$R_{K^{(*)}}$ and the origin of Yukawa couplings

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ABSTRACT: We explore the possibility that the semi-leptonic B decay ratios $R_{K^{(*)}}$ which violate $\mu - e$ universality are related to the origin of the fermion Yukawa couplings in the Standard Model. Some time ago, a vector-like fourth family (without a Z') was used to generate fermion mass hierarchies and mixing patterns without introducing any family symmetry. Recently the idea of inducing flavourful Z' couplings via mixing with a vector-like fourth family which carries gauged U(1)$'$ charges has been proposed as a simple way of producing controlled flavour universality violation while elegantly cancelling anomalies. We show that the fusion of these two ideas provides a nice connection between $R_{K^{(*)}}$ and the origin of Yukawa couplings in the quark sector. However the lepton sector requires some tuning of Yukawa couplings to obtain the desired coupling of Z' to muons.

Contents

1   Introduction

Recently, the phenomenological motivation for considering non-universal Z' models has increased due to mounting evidence for semi-leptonic B decays which violate $\mu - e$ universality at rates which exceed those predicted by the SM [2–4]. In particular, the LHCb Collaboration and other experiments have reported a number of anomalies in $B \to K^{(*)} l^+ l^-$ decays such as the R_K [5] and R_{K^*} [6] ratios of $\mu^+\mu^-$ to e^+e^- final states, which are observed to be about 70% of their expected values with a 4σ deviation from the SM, and the P_5' angular variable, not to mention the $B \to \phi\mu^+\mu^-$ mass distribution in $m_{\mu^+\mu^-}$.

Following the measurement of R_{K^*} [6], a number of phenomenological analyses of these data, see e.g. [7–16] favour a new physics operator of the form $\bar{b}_L \gamma^\mu s_L \, \bar{\mu}_L \gamma_\mu \mu_L$, or of the form, $\bar{b}_L \gamma^\mu s_L \, \bar{\mu} \gamma_\mu \mu$, each with a coefficient Λ^{-2} where $\Lambda \sim 31.5\,\mathrm{TeV}$, or some linear combination of these two operators. For example, in a flavourful Z' model, the new physics operator will arise from tree-level Z' exchange, where the Z' must dominantly couple to $\mu\mu$ over ee, and must also have the quark flavour changing coupling $b_L s_L$ which must dominate over $b_R s_R$. There is a large and growing body of literature on non-universal Z' models as applied to explaining these anomalies [17–36], with more recent papers following the R_{K^*} measurement in [37–47]. One of the challenges facing such models is the requirement that they be anomaly-free.

In the present paper we will investigate the possible connection between the experimental signal for new physics in R_{K^*} and the origin of fermion Yukawa couplings. Remarkably, despite the huge recent literature on R_{K^*}, relatively few papers are concerned with its possible connection with Yukawa couplings [48–52]. Here we shall take the point of view

that the observation of R_{K^*} is the first hint that the origin of Yukawa couplings is just round the corner close to the TeV scale. We shall focus on Z' models where the physics responsible for the Z' mass and couplings is also responsible for generating the effective Yukawa couplings. This is the first time that such a connection has been considered in the literature to our knowledge.

We consider a scenario in which the Standard Model (SM) at the electroweak scale is an effective theory resulting from some theory at some higher scale(s) which may be as low as the TeV scale. We require that all fermion Yukawa couplings must result from higher dimension operators, so that the effective Yukawa couplings of the SM can be expressed in terms of the left-handed fermion electroweak doublets, $\psi_i = L_i, Q_i$, where $i = 1, 2, 3$, and the CP conjugated right-handed electroweak singlets, $\psi_j^c = u_j^c, d_j^c, e_j^c, \nu_j^c$,

$$\mathcal{L}_{\text{eff}}^{Yuk} = \left(\frac{\langle \phi_i \rangle}{\Lambda_{i,n}^\psi} \right)^n \left(\frac{\langle \phi_j \rangle}{\Lambda_{j,m}^{\psi^c}} \right)^m H \psi_i \psi_j^c, \tag{1.1}$$

plus $H.c.$, summed over fields, families and powers of n, m. Eq. (1.1) involves new SM singlet fields ϕ_i which develop VEVs, leading to effective Yukawa couplings suppressed by powers of $\langle \phi_i \rangle / \Lambda$. Our scenario also involves a massive Z' under which the three SM families ψ_i have zero charge, and which only couples to it via the same singlet fields ϕ_i which have non-zero charge under the associated $U(1)'$ gauge group,

$$\mathcal{L}_{\text{eff}}^{Z'} = \left(\frac{\langle \phi_i \rangle}{\Lambda_{i,n}^{\prime\psi}} \right)^n \left(\frac{\langle \phi_j \rangle}{\Lambda_{j,m}^{\prime\psi}} \right)^m g' Z'_\mu \psi_i^\dagger \gamma^\mu \psi_j + (\psi \to \psi^c) \tag{1.2}$$

summed over fields, families and powers of n, m, where g' is the $U(1)'$ gauge coupling and we allow for different coupling strengths in the gauge coupling denominator factors Λ' as compared to Λ. The absence of a coupling at a given order corresponds to a particular Λ or Λ' becoming formally infinite. In a given model, such as the example discussed in this paper, the various Λ and Λ' may be simply related. The key feature of this scenario is that the same numerator factors of $\langle \phi_i \rangle$ control both the Yukawa couplings in eq. (1.1) and the Z' couplings in eq. (1.2).

Another key feature of the above scenario is that the Z' mass is also generated by the VEVs $\langle \phi_i \rangle$, so that $M_{Z'} \sim g' \langle \phi_i \rangle$. This implies that the observation of R_{K^*}, which sets the scale of the Z' mass and couplings, also sets the scale of the theory of flavour, which must both be not far from the TeV scale, $\langle \phi_i \rangle \sim$ TeV. This does not happen in scalar leptoquark models, for example, since the scalar mass can be written down by hand and it is not linked to the flavour scale. Thus although the observation of R_{K^*} fixes the mass of the mediator Z' or leptoquark to be at the TeV scale, it is only in the case of the Z' that the VEV of $\langle \phi_i \rangle$ is necessarily also fixed to be at the TeV scale, since its mass is given by $M_{Z'} \sim g' \langle \phi_i \rangle$. By contrast, a TeV leptoquark is consistent with $\langle \phi_i \rangle$ and Λ being much higher than the TeV scale, providing that their ratio is held fixed, since its mass is not related to $\langle \phi_i \rangle$. For this reason we prefer to consider Z' models.

In the scenario of eqs. (1.1), (1.2), in the limit that $\langle \phi_i \rangle = 0$, there are no Yukawa couplings and also no couplings of SM fermions to the Z' since we assume they are not charged

under the associated U(1)′ gauge group. When $\langle\phi_i\rangle/\Lambda_i$ are switched on then Yukawa couplings and small non-universal and flavour dependent couplings of SM fermions to the Z' are generated simultaneously, as well as the Z' mass itself. The above framework then provides a link between flavour changing observables and the origin of Yukawa couplings of the kind that we are interested in.

In particular, there will be a connection between the experimental signal for new physics in R_{K^*} due to Z' exchange and the origin of Yukawa couplings. Since the small Yukawa couplings are known (or at least their eigenvalues), this constrains the values of $\langle\phi_i\rangle/\Lambda_i$, and since we wish to explain R_{K^*} via non-universal Z' exchange, then this will also constrain the Z' mass to be around the few TeV scale. Then we have $M_{Z'} \sim \langle\phi_i\rangle \sim$ few TeV, and hence the theory of flavour is at the few TeV scale. In general one might expect severe experimental flavour and collider constraints such an effective theory of flavour at the TeV scale. For example there could be dangerous flavour changing neutral currents (FCNCs) from higher order operators with dimensional scale $\Lambda_i \sim$ few TeV. However in such a scenario, such FCNC operators are expected to have coefficients suppressed by small Yukawa couplings (generated by powers of $\langle\phi_i\rangle/\Lambda_i$). Of course this is not obvious from a general analysis, so one is motivated to provide some explicit renormalisable examples to substantiate this claim.

In fact there is a threefold motivation for going beyond the effective description in eqs. (1.1), (1.2). Firstly, the treatment of the third family Yukawa couplings, especially the top quark coupling, which must be considered carefully. Secondly, for the Yukawa couplings, the scales $\langle\phi_i\rangle$ and Λ_i must be of order a few TeV, in order to explain R_{K^*}, and so the effective theory in eqs. (1.1), (1.2) must break down not far above the TeV scale in any case. Thirdly, in a concrete model capable of describing all the quark and lepton masses and mixings, the relation between the Yukawa and Z' couplings becomes very clear and it is more straightforward to see whether such a model can evade all the experimental constraints while providing an explanation of R_{K^*}. In particular we can clearly investigate whether FCNC operators may be adequately suppressed by small Yukawa as claimed above. The main goal of this paper, then, is to show that the effective theory in eqs. (1.1), (1.2), with Z' mass $M_{Z'} \sim g'\langle\phi_i\rangle$, can be realised by some ultraviolet complete renormalisable theory.

Recently we have proposed an explanation of R_{K^*} based on a fourth vector-like family which carries U(1)′, where the anomalies cancel between conjugate representations in the fourth family [53]. In the "fermiophobic" example, we showed that even if the three chiral families do not carry U(1)′ charges, effective non-universal couplings, as required to explain R_{K^*}, can emerge indirectly due to mixing with the fourth family [53]. A closely related idea was also considered some time later [54] in which the third chiral family and the conjugate half of the fourth vector-like family carried the U(1)′ charges, but otherwise the mechanism for R_{K^*} works very similar to [53]. Other phenomenological implications such as the muon $g - 2$ and $\tau \to \mu\gamma$ were also considered [54]. We have pursued this idea in several subsequent papers, including: F-theory models with non-universal gauginos [55]; SO(10) models (including the question of neutrino mass) [56]; SU(5) models (focussing on the Yukawa relation $Y_e \neq Y_d^T$) [57]; and flavourful Z' portal models with a coupling to a fourth-family singlet Dirac neutrino Dark Matter, including an extensive discussion of all

phenomenological constraints [58]. In contrast to our previous work, here we shall require that small Yukawa couplings are not put in by hand, but are instead generated by couplings to the fourth family, leading to the effective operators in eqs. (1.1) and (1.2).

In this paper, then, we will construct an explicit renormalisable model with a vector-like family as an ultraviolet completion of eqs. (1.1) and (1.2). Our starting point is the model proposed in [59, 60], which involves one vector-like family distinguished by a discrete Z_2. We introduce a new $U(1)'$ gauge group, spontaneously broken at the TeV scale, under which the three chiral families are neutral but the vector-like fourth family is charged.[1] The mixing between the fourth family and the three chiral families provides the Yukawa couplings as in eq. (1.1), as well as the non-universal effective Z' couplings involving the three light families in eq. (1.2), providing a link between the Yukawa and Z' couplings. However, while the fusion of these two ideas provides a nice connection between $R_{K^{(*)}}$ and the origin of Yukawa couplings in the quark sector, the lepton sector requires some tuning of Yukawa couplings to obtain the desired coupling of Z' to muons.

The layout of the remainder of the paper is as follows. In section 2 we consider the model with the vector-like family, where the Higgs is charged under the $U(1)'$ and effective Yukawa and Z' couplings are generated by the same mixing. Section 3 concludes the paper.

2 The model

2.1 The renormalisable Lagrangian

The model we consider here is defined in table 1. The model involves three chiral families $\psi_i(0), \psi_i^c(0)$, plus a fourth vector-like family consisting of $\psi_4(1), \psi_4^c(1)$ plus the conjugate representations $\overline{\psi_4}(-1), \overline{\psi_4^c}(-1)$, where the $U(1)'$ charges are shown in parentheses. The gauged $U(1)'$ is broken by the singlet scalars $\phi(1)$, with vacuum expectation values (VEVs) around the TeV scale, yielding a massive Z' at this scale. Since the Higgs doublets $H(-1)$ are charged under the $U(1)'$, this forbids all Yukawa couplings, except those which couple the first three families to the fourth family.

A similar model was proposed as a model of effective Yukawa couplings in [59, 60]. The main difference is that the model here involves a gauged $U(1)'$ resulting in effective Yukawa and flavourful Z' couplings as in eqs. (1.1) and (1.2) which are related, while in [59, 60] only the effective Yukawa couplings were considered. A welcome consequence of this is that, unlike [59, 60], we shall not require an additional Z_2 symmetry to forbid renormalisable Yukawa couplings. Instead such couplings are forbidden by the gauged $U(1)'$ under which the fourth vector-like family and Higgs doublets are charged. In addition, we shall go beyond the mass insertion approximation of [59, 60], which breaks down for the top quark Yukawa coupling. Another difference is that the model in [59, 60] was supersymmetric, while our model here is not. However we require two Higgs doublets H_u, H_d, both with negative $U(1)'$ charge.

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[1]A nice consequence of this is that it is no longer necessary to introduce the discrete Z_2 symmetry.

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)'$
Q_i	**3**	**2**	1/6	0
u_i^c	**$\overline{3}$**	**1**	$-2/3$	0
d_i^c	**$\overline{3}$**	**1**	1/3	0
L_i	**1**	**2**	$-1/2$	0
e_i^c	**1**	**1**	1	0
ν_i^c	**1**	**1**	0	0
Q_4	**3**	**2**	1/6	1
u_4^c	**$\overline{3}$**	**1**	$-2/3$	1
d_4^c	**$\overline{3}$**	**1**	1/3	1
L_4	**1**	**2**	$-1/2$	1
e_4^c	**1**	**1**	1	1
ν_4^c	**1**	**1**	0	1
$\overline{Q_4}$	**$\overline{3}$**	**$\overline{2}$**	$-1/6$	-1
$\overline{u_4^c}$	**3**	**1**	2/3	-1
$\overline{d_4^c}$	**3**	**1**	$-1/3$	-1
$\overline{L_4}$	**1**	**$\overline{2}$**	1/2	-1
$\overline{e_4^c}$	**1**	**1**	-1	-1
$\overline{\nu_4^c}$	**1**	**1**	0	-1
ϕ	**1**	**1**	0	1
H_u	**1**	**2**	1/2	-1
H_d	**1**	**2**	$-1/2$	-1

Table 1. The model consists of three left-handed chiral families $\psi_i = Q_i, L_i$ and $\psi_i^c = u_i^c, d_i^c, e_i^c, \nu_i^c$ ($i = 1, 2, 3$), plus a fourth vector-like family consisting of ψ_4, ψ_4^c plus $\overline{\psi_4}, \overline{\psi_4^c}$, together with the U(1)$'$ breaking scalar fields ϕ and the two Higgs scalar doublets H_u, H_d which are both negatively charged under U(1)$'$.

The allowed renormalisable Yukawa couplings and explicit masses allowed by U(1)$'$ are,

$$\mathcal{L}^{ren} = y_{i4}^\psi H \psi_i \psi_4^c + y_{4i}^\psi H \psi_4 \psi_i^c + x_i^\psi \phi \psi_i \overline{\psi_4} + x_i^{\psi^c} \phi \psi_i^c \overline{\psi_4^c} + M_4^\psi \psi_4 \overline{\psi_4} + M_4^{\psi^c} \psi_4^c \overline{\psi_4^c} \quad (2.1)$$

plus $H.c.$, summed over fields and families, where x, y are dimensionless coupling constants ideally of order unity, while M are explicit mass terms of order a few TeV.

2.2 Mass insertion approximation

Although the usual Yukawa couplings $y_{ij}^\psi H \psi_i \psi_j^c$ are forbidden for $i, j = 1, \ldots 3$ (since H are charged under U(1)$'$) effective 3×3 Yukawa couplings may be generated by the two
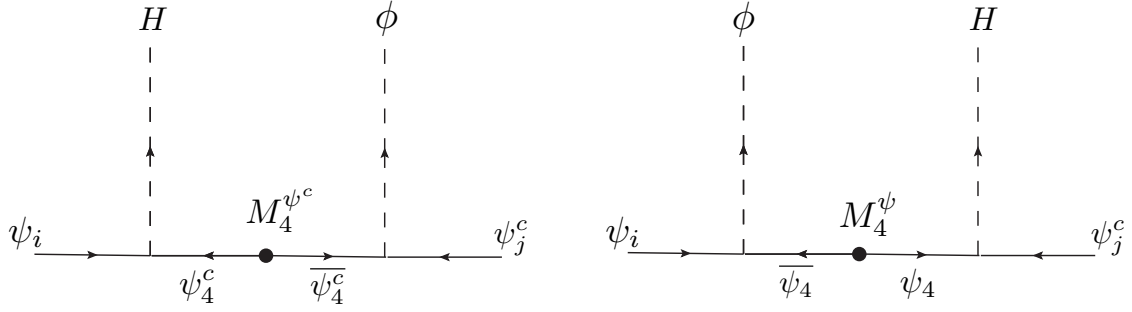
Figure 1. Diagrams in the model which lead to the effective Yukawa couplings in the mass insertion approximation, where $H = H_u, H_d$.
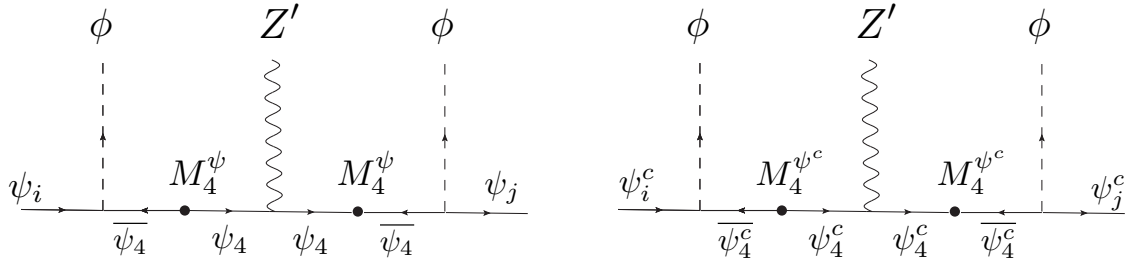

Figure 2. Diagrams in the model which lead to the effective Z' couplings in the mass insertion approximation.

mass insertion diagrams in figure 1 (up to an irrelevant minus sign),

$$\mathcal{L}_{\text{eff}}^{Yuk} = \frac{x_j^{\psi^c} \langle \phi \rangle}{M_4^{\psi^c}} y_{i4}^{\psi} H \psi_i \psi_j^c + \frac{x_i^{\psi} \langle \phi \rangle}{M_4^{\psi}} y_{4j}^{\psi} H \psi_i \psi_j^c \tag{2.2}$$

plus $H.c.$, summed over fields and families, which can be compared to eq. (1.1).

The model also involves a massive Z' under which the three SM families ψ_i, ψ_i^c have zero U(1)$'$ charge. Although the usual Z' couplings $g' Z'_\mu \psi_i^\dagger \gamma^\mu \psi_j$ are forbidden for $i, j = 1, \dots 3$, the fourth vector-like family has non-zero U(1)$'$ charge, and effective Z' couplings may be generated by the two mass insertion diagrams in figure 2,

$$\mathcal{L}_{\text{eff}}^{Z'} = \frac{x_i^{\psi} \langle \phi \rangle}{M_4^{\psi}} \frac{x_j^{\psi} \langle \phi \rangle}{M_4^{\psi}} g' Z'_\mu \psi_i^\dagger \gamma^\mu \psi_j \; + \; \frac{x_i^{\psi^c} \langle \phi \rangle}{M_4^{\psi^c}} \frac{x_j^{\psi^c} \langle \phi \rangle}{M_4^{\psi^c}} g' Z'_\mu \psi_i^{c\dagger} \gamma^\mu \psi_j^c \tag{2.3}$$

summed over fields and families, which can be compared to eq. (1.2). The above model is therefore an example of a renormalisable model which can lead to the effective theory of the

kind discussed in the Introduction, namely one in which Yukawa and Z' couplings are both controlled by the same physics, in this case the VEVs $\langle \phi \rangle$ and the fourth family vector-like masses M_4^ψ and $M_4^{\psi^c}$. Moreover, the mass of the Z' is given by $M_{Z'} = g'\langle \phi \rangle$, which is the same scale at which the Yukawa couplings are generated. However, while the Yukawa couplings are generated at first order, the Z' couplings are generated at second order in the mass insertion approximation. We shall discuss the phenomenological implications of this later. For the moment, let us return to the Yukawa couplings and discuss them in some more detail.

There is a such a Yukawa matrix as in eq. (2.2) for each of the four charged sectors $\psi = u, d, e, \nu$. In the case of neutrinos, this refers to the Dirac Yukawa matrix, and there will be a further Majorana mass matrix for the singlet neutrinos $M_{ij}^{\nu^c} \nu_i^c \nu_j^c$. Since nothing prevents the Majorana masses $M_{ij}^{\nu^c}$ being arbitrarily large, well above the U(1)$'$ breaking scale, this will lead to a conventional seesaw mechanism for small neutrino masses. On the other hand we are assuming that the vector-like masses M_4^ψ and $M_4^{\psi^c}$ to be close to the U(1)$'$ breaking scale of order the TeV scale.

2.3 The 5×5 matrix

Since the large top quark Yukawa coupling y_t is not present at renormalisable level, it must also arise from mixing with the fourth vector-like family. Clearly the mass insertion approximation in eq. (2.2) breaks down in the case $y_t \sim y_{33}^u \sim 1$. The large top quark Yukawa coupling therefore motivates us to go beyond the mass insertion approximation used in [59, 60]. To proceed, we first arrange the masses and couplings in eq. (2.1) into 5×5 matrices, one for each charge sector $\psi = u, d, e, \nu$,

$$
M^\psi = \begin{pmatrix}
 & \psi_1^c & \psi_2^c & \psi_3^c & \psi_4^c & \overline{\psi_4} \\
\hline
\psi_1| & 0 & 0 & 0 & y_{14}^\psi H & x_1^\psi \phi \\
\psi_2| & 0 & 0 & 0 & y_{24}^\psi H & x_2^\psi \phi \\
\psi_3| & 0 & 0 & 0 & y_{34}^\psi H & x_3^\psi \phi \\
\psi_4| & y_{41}^\psi H & y_{42}^\psi H & y_{43}^\psi H & 0 & M_4^\psi \\
\overline{\psi_4^c}| & x_1^{\psi^c} \phi & x_2^{\psi^c} \phi & x_3^{\psi^c} \phi & M_4^{\psi^c} & 0
\end{pmatrix} .
\tag{2.4}
$$

There are three distinct mass scales in these matrices: the Higgs VEVs $\langle H \rangle$, the ϕ VEVs $\langle \phi \rangle$ and the vector-like fourth family masses M_4^ψ, $M_4^{\psi^c}$. If all these mass scales are of the same order then the correct procedure is to diagonalise the full 5×5 matrices in each of the charge sectors (apart from neutrinos which must be treated differently due to the Majorana masses and the seesaw mechanism). Then unitarity violation will play a role. However, in the approximation $\langle H \rangle \ll \langle \phi \rangle$ (physically $M_Z \ll M_{Z'}$), it will not be necessary to diagonalise the full matrix in one step, as we shall see.

2.4 A convenient basis for quarks

Since the upper 3×3 block of eq. (2.4) contains zeros we are free to rotate the first three families as we wish without changing the upper 3×3 block. Similarly the Z' couplings to

the first three families remain zero under such rotations. For example, in the quark sector we are allowed to go to a particular basis in Q_i, d_i^c, u_i^c ($i = 1, \ldots 3$) flavour space where $x_{1,2}^Q = 0$, $y_{41,42}^u = 0$, $y_{41,42}^d = 0$. Then we can further rotate the first and second families to set $x_1^{u^c} = 0$, $x_1^{d^c} = 0$ and $y_{14}^u = 0$ (but in general $y_{14}^d \neq 0$ since the quark doublet rotations have already been used up). In this basis the 5×5 quark matrices M^u, M^d from eq. (2.4) become, respectively,

$$
\left(
\begin{array}{c|ccccc}
 & u_1^c & u_2^c & u_3^c & u_4^c & \overline{Q_4} \\
\hline
Q_1| & 0 & 0 & 0 & 0 & 0 \\
Q_2| & 0 & 0 & 0 & y_{24}^u H^u & 0 \\
Q_3| & 0 & 0 & 0 & y_{34}^u H^u & x_3^Q \phi \\
Q_4| & 0 & 0 & y_{43}^u H^u & 0 & M_4^Q \\
\overline{u_4^c}| & 0 & x_2^{u^c}\phi & x_3^{u^c}\phi & M_4^{u^c} & 0
\end{array}
\right),
\left(
\begin{array}{c|ccccc}
 & d_1^c & d_2^c & d_3^c & d_4^c & \overline{Q_4} \\
\hline
Q_1| & 0 & 0 & 0 & y_{14}^d H^d & 0 \\
Q_2| & 0 & 0 & 0 & y_{24}^d H^d & 0 \\
Q_3| & 0 & 0 & 0 & y_{34}^d H^d & x_3^Q \phi \\
Q_4| & 0 & 0 & y_{43}^d H^d & 0 & M_4^Q \\
\overline{d_4^c}| & 0 & x_2^{d^c}\phi & x_3^{d^c}\phi & M_4^{d^c} & 0
\end{array}
\right)
\tag{2.5}
$$

Note that the fifth column is identical for both matrices, but, after electroweak symmetry breaking, the quark doublets Q are split apart into u, d which become part of M^u, M^d, respectively.

For purposes of later comparison, it is instructive to show the result for the effective 3×3 Yukawa matrices for the quarks, $y_{ij}^u H_u Q_i u_j^c$ and $y_{ij}^d H_d Q_i d_j^c$, obtained from the 5×5 matrices in eq. (2.5) (in the above basis) using the mass insertion approximation in eq. (2.2),

$$
y_{ij}^u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_{24}^u x_2^{u^c} & y_{24}^u x_3^{u^c} \\ 0 & y_{34}^u x_2^{u^c} & y_{34}^u x_3^{u^c} \end{pmatrix} \frac{\langle\phi\rangle}{M_4^{u^c}} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x_3^Q y_{43}^u \end{pmatrix} \frac{\langle\phi\rangle}{M_4^Q}
$$

$$
y_{ij}^d = \begin{pmatrix} 0 & y_{14}^d x_2^{d^c} & y_{14}^d x_3^{d^c} \\ 0 & y_{24}^d x_2^{d^c} & y_{24}^d x_3^{d^c} \\ 0 & y_{34}^d x_2^{d^c} & y_{34}^d x_3^{d^c} \end{pmatrix} \frac{\langle\phi\rangle}{M_4^{d^c}} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x_3^Q y_{43}^d \end{pmatrix} \frac{\langle\phi\rangle}{M_4^Q}
\tag{2.6}
$$

The effective Yukawa matrices above consist of the sum of two rank 1 matrices, so the first family will be massless without further modification. The simplest modification is to include Higgs messengers as in [59, 60], where such Higgs messengers are not charged under U(1)′ and so will not induce any Z' couplings to the first family. Indeed the dominance of the fourth family diagrams over the Higgs exchange diagrams provides a nice explanation of the smallness of the first family masses. Here we shall not consider the Higgs messenger diagrams explicitly and instead work in the massless first family limit, which is a good approximation and is sufficient to illustrate the connection with Z' couplings which are only induced by mixing with the fourth vector-like family.

If one of the two Yukawa terms in each of the expressions in eq. (2.6) is dropped, then the second family masses will become zero as well. This suggests a natural explanation of the smallness of the second family compared to the first family, namely that one of the two terms dominates over the other one. This assumption was called "messenger dominance" in [59, 60]. In order to account for the smallness of the CKM element V_{cb} in the

quark sector, it is natural to assume that the left-handed quark messengers dominate over the right-handed messengers, $M_4^Q \ll M_4^{d^c}, M_4^{u^c}$, which was called "left-handed messenger dominance" in [59, 60], with the further assumption $M_4^Q \ll M_4^{d^c} \ll M_4^{u^c}$ reproducing the more pronounced mass hierarchy in the up sector than the down sector. Assuming all this leads to $|V_{cb}| \sim m_s/m_b$ with V_{ub}, though naturally small, being unconstrained [59, 60]. However to explain the smallness of the Cabibbo angle requires further model building such as an $SU(2)_R$ symmetry [59, 60], although here we assume its smallness is accidental.

2.5 A basis for decoupling the heavy fourth family

As already remarked, given the large top Yukawa coupling, we need to go beyond mass insertion approximation $\langle \phi \rangle \ll M_4^\psi$, so we must return to the full 5×5 mass matrices. However we shall still assume that $\langle H \rangle \ll \langle \phi \rangle$ so that we may switch off the Higgs VEVs all together in the first instance, and so obtain an effective SM after integrating out the heavy fourth family. A first step in this direction is to work in a basis where the 5×5 mass matrices have the form

$$
M^\psi = \begin{pmatrix}
 & \psi_1^c & \psi_2^c & \psi_3^c & \psi_4^c & \overline{\psi_4} \\
\hline
\psi_1| & & & & & 0 \\
\psi_2| & & & & & 0 \\
\psi_3| & & & \tilde{y}'^\psi_{\alpha\beta}H & & 0 \\
\psi_4| & & & & & \tilde{M}_4^\psi \\
\overline{\psi_4^c}| & 0 & 0 & 0 & \tilde{M}_4^{\psi^c} & 0
\end{pmatrix}. \tag{2.7}
$$

where $\tilde{y}'^\psi_{\alpha\beta}$ with $\alpha, \beta = 1, \ldots 4$ are the 4×4 upper block Yukawa matrices in this basis. The key feature of eq. (2.7) are the zeros in the fifth row and column which are achieved by rotating the first four families by the unitary 4×4 transformations introduced in [53],

$$
V_Q = V_{34}^Q V_{24}^Q V_{14}^Q, \quad V_{u^c} = V_{34}^{u^c} V_{24}^{u^c} V_{14}^{u^c}, \quad V_{d^c} = V_{34}^{d^c} V_{24}^{d^c} V_{14}^{d^c}, \tag{2.8}
$$

where each of the unitary matrices V_{i4} are parameterised by a single angle θ_{i4} describing the mixing between the ith chiral family and the 4th vector-like family.

As emphasised in [53], the basis in eq. (2.7) is very useful for decoupling the heavy states since the first three rows and columns of the matrices do not involve any large mass terms. Then we can decouple the heavy fourth vector-like family, in the approximation $\langle H \rangle \ll \langle \phi \rangle$, by simply striking out the fourth and fifth rows and columns of these matrices. The Yukawa matrices of the SM, $\tilde{y}'^\psi_{ij}$, then correspond to the remaining 3×3 upper blocks of the 4×4 Yukawa matrices, $\tilde{y}'^\psi_{\alpha\beta}$. However, it should be remembered that the undecoupled three families in this basis contain admixtures of the original fourth vector-like family due to the mixing, and so will have modified Yukawa couplings, as compared to the original (unmixed) three chiral families, as follows.

In the above basis, where only the fourth components of the fermions are very heavy, the 4×4 Yukawa couplings are,

$$
\tilde{y}'^u_{\alpha\beta} = V_Q \tilde{y}^u_{\alpha\beta} V_{u^c}^\dagger, \quad \tilde{y}'^d_{\alpha\beta} = V_Q \tilde{y}^d_{\alpha\beta} V_{d^c}^\dagger, \tag{2.9}
$$

and $\tilde{y}^u_{\alpha\beta}, \tilde{y}^d_{\alpha\beta}$ are identified with the 4×4 upper blocks of eq. (2.5). The effective SM Yukawa couplings for the quarks then correspond to the 3×3 upper blocks of $\tilde{y}'^u_{\alpha\beta}, \tilde{y}'^d_{\alpha\beta}$, namely

$$y^u_{ij}H_uQ_iu^c_j, \quad y^d_{ij}H_dQ_id^c_j, \quad \text{with} \quad y^u_{ij} \equiv \tilde{y}'^u_{ij}, \quad y^d_{ij} \equiv \tilde{y}'^d_{ij}, \quad (i,j=1,\ldots3). \tag{2.10}$$

The effective SM Yukawa couplings have non-zero elements due to the mixing, even though originally they were all zero. This is the origin of flavour in the low energy effective SM theory.

2.6 Effective quark Yukawa couplings revisited

We now calculate the effective SM Yukawa matrices for quarks in this particular basis, following the above more accurate procedure, and then compare to the results to the mass insertion approximation in eq. (2.6). This treatment still assumes that $\langle H \rangle \ll \langle \phi \rangle$ but relaxes the assumption $\langle \phi \rangle \ll M^Q_4$, which is not valid due to the large top quark Yukawa coupling. Therefore the treatment in this subsection is necessary for consistency.

From eq. (2.5) we read off the upper 4×4 blocks,

$$\tilde{y}^u_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & y^u_{24} \\ 0 & 0 & 0 & y^u_{34} \\ 0 & 0 & y^u_{43} & 0 \end{pmatrix}, \quad \tilde{y}^d_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & y^d_{14} \\ 0 & 0 & 0 & y^d_{24} \\ 0 & 0 & 0 & y^d_{34} \\ 0 & 0 & y^d_{43} & 0 \end{pmatrix} \tag{2.11}$$

Eq. (2.5) also shows that we need $(3,4)$ mixing in the Q sector and both $(2,4)$ and $(3,4)$ mixing in the u^c, d^c sectors to go to the decoupling basis in eq. (2.7). The unitary matrices in eq. (2.8) are then given by,

$$V_Q = V^Q_{34} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c^Q_{34} & s^Q_{34} \\ 0 & 0 & -s^Q_{34} & c^Q_{34} \end{pmatrix}, \quad s^Q_{34} = \frac{x^Q_3 \langle \phi \rangle}{\sqrt{(x^Q_3 \langle \phi \rangle)^2 + (M^Q_4)^2}}, \tag{2.12}$$

where we anticipate that $s^Q_{34} \sim 1$ due to the large top quark Yukawa coupling. For V_{u^c}, V_{d^c} only the angles $\theta^{u^c}_{24}, \theta^{u^c}_{34}$ and $\theta^{d^c}_{24}, \theta^{d^c}_{34}$ may be non-zero, and indeed must be to generate the second family masses and the mixing angles (although $\theta^{u^c}_{34}$ is not necessary). However these angles are all small, since we assume $M^Q_4 \ll M^{d^c}_4 \ll M^{u^c}_4$, so here we are allowed to approximate,

$$V_{u^c} = V^{u^c}_{34}V^{u^c}_{24} \approx \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \theta^{u^c}_{24} \\ 0 & 0 & 1 & \theta^{u^c}_{34} \\ 0 & -\theta^{u^c}_{24} & -\theta^{u^c}_{34} & 1 \end{pmatrix}, \quad \theta^{u^c}_{24} \approx \frac{x^{u^c}_2 \langle \phi \rangle}{M^{u^c}_4}, \quad \theta^{u^c}_{34} \approx \frac{x^{u^c}_3 \langle \phi \rangle}{M^{u^c}_4} \tag{2.13}$$

and similarly for V_{d^c}.

Given the results in eqs. (2.11), (2.12)), (2.13), the 4×4 matrices in the basis of eq. (2.9) may be readily computed and the Yukawa couplings of the SM in eq. (2.10) may then be identified as their upper 3×3 blocks,

$$
y_{ij}^u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \theta_{24}^{u^c} y_{24}^u & \theta_{34}^{u^c} y_{24}^u \\ 0 & c_{34}^Q \theta_{24}^{u^c} y_{34}^u & c_{34}^Q \theta_{34}^{u^c} y_{34}^u \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & s_{34}^Q y_{43}^u \end{pmatrix},
$$

$$
y_{ij}^d = \begin{pmatrix} 0 & \theta_{24}^{d^c} y_{14}^d & \theta_{34}^{d^c} y_{14}^d \\ 0 & \theta_{24}^{d^c} y_{24}^d & \theta_{34}^{d^c} y_{24}^d \\ 0 & c_{34}^Q \theta_{24}^{d^c} y_{34}^d & c_{34}^Q \theta_{34}^{d^c} y_{34}^d \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & s_{34}^Q y_{43}^d \end{pmatrix} \tag{2.14}
$$

The effective Yukawa matrices in eq. (2.14) reduce to those in the mass insertion approximation in eq. (2.6) in the small θ_{34}^Q angle limit. However such an approximation is not justified since the top quark Yukawa coupling is identified as $y_t \approx s_{34}^Q y_{43}^u$ which implies that $s_{34}^Q \sim 1$ as already anticipated. We must use the more reliable expressions for the Yukawa matrices in eq. (2.14). The bottom quark Yukawa coupling $y_b \approx s_{34}^Q y_{43}^d$ may also be large if $\tan\beta = v_u/v_d$ is large. The small values of the Yukawa coupling of the charm quark $y_c \approx \theta_{24}^{u^c} y_{24}^u$ and strange quark $y_s \approx \theta_{24}^{d^c} y_{24}^d$, are accounted for by the assumption $M_4^Q \ll M_4^{d^c} \ll M_4^{u^c}$, which justifies the small angle approximations for the other angles.

2.7 Effective Z' couplings revisited

As discussed in [53], there is an approximate GIM mechanism in the electroweak sector involving $W^\pm, Z$ boson exchange, because all four families $\psi_\alpha, \psi_\alpha^c$, transform identically under the SM gauge group, so their mixing does not induce any such flavour violation. However, this is not true for the Z' gauge bosons, which only couple to the fourth family. After the mixing with the fourth family, the three light families have induced non-universal and flavour violating couplings to the Z', which depend on the mixing angles of the 4×4 unitary matrices in eq. (2.8), as we now discuss.

After U(1)$'$ breaking, we have a massive Z' gauge boson which only couples to the fourth family in the basis of eq. (2.5), ignoring the heavy neutrino singlets,

$$
\mathcal{L}_{Z'}^{\text{gauge}} = g' Z'_\mu \left(Q_\alpha^\dagger D_Q \gamma^\mu Q_\beta + u_\alpha^{c\dagger} D_{u^c} \gamma^\mu u_\beta^c + d_\alpha^{c\dagger} D_{d^c} \gamma^\mu d_\beta^c + L_\alpha^\dagger D_L \gamma^\mu L_\beta + e_\alpha^{c\dagger} D_{e^c} \gamma^\mu e_\beta^c \right) \tag{2.15}
$$

where $\alpha, \beta = 1, \ldots 4$ and the diagonal 4×4 charge matrices are

$$
D_Q = D_{u^c} = D_{d^c} = D_L = D_{e^c} = \text{diag}(0, 0, 0, 1). \tag{2.16}
$$

Going to the basis of eq. (2.7), we have

$$
\mathcal{L}_{Z'}^{\text{gauge}} = g' Z'_\mu \left(Q_\alpha^{'\dagger} D_Q' \gamma^\mu Q_\beta' + u_\alpha^{'c\dagger} D_{u^c}' \gamma^\mu u_\beta^{'c} + d_\alpha^{'c\dagger} D_{d^c}' \gamma^\mu d_\beta^{'c} + L_\alpha^{'\dagger} D_L' \gamma^\mu L_\beta' + e_\alpha^{'c\dagger} D_{e^c}' \gamma^\mu e_\beta^{'c} \right) \tag{2.17}
$$

where the rotated charge matrices are,

$$
D_Q' = V_Q D_Q V_Q^\dagger, \ D_{u^c}' = V_{u^c} D_{u^c} V_{u^c}^\dagger, \ D_{d^c}' = V_{d^c} D_{d^c} V_{d^c}^\dagger \tag{2.18}
$$

where V_Q, V_{u^c}, V_{d^c} are given in eqs. (2.12), (2.13).

Focussing on the upper 3×3 blocks of D', the Z' coupling matrices of the SM quarks are

$$\mathcal{L}^{\text{gauge}}_{Z',3\times3} = D'^{Q}_{ij} g' Z'_\mu Q_i^\dagger \gamma^\mu Q_j + D'^{u^c}_{ij} g' Z'_\mu u_i^{c\dagger} \gamma^\mu u_j^c + D'^{d^c}_{ij} g' Z'_\mu d_i^{c\dagger} \gamma^\mu d_j^c, \qquad (2.19)$$

where the charge matrices may then be computed from the upper 3×3 blocks of eq. (2.18),

$$D'^{Q}_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (s^{Q}_{34})^2 \end{pmatrix}, \; D'^{u^c}_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & (\theta^{u^c}_{24})^2 & \theta^{u^c}_{24}\theta^{u^c}_{34} \\ 0 & \theta^{u^c}_{24}\theta^{u^c}_{34} & (\theta^{u^c}_{34})^2 \end{pmatrix}, \; D'^{d^c}_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & (\theta^{d^c}_{24})^2 & \theta^{d^c}_{24}\theta^{d^c}_{34} \\ 0 & \theta^{d^c}_{24}\theta^{d^c}_{34} & (\theta^{d^c}_{34})^2 \end{pmatrix} \quad (2.20)$$

The important point is that the Z' couplings of the SM quarks in eqs. (2.19), (2.20) are controlled by the same mixing angles that control their Yukawa couplings, in the same basis eq. (2.14), but are second order in these mixing angles. Thus while the large top quark Yukawa coupling implies that $s^{Q}_{34} \sim 1$ and hence the Z' couples in an unsuppressed way to the third family quark doublet $Q_3 = (t_L, b_L)$, there are no couplings to the first or second family quark doublets $Q_1 = (u_L, d_L)$, $Q_2 = (c_L, s_L)$ in the basis of eq. (2.14). Moreover, the small value of the Yukawa coupling of the charm quark $y_c \sim \theta^{u^c}_{24} \sim m_c/m_t$ implies that the c_R coupling to Z' is suppressed by $(\theta^{u^c}_{24})^2 \sim (m_c/m_t)^2 \sim 10^{-4}$ in this basis. The s_R coupling to Z' is similarly suppressed, so there is a negligible contribution to $K_0 - \bar{K}_0$ mixing for $M'_Z \sim 1\,\text{TeV}$.

2.8 Phenomenology

One way to explain the muon anomalies especially $R_{K^{(*)}}$ is via the couplings

$$\mathcal{L} \supset Z'_\mu \left(g_{bs} \bar{s}_L \gamma^\mu b_L + g_{\mu\mu} \bar{\mu}_L \gamma^\mu \mu_L \right), \qquad (2.21)$$

where in our model of quarks the above Z' coupling originates from $g_{bb} \bar{b}_L \gamma^\mu b_L$ where $g_{bb} = g'(s^{Q}_{34})^2$ from eq. (2.20), where this coupling is in the basis where the quark Yukawa matrices are given by eq. (2.6) (or equivalently eq. (2.14)). The CKM matrix for the quarks may be constructed in the usual way, by diagonalising these Yukawa matrices,

$$V_{uL} y^u V_{uR}^\dagger = \text{diag}(y_u, y_c, y_t), \quad V_{dL} y^d V_{dR}^\dagger = \text{diag}(y_d, y_s, y_b) \qquad (2.22)$$

to yield the unitary 3×3 CKM matrix,

$$V_{\text{CKM}} = V_{uL} V_{dL}^\dagger. \qquad (2.23)$$

The previous assumption of this model that $M_4^Q \ll M_4^{d^c} \ll M_4^{u^c}$ implies that the CKM mixing originates predominantly from the down sector, hence to good approximation,

$$V_{\text{CKM}} \approx V_{dL}^\dagger. \qquad (2.24)$$

This implies that in the diagonal quark mass basis, the off-diagonal quark coupling in eq. (2.21) is generated with

$$g_{bs} = g'(s^{Q}_{34})^2 (V'^\dagger_{dL})_{32} \approx g'(s^{Q}_{34})^2 V_{ts}. \qquad (2.25)$$

In our model, we expect $s_{34}^Q \sim 1/\sqrt{2}$, say, due to the large top Yukawa coupling, with the gauge coupling $g' \sim 1$ and $V_{ts} \sim -0.04$ (in the usual PDG convention for V_{CKM}) and so from eq. (2.25),

$$g_{bs} = g'(s_{34}^Q)^2 V_{ts} \sim -\frac{1}{50}. \qquad (2.26)$$

We have not yet specified the lepton sector so we do not yet know the value of $g_{\mu\mu}$. In fact we will be guided by these anomalies in our construction of the lepton sector. However we remark that $g_{\mu\mu}$ will have the same relative sign as g_{bb} (positive in our convention), hence the model predicts that $g_{\mu\mu}$ and g_{bs} will have the opposite relative sign as required to account for the R_K and R_{K^*} measurements [61].

As discussed in the Introduction, one possible explanation of the R_K and R_{K^*} measurements in LHCb is that the low-energy Lagrangian below the weak scale contains an additional contribution to the effective 4-fermion operator with left-handed muon, b-quark, and s-quark fields:

$$\Delta \mathcal{L}_{\text{eff}} \supset G_{bs\mu}(\bar{b}_L \gamma^\mu s_L)(\bar{\mu}_L \gamma_\mu \mu_L) + \text{h.c.}, \qquad G_{bs\mu} \approx \frac{1}{(31.5 \text{ TeV})^2}. \qquad (2.27)$$

The positive sign of $G_{bs\mu}$ matches the required negative value of $C_9^\mu < 0$ with $C_9^\mu = -C_{10}^\mu$. The coefficient $G_{bs\mu}$ is due to Z' exchange at tree-level and is given as function of the couplings in eq. (2.21),

$$G_{bs\mu} = -\frac{g_{bs}g_{\mu\mu}}{M_{Z'}^2} \approx \frac{1}{(31.5 \text{ TeV})^2} \qquad (2.28)$$

where the opposite relative sign of $g_{\mu\mu}$ and g_{bs} is crucial to account for the R_K and R_{K^*} measurements [61].

The Z' coupling to bs leads to an additional tree-level contribution to $B_s - \overline{B}_s$ mixing due to the effective operator arising from Z' exchange at tree level:

$$\Delta \mathcal{L}_{\text{eff}} \supset -\frac{g_{bs}^2}{2M_{Z'}^2}(\bar{s}_L \gamma^\mu b_L)^2 + \text{h.c}, \qquad (2.29)$$

Such a new contribution is highly constrained by the measurements of the mass difference ΔM_s of neutral B_s mesons as discussed in 2015 leading to the bound of $M_{Z'}/|g_{bs}| \gtrsim 150 \text{ TeV}$ [62], with a very recent 2017 bound from updated lattice results of $M_{Z'}/|g_{bs}| \gtrsim 500 \text{ TeV}$ [61]. However the stronger 2017 bound arises from a discrepancy with the Standard Model which could in principle disappear.

If we take the milder $B_s - \overline{B}_s$ mixing bound then this constrains,

$$\frac{|g_{bs}|}{M_{Z'}} \lesssim \frac{1}{150 \text{ TeV}}. \qquad (2.30)$$

Since g_{bs} is approximately known in our model, then using eq. (2.25), this leads to a lower bound on the Z' mass in this model,

$$M_{Z'} \gtrsim 3 \text{ TeV}, \qquad (2.31)$$

where the bound would increase by a factor of three for the stronger 2017 bound on $B_s - \overline{B}_s$ mixing.

Since the Higgs doublets are charged under U(1)′, they will induce $Z - Z'$ mixing which will affect the SM prediction of M_W/M_Z, leading to corrections to the well determined electroweak precision parameter ρ or T. One may estimate the lower bound,[2]

$$\frac{M_{Z'}}{g'k} \gtrsim 9.5 \text{ TeV},\qquad(2.32)$$

where k is the model dependent normalisation of the U(1)′ charge of the Higgs doublets.

Taking the ratio of eqs. (2.30) to (2.28) we find the relatively model independent bound,

$$\frac{|g_{bs}|}{g_{\mu\mu}} \lesssim \frac{1}{25}.\qquad(2.33)$$

However this would reduce by an order of magnitude for the stronger 2017 bound on $B_s - \overline{B}_s$ mixing. Since g_{bs} is approximately known in our model, then eqs. (2.33) and (2.25) imply the bound

$$g_{\mu\mu} \gtrsim \frac{1}{2}.\qquad(2.34)$$

If the U(1)′ gauge charges corresponded to a fourth family $B - L$ then this bound would become $g_{\mu\mu} \gtrsim 1/6$ but the coupling $g_{\mu\mu}$ must still be quite sizeable in any case. The message is clear that in order to account for $R_{K^{(*)}}$ we need to generate an unsuppressed coupling of Z' to muons. In particular we need a large value of θ_{24}^L, as observed in [58]. Assuming this, then we satisfy the phenomenological constraints from Z' searches, $B_s - \overline{B}_s$ mixing, trident events and so on as fully discussed in [58] to which we refer the reader for more details. We now return to the question of how it is possible to obtain large muon couplings to Z' in this model, and show that this may only be achieved at the expense of tuning of Yukawa couplings.

2.9 Leptons

The leptons can be treated by exactly the same methods as developed for the quarks. However, the situation for the leptons is slightly different since there will be a further Majorana mass matrix for the singlet neutrinos $M_{ij}^{\nu^c} \nu_i^c \nu_j^c$. Since nothing prevents the Majorana masses $M_{ij}^{\nu^c}$ being arbitrarily large, well above the U(1)′ breaking scale, this will lead to a conventional seesaw mechanism for small neutrino masses. In principle, large lepton mixing can originate from the seesaw mechanism due to the arbitrary neutrino Yukawa matrix and the heavy Majorana mass matrix $M_{ij}^{\nu^c}$, so we are free to treat the charged lepton Yukawa matrices any way we wish.

Motivated by the $R_{K^{(*)}}$ anomalies, we work in a new basis for the charged leptons, by performing suitable rotations on the fields L_i, e_i^c, to give,

$$\begin{pmatrix}
 & e_1^c & e_2^c & e_3^c & e_4^c & \overline{L_4} \\
\hline
L_1| & 0 & 0 & 0 & 0 & 0 \\
L_2| & 0 & 0 & 0 & y_{24}^e H^d & x_2^L \phi \\
L_3| & 0 & 0 & 0 & y_{34}^e H^d & 0 \\
L_4| & 0 & y_{42}^e H^d & 0 & 0 & M_4^L \\
\overline{e_4^c}| & 0 & x_2^{e^c} \phi & x_3^{e^c} \phi & M_4^{e^c} & 0
\end{pmatrix}.\qquad(2.35)$$

Now assuming all angles to be small apart from θ_{24}^L, motivated by the discussion in the previous subsection [58], we obtain,

$$
y_{ij}^e = \begin{pmatrix} 0 & 0 & 0 \\ 0 & c_{24}^L \theta_{24}^{e^c} y_{24}^e & c_{24}^L \theta_{34}^{e^c} y_{24}^e \\ 0 & \theta_{24}^{e^c} y_{34}^e & \theta_{34}^{e^c} y_{34}^e \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & s_{24}^L y_{42}^e & 0 \\ 0 & 0 & 0 \end{pmatrix} , \tag{2.36}
$$

The Z' coupling matrices in this basis (ignoring the heavy Majorana neutrinos) are,

$$
D_{ij}^L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & (s_{24}^L)^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \; D_{ij}^{e^c} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & (\theta_{24}^{e^c})^2 & \theta_{24}^{e^c}\theta_{34}^{e^c} \\ 0 & \theta_{24}^{e^c}\theta_{34}^{e^c} & (\theta_{34}^{e^c})^2 \end{pmatrix} . \tag{2.37}
$$

So far this looks very similar to the basis used for the up type quarks, in eqs. (2.14), (2.20) with the replacements $Q \to L$, with $u^c \to e^c$ and a relabelling $L_2 \leftrightarrow L_3$ and $e_2^c \leftrightarrow e_3^c$. Indeed, without further assumption, the Yukawa matrices seem to be dominated by the element with the largest angle, which would imply that the second family charged lepton is the heaviest, so we would interpret that as the τ lepton. However, let us suppose that we tune the Yukawa coupling y_{42}^e to be very small in this basis, with the hierarchy $y_{42}^e \ll y_{34}^e$ so that the charged lepton Yukawa matrix is in fact dominated by the first matrix in eq. (2.36), even though the angles are assumed to be small. This may also lead to large left-handed charged lepton mixing, providing an explanation of large atmospheric lepton mixing in neutrino oscillations. Moreover, since the first matrix is rank one, the smaller muon mass is controlled by y_{42}^e from the second matrix in eq. (2.36).[3] We conclude that this explanation of $R_{K^{(*)}}$ requires the Yukawa coupling y_{42}^e to be tuned to be small.

3 Conclusions

We have explored the possibility that the semi-leptonic B decay ratios $R_{K^{(*)}}$ which violate $\mu-e$ universality are related to the origin of the fermion Yukawa couplings in the Standard Model. We have constructed an explicit renormalisable model with a vector-like family as an ultraviolet completion of eqs. (1.1) and (1.2). The considered model involves a new U(1)$'$ gauge group, spontaneously broken at the TeV scale, under which the three chiral families are neutral but the vector-like fourth family is charged. The mixing between the fourth family and the three chiral families then provides the effective Yukawa couplings, as in figure 1, as well as the non-universal effective Z' couplings involving the three light families, in figure 2, which are both generated by the same physics. However, while the model provides a nice connection between $R_{K^{(*)}}$ and the origin of Yukawa couplings in the quark sector, the lepton sector requires some tuning of Yukawa couplings to obtain the desired coupling of Z' to muons.

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[3]Alternatively, we may also tune $y_{24}^e \ll y_{34}^e$ so there is small left-handed charged lepton mixing. Most of the large lepton mixing would then originate in the neutrino sector.

Acknowledgments

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