Optimizing the Chase: Scalable Data Integration under Constraints

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ABSTRACT
We are interested in scalable data integration and data exchange under constraints/dependencies. In data exchange the problem is how to materialize a target database instance, satisfying the source-to-target and target dependencies, that provides the certain answers. In data integration, the problem is how to rewrite a query over the target schema into a query over the source schemas that provides the certain answers. In both these problems we make use of the chase algorithm, the main tool to reason with dependencies. Our first contribution is to introduce the frugal chase, which produces smaller universal solutions than the standard chase, still remaining polynomial in data complexity. Our second contribution is to use the frugal chase to scale up query answering under views using weakly-acyclic target constraints, a useful language capturing RDF/S. The latter problem can be reduced to query rewriting under constraints. We construct a compact graph-based representation of the mappings and the constraints and develop an efficient algorithm to run the frugal chase on this representation. We show experimentally that our approach scales to large problems, speeding up the compilation of the dependencies into the mappings by close to 2 and 3 orders of magnitude, compared to the standard and the core chase, respectively. Compared to the standard chase, we improve online query rewriting time by a factor of 3, while producing equivalent, but smaller, rewritings of the original query.

1. INTRODUCTION
We witness an explosion of available data in all areas of human activity, from large scientific experiments, to medical data, to distributed sensors, to social media. Integrating data from disparate sources can lead to novel insights across scientific, industrial, and governmental domains. This integration is achieved by either creating a data warehouse, that is, by copying/transferring the data to a centralized site under a single schema for subsequent analysis (data exchange [14]), or by leaving the data at their original sources and querying the data at analysis time (virtual data integration [20]).

Formally, a data integration/exchange setting, \(<S,T,\Sigma_{st},\Sigma_{t}>\), is a tuple where \(S\) is a set of heterogeneous schemas from multiple sources, \(T\) is the target schema (aka the domain, global, or mediated schema), which must satisfy a set of target constraints/dependencies \(\Sigma_{t}\), and \(\Sigma_{st}\) is a set of schema mappings that transform the data in schemas \(S\) into schema \(T\). The main problem in this setting, and the focus of this paper, is conjunctive query answering (aka answering queries using views under dependencies [19], or computing certain answers under constraints [14, 1, 11]): the user poses a query over target schema \(T\) and the system needs to provide the certain answers to the query using the data from the sources \(S\) such that all constraints \(\Sigma_{st}\) and \(\Sigma_{t}\) are satisfied. Data exchange and virtual integration are the two main approaches for getting the certain answers. Data exchange achieves this with a target database, by obtaining data from the sources using (satisfying) the schema mappings \(\Sigma_{st}\), and satisfying the target constraints \(\Sigma_{t}\). Then, the certain answers to a user query can be computed by directly evaluating the query over the materialized target database. In virtual integration, a mediator uses the schema mappings and target constraints to rewrite the target user query into a query that only uses terms from the source schemas but produces all the certain answers.

Mappings in \(\Sigma_{st}\), and target constraints in \(\Sigma_{t}\), are usually given in the form of logical implications of conjunctive formulas, called tuple-generating dependencies (TGDs) [6] or Global-Local-as-View (GLAV) rules [15]. TGDs are very expressive, but query answering (and rewriting) under general target TGDs is undecidable [6]. However, some syntactic restrictions, such as weakly-acyclic TGDs [14], are decidable (even data-complexity tractable) in query answering.

In data exchange, the main technique to reason with constraints is the chase [6], a form of forward chaining that “fetches” data from the sources through the mappings to a target database, and also “completes” this database w.r.t. the target constraints. One can then disregard the constraints and do query answering over the completed database. In this paper, we present an optimization of the chase algorithm usable in data exchange with GLAV mappings and standard-chase terminating (e.g., weakly-acyclic) target TGDs.

In virtual integration (which is achieved through query rewriting), the initial focus has been on settings without target constraints, i.e., \(\Sigma_{t} = \emptyset\), and mappings with only one source predicate in the rule antecedent, called Local-as-view (LAV) mappings [27, 12]. LAV mappings expose the core challenges in query rewriting, since they contain joins of existential variables. Query rewriting with LAV mappings and no constraints, aka answering queries using views, has been extensively studied in query optimization, data integration and other areas [20, 26]. In previous work, we developed a scalable query rewriting algorithm for LAV mappings and no target constraints, named GQR [24], that is about two orders of magnitude faster than the previous state-of-the-art (Minicon [31], MCDSAT [5]), and rewrites a conjunctive query in the presence of...
10,000 LA V mappings in under 1 second. For virtual integration with target constraints, Afrati and Kiourti [1] used the chase algorithm in a novel way by “compiling” the target constraints (specifically, LA V weakly-acyclic TGDs) into the LA V mappings to reduce the problem to view-based query rewriting without constraints. In this paper, we present an algorithm for query rewriting under LA V weakly-acyclic target TGDs, building on [1, 24], and our optimized chase. This type of constraints is first-order rewritable, includes practically interesting languages like RDF/S, and has good computational properties in data integration, data exchange [16, 1], and in inconsistent and incomplete databases [2].

In particular, our paper presents two main contributions:

1. The frugal chase. We develop a novel, optimized version of the standard chase, usable in data integration, data exchange, or incomplete database settings, with GLA V mappings and GLA V target constraints. Instead of adding the entire consequent to a solution when a chase rule is applicable, as in the standard chase, the frugal chase avoids adding provably redundant atoms (Sect. 3). We prove that the frugal chase results in equivalent, yet smaller in size (number of tuples), universal solutions with the same data complexity as the standard chase. We also present a procedural version of the frugal chase (Sect. 4), and a compact version of the latter adapted to our GQR rewriting algorithm (Sect. 5).

2. A scalable conjunctive query rewriting algorithm for LA V mappings under weakly-acyclic LA V target constraints. This algorithm uses the compact frugal chase to efficiently compile the constraints into the mappings (à la [1]), and then efficiently do query rewriting (using [24]). Our compact frugal chase identifies common patterns across the mappings and “factorizes” them using a graph representation. This allows us to trigger a constraint for multiple views at the same time, and to add consequent predicates across multiple views at once. Identifying and indexing common patterns and chasing the mappings can be performed offline in a precompilation phase, independently of user queries, thereby speeding up system’s online query performance. Our compact graph representation of the mappings is particularly tailored for GQR, optimizing our solution even more. Our algorithm experimentally performs about 2 orders of magnitude faster than running the standard chase on each view individually and then applying query rewriting using views. Our approach scales to larger numbers of constraints and views and produces smaller, but equivalent, UCQ rewritings (containing less and shorter conjunctive queries), that compute a solution in a scalable way, by 


compile the standard chase on each view individually and then applying query rewriting using views. Our approach scales to larger numbers of constraints and views and produces smaller, but equivalent, UCQ rewritings (containing less and shorter conjunctive queries), that compute the certain answers. For our experimental setting the size of the frugal chased mappings is very close to the core [13, 10] (i.e., the globally minimized chased mappings). Nevertheless, our compact algorithm achieves this output almost 3 orders of magnitude faster than the core chase [10], since we do not rely on minimization.

2. BACKGROUND

Queries. We use rule notation for conjunctive queries. The following query asks for doctors responsible for discharging patients:

\[ q(d) = \text{Doctor}(d), \text{DischargesPatientFromClinic}(d,p) \]

The antecedent of the query is called the body of the query while \( q(d) \), the consequent, is the head. Variables in the head are called distinguished (e.g., \( d \)); the rest are existential variables (e.g., \( p, c \)). A union of conjunctive queries (UCQ) is a set of same-head rules. We denote \( var(S) \) (or \( cons(S) \)) the set of variables (or constants) of a query or a set of predicates \( S \).

Query Containment. Query \( Q_1 \) is contained in query \( Q_2 \), \( Q_1 \subseteq Q_2 \), iff for all databases \( D \) the result of evaluating \( Q_1 \) on \( D \), \( Q_1(D) \), is contained in the result of evaluating \( Q_2 \), that is, \( Q_1(D) \subseteq Q_2(D) \).

Homomorphism. Given two sets of atoms \( S_1, S_2 \), a homomorphism from \( S_1 \) to \( S_2 \) is a function \( h: var(S_1) \cup cons(S_1) \rightarrow var(S_2) \cup cons(S_2) \), such that: (1) \( h \) is the identity on constants, and (2) for all atoms \( A \in S_1 \), \( h(A) \in S_2 \) (a homomorphism \( h \) is extended over atoms, sets of atoms, and queries in the obvious manner). Given two conjunctive queries \( Q_1, Q_2 \), a containment mapping from \( Q_2 \) to \( Q_1 \) is a homomorphism \( h:\text{body}(Q_2) \rightarrow \text{body}(Q_1) \) s.t. \( h(\text{head}(Q_2)) = \text{head}(Q_1) \). It holds that \( Q_1 \subseteq Q_2 \) iff there is a containment mapping from \( Q_2 \) to \( Q_1 \) [9].

Dependencies are used to define schema mappings and target schema constraints. Tuple-generating dependencies (or GLA V rules), are formulas of the form \( \forall \vec{x}, \vec{y} \phi(\vec{x}, \vec{y}) \rightarrow \exists \vec{z} \psi(\vec{x}, \vec{z}) \), with \( \phi \) and \( \psi \) conjunctive formulas over sets of predicates \( A \) and \( B \), and \( \vec{x}, \vec{y}, \vec{z} \) tuples of constants or variables. A TGD with a single predicate in the consequent is called a Global-as-View (GAV) rule. A TGD with a single predicate in the antecedent is called a Local-as-View (LAV) rule [20]. When \( A = B \) are the set of source and target predicates, respectively, the formulas are called source-to-target tuple-generating dependencies (st-TGDs) [14]. When \( A = B \) is the set of target predicates, the formulas are called target TGDs. A second important class of constraints (not considered in this paper) involves equality-generating dependencies (EGDs), which are generalizations of functional dependencies.

Containment under Dependencies. Query \( Q_1 \) is contained in query \( Q_2 \) under a set of constraints \( \Sigma \), denoted \( Q_1 \preceq_{\Sigma} Q_2 \), iff for all databases \( D \) that are consistent with \( \Sigma \), \( Q_1(D) \subseteq Q_2(D) \).

Weakly Acyclic TGDs (wa-TGDs) [14]. Let \( \Sigma \) be a set of TGDs over scheme \( \mathcal{R} = \{ R_1, \ldots, R_n \} \). Construct a directed graph, called the dependency graph, as follows: (1) there is a node for every pair \((R_i, A_i)\) with \( A_i \) an attribute of \( R_i \), call such pair \((R_i, A_i)\) a position; (2) add edges as follows: for every TGD \( \forall \vec{x}, \vec{y} \phi(\vec{x}, \vec{y}) \rightarrow \exists \vec{z} \psi(\vec{x}, \vec{z}) \) in \( \Sigma \), for every \( x \in \vec{x} \) that occurs in \( \phi \) and for every occurrence of \( x \) in \( \phi \) in position \((R_i, A_i)\): (2.1) for every occurrence of \( x \) in \( \phi \) in position \((R_i, B_k)\), add an edge \((R_i, A_i) \rightarrow (R_i, B_k)\) (if it does not already exist), and (2.2) for every existentially quantified variable \( z \in \vec{z} \) and for every occurrence of \( z \) in \( \phi \) in position \((R_i, C_m)\), add a special edge \((R_i, A_i) \rightarrow (R_i, C_m)\) (if it does not already exist). Then \( \Sigma \) is weakly acyclic (wa) if the dependency graph has no cycle with a special edge. LA V Weakly Acyclic TGDs are wa-TGDs that have a single predicate in the antecedent. The Chase is useful to reason with dependencies. Given a conjunction of atoms \( B \) (we will run the chase on both database instances and on consequents of mappings) and a TGD \( \sigma = \forall \vec{x}, \vec{y} \phi(\vec{x}, \vec{y}) \rightarrow \exists \vec{z} \psi(\vec{x}, \vec{z}) \), then \( \sigma \) is applicable to \( B \) with antecedent homomorphism \( h \) iff \( h \) is a homomorphism from \( \phi \) to \( B \) (intuitively, the antecedent holds in \( B \)), such that \( h \) cannot be extended to cover \( \psi \) (i.e., the consequent is not already satisfied). We apply the TGD \( \sigma \) by adding its consequent to \( B \). Formally, a chase step adds \( \psi(h(\vec{x}), f(\vec{z})) \) to \( B \), where \( h \) is the antecedent homomorphism and \( f \) creates “fresh” variables, known as skolens or labeled nulls, for all the existential variables \( \vec{z} \). The standard chase is an exhaustive series of chase steps, and may be finite or infinite depending on the constraints. We denote \( chase_\Sigma(B) \) the result of chasing \( B \) with all constraints in \( \Sigma \). Variations of the chase are the parallel and core chase [10]. Every parallel chase step decides all applicable constraints on \( B \) first, and then adds to \( B \) their consequents. Each core chase step is a parallel step followed by minimization of the result.

Data Exchange. Given a finite source database instance \( I \), a set of st-TGDs \( \Sigma_{st} \), and a set of target dependencies \( \Sigma_t \), the data exchange problem [14] is to find a finite target database instance \( J \), called a solution, such that \( I, J \) satisfy \( \Sigma_{st} \) and \( J \) satisfies \( \Sigma_t \). The certain answers of a query \( q \) on the target schema obtained using the source instance \( I \), denoted by \( certain(q, I) \), is the set of all tuples \( t \) of constants from \( I \) such that for every solution \( J, t \in q(J) \) [14]. For certain classes of TGDs we can reach a rep-
representative solution for the entire space of solutions, called a **universal solution**, which has homomorphisms to all other solutions, and **certain** queries can be computed by issuing q on it [14]. The chase, chases(I), is a sound algorithm for finding universal solutions. The chase with general target TGDs might not terminate, so relevant research has focused on identifying chase-terminating classes of TGDs, such as wa-TGDs, for which the chase is both a sound and a complete algorithm for computing universal solutions.

**(Virtual) Data Integration (VDI)** is the problem of computing the certain answers of a target query by rewriting it as a query over the source schemas and querying the sources directly. Given a query Q, a set of GLAV schema mappings \( M = \{ M_1, \ldots, M_n \} \), and no target constraints, \( Q' \) is a **maximally-contained rewriting** [27] of Q using M if: (1) \( Q' \) only contains source predicates, (2) \( Q' \subseteq Q \), and (3) there is no query \( Q'' \) only containing source predicates, such that \( Q' \subseteq Q'' \subseteq Q \) and \( Q'' \not\subseteq Q' \). When considering target constraints, this definition involves containment under dependencies (\( \subseteq \Sigma \)). In this paper we reduce the query rewriting with constraints problem to query rewriting without target constraints. Our algorithm produces a shorter, yet equivalent, mapping: \( \Sigma' \). Using this query will give the UCQ rewriting:

\[
\text{TreatsPatient}(d, p), \text{HasChronicDisease}(p, s) \quad \text{Doctor}(d), \text{Patient}(p)
\]

**Data Integration with weakly acyclic LAV TGDs.** We are interested in first-order rewritings (i.e., expressible in SQL) since we want to develop a practical solution that leverages scalable relational technology. When considering target constraints in data integration, maximally-contained first-order rewritings do not always exist. We focus on weakly-acyclic LAV constraints, which are first-order rewritable [16, 1]. We discuss other cases of first-order rewritable constraints in section 7. LAV wa-TGDs is a subset of the useful class of weakly-acyclic inclusion dependencies, as well as the class of LAV TGDs with no existential variables (LAV full TGDs), which can express important web ontology languages such as RDF/S. A maximally-contained UCQ rewriting under LAV wa-TGDs, which computes the certain answers, can be obtained by first chasing the views with the target constraints, and then applying a query answering using views algorithm (without constraints) [1].

**Theorem 1. Chasing the Views [1]** Given a query Q on a schema R, a set of LAV schema mappings \( V = \{ V_1, \ldots, V_n \} \) on R, a source instance I under V and a set of weakly-acyclic LAV TGD constraints \( \Sigma \) on R, the set of certain answers \( \text{certain}(Q, \Sigma) \) is computed by the UCQ maximally-contained rewriting of Q using \( \{ V'_1, \ldots, V'_n \} \), where each \( V'_i \in \{ V'_1, \ldots, V'_n \} \) is produced by running the standard chase on the consequent of \( V_i \) using \( \Sigma \).

**Motivating Example.** Consider the following LAV rules describing sources \( S_1 \rightarrow S_2 \) of medical data. \( S_1 \) contains physicians that treat patients with a chronic disease. \( S_2 \) records the physician responsible for discharging a patient from a clinic. \( S_3 \) is the same to \( S_1 \) but physicians are typed as Doctors. \( S_4 \) provides Surgeons.

\[
\begin{align*}
S_1&(d, s) \rightarrow \text{TreatsPatient}(d, p), \text{HasChronicDisease}(p, s) \\
S_2&(d, p, c) \rightarrow \text{DischargesPatientFromClinic}(d, p, c) \\
S_3&(d, s) \rightarrow \text{TreatsPatient}(d, p), \text{HasChronicDisease}(p, s), \text{Doctor}(d) \\
S_4&(d) \rightarrow \text{Surgeon}(d)
\end{align*}
\]

The maximally contained (with no constraints) rewriting of q (presented in the beginning of this section) is: \( q'(d) \rightarrow S_1(d, s), S_2(d, z, c) \).

Now, consider the following (RDF/S) constraints that capture “domain” and “range” properties, and “subclass” relations. Constraint \( c_1 \) states that the domain of TreatsPatient is Doctor and the range is Patient. Constraint \( c_2 \) states that Surgeons are Doctors (as for queries our notation for constraints and views omits quantifiers).

\[
\begin{align*}
c_1 &: \text{TreatsPatient}(x, y) \rightarrow \text{Doctor}(x), \text{Patient}(y) \\
c_2 &: \text{Surgeon}(x) \rightarrow \text{Doctor}(x)
\end{align*}
\]

**Theorem 1** guarantees that we can answer query q, using sources \( S_1 \rightarrow S_4 \), by first chasing the consequents of the views and then looking for maximally-contained rewritings of the query using the chased views. Running the chase on \( S_1 \rightarrow S_4 \) using \( c_1 \) and \( c_2 \) yields:

\[
\begin{align*}
S_1&(d, s) \rightarrow \text{TreatsPatient}(d, p), \text{HasChronicDisease}(p, s), \text{Doctor}(d), \text{Patient}(p) \\
S_2&(d, p, c) \rightarrow \text{DischargesPatientFromClinic}(d, p, c) \\
S_3&(d, s) \rightarrow \text{TreatsPatient}(d, p), \text{HasChronicDisease}(p, s), \text{Doctor}(d), \text{Patient}(p) \\
S_4&(d) \rightarrow \text{Surgeon}(d), \text{Doctor}(d)
\end{align*}
\]

The maximally-contained rewriting of q using \( S_1 \rightarrow S_4 \) is the UCQ:

\[
q'(d) \rightarrow (S_1(d,x), S_2(d,y,z)) \lor (S_3(d,u), S_2(d,w,v)) \lor (S_4(d), S_5(d,s,t)).
\]

**3. OPTIMIZING THE CHASE**

Often, in the presence of existential variables/labeled nulls, some of the consequences of a chase rule are already implied in the database. That is, although the rule is not satisfied (i.e., there is no homomorphism from the consequent to the database instance that is an extension of the antecedent homomorphism), the consequent might be partially satisfiable; adding a subset of the consequent’s atoms can construct such a homomorphism from the entire consequent. In this section we introduce the *frugal* chase, which is equivalent to the standard chase, but results in smaller universal solutions.

Consider the simple data exchange scenario of Fig. 1 with a source described by \( S_1 \) and a target constraint \( c_3 \) (cf. Sect. 2). Existential variables in the constraint introduce additional labeled null tuples which are redundant (nulls do not participate in the certain answers of a query). The bottom 3 rows of TreatsPatient and Doctor can be removed. Our frugal chase avoids adding such redundant facts.

Before we define the frugal chase, let us introduce some useful notions. Let \( B \) a database instance (or any other conjunction of atoms such as the consequent of a TGD). The *Gaifman graph of facts* [13] is an undirected graph with nodes being the existential variables (labeled nulls) of \( B \); edges between nodes exist if the variables co-exist in an atom in \( B \). Dually, the *Gaifman graph of facts* of \( B \), denoted \( grf(B) \), is the graph whose nodes are the atoms of \( B \) and two nodes are connected if they share existential variables/labeled nulls. Note that parts of \( B \) that are connected only through constants (as well as distinguished variables for TGDs), constitute different connected components of the Gaifman graph of facts. For \( G_t \) a connected component of \( grf(B) \), \( V(G_t) \) is the
Figure 1: Standard vs Frugal Chase.

set of all the facts (i.e., the nodes) in $G_1$. For any conjunction of atoms $B$ we denote the decomposition of $B$ to (the facts of) its connected components $\{G_1, \ldots, G_n\}$ as the set of sets (of facts) $\text{dec}(B) = \{V(G_1), \ldots, V(G_n)\}$. Fig. 2 shows in dotted circles the different connected components of TGD c and instance B.

A constraint can be decomposable to an equivalent set of “simpler” constraints, each with a different element of “simulation” as consequent. For example, $C_3$ in Fig. 1 can be broken down to two constraints with consequents $\{\text{TreatsPatient}(d,p) , \text{Doctor}(d)\}$ and $\{\text{Patient}(p)\}$ respectively. For Fig. 1, applying the standard chase using the new set of constraints would also avoid the redundancies presented. Nevertheless, our frugal chase produces smaller chase results, even with non-decomposable constraints.

Informally, a set of predicates is partially satisfiable, and not added during the frugal chase application, in two cases: if it is a connected component of the constraint and is mapped to the instance as a whole, or if its image on the database instance is a connected component of new “fresh” labeled nulls. In the example of Fig. 2, if we choose to satisfy $\{P_1(x_1, z_1), P_2(x_1, z_2), P_3(x_2, z_3)\}$, which maps $z_3$ to a different variable, $n_5$ (according to Def. 1) there needs to be a single satisfying homomorphism $h'$ which maps all elements of a partially satisfiable set. Hence, the set of partially satisfiable predicates of a constraint might not be unique; in Fig. 2 there are two equivalent alternative frugal chases, given by $h'$ and $h''$. Function $h$ in $h''$ maps variables that are not in the domain of the satisfying homomorphism to fresh names. This non-uniqueness does not cause a problem for the end chase result; as we prove later, all frugal chase results are universal solutions. Currently our algorithms choose one arbitrary partially satisfiable set (our implementation chooses the first it discovers). Nevertheless, one could develop heuristics to materialize the preferred alternative, e.g., the shorter chase or one depending on the application.

The frugal chase is applicable whenever not all atoms of the consequent are in a partially satisfiable set; by Lemma 1 this is equivalent to the standard chase applicability (proof omitted due to space).

**Lemma 1.** Let $\sigma$ be a TGD constraint $\forall \vec{x}, \vec{y} . \phi(\vec{x}, \vec{y}) \rightarrow \exists \vec{z} . \psi(\vec{x}, \vec{z})$ and $B$ an instance $\sigma$. $\sigma$ is applicable to $B$, per the standard chase, iff there exists an antecedent homomorphism $h$ that maps $\phi(\vec{x}, \vec{y})$ to $B$. A set of atoms $S \subseteq \psi$ is partially satisfiable for $h$ if there exists an extension of $h$, $h'$, called a satisfying homomorphism for $S$, s.t. $h'(S) \subseteq B$ and for each $S_i \in \text{dec}(S)$, either:

1. $(a)$ for all existential variables $\vec{z}$ in $S_i$, $h'(z_i)$ is an existential variable (or labeled null) in $B$, and
2. $S_i \in \text{dec}(\psi)$, i.e., the mapped set $S_i$ is actually an entire connected component of the constraint consequent.

We illustrate Def. 1 with the example in Fig. 2, which shows how the frugal chase applies constraint $c$ on instance $B$ ($\Phi$ is a conjunction of atoms, $c$ and $d$ are constants, $x_1$ and $z_1$ are variables, $n_1$ are labeled nulls, and $P_i$ are relations). Consider whether the set of predicates $S = \{P_1(x_1, z_1), P_2(x_1, z_2), P_3(x_1, z_3), P_4(x_2, z_4)\}$, a subset of the consequent of constraint $c$, is partially satisfiable. The decomposition $\text{dec}(S)$ has two elements (connected components): $S_1 = \{P_1(x_1, z_1), P_2(x_1, z_2)\}$ and $S_2 = \{P_3(x_1, z_3), P_4(x_2, z_4)\}$. Per Def. 1, for our $S$ there exists a homomorphism $h'$ which extends the one from the antecedent ($h$), such that $h'(S) \subseteq B$. Moreover, $S_1$ is an entire connected component of the constraint (falling in case 2 of Def. 1), while for $S_2$ all its existential variables are mapped through $h'$ (and in particular $z_2$) to existential variables (case Def. 1.1(a)), and $h'(S_2)$ is a connected component of $B$ (case Def. 1.1(b)). So $S$ is partially satisfiable. Note that $P_4(x_2, z_4)$ in the constraint could be partially satisfiable by mapping it to

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**Figure 2: Frugal Chase: Partially Satisfiable Set.**

$P_4(x_1, z_4)$ in $B$: however not at the same time as $\{P_1(x_1, z_1), P_2(x_1, z_2), P_3(x_1, z_3), P_4(x_2, z_4)\}$, which maps $z_4$ to a different variable, $n_5$ (according to Def. 1) there needs to be a single satisfying homomorphism $h'$ which maps all elements of a partially satisfiable set. Hence, the set of partially satisfiable predicates of a constraint might not be unique; in Fig. 2 there are two equivalent alternative frugal chases, given by $h'$ and $h''$. Function $h$ in $h''$ maps variables that are not in the domain of the satisfying homomorphism to fresh names. This non-uniqueness does not cause a problem for the end chase result; as we prove later, all frugal chase results are universal solutions. Currently our algorithms choose one arbitrary partially satisfiable set (our implementation chooses the first it discovers). Nevertheless, one could develop heuristics to materialize the preferred alternative, e.g., the shorter chase or one depending on the application.

The frugal chase is applicable whenever not all atoms of the consequent are in a partially satisfiable set; by Lemma 1 this is equivalent to the standard chase applicability (proof omitted due to space).

**Lemma 1.** Let $\sigma$ be a TGD constraint $\forall \vec{x}, \vec{y} . \phi(\vec{x}, \vec{y}) \rightarrow \exists \vec{z} . \psi(\vec{x}, \vec{z})$ and $B$ an instance $\sigma$. $\sigma$ is applicable to $B$, per the standard chase, iff there exists an antecedent homomorphism $h$ that maps $\phi(\vec{x}, \vec{y})$ to $B$. A set of atoms $S \subseteq \psi$ is partially satisfiable for $h$ if there exists an extension of $h$, $h'$, called a satisfying homomorphism for $S$, s.t. $h'(S) \subseteq B$ and for each $S_i \in \text{dec}(S)$, either:

1. $(a)$ for all existential variables $\vec{z}$ in $S_i$, $h'(z_i)$ is an existential variable (or labeled null) in $B$, and
2. $S_i \in \text{dec}(\psi)$, i.e., the mapped set $S_i$ is actually an entire connected component of the constraint consequent.

We illustrate Def. 1 with the example in Fig. 2, which shows how the frugal chase applies constraint $c$ on instance $B$ ($\Phi$ is a conjunction of atoms, $c$ and $d$ are constants, $x_1$ and $z_1$ are variables, $n_1$ are labeled nulls, and $P_i$ are relations). Consider whether the set of predicates $S = \{P_1(x_1, z_1), P_2(x_1, z_2), P_3(x_1, z_3), P_4(x_2, z_4)\}$, a subset of the consequent of constraint $c$, is partially satisfiable. The decomposition $\text{dec}(S)$ has two elements (connected components): $S_1 = \{P_1(x_1, z_1), P_2(x_1, z_2)\}$ and $S_2 = \{P_3(x_1, z_3), P_4(x_2, z_4)\}$. Per Def. 1, for our $S$ there exists a homomorphism $h'$ which extends the one from the antecedent ($h$), such that $h'(S) \subseteq B$. Moreover, $S_1$ is an entire connected component of the constraint (falling in case 2 of Def. 1), while for $S_2$ all its existential variables are mapped through $h'$ (and in particular $z_2$) to existential variables (case Def. 1.1(a)), and $h'(S_2)$ is a connected component of $B$ (case Def. 1.1(b)). So $S$ is partially satisfiable. Note that $P_4(x_2, z_4)$ in the constraint could be partially satisfiable by mapping it to
**Proof.** Let $c_1, c_2, \ldots$ be an ordering of constraints (possibly repeating with different antecedent homomorphisms) in the frugal chase application. Let $f\text{chase}_{c_2}(B)$ and $\text{chase}_{c_1}(B)$ the results of the frugal and the standard chase resp. on $B$ with $c_1, f\text{chase}_{c_2}(B)$ and $\text{chase}_{c_1}(B)$ are homomorphically equivalent, by Th. 2. Let $g_1$ the homomorphism from $\text{chase}_{c_1}(B)$ to $f\text{chase}_{c_2}(B)$. Let a constraint $c_1$ satisfied, per the standard chase, on $\text{chase}_{c_2}(B)$, with satisfying homomorphism $g_2$. Composing $g_2$ with $g_1$, proves that the constraint is also satisfied on $f\text{chase}_{c_2}(B)$. Hence, $f\text{chase}_{c_2}(B)$ triggers at most as many constraints as $\text{chase}_{c_1}(B)$ does. Also, assuming $c_2$ is applicable to $f\text{chase}_{c_2}(B)$, it is also to $\text{chase}_{c_1}(B)$, with the same antecedent homomorphism. Moreover, after the application of $c_2$, $f\text{chase}_{c_2}(f\text{chase}_{c_2}(B))$ is homomorphically equivalent to $\text{chase}_{c_2}(\text{chase}_{c_1}(B))$ (proof is omitted due to space, but is very similar to our appendix proof). Hence, inductively, if the standard chase terminates on $B$ with $\Sigma$ so does the frugal chase with $f\text{chase}_{c_2}(B)$ satisfying all constraints and being a solution. In order to show that it is universal, i.e. has homomorphisms to all other solutions, we note that it has a homomorphism to $\text{chase}_{c_2}(B)$ which in turn has homomorphisms to all other solutions.

The intuition behind our frugal chase is taking care that when adding a non-partially-satisfiable predicate, we will not introduce a join among the constraint and some relation, or some constant, in $B$ which is not in the constraint (such a join would not be introduced by the standard chase). This case is avoided when satisfying an entire connected component of the constraint since there is nothing else (existentially) joining with that component in the consequent to be added (e.g., predicates $P_1$ and $P_2$ of $c$ in Fig. 2). If our set of satisfied predicates (e.g., $P_4$ and $P_5$ of $c$ in Fig. 2) is not an entire connected component of the consequent, its image has to be; otherwise whatever these predicates join with in the constraint (which we will add to our instance), will “accidentally” join with whatever joins with their images. For example, consider:

$$c_4: P(x), R(x, z) \rightarrow P_1(x, y, z), P_2(y, w), P_3(y)$$

$$B = \{P(d), R(d, c) \rightarrow P_1(d, y, c), P_2(y, w), P_3(y)\}$$

Running the standard chase with $c_4$ on $B$ produces:

$$B' = \{P(d), R(d, c) \rightarrow P_1(d, y, c), P_2(y, w), P_3(y)\}$$

Had we assumed partial satisfiability for both $P_1, P_2$ we would get:

$$B'' = \{P(d), R(d, c) \rightarrow P_1(d, y, c), P_2(y, w), P_3(y)\}$$

$B'$ and $B''$ are not equivalent since there is no homomorphism from $B'$ to $B''$, since the join of $P_2P_3$ in $B''$ cannot be preserved. If $P_3$ was missing from $B$ (we would fall in case 1: all joins of $y_1$ in $B$ would be with images of partially satisfiable predicates) or if $P_3$ was missing from $c_4$ (we would fall in case 2: the entire connected component in the constraint is partially satisfiable), then we would end up in $B''$ and $B'$ either both missing $P_3$ or both missing $P_4$ in which case they would be homomorphically equivalent.

**Complexity.** The problem of deciding whether the standard chase is applicable on an instance is polynomial in data complexity [14]. The difference to our case is essentially case 1(b) of Def. 1, which introduces a polynomial traversal of the corresponding connected component of the database, for all $S_k$. So, the frugal chase remains polynomial in data complexity. Notice that our definitions care for only one (arbitrarily chosen) partially satisfiable set. Potentially, we could exhaustively examine all subsets of a constraint’s consequent; still polynomial in data complexity. Moreover, as our experiments attest for LAV constraints, checking partial satisfiability and running the frugal chase is faster than the standard in practice.

4. **PROCEDURAL FRUGAL CHASE**

In preparation to use the frugal chase in query rewriting using GQR, we present an alternative definition that examines each atom separately and decides whether it is partially satisfiable.

**DEF. 2.** Let $\sigma$ be a TG constraint $\forall x, y, z, \phi(x, z, y) \rightarrow \exists w \psi(x, z, 2)$, and $B$ an instance $s.t.$ there exists a homomorphism $h: \phi(x, z, y) \rightarrow B$. For all atoms $P(x, z, y) \in \psi(x, z, 2)$, $P(x, z, y)$ is partially satisfiable for $h$ if there exists an extension of $h$, $h'$, called a satisfying homomorphism for $P$, s.t. $h'(P) \subseteq B$ and for all $z \in 2$:

1. if $h'(z)$ is an existential variable (labeled null for instances) then for every atom $R_2$ in $B$ that contains $h'(z)$, there is an atom $R_2$ in the constraint that contain $z$, in the same argument positions, s.t.: (a) $R_2$ is partially satisfiable for $h'$ (its satisfying homomorphism is an extension of $h'$), (b) $R_2$ is the image of $R_2$ through the satisfying homomorphism for $R_2$, and (c) for all $R(z, y) \in \psi(z, 2)$ that contains $z$, if $R$ is partially satisfiable for $h$, it is also partially satisfiable for $h'$ (which, recursively, means it uses an extension of $h'$);

2. if $h'(z)$ is a constant (for instances or mappings), or (b) a distinguished variable (for mappings), or (c) an existential variable (labeled null for instances) which does not fall into case 1(iii) above (i.e., it joins with at least one atom which is not the image of a partially satisfiable atom that joins with $P$ on $z$ in the same argument positions), then all atoms in the connected component of $grf(\psi)$ that is in, are partially satisfiable for $h'$ (which means they use extensions of $h'$).

**THEOREM 4.** A set $S$ of atoms is partially satisfiable per Def. 1, iff every atom in $S$ is partially satisfiable per Def. 2.

**Proof.** It is not hard to see that cases 1(a), 1(b) and 2 of Def. 2 correspond to the same cases of Def. 1. However Def. 2 considers “atomic” satisfying homomorphisms over single atoms. We need to make sure that these homomorphisms can be unified to construct one from the entire set of partially satisfiable atoms (i.e., $h'$ of Def. 1). Since every atomic satisfying homomorphism in Def. 2 extends $h$ (that maps the antecedent variables to $B$) we really need to examine only existential variables. Notice that all partial satisfiable atoms (per Def. 2) that share existential variables need to fall into the same case of Def. 2. For partially satisfiable atoms in the constraint, that share existential variables and fall in case 2 of Def. 2, their satisfying homomorphisms agree and are essentially the same, since the predicates belong in the same connected component of $grf(\psi)$. For partially satisfiable atoms, that share existential variables, and fall in case 1 of Def. 2, case 1(c) takes care that their homomorphisms agree on their common values.

With respect to Def. 2, we define an applicable homomorphism, the frugal chase step and the frugal chase similarly to Def. 1.

To illustrate Def. 2, we run the frugal chase on a view (instead of an instance). For views, distinguished variables are interpreted as constants in our homomorphisms, for both the standard and the frugal chase. Consider the following non-decomposable constraint:

$$c_5: P(x, v), R(v, t) \rightarrow P_1(x, y, z), P_2(y, w)$$

And view $V_5 = S_5(x_1) \rightarrow P_1(x_1, y_1, z_1), P_2(x_1, w_1)$.

The standard chase algorithm run with $c_5$ on $V_5$ will produce:

$$S_5''(x_1) \rightarrow P_1(x_1, y_1, z_1), P_2(x_1, w_1), P_3(x_1, y_1, z_1), P_4(y, w)$$

Nevertheless a shorter, equivalent mapping per our chase is:

$$S_5''(x_1) \rightarrow P_1(x_1, y_1, z_1), P_2(x_1, w_1), P_3(x_1, y_1, z_1), P_4(y, w)$$

When considering whether $P_1(x, y, z)$ in $c_5$ is partially satisfiable, we notice that $y$ and $z$ can only map to don’t care variables in $S_5$, namely $y_1$ and $z_1$ respectively (hence case 1(b) of Def 2 trivially applies). Case 1(c) of Def 2 dictates that we can partially satisfy $P_1(x, y, z)$ as long as $y$ and $z$ are not being mapped to different variables than $y_1$ and $z_1$, in some other atom’s satisfying homomorphism. This means that $P_2(y, w)$ in $c_5$ cannot be partially satisfiable when $P_1(x, y, z)$ is, since in order to partially satisfy...
we examine $x$. In fact if we examine $P_2(y,w)$ for partial satisfaction before we examine $P_1(x,y,z)$, we find another reason for which $P_2(y,w)$ cannot be partially satisfiable: variable $y$ can only map to the distinguished variable $x$ in $S_2$ and, per case 2, we check whether this mapping can be extended to cover the atoms of $c_5$ joining (directly or indirectly) with the existential variables of $P_2(y,w)$, in effect $P_1(x,y,z)$, such an extension cannot happen as for $P_1$ in $c_5$, $y$ has to map to $y_1$ in $S_3$ rather than $x$. Hence, $P_2(y,w)$ in $c_5$ cannot be partially satisfiable for $S_2$; only $P_1(x,y,z)$ can and $P_2$ needs to be added explicitly, when applying the rule.

The frugal chase is applicable to all cases of TGD languages in which the standard chase applies. It yields smaller universal solutions in data exchange, smaller chased mappings in data integration (which lead to smaller rewritings using these mappings), smaller database instances in incomplete databases, and smaller chased queries for query containment (ucq containment under constraints) can sometimes be done by chasing one ucq and relying on classic containment [22, 25]). However, our chase does not produce a minimal solution, i.e., the core [13]. Using the frugal chase, one could explore all combinations of partially satisfiable atoms and keep the maximum set, but even then, pre-existing redundancies in the instance or the constraints are not accounted for.

5. USING A COMPACT CHASE FOR QUERY REWRITING UNDER CONSTRAINTS

In this section, we present our approach to compute the maximally-contained UCQ rewriting of a target query under target LAV weakly-acyclic dependencies and LAV views (i.e., the perfect rewriting [4]), using the frugal chase. First, we show (Th. 5) that our algorithm computes the certain answers (this is the equivalent of Th. 1 [11] for the frugal chase). Second, we describe a graph representation for queries, mappings (views), and now constraints, that we introduced in GQR [24]. This representation allows us to compactly represent common subexpressions in the views and constraints, and efficiently reason with them. Third, we briefly describe the GQR query rewriting approach. Finally, we describe a compact graph-based version of the frugal chase that radically improves the compilation of the constraints into the (graph representation of) the views.

**Theorem 5.** Given a query $Q$ on schema $T$, a set of LAV schema mappings $V = \{V_1, ..., V_n\}$, a source instance $I$ under $V$, and a set of LAV weakly-acyclic TGD constraints $\Sigma$, the set of certain answers $\text{cert}(Q, I)$ is computed by the UCQ maximally-contained rewriting of $Q$ using $\{V_1', ..., V_n'\}$, where each $V_i'$ is produced by running the frugal chase on $V_i$ using $\Sigma$.

**Proof.** Consider the set of views $V'' = \{V'_1, ..., V'_n\}$, which are taken by running the standard chase on the consequent of $V_i$ using $\Sigma$. We know by Th. 1 that a maximally-contained rewriting of $Q$ using $V''$ (denoted $\text{MCR}(Q, V'')$) produces the certain answers of the query. The view sets $V''$ and $V'$ are equivalent since for every view in one set there is an equivalent one in the other, by Th. 3. Two equivalent sets of views produce equivalent maximally contained rewritings, hence in our case $\text{MCR}(Q, V'') \equiv \text{MCR}(Q, V')$. This means that for each conjunctive query $r'' \in \text{MCR}(Q, V'')$ there is a $r' \in \text{MCR}(Q, V')$, s.t. $r'' \subseteq r'$. Let a tuple $t \in \text{cert}(Q, I)$ and so $t \in \text{MCR}(Q, V'')(I)$ and in particular $t \in r''(I)$. Then $t \in r'(I)$ and hence $t \in \text{MCR}(Q, V')$. So every certain answer is computed by $\text{MCR}(Q, V')$. Symmetrically, for any tuple $t$ we obtain from $\text{MCR}(Q, V')$, with the same reasoning it holds that $t \in \text{MCR}(Q, V'')$ and hence is a certain answer.

5.1 Graph Modeling

We represent queries, mappings, and constraints as graphs (extending [24]). Predicates and their arguments map to graph nodes. Predicate nodes are labeled with the name of the predicate, and are connected through edges to their arguments. Shared variables between atoms result in shared variable nodes, directly connected to predicate nodes. Edges have integer labels that represent the argument positions. We discard variables’ names, since to decide on a partial satisfiability and rewriting we only need the type (distinguished or existential) of the variables involved. For example, Figs. 3(a), (b) and (e) show the graph representation of a query $q$ ($q(d,e)\rightarrow d(d), DPFC(d,z,e)$), LAV views $S_1$, $S_2$ and $S_3$, and constraint $c_1$ (cf. Sect. 2), respectively. Distinguished variable nodes are depicted with a circle, and existential ones with the symbol $\ominus$.

Our algorithms consist of mapping subgraphs of the query/constraints to subgraphs of the views. The smallest subgraphs we consider consist of one central predicate node and its (existential or distinguished) variable nodes. We call these primitive graphs predicate join patterns (PJs). Figs. 3(d) shows all PJs for sources $S_1$, $S_2$ and $S_3$. Because the join conditions for a particular PJ within each view are different, some “bookkeeping” is needed to capture these joins. For each variable node, we record the joins it participates in across the views in a “joinbox”. Figure 3(c) shows the joinbox for the second variable of the TP (TreatsPatient) PJ, which records that this variable joins with the first argument of HCD in $S_1$ and in $S_3$. A critical feature of this representation is that the graph patterns of predicates repeat in multiple views, so we compactly represent all the occurrences of a pattern across different views with one PJ. This has a tremendous advantage; mappings from a query/constraint PJ to a view one are computed just once instead of every time this predicate is met in a view. For example, the 6 predicates in the sources of Fig. 3(b) correspond to only 4 PJs, in Fig. 3(d), PJs can be constructed straightforwardly in polynomial time by scanning each LAV view and its joins, and hashing the different patterns encountered. We also index the patterns to retrieve them efficiently.

5.2 Graph-based Query Rewriting (GQR)

In its original version [24], GQR has a pre-processing/off-line phase, where it extracts the PJs from the views and indexes them. At query time, GQR (cf. [24] for a detailed description) processes the user query one subgoal at-a-time. It retrieves the view PJs that match each query subgoal and incrementally combines the retrieved view PJs to form larger subgraphs that cover larger portions of the query, building the maximally-contained rewriting incrementally. For example, given the query in Fig. 3(a), and the PJs in Fig. 3(b), Fig. 4(a) shows GQR retrieving the PJs corresponding to query predicates $D$ and $DPFC$, and combining them into a single graph (Fig. 4(c)). Since the combined graph covers the query, the process terminates and outputs the logical rewritings (Fig. 4(c)).

In this paper, GQR takes as input not the original PJs from the sources, but the PJs chased with the target constraints using the compact frugal chase described next. As discussed, we chase the LAV wa-TGDs into the mappings and reduce the problem to query rewriting (without constraints) using the chased views. Fig. 3(h) shows the view PJs (resulting from the frugal chase) of views $S_1'$, $S_2'$ and $S_3'$ with constraint $c_1$ (from Sect. 2). The overall result is an efficient algorithm for query rewriting under constraints.

5.3 Compact frugal chase

This section presents our compact frugal chase implementation for chasing a set of LAV mappings using a set of LAV weakly acyclic dependencies. Instead of considering every view subgoal, our compact frugal chase considers the distinct patterns (PJs) that
all views contain. We start by finding mappings of the antecedent
to atomic PJs, that repeat themselves across views and hence com-
 pactly represent pieces of multiple views (which imply the same
constraint antecedent). Finding a homomorphism to such a pattern
means triggering the constraint for all the views represented by
this pattern. Then we recursively map all partially satisfiable sub-
goals of a rule’s consequent, to the compact graph representation
of these atoms in the mappings. When we apply the chase step we
add compact consequent patterns (constraint PJs) that represent
the addition of a predicate to multiple views simultaneously. We end
up in a compact PJ representation of the chased mappings, shorter
than the standard chased ones, which leads to smaller yet equiva-
 lent rewritings of the input queries using these mappings. As noted,
the output of our compact chase algorithm is using the same PJ rep-
resentation as its input, and is particularly tailored for GQR [24],
improving our solution even more. Our initial implementation does
not include constants in the queries, the views or the constraints.
Alg. 1 executes all constraints repeatedly, until it converges and
stops adding new predicates. It takes as inputs the view PJs, con-
structed and indexed, representing our original set of views, and the
set of constraints. The output of the algorithm is the set of chased
PJs. The algorithm first maps the antecedent predicate to a view PJ
in order to trigger the constraint for all views in this view PJ. Then,
it examines each consequent’s predicate, for all applicable views
triggering the constraint; some existing view PJ (that matches a
consequent PJ’s pattern) might contain some of the relevant views
and might prove the corresponding consequent predicate already
partially satisfiable. For the applicable views that are left not sats-
ifying a consequent predicate, we will have to add a copy of the
consequent PJ as a new view PJ to our index (our set of view PJs
will now correspond to the longer, frugal-chased mappings). Us-
ing this control flow, Alg. 1 is our high-level algorithm and keeps
track of the constraints, and the PJs that satisfy them, for specific
views. To check if a specific view PJ is the image of a constraint
PJ, for one of the views it contains, it calls upon Alg. 2, which in
turn checks the cases of Def. 2, by calling Alg. 4; the latter iterates
over connected components of the views or the constraint respec-
tively. Algorithm 4 calls Alg. 3 which, after examining whether a
constraint and a view PJ have been processed before, calls again
Alg. 2. Hence, algorithms 2, 4, and 3 are mutually recursive.
In line 2, Alg. 1 iterates over all constraints, and in line 3 finds all
the different view PJs that match the antecedent (denoted \( P_{\text{ant}} \)),
for a specific constraint. For each \( P_{\text{ant}} \), line 5 constructs new PJs
for all constraint consequents. For each consequent PJ, “initializa-
tion” means the following (pseudo-code omitted due to space):
1) Depending on the pattern of \( P_{\text{ant}} \), the variable nodes in a con-
sequent PJ become distinguished or existential (initially constraint
node types are undetermined). Fig. 3(f) shows that when we trig-
er constraint \( c_i \) for the antecedent PJ for \( TP \), the variables of the
constraint (originally of unknown (?) type) take a specific type. We
change to distinguished all variable nodes that have been
“unified” with distinguished variable nodes; the rest are existential.
2) For all views triggering the constraint (\( \text{ViewSet} \)), we include
joins in the consequent PJ. Such joins include all joins to
other consequents (which eventually become part of every view),
as well as the joins inherited from any variable shared with \( P_{\text{ant}} \).
In effect, when we trigger constraint \( c_i \), in Fig.3(e), for the view
PJ TP in Fig.3(d), after line 5 we have constructed the two con-
sequent PJs shown in Fig.3(f), which capture the addition of pred-
icates “P” and “D” in the views they contain. Subsequently we
need to examine if for some of these views there are pre-existing
PJs satisfying the consequent PJ’s predicates. To check this, for
every consequent PJ, \( P_{\text{con}} \), we query our index for view PJs that
capture its pattern (it could be a “more” general pattern, i.e., one
that has distinguished variables in places that the consequent PJ
has existential ones) (lines 6-7). Retrieving such a view PJ, we
need to examine it for all views common to the “triggering” views
(line 8). The main method to check whether the predicate of \( P_{\text{con}} \)
is partially satisfied by a relevant view \( s \) in a view PJ, \( P_P^{TV} \), is
\( \text{isPredSatisfiable} \) (line 16), which is a recursive implementation
of Def. 2. Due to its recursive nature, some previous calls of
\( \text{isPredSatisfiable} \) might have traversed the connected component
of the constraint and might have already memoized the specific
combination, of PJs and view currently in question, as being
“satisfiable” or “unsatisfiable”.
If we find this combination satisfiable, we avoid calling the method
again (line 9), and we call updateJoinboxes (line 10) which up-
dates several join pointers on our view and constraint PJs. It first
transfers the joinbox information for the specific view \( s \) from the

Figure 3: (a) Query \( q \). (b) Sources \( S_1-S_3 \) as graphs. (c) Joinbox for a variable node. (d) View PJs for \( S_1-S_2 \) with their joinboxes. (e) Constraint \( c_1 \) as a graph. (f) PJs resulting from chasing PJ TreatsPatient (TP) with the constraint \( c_1 \). (g) Merging existing and inferred PJs. (h) Chased view PJs.

Figure 4: Graph-based Query Rewriting.
Algorithm 1: Compact Frugal Chase

**Input:** An indexed set of view PJs, a set of LAV-WA constraints $\Sigma$

**Output:** Chased PJs after the frugal chase converges for $\Sigma$

1: while Index of view PJs changes do
2:   for all $\sigma \in \Sigma$ : $P(x, y) \rightarrow C_1(x, z), ..., C_n(x, z)$ do
3:      if $P J'_{\text{ant}}$ ← antecedent PJ in views do
4:         $\text{ViewSet} \leftarrow$ views in $P J'_{\text{ant}}$ not used to trigger $\sigma$
5:      initialize constraint PJs for $\text{ViewSet}$
6:   for all constraint PJs $P J^C_{\text{ci}}$ do
7:      for all $P J_{\text{ci}}$ ← view PJs capturing pattern $P J^C_{\text{ci}}$ do
8:         for all view $s$ common to $\text{ViewSet}$ and $P J_{\text{ci}}$ do
9:            if $P J^V_{\text{ci}}$ is marked as satisfiable for $\sigma$, $P J^C_{\text{ci}}$, and $s$ then
10:               updateJoinboxes($P J^V_{\text{ant}}, P J^C_{\text{ci}}, P J^C_{\text{ci}}, \sigma, s$)
11:         if $P J^C_{\text{ci}}$ “satisfied” for all views in $\text{ViewSet}$ then
12:            continue to the next consequent PJ (line 6)
13:      continue (line 8)
14:   mark $P J^C_{\text{ci}}$ as satisfiable for $\sigma$, $P J^C_{\text{ci}}$, and $s$
15:   if $\text{isPJ}Satisfiable}(P J^V_{\text{ant}}, P J^C_{\text{ci}}, P J^C_{\text{ci}}, \sigma, s, ST)$ then
16:      updateJoinboxes($P J^V_{\text{ant}}, P J^C_{\text{ci}}, P J^C_{\text{ci}}, \sigma, s$)
17:   if $P J^C_{\text{ci}}$ has been “satisfied” for all views in $\text{ViewSet}$ then
18:      continue to the next consequent PJ (line 6)
19:   continue (line 8)
20: else mark $P J^C_{\text{ci}}$ as unsatisfiable for $\sigma$, $P J^C_{\text{ci}}$, and $s$
21: add $P J^C_{\text{ci}}$ to Index, update joinboxes of affected PJs

Algorithm 2: isPJ\text{Satisfiable}(P J^V, P J^C, s, ST)

**Input:** A pre-existing view PJ $P J^V$, a constraint PJ $P J^C$, view $s$, the state $ST$ of this recursion so far

**Output:** true if the predicate in $P J^V$ for view $s$ is partially satisfiable in $P J^C$

1: if $P J^V$ is marked unsatisfiable for $\sigma$, $P J^C$ and view $s$ then
2:   return false
3: for all $V_k$ ← node on edge $k$ of $P J^V$ do
4:   $\text{joins}_{V_k} \leftarrow$ the joinbox joins for $s$ from $V_k$
5: $C_k$ ← node on edge $k$ of $P J^C$
6: $\text{joins}_{C_k} \leftarrow$ the joinbox joins in $C_k$
7: if $C_k$ is an antecedent variable in the constraint then
8:   if joins with antecedent in $\text{joins}_{C_k} \not\subseteq \text{joins}_{V_k}$ then
9:      return false
10: else
11:   if $V_k$ is a distinguished variable then
12:      if $ST = 1$ then
13:         return false
14:      $ST \leftarrow 2$
15:      if $\text{checkCase}2(\text{joins}_{V_k}, \text{joins}_{C_k}, s, ST)$ then
16:         return false
17: else
18:   switch (ST)
19:   case(2):
20:      if $\text{checkCase}2(\text{joins}_{V_k}, \text{joins}_{C_k}, s, ST)$ then
21:         return false
22:      case(1):
23:         if $\text{checkCase}1(\text{joins}_{V_k}, \text{joins}_{C_k}, s, ST)$ then
24:         return false
25:      case(UNDECIDED):
26:         $ST \leftarrow 1$
27:      if $\text{checkCase}2(\text{joins}_{V_k}, \text{joins}_{C_k}, s, ST)$ then
28:         if $ST = 1$ then
29:            for all $P J'_{\text{ci}}$ ← joined predicates in $\text{joins}_{C_k}$ do
30:               if $P J'_{\text{ci}}$ is marked as satisfiable for $s$, but maps $V_k$ to a different variable than $C_k$ then
31:                  return true
32:      return true

Checking partial satisfiability. In order to check whether $P J^V$ can partially satisfy $P J^C$ for $s$, Alg. 2 is called. This returns false if at some other point this combination has been marked as unsatisfiable (lines 1-2). Dictated by Def. 2, the partial satisfiability check happens on a variable per variable basis (line 3), for the specific view $s$, and Alg. 2 returns true if no variable check fails (line 35).

If the constraint variable, $C_k$, is an antecedent variable, we only need to check that the corresponding view PJ variable, $V_k$, joins with the antecedent view PJ in the positions that $C_k$ joins with the antecedent constraint PJ (lines 7-9) (i.e., the variables satisfy the constraint antecedent homomorphism). If $C_k$ is existential, but $V_k$ is distinguished we have to be in case 2 (lines 11-16). We check this by calling, in line 15, checkCase2 (shown in Alg. 4) which essentially checks that the joined predicates, to the constraint variable, can be mapped to some of the joined predicates of $V_k$. This ends up in recursively considering the connected component of the constraint that $C_k$ is in. On the other hand, checkCase1, omitted due to space but almost identical to Alg. 4 (just the outer for loops are reversed), considers that all joined predicates of $V_k$, i.e., the corresponding connected component of the view, are images of constraint predicates joining with $C_k$.

If $V_k$ is existential then depending on the current state (lines 18-24), we will check to make sure that $P J^V_{\text{ci}}$ can still partially satisfy area/consequent due to the view PJ. Fig. 3(g) shows that the pre-existing PJ for Doctor in Fig. 3(d) already satisfies the Doctor predicate for $S_3$, and hence $S_3$ will be deleted from the consequent PJ of Fig. 3(f) (an additional optimization of our algorithm, shown in the figure, is that if the Doctor PJ of Fig. 3(f) remains unsatisfiable until the end, for some views (e.g., $S_1$), instead of adding it to our index as an additional PJ we can merge it with an existing one (Fig. 3(g))). Moreover, updateJoinboxes updates the joinboxes of all other consequent PJs, s.t. if they are not satisfied and end up as new PJs in our index, their joins already point to $P J^V_{\text{ci}}$ for this predicate and view.

If after updateJoinboxes, which has just deleted the satisfiable view from the consequent PJ, the consequent PJ, $P J^C_{\text{ci}}$, becomes empty, this means that the corresponding consequent predicate is partially satisfiable by existing view PJs for all applicable views. Hence, in line 12, the algorithm jumps to the next consequent PJ. Otherwise, in line 13, the algorithm goes on examining the next applicable view $s$ for $P J^V_{\text{ci}}$ and $P J^C_{\text{ci}}$, i.e. jumps to line 8. If we have no prior information for $s$, $P J^C_{\text{ci}}$ and $P J^V_{\text{ci}}$ we call $\text{isPJ}Satisfiable$, in line 16. A global variable $ST$ (initially set ’undecided’ at line 15) maintains which of the two cases of Def. 2 we are currently checking. If we find a partial satisfaction, lines 17-20 repeat the steps described in lines 10-13. Since our algorithms visit joined predicates recursively, we wouldn’t like to fall in an infinite cycle and visit the same PJs again for the same view, such as the one currently considering. In line 14, starting in good faith, we set our pair of PJs and view as already visited and satisfiable, in order to avoid an infinite recursion. If the pair fails to prove partial satisfiability we reverse this decision in line 22. At the end of Alg. 1 (line 23), we will add to our index each consequent PJs with the information for the views left in it (i.e., those views not partially satisfying the consequent predicate). In Fig. 3(h) both sources remain in the consequent PJ for Patient, and this entire PJ is added to our output (Fig. 3(h)). If a new PJ gets added to our index, all view PJs containing the same views (and joining with the new predicate) update their joinboxes accordingly (Fig. 3(h)).
Algorithm 3 joinsSatisfiedRecursively(jdv, jdc, s, ST)
Input: Join description jdv of view PJ variable, join description jdc of constraint variable, source s, state ST of the recursion
Output: true if the joined predicate described by jdv is the satisfying homomorphism's image of the joined predicate jdc
1: if jdv == jdc // Joins are to the same predicate name on the same position then
2: neighborVPJ ← joined PJ in view described by jdv
3: neighborCPJ ← joined PJ in constraint described in jdc
4: if neighborVPJ has been marked as satisfiable for neighborCPJ for view s then
5: return true
6: if neighborVPJ is marked as unsatisfiable for neighborCPJ for source s then
7: return false
8: mark neighborVPJ as satisfiable for neighbourCPJ then
9: if isPJSatisfiable(neighbourVPJ, neighbourCPJ, s, ST) then
10: return true
11: mark neighborVPJ as unsatisfiable for neighbourCPJ for view s
12: return false

Algorithm 4 checkCase2(joinsv, joinsc, s, ST)
Output: true if every joined predicate in joinsv is partially satisfiable with a predicate in joinsc
1: for all join descriptions jdc in joinsc do
2: for all join descriptions jdv in joinsv do
3: if joinsSatisfiedRecursively(jdv, jdc, s, ST) then
4: continue outer for (line 1)
5: return false //there is one join in constraint not satisfied by any join in source
6: return true

6. EXPERIMENTAL EVALUATION

To evaluate our approach we compare our compact frugal chase algorithm to the standard chase, as well as the parallel and the core chase, in the context of compiling LAV wa-TGD constraints into LAV mappings. First, we run our compact graph chase on a set of conjunctive LAV mappings using a set of LAV weakly acyclic constraints, feeding the resulting PJIs to QGR [24] to rewrite a conjunctive query (using the PJIs representing the chased mappings). Second, we have implemented and run the standard, parallel and core chase algorithms in order to compile the same sets of constraints into our set of mappings, as in [1]. We compute PJIs for the standard, parallel and core chased mappings. In all cases the chased PJIs are inputted directly to QGR for query rewriting. Our compact chase outperforms the standard and the core chase by close to 2 and 3 orders of magnitude resp., while our output remains very close to the core. For producing our queries, constraints, and views, we wrote a random query generator, extending the ones used in Minicon [31], which is highly customizable as we discuss next. In fact, it can generate queries and views (and constraints) which capture most cases of the mapping scenarios identified in [3] (note that our prototype implementation, however, does not consider constants).

6.1 Chain queries, constraints and views

Initially we generated 20 datasets of chain queries, chain constraints and chain views. We used a space of 300 predicates to generate our LAV weakly acyclic constraints and views with each constraint/view having 5 predicates joined on a chain. The first atom in our constraints is the antecedent. Each predicate has length 4 and each constraint/view can have up to 3 repeated predicates. Each view has 4 distinguished variables. Additionally in order to get more relevant constraints to views, we generated 10% of our constraints from a smaller subspace of our space of predicates, of size 60. Also, we constructed around 10% of our views by taking one constraint and dropping one of its atoms, causing our constraints to most likely have all atoms except this one partially satisfiable on these views (unless they map their existential variables to distinguished view variables causing case 2 of Def. 2). Lastly, we generated our 20 queries by randomly selecting 3 constraint antecedent atoms and one of these extra non-partially satisfiable atoms that the constraints contain; we do so as to not penalize the standard and parallel chases by querying redundant atoms that they will add (the “extra” atom that the query contains will be probably added by all chase algorithms, including the frugal).

We run our experiments on a cluster of 2GHz processors each with 2Gb of memory. We allocate to each processor 300 views and 300 constraints and one hour of wall time for that job to be finished. Due to the density of our data in this setting, most of the standard/parallel and core chases runs died after reaching 100 constraints (only 7 made it to 140), while for the compact chase all of them reached 140 constraints and 14/20 completed with 180 constraints, which indicates that the compact chase scaled almost twice as much. Figure Fig. 5(a) shows the average total time for running the chases and rewriting the queries (note that the figure averages over the successful queries at each point as discussed above). As seen in the figure, the compact algorithm outperforms the standard and parallel chases by close to two orders of magnitude and the core by close to three. Moreover as Fig. 5(b) shows, the number of PJIs that the compact chase produces for 100 constraints, for which all queries succeeded in all frameworks, is the same as the core and is consistently less than the standard/parallel as the number of views increases (for 100 constraints and 300 views the compact chased mappings have 5895 predicates while the standard chased ones have 6232). In fact, the frugal chase computes almost the
same output as the core chase, and this is done very efficiently without minimization. This leads to (equivalent but) fewer conjunctive rewritings for both the frugal and the core chase as output of the query rewriting problem as Fig. 5(d) shows. In particular, for 300 views and 100 constraints our system produced around 30% less query rewritings than the standard/parallel chases, and only 13% more than the core chase. Lastly, the sets of PJs in Figure 5(b) are rather small for GQR, so it reformulates the queries extremely fast (in milliseconds). Fig. 5(c) shows that the reformulation time for the compact chased PJs (and the core chased PJs) is considerably (more than 3x) faster than the standard/parallel chased ones, with the gap between them increasing with the number of views (the core time in Fig. 5(c) for 250 views seems to be an outlier). A side note of our setting is that the parallel chase, which is just a specific ordering on the standard chase execution steps, is slower than the standard chase (see related work for the relevant discussion).

6.2 Star constraints and views

For our second experiment we evaluated our algorithm when it degenerates to the standard chase, and has the same output, i.e. no predicate is partially satisfiable. We did not measure query rewriting; since both compact and standard chase produce the same chased mappings in the form of PJs, GQR takes the same time to rewrite these for any query, yielding identical rewritings.

We created a space with 500 predicate names, out of which we populate our views and constraints. Each atom has 5 variables and each view/constraint can have up to 3 occurrences of the same predicate. In each formula, one predicate is the “star” joining with all other predicates which don’t join directly to each other. Using this setting, we created 20 datasets of 1000 views and 300 constraints. Each formula has 5 predicates (one of these five predicates in a constraint is the antecedent). Additionally each view has 5 distinguished variables, choosing most of them from the star predicate; this fact essentially introduces distinguished variables in the connected component of the view and oblige our algorithm to fall in case 2 of our definition, i.e., map the constraint consequent in its entirety or not at all (since the consequents are “shorter” than the source descriptions). Since the space of predicates is rather sparse this setting causes the constraints to be unsatisfiable. Moreover it is also difficult for this dataset to produce redundant predicates. This means that the minimization the core chase runs is wasted time.

Each dataset is tested for up to 1000 views and 300 constraints and each processor was allocated a specific number of constraints, scaling from 20 to 300, and all the 1000 views, and we enforced 1 hour wall time. Fig. 5(e) shows the average time for the compact chase versus the standard the parallel and the core chase on all 1000 views. Note that for this experiment the size of the output blows up exponentially as the number of constraints grows: the number of predicates goes from around 5,500 in the original mappings to approximately 180,000 for 250 constraints. This exhausts all algorithms: for the compact chase, all experiments run out of time/memory, for 1000 views, between 250 and 300 constraints, while the other chases crash around 180 constraints. As the figure shows, the compact frugal chase scales to around 30% more constraints while being close to two order of magnitude faster than the standard chase and almost three compared to the core. Additionally note figure Fig. 5(f) which shows the compact frugal chase performance in the same setting but scaling the constraints for only 20 views. Such a small database essentially reduces the advantage of our compact representation: the 20 view formulas generated from our sparse domain, have negligible overlap. Nevertheless our atom oriented implementation of the frugal chase gives a speedup of about one order of magnitude compared to standard/parallel and close to 2 compared to the core. The size of the chased mappings ranges from hundreds for 180 constraints to thousands for 250 constraints.

7. RELATED WORK

Early approaches dealing with relational query rewriting, with no constraints, involve algorithms such as the bucket [20], and the inverse rules [12]. Modifications of the inverse rules algorithm for full dependencies or functional dependencies where presented in [12]. Koch [23] presented another variation of inverse rules for answering queries using views under GLAV dependencies (for dependencies common to both source and global schemas). A more
efficient approach to view-based query answering was proposed in 2001, by the MiniCon [31] algorithm. MCDSAT [5] exhibited a better performance than MiniCon, and is essentially the MiniCon algorithm cast as a satisfiability problem. In this paper we employ GQR [24] which significantly outperformed MCDSAT.

There are approaches to data integration and query answering under constraints for which the chase does not terminate, but rather one has to do query expansion, using the constraints. The family of DL-Lite languages [4] is a famous such first-order rewritable class of constraints, which underlies the semantic web language OWL2-QL. Extensions of DL-Lite include flavors of the Datalog+/-/ family of languages [8]. Query expansion rewrites the query using the constraints into a UCQ, before even considering the views or data. Although the LA V wa-TGDs used in this work are both FO-rewritable and chase-terminating, we focus on running the chase on the views without needing to take the query into account, in order to speed up the system’s online performance. There are flavors of Datalog+/-, such as the guarded fragment [7], for which the chase does not terminate, but there is a finite part of the infinite chase sufficient for query answering, but again, once a specific query is given. Our frugal chase could substitute the chase algorithms in these fragments.

Chase&Backchase (C&B) [11] is a technique for query mini-
mization and for finding equivalent rewritings, by first chas-
ing all constraints and mappings and then minimizing the chase.

Afrati et. al [1] presented an optimized version for finding equivalent rewritings, as well as maximally-contained, as discussed. In this paper we outperformed the latter approach (running the standard chase and then doing query rewriting). Additionally, the frugal chase can replace the chase in the (C&B) algorithm, for finding equivalent rewritings and be even more advantageous (being shorter than the standard chase) for query minimization. Our frugal chase can be used for query containment; in [25] we used the standard chase and a compact graph-based algorithm for UCQ containment.

The chase algorithm has been studied in multiple variations in the recent literature (for overviews see [30, 18]), which guarantee chase termination for one or all instances, under different classes of constraints, producing universal solutions. These include the naive oblivious chase [7], the skolem oblivious chase [28] and the parallel and core chase [10]. Whenever the oblivious and naive chases terminate: 1) the standard chase also terminates and 2) they produce longer solutions than the standard chase [30, 18]. This justifies our choice of optimizing and implementing the standard chase since we focus on producing shorter chased mappings. The parallel chase is just a specific ordering of standard chase step executions; in particular, each parallel chase step decides all applicable constraints on the instance as it has been formed so far and, only after finishing considering all constraints, adds all consequents before going to the next step. Notice that this might penalize the parallel chase as evident by our experiments; the “incremental” addition of consequents that the standard chase performs might prove some “parallel” rules satisfiable, and hence avoid applying them. Each core chase step is a parallel chase step followed by minimization of the produced instance. As its name suggests the core chase leads to a minimal universal solution, i.e., the core, as opposed to the frugal chase which leads to smaller but not necessarily minimal solutions. The prize paid however by the core chase is extra minimization (whose equivalent decision problem is NP-hard), as indicated by our experiments. Nevertheless the core chase is more “general” than the standard and the frugal chase since it is a complete algorithm for producing universal solutions under more general classes of constraints [10]. Notice that the “building block” of the core and parallel chases is the standard chase which we could replace with the frugal chase in order to implement these algorithms. This can be very useful as the frugal chase step is not only faster but produces smaller intermediate instances; the minimization step of the core chase would also run faster on such instances. Other data exchange literature that computes the core universal solution, includes [13, 16, 17]. The space of universal solutions (such as our frugal chase) which lie in between the core and the standard chase has been discussed by Henrich et al. [21]. Approaches in [32, 29] construct mappings and constraints that use negation to avoid adding redundant predicates to a solution. We plan to evaluate our approach in these settings and extend our algorithms to produce the core by leveraging our compact graph-based format.

8. DISCUSSION

We have presented several contributions. First, we introduced the frugal chase, which produces smaller universal solutions than the standard chase. Second, we developed an efficient compact version of the frugal chase. Third, we used this frugal chase to scale up query rewriting under constraints, for the case of LA V mappings and LA V weakly acyclic constraints. As our experiments show, we gain the additional expressivity of using dependencies very cheap: in essence, our algorithm only pays the cost of relational query answering using our preprocessed views, which due to our optimized, shorter chase, our compact format, and our indexing, becomes very efficient. In future work, we plan to explore extensions of our chase that would compute the core solution, as well as evaluate our system both in data exchange scenarios (i.e., chase instances rather than mappings) and with other chase-terminating cases of constraints, including limited interactions of TGDs and EGDs.

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10. REFERENCES

that there exist homomorphisms that map $B'$ to $B''$ and vice versa, assuming $\sigma$ is applicable; which means there is a homomorphism $h$ from the antecedent of $\sigma$, say $\phi(\vec{x}, \vec{z})$ to $B$ s.t. it cannot be extended over the consequent, $\psi(\vec{x}, \vec{y})$. We know by Lemma 1 that there is a conjunct with at least one atom in $\sigma$, $\psi_{1}(\vec{x}, \vec{y})$, that is not partially satisfiable for $h$. If $\psi_{1}$ is the entire consequent of the constraint, then the applicable homomorphism by definition will be the same as the standard chase application homomorphism (modulo names of fresh variables). This means that $B'$ and $B''$ are homomorphically equivalent (in fact, they are isomorphic since there is no essential optimization to the result of the frugal chase step).

We now examine the case where a part of the constraint, say $\psi_{1}$ is partially satisfiable, while another part $\psi_{2}$ is not. Conject $\psi_{1}$ can have three categories of variables/constants: (1) Those that are antecedent variables or constants in the constraint, i.e., belong in $\vec{x}$ (we abuse notation and regard that constants are in $\vec{x}$). (2) Those existential variables, $e_{\psi_{2}}$, that (a) fall under case 2 of our definition, i.e., they map to constants/distinguished variables or existential variables that join with a part of the database which is not a partially satisfiable image, together with variables that (b) belong in the same gaifman connected component with the variables in $\vec{e}_{\vec{x}}$. We denote by $e_{\psi_{2}}B$ the images of $e_{\psi_{2}}$ in the database. All atoms containing $e_{\psi_{2}}B$ in the constraint should be partially satisfiable, hence the variables of $e_{\psi_{2}}B$ exist only in the conjunction $\psi_{1}$ in the constraint. (3) Those existential variables that map to variables that only exist in images of partially satisfiable atoms and are not in $e_{\psi_{2}}B$, i.e., they are not contained in any “case 2 predicate” above; we denote these variables in the constraint as $e_{\psi_{2}}A$. We denote by $e_{\psi_{2}}A$ the images of $e_{\psi_{2}}A$ in the database. All atoms containing $e_{\psi_{2}}A$ are images of partially satisfiable atoms, hence $e_{\psi_{2}}A$ exists only in the image of $\psi_{1}$ in the database.

Hence, our constraint has the form: $\sigma: \phi(\vec{x}, \vec{y}) \rightarrow \psi_{1}(\vec{x}, e_{\psi_{1}}B, e_{\psi_{1}}A), \psi_{2}(\vec{x}, e_{\psi_{2}}B, \vec{r})$ with $\vec{r}$ the rest of the variables belonging solely in $\psi_{2}$.

In general, our database instance has the form (reusing $\phi$ and $\psi_{1}$ as the images of the antecedent and the partially satisfiable conjunction, respectively): $\{\phi(\vec{x}_{1}, \vec{y}_{1}), \psi_{1}(\vec{x}_{1}, e_{\psi_{1}}B, e_{\psi_{1}}A), \psi_{2}(\vec{x}_{2}, e_{\psi_{2}}B, \vec{r}_{1}, \vec{y}_{2})\}$ with $\psi_{2}$ denoting the rest of the atoms in the database (if any). Terms in $\vec{x}_{1}$ and $\vec{y}_{2}$ are the images of $\vec{x}$ and $\vec{y}$ respectively, and $\vec{r}_{1}$ contains the rest of the terms in the database.

Moreover, by definition there is no constraint predicate that contains variables from both $e_{\psi_{1}}B$ and $e_{\psi_{2}}A$. Hence, we can “break” $\psi_{1}$ in the constraint into two parts, one containing $e_{\psi_{1}}B$ and one containing $e_{\psi_{2}}A$ (exactly one of the two parts might be empty).

Our constraint now can be written as: $\sigma: \phi(\vec{x}, \vec{y}) \rightarrow \psi_{11}(\vec{x}, e_{\psi_{1}}B_{1}), \psi_{12}(\vec{x}, e_{\psi_{2}}A_{1}, \vec{r}_{1})$. Similarly our instance becomes: $\{\phi(\vec{x}_{1}, \vec{y}_{1}), \psi_{11}(\vec{x}_{1}, e_{\psi_{1}}B_{1}), \psi_{12}(\vec{x}_{2}, e_{\psi_{2}}A_{2}, \vec{r}_{1}, \vec{y}_{2})\}$.

The standard chase run on this database with $\sigma$ will produce the instance: $\{\phi(\vec{x}_{1}, \vec{y}_{1}), \psi_{11}(\vec{x}_{1}, e_{\psi_{1}}B_{1}), \psi_{12}(\vec{x}_{2}, e_{\psi_{2}}A_{2}, \vec{r}_{1}, \vec{y}_{2})\}$. The frugal chase will produce $\{\phi(\vec{x}_{1}, \vec{y}_{1}), \psi_{11}(\vec{x}_{1}, e_{\psi_{1}}B_{1}), \psi_{12}(\vec{x}_{1}, e_{\psi_{2}}A_{2}, \vec{r}_{1}, \vec{y}_{2})\}$. To verify that the two instances are equivalent, the only non-obvious fact is that $e_{\psi_{1}}B$ can map to $e_{\psi_{2}}A$. Since case 1 of Def. 2, states that in the same positions that two atoms in the database contain $h'(z)$, the corresponding atoms in the constraint contain $z$, it means that the restriction of $h'(z)$ on these variables is one-to-one, and the inverse can map $h'(z)$ to $z$. Hence, the two instances are homomorphically equivalent. Moreover, if the databases are views, the part containing distinguished variables ($\psi_{12}$) maps to itself and the homomorphisms that prove equivalence are containment mappings. Lastly, the fact that $B''$ satisfies the constraint, is directly proven by using the applicable homomorphisms that we used to construct $B''$. 

**APPENDIX**

**PROOF OF THEOREM 2.** We prove the theorem for Def. 2 since it is equivalent to Def. 1 by Th. 4. The frugal chase step is applicable iff the standard chase step is, by Lemma 1. We first show