Euclidean Distance (MED). Furthermore, a Hybrid Mapping GSM (HM-GSM) scheme operating without CSI knowledge is investigated, where the TAC selection and bit-to-TAC mapping are both taken into consideration for optimizing the Average Hamming Distance (AHD). Finally, an Enhanced High Throughput GSM (E-HT-GSM) scheme is developed, which makes full use of all the TACs. This scheme is capable of achieving an extra one bit transmission rate per time slot. Moreover, rotated phase is employed and optimized for the reused TACs. Our simulation results show that the proposed E-GSM system and HM-GSM system are capable of outperforming the C-GSM system. Furthermore, the E-HT-GSM system is capable of obtaining one extra bit transmission rate per time slot compared to the C-GSM system.

Index Terms—Generalized Spatial Modulation (GSM), Transmit Antennas Combination (TACs), Average Hamming Distance (AHD).

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<th>Symbol</th>
<th>Description</th>
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<tr>
<td>B</td>
<td>Transmission rate</td>
</tr>
<tr>
<td>$B_1$</td>
<td>The number of bits that one TAC index carried</td>
</tr>
<tr>
<td>$B_2$</td>
<td>The number of bits that APM symbols carried</td>
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<tr>
<td>$C_{N_t}$</td>
<td>The total number of TACs</td>
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<td>$C_{C-GSM}$</td>
<td>The complexity of C-GSM system</td>
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<td>$C_{E-GSM}$</td>
<td>The complexity of optimal TAC selection</td>
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<td>$D(\bar{\varsigma})$</td>
<td>The value of AHD</td>
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<td>The value of PEP</td>
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<tr>
<td>$H$</td>
<td>The channel matrix</td>
</tr>
<tr>
<td>$I_q$</td>
<td>The q-th TAC set</td>
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<tr>
<td>$I_{\text{all}}$</td>
<td>The TAC set including all the TACs</td>
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<td>$I_j$</td>
<td>The j-th TAC</td>
</tr>
<tr>
<td>$M$</td>
<td>The modulation order of APM</td>
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<tr>
<td>$N_t$</td>
<td>The total number of TAs</td>
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<tr>
<td>$N_a$</td>
<td>The number of activated TAs</td>
</tr>
<tr>
<td>$N_{\text{all}}$</td>
<td>The total number of TACs</td>
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<td>$N$</td>
<td>The number of one legitimate TAC set</td>
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<td>The number of discarded TACs</td>
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<tr>
<td>$N_r$</td>
<td>The number of receiver antennas</td>
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<tr>
<td>$n$</td>
<td>The noise matrix</td>
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<tr>
<td>$P_b$</td>
<td>The ABEP of GSM system</td>
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<td>$y$</td>
<td>The receive signal</td>
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I. INTRODUCTION

GENERALIZED Spatial Modulation (GSM) [1]-[2] is a novel low-complexity high-efficiency scheme relying on a reduced number of Radio Frequency (RF) chain...
early contributions on GSM have been mainly focused on the low-complexity receiver design [12]-[19], on the performance and achieve rate analysis [20]-[22], and on their applications for specific communication scenarios [23]. Specifically, the authors of [12] proposed a low-complexity near-Maximum Likelihood (ML) detector. The detectors of [13]-[16] exhibited a considerably reduced complexity, since the sparsity of the activated antennas was exploited based on the classic Compressive Sensing (CS) algorithms. The threshold-aided CS detector of [16] is capable of striking a better trade-off between the performance and complexity than other detection schemes. Moreover, the detector of [17] employed both forward error correction codes and turbo equalization to achieve the highest possible coding gain. Additionally, the low-complexity GSM conceived receiver design for dispersive channels have been investigated in [18] and [19]. On the other hand, the capacity of GSM system was analyzed in [20]-[22] and the achievable rate was quantified in [21]. Since its high capacity and energy efficiency, GSM is a promising candidate for millimeter-wave communications [23]. However, the above research did not consider the Channel State Information (CSI) at the transmitter.

Recently, some transceiver designs of SM-based systems were developed in [24]-[32], as shown in Table I. Moreover, the CSI based link adaptation, antenna selection-aided distributed/cooperative protocols and precoding were investigated in [33]-[40] for SM, as shown in Table II. However, to the best of our knowledge how to select the TAC of GSM has not been investigated to date. Against the above background, the major contributions of this paper are summarized as follows:

1) An Enhanced GSM (E-GSM) system is proposed, where the TAC set is selected by exploiting the knowledge of the CSI based on maximizing the Minimum Euclidean Distance (MED). A low-complexity optimal TAC selection algorithm is developed for the E-GSM system. Our simulation results show that the proposed E-GSM scheme is capable of providing considerable performance gain over the Conventional GSM (C-GSM) system.

2) A Hybrid Mapping based GSM (HM-GSM) system is introduced and a low-complexity TAC selection algorithm is proposed. In Section IV, a hybrid bit-to-TAC mapping method dispensing with CSI knowledge is proposed for the HM-GSM system to achieve a better ABEP. In Section V, an E-HT-GSM scheme is proposed, where all the TACs are exploited and the phase optimization as well as its performance analysis is offered in Section V. Section VI presents the simulation results, while Section VI concludes this paper. For reader’s convenience, the full table of contents is include here.

Notation: $\| \cdot \|_F$ denotes the Frobenious norm of a matrix; $| \cdot |$ represents the magnitude of a complex quantity; $(\cdot)^T$ and $(\cdot)^H$ stand for the transpose and the Hermitian transpose of a vector/matrix, respectively.

II. CONVENTIONAL GSM

In C-GSM systems, $N_u$ out of $N_t$ ($N_u < N_t$) TAs are activated for data transmission, so that $CN_u$ legitimate TACs are available, and $N = 2^{\lceil \log_2(CN_u) \rceil}$ TACs are discarded, hence resulting in wasting some resource. Therefore, how to choose the optimal TAC set and exploiting the remaining TACs become challenging research problems.
It follows from Eq. (2) that, the optimal ML-based demodulator can be formulated as

$$\hat{(I, S)}_{ML} = \arg \min_{I \in \mathcal{I}, S \in \mathcal{S}} \| y - H_I S \|_F^2,$$  

(3)

where $\mathcal{I} = \{I_1, I_2, ..., I_N\}$ is the set of TAC, and $\mathcal{S}$ is the set of $N_u$-element symbol vector.

III. TRANSMIT ANTENNA COMBINATION SELECTION FOR THE GSM SYSTEM RELYING ON CHANNEL STATE INFORMATION

In the C-GSM system, $N$ legitimate TACs are selected randomly from $C_{N_t}^{N_u}$, whose performance can be further improved by selecting the optimal TAC set. In this section, an optimal TAC set based E-GSM system is proposed, where $N$ legitimate TACs are obtained utilizing the CSI instead of selecting them randomly based on maximizing the MED of GSM system. Then, a low-complexity optimal algorithm is introduced.

A. Optimal TAC Set Based E-GSM

For the C-GSM system having $N_t$ TAs and $N_u$ activated TAs, there are a total of $N_{\text{all}} = C_{N_t}^{N_u}$ TACs, and $N$ out of $N_{\text{all}}$ TACs are selected randomly. Hence, there are $C_{N_{\text{all}}}^N$ possible TAC sets $\mathcal{I}_q, q = (1, ..., C_{N_{\text{all}}}^N)$. In the C-GSM system, a specific TAC set is selected randomly from $\{1, ..., I_{N_{\text{all}}}^{N_t} \}$. In this section, the optimal TAC set having the largest MED is selected from the $C_{N_{\text{all}}}^N$ TAC sets, as shown in Fig. 1. More specifically, for each TAC set $\mathcal{I}_q$, the MED of the specific channel matrix $H$ between the two GSM symbols $x_m$ and $x_n$ can be defined as [31]

$$d_{\text{min}}^q = \min_{x_m \neq x_n} \| H_{I_m} x_m - H_{I_n} x_n \|_F^2.$$  

(4)

where $I_m = (m_1, m_2, ..., m_{N_u})$ and $I_n = (n_1, n_2, ..., n_{N_u})$ denote two TACs. For $C_{N_{\text{all}}}^N$ TAC sets we have $C_{N_{\text{all}}}^N$ MEDs and the optimal TAC set index having the largest MED can be obtained as

$$\hat{q} = \arg \max_{q \in \{1, 2, ..., C_{N_{\text{all}}}^N\}} (d_{\text{min}}^q).$$  

(5)

Then the optimal TAC set for the specific channel matrix is $\mathcal{I}_{\hat{q}}$. In the optimal TAC set selection, we have to calculate $C_{N_{\text{all}}}^N$ MEDs. For each MED, $NM_{N_u}$ Euclidean distances have to be calculated to get the minimal one. Hence, a total of $C_{N_{\text{all}}}^N \cdot NM_{N_u}$ Euclidean Distances (ED) should be computed for the optimal TAC set selection. In fact, there are too many MEDs that are repeatedly calculated. Hence, the optimal TAC set selection can be simplified by computing each ED only once. Assuming that the set $\mathcal{I}_q$ contains all the TACs as $\mathcal{I}_q = \{I_1, ..., I_{N_{\text{all}}}, I_{N_{\text{all}}}\}$, each TAC corresponding to $MN_u$ GSM symbols, the set $\mathcal{X}_{\text{all}}$ containing $N_{\text{all}} \cdot MN_u$ possible GSM vectors is expressed as

$$\mathcal{X}_{\text{all}} = \{x_1, ..., x_{MN_u}, ..., x_{N_{\text{all}} \cdot MN_u}\}.$$  

(6)

Then, the ED of a specific channel matrix $H$ between the two different GSM symbols $x_m \in \mathcal{X}_{\text{all}}$ and $x_n \in \mathcal{X}_{\text{all}}$ can be defined as

$$d_{m,n} = \| H_{I_m} x_m - H_{I_n} x_n \|_F^2, m \neq n.$$  

(7)

There are a total of $(N_{\text{all}} \cdot MN_u - 1)^2 \cdot (d_{m,n})_{n=1}^{MN_u}$ values and we only have to calculate $(N_{\text{all}} \cdot MN_u - 1)^2 / 2$ values due to the symmetry of the GSM symbols $(d_{m,n} = d_{n,m})$. The simplified optimal TAC set selection works follows.

Step 1: Obtain the $(N_{\text{all}} \cdot MN_u - 1)^2$ values of $d_{m,n}$ according to (7), which is shown in Table III, where we have $N_M = MN_u$ and $d_{m,n} = d_{n,m}$.

Step 2: Obtain the legitimate TAC sets $\mathcal{I}_q, q = (1, ..., C_{N_{\text{all}}}^N)$.

Step 3: For each TAC set $\mathcal{I}_q$, find the $d_{\text{min}}^q$ based on Table III. Specifically, if we have $I_q = [I_1, ..., I_{N_{\text{all}}}]$, we find the MED $d_{\text{min}}^q$ from the values of $d_{m,n}$ corresponding to $I_q$ based on Table III.

Step 4: Get the optimal TAC set index by (5).

Although the simplified optimal TAC set can be obtained by only calculating $(N_{\text{all}} \cdot MN_u - 1)^2 / 2$ EDs, its complexity is still excessive for large values of $N_t$ and $N_u$. Next, a simplified optimal TAC set selection algorithm is introduced.

B. Low-Complexity Optimal TAC Selection Based E-GSM

In this section, low-complexity optimal TAC selection algorithms are developed for $M = 1$ and $M > 1$ respectively. Specifically, for the case of $M = 1$, the activated antenna indices transmit the symbol ‘1’, which is also termed as Generalized Space Shift Keying (GSSK) scheme [41].

1) TAC Design for $M = 1$:

For the case of $M = 1$, the activated TAC transmits the symbol ‘1’, so that the ED matrix can be defined as

$$W = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1N_{\text{all}}} \\ w_{21} & w_{22} & \cdots & w_{2N_{\text{all}}} \\ \vdots & \vdots & \ddots & \vdots \\ w_{N_{\text{all}}1} & w_{N_{\text{all}}2} & \cdots & w_{N_{\text{all}}N_{\text{all}}} \end{bmatrix},$$  

(8)

where the element $w_{mn}, m = (1, ..., N_{\text{all}}), n = (1, ..., N_{\text{all}})$ denotes the ED between two GSM symbols as

$$w_{mn} = \| H_{I_m} - H_{I_n} \|_F^2,$$  

$$= \| (h_{m_1} + h_{m_2} + \cdots + h_{m_{N_u}}) - (h_{n_1} + h_{n_2} + \cdots + h_{n_{N_u}}) \|_F^2,$$  

(9)

where $D_m$ and $D_n$ represent the different elements between $I_m$ and $I_n$. Assuming that the identical element set between $I_m$ and $I_n$ is $\Lambda = \text{intersect}(I_m, I_n)$, the values of $D_m$ and $D_n$ are obtained by

$$D_m = \text{setdiff}(I_m, \Lambda), D_n = \text{setdiff}(I_n, \Lambda),$$  

(10)

where intersect($A$, $B$) denotes the function returning the identical elements between $A$ and $B$, while setdiff($A$, $B$)
denotes the function returning different elements between A and B. As a result, (5) can be reformulated as

\[ d_{\text{min}}^q = \min_{x_n \neq x_n} \|H_{x_n} - H_{x_n}\|^2_F \]

For \( q = (1, \ldots, C_{N_{\text{all}}}^C) \), there are a lot of repeated calculations of \( \|H_{D_m} - H_{D_n}\|^2_F \). To reduce the complexity of finding the optimal set based TAC selection, only the different values of \( \|H_{D_m} - H_{D_n}\|^2_F \) are calculated.

For the generalized cases, there are a total of \( (N_u + 1) \) TACs and \( (N_{\text{all}} - N) \) TACs have to be removed from \( \mathbb{I}_{\text{all}} \). After obtaining the \( N_{ED} \) different EDs \( \mathbf{w} = [w_1, \ldots, w_{N_{ED}}] \), the TAC selection for \( M = 1 \) operates as follows.

**Step 1:** Obtain the ED matrix \( \mathbf{W} \) based on \( N_{ED} \) different EDs \( \mathbf{w} = [w_1, \ldots, w_{N_{ED}}] \).

**Step 2:** Sort the \( N_{ED} \) EDs in an ascending order as

\[ [v_1, v_2, \ldots, v_{N_{ED}}] = \text{sort}(\mathbf{w}, '\text{ascend}') \]

**Step 3:** Remove all the values of \( v_1, v_2, \ldots, \) in the sequence from the matrix \( \mathbf{W} \) until we removed \( N_{\text{left}} \) TACs defining \( \mathbb{I}_{\text{Re}} \).

**Step 4:** Obtain the optimal TAC set as \( \mathbb{I}_o = \mathbb{I}_{\text{all}} \setminus \mathbb{I}_{\text{Re}} \).

Considering \( N_t = 4 \), \( N_u = 2 \) for example, the EDs between any two GSM symbols are presented in Table IV and we have

\[ \begin{align*}
   w_{12} &= w_{21} = w_{56} = w_{65} = \|h_2 - h_3\|^2_F; \\
   w_{13} &= w_{34} = w_{46} = w_{64} = \|h_2 - h_4\|^2_F; \\
   w_{14} &= w_{43} = w_{36} = w_{63} = \|h_1 - h_3\|^2_F; \\
   w_{15} &= w_{52} = w_{25} = \|h_1 - h_4\|^2_F; \\
   w_{16} &= w_{61} = \| (h_1 + h_2) - (h_3 + h_4) \|^2_F; \\
   w_{23} &= w_{32} = w_{45} = w_{54} = \|h_3 - h_4\|^2_F; \\
   w_{24} &= w_{42} = w_{35} = w_{53} = \|h_1 - h_2\|^2_F; \\
   w_{25} &= w_{52} = \| (h_1 + h_3) - (h_2 + h_4) \|^2_F; \\
   w_{34} &= \| (h_1 + h_4) - (h_2 + h_3) \|^2_F.
\end{align*} \]

As a result, there are only a total of 9 EDs \( \mathbf{w} = [w_{12}, w_{13}, w_{14}, w_{16}, w_{23}, w_{24}, w_{25}, w_{34}] \) to be computed for TAC selection. If the minimum value of \( \mathbf{w} \) is \( w_{12} \), the removed TAC set \( \mathbb{I}_{\text{Re}} \) may represent any of the combinations \( \{(I_1, I_2), (I_1, I_6), (I_2, I_5), (I_2, I_6)\} \).

2) **TAC Design for** \( M > 1 \):

For the case of \( M > 1 \), the elements in \( \mathbf{W} \) of (8) can be defined as

\[ w_{mn} = \min_{s \neq m} \|H_{I_m} s_m - H_{I_n} s_n\|^2_F = \begin{cases} \\
   \min_{s \neq m} \|H_{I_m} s_m - s_n\|^2_F, & \text{if } I_m = I_n, s_m \neq s_n; \\
   \min_{s \neq m} \|H_{I_m} s_m - H_{D_m} s_m\|^2_F, & \text{if } I_m \neq I_n.
\end{cases} \]

(16)

where \( s_m^A \) and \( s_n^A \) represent the specific subsets of \( s_m \) and \( s_n \) corresponding to the \( A \) index set and \( s_m^\Lambda \), \( s_n^\Lambda \). Due to the associated symmetry, each \( w_{mn} \) can be obtained by computing \( \| (1 + M N_u) M N_u \|^2 / 2 \) EDs in (16).

Since we have \( w_{mn} = w_{nm} \), the ED matrix \( \mathbf{W} \) of (8) can be obtained by computing \( N_{ED}^B = (N_{\text{left}} + 1) M N_u / 2 \) different values \( w_{mn} \). After obtaining the ED matrix \( \mathbf{W} \), the TAC selection can be completed by the Steps 1-4 of the TAC design of \( M = 1 \).

Considering \( N_t = 4 \), \( N_u = 2 \) for example, the ED matrix \( \mathbf{W} \) is presented in Table V, where \( w_{mn} \) can be obtained by computing 10 EDs. Observe from Eq. (15) and Tables IV-V that when the number of common minimum values of ED matrix \( \mathbf{W} \) satisfied that \( N_{\text{same}} / 2 > N_{\text{left}} \), the minimum value will still exist after removing \( N_{\text{left}} \) TACs. In this case, the value of (11) is the same as that of the conventional TAC set.

C. **Complexity Analysis of** \( E \)-**GSM Systems**

In this section, the complexity orders of the ML-aided C-GSM and of the proposed E-GSM are compared in terms of the numbers of real-valued multiplications and additions. For the specific matrices of \( \mathbf{A} \in \mathbb{C}^{m \times n}, \mathbf{B} \in \mathbb{C}^{n \times p}, \mathbf{c} \in \mathbb{C}^{n \times 1} \) and \( \mathbf{d} \in \mathbb{C}^{n \times 1} \), the operations of \( \mathbf{AB}, ||\mathbf{c}||_F^2 \) and \( \mathbf{c} \pm \mathbf{d} \) require \( mnp \), \( 2mp \), \( 4n - 1 \), and \( 2n \) Floating-point operations (Flops), respectively. Accordingly, the complexity order of the C-GSM relying on the ML detector becomes

\[ C_{\text{C-GSM}} = (8N_r N_u + 4N_t - 1) N M N_u, \]

(17)

since the operation \( || \mathbf{y} - \mathbf{H}_s \|^2_F \) requires \( CF = (8N_r N_u + 4N_t - 1) \) Flops, and this operation is computed \( 2^B \) times.

The complexity order of the low-complexity optimal TAC set based E-GSM system with ML detector is

\[ C_{\text{E-\text{GSM}}} = \begin{cases} \\
   C_{\text{C-GSM}} + CF N_{ED}, & \text{if } M = 1; \\
   C_{\text{C-GSM}} + CF N_{ED} (1 + M N_u) M N_u / 2, & \text{else}
\end{cases} \]

(18)

since \( C_{N_t}^B (M N_u - 1)^2 / 2 \) EDs are calculated in the TAC set selection.

IV. **TRANSMIT ANTENNA COMBINATION SELECTION FOR THE GSM SYSTEM OPERATING WITHOUT CHANNEL STATE INFORMATION**

In this selection, the ABEP assisted TAC selection conceived for the HM-GSM system operating without CSI knowledge is investigated. Firstly, the ABEP analysis of
the GSM system is carried out. Then, its TAC selection is developed. Finally, a novel bit-to-TAC mapping approach is proposed for further improving the GSM system’s performance.

A. ABEP Analysis of the GSM System

In this section, the ABEP performance of the GSM system is derived. Let us denote the transmit and receive signal of GSM by \( x_i \) and \( x_j \), respectively. Then the ABEP upper bound is given by

\[
P_b = \sum_{x_i} \sum_{x_j \neq x_i} \frac{d_{x_i,x_j} P_p(x_i \rightarrow x_j)}{B2^B},
\]

(19)

where \( P_p(x_i \rightarrow x_j) \) is the Pairwise Error Probability (PEP) and \( d_{x_i,x_j} \) is the number of bit errors associated with the corresponding PEP event. According to [24], \( P_p(x_i \rightarrow x_j) \) may be expressed as

\[
P_p(x_i \rightarrow x_j) = \gamma(\bar{s}) \sum_{k=0}^{N_r-1} C_{N_r-1+k}[1 - \gamma(\bar{s})]^k,
\]

(20)

where \( \gamma(\bar{s}) = \frac{1}{2} \left( 1 - \frac{\sqrt{2}}{1+\sqrt{2}} \right) \) and \( \bar{s} \) is the mean value of \( \bar{s} = \frac{1}{|H(X_i-x_j)|} \sum_{i=1}^{N_r} 2 \sigma^2 \) with \( N_r = 1 \). For the GSM system, assuming that the antenna indices of the transmit signal \( x_i \) and the estimated signal \( x_j \) are \( (l_1, l_2, ..., l_{N_u}) \) and \( (\hat{l}_1, \hat{l}_2, ..., \hat{l}_{N_u}) \), respectively, and the corresponding symbol vectors are \( s = [s_1, s_2, ..., s_{N_u}]^T \) and \( \hat{s} = [\hat{s}_1, \hat{s}_2, ..., \hat{s}_{N_u}]^T \). Then the value of \( \bar{s} \) for the GSM system is given by

\[
\bar{s} = \begin{cases} 
|s_1 - \hat{s}_1|^2 + \cdots + |s_{N_u} - \hat{s}_{N_u}|^2, & \text{if } m = N_u \\
\frac{|s_1 - \hat{s}_1|^2 + \cdots + |s_m - \hat{s}_m|^2 + 2(N_u - m)}{2 \sigma^2}, & \text{if } 0 < m < N_u \\
\frac{2N_u}{2 \sigma^2}, & \text{if } m = 0
\end{cases}
\]

(21)

where \( m \) is the number of identical antenna indices between \( (l_1, l_2, ..., l_{N_u}) \) and \( (\hat{l}_1, \hat{l}_2, ..., \hat{l}_{N_u}) \). Based on the values of \( \bar{s} \) obtained in Eq. (21), the ABEP of the GSM system can be evaluated using (19).

Observe from (21) that there are lots of identical \( \bar{s} \) values and the number of different values of \( \bar{s} \) is finite for a fixed constellation size \( M \), \( N_r \) and SNR variance \( \sigma^2 \). Hence, the above ABEP expressions can be further simplified. Assuming that the GSM systems have \( n \) different values \( \bar{s} \) as \( \bar{s}_1, ..., \bar{s}_{N_u} \), the corresponding PEP values \( F(\bar{s}_1), ..., F(\bar{s}_{N_u}) \) can be obtained from (20), so that (19) can be represented as

\[
P_b = \frac{\sum_{i=1}^{\lambda_1} d_{\bar{s}_1} F(\bar{s}_1) + \sum_{i=1}^{\lambda_2} d_{\bar{s}_2} F(\bar{s}_2) + \cdots + \sum_{i=1}^{\lambda_n} d_{\bar{s}_n} F(\bar{s}_n)}{B2^B},
\]

(22)

with

\[
D(\bar{s}_p) = \sum_{i=1}^{\lambda_p} d_{\bar{s}_p}^i / B2^B, \quad p = 1, ..., n,
\]

(23)

where \( \lambda_p \ p \in \{1, ..., n\} \) is the total number of candidate \( \bar{s}_p \) for all the \( x_i \) and \( x_j \), while \( d_{\bar{s}_p}^i \) is the corresponding Hamming Distance (HD), and \( D(\bar{s}_p) \) is the Average HD (AHD) for the value \( \bar{s}_p \).

Next, AHD based TAC selection is developed. Since only the TACs are selected, we employ \( M = 1 \) for our GSM system for simplicity, so that Eq. (21) can be further simplified to

\[
\bar{s} = \begin{cases} 
0, & \text{if } m = N_u \\
\frac{2(N_u - m)}{2 \sigma^2}, & \text{if } 0 < m < N_u \\
\frac{2N_u}{2 \sigma^2}, & \text{if } m = 0
\end{cases}
\]

(24)

As a result, there are a total of \( N_u \) different values \( \bar{s} \) as \( \bar{s}_1, ..., \bar{s}_{N_u} \) as

\[
\bar{s}_1 = \frac{1}{\sigma^2}, \quad m = N_u - 1 \\
\bar{s}_2 = \frac{1}{\sigma^2}, \quad m = N_u - 2 \\
\vdots \\
\bar{s}_{N_u} = \frac{N_u}{\sigma^2}, \quad m = 0
\]

(25)

Hence, the ABEP of the GSM system associated with \( M = 1 \) is dominated by the values of \( \bar{s}_m \) and \( F(\bar{s}_m) \). The relationship between \( \bar{s}_m \) and \( F(\bar{s}_m) \) is presented graphically in Fig. 2. Observe from Fig. 2 that we have

\[
F(\bar{s}_1) \gg F(\bar{s}_2) > \cdots > F(\bar{s}_{N_u}).
\]

(26)

where \( x \gg y \) denotes the value of \( x \) is much larger than that of \( y \). Hence, the associated optimization principle can be invoked for designing a GSM system having a small value of \( D(\bar{s}_1) \). According to (23), the value of \( D(\bar{s}_1) \) is dominated by two parts: the total number \( \lambda_1 \) of
candidate $\zeta_i$ ($m = N_u - 1$) and its corresponding HD of $d_{\zeta_i}^t$ ($t = 1, ..., \lambda_1$). Next, the TAC selection aims for attaining a small value of $\lambda_1$ and $d_{\zeta_i}^t$ ($t = 1, ..., \lambda_1$), which are introduced as follows.

**B. TAC Selection**

For each TAC $I_j, j \in [1, N_{all}]$, the set $\psi_j = I_{all} \setminus I_j$ consists of $N_u$ parts $\psi_{j}^{N_u - 1}$, $\psi_{j}^{N_u - 2}$, and $\psi_{j}^{0}$, where $\forall I_r \in \psi_{j}^{m}$, $m = 0, ..., N_u - 1$ indicates that there are $m$ common indices between $I_r$ and $I_j$, while $A \setminus B$ represents removing $B$ from $A$. Next, the TAC selections invoked for specific antenna setups are introduced as follows.

1) **TAC Selection for** $N_t = 4, N_u = 2$:

Fig. 3 portrays the TAC selection for the case of $N_t = 4$ and $N_u = 2$. In this scenario, we have $I_{all} = \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4)\}$. For any two TACs from $I_{all}$, the two TACs are connected by the curves seen in Fig. 3, if there are $N_u - 1$ common indices between the two TACs. As a result, the TAC selection problem has been transformed as to how we remove TACs having the lowest number of curves. For the case of $N_t = 4$ and $N_u = 2$, we have

$$
\begin{align*}
(1, 2) & \rightarrow \psi_1^1 = \{(1, 3), (1, 4), (2, 3), (2, 4)\}; \psi_1^0 = \{(3, 4)\} \\
(1, 3) & \rightarrow \psi_2^1 = \{(1, 2), (1, 4), (2, 3), (3, 4)\}; \psi_2^0 = \{(2, 4)\} \\
(1, 4) & \rightarrow \psi_3^1 = \{(1, 2), (1, 3), (2, 3), (3, 4)\}; \psi_3^0 = \{(2, 3)\}
\end{align*}
$$

Observe from Fig. 3 that by removing $(I_1, \psi_1^0)$, $(I_2, \psi_2^0)$ and $(I_3, \psi_3^0)$, we can obtain a smaller value of $\lambda_1$.

2) **TAC Selection for** $N_t = 5, N_u = 2$:

Fig. 4 portrays the TAC selection for the case of $N_t = 5$ and $N_u = 2$. For the case of $N_t = 5$ and $N_u = 2$, we have $I_{all} = \{(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5)\}$. $N = 8$ TACs should be selected from the set $I_{all}$. For the case of $N_t = 5$ and $N_u = 2$, we have

$$
\begin{align*}
(1, 2) & \rightarrow \psi_1^1 = \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}; \psi_1^0 = \{(3, 4), (3, 5), (4, 5)\} \\
(1, 3) & \rightarrow \psi_2^1 = \{(1, 2), (1, 3), (1, 5), (2, 3), (2, 4), (3, 4), (4, 5)\}; \psi_2^0 = \{(3, 5)\} \\
(1, 4) & \rightarrow \psi_3^1 = \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5)\}; \psi_3^0 = \{(3, 5)\}
\end{align*}
$$

Observe from Fig. 4 that by removing $(I_1, \psi_1^0)$, $(I_2, \psi_2^0)$, $(I_3, \psi_3^0)$, and $(I_1, I_{10})$, we arrive at a reduced value of $\lambda_1$.

$$
\begin{align*}
\psi_1^1 = \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\} \\
\psi_1^0 = \{(3, 4), (3, 5), (4, 5)\}
\end{align*}
$$

Then we can get the TAC set as

$$
\tilde{I}_{o} = \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 4), (3, 5)\}, \quad \psi_1^1 \subseteq \tilde{I}_{o}
$$

3) **Generalized TAC Selection for** $N_u = 2$:

Based on the above TAC selection examples, the generalized TAC selection design is introduced as follows.

**Step 1:** For the specific TAC $I_j, j \in [1, N_{all}]$, obtain the corresponding TAC set $\psi_j^{N_u - 1}, \psi_j^{N_u - 2}, ..., \psi_j^{0}$.

**Step 2:** Remove $N_u - N + 1$ TACs $I_j$ from the TAC set $\psi_j^{N_u - 2}, ..., \psi_j^{0}$ and obtain the $I_{left} = \{\psi_j^{N_u - 2}, ..., \psi_j^{0}\}\setminus I_j$.

**Step 3:** Obtain the final TAC set $I_o = \{\psi_j^{N_u - 1}, I_{left}\}$.

Due to the associated symmetry, the TAC selection is the same for each $I_j$. For simplicity, we introduce the TAC selection process for $I_3 = [1, 2, ..., N_u]$ in detail as follows.

For the case of $N_u = 2$, the TAC set can be expressed...
Based on the generalized TAC selection principle, the final TAC set can be obtained as

$$I_o = \{ \psi_1^{N_u-1}, \mathcal{I}_{\text{left}} \}$$  \hspace{1cm} (34)
1) Bit-to-TAC Mapping for $N_t = 4, N_u = 2$:

Fig. 5 presents the bit-to-TAC mapping for the case of $N_t = 4$ and $N_u = 2$. In this case, we have $B = 2$ and

\[
\begin{align*}
(1, 3) & \rightarrow \psi^1_{o,1} = \{(1, 4), (2, 3)\} \\
(1, 4) & \rightarrow \psi^1_{o,2} = \{(1, 4), (2, 4)\} \\
(2, 3) & \rightarrow \psi^1_{o,3} = \{(1, 3), (2, 4)\} \\
(2, 4) & \rightarrow \psi^1_{o,4} = \{(1, 4), (2, 3)\}
\end{align*}
\]

(39)

According to (35)-(38), the HD set $H_{\lambda_i}$ can be obtained by

\[
\begin{align*}
(1, 3) & \rightarrow \lambda^1 = 2 \rightarrow H_{\lambda_i} = (1, 1) \\
(1, 4) & \rightarrow \lambda^2 = 2 \rightarrow H_{\lambda_i} = (1, 1) \\
(2, 3) & \rightarrow \lambda^3 = 2 \rightarrow H_{\lambda_i} = (1, 1) \\
(2, 4) & \rightarrow \lambda^4 = 2 \rightarrow H_{\lambda_i} = (1, 1)
\end{align*}
\]

(40)

As seen from Fig. 5, the bit-to-TAC mapping can be obtained as

\[
00 \rightarrow (1, 3), 01 \rightarrow (1, 4), 10 \rightarrow (2, 3), 11 \rightarrow (2, 4).
\]

(41)

2) Bit-to-TAC Mapping for $N_t = 5, N_u = 2$:

Fig. 6 presents the bit-to-TAC mapping for the case of $N_t = 5$ and $N_u = 2$. For the scenario of $N_t = 5$ and $N_u = 2$, we have $B = 3$ and

\[
\begin{align*}
(1, 4) & \rightarrow \psi^1_{o,1} = \{(1, 3), (1, 5), (2, 4), (3, 4)\} \\
(1, 5) & \rightarrow \psi^1_{o,2} = \{(1, 3), (1, 4), (2, 5), (3, 5)\} \\
(2, 4) & \rightarrow \psi^1_{o,3} = \{(1, 4), (2, 3), (2, 5), (3, 4)\} \\
(2, 5) & \rightarrow \psi^1_{o,4} = \{(1, 5), (2, 3), (2, 4), (3, 5)\} \\
(2, 3) & \rightarrow \psi^1_{o,5} = \{(1, 3), (2, 4), (2, 5), (3, 4), (3, 5)\} \\
(1, 3) & \rightarrow \psi^1_{o,6} = \{(1, 4), (1, 5), (2, 3), (3, 4), (3, 5)\} \\
(3, 4) & \rightarrow \psi^1_{o,7} = \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\} \\
(3, 5) & \rightarrow \psi^1_{o,8} = \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 4)\}
\end{align*}
\]

(42)

According to (36)-(38), the set $H_{\lambda_i}$ for each TAC $I_{o,i}$ can be expressed as

\[
\begin{align*}
(1, 4) & \rightarrow \lambda^1 = 4 \rightarrow H_{\lambda_i} = (1, 1, 1, 2) \\
(1, 5) & \rightarrow \lambda^2 = 4 \rightarrow H_{\lambda_i} = (1, 1, 1, 2) \\
(2, 4) & \rightarrow \lambda^3 = 4 \rightarrow H_{\lambda_i} = (1, 1, 1, 2) \\
(2, 5) & \rightarrow \lambda^4 = 4 \rightarrow H_{\lambda_i} = (1, 1, 1, 2) \\
(1, 3) & \rightarrow \lambda^5 = 5 \rightarrow H_{\lambda_i} = (1, 1, 1, 2, 2) \\
(2, 3) & \rightarrow \lambda^6 = 5 \rightarrow H_{\lambda_i} = (1, 1, 1, 2, 2) \\
(3, 4) & \rightarrow \lambda^7 = 5 \rightarrow H_{\lambda_i} = (1, 1, 1, 2, 2) \\
(3, 5) & \rightarrow \lambda^8 = 5 \rightarrow H_{\lambda_i} = (1, 1, 1, 2, 2)
\end{align*}
\]

(43)

Hence, the general bit-to-TAC mapping principle is based on the values of $H_{\lambda_i}$ in (43), which is detailed as follows.

- Initialize $(1, 3) \rightarrow 000, (1, 4) \rightarrow 001.$
- **Step 1:** The bit-to-TAC mapping begins from $\psi^1_{o,1}.$ Since the TAC mapping should satisfy Eq. (43), we have $(1, 5) \in \{010, 011, 100, 101\}.
- **Step 2:** If $(1, 5) \rightarrow 010,$ we have $H_{\lambda_i} = (1, 2)$ and $H_{\lambda_i} = (1, 2).$ According to Eq. (43), we have $(2, 4), (3, 4) \in \{101, 011\}$ and $(2, 5), (3, 5) \in \{011, 110\}.$
- **Step 3:** If $(1, 5) \rightarrow 011,$ we have $H_{\lambda_i} = (1, 1) \text{ and } H_{\lambda_i} = (1, 2).$ According to (42)-(43), we have $(2, 5), (3, 5) \in \{010, 111\}$ and the following expressions can be formulated

\[
\begin{align*}
(1, 4) & \rightarrow \psi^1_{o,1} = \{(1, 3), (1, 5), (2, 4), (3, 4)\} \\
(1, 5) & \rightarrow \psi^1_{o,2} = \{(1, 3), (1, 4), (2, 5), (3, 5)\} \\
(2, 4) & \rightarrow \psi^1_{o,3} = \{(1, 4), (2, 3), (2, 5), (3, 4)\} \\
(2, 5) & \rightarrow \psi^1_{o,4} = \{(1, 5), (2, 3), (2, 4), (3, 5)\}
\end{align*}
\]

(44)

- **Step 4:** The bit-to-TAC mapping continues from the TAC $(2, 5).$ If $(2, 5) \rightarrow 010,$ we have

\[
(2, 5) \rightarrow \psi^1_{o,4} = \{(1, 5), (2, 3), (2, 4), (3, 5)\}.
\]

(45)

Then we should have $(2, 4) \rightarrow 000$ in order to satisfy Eq. (43), which is the same as $(1, 3) \rightarrow 000.$ Hence, $(2, 5)$ cannot be mapped to $010$ and the bit-to-TAC mapping starts from $(2, 5) \rightarrow 111$ as

\[
(2, 5) \rightarrow \psi^1_{o,4} = \{(1, 5), (2, 3), (2, 4), (3, 5)\}.
\]

(46)

- **Step 5:** According to (46), the bit-to-TAC mapping continues from the TAC $(2, 4).$ If $(2, 4) \rightarrow 110,$ the HD
the complexity of the bit-to-TAC mapping is determined and the following expressions can be formulated

$$\psi^{N_u-1}_{o,i}, \psi^{N_u-1}_{o,i}, \lambda^1_i, H^{x}_i, i = 1, ..., N, D^i_\text{max}, b_{o,1} = b_1, b_{o,2} = b_2$$

Output: \( \hat{b} \).

1: For the TAC \( I_{0,3} \), obtain the possible bits set as \( \mathbb{B}_{o,3} \) with number of \( n_{o,3} \);
2: for \( t_3 \in (1, n_{o,3}) \) do
3: \( b_{o,3} = \mathbb{B}_{o,3}^{t_3}, \mathbb{B}_{o,3}^{t_3} \) is the \( t_3 \)-th element of \( \mathbb{B}_{o,3} \);
4: Update \( H^{x}_1, H^{x}_2, H^{x}_3 \), where \( H^{x}_i \) denotes the HDs between two TACs inside the set \( \psi^{N_u-1}_{o,i} \).
5: if \( \max(H^{x}_i) > D^i_\text{max} \parallel \sum H^{x}_i > D^i_\text{max} \) then
6: break;
7: else
8: \( b_{o,3} = \mathbb{B}_{o,3}^{t_3} \)
9: end if
10: for the TAC \( I_{0,4} \), obtain the possible bits set as \( \mathbb{B}_{o,4} \) with number of \( n_{o,4} \);
11: for \( t_4 \in (1, n_{o,4}) \) do
12: \( b_{o,4} = \mathbb{B}_{o,4}^{t_4} \),
13: Update \( H^{x}_1, H^{x}_2, H^{x}_3 \) and \( H^{x}_4 \).
14: if \( \max(H^{x}_i) > D^i_\text{max} \parallel \sum H^{x}_i > D^i_\text{max} \) then
15: break;
16: else
17: \( b_{o,4} = \mathbb{B}_{o,4}^{t_4} \)
18: end if
19: end for
20: Get the possible bits set \( \mathbb{B}_{o,5} \) with number of \( n_{o,5} \);
21: for \( t_5 \in (1, n_{o,5}) \) do
22: ... 
23: end for
24: ... 
25: Get the possible bits set \( \mathbb{B}_{o,N} \) with number of \( n_{o,N} \);
26: for \( t_N \in (1, n_{o,N}) \) do
27: ... 
28: end for
29: end for
30: \( \hat{b} = \{b_1, b_2, \mathbb{B}_{o,3}^{t_3}, \mathbb{B}_{o,3}^{t_4}, ..., \mathbb{B}_{o,N}^{t_N} \} \)

Finally, as shown in Fig. 6, the bit-to-TAC mapping for the case of \( N_t = 5 \) and \( N_u = 2 \) is given by

\[
\begin{align*}
000 & \rightarrow (1,3), 001 \rightarrow (1,4), 010 \rightarrow (3,5), 011 \rightarrow (1,5) \nonumber \\
100 & \rightarrow (2,4), 101 \rightarrow (2,3), 110 \rightarrow (2,3), 111 \rightarrow (2,5), \nonumber \\
\end{align*}
\]

$$\text{(48)}$$

3) Bit-to-TAC Mapping for Generalized Cases:

Based on the bit-to-TAC mapping of the above specific examples, the generalized bit-to-TAC mapping is formulated as follows.

**Step 1:** Obtain the TAC set \( \mathbb{I}_o \) and the HD set \( \mathbb{H} \) according to (34) and (38).

**Step 2:** For each TAC \( I_{o,i} \in \mathbb{I}_o \), obtain the set \( \psi^{N_u-1}_{o,i} \) and the value \( \lambda^1_i \) according to (34).

**Step 3:** Obtain the HD set \( H^{x}_i \) according to (38).

**Step 4:** Complete the bit-to-TAC mapping based on \( I_{o,i} \in \mathbb{I}_o, \psi^{N_u-1}_{o,i}, H^{x}_i \) with \( D_{\xi_i} \).

Assuming that \( \mathbb{B} = \{b_1, ..., b_N\} \) is the set of information bits, the bit-to-TAC mapping can be detailed as in Algorithm 1. As shown in Algorithm 1, there are lots of mapping options, which have the smallest value \( d^i_{N^{x}_1} \). In fact, we only have to find a single appropriate mapping, hence the complexity of the bit-to-TAC mapping is determined by \( \mathbb{I}_o \) and by the specific technique we select for mapping. Taking \( N_t = 6 \) and \( N_u = 2 \) as an example, we have \( \mathbb{I}_o = \{(1,3), (1,4), (1,5), (1,6), (2,3), (2,4), (2,5), (2,6)\} \).

Algorithm 1: Bit-to-TAC mapping based on the obtained TAC set \( \mathbb{I}_o \)

**Input:** \( \mathbb{I}_o, \psi^{N_u-1}_{o,i}, \lambda^1_i, H^{x}_i, i = 1, ..., N, D^i_\text{max} \), \( b_{o,1} = b_1 \), \( b_{o,2} = b_2 \)

**Output:** \( \hat{b} \).

1: For the TAC \( I_{o,3} \), obtain the possible bits set as \( \mathbb{B}_{o,3} \) with number of \( n_{o,3} \);
2: for \( t_3 \in (1, n_{o,3}) \) do
3: \( b_{o,3} = \mathbb{B}_{o,3}^{t_3}, \mathbb{B}_{o,3}^{t_3} \) is the \( t_3 \)-th element of \( \mathbb{B}_{o,3} \);
4: Update \( H^{x}_1, H^{x}_2, H^{x}_3 \), where \( H^{x}_i \) denotes the HDs between two TACs inside the set \( \psi^{N_u-1}_{o,i} \).
5: if \( \max(H^{x}_i) > D^i_\text{max} \parallel \sum H^{x}_i > D^i_\text{max} \) then
6: break;
7: else
8: \( b_{o,3} = \mathbb{B}_{o,3}^{t_3} \)
9: end if
10: for the TAC \( I_{o,4} \), obtain the possible bits set as \( \mathbb{B}_{o,4} \) with number of \( n_{o,4} \);
11: for \( t_4 \in (1, n_{o,4}) \) do
12: \( b_{o,4} = \mathbb{B}_{o,4}^{t_4} \),
13: Update \( H^{x}_1, H^{x}_2, H^{x}_3 \) and \( H^{x}_4 \).
14: if \( \max(H^{x}_i) > D^i_\text{max} \parallel \sum H^{x}_i > D^i_\text{max} \) then
15: break;
16: else
17: \( b_{o,4} = \mathbb{B}_{o,4}^{t_4} \)
18: end if
19: end for
20: Get the possible bits set \( \mathbb{B}_{o,5} \) with number of \( n_{o,5} \);
21: for \( t_5 \in (1, n_{o,5}) \) do
22: ... 
23: end for
24: ... 
25: Get the possible bits set \( \mathbb{B}_{o,N} \) with number of \( n_{o,N} \);
26: for \( t_N \in (1, n_{o,N}) \) do
27: ... 
28: end for
29: end for
30: \( \hat{b} = \{b_1, b_2, \mathbb{B}_{o,3}^{t_3}, \mathbb{B}_{o,3}^{t_4}, ..., \mathbb{B}_{o,N}^{t_N} \} \)

then the bit-to-TAC mapping can be found without any extra complexity as

\[
\begin{align*}
(1,3) & \rightarrow 000, (1,4) \rightarrow 001, (1,5) \rightarrow 010, (1,6) \rightarrow 011 \\
(2,3) & \rightarrow 100, (2,4) \rightarrow 101, (2,5) \rightarrow 110, (2,6) \rightarrow 111, \nonumber \\
\end{align*}
\]

$$\text{(49)}$$

In order to provide further insights, we compare the AHD of the proposed HM-GSM system to that of the C-GSM system in Table IV. Observe from Table IV that the proposed HM-GSM scheme is capable of attaining a significantly lower AHD \( D(\xi_i) \) than the C-GSM system.

V. ENHANCED HIGH THROUGHPUT GSM

In the above section, we have analyzed how to choose an optimal TAC set from the entire TAC set space, which is capable of providing a performance gain over the C-GSM system. In this section we discuss, how to exploiting the remaining TACs to increase the throughput.
Then the total number of TACs required by the proposed HGM-GSM are wasted. Here we propose reusing the remaining legitimate TACs for high bit-to-TAC mapping to convey, if $I^\theta (x_i)$ conveys as

$$I^\theta (x_i) = HM-GSM,$$

Naturally, we can only arrange for $N_t = 4, N_u = 2$ bits conveyed to a single TAC. However, to convey an extra bit per time slot, we need to extend the number of bits conveyed to Step 2: Extend the number of bits conveyed to

$$R_e = \lfloor \log_2 (C_{N_i}^{N_u}) \rfloor$$

Then the total number of TAC required by the proposed E-HT-GSM scheme becomes $N_e = 2^{R_e}$.

**Step 3:** Naturally, we can only arrange for $N_p$ bits be conveyed, if $(N - C_{N_i}^{N_u})$ TACs are reused randomly and distinguishing rotated phase $\theta$ is employed for the reused TACs. The extended TAC set becomes $I_{\text{high}} = I_{\text{all}} \cup I_{\text{repeat}}$.

**Step 4:** GSM mapping utilizing the extended TAC set $I_{\text{high}}$.

Let us consider $N_t = 4$ for example, then the set of legitimate TACs for $N_t = 4$ is expressed as

$$I_{\text{all}} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}.$$

When reusing the two TACs $[110]^{T}$ and $[0011]^{T}$, the repeated TACs are given by

$$I_{\text{repeat}} = \left\{ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} e^{j\theta} \right\}.$$

According to (20) and (21), the extended TAC set is given by

$$I_{\text{high}} = \left\{ \begin{bmatrix} 1 & 0 & 0 & 0 \\ e^{j\theta} & 0 & 0 & 0 \\ 0 & e^{j\theta} & 0 & 0 \\ 1 & 0 & e^{j\theta} & 0 \end{bmatrix} \right\}.$$

At the receiver, the ML detector and a range of low-complexity detectors designed for C-GSM systems can also be applied to the proposed E-HT-GSM systems.

### A. Proposed E-HT-GSM System

In the C-GSM system, only $N = 2^{\lfloor \log_2 (C_{N_i}^{N_u}) \rfloor}$ TACs represent antenna index, while the remaining $C_{N_i}^{N_u} - N$ TACs are wasted. Here we propose reusing the remaining $(2^{\lfloor \log_2 (N) \rfloor + 1} - C_{N_i}^{N_u})$ TACs to convey an extra bit per time slot. The detailed process of the proposed E-HT-GSM is as follows.

**Step 1:** Determine the number of bits that the conventional TAC index conveys as

$$R_e = \lfloor \log_2 (C_{N_i}^{N_u}) \rfloor$$

**Step 2:** Extend the number of bits conveyed to

$$R_p = \lfloor \log_2 (C_{N_i}^{N_u}) \rfloor + 1.$$ (51)

Then the total number of TAC required by the proposed E-HT-GSM scheme becomes $N_e = 2^{R_p}$.

**Step 3:** Naturally, we can only arrange for $N_p$ bits be conveyed, if $(N - C_{N_i}^{N_u})$ TACs are reused randomly and distinguishing rotated phase $\theta$ is employed for the reused TACs. The extended TAC set becomes $I_{\text{high}} = I_{\text{all}} \cup I_{\text{repeat}}$.

**Step 4:** GSM mapping utilizing the extended TAC set $I_{\text{high}}$.

Let us consider $N_t = 4$ for example, then the set of legitimate TACs for $N_t = 4$ is expressed as

$$I_{\text{all}} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}.$$

When reusing the two TACs $[110]^{T}$ and $[0011]^{T}$, the repeated TACs are given by

$$I_{\text{repeat}} = \left\{ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} e^{j\theta} \right\}.$$

According to (20) and (21), the extended TAC set is given by

$$I_{\text{high}} = \left\{ \begin{bmatrix} 1 & 0 & 0 & 0 \\ e^{j\theta} & 0 & 0 & 0 \\ 0 & e^{j\theta} & 0 & 0 \\ 1 & 0 & e^{j\theta} & 0 \end{bmatrix} \right\}.$$

At the receiver, the ML detector and a range of low-complexity detectors designed for C-GSM systems can also be applied to the proposed E-HT-GSM systems.

### B. The Optimization of $\theta$

In this section, the value of $\theta$ is optimized by maximizing the Minimum Distance (MD) $[42]$ associated with $\theta$ between the pair of E-HT-GSM symbols. Assuming that $x_i$ and $x_j$ are the transmit and estimated E-HT-GSM symbols, respectively, the MD between $x_i$ and $x_j$ can be expressed as

$$\delta_{\min}(x_i, x_j) = \min_{x_i, x_j} \det((x_i - x_j)(x_i - x_j)^H).$$ (55)

The optimization principle is based on maximizing the value of $\delta_{\min}(x_i, x_j)$. For simplicity, let us consider $N_u = 2$ for example. Specifically, the calculation is based on four scenarios:

1) Both $x_i$ and $x_j$ are independent of $\theta$. In this case, the value of $\delta_{\min}(x_i, x_j)$ is independent of $\theta$ as well;
2) Both $x_i$ and $x_j$ are associated with $\theta$. In this case, the calculation is the same as the scenario 1);
3) $x_i$ is independent of $\theta$ and $x_j$ is associated with $\theta$;
4) $x_i$ is associated with $\theta$ and $x_j$ is independent of $\theta$.

The calculation process of scenario 3) and scenario 4) is the same due to their symmetry. As a result, we can focus on scenario 3) for our analysis. Assuming that the TAC and symbol vectors of $x_i$ and $x_j$ are given as $(l_1, l_2, s = [s_1, s_2]^T$, and $(\hat{l}_1, \hat{l}_2, \hat{s}) = [\hat{s}_1, \hat{s}_2]^T$, respectively, the value of $\delta_{\min}(x_i, x_j)$ can be calculated for three independent cases.

**Case 1:** All the activate antenna indices of $x_i$ and $x_j$ are the same. Assuming $l_1 = \hat{l}_1 = 1, l_2 = \hat{l}_2 = 2, x_i = .
\[ [s_1 \ s_2 \ 0_{2 \times (N_r - 2)}]^T, \quad \text{and} \quad x_j = [\hat{s}_1 \ \hat{s}_2 \ 0_{2 \times (N_r - 2)}]^T e^{j\theta} \]

for example, the MD between \( x_i \) and \( x_j \) obeys

\[
\delta_{\text{min}}(x_i, x_j) = \min_{x_i, x_j} \det(x_i - x_j)(x_i - x_j)^H
\]

\[
= \min_{s_1, \hat{s}_1, s_2, \hat{s}_2} \det(s_1 - \hat{s}_1 e^{j\theta} s_2 - \hat{s}_2 e^{j\theta} 0_{2 \times (N_r - 2)}) \times (s_1 - \hat{s}_1 e^{j\theta} s_2 - \hat{s}_2 e^{j\theta} 0_{2 \times (N_r - 2)})^H
\]

\[
= \min_{s_1, \hat{s}_1, s_2, \hat{s}_2} \det(||s_1||^2 + ||s_2||^2 + ||\hat{s}_1||^2 + ||\hat{s}_2||^2 - (s_1 \hat{s}_1 + s_2 \hat{s}_2)e^{j\theta}). \quad (57)
\]

Case 2: Only a single activated antenna index of \( x_i \) and \( x_j \) is the same. Assuming \( l_1 = 1, \hat{l}_1 = 2, l_2 = 1, \hat{l}_2 = 3, x_i = [s_1 \ s_2 \ 0_{2 \times (N_r - 2)}]^T, \) and \( x_j = [\hat{s}_1 \ 0 \ \hat{s}_2 \ 0_{2 \times (N_r - 3)}]^T e^{j\theta} \)

for example, the MD between \( x_i \) and \( x_j \) obeys

\[
\delta_{\text{min}}(x_i, x_j) = \min_{x_i, x_j} \det(x_i - x_j)(x_i - x_j)^H
\]

\[
= \min_{s_1, \hat{s}_1, s_2, \hat{s}_2} \det(s_1 - \hat{s}_1 e^{j\theta} s_2 - \hat{s}_2 e^{j\theta} 0_{2 \times (N_r - 2)}) \times (s_1 - \hat{s}_1 e^{j\theta} s_2 - \hat{s}_2 e^{j\theta} 0_{2 \times (N_r - 3)})^H
\]

\[
= \min_{s_1, \hat{s}_1, s_2, \hat{s}_2} \det(||s_1||^2 + ||s_2||^2 + ||\hat{s}_1||^2 + ||\hat{s}_2||^2 - s_1 \hat{s}_1 e^{j\theta} + s_2 \hat{s}_2 e^{j\theta}). \quad (58)
\]

Case 3: None of the activated antenna indices are the same. Considering \( l_1 = 1, \hat{l}_1 = 2, l_2 = 3, \hat{l}_2 = 4, x_i = (s_1 \ s_2 \ 0_{2 \times (N_r - 2)}) e^{j\theta}, \) and \( x_j = (0 \ 0 \ \hat{s}_1 \ \hat{s}_2 \ 0_{2 \times (N_r - 4)}) e^{j\theta} \)

for example, the MD between \( x_i \) and \( x_j \) obeys

\[
\delta_{\text{min}}(x_i, x_j) = \min_{x_i, x_j} \det(x_i - x_j)(x_i - x_j)^H
\]

\[
= \min_{s_1, \hat{s}_1, s_2, \hat{s}_2} \det(s_1 - \hat{s}_1 e^{j\theta} s_2 - \hat{s}_2 e^{j\theta} 0_{2 \times (N_r - 4)}) \times (s_1 - \hat{s}_1 e^{j\theta} s_2 - \hat{s}_2 e^{j\theta} 0_{2 \times (N_r - 4)})^H
\]

\[
= \min_{s_1, \hat{s}_1, s_2, \hat{s}_2} \det(||s_1||^2 + ||s_2||^2 + ||\hat{s}_1||^2 + ||\hat{s}_2||^2). \quad (59)
\]

As a result, the values of \( \delta_{\text{min}}(x_i, x_j) \) for the three cases can be summarized as (59), which is on the top of next page. According to (59), we have

\[
\delta_{\text{min}}(x_i, x_j) = \min_{x_i, x_j} \det(||s_1||^2 + ||s_2||^2 + ||\hat{s}_1||^2 + ||\hat{s}_2||^2 - (s_1 \hat{s}_1 + s_2 \hat{s}_2)e^{j\theta}). \quad (60)
\]

To provide further insights, Fig. 7 shows the value of \( \delta_{\text{min}}(x_i, x_j) \) in (60) for BPSK, QPSK, 16 QAM and 64-QAM in conjunction with different \( \theta \). If \( \theta \in [0, \pi/2] \), as seen from Fig. 7, \( \theta \) can be optimized by maximizing the value of \( \delta_{\text{min}}(x_i, x_j) \) in (60) as

\[
\theta = \begin{cases} 
\frac{\pi}{4} & \text{BPSK} \\
\frac{\pi}{4} & \text{QPSK} \\
\frac{\pi}{16} & \text{16QAM} \\
\frac{\pi}{16} & \text{64QAM}
\end{cases}
\]

VI. SIMULATION RESULTS

In this section, the performances of the proposed E-GSM, HM-GSM and E-HT-GSM schemes are presented and compared under different antenna configurations. In all the simulation results, perfect channel state information is assumed and the values of the specific \( \theta \) are selected based on Eq. (59). ML detectors are employed at the receiver for Figs. 8-14. Moreover, the analytical ABEP performances of the proposed systems are added as benchmarks, which can be obtained by (19). As seen from Fig. 8-14, the upper bound becomes very tight upon increasing the SNR values, which is helpful for evaluating the BER performances of the proposed systems.

Specifically, Figs. 8-10 compare the performances of both the proposed E-GSM, and of the HM-GSM systems to the C-GSM system using \( N_t = 4, N_r = 4, N_u = 2 \) at different values of \( M \). Fig. 11 compares the complexity of the proposed E-GSM and HM-GSM systems to that of the C-GSM system for the same setups as in Figs. 8-10. The
mapping principles of the HM-GSM and C-GSM system are shown in Table VI. Observe from Figs. 8-11 that the proposed E-GSM system is capable of outperforming the C-GSM system by 5 dB, 3.5 dB and 2.5 dB for $M = 1$, $M = 2$ and $M = 4$ respectively at the BER=10$^{-5}$ at an acceptable extra complexity. The proposed HM-GSM scheme is capable of outperforming C-GSM system by 1 dB, 0.6 dB and 0.4 dB for $M = 1$, $M = 2$ and $M = 4$ at the BER=10$^{-5}$ at the same complexity. Moreover, observe from Fig. 10 that the proposed E-GSM and HM-GSM systems also perform better than the scheme in [2].

In order to provide further insights, Figs. 12-13 compare the performances of the proposed E-GSM, HM-GSM to the C-GSM system using $N_t = 5, N_r = 4, N_u = 3$ and $N_t = 6, N_r = 4, N_u = 2$, respectively. Observe from Fig. 12 that the performance advantage of the E-GSM over the C-GSM system becomes modest. This is because for the case of $N_t = 5, N_u = 3$, we have $N_{\text{left}} = 2$, while $N_{\text{same}}/2 = 3$ for $\beta = 2$, $N_{\text{same}}/2 = 1$ for $\beta = 1$, $N_{\text{same}}/2 = 1$ for $\beta = 0$ remain valid according to (12). When the number of common minimum values in $W$ satisfies $N_{\text{same}}/2 > N_{\text{left}} = 2$, the performance of the optimal TAC set is the same as that of the conventional TAC set. For the case of $N_t = 6, N_r = 4, N_u = 2, N_{\text{left}} = 7$, $N_{\text{same}}/2 = 4$ for $\beta = 1$ and $N_{\text{same}}/2 = 1$ for $\beta = 0$. As a result, $N_{\text{same}}/2 < N_{\text{left}}$ always remains true and the proposed E-GSM system using $N_t = 6, N_r = 4, N_u = 2$ is capable of providing significant performance gain over the C-GSM system, as observed in Fig. 13.

Finally, Fig. 14 compares the performance of the proposed E-HT-GSM system to that of the C-GSM system at different transmission rates. Specifically, for the C-GSM system, we employ QPSK modulation for the cases of $N_t = 4, N_r = 4, N_u = 2$ and $N_t = 8, N_r = 8, N_u = 3$ to obtain the normalized throughput of 6 bits/symbol.

Fig. 9. Performance comparison of the proposed schemes and of the conventional GSM systems using $N_t = 4, N_r = 4, N_u = 2$ and $M = 2$ at 3 bits/symbol.

Fig. 10. Performance comparison of the proposed schemes and of the conventional GSM systems using $N_t = 4, N_r = 4, N_u = 2$ and $M = 4$ at 4 bits/symbol.

Fig. 11. Complexity comparison of the proposed schemes and of the conventional GSM system using $N_t = 4, N_r = 4$ and $N_u = 2$.

Fig. 12. Performance comparison of the proposed schemes and of the conventional GSM systems using $N_t = 5, N_r = 4, N_u = 3$.
and 11 bits/symbol. For the proposed E-HT-GSM systems relying on the above setups, we achieve an extra bit of throughput, yielding 7 bits/symbol and 12 bits/symbol, respectively. Observe from Fig. 14 that the proposed E-HT-GSM system is capable of increasing the throughput by one bit per channel use at a 0.5 dB performance loss at BER= $10^{-4}$ for both the above mentioned setups.

VII. CONCLUSIONS

The problem of TAC set optimization has been investigated. Firstly, a low-complexity TAC selection algorithm relying on CSI knowledge was designed for our E-GSM systems. Then, hybrid bit-to-TAC mapping based TAC optimization operating without CSI knowledge was designed for the GSM system. The proposed E-GSM system and HM-GSM system are capable of outperforming the C-GSM system at a negligible extra complexity, while the proposed E-HT-GSM system conceived is capable of increasing the throughput per time slot by one bit at a negligible performance loss.

References


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