

Effect of Tm³⁺ concentration on the emission wavelength shift in Tm³⁺-doped silica microsphere lasers

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In this work, a Tm³⁺-doped sol-gel silica microsphere lasing at 2.0 μm is reported. Microspheres with different Tm³⁺ concentrations are fabricated by overlaying different Tm³⁺ concentration sol-gel solutions on the surface of a pure silica microsphere resonator and then annealing the sample with a CO₂ laser. Based on a traditional fiber taper-microsphere coupling method, single and multimode microsphere lasing in the wavelength range 1.8-2.0 μm is observed if an 808 nm laser diode is used as a pump source. A relatively low threshold pumping power of 1.2 mW is achieved using this arrangement. This sol-gel method allows for an easy varying of the Tm³⁺ doping concentration. The observed laser output shifts to longer wavelengths when the Tm³⁺ doping concentration increases. This has been explained by the larger Tm absorption at shorter wavelengths. The ability to fabricate sol-gel co-doped silica glass microlasers represents a new generation of low threshold and compact infrared laser sources for use as miniaturized photonic components for a wide range of applications including gas sensing and medical surgery.

OCIS codes: (140.3460) Lasers; (140.3945) Microcavities; (140.3510) Lasers, fiber.

Whispering gallery mode (WGM) optical microcavities including microdroplets, microdisks, microspheres and microtoroids have become established as attractive photonic building blocks for use in optoelectronic systems. Devices based on optical microcavities are already considered indispensable for a wide range of applications [1-3]. The combination of high quality factors (up to 10¹⁰) and small mode volumes (of the order of 100 μm³) in WGM resonators

significantly enhances light matter interactions, resulting in excellent optical laser cavities with low threshold and narrow linewidth lasers [4-7]. To date, a large number of laser emissions based on rare earth ion doped glass microspheres have been investigated. Yb³⁺-Er³⁺ co-doped visible microlasers have been widely reported and largely based on glass microsphere resonators [8, 9]. Nd³⁺-doped and Er³⁺-doped microsphere resonators have been demonstrated predominantly for emission in the near-infrared range [10, 11].

Tm³⁺-doped microsphere laser cavities with wavelengths around 2.0 μm have recently attracted much interest owing to their extensive potential applications in laser medicine surgery, military, remote chemical sensing and monitoring of atmospheric pollution. A single mode microsphere laser at 2 μm was demonstrated from a Tm³⁺-doped tellurite or silica glass microsphere [12, 13]. Tm³⁺-doped silica microtoroid lasers with a central wavelength of 2 μm fabricated on silicon chips using a sol-gel synthesis process have also been demonstrated [14].

In this work, Tm³⁺-doped microspheres were prepared using the well-established sol-gel method, which has been verified as a low cost, efficient and flexible method for fabricating many active-ion doped gain microspheres [15, 16]. Infrared lasers with an output central wavelength around 2.0 μm were fabricated and observed by optically coupling the Tm³⁺-doped microsphere to a silica tapered fiber and using an 808 nm laser diode as pump source. The lasing characteristics were studied in detail by adjusting the coupling position and distance between the tapered fiber and the microsphere. Furthermore, compared with the lasing output at 2 μm only in Ref. [12], the lasing wavelength can be experimentally observed as shifting from 1.83 μm to 2.05 μm when varying the doping concentration in the range 0.05% to 1.5%.

Silica microspheres with diameters ranging from 40 to 200 μm have been fabricated, using standard single-mode fibers (SMF-28,

Thorlabs). These silica microspheres were made in the standard manner using a circular ZnSe lens focused CO₂ laser beam (power ~3 W) which was directed onto a section of a silica single-mode optical fiber. A small weight attached to the bottom of the silica fiber upon heating ensured the formation of a very thin tapered region (diameter circa 3 μm), which acts as the stem of the microsphere. The same CO₂ laser was then used to cut the fiber and to reheat the remaining glass at the tip. The surface tension of the molten silica at the fiber tip under a high temperature caused the fiber tip to assume a spherical shape under the effect of gravity. Finally, a further broad-focus laser heating process was performed to reduce geometric irregularities on the microsphere.

The Tm³⁺-doped glass was prepared using the well-known sol-gel method [17]. The sol-gel solution was prepared using 6.5 ml tetraethoxysilane (TEOS) in 0.7 ml water and 0.5 ml hydrochloric acid, where 8.3 ml isopropanol acted as the co-solvent. TmCl₃ was added to the solution to introduce Tm³⁺ ions. The solution was mixed in a silica beaker and placed on a hotplate, stirred at 70 °C and 400 rpm for 2 hours. This ensured that the Tm³⁺ and ions were evenly dispersed in the solution. Following aging at room temperature for a further 10 h, the solution was ready to use. The undoped microsphere was immersed in the solution for 5 min and the microsphere was coated using the Tm³⁺-doped solution. Finally, the coated microsphere was heated using a CO₂ laser beam to soften the silica glass (~2000 °C) and hence fabricate the Tm³⁺-doped microsphere. When heated at this temperature, the organic solvents were completely evaporated and the rare earth ions dissolved into the surface of silica microsphere. This process cycle was repeated 3 times in order to ensure sufficient active ions were introduced into the microsphere. The image of the Tm³⁺-doped microsphere (with 157 μm diameter) is shown in the inset of Fig. 1.

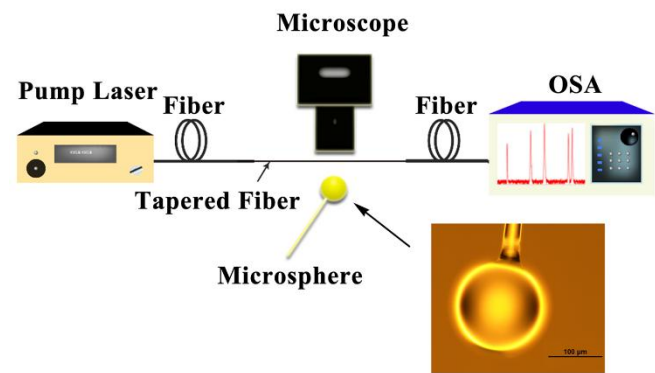


Fig. 1. Experimental setup for characterizing the Tm³⁺-doped microsphere lasers. Inset: microscope image of the Tm³⁺-doped microsphere.

The experimental setup for measuring the microsphere laser characteristics is shown in Fig. 1. The tapered fiber used for light coupling was fabricated by heating with a ceramic microheater (CMH-7-19, NTT-AT) a strand of 1060XP single-mode fiber under tension [18]. In this work, the fabrication parameters could be controlled such that the waist diameter of the tapered silica fiber could be accurately controlled in the range 1.0 to 2.0 μm. Light from an 808 nm laser pump diode (LE-LS-808-200TFCS-LH, Leoptics, China) was launched into one end of the taper and then coupled into

the microsphere. The transmitted spectrum was collected using an optical spectrum analyzer (OSA) (AQ-6375, Yokogawa, Japan). The coupling positions between the taper and microsphere were monitored from both directions using two 20X microscope objectives which were optically coupled to CCD cameras.

As the position of the microsphere was aligned with the fiber taper, the pump laser light was coupled into the Tm³⁺-doped microsphere and the resulting 2 μm emission from the microsphere was coupled out of it and transmitted through the fiber taper. The inset of Fig. 2 (a) shows the output optical spectrum from the Tm³⁺-doped microsphere when the pump power is below the laser threshold. The spectrum was recorded on the OSA with a spectral resolution of 1 nm. When the 808 nm pump laser power reached 1.2 mW, a relatively broad emission in the range 1600 to 2100 nm was observed (In this letter, the meaning of pump power is the input to the fiber taper.). This is the characteristic emission attributed to the ³F₄→³H₆ transitions of Tm³⁺ ions as previously reported [19]. As the pump power increased to 7.0 mW, the characteristic multimode laser peaks were observed at 2.0 μm on the OSA with a spectral resolution of 0.05 nm (Fig. 2 (a)). As explained by the schematic of the energy-level diagrams in inset of Fig. 2 (b), Tm³⁺ ions are excited to the ³H₄ energy level through the absorption of the 808 nm wavelength light of the pump laser. The depopulation process exists through the rapid non-radiative relaxation from the Tm³⁺: ³H₄ to the ³F₄ level and transfer to the ³H₆ level with the infrared emission occurring at around 2.0 μm.

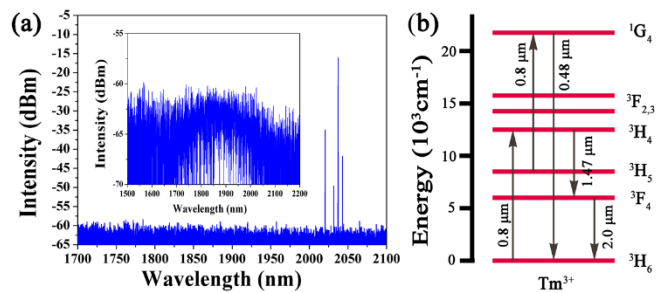


Fig. 2. (a) Laser emission spectra from the Tm³⁺-doped microsphere when the pump power was set to 7.0 mW. The inset is output spectrum from the Tm³⁺-doped microsphere excited by an 808 nm laser with a pump power of 1.2 mW. (b) The energy-level diagrams of Tm³⁺ ions and relative transitions.

Changing the position of the coupling between Tm³⁺-doped microsphere and silica fiber taper, a single mode lasing emission from the microsphere was observed and the resulting output spectrum is shown in Fig. 3 (a). The spectrum was recorded using the same OSA with a spectral resolution of 0.05 nm. The wavelength of the single mode lasing peak is centred at 2037.1 nm, the peak power is 0.028 mW when the pump power reaches 7.0 mW. The output power of the microsphere laser as function of pump power is shown in Fig. 3 (b). Fig. 3 (c) shows the WGMs obtained around 1550 nm, as measured through the fiber taper coupler. The quality factor of the microsphere was measured as high as 10⁵ around the wavelength of λ~1550 nm using the well-known formula $Q = \lambda / \text{FWHM}$, where FWHM is the full width at half-maximum. This IR microsphere laser with an output emission at around 2.0 μm and low threshold level is ideal as an integrated optical source for a large

number of applications including gas sensing and surgery in medicine.

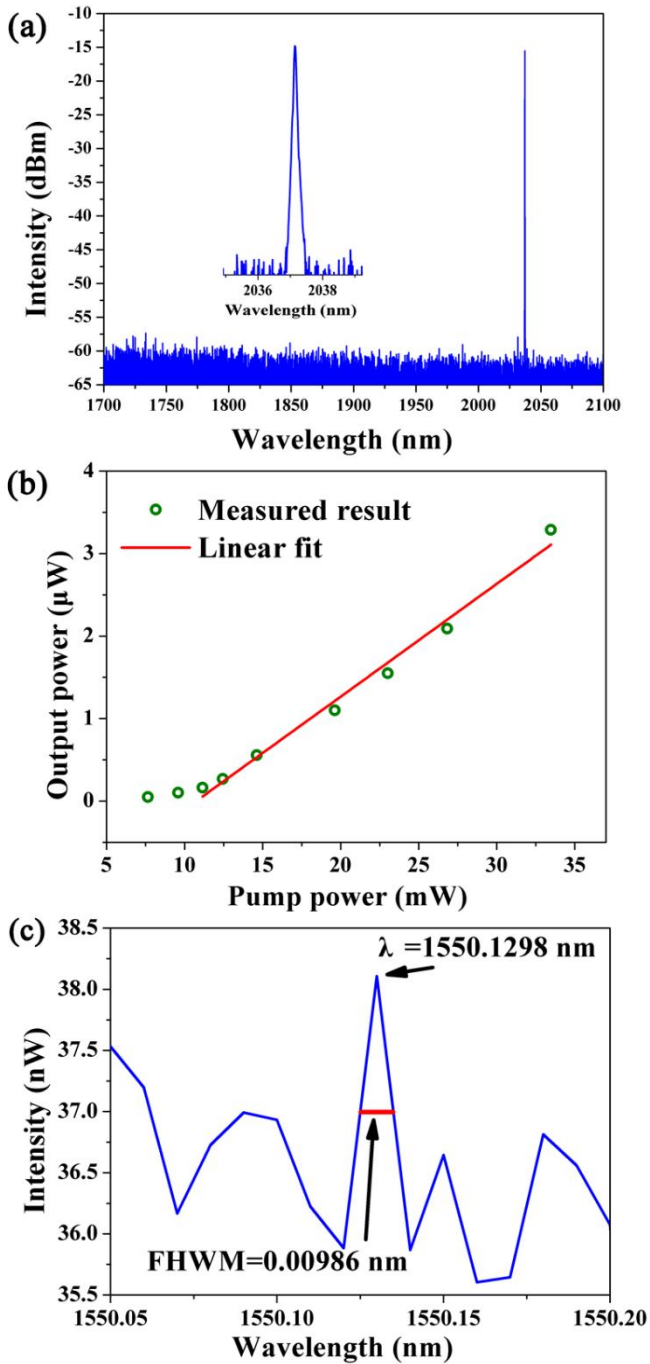


Fig. 3. (a) Single mode laser emission spectrum from the Tm^{3+} -doped microsphere when the pump power reaches 7.0 mW. The inset is the enlargement of the laser peak; (b) Output power of the microsphere laser at 2037.1 nm as a function of pump power. (c) WGMs observed when light is coupled into doped microsphere via the fiber taper coupler.

The characteristic emission attributed to ${}^3\text{F}_4 \rightarrow {}^3\text{H}_6$ transitions of Tm^{3+} ions, is observed at around 1.8 μm [20]. Previously, the Tm^{3+} -doped laser was found to have occurred near 2.0 μm . An

explanation of the phenomenon that the lasing wavelength at 2.0 μm is a shifted version (by +200 nm) from the emission peak at 1800 nm has been ascribed to the re-absorption in the Tm^{3+} microsphere resulting from two factors: the pumping wavelength is not exactly at the peak of the WGM; and the mode distribution of the 793 nm pump light and 2.0 μm signal light do not exactly match which has been reported in [12]. The sol-gel method is very convenient for ease of changing the doping concentration, and this allowed the characteristics of the Tm^{3+} -doped laser to be investigated with ease for various doping concentrations in the range 0.05 mol% to 1.5 mol%. It was found that the concentration of Tm^{3+} affects the laser emission peak wavelength. In the experiment, the diameter of the sphere was accurately controlled at $150 \mu\text{m} \pm 10 \mu\text{m}$. Fig. 4 (a) shows the laser output for the three different Tm^{3+} doping level microspheres when operated with the same pump power. From Fig. 4 (a), it is clear that as the doping concentration of Tm^{3+} increases, the emission position of the laser shifts to longer wavelengths. When the concentrations of Tm^{3+} are 0.05 mol%, 0.1 mol% and 1.5 mol%, the lasers peaks were centred around 1.85 μm , 1.95 μm and 2.05 μm respectively. However the effect of slight geometrical variations on the resonant wavelength can be very important [11, 21, 22]. In the experiment, the effect of different WGM coupling, different microsphere size and different pump power on the emission wavelength shift in Tm^{3+} -doped silica microsphere lasers is demonstrated (with 1.5 mol% concentrations). Fig. 4 (b) shows the laser output for the three different WGM coupling cases when operated with the same microsphere (with 134 μm diameter) and pump power (4.4 mW). Note that some coupling conditions, such as the fiber-sphere air-gap and coupling position may change within the measurements here. The Fig. 4 (c) shows the laser output for the three different pump powers (4.8 mW, 4.2 mW and 5.4 mW) when operated with the same microsphere (with 60 μm diameter). Fig. 4 (d) shows the laser output for the three different microsphere sizes (with 134 μm , 60 μm and 100 μm diameters) when operated with the same pump power. From Fig. 4 (b), (c) and (d), it is clear that different WGM coupling, microsphere size and pump power have effect on the emission wavelength shift. However, these reasons are not enough to account for the observed wavelength shifts from 1.8 μm to 2.0 μm . The absorption and the emission spectra of Tm^{3+} have an overlap over the wavelength of 1.8-2.0 μm . Due to this overlap, high-energy photons may be reabsorbed and then undergo multiple reabsorption and re-emission processes [23, 24]. As the concentration of Tm^{3+} increases, Tm^{3+} ions are in proximity of each other and high-energy photons are more easily reabsorbed. The laser transition of Tm^{3+} is a three-level system, in which the reabsorption of the un-pumped ions in the laser propagation area forces the laser output spectrum to shift to longer wavelengths. This is probably why the emission wavelength of the laser shifts to the wavelength higher value (2.0 μm) as the concentration of Tm^{3+} increases. Actually lasing in the microsphere is rather complex, owing to the large number of high-Q modes that are present in the silica microsphere used in the experiments, the spatial selectivity of the pump source, the loading of the silica microsphere as a result of the fiber taper position, the large spectral gain bandwidth, the variations in the emission and absorption cross sections versus wavelength in the Tm^{3+} doped sol-gel solution materials [25]. Although the mechanism behind the wavelength shifts is much more complex here, it can be primarily justified by the increased contribution of the absorption band at shorter wavelengths (Stokes

shift), which at higher concentrations reduces the gain at shorter wavelengths by increasing the overall losses, thus shifting the emission centre at longer wavelengths.

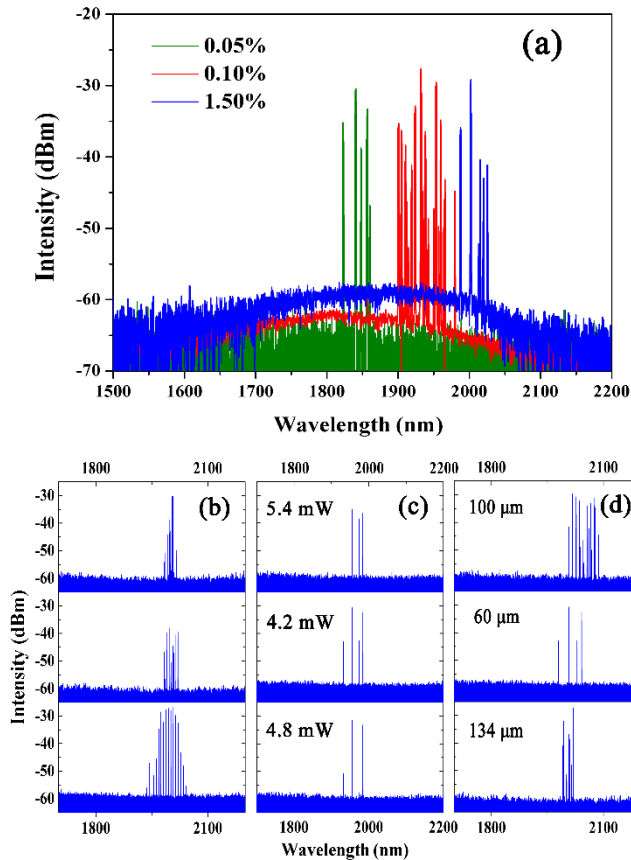


Fig. 4. (a) Laser emission wavelengths with varying Tm^{3+} doping concentrations; (b) Laser emission wavelengths with varying WGMC coupling; (c) Laser emission wavelengths with varying pump power; (d) Laser emission wavelengths with varying microsphere size.

In conclusion, Tm^{3+} -doped silica microsphere lasers with emissions around 1.8-2.0 μm in a fiber tapered coupled system has been successfully fabricated and characterized. The source was pumped using a commercial 808 nm laser diode, and this resulted in observable single mode operation of the microsphere at around 2037.1 nm. The threshold pump power could be used as low as 1.2 mW. This Tm^{3+} -doped microsphere resonator therefore provides a low threshold laser source as an integrated optical device for operation in the IR spectral region. This laser is potentially useful for gas sensing, military and medical applications. The sol-gel based fabrication method is low cost, efficient and a highly flexible means of fabricating high gain microspheres as laser sources. It has been shown that the sol-gel method utilized in this study can easily accommodate changing the resulting Tm^{3+} doping concentration. The resulting lasing wavelength also shifts from 1.8-2.0 μm with different doping concentrations. This has been ascribed to the competition between reabsorption and emission, which shifts the lasing emission to longer wavelengths for higher Tm^{3+} ion concentrations.

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