Large eddy simulation of flow past a forced oscillating square cylinder

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INTRODUCTION

A finite volume incompressible flow solver has been developed to simulate a moving solid body in fluid. A square cylinder forced to oscillate with a prescribed sinusoidal motion is studied by using large eddy simulation (LES). Although the geometry of the square cylinder is simple, it is very challenging to LES, for example, the estimation of transition and the vortex formation length in the wake region.

In the past decades, with the advancement of measurement and computer techniques, a lot of work has been done on the vortex induced vibration (VIV) of a square or rectangular cylinder. Bearman et. al. [1] conducted an experiment of a forced oscillating square cylinder at a range of reduced velocity and motion amplitude ratio (A/D, where A is amplitude and D is side length of cylinder). In their experiment, they focused on surface pressure fluctuations. They found surface fluctuating pressure is correlated across the span when the cylinder is at lock-in region. Daniels et. al. [2] used LES to analyse VIV of an elongated rectangular cylinder with side ratio B/D = 4 (breadth/thickness) in one degree of freedom, i.e. pitching or heaving. In their simulations, the upstream turbulence with turbulence intensity (TI) of 6% was imposed. Results are compared between uniform incoming flow and flow with free upstream turbulence. It is found that the upstream turbulence can reduce the span-wise correlations and subsequently reduce the oscillation amplitude compared to the smooth incoming flow.

In this paper, the Reynolds number based on a free stream velocity and a side length D of cylinder ranges from 7.5×10^3 to 1.65×10^4. The reduced velocity is defined as \( U_r = U/ND \) and is chosen from 5 to 11, where \( N \) is the forced oscillation frequency. The oscillation amplitude ratio A/D is set to 0.05 and 0.10. The vortex shedding frequency versus reduced velocity relationship and time averaged surface pressure fluctuation distribution are validated. Additionally, phase average of flow field and surface pressure coefficient distribution at different phase angles are calculated. The phase averaged flow field is used to identify and analyse the span-wise vortical structures. Overall, the results from current study are in good agreement with the experimental measurements.

NUMERICAL METHOD

The developed incompressible flow solver uses second order scheme in both time and space. For spatial discretisation, the central different scheme is used. For temporal discretisation, the Runge-Kutta 2\(^{nd}\) order explicit integration is used. The mixed time scale model (MTS) [3] is adopted for the sub-grid scale (SGS) term. The MTS model parameters \( C_{MTS} \) and \( C_T \) are set as default to 0.05 and 10, respectively.

To construct the solid body, an efficient immersed boundary method (IBM) by Yang et. al. [4] is used. IBM is very useful and efficient in many applications [5]. However, it is very challenging to simulate bluff body in turbulent flow accurately. To validate the current solver and implemented IBM, we simulated flow past a stationary square cylinder at Reynolds number 21,400 based on free stream velocity \( U \) and cylinder side length \( D \). The global integral quantities such as lift and drag coefficients and Strouhal number are in good agreement with experimental measurements. The turbulent statistics, in particular in the shear layer and wake regions are compared rigorously as well. We also found that the developed solver can accurately predict surface pressure fluctuations, which are well known to be extremely challenging for LES.

Following our previous work, a square cylinder with a prescribed sinusoidal motion in uniform incoming flow is studied. The prescribed motion perpendicular to free-stream can be described as

\[ y(t) = A\sin(2\pi Nt) \]  \hspace{1cm} (1)
where \( t \) is the time, \( N \) is the frequency of oscillating cylinder, and \( A \) is the oscillation amplitude. To validate the simulation, similar parameters of experimental measurement by Bearman et. al. [1] are used. In this case, the Reynolds number is set to vary from 7500 to 16500. The oscillating amplitude ratio \( A/D \) is set to 0.05 and 0.1. The forced oscillation frequency \( N \) is fixed, while the freestream velocity is changed over different test cases. The reduced velocity \( U_r \) is chosen from 5 to 11. The ratio of Reynolds number to reduced velocity \( ND^2/\nu \) is fixed at 1500. The simulations are performed on the Cartesian grid. The centre of the square cylinder in the neutral position is located at the origin of the coordinate. The computational domain is \( 25D \times 20D \times 2D \) in the stream-wise, cross-stream and span-wise directions respectively. The upstream and downstream boundaries to the cylinder are \( 5D \) and \( 20D \), and the distance from neutral position to upper and lower boundaries is \( 10D \). The span-wise length is \( 2D \). A fine and uniform mesh is used in the body motion region, and 160 cells are used to resolve per side of cylinder. The mesh is stretched toward to the far regions. As for the boundary condition, a uniform incoming velocity is imposed at the inlet, \( u=(U,0,0) \). The outlet condition is zero gradient, \( \partial u/\partial x=0 \). Symmetric boundary condition is used at top and bottom planes, \( \partial u/\partial y=\partial w/\partial y=0 \) and \( v=0 \). Zero gradient boundary condition is used for pressure at these planes. Both velocity and pressure are periodic in span-wise direction. For all the cases, 20 complete cycles are simulated and the non-dimensional time unit \( t^*=tU/D \) varies from 100 to 200, which is sufficient in current simulations.

RESULT AND DISCUSSION

We first validate the relationship between non-dimensional shedding frequency and reduced velocity as shown in Figure 1. The vortex shedding frequency is denoted as \( n \) and it is computed from the principal frequency of vertical velocity time series in the downstream of neutral position of the cylinder. It shows the lock-in region where the \( n/N \) is 1 is at around reduced velocity \( U_r \) from 6 to 8 for two amplitude ratios. Bearman et. al. [6] conducted experimental measurement to show that the Strouhal number of a stationary square cylinder at this Reynolds number range is 1.30. This shows that the current results are in a good agreement with experimental data.

![Figure 1. Vortex shedding frequency versus reduced velocity. Left: A/D=0.05; Right: A/D=0.1. The line represents the Strouhal number of stationary square cylinder vortex shedding.](image)

In the forced oscillating square cylinder cases, the effective vortex shedding has two sources. One is due to flow past the bluff body to generate asymmetric vortex shedding and the other one is due to the motion of bluff body. The transversal motion of cylinder generates vortices to shed and propagate in the transversal direction. The transversal vortex shedding frequency is governed by the body motion and it has an equivalent or close frequency to body motion. Due to the free-stream, the transversal vortices are convected to downstream and interact with vortices generated by the flow past the cylinder. When the frequency of body motion is close to vortex shedding frequency, the effective vortex shedding frequency is equivalent to frequency of cylinder motion, i.e. lock-in phenomenon occurs. The common part of lock-in region is observed in simulations of two amplitude ratios (\( A/D=0.05 \) and 0.1) at reduced velocity from 6 to 8. The free-stream velocity determines the flow convection intensity. For a high reduced velocity, the fluid convection effect is dominant over the body motion, which causes the vortex shedding of the oscillating cylinder to be similar to the flow past a stationary cylinder. Oscillating cylinder cases at reduced velocity over 8 for both amplitude ratios have a good agreement with experimental measurements. For the low reduced velocity cases (\( U_r<6 \)) at \( A/D=0.1 \), the shedding frequency is overestimated, and \( n/N \) is kept at 1. It indicates that this amplitude ratio also dominates vortex shedding at low reduced velocity.
Figure 2. Pressure coefficient $C_p$ distribution the cylinder surface. Left: Comparison of $C_p$ distribution of stationary square cylinder at $Re=20,000$ in experiment and $Re=21,400$ in CFD; Right: Comparison of $C_p$ distribution of an oscillating cylinder at amplitude ratio $A/D=0.1$ and $U_r=7.5$ and 8.4.

Figure 2 shows the comparison of pressure coefficient $C_p$ distribution along circumferential direction of square cylinder for both stationary and oscillating cases at amplitude ratio $A/D=0.1$. The presented results are span-wise averaged. For the flow past a stationary cylinder, the small difference of Reynolds number does not affect the results. The LES result shows a very good agreement. For the oscillating cylinder, cases with reduced velocity at 7.5 and 8.4 are used to validate. As can be seen, the pressure coefficient has a greater absolute value at central part of side surface ($s/D$ from 1 to 2) in oscillating cases than the stationary case. The reduced velocity at 7.5 and 8.4 correspond to the lock-in region where the vortex shedding is at the same pace as cylinder oscillation. The cylinder oscillates in lock-in region can amplify the vortex shedding procedure and the surface pressure is amplified correspondently. It is noted that the resonance of square cylinder is at reduced velocity 7.7 in the experiment measurement [1]. It shows that the experiment with a closer value to resonance reduced velocity ($U_r=7.5$) has a greater absolute pressure over the other one ($U_r=8.4$) on the side surface. This feature is well captured in the current study. The comparison shows that LES gets a near constant $C_p$ on the rear surface ($s/D$ ranges from 2 to 2.5), while experimental measurement shows a more variant distribution. It should be noted that in experiment, since only several probes installed on the rear surface, hence the sampling resolution is not sufficient to resolve the variation. The flat distribution of pressure coefficient on the rear surface is consistent between stationary and oscillation cases of LES. Moreover, DNS study of flow past a stationary square cylinder [7] shows a similar flat distribution on the rear surface, which provides some evidence to support the present LES results.

Figure 3. Fluctuating pressure coefficient $C_p'$ distribution on a square cylinder at amplitude ratio $A/D=0.1$.

Figure 3 shows the coefficient of fluctuating pressure $C_p'$ distribution on both stationary and oscillating square at amplitude ratio $A/D=0.1$. The cases with reduced velocity at lock-in and post lock-in region ($U_r>8$) are chosen to compare with experimental measurement. It can be seen in post lock-in region, the $C_p'$ has a similar trend and magnitude as the distribution of flow past a stationary cylinder. However,
when the reduced velocity is close to resonance reduced velocity, i.e. $U_r=7.8$, the pressure fluctuation is increased in the side surface. This indicates resonance also increases the surface pressure fluctuation.

**Figure 4.** Phase and span-wise averaged flow field at different phase angles at amplitude ratio $A/D=0.1$ and reduced velocity $U_r=7.8$. Top: non-dimensional span-wise vorticity contour $\omega_z D/U$ and streamline; Bottom: The correspondent pressure coefficient distribution on the lower surface. From left to right, phase angles are 36°, 54° and 72° in upstroke of a sinusoidal motion (1).

Figure 4 shows phase and span-wise averaged flow field in three consecutive phase angles in upstroke motion with the correspondent pressure coefficient distribution on the lower surface. As can be seen, during upstroke motion, a clockwise-rotated vortex is generated and shed from lower rear corner. At the initial step, a small vortex is formed in the lower rear corner and a drop of pressure coefficient distribution is found in near $x/D=0.5$ position. With time evolved, this vortex is developed in the flow perpendicular direction. The pressure coefficient distribution shows the effect of corner vortex moves toward upstream. Meanwhile, due to counter-clockwise rotated vortex generated from the leading edge corner, the corner vortex is neutralized and convected to downstream. The pressure coefficient distribution shows the effect of corner vortex is smeared and the distribution is flattened in the end.

**CONCLUSION**

In this paper, a forced oscillating square cylinder by immersed boundary method at reduced velocity from 5 to 11 at amplitude ratio $A/D=0.05$ and 0.1 in uniform incoming flow is studied and validated. Vortex shedding frequency is computed and the lock-in phenomenon is observed. The surface pressure and fluctuating pressure coefficients distribution are examined. The LES data confirms that for both pressure and fluctuating pressure on side surface of oscillating square cylinder are evidently amplified, which indicates that the turbulent flow associated with body oscillation can increase the intensity of surface force and fluctuations.

**REFERENCES**


